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Argonne National Laboratory

REACTOR OSCILLATOR DETERMINATION OF
CAPTURE-TO-FISSION RATIO IN AN
UNDERMODERATED CRITICAL ASSEMBLY

by

W. C. Redman and M. M. Bretscher

RELEASED FOR ANNOUNCEMENT

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Reactor Physics Division

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REACTOR OSCILLATOR DETERMINATION OF CAPTURE-TO-FISSION RATIO IN AN UNDERMODERATED CRITICAL ASSEMBLY

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SUMMARY

The feasibility of a proposed experimental method for the determination of the effective capture-to-fission ratio, $\bar{\alpha}$, for materials inserted in a low-flux reactor has been demonstrated. The procedure involves a comparison of reactor response to oscillated samples of a fissile material, an absorber, and a spontaneous-fission neutron source, plus an experimental determination of fission rate for the fissile material and capture rate for the absorber. In addition, it is necessary that the neutron source be calibrated. These experimental results, combined with a knowledge of the number of neutrons per fission for the fissile material, yield a value of the quantity $1 + \bar{\alpha}$. Since $\bar{\alpha}$ for systems of particular interest in the current power reactor development program is appreciably less than unity, extreme accuracy is required for each of the experimental determinations on which the derived value of $1 + \bar{\alpha}$ is based, if even moderate accuracy is to be obtained in the resulting value for $\bar{\alpha}$.

This method has been tested in Hi-C Core No. 8, a critical assembly of 3% enriched UO_2 fuel pins, moderated and reflected by light water, in a lattice spacing which yields a hydrogen-to- U^{238} atom ratio of 2.91. The oscillator and absolute counting data yield a value of 0.217 for $\bar{\alpha}$, with a probable error of ± 0.010 (4.6%). This result is in reasonable agreement with a calculated value of 0.233 and values of 0.215 and 0.220 derived from measured U^{235} fission cadmium ratios and calculated thermal and epithermal values for $\bar{\alpha}$.

I. MOTIVATION

The ratio of neutron capture to fission by fissile materials, α , has a strong influence on the economy of both thermal and fast reactors. The high resistance of oxide fuel elements to radiation damage has focused attention on long-term reactivity effects in thermal reactor systems, but the limited knowledge of effective neutron spectra in the various thermal-reactor designs makes difficult the application of measured differential cross sections to the calculation of burnup of the fissile material. In fast reactor systems, a knowledge of α is required to establish the efficiency of the breeding process.

At present there is one established technique for the determination of α , but its usefulness is quite limited because it requires a long,

high-flux irradiation. This method requires the production of a quantity of neutron capture product sufficient for the determination of its abundance relative to the parent isotope by a mass-spectrometric measurement. Recent developments in instrumentation have resulted in a significant improvement in the sensitivity of this method. A 100-in. mass spectrograph operated by Argonne's Chemistry Division can detect 10 parts per million for heavy mass materials and requires only a 10^{-16} -g sample. A special spectrometer in the Chemistry Division can achieve a thousandfold improvement in isotopic content per pass, with a 10% collection efficiency. Thus it might be feasible, through a combined use of these devices for sample preparation before irradiation and treatment of the irradiated material after irradiation, to make a precise determination of the capture product production with only a 0.01% burnup. (This corresponds to an increase of the capture product by 10 ppm, for $\alpha = 0.1$.) Unfortunately, this burnup requires 0.5 MWd for a reactor containing 5 kg of fissile material, and 50 MWd for 500 kg, if the sample is located at a position of average flux. In addition, this procedure for the determination of alpha requires a radiochemical determination of the number of fissions occurring in the sample. Uncertainty in the yields of fission fragments, particularly as a function of energy, introduces a significant error in this procedure for the determination of alpha.

A recent series of measurements¹ has demonstrated the current capabilities of this technique. Small samples of fissile materials having extremely small concentrations of interfering isotopes were irradiated for 33 MWd at several locations in the core and blanket of the fast reactor EBR-I. Capture rates for the U^{233} , U^{235} , and Pu^{239} samples were determined by mass-spectrometric analysis and for U^{238} by alpha particle counting of the Pu^{239} resulting from U^{239} decay. The fission rate was established by beta counting the Cs^{137} obtained by a radiochemical extraction procedure with a calibrated end-window proportional counter. The resulting values for the effective capture-to-fission ratio, $\bar{\alpha}$, ranged from 0.05 to 0.24, with a typical standard error of $\pm 7\%$.

The nuclear design of a power reactor normally proceeds from experimental data acquired through systematic exploration of the characteristics of a flexible low-power assembly of the materials and geometry of interest. These so-called zero-power reactors or critical assemblies provide a proving ground for evaluation of reactor concepts at a fraction of the cost of constructing a power reactor. Unfortunately, there has existed no direct means to determine the loss of neutrons by radiative capture in the fissile materials in such assemblies. This is particularly disadvantageous for the fast reactor development program, since there exist only a few power reactors for which this information can be obtained by the existing high-flux procedure.

The present and anticipated critical-assembly programs at Argonne emphasize the need for a direct, accurate method for determining the

effective capture-to-fission ratio, $\bar{\alpha}$, in low-flux systems. Recently, the initial conversion ratio in undermoderated systems has been investigated indirectly in the Hi-C program, a series of critical experiments intended to establish the conversion capabilities of undermoderated thermal reactors. Critical experiments on a plutonium-fueled thermal reactor assembly are underway in EBWR. Because of the variation of alpha with effective neutron energy among such systems, it is desirable that the variation of alpha with neutron spectrum, or moderator-to-fuel ratio, be established experimentally. Furthermore, the ZPR-6 facility is being used for a long-term program of basic studies on the properties of fast reactor assemblies. Since breeding is one of the major motivations for the entire civilian fast reactor program, a need exists for a convenient means to establish $\bar{\alpha}$ in fast critical assemblies. This is the ultimate objective of this investigation of the feasibility of a reactor oscillator method for the direct determination of capture-to-fission ratio in zero-power systems.

As a matter of convenience, and because of the more highly developed state of knowledge about such reactors, the initial exploratory studies with this proposed technique² were undertaken in a thermal reactor assembly.

II. METHOD

A. Derivation of Basic Relation

The reactivity associated with the insertion of a fissile sample in a critical reactor results from the combined effect of production of fission neutrons and the loss of reactor neutrons by both the fission and capture processes. A two-group perturbation-theory treatment, summarized in Appendix A, yields the following expression for this reactivity change:

$$\rho_U = \frac{R_f}{N_U} \bar{\nu} [W_f - (1 + \bar{\alpha}) W_a], \quad (1)$$

where

$$N_U = \bar{\nu} \int_{\text{core}} \phi_1^\dagger(\underline{r}) \Sigma_{fc} \phi_2(\underline{r}) d\underline{r},$$

$$R_f = \text{fission rate in sample} = \int_{\text{sample}} \Sigma_{fs} \phi_2(\underline{r}) d\underline{r},$$

$$W_f = \int_{\text{sample}} \phi_1^\dagger(\underline{r}) \Sigma_{fs} \phi_2(\underline{r}) d\underline{r} \int_{\text{sample}} \Sigma_{fs} \phi_2(\underline{r}) d\underline{r},$$

and

$$W_a = \int_{\text{sample}} \phi_2^\dagger(\underline{r}) \Sigma_{as} \phi_2(\underline{r}) d\underline{r} \int_{\text{sample}} \Sigma_{as} \phi_2(\underline{r}) d\underline{r}.$$

W_f and W_a are the average values of the fast and slow adjoint fluxes, $\phi_1^\dagger(\underline{r})$ and $\phi_2^\dagger(\underline{r})$, and physically correspond to the weighting function for the importance of the neutrons produced within the sample by fission and those absorbed within the sample, respectively. Σ_{fc} is the macroscopic fission cross section of the core; Σ_{fs} and Σ_{as} are the fission and absorption cross sections of the sample. $\bar{\nu}$ is the average number of neutrons released per fission in the sample; $\phi_2(\underline{r})$ corresponds to the slow flux.

Equation (1) is based on assumptions that the sample is placed within a void in the reactor (otherwise Σ_{fs} and Σ_{as} are not the macroscopic cross sections of the sample), that the sample is located in a region where flux gradients are essentially zero, and that reactivity changes due to elastic and inelastic scattering effects within the sample are negligible.

It is necessary, through supplemental experiments, to establish the values for W_f and W_a which physically correspond to the weighting function or importance of the fission neutrons produced and reactor neutrons absorbed by the sample. Use of a spontaneous fission source of Cf^{252} provides a convenient way to determine the fission-neutron importance from observation of the apparent reactivity associated with the insertion of a sample of known neutron-emission rate S . From such a measurement

$$W_f = \rho_{\text{Cf}} N_{\text{Cf}} / S. \quad (2)$$

The quantity N_{Cf} , defined similarly to N_U , depends on the fission rate existing within the reactor core during the measurements with the Cf^{252} source. It is assumed that the quantity of Cf^{252} is sufficiently small that absorption is negligible.

In a similar way, the importance for absorbed neutrons is established by use of a material which responds in the same way to the reactor neutrons as does the fissile sample. In the measurements described here, manganese has been used as a nuclear absorption equivalent for U^{235} . From reactivity measurements with a manganese sample having an absorption rate R_{Mn} ,

$$W_a = -\rho_{\text{Mn}} N_{\text{Mn}} / R_{\text{Mn}}. \quad (3)$$

Inserting the values for the worths of fission and reactor neutrons in Eq. (1) yields the following expression:

$$\rho_U = \frac{R_f}{N_U} \left[\bar{\nu} \frac{\rho_{\text{Cf}} N_{\text{Cf}}}{S} + (1 + \bar{\alpha}) \frac{\rho_{\text{Mn}} N_{\text{Mn}}}{R_{\text{Mn}}} \right]. \quad (4)$$

In the experimental procedure, the samples of interest are oscillated between a position at the center of the reactor and one almost completely removed from the reactor environment, and the resulting periodic variation in power level is observed. For this condition, the reactivity is proportional to the change in power level ΔV (amplitude of power level oscillation) divided by the power level V . Since each term in Eq. (4) contains a reactivity term, the quantity $\Delta V/V$ may be substituted for ρ , yielding the expression

$$(\Delta V/V)_U = \frac{R_f}{N_U} \left[\frac{\bar{\nu} N_{\text{Cf}}}{S} \left(\frac{\Delta V}{V} \right)_{\text{Cf}} + (1 + \bar{\alpha}) \frac{N_{\text{Mn}}}{R_{\text{Mn}}} \left(\frac{\Delta V}{V} \right)_{\text{Mn}} \right]. \quad (5)$$

In the experimental procedure followed, all quantities are either determined at the same power level or normalized to the same level. Then $N_U = N_{\text{Cf}} = N_{\text{Mn}}$ and Eq. (5) may be written in terms of the measured quantities and available data as follows:

$$(\Delta V/V)_U = R_f \left[\frac{\bar{\nu}}{S} \left(\frac{\Delta V}{V} \right)_{Cf} + \frac{1 + \bar{\alpha}}{R_{Mn}} \left(\frac{\Delta V}{V} \right)_{Mn} \right]. \quad (6)$$

Rearrangement yields

$$1 + \bar{\alpha} = \frac{R_{Mn}}{(\Delta V/V)_{Mn}} \left[\frac{(\Delta V/V)_U}{R_f} - \frac{\bar{\nu}}{S} \left(\frac{\Delta V}{V} \right)_{Cf} \right]. \quad (7)$$

The experimental procedure involves oscillation of samples of U^{235} , Cf^{252} , and manganese plus irradiation and subsequent absolute counting for samples of manganese and U^{235} . In addition, it is necessary that the Cf^{252} neutron source be calibrated. These experimental results, combined with a knowledge of the number of neutrons per fission of U^{235} , yield a value for $1 + \bar{\alpha}$. Since $\bar{\alpha}$ for systems of particular interest in the civilian power reactor development program is appreciably less than unity, extreme accuracy is required for each of the experimental determinations on which the derived value of $1 + \bar{\alpha}$ is based, if even moderate accuracy is to be obtained in the resulting value for $\bar{\alpha}$.

B. Previous Work

Several determinations of $\bar{\alpha}$, based on a relation similar to Eq. (7), were made in the course of the critical assembly program for the Intermediate Power Breeder Project at KAPL.³ The reactivity effect produced by insertion of a fuel sample in PPA-5 was determined from the displacement of a calibrated control rod required to maintain criticality. The fission rates in both the sample and the entire core were established by gamma counting relative to a sample irradiated in a calibrated thermal-neutron flux. The beta activity of calibrated foils of manganese and gold provided information on the absorption rate and, in combination with a reactivity measurement, the importance function for reactor neutrons. The importance function for fission neutrons was established by separating the beneficial effect of fuel from the absorption effect through enclosing bare and cadmium-covered fuel samples in a cadmium cylindrical shell filled with paraffin. The limitations in this earlier work are evident from the estimated error of ± 0.10 in $\bar{\alpha}$ for values in the range from 0.32 to 0.47.

The results of some preliminary measurements made with a weaker Cf^{252} source contained in Teflon have been reported.⁴

III. EXPERIMENTAL DETAILS

The experimental procedures have benefited significantly from equipment and techniques developed for previous experimental activities in Argonne's reactor physics program.

A. Description of Critical Assembly

The ZPR-7 critical facility⁵ was used for the verification of this proposed reactor oscillator method for the determination of α . The initial tests were made in a core geometry studied earlier as a part of the high-conversion critical assembly program.⁶ The core, identified as Hi-C Core No. 8, was composed of a cylindrical array of 948 aluminum-clad Hi-C fuel pins, arranged in a square lattice having a 1.24-cm pitch. The fuel was 3.04 w/o enriched UO_2 in the form of 0.94-cm-diameter pellets stacked to a height of 122 cm. The core was moderated and reflected by light water. The arrangement of fuel pins was uniform except at the center of the core, where five lattice positions in a cross arrangement were left vacant to provide space for the insertion of the voided aluminum tube in which samples contained in 5/8-in.-OD aluminum cans were oscillated.

A schematic representation of a typical Hi-C core is shown in Fig. 1. The uniform lattice of fuel pins which comprise the core is sometimes perturbed by as many as four finger-type arrays of B_4C safety elements which occupy fuel lattice locations in groups of five adjacent positions. Four cadmium blades at the core-reflector interface function as safety or control elements. Figure 2 shows the Hi-C Core No. 8 with the oscillator and sample changer mechanisms installed above the 21.6-cm-radius core. The cadmium box which surrounds the turret containing the samples obscures much of the top of the core.

B. Oscillated Samples

1. Description

Significant properties of the samples used for the oscillator measurements are presented in Table I. Sample sizes were chosen to give comparable reactivity effects, subject to the availability of materials and maintaining similar geometrical conditions. The main samples were foils of 93 w/o U^{235} , 90 w/o Mn-10 w/o Cu, and approximately 1 μg of Cf^{252} in a quartz container. In addition, samples of copper and aluminum, and an empty can were used in order to determine the net signals from the materials of principal interest.

A variety of U^{235} samples were oscillated and later irradiated in order to investigate a possible dependence of the response on the dimensions of the sample, in particular, its thickness and lateral extent. The first

112-335

Fig. 1. Hi-C Critical Cutaway

112-5664

Fig. 2. Hi-C Core No. 8 with Oscillator Installed

TABLE I. Description of Oscillated Samples

Sample	Mass, g	Dimensions, in.	Aluminum Sample Can Mass, g	Comments
U #1; 93.2 w/o U ²³⁵	0.5548	5 x 0.25 x 0.002	17.958	Wrapped in 0.1131 g aluminum foil
U #2; 93.2 w/o U ²³⁵	0.5650	1½ x 3/4 x 0.002	17.926	Wrapped in 0.1001 g aluminum foil
U #3; 93.2 w/o U ²³⁵	0.7836	5 x 0.1 x 0.005	17.958	Wrapped in 0.0749 g aluminum foil
Mn #1; 90 w/o Mn-10 w/o Cu	1.1616	5 x 1.0 x 0.002	18.105	
Mn #2; 90 w/o Mn-10 w/o Cu	1.1613	5 x 1.0 x 0.002	18.105	
Mn #3; 90 w/o Mn-10 w/o Cu	1.1605	5 x 1.0 x 0.002	18.027	
Cf ²⁵²	≈10 ⁻⁶	Point, deposited in quartz, supported by aluminum	17.555	1.4581 g quartz vial, 3.4171 g aluminum
Cu	5.0498	5 x 3.6 x 0.002	18.078	Rolled
Empty Al can			18.283	
Al	20.6835	5 x 0.56 OD x 1/16 wall	17.926	2S aluminum sleeve

uranium sample was 2 mil thick and extended over the entire height of the sample can. The second uranium sample was also a 2-mil-thick foil, with significantly altered height and width. This was used to determine whether there was a detectable difference in response when the sample was located at the center of the can (as in the case of the californium sample) rather than along the entire 5-in. length. The third uranium sample also extended the full height of the sample can, but had a 250% greater thickness (5 mils).

Because manganese is extremely brittle in pure form, it was necessary to work with an alloy. A nominal 90 w/o manganese, 10 w/o copper alloy was used throughout the measurements reported here.

The manganese-copper foils used to derive the oscillatory response were also irradiated subsequently. For this reason, three such foils were oscillated. The manganese-copper samples were in the form of 1-in.-wide by 5 in.-long by 2-mil-thick strips matched closely in weights. The foils were shaped to maintain contact with the inner wall of the sample can along the 1-in. width. It was necessary to correct for the contribution of copper to the reactivity and so a copper sample was also oscillated. The width of the copper sample required rolling to form a two-layer sleeve with an additional overlap of approximately 0.1 in.

The Cf²⁵² source was contained in a quartz vial positioned by a low-mass aluminum structure inside a standard aluminum sample can at the approximate center of the can. In addition, an empty sample can was also oscillated as well as a can containing a sleeve of aluminum in order to obtain a correction for the variation in aluminum content of the various sample cans.

2. Composition of Manganese-Copper Foils

Numerous chemical analyses have been made of various samples of the 2-mil-thick manganese-copper alloy. Most of the samples used

in this and other oscillator experiments were obtained from two long strips of manganese-copper alloy, a part of the batch of material supplied on one purchase order. Hence, it is reasonable to assume that within a single strip the composition is constant, and probably this holds true from strip to strip. However, for purposes of correction of the manganese-copper response for the contribution of copper and other impurities, reliance was placed mainly on the specific analyses for the actual samples used. In addition to manganese and copper, it has been found that significant quantities of nitrogen, hydrogen, oxygen, and carbon are present in these samples. Spectrochemical analyses have indicated that the quantity of other materials is quite small.

A chemical analysis of the actual manganese-copper samples oscillated and later irradiated was performed for the two major constituents and the most significant impurity, based on knowledge of the samples' content obtained from earlier analyses. The results are as follows:

Mn Sample No.	w/o		
	Mn	Cu	N
1	88.2 $\pm$ 0.2	10.4 $\pm$ 0.1	0.87 $\pm$ 0.09
2	88.6 $\pm$ 0.2	9.85 $\pm$ 0.1	0.78 $\pm$ 0.08
3	88.7 $\pm$ 0.2	11.1 $\pm$ 0.1	0.85 $\pm$ 0.09

From analyses of other manganese-copper samples it is known that there is also present approximately 0.5 w/o of oxygen, 0.1 w/o of carbon, and 0.01 w/o of hydrogen. In regard to other impurities, various spectrochemical analyses have indicated the following "order of magnitude" abundances:

< 0.2%	P
< 0.1%	As, Hg, K, Sr, V, Zn, Zr
< 0.01%	Ag, Al, Ba, Be, Bi, Ca, Co, Cr, Fe, Mo, Na, Ni, Pb, Sb, Si, Sn, Ti
< 0.01%(?)	B
< 0.001%	Li
< 0.0004%	Cd

Among these materials detected spectrochemically, only mercury could be expected to make a significant contribution to the reactivity, due to its high thermal cross section. Supplemental analyses yielded values of 13 and 15 ppm of mercury, indicating the conservatism in the upper limit set by spectrochemical analysis for mercury and presumably also for the other elements indicated above.

3. Properties of the Californium Source

The californium sample was supplied by the Transplutonium Isotopes Group in the Chemistry Division. Its isotopic composition, as of August 10, 1964, is indicated in Table II. Recent data^{7,8} on half-lives and thermal-neutron cross sections for the various isotopes are also included.

TABLE II. Properties of the Californium Source

Isotope	atom %, 8/10/64	Half-life	Thermal-neutron Cross Section, barns	
			Capture	Fission
249	7.24	360 yr	270 ± 100	1735 ± 70
250	16.2	13.2 ± 0.5 yr	2000	
251	5.04	~ 800 yr	3800	3000 ± 260
252	71.5	2.646 ± 0.004 yr	8.5	
253	0	17.6 ± 0.2 day		
254	0	60.5 ± 0.2 day	<2	

The sample is known to have calcium, iron, and aluminum contaminants, but was clean of curium at the time of preparation. Decay of Cf^{252} by alpha-particle emission is 31.3 ± 0.2 times more probable than its disappearance by spontaneous fission, and the resulting Cm^{248} exhibits spontaneous fission 11% of the time. However, the total half-life is quite long, approximately 5×10^5 years, so its contribution is negligible. A measurement made with the sample on June 30, 1965, indicated 2.51×10^7 fissions per minute, with an estimated error of approximately 4%.

C. Oscillator and Sample Changer

The construction and operation of the pneumatically driven oscillator and sample-changing mechanism have been described previously.⁹ Figure 3 shows the construction of the sample-changing features which allow the selection for oscillation of any of up to twelve samples during each reactor run, and a typical sample can.

This device was located above and on the axis of the core. The period of oscillation used throughout these measurements was 15 sec, with about 2.2 sec required for the 32-in. stroke. The samples were contained in 5/8-in.-OD aluminum cans. These were oscillated inside a perforated aluminum tube from which water was excluded by a 15-mil thickness of shrunken plastic tubing. The tube plus tubing was 0.79 in. in diameter and extended approximately 20 in. below the midplane of the reactor core. The nylon clip used to attach the sample cans to the oscillator-rod mechanism

was sized so that its signal almost balanced out the negative reactivity contribution of the magnesium rod and an empty aluminum sample can. In order to reduce the activation of samples while stored in the turret of the sample changer, a cadmium cover was placed over the sample-changer mechanism.

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Fig. 3. Detail of the Sample Changer

D. Signal Detection and Recording

The procedure used to record and process the data resulting from oscillation of samples in the critical assemblies has been described previously.¹⁰ The reactor power level is monitored by a boron-lined, uncompensated ionization chamber located in the reflector on the reactor midplane. The signal is measured with a vibrating-reed electrometer which is provided with a calibrated bucking circuit to bias out a major fraction of the average signal. The power-level fluctuations are thus amplified for recording on a 0-10-mV strip chart recorder. Typical signals for decade increments in power level over a range from 10 mV to 10 V are reproduced in Fig. 4.

112-5954

Fig. 4. Reactor Response to Oscillated Samples

An encoder, attached to the recorder, permits digital readout of the electrometer on a punched data card. This is actuated every twentieth of a cycle by a pulse generator which also initiates sample insertion or removal after every ten readings. Data for each cycle are recorded on a separate card, together with sample identification, run number, and cycle sequence. The number of cycles recorded for each sample is determined by the operator. Data taking is interrupted when a new sample is selected and is resumed after the reactor has been relevelled with the new sample oscillating.

E. Experimental Conditions

Table III presents the experimental conditions existing during the various runs from which the data reported here were obtained. All experiments were performed within a $3\frac{1}{2}$ -week period; thus, there should be but small change in experimental conditions due to aging of the detector circuitry, temperature variations, and other sources of drift.

TABLE III. Description of Experiments (Hi-C Core No. 8 in ZPR-7 Facility)

Date	Run No.	Purpose	Average Channel 4, mV (with $10^7 \Omega$ reactor)	Power Level Channel 5, Amp $\times 10^8$	Dimension, in.	Oscillator Period, sec
7/6/65	1	($\Delta V/V$) _{Cf, Cu}	3.7-225	0.045-5.1		14.96
	2	($\Delta V/V$) _{Cf, Cu}	3.9-224	0.042-5.1		14.96
7/7/65	3	($\Delta V/V$) _{Cf, Cu}	3.5-226	0.043-5.1		14.93
7/8/65	4	($\Delta V/V$) _{all}	225	5.1		15.00
	5	($\Delta V/V$) _{all}	226	4.8		14.99
7/9/65	6	($\Delta V/V$) _{all}	226	5.0		14.99
7/12/65	7	R _{Mn#1} , R _U , ($\Delta V/V$) _{Cu}	226	5.1	5 x 1/4 x 0.002 (U)	14.99
7/13/65	8	R _U , ($\Delta V/V$) _{Cu}	226	4.6	5 x 1/4 x 0.002	15.00
	9	R _U , ($\Delta V/V$) _{Cu}	226	4.3	5 x 1/4 x 0.002	15.01
7/14/65	10	R _U , ($\Delta V/V$) _{Cu}	226	3.9	5 x 1/4 x 0.002	15.01
	11	R _{Mn#2} , ($\Delta V/V$) _{Cu}	226	4.2	1 1/2 x 3/4 x 0.002	15.00
	12	R _{Mn#3} , ($\Delta V/V$) _{Cu}	225	4.2		15.01
7/15/65	13	R _U , ($\Delta V/V$) _{Cu}	225	4.0		15.01
	14	Mn Cd R	224	4.2		-
	15	U ²³⁵ Cd R	224	4.2		-
	16	R _U , ($\Delta V/V$) _{Cu}	defective	4.1	1 1/2 x 3/4 x 0.002	14.99
7/21/65	17	R _U , ($\Delta V/V$) _{Cu}	defective	4.1	1 1/2 x 3/4 x 0.002	14.98
	18	R _U , ($\Delta V/V$) _{Cu}	226	4.5	5 x 1/4 x 0.002	15.01
	19	R _U , ($\Delta V/V$) _{Cu}	226	4.4	5 x 1/4 x 0.002	15.02
	20	R _U , ($\Delta V/V$) _{Cu}	225	4.1	5 x 1/4 x 0.002	15.01
7/22/65	21	U ²³⁵ Cd R	245	4.1		-
7/27/65	22	($\Delta V/V$) _{Cf, U#3, Cu}	248	3.5		15.00
7/28/65	23	R _U , ($\Delta V/V$) _{Cu}	256	3.9	5 x 0.1 x 0.005	14.95
	24	R _U , ($\Delta V/V$) _{Cu}	226	3.9	5 x 0.1 x 0.005	15.01
	25	R _U , ($\Delta V/V$) _{Cu}	251	3.5	5 x 0.1 x 0.005	15.02

The channel used for the oscillator measurements provided the basis for reproducing power levels in the various experiments. In addition, the power level as read from two of the operational instruments,

Channels 4 and 5, was usually recorded. Generally, the current level indicated by the linear channel (No. 4) was reproducible. Channel 5, a logarithmic channel, showed less stability.

The standard conditions for the instrument channel used for the recording of the flux level variations were $10^7\text{-}\Omega$ input resistor, 10-mV full-scale sensitivity for the strip-chart recorder, time constant setting of 1/8, 15-sec cycle for sample oscillation, and 950-mV bucking signal. The californium sample was oscillated at several lower power levels as well, down to about 1/100 the level at which all other measurements were performed. For these measurements the bucking signal was as low as 5 mV.

In identifying the purpose of the reactor run, $(\Delta V/V)$ designates a reactivity measurement, with the subscript identifying the samples oscillated. The symbol R indicates an irradiation, either of manganese or uranium. The irradiations for a determination of the cadmium ratio at the location of the inserted samples are indicated by CdR. During all irradiations except the latter, and for all reactivity measurements, the copper sample was oscillated as a monitor to check that there had been no significant change in experimental conditions from run to run.

Included in Table III is an indication of the sample size for the various irradiations of uranium samples. Although the actual manganese samples oscillated were subsequently irradiated, it was not believed practical to do this for the uranium samples for two reasons. First, previous experience had indicated the desirability of making several determinations of the U^{235} fission rate by the radiochemical procedure. In addition, the half-life of the isotope separated in that procedure (Mo^{99}) is approximately 66 hr, compared with the 2.58-hr half-life of manganese and the 5.1-min and 12.9-hr half-lives of copper present in the manganese sample. Thus, it was reasonable to assume that the residual activity from previous oscillations of the manganese sample was negligible, whereas this assumption would not be justified in the case of the U^{235} samples. The table indicates that seven samples similar in dimensions to uranium No. 1 were irradiated, three similar to uranium No. 2 and three similar to uranium No. 3.

From the measured U^{235} fission rate at the center of the reactor, a thermal-neutron flux of $6 \times 10^6 \text{ n/cm}^2\text{-sec}$ is established for the standard power level. Combining this value with the measured radial and axial flux distributions yields a value of $3\frac{1}{2} \text{ W}$ for the reactor power level existing during these measurements.

IV. SPECIFIC MEASUREMENTS

The oscillator measurements comprise the bulk of the experimental data. In addition, absolute determination of the manganese absorption rate by coincidence counting, determination of the fission rate by radiochemical procedures involving Mo^{99} , and a manganese bath calibration of the Cf^{252} source are significant related measurements. Each of these is discussed in turn.

A. Relative Reactivities

1. Procedure

The copper sample was used as a reference standard for all measurements. Several distinct sets of measurements were made for each oscillated sample. All measurements were performed under identical experimental conditions insofar as could be determined from the daily calibration of the oscillatory signal circuitry and the power-level channels on the ZPR-7 facility.

An identical procedure was followed in each set of measurements. Five sets of the reactor response to eleven 15-sec cycles were obtained on punched cards for each of the samples oscillated. For reasons to be explained later, the Cf^{252} sample was oscillated at several lower power levels in addition to that used for all of the other measurements. The copper reference sample was measured at both the beginning and the end of each run, except for the californium measurements, which began at a power level so low that the effect of the copper sample would be difficult even to detect.

As explained earlier, the readout equipment produces a three-digit reading of the recorder location every twentieth of a cycle, with the values for each cycle recorded on a separate card. Twelve cards for eleven successive cycles plus the first value for a twelfth constitute the usual data deck. These data are processed on an IBM-704 computer utilizing a FORTRAN-II program, RP-328X,¹⁰ which yields the coefficients for a third-order Fourier series fit to the observed oscillations in the reactor power. Since the shape of the reactivity input is constant, varying only in magnitude for the different samples, each Fourier coefficient is proportional to the reactivity. For the data treatment reported here, the calculated amplitude of the first-order term, ΔV , relative to the average level, V , is used as a measure of the reactivity of the sample.

2. Oscillator Results

Tables IV, V, and VI present the experimental observations and results derived from them for the various samples oscillated. As mentioned

previously, each sample was oscillated for five sets of eleven oscillations each. The values reported in Table IV represent unweighted averages of the five separate determinations, with the standard deviation from the mean as the reported error.

A majority of the oscillator data was obtained in Runs 4, 5, and 6. At a later date, Run 22 was made to extend the measurements to include a thicker uranium sample. In addition, the copper monitor data for the various irradiations are also included in Table IV. Toward the end of this period (Runs 22-25) there appeared to be a decrease in the reactivity effect of the copper. For this report, no normalization has been introduced since the values of $R_{U\#3}$ (Runs 23-25) and $(\Delta V/V)_{U\#3}$ (Run 22) occur as a ratio in the calculation of $1 + \bar{\alpha}$, and the values for californium from Run 22, after suitable correction for decay, were consistent with the other californium data.

TABLE IV. Observed Average Reactor Response for Various Samples

Sample	$(\Delta V/V) \times 10^3$ for Run No.			
	4	5	6	22
Cu	$-(2.3808 \pm 0.0253)$ $-(2.3121 \pm 0.0191)$	$-(2.3896 \pm 0.0328)$ $-(2.4099 \pm 0.0256)$	$-(2.3315 \pm 0.0276)$ $-(2.3937 \pm 0.0481)$	$-(2.2686 \pm 0.0324)$ $-(2.2889 \pm 0.0398)$
Empty Can	$-(0.0351 \pm 0.0173)$	$-(0.0696 \pm 0.0234)$	$-(0.0757 \pm 0.0332)$	
Al Sleeve	$-(1.4226 \pm 0.0448)$	$-(1.4004 \pm 0.0252)$	$-(1.3608 \pm 0.0378)$	
Mn #1	$-(2.0875 \pm 0.0336)$	$-(2.0748 \pm 0.0238)$	$-(2.1212 \pm 0.0384)$	
Mn #2	$-(2.1777 \pm 0.0160)$	$-(2.2118 \pm 0.0345)$	$-(2.1843 \pm 0.0493)$	
Mn #3	$-(2.1571 \pm 0.0461)$	$-(2.1376 \pm 0.0160)$	$-(2.1389 \pm 0.0307)$	
U #1	$+(2.9983 \pm 0.0451)$	$+(3.0637 \pm 0.0590)$	$+(3.1309 \pm 0.0324)$	
U #2	$+(2.8021 \pm 0.0097)$	$+(2.8578 \pm 0.0254)$	$+(2.9677 \pm 0.0414)$	
U #3				$+(3.4857 \pm 0.0409)$ $+(3.4378 \pm 0.0669)$ $+(3.4399 \pm 0.0290)$

Run No.	$(\Delta V/V)_{Cu} \times 10^3$	Run No.	$(\Delta V/V)_{Cu} \times 10^3$
7	$-(2.3789 \pm 0.0332)$	17	$-(2.3639 \pm 0.0250)$
8	$-(2.4399 \pm 0.0198)$	18	$-(2.4099 \pm 0.0208)$
9	$-(2.4292 \pm 0.0581)$	19	$-(2.4131 \pm 0.0273)$
10	$-(2.4383 \pm 0.0379)$	20	$-(2.3333 \pm 0.0248)$
11	$-(2.3284 \pm 0.0244)$	23	$-(2.2382 \pm 0.0214)$
12	$-(2.3712 \pm 0.0229)$	24	$-(2.3129 \pm 0.0433)$
13	$-(2.4125 \pm 0.0185)$	25	$-(2.3296 \pm 0.0343)$
16	$-(2.3985 \pm 0.0287)$		

TABLE V. Weighted Average Reactor Response for Various Samples

Sample	$\Delta V/V \times 10^3$ Weighted Average
Empty Al Can	$-(0.0515 \pm 0.0128)$
Al Sleeve	$-(1.3944 \pm 0.0190)$
Mn #1	$-(2.0876 \pm 0.0173)$
Mn #2	$-(2.1838 \pm 0.0139)$
Mn #3	$-(2.1395 \pm 0.0136)$
U #1	$+(3.0821 \pm 0.0240)$
U #2	$+(2.8165 \pm 0.0089)$
U #3	$+(3.4533 \pm 0.0223)$
Cu. (Runs 4 - 6 only)	$-(2.3578 \pm 0.0109)$

TABLE VI. Net Specific Reactivities

Material	$(\Delta V/V)/g, \times 10^3$
Al	$-(0.0661 \pm 0.0011)$
Cu	$-(0.4594 \pm 0.0033)$
Mn #1	$-(1.9346 \pm 0.0210)$
Mn #2	$-(2.0220 \pm 0.0184)$
Mn #3	$-(1.9773 \pm 0.0182)$
U ²³⁵ #1	$+(6.0394 \pm 0.0526)$
U ²³⁵ #2	$+(5.4197 \pm 0.0296)$
U ²³⁵ #3	$+(4.7815 \pm 0.0352)$

The average reactor response for the various samples reported in Table V is based on a weighting inversely proportional to the square of the respective standard deviations. Thus, the weighted average of the individual values, $\Delta V/V$, is given by

$$\overline{\Delta V/V} = \frac{\sum_i w_i (\Delta V/V)_i}{\sum_i w_i},$$

where

$$w_i = 1/s_i^2.$$

The standard deviation of this weighted average is given by

$$\bar{s} = \sqrt{\sum_i \left(\frac{w_i}{\sum_i w_i} \right)^2 s_i^2}.$$

The data of Table V have been used to derive the values of the reactivity per gram of pure material, as reported in Table VI. The first step involves a correction for the effect of the empty can and a smaller correction for variation in the amount of aluminum associated with each of the oscillated samples. The latter is obtained from the reactivity worth for aluminum derived from the data for the empty sample can and that containing the aluminum sleeve.

The values reported for manganese were derived by correcting the manganese-copper data for the response of the copper present and an equivalent response for other significant impurities. This procedure is illustrated by Table VII, which records the composition assumed for each of the three manganese-copper samples, based on the chemical analyses mentioned earlier. Table VII also shows how the impurities were converted to copper equivalents so that the observed reactivity for a copper sample could be used to correct the response of the manganese-copper samples for materials other than manganese. The reactivity contribution of the small amounts of oxygen and carbon present is trivial because of their very small absorption cross section and the moderate degree to which they reduce the energy of neutrons through scattering. By using the thermal absorption cross section per gram of material relative to that for copper, the nitrogen and hydrogen contents were converted to equivalent copper content in order to correct the response of the manganese-copper foil for materials other than manganese. This procedure was reasonable since the relative effectiveness of nitrogen is only 2.22 compared with copper and for hydrogen 5.40. These factors are weighted by the relatively small amount of these two materials present compared with copper which in turn has a significantly smaller reactivity effect per gram than does the manganese. This correction for the contribution of copper and other impurities to the response for the manganese-copper samples amounted to an average of only 3.1%. Hence, any errors in the assumed composition of the manganese-copper alloy do not contribute significantly to the derived value for the net specific reactivity of manganese.

TABLE VII. Copper Equivalent in Manganese Foils

Material	Assumed Composition, w/o			$\left(\sigma_a^{th}/g\right)_x$	Copper Equivalent, w/o		
	Mn #1	Mn #2	Mn #3	$\left(\sigma_a^{th}/g\right)_{Cu}$	Mn #1	Mn #2	Mn #3
Mn	88.2	88.6	88.7				
Cu	10.4	9.85	11.1	1	10.4	9.85	11.1
N	0.87	0.78	0.85	2.22	1.93	1.73	1.89
H	0.0067	0.0067	0.0067	5.40	0.036	0.036	0.036
O	0.52	0.52	0.52	2.1×10^{-3}	0.0011	~0	~0
C	0.09	0.09	0.09	5.5×10^{-3}	0.0005	~0	~0
Other*	-0.09	0.15	1.27				
Total	100	100	100		12.37	11.62	13.03

*By subtraction.

The uranium samples contained 93.15 w/o U^{235} . A comparison of the thermal cross sections weighted by the amount of the U^{234} , U^{236} , and U^{238} isotopes present in the sample indicates that less than 0.1% of the

thermal absorptions in the uranium samples would be due to isotopes other than U^{235} . Thus the values given in Table VI were obtained by dividing the response for the uranium samples, after correction for the contributions of the aluminum wrapper and sample can, by the mass of the samples and the isotopic abundance of U^{235} .

3. Apparent Reactivity of Californium

The Cf^{252} source is unique among the samples oscillated in that the spontaneously emitted neutrons contribute an apparent reactivity effect which is independent of reactor power level in addition to the absorption effects which are directly proportional to the power. The amount of californium present in the source was less than a microgram. Hence, it would not be expected to contribute an observable reactivity effect due to absorption of the sample. Nonetheless, the effect due to the quartz vial, aluminum support structure, sample can, nylon sample can clip, and magnesium insertion rod would, at significant power levels, add a perturbing signal to that due to the neutron emission. Consequently, the Cf^{252} sample was oscillated at powers as low as 1/100 that used for the other measurements in order to emphasize the effect due to introduction and removal of spontaneously emitted neutrons.

The results of oscillating the Cf^{252} source over a range of power levels are presented in Table VIII. A least-squares fit of the amplitude of response as a function of power level provides a separation of the signal into the effect of the spontaneously emitted neutrons (intercept) and the negative reactivity effect of the associated material and possible absorption by the Cf^{252} sample (slope). This is illustrated by Fig. 5 which displays the data obtained in Runs 1, 2, and 3 for a power range from 50 to 1000 mV.

TABLE VIII. Reactor Response to Cf^{252} as a Function of Power Level
(All values in mV)

Reactor Run No.:	1		2		3		22	
	ΔV	V	ΔV	V	ΔV	V	ΔV	V
	1.1743 ± 0.0077	12.06	1.1521 ± 0.0051	10.71	1.1567 ± 0.0059	11.28	1.1537 ± 0.0081	12.34
	1.2453 ± 0.0056	60.65	1.2444 ± 0.0089	59.56	1.2402 ± 0.0111	60.63	1.1855 ± 0.0060	20.71
	1.2405 ± 0.0099	110.1	1.2348 ± 0.0096	110.5	1.2649 ± 0.0147	109.7	1.2031 ± 0.0045	40.93
	1.2193 ± 0.0097	210.4	1.2296 ± 0.0202	210.8	1.2215 ± 0.0163	209.6	1.2181 ± 0.0091	61.64
	1.2286 ± 0.0183	310.3	1.2224 ± 0.0302	310.5	1.1924 ± 0.0193	310.6	1.2286 ± 0.0092	81.36
	1.2143 ± 0.0194	409.8	1.2055 ± 0.0218	409.5	1.2288 ± 0.0218	409.7	1.2194 ± 0.0059	101.38
	1.1300 ± 0.0127	509.7	1.2008 ± 0.0189	509.9	1.1576 ± 0.0095	509.8	1.2075 ± 0.0090	111.37
	1.0877 ± 0.0213	956.1	1.0954 ± 0.0426	954.8	1.0498 ± 0.0418	955.3		
$(\Delta V/V)_{Cu} \times 10^3$	-2.427 ± 0.041		-2.342 ± 0.024		-2.367 ± 0.042		-2.269 ± 0.032	
							-2.289 ± 0.040

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Fig. 5. Components of the Signal from the Cf^{252} Source

confirmed by the traces reproduced in Fig. 4 for the reactor response to the californium source at successive decade increments in reactor power level. For this reason, measurements at the lower end of the power range were emphasized for the californium sample.

When the californium source was oscillated, it was noted that the response at extremely low power levels was significantly less than that at intermediate levels. In order to better define this unexpected behavior at very low power levels, additional detail was obtained for the response in the interval from 10 to 110 mV in a subsequent series of measurements (Run 22).

The results of the least-squares fitting of the californium data are presented in Table IX. The actual fitting used an Applied Mathematics Division Code ANE-209 on the IBM-704, with weighting of individual values inversely proportional to their standard error. Since it was obvious that the use of data at the lowest power level would significantly distort the fit, only values for the power range from 60 mV upward were used. Two fits were made for each of the sets of the data, the first including the value at 60 mV, the second omitting this value. As seen from the results assembled in Table IX, no significant difference occurs when the value at 60 mV is omitted. Note that these data were all obtained on two successive days and so no correction was made for the 2.646-yr half-life for the californium decay.

The amplitude of the reactor response to a true reactivity effect is proportional to the power level, while the fluctuations due to reactor noise and the statistics of neutron detection vary as the square root of the flux. Hence, the signal-to-noise ratio is proportional to the square root of the flux. Thus for the usual reactivity measurements, the error diminishes as the flux level is increased. When the signal is independent of the power level, as in the case of spontaneous neutron emission by the californium source, the signal-to-noise ratio is proportional to $1/\sqrt{\text{flux}}$. This behavior is

TABLE IX. Weighted Least-squares Fit of Cf^{252} Data

Reactor Run No.	No. of Data Points	Intercept, mV	Slope, $\times 10^3$
1	7	1.2590 ± 0.0092	$-(0.1936 \pm 0.0335)$
	6	1.2635 ± 0.0181	$-(0.2025 \pm 0.0480)$
2	7	1.2518 ± 0.0048	$-(0.1227 \pm 0.0194)$
	6	1.2519 ± 0.0082	$-(0.1229 \pm 0.0261)$
3	7	1.2657 ± 0.0142	$-(0.2040 \pm 0.0393)$
	6	1.2847 ± 0.0197	$-(0.2436 \pm 0.0475)$
Weighted mean:	7	1.2544 ± 0.0041	$-(0.1502 \pm 0.0256)$
	6	1.2577 ± 0.0070	$-(0.1604 \pm 0.0354)$

The low-power portion of the three initial sets of values for the californium source are plotted together with the results of Run 22 with this source, suitably corrected for decay, in Fig. 6. The evidence for a deviation at low power from the linear dependence on power level is quite strong, based on these data and the average least-squares fit indicated in the figure.

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Fig. 6. Reactor Response at Low Power to Oscillated Cf^{252} Source

The errors shown are the standard deviation from the mean.

It was not possible to extend these measurements to even lower power levels because the rapid insertion of the strong californium source would have resulted in a period "scram" of the reactor at only a slightly lower power level than the approximate 10 mV at which measurements were made. This suggests a possible explanation for the marked variation of response at extremely low power levels. For the oscillator experiments, the source is in the reactor

half the time, and so the rod setting required to maintain a constant average power level actually corresponds to a subcritical condition due to the neutron contribution of the californium source. In fact, it was evident that the rod setting for constant power was strongly dependent on power level for the range covered in Fig. 6. Hence, for this range of power, k_{eff} was less than unity and varied with power. Thus, over this range of measurements, the source introduced is multiplied in a different manner at different power levels.

A consideration of the reactor transfer functions for a critical and a subcritical system provides a means to correct the californium data to conditions equivalent to those for the other oscillated samples. If the oscillating and steady components of the neutron density are represented by $\delta n(t)$ and n , then they are related to the amplitude of the reactivity oscillation in dollars, $\delta k(t)/\bar{\gamma}\beta$, through the reactor transfer function $W(j\omega)$ as follows:

$$\frac{\delta n(t)}{n} = \frac{\delta k(t)}{\bar{\gamma}\beta} W(j\omega).$$

Here ω is the angular frequency of the oscillation.

It has been shown¹¹ that the transfer function for a subcritical reactor ($\delta k_0 < 0$) held at an equilibrium neutron density by an extraneous source is given by

$$W(j\omega) = \frac{\bar{\gamma}\beta}{j\omega \left(\ell + \bar{\gamma}\beta \sum_i \frac{a_i}{j\omega + \lambda_i} \right) - \delta k_0} \delta k_0$$

This differs from the functional form for a critical assembly only by the reactivity term in the denominator, and indicates that the response to a sample oscillated in a subcritical system is smaller than for a critical system.

The californium data should be multiplied by the ratio of transfer functions for a critical and subcritical system, W_c/W_s . By some simple algebra,

$$\frac{W_c}{W_s} = 1 - \frac{\delta k_0}{\bar{\gamma}\beta} W_c.$$

The amount by which the reactor was subcritical during each set of oscillations is readily determined from the location and calibration of the control rod used to keep the average neutron level constant. The critical transfer function can be calculated from the properties of the delayed neutrons and the neutron lifetime.

During all of these measurements, the reactor power level was held steady by adjustment of a single, weakly absorbing control rod. It had a maximum differential worth of approximately two cents per inch, and its location was determined to the nearest 0.01 in. The amount by which the reactor was subcritical, in cents, for each of the apparent reactivity measurements for the californium source is recorded in Table X. Also included are the amounts by which each amplitude must be increased to correct to

criticality, being the product of the subcriticality in dollars and the critical transfer function, and the corrected amplitudes.

TABLE X. Correction of the Cf^{252} Data to Criticality

Run No.	V, mV	$-\rho, \%$	$\$ \rho \times W_C, \%$	$\Delta V_C, \text{mV}$
1	12.06	7.87	10.88	1.3021 ± 0.0085
	60.65	1.72	2.38	1.2749 ± 0.0057
	110.1	0.99	1.37	1.2575 ± 0.0100
	210.4	0.65	0.87	1.2302 ± 0.0098
	310.3	0.46	0.63	1.2363 ± 0.0184
	409.8	0.32	0.44	1.2196 ± 0.0195
	509.7	0.23	0.32	1.1336 ± 0.0127
	956.1	0.08	0.11	1.0889 ± 0.0213
2	10.71	8.62	11.91	1.2893 ± 0.0057
	59.56	1.65	2.28	1.2728 ± 0.0091
	110.5	0.95	1.31	1.2510 ± 0.0097
	210.8	0.49	0.68	1.2380 ± 0.0203
	310.5	0.41	0.56	1.2292 ± 0.0304
	409.5	0.25	0.35	1.2097 ± 0.0219
	509.9	0.23	0.32	1.2046 ± 0.0190
	954.8	0.06	0.09	1.0964 ± 0.0426
3	11.28	7.81	10.79	1.2815 ± 0.0065
	60.63	1.46	2.01	1.2651 ± 0.0113
	109.7	0.85	1.17	1.2797 ± 0.0149
	209.6	0.53	0.74	1.2305 ± 0.0164
	310.6	0.42	0.58	1.1993 ± 0.0194
	409.7	0.33	0.46	1.2345 ± 0.0219
	509.8	0.23	0.32	1.1613 ± 0.0095
	955.3	0.08	0.11	1.0510 ± 0.0418
22	12.34	6.76	9.34	1.2615 ± 0.0089
	20.71	4.42	6.11	1.2579 ± 0.0064
	40.93	2.34	3.24	1.2421 ± 0.0046
	61.64	1.58	2.19	1.2448 ± 0.0093
	81.36	1.33	1.84	1.2512 ± 0.0094
	101.38	1.05	1.45	1.2371 ± 0.0060
	111.37	0.93	1.28	1.2230 ± 0.0091

A value of 1.382 was used for the critical transfer function for oscillation in Hi-C Core No. 8 with a 15-sec period ($\omega = 0.42$ rad/sec). Interpolation of the data in Table C-1 of Reference 11 gave a value of $W_C = 1.318$ for a U^{235} -fueled core having a 17- μsec prompt-neutron lifetime.¹²

However, the effect of U^{238} fissions is not negligible, and the value of W_c for U^{238} at this frequency is about 25% larger. A U^{238}/U^{235} fission ratio of 0.1014 ± 0.0007 was measured⁶ for Hi-C Core No. 7, an assembly differing in composition from the core used here only in a fuel cladding of stainless steel instead of aluminum. The effect of the U^{238} fissions is enhanced by the significantly higher yield of delayed neutrons. This is the basis for the value mentioned above.

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Fig. 7. Cf^{252} Source Data at Low Power Levels, Corrected for Subcriticality

The corrected data at the lower power levels are plotted in Fig. 7 to show that the correction based on a difference in transfer functions adequately accounts for the anomolous behavior at the very low levels. It is seen that even the values at the lowest level are consistent with a linear relationship between amplitude and power level. The results of least-squares fit of the corrected sets of data, assembled in Table XI, reveal that there is only a slight difference if the lowest values are included.

TABLE XI. Weighted Least-squares Fit of the Corrected Cf^{252} Data

Reactor Run No.	No. of Data Points	Intercept, mV	Slope, $\times 10^3$
1	8	1.2918 ± 0.0084	$-(0.2492 \pm 0.0330)$
	7	1.2864 ± 0.0098	$-(0.2364 \pm 0.0350)$
2	8	1.2865 ± 0.0041	$-(0.1918 \pm 0.0234)$
	7	1.2772 ± 0.0052	$-(0.1662 \pm 0.0210)$
3	8	1.2854 ± 0.0066	$-(0.2248 \pm 0.0241)$
	7	1.2880 ± 0.0130	$-(0.2407 \pm 0.0358)$
22	7	1.2781 ± 0.0063	$-(0.2502 \pm 0.0946)$
	6	1.2760 ± 0.0080	$-(0.2245 \pm 0.1440)$
Weighted means	all	1.2851 ± 0.0029	$-(0.2209 \pm 0.0148)$
	>20 mV	1.2792 ± 0.0038	$-(0.1964 \pm 0.0160)$

B. Absolute Absorption Rate of Manganese

Each of the three manganese-copper foils previously oscillated in Runs 4, 5, and 6 was irradiated in order to determine the absolute absorption rate of its manganese content. For the first two irradiations, three samples were placed at symmetrical positions in the turret of the sample changer. The first was the copper sample used as the reference material in the oscillator experiments. Five sets of oscillations were made with this sample. The second was the manganese No. 1 or manganese No. 2 sample, located four positions from the first. It was then inserted into the reactor for a 30-min irradiation. The third sample, a manganese-copper foil of comparable dimensions and weight, provided an indication of the amount of activity obtained by the irradiated sample during its residence time in the turret. The background foil was omitted in the third irradiation.

All three irradiations were performed under quite similar conditions. Typically, the reactor was started up and made supercritical by withdrawing the last control rod, resulting in a doubling time of approximately 35 sec. About 7 min were required for the reactor power to increase by a factor of 1000 to the level at which the oscillations and irradiations were performed, and approximately 16 min were required for the oscillations. After the 30-min irradiation of this sample, the reactor was shut down. The current level read on the oscillation detector channel, and confirmed by Channels 4 and 5, was used to duplicate the power level in the succeeding runs. Pertinent characteristics of these three irradiations are presented in Table XII.

Following each irradiation, five 1/2-in.-diameter foils were punched at 1-in. intervals along the lengths of both the irradiated and background samples. These foils were first gamma counted with 2-in.-diameter NaI crystals on opposite sides (Channels 3 and 4 of the automatic counting facility) in order to provide data from which to derive the required background correction. The bias was set with a Cs^{137} source to reject all pulses having an energy less than 662 keV. The five foils from each of the irradiated samples were then counted in the beta-gamma coincidence arrangement used in connection with the manganese bath procedure for source calibration.¹³ This yielded an absolute disintegration rate for the foils.

Table XIII indicates the axial variation of flux within the turret as determined from the gamma counting of the background foils. Similar activities of the irradiated foils are included for comparison. Since the background and irradiated foils have different irradiation histories, it is necessary to apply a correction factor to the background foils before using this information in connection with the irradiated ones. The turret foils received typically successive 16- and 30-min irradiations at the same location, and the irradiated ones a 16-min exposure in the turret followed by 30 min at the core position used for the oscillator experiments. Thus a fraction

$$\frac{(1 - e^{-16\lambda}) e^{-30\lambda}}{1 - e^{-46\lambda}}$$

of the turret foil activity is due to events occurring in the first 16 min. (Here λ is the disintegration constant for the Mn^{56} , in min^{-1} .) This amount must be subtracted from the activity of the irradiated foil to yield the activity occurring from the 30-min exposure in the core. By application of this correction, an amount varying from 0.18 to 0.39% of the core foil activity was attributed to background, as given in Table XII. The variation with position is consistent with the location of the cadmium-covered sample changer in a flux gradient just above the 4-in.-thick top water reflector.

TABLE XII. Determination of Absolute Absorption Rate of Manganese

Run No.	7	11	12
Doubling time, sec	33	40	38
Time to reach power, min	5	8	7
Time for oscillations, min	17	16	16
Irradiation time, min	30	30	30
Scram time	1110	1015	1204
Irradiated sample, g	Mn #1: 1.16155	Mn #2: 1.16134	Mn #3: 1.16051
Al sample can, g	18.10454	18.12910	18.02745
$(\Delta V/V)_{\text{Cu}} \times 10^3$	2.3789 ± 0.0332	2.3284 ± 0.0244	2.3712 ± 0.0229
Power level			
Oscillator channel, mV	955 ± 1	955 ± 1	955 ± 1
Channel 4, mV ($10^7 \Omega$)	226	226	225
Channel 5, Amp $\times 10^8$	5.1	4.2	4.2
<u>β-γ Coincidence Counting</u>	$\bar{t} \quad \frac{1}{M} \left \frac{dN}{dt} \right _{t=0} \times 10^{-5}$ (20-min counts)	$\bar{t} \quad \frac{1}{M} \left \frac{dN}{dt} \right _{t=0} \times 10^{-5}$ (10-min counts)	$\bar{t} \quad \frac{1}{M} \left \frac{dN}{dt} \right _{t=0} \times 10^{-5}$ (10-min counts)
Foil A	1339 1.773	1105 1.800	1307 1.780
B	1403 1.760	1118 1.792	1321 1.775
C	1427 1.762	1132 1.790	1336 1.746
D	1452 1.745	1146 1.783	1352 1.744
E	1516 1.749	1159 1.773	1407 1.729
	1.759	1.787	1.755
<u>Weighted Average:</u>			
20-min recount, next day, Foil A	944 1.754	910 2.254	935 1.843
<u>Background Correction Factor</u>			
Foil A	1.0039	1.0036	1.0036
B	1.0030	1.0026	1.0026
C	1.0024	1.0025	1.0025
D	1.0020	1.0020	1.0020
E	1.0020	1.0018	1.0018
Abs/sec/g Mn $\times 10^{-6}$, R_a (normalized to 1000 mV)	1.654 ± 0.018	1.675 ± 0.004	1.643 ± 0.023

TABLE XIII. Relative Manganese Activities in the Core and Turret

	Foil No.:	A	B	C	D	E
	Run No.					
Position from bottom of can, in.		0.50	1.50	2.50	3.50	4.50
Specific count rate, core, N_{0C}	7	719.0	720.9	716.4	714.0	710.0
	11	726.4	725.5	723.0	717.3	721.6
Specific count rate, turret, N_{0T}	7	4.637	3.567	2.838	2.318	2.306
	11	3.012	2.159	2.052	1.614	1.507
Ratio, $(N_{0T}/N_{0C}) \times 10^3$	7	6.44	4.94	3.96	3.24	3.24
	11	4.14	2.97	2.83	2.25	2.08
Flux Ratio, $(\phi_T/\phi_C) \times 10^3$	7	2.55	1.95	1.56	1.28	1.28
	11	2.60	1.87	1.78	1.41	1.31
$(\phi_T/\phi_C)_{ave} = \frac{\int_0^{5.0} (\phi_T/\phi_C) dx}{\int_0^{5.0} dx} = 1.766 \times 10^{-3}$						

In the beta-gamma coincidence counting, each foil was counted in the interval from 1/2 to 4 hr after irradiation, for a time sufficient to give a minimum of 40,000 counts for each of the five foils from each of the three irradiations. For each set, one foil was recounted on the following day. The calculated initial disintegration rates per gram of foil material are included in Table XII. These values have not been corrected for the small amount of activity introduced during location in the turret. The three foils counted on the following day, some eight to nine half-lives after irradiation, had appreciably less activity and hence a greater uncertainty of statistical origin. Yet they are not in significant disagreement with the assumption that only manganese activity is detected in the beta-gamma coincidence counting. This agreement, combined with the known cross sections, half-lives, and decay schemes for the copper isotopes argues against any contribution of copper activity to the measured counting rates.

The quantity $|dN/dt|_{t=0}/M$ is computed from the coincidence counting data by a computer code¹⁴ which corrects for background and random coincidences, and also calculates the efficiencies for β and γ detection. This is related to the desired absorption rate $\phi \Sigma_a V/M$ in the following way:

$$\frac{|dN/dt|_{t=0}}{M} = \frac{\phi \Sigma_a V}{M} (1 - e^{-\lambda T}).$$

Here T is the irradiation time, and $\left| \frac{dN}{dt} \right|_{t=0}$ is the disintegration rate for the foil of mass M at the end of the irradiation. Thus the values reported in Table XII require division by the quantity $(1 - e^{-\lambda T})$ as well as the background correction factor.

Since the foil was not 100% manganese, the absorptions per second per gram of manganese were obtained by dividing by the manganese content. The irradiations were performed at a power level of 955 mV on the oscillator channel, but a normalization to a reference power level of 1000 mV was made for convenience. The resulting values for the absorptions per second per gram of manganese, so normalized, are included in Table XII.

Additional corrections are required because of the complexity of the Mn^{56} decay scheme and the possibility of gamma-gamma rather than beta-gamma coincidences. Self-absorption by the foil results in a different efficiency for detection of each of the beta groups by the plastic scintillator. Branching in the gamma emission from the excited levels together with the use of a gate to detect preferentially the 0.845-MeV gamma with the NaI(Tl) crystal adds to the complication in the coincidence counting. This is considered in detail in Appendix B, where it is shown that the values for the absorption rate given in Table XII should be reduced by 0.968% to allow properly for the peculiarities of the Mn^{56} decay and the characteristics of the detectors used.

A coincidence count can result even when no beta ray is detected by the plastic scintillator, due to gamma interactions therein. The probability of gamma-gamma coincidences is very low, and consideration of the various ways that these can occur is also included in Appendix B. There the basis is presented for a token reduction of the absorption rate by 0.05% for the contribution from gamma-gamma coincidences.

C. Absolute Fission Rate of U^{235}

Initially a procedure very similar to that used for the determination of the manganese absorption rate was followed for the determination of U^{235} fission rates. The copper sample and two nearly identical samples of U^{235} were placed at three symmetrical locations within the turret of the sample changer. The standard copper sample was oscillated to obtain five sets of values. The first U^{235} sample was irradiated for 30 min and the second was intended to provide background information.

Values obtained from gamma counting of the irradiated and background U^{235} samples in the automated counting facility prior to chemical processing of the irradiated sample were consistent with the values obtained with the manganese samples. However, the large number of different half-lives involved makes very difficult the use of the time-dependent ratio of

activities to deduce a background correction from those data. It is feasible to derive the required background information from the measurements made in connection with the manganese irradiations. From those data, it is possible to determine the magnitude of the flux within the sample changer relative to that at the position of irradiation within the core. Consequently, the exposure of a second uranium sample in the turret was discontinued.

The activity for the irradiated (C) and background (T) manganese samples may be expressed as follows:

$$N_{0C} = \frac{\phi_T \Sigma_a V}{\lambda} \left(1 - e^{-\lambda t_T}\right) e^{-\lambda t_I} + \frac{\phi_C \Sigma_a V}{\lambda} \left(1 - e^{-\lambda t_I}\right); \quad (8)$$

$$N_{0T} = \frac{\phi_T \Sigma_a V}{\lambda} \left[1 - e^{-\lambda(t_T + t_I)}\right]; \quad (9)$$

Here t_T is the effective time at full power prior to the insertion of the sample for irradiation and t_I is the irradiation time. Dividing Eq. (9) by Eq. (8) and solving for the ratio of core to turret flux yield the following relationship:

$$\frac{\phi_C}{\phi_T} = \frac{1 - e^{-\lambda(t_T + t_I)}}{1 - e^{-\lambda t_I}} \left[\frac{N_{0C}}{N_{0T}} - \frac{\left(1 - e^{-\lambda t_T}\right) e^{-\lambda t_I}}{1 - e^{-\lambda(t_T + t_I)}} \right]$$

From the information presented in Table XIII, the relative values for the flux at various positions in the turret were derived. These are

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Fig. 8. Turret Flux Relative to Central Core Flux

short-lived that they can be ignored, i.e., it is realistic to assume that the fission process results in the direct formation of Mo^{99} . Then from relations similar to Eqs. (8) and (9) above,

plotted in Fig. 8. It is necessary to extrapolate the curve drawn through these points in order to arrive at an average value for the flux in the turret, since the entire foils were dissolved and processed in the radiochemical procedure for the determination of fission rate. For the curve shown, ϕ_T/ϕ_C is 1.766×10^{-3} .

The observed fission rates are based on the measured concentration of Mo^{99} ($t_{1/2} = 66$ hr). The Mo^{99} precursors are sufficiently

$$\frac{|dN/dt|_{t=0}}{M} = \frac{\phi_C Y \Sigma_f V (1 - e^{-\lambda T})}{M} \left[\frac{1 + \phi_T / \phi_C (1 - e^{-\lambda T'}) e^{-\lambda T}}{1 - e^{-\lambda T}} \right] \quad (10)$$

Here Y is the yield of the Mo^{99} in the fission process, T' is the time that the irradiated sample spent in the turret during the oscillation measurements plus a time equal to the startup period (to account for the fissions occurring during the approach to power), and T is the irradiation time.

The total number of fissions per gram,

$$T \cdot \frac{|dN/dt|_{t=0}}{MY(1 - e^{-\lambda T})}$$

was obtained from the procedure followed by the Radiochemical Group.¹⁵ This involves dissolving the foil in HCl , followed by recovery of the Mo^{99} by solvent extraction. The material is then prepared in a form suitable for counting, and the activity is determined with a calibrated end-window flow counter. Thus to obtain the background-corrected value for the fission rate per gram, $\phi_C \Sigma_f V / M$, the reported quantity must be divided by T and the part of Eq. (10) within the brackets. The magnitude of this background correction factor is indicated in Table XIV, together with the values obtained by the radiochemical group.

TABLE XIV. Absolute Fission Rates of U^{235}

Run No.:	5 x 0.25 x 0.002-in. Samples						
	7	8	9	10	18	19	20
Irradiated sample, g	0.53932	0.53864	0.55057	0.56782	0.55490	0.55620	0.55921
Fissions/g x 10^{-10}	1.959	1.996	1.838	1.826	1.8505	1.8590	1.8156
Doubling time, sec	33	38	38	36	33	33	35
Time to reach power, min	5	7	6	5	4	5	6
Time for oscillations, min	17	13	19	17	16	18	15
Background correction factor	1.00324	1.00123	1.00123	1.00123	1.00123	1.00123	1.00123
Fissions/sec/g U^{235} x 10^{-6} (normalized to 1000 mV)	12.20	12.46	11.47	11.40	11.54	11.60	11.33
Run No.:	1.5 x 0.75 x 0.002-in. Samples			5 x 0.1 x 0.005-in. Samples			
	13	16	17	23	24	25	
Irradiated sample, g	0.54580	0.54241	0.54882	0.79115	0.75307	0.73825	
Fissions/g x 10^{-10}	1.751	1.739	1.761	1.5229	1.5425	1.5091	
Doubling time, sec	38	41	43	40	38	39	
Time to reach power, min	6	10	8	7	5	7	
Time for oscillations, min	16	16	16	16	16	16	
Background correction factor	1.00123	0.00123	1.00123	0.00129	0.00129	0.00129	
Fissions/sec/g U^{235} (normalized to 1000 mV)	10.93	10.85	10.99	9.503	9.626	9.418	

Table XIV summarizes the pertinent information for the numerous irradiations of U^{235} samples. There exist seven values for the 5-in. x 1/4-in. x 2-mil uranium samples (Runs 7-10 and 18-20). The initial determinations were repeated three additional times, since two of the first four measurements gave values higher than the other two. Three values are available for samples similar in dimensions to uranium

No. 2, namely, 1 1/2 in. x 3/4 in. x 2 mil (Runs 13, 16, and 17). Three values are also available for a 5-mil-thick sample (Runs 23-25). For all of these

measurements the samples were contained in a 17.78-g aluminum can, and the irradiation was for 30 min at a level of 955 ± 1 mV on the oscillator channel ($10^7 \Omega$). The average fission rates, background corrected and normalized to a power level of 1000 mV, are as follows:

Sample Size, in.	R_f , Average Fission Rate, fissions/sec/g $U^{235} \times 10^{-6}$
$5 \times 0.25 \times 0.002$	11.70 ± 0.24
$1.5 \times 0.75 \times 0.002$	10.92 ± 0.19
$5 \times 0.1 \times 0.005$	9.515 ± 0.100

These are equivalent to uranium No. 1, uranium No. 2, and uranium No. 3, respectively. Note that the values are expressed as fission rates per gram of U^{235} for consistency with the derived value for $(\Delta V/V)_U$. Since these occur as a ratio, any error in the 93.15% enrichment cancels.

One additional factor which should be considered in connection with the determinations of U^{235} fission rate is the contribution of U^{238} fissions to the Mo^{99} activity. There exist results of measurements with Core No. 7 of the Hi-C program which are useful for estimating the contribution of U^{238} fissions. Core No. 7 had the same hydrogen-to- U^{238} ratio as Core No. 8 but had a stainless steel clad for the fuel rather than the aluminum clad present during these measurements. For Core No. 7, a U^{238}/U^{235} fission ratio of 0.1014 ± 0.007 was observed.⁶ This measurement was made within a fuel pin and applies to the 3 w/o U^{235} fuel, which had a U^{238}/U^{235} ratio of approximately 33. The ratio of U^{238}/U^{235} for the samples oscillated and irradiated is 0.060, so the U^{238} -to- U^{235} fission ratio here is approximately 1.8×10^{-4} . This is negligible compared with other uncertainties. Such a correction would also be applied to $(\Delta V/V)_U$ and, since this and R_f enter the calculation of $\bar{\alpha}$ as a ratio, the effect is further reduced.

D. Strength of the Cf^{252} Source

The californium source oscillated in Runs 1, 2, 3, and 22 was calibrated by the manganese bath technique.¹⁶ Samples of a manganous sulfate solution were withdrawn nine times on five successive days after an exposure sufficiently long that a saturated activity had been achieved. Table XV summarizes the observations. Here N_t is disintegration rate for the manganese in the entire bath at the time that the liquid sample was withdrawn for gamma-gamma coincidence counting. It is derived from the observed counting rates, with appropriate corrections for counting channel deadtimes and background rates, accidental coincidences, Mn^{56} decay, and the fraction of the total solution counted. The results of correction for decay of the Cf^{252} source are given in the final column of Table XV, yielding an average value and standard error of $(3.810 \pm 0.022) \cdot 10^6$ disintegrations per second.

TABLE XV. Calibration of Cf^{252} Source

Aliquot Date	Time Withdrawn	$N_t, \times 10^{-6}$	$N_0, \times 10^{-6}$ $t = 0: 7/6/65$ $T_{1/2} = 2.646 \text{ yr}$
7/19/65	0918	3.769	3.804
	1157	3.764	3.799
7/20/65	0919	3.783	3.821
	1130	3.797	3.835
7/21/65	0942	3.764	3.804
	1127	3.753	3.794
7/22/65	0954	3.806	3.850
	1138	3.758	3.801
7/23/65	0912	3.737	3.783
Average:			3.810 ± 0.022

Since a saturated condition existed, this is also the neutron absorption rate for the manganese in the bath. Actually the rate is just one-half this value, because of a double efficiency in the gamma-gamma counting. The counting channels are gated so that both the 1.8- and 2.1-MeV peaks of the Mn^{56} decay can be accepted by either crystal. This factor is almost precisely offset by the ratio, R , of the total absorption of neutrons by the solution to the manganese absorptions for the particular concentration used in these measurements. By use of reported values for the thermal and epithermal cross sections of manganese, hydrogen, sulfur, and oxygen, suitably weighted for the hydrogen-to-manganese atom ratio of 42.51 existing during these measurements, a ratio of 2.102 ± 0.004 resulted.

Two additional factors, both a function of the concentration of the solution, must be considered in order to convert the observed disintegration rate to a source strength. The first is the efficiency, E , of the bath for the absorption of source neutrons. The bath efficiency factor includes the product of small correction factors for neutron leakage from the bath, streaming in the beam tube, and absorption by the tube and source, divided by the calibration of the gamma-gamma counter relative to the beta-gamma counter and the sample volume fraction. The second, a correction for attenuation of the gamma radiation within the counted solution, δ , was determined by counting samples of the solution and pure water to which was added a small quantity of a manganous sulfate solution of high specific activity. For the concentration used in these measurements, $E = 0.4039 \pm 0.0020$ and $\delta = 1.027 \pm 0.001$.

From these values, the strength for the Cf^{252} source on the reference date (July 6, 1965) was

$$\begin{aligned}
 S_0 &= \frac{ER}{2\delta} N_0 \\
 &= \frac{(0.4039 \pm 0.0020)(2.102 \pm 0.004)}{2(1.027 \pm 0.001)} (3.810 \pm 0.022) 10^6 \\
 &= (1.574 \pm 0.008) 10^6 \text{ neutrons/sec.}
 \end{aligned}$$

A large measure of confidence is placed in this value, since a calibration of the National Bureau of Standards No. II radium-beryllium (γ, n) source in this bath resulted in a value of $(1.186 \pm 0.008) 10^6 \text{ n/sec}$, in excellent agreement with its quoted value of $(1.180 \pm 0.013) 10^6 \text{ n/sec}$.

Additional confirmation for the above value comes from fission counting of the Cf^{252} sample just prior to its insertion in the quartz container. On June 30, 1965, $(2.51 \pm 0.09) 10^7$ fissions/min were measured. The use of a value of 3.78 neutrons/spontaneous fission and a correction for decay to July 6 yielded a source strength of $(1.57 \pm 0.06) 10^6 \text{ n/sec}$, in excellent agreement with the value from the manganese bath calibration.

V. CAPTURE-TO-FISSION RATIO

A. Experimental Value

When substituted into Eq. (7), the results presented in the preceding section, together with the recommended¹⁷ value of $\bar{\nu}$ for thermal fission of U^{235} , yield a value of 1.217 ± 0.015 for $1 + \bar{\alpha}$. Here the indicated error is computed from the standard deviations, based on the reproducibility of the individual measurements, except for the Cf^{252} source strength, which includes the uncertainty in the various factors required to convert the counting rate of the solution to a neutron intensity. Table XVI identifies the specific data on which this result is based.

TABLE XVI. Experimental Data for Capture-to-Fission Ratio

Quantity	Source--Value			Standard Error, %
R_{Mn}	<u>Mn No. 1</u>	<u>Mn No. 2</u>	<u>Mn No. 3</u>	
$(\Delta V/V)_{Mn}$	$(1.637 \pm 0.018) 10^6$	$(1.658 \pm 0.004) 10^6$	$(1.626 \pm 0.023) 10^6$	
$[R/(\Delta V/V)]_{Mn}$	$-(1.9346 \pm 0.0210) 10^{-3}$	$-(2.0220 \pm 0.0184) 10^{-3}$	$-(1.9773 \pm 0.0182) 10^{-3}$	
	$-(8.462 \pm 0.096) 10^8$	$-(8.200 \pm 0.078) 10^8$	$-(8.223 \pm 0.087) 10^8$	
	Weighted Average: $-(8.278 \pm 0.050) 10^8 \text{ sec}^{-1}$			0.60
R_f	<u>U^{235} No. 1</u>	<u>U^{235} No. 2</u>	<u>U^{235} No. 3</u>	
$(\Delta V/V)_{U^{235}}$	$(11.70 \pm 0.24) 10^6$	$(10.92 \pm 0.19) 10^6$	$(9.515 \pm 0.100) 10^6$	
$\left(\frac{\Delta V/V}{R_f}\right)_{U^{235}}$	$(6.0394 \pm 0.0526) 10^{-3}$	$(5.4197 \pm 0.0296) 10^{-3}$	$(4.7815 \pm 0.0352) 10^{-3}$	
	$(5.155 \pm 0.086) 10^{-10}$	$(4.961 \pm 0.032) 10^{-10}$	$(5.025 \pm 0.049) 10^{-10}$	
	Weighted Average: $(4.996 \pm 0.026) 10^{-10} \text{ sec}$			0.52
$(\Delta V/V)_{Cf}$	Table XI--Intercept, $P \geq 20 \text{ mV}$: $(1.2792 \pm 0.0038) 10^{-3}$			
S_0	7/6/65: $(1.574 \pm 0.008) 10^6$			
$\left(\frac{\Delta V/V}{S_0}\right)_{Cf^{252}}$	$(8.127 \pm 0.059) 10^{-10} \text{ sec}$			0.73
$\bar{\nu}$	IAEA: 2.430 ± 0.008			0.33
$1 + \bar{\alpha}$	1.217 ± 0.015			1.23
$\bar{\alpha}$	0.217 ± 0.015			6.91

Each of the three values for R_{Mn} was derived from measurements of the activity of five foils punched from the irradiated samples. They exhibited very good agreement within each set, and among the three sets. The values are the normalized ones given in Table XII, reduced by approximately 1% to allow for the complexity of the Mn^{56} decay scheme and the occurrence of gamma-gamma coincidences. The relative specific reactivities for the three manganese samples are repeated from Table VI. The ratio of absorption rate to reactivity for manganese has appreciably less than a 1% standard error.

The similar ratio for U^{235} , in spite of the spread of the individual measurements due to differences in flux depression and self-shielding for the three sizes of samples, has only a half percent standard error. It was derived from values reported previously in Tables VI and XIV.

Table XI reported the results of least-squares fitting of the data from oscillations of the californium source at several power levels, suitably corrected for the subcritical state of the Hi-C Core No. 8 during these measurements. The weighted average of those fits in which the results at approximately 10 mV were omitted was chosen for the calculation of effective capture-to-fission ratio. This has been done, since the correction for subcriticality amounted to approximately 10% for these measurements at the lowest power level, P, yet in the least-squares fit there was no reduction in the weight assigned to these values for any uncertainty in this correction. The value of the intercept based on values greater than 20 mV is less than 1/2% lower than that obtained from fitting all data (1.2792 versus 1.2851). For normalization to the reference level of 1000 mV, the intercept has been divided by 10^3 . The value for the apparent reactivity of the californium source, relative to its strength on the date to which the oscillator measurements have been corrected, also has a standard error less than 1%.

In the calculation of $1 + \bar{\alpha}$, the value for number of neutrons per fission reported in the table was reduced by 0.006 to account for the 38% of the delayed neutrons which are emitted when the oscillating sample is in the "out" position. The basis for this is discussed in Appendix C.

The standard deviation reflects only the reproducibility of the measurements and does not reflect any additional uncertainty due to possible systematic errors. At the start of these experiments it was realized that the errors in the individual measurements had to be held in the vicinity of 1% if a useful value for alpha was to be obtained. This has been accomplished.

The quantity $\bar{\alpha}$, rather than $1 + \bar{\alpha}$, is of principal interest, so the uncertainty is magnified when unity is subtracted from the calculated value of $1 + \bar{\alpha} = 1.217 \pm 0.015$. By use of the more common probable error, equal to 0.6745 times the standard deviation, a value of $\bar{\alpha} = 0.217 \pm 0.010$ results.

B. Semiempirical and Calculated Values

In the Hi-C program, values for alpha for several cores of stainless steel-clad fuel have been obtained from a combination of measured cadmium ratios for U^{235} fission and calculated values for thermal and epithermal alphas.⁶ A comparison of certain characteristics of some of the Hi-C cores with the core used for the measurements reported here will serve to provide an approximate value for alpha. Measured cadmium ratios for two of the Hi-C cores and in the oscillator tube are presented in Table XVII.

TABLE XVII. Measured Cadmium Ratios in
Hi-C Cores of 1.24-cm Square Pitch

Core No.	Clad	Location	Mn		U ²³⁵	
			Foil mg/cm ²	CdR	Foil mg/cm ²	CdR
7	SS	Fuel	38	3.00 ± 0.03		4.49 ± 0.11
		Moderator	38	3.19 ± 0.05		5.19 ± 0.15
8	Al	Fuel	38	3.12 ± 0.03		
		Moderator	38	3.37 ± 0.02		
		Oscillator tube	42	5.15	249	6.88
		Oscillator tube	35	5.20	78	7.95

The only significant difference between the core used for the oscillator experiment and Hi-C Core No. 8 was the absence of the central five fuel pins and their replacement by moderator and void. Measurements with 0.82-cm-diameter, approximately 2-mil-thick, manganese-copper foils in 20-mil-thick cadmium or aluminum covers placed between fuel pins in Core No. 8 gave a cadmium ratio for manganese of 3.37 ± 0.02 . Within a fuel pin the value was 3.12 ± 0.03 . The same measurement at the center of the oscillator tube gave values of 5.15 and 5.20, indicating a softer spectrum as would be expected because of removal of the fuel pins and their partial replacement by water. Details of the cadmium-ratio measurement for both manganese and U²³⁵ at the oscillator position are given in Appendix D.

No values exist for direct comparison with the U²³⁵ cadmium ratios of 6.89 and 7.95 measured in the oscillator tube with 5- and 2-mil-thick foils, respectively, but measurements with Hi-C Core No. 6 are consistent with this value. Core No. 6 had the same 1.24-cm square pitch as Core No. 8 and the assembly used for the oscillator measurements, but it was composed of the stainless steel-clad Hi-C fuel pins. A cadmium ratio for manganese of 3.19 ± 0.05 in the moderator between fuel pins was measured for the same sizes of foils used for the measurements reported above, indicating a hardened spectrum when the aluminum cladding is replaced by stainless steel. With foils placed between pellets inside a fuel pin, the cadmium ratio for manganese was measured as 3.00 ± 0.03 . Measured cadmium ratios for U²³⁵ fission of 5.19 ± 0.15 and 4.49 ± 0.11 were obtained in the moderator and within a fuel pin, respectively. Although the foils used for these U²³⁵ measurements were not similar to those used in the oscillator tube, comparison with the various cadmium ratios for manganese indicates that the values of 6.89 and 7.95 obtained in the oscillator tube are not unreasonable.

From THERMOS¹⁸ and GAM-I¹⁹ calculations for Hi-C Core No. 8, values of 0.1785 for α_{th} and 0.4667 for α_{epi} Cd are available.¹² Combining

these with the measured cadmium ratios for U^{235} fissions in the oscillator tube yields the following semiempirical values for the effective α in the oscillator tube:

$$\begin{aligned}\bar{\alpha}_{25} &= \alpha_{th} \frac{(CdR)_f - 1}{(CdR)_f} + \frac{\alpha_{epi} Cd}{(CdR)_f} \\ &= 0.215 \text{ (2-mil } U^{235}) \text{ and } 0.220 \text{ (5-mil } U^{235}).\end{aligned}$$

The differences in cadmium ratios for U^{235} fissions is consistent, at least qualitatively, with the reduction in self-shielding associated with the use of the nominal 2-mil-thick material compared with the 5-mil foil. Note that the semiempirical value for $\bar{\alpha}$ derived from this ratio is not very sensitive to the experimental values, being 0.215 and 0.220, respectively.

These values are in excellent agreement with the value derived from the oscillator experiment. The value for $\alpha_{epi} Cd$ is not very sensitive to the degree of moderation, and it is in reasonably good agreement with an average value of 0.464 obtained with a 20-mil thickness of cadmium.²⁰ This is not surprising, since the data of GAM-I have been adjusted to agree with the results of this and other integral-type experiments. However, the neutron energy in the GAM-I calculation extends from 0.415 eV upward, whereas the effective cadmium cutoff for isotropic incidence on a 20-mil sheet is nearer 0.5 eV.²¹ A downward shift in the effective cadmium cutoff would decrease the cadmium ratio, with an attendant increase in the semiempirical value for α_{25} .

An additional value is available for comparison with the result of this experiment. Recent calculations²² have yielded a value of $\alpha_{25} = 0.2326$ for Hi-C Core No. 8, in reasonable agreement with the slightly lower experimental value determined at a location having a slightly higher H/U ratio than the average for Core No. 8.

VI. COMMENTS ON METHOD FOR $\bar{\alpha}$ DETERMINATION

The development of a procedure for the direct measurement of capture-to-fission ratio in low-flux systems is a significant addition to the techniques available for experimentation with zero-power reactors. Yet it would be premature to claim that this procedure is now available for application routinely without additional testing and further investigation of several potential sources of error. It is appropriate to enumerate several limitations, experimental difficulties, and known, suspected, or possible systematic errors which warrant additional consideration.

The determination of the worth or importance of reactor and fission neutrons has involved the use of substitute materials for U^{235} . The cadmium ratio of 5.18 for 2-mil-thick manganese, compared with cadmium ratios for U^{235} fissions of 6.88 and 7.95 for 5- and 2-mil-thick foils, respectively, is a measure of the extent to which manganese adequately samples the reactor neutrons. It is known that manganese has a resonance at 337 eV and that U^{235} has numerous capture and fission resonances in the epithermal region, so that their relative response as a function of neutron energy varies markedly. The cadmium ratios are of course only coarse indicators of their average response to reactor neutrons divided into two energy groups.

A comparison of the cadmium ratio for U^{235} absorption $(CdR)_a$ with the value for manganese is more pertinent. Although a direct measurement of this quantity is not practical in low-flux systems, it can be obtained from the measured fission cadmium ratio $(CdR)_f$ and the calculated values for the effective thermal and epithermal capture-to-fission ratios. The relationship is as follows:

$$(CdR)_a = [(CdR)_f - 1] \frac{1 + \bar{\alpha}_{th}}{1 + \bar{\alpha}_{epi\ Cd}} + 1.$$

This yields absorption cadmium ratios of 5.73 for the 5-mil-thick and 6.58 for the 2-mil U^{235} foils, in reasonable agreement with the $(CdR)_{Mn}$ of 5.18.

The neutrons from spontaneous fission of Cf^{252} have been used as a substitute for U^{235} fission neutrons. The average fission-neutron energy for U^{235} is 1.94 ± 0.06 MeV and for the spontaneous fission of Cf^{252} is 2.2 ± 0.1 MeV. For both manganese and Cf^{252} , their use as substitutes is warranted for lack of a more suitable material. In fact, the high rate of spontaneous fission of Cf^{252} is a key factor in this measurement, since absorption effects for the minute quantity of material used as a source of fission neutrons are negligible.

As a matter of interest, the relative worths of fission and absorbed reactor neutrons is given by

$$\frac{W_f}{W_a} = - \left(\frac{\Delta V/V}{S_0} \right)_{Cf} \left(\frac{R}{\Delta V/V} \right)_{Mn}$$

From the experimental data of Table XVI, the value is 0.673. This is realistic in view of the migration of fission neutrons to a region of lower effectiveness and their chance of leakage from the relatively small core during their slowing down and diffusion.

The Cf^{252} source was at the center and the foils used for the cadmium ratio determinations were located near the bottom of the sample cans, while the other samples were distributed along the length at the wall of the can. If any of the effects produced are position-sensitive, then the various measurements were not performed under equivalent conditions. The activities of the foils punched along the length of the irradiated manganese samples, as reported in Table XII, indicate that the flux gradient over the height of the sample can is quite small.

Another limitation involves the possibility of effects due to inelastic scattering and moderation. This was investigated only in a cursory fashion. For example, a 33.6-g sample of bismuth produced a signal barely perceptible above the noise level, giving a $\Delta V/V$ of $\sim 9 \times 10^{-6}/g$, and so no further measurements of this type were made.

Because of possible self-shielding effects, the derived value of alpha is strictly applicable only for the range of U^{235} foil thicknesses for which measurements were made. From the slight variation of the quantity $[(\Delta V/V)/R_f]_{U^{235}}$ with foil thickness, as indicated by the data for U^{235} samples Nos. 1 and 3 in Table XVI, together with the relative insensitivity of the derived value of $\bar{\alpha}$ on this quantity, it is inferred that any sensitivity to U^{235} sample thickness is slight.

Differentiation of the expression for $1 + \bar{\alpha}$ in terms of the experimentally determined quantities reveals that the sensitivity of the value of $1 + \bar{\alpha}$ to errors in the individual quantities varies markedly. This is fortunate, since the particular quantity which has been the most troublesome, R_f , is among those on which the value of $1 + \bar{\alpha}$ depends in the least sensitive way. A 1% error in $\bar{v}$, $(\Delta V/V)_{Cf}$, and S_0 each results in a $1\frac{1}{3}\%$ error in $1 + \bar{\alpha}$, whereas a 1% error in R_{Mn} and $(\Delta V/V)_{Mn}$ results in a comparable error in $1 + \bar{\alpha}$; a similar error in $(\Delta V/V)_U$ and R_f leads to only a 1/3% error.

Difficulty was experienced in obtaining reproducible values for the determination of the fission rate in irradiated U^{235} samples by the radiochemical technique. For one of the sample sizes, seven separate

determinations were made in order to reduce the standard error. In a separate experiment, fourteen samples were irradiated on a rotating wheel (to insure identical exposures), yet the radiochemical determinations for these samples had a standard error of 0.8%. In addition, the possibility of a systematic error exists in this determination, since the fission yield of Mo^{99} enters in this determination.

Finally, in the data treatment certain quantities have been normalized to a power level of 1000 mV on the oscillator channel, and the isotopic content of the uranium foils and the manganese content of the manganese-copper foils have been introduced. This has been done only for convenience. It is immaterial whether the values for R_{Mn} , R_f , and $(\Delta V/V)_{\text{Cf}}$ are each normalized to 1000 mV or only the value for $(\Delta V/V)_{\text{Cf}}$ is normalized to the 955-mV power level at which the samples were irradiated. Since the U^{235} samples oscillated and irradiated were prepared from the same batch of material, the presence of values based on measurements with these samples as a ratio in Eq. (7) means that the isotopic abundance of U^{235} cancels. Similarly the oscillated and irradiated manganese-copper samples were identical and so the manganese content cancels in the ratio $[R/(\Delta V/V)]_{\text{Mn}}$ which occurs in Eq. (7).

APPENDIX A

Reactor Response to Fissile Sample

In order to review briefly the fundamental theory of reactor oscillator measurements, the problem will be treated in a two-group approximation by means of standard perturbation techniques. Since excellent treatments of the perturbation method are readily available,^{23,24} most of the mathematical details are omitted.

Suppose the reactor is just critical and that the quantities appearing in the neutron-balance equations are such functions of the spatial coordinates that the equations may be used to describe all regions of the reactor. The two-group balance equations may then be written as

$$-\nabla \cdot [D_1(\underline{r}) \nabla \phi_1(\underline{r})] + \Sigma_{R,1}(\underline{r}) \phi_1(\underline{r}) = \bar{\nu} \Sigma_{f,2}(\underline{r}) \phi_2(\underline{r}) \quad (A-1)$$

and

$$-\nabla \cdot [D_2(\underline{r}) \nabla \phi_2(\underline{r})] + \Sigma_{a,2}(\underline{r}) \phi_2(\underline{r}) = p_c(\underline{r}) \Sigma_{R,1}(\underline{r}) \phi_1(\underline{r}). \quad (A-2)$$

Here the subscripts 1 and 2 refer to the fast and slow groups, respectively, and D , ϕ , and Σ are the diffusion coefficient, the neutron flux, and the macroscopic cross section. $\Sigma_{R,1}$ is the removal cross section (due to elastic and inelastic scattering) of neutrons from the fast to the slow group. The function $p_c(\underline{r})$ is introduced to account for actual absorption during slowing down; it is the fraction of neutrons leaving the fast group which reach the slow group at position $\underline{r}$. The fluxes and their derivatives are assumed to be finite and continuous throughout the reactor and to vanish on the extrapolated boundary. In writing the above equations, it was also assumed that only low-energy neutrons contribute to the absorption and fission rates and that the fission neutrons constitute the high-energy group.

Consider the situation of a small lump of absorbing, fissile material introduced at some point in the critical system. In order to maintain criticality, it would be necessary to adjust the control rods, thereby adding or reducing reactivity. The problem is to calculate the reactivity change resulting from the introduction of the fissile sample. For mathematical convenience, assume that nonuniform changes in the neutron parameters of the reactor system can be offset by a uniform change in the average number of neutrons released per fission, $\bar{\nu}$, so that the system remains critical. Then, following Meghrebian and Holmes,²⁴ perturbation methods yield

$$\begin{aligned}
 \rho = \frac{\delta k}{k} = -\frac{\delta \bar{\nu}}{\bar{\nu}} = \frac{1}{\bar{\nu} \int_V \phi_1^\dagger \Sigma_{f,2} \phi_2 \, d\mathbf{r}} & \left[\bar{\nu} \int_V \phi_1^\dagger \delta \Sigma_{f,2} \phi_2 \, d\mathbf{r} - \int_V \phi_2^\dagger \delta \Sigma_{a,2} \phi_2 \, d\mathbf{r} \right. \\
 & - \int_V \phi_1^\dagger \delta \Sigma_{R,1} \phi_1 \, d\mathbf{r} + \int_V \phi_2^\dagger [\rho_c \Sigma_{R,1}] \phi_1 \, d\mathbf{r} \\
 & \left. - \int_V \delta D_1 \nabla \phi_1^\dagger \cdot \nabla \phi_1 \, d\mathbf{r} - \int_V \delta D_2 \nabla \phi_2^\dagger \cdot \nabla \phi_2 \, d\mathbf{r} \right]. \quad (A-3)
 \end{aligned}$$

The integrals extend over the extrapolated boundaries of the reactor. However, the changes in the reactor parameters ($\delta \Sigma_f$, $\delta \Sigma_a$, δD , etc.) are zero everywhere except over the region of the sample. Therefore, the integrals in the brackets of Eq. (A-3) may be considered as extending only over the sample volume. If the sample is oscillated within a void in the reactor, as is usually the case, the δ -terms refer simply to the parameters of the sample. $\phi_1^\dagger$ and $\phi_2^\dagger$ are the fast and slow adjoint fluxes.

To simplify Eq. (A-3), the following assumptions are made:

- (1) The sample is placed within a void in the reactor. Therefore, $\delta \Sigma_{f,2}$, $\delta \Sigma_{a,2}$, δD_1 , δD_2 , and $\delta \Sigma_{R,1}$ are just the parameters of the sample.
- (2) The sample is placed at the center of the core where the flux gradients are essentially zero. This eliminates the last two integrals in Eq. (A-3).
- (3) The removal cross section $\Sigma_{R,1}$ is essentially unchanged by the presence of the sample. This assumption implies that the effects of slowing down and inelastic scattering are negligible. Consequently, the middle two terms in Eq. (A-3) vanish.
- (4) The presence of the sample does not significantly change the nonleakage probability, that is, a negligible number of neutrons scattered from the sample leak out of the system through the void in which the sample is oscillated.

Equation (A-3) may now be written as

$$\rho = \frac{1}{N} [\bar{\nu} R_f W_f - R_a W_a], \quad (A-4)$$

where

$$N = \bar{\nu} \int_V \phi_1^\dagger(\underline{r}) \Sigma_{f,2}(\underline{r}) \phi_2(\underline{r}) d\underline{r};$$

$$R_f = \text{fission rate in sample} = \int_{\text{sample}} \delta \Sigma_{f,2}(\underline{r}) \phi_2(\underline{r}) d\underline{r};$$

$$R_a = \text{absorption rate in sample} = \int_{\text{sample}} \delta \Sigma_{a,2}(\underline{r}) \phi_2(\underline{r}) d\underline{r};$$

W_f = the "worth" of fast fission neutrons

$$= \frac{1}{R_f} \int_{\text{sample}} \phi_1^\dagger(\underline{r}) \delta \Sigma_{f,2}(\underline{r}) \phi_2(\underline{r}) d\underline{r};$$

and

W_a = the "worth" of the absorbed slow neutrons

$$= \frac{1}{R_a} \int_{\text{sample}} \phi_2^\dagger(\underline{r}) \delta \Sigma_{a,2}(\underline{r}) \phi_2(\underline{r}) d\underline{r}.$$

Now $R_a = R_f(1 + \bar{\alpha})$, where $\bar{\alpha}$ is the effective capture-to-fission ratio. Therefore Eq. (A-4) becomes

$$\rho = \frac{R_f}{N} \left[\bar{\nu} W_f - (1 + \bar{\alpha}) W_a \right]. \quad (\text{A-5})$$

This is the equation introduced in Section II for the reactivity effect produced by insertion of a fissile sample in a critical reactor.

APPENDIX B

Corrections in Coincidence Counting of Mn⁵⁶

Obviously, not all decaying atoms are detected in a counting procedure. The efficiency depends upon the sample's self-absorption of the radiation detected, the sample-detector geometry, and the efficiency of the detector. For a fixed sample size, these factors are independent of the manner in which the radioactive atoms are formed, except insofar as the distribution of active atoms within the sample is nonuniform.

A coincidence-counting procedure allows the determination of absolute disintegration rates in a straightforward manner, as follows. Assume an irradiated sample which contains a single radioactive species undergoing beta decay at a rate of D disintegrations per second, a fraction of which decay through an excited state of the product nucleus with accompanying emission of gamma radiation of yield Y_γ . If beta and gamma detectors are placed nearby, then the single- and coincident-counting rates observed are:

$$C_\beta = D S_\beta \Omega_\beta(r) \epsilon_\beta;$$

$$C_\gamma = D Y_\gamma S_\gamma \Omega_\gamma(r) \epsilon_\gamma;$$

$$C_{\beta,\gamma} = D S_\beta \Omega_\beta(r) \epsilon_\beta Y_\gamma S_\gamma \Omega_\gamma(r) \epsilon_\gamma.$$

Here C designates the observed counting rate after correction for background and accidentals, S is the transparency factor for sample, $\Omega(r)$ is the solid angle subtended by the detector, and ϵ is the efficiency of the detector. The absolute disintegration rate can be obtained directly from the observed counting rates and a knowledge of the disintegration scheme, as follows:

$$\frac{C_\beta C_\gamma}{C_{\beta,\gamma}} = \frac{D^2 S_\beta \Omega_\beta(r) \epsilon_\beta Y_\gamma S_\gamma \Omega_\gamma(r) \epsilon_\gamma}{D S_\beta \Omega_\beta(r) \epsilon_\beta Y_\gamma S_\gamma \Omega_\gamma(r) \epsilon_\gamma} = D. \quad (B-1)$$

The decay scheme for 2.58-hr Mn⁵⁶, based on recent measurements by Bienlein and Dinter,²⁵ is shown in Fig. 9. In the coincidence counting for the determination of the Mn⁵⁵ neutron-absorption rate, the 0.818-cm-diameter, 2-mil-thick manganese-copper foils punched from the 5-in.-long strip were placed between halves of a 1 $\frac{7}{8}$ -in.-diameter by 5/8-in. right circular cylinder of a Pilot B plastic scintillator. This was located adjacent to a 3-in.-diameter NaI(Tl) crystal. Beta rays detected in the former, in coincidence with gamma events detected by the latter in the vicinity of a window at 0.845-MeV energy, were used as a measure of the disintegration rate. It is necessary that the coincidence-counting rate, as

calculated by the COINC computer program,¹⁴ be corrected for the yields of the beta and gamma radiation detected as well as the small probability of gamma-gamma coincidences.

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Fig. 9. Decay Scheme for Mn⁵⁶

the *i*th beta group, and $\epsilon_{\gamma j}$ is the corresponding quantity for the gamma rays in the NaI(Tl) crystal. It is necessary to evaluate the factor $1 + k_{\text{Mn}}$ by which the ratio of the product of the net counting rates in the separate channels to the coincidence rate must be divided, to obtain the absolute disintegration rate, in terms of the appropriate yields and efficiencies.

Bienlein and Dinter's data for the energy and yields for the Mn⁵⁶ → Fe⁵⁶ transition are presented in Table XVIII, together with the detection efficiencies for each. The calculation of the latter is described in the material which follows.

It has been observed that a large portion of the absorption curve for a specific beta source can be represented by an exponential relationship of the form

$$I = I_0 e^{-\mu \rho t}$$

Here μ is the mass absorption coefficient (cm²/g), ρ the density (g/cm³), and t is the absorber thickness (cm). For this type of dependence, the fraction of beta particles escaping a foil of thickness t is²⁷

$$(1 - e^{-\mu \rho t}) / \mu \rho t.$$

1. Correction for Yields in Beta-Gamma Coincidences

Derivation of the correction for the complexity of the Mn⁵⁶ decay scheme is patterned after Axton's evaluation.²⁶ Since for Mn⁵⁶ there is no decay directly to the ground state, $f = 1$ in Eq. (B-1) and so

$$\frac{C_{\beta} C_{\gamma}}{C_{\beta, \gamma}} = D \frac{\sum_i B_i \epsilon_{\beta_i} \cdot \sum_j \Gamma_j \epsilon_{\gamma_j}}{f(B_i, \Gamma_j, \epsilon_{\beta_i}, \epsilon_{\gamma_j})} = D(1 + k_{\text{Mn}}). \quad (\text{B-2})$$

Here B_i is the fractional yield of the *i*th beta particle, Γ_j is that for the gamma ray, ϵ_{β_i} is the efficiency of the Pilot B plastic scintillator for the detection of

TABLE XVIII. Yields and Detection Efficiencies for Mn⁵⁶ Decay

Radiation	Energy, MeV	Yield, %	Efficiency, %
β_1	2.838	47	96
β_2	1.028	34	81
β_3	0.718	18	70
β_4	0.30	1	40
			$\% \div \sum_j \Gamma_j \epsilon_{\gamma_j}$
γ_1	0.845	98.63	89.22
γ_2	1.80	33.32	25.59
γ_3	2.11	17.46	18.44
γ_4	2.52	0.85	13.26
γ_5	2.67	0.68	11.63
γ_6	3.00	0.54	9.31
γ_7	3.38	0.15	7.41

The energy dependence of the absorption coefficient has been investigated in several transmission-type experiments. By use of the form suggested by Barnard,²⁸ namely, $\mu(\text{cm}^2/\text{g}) = 15.63/E^{1.2}$, where E is the maximum beta-particle energy in MeV, detection efficiencies of 92.3, 77.0, 67.5, and 37.8% were found for the four groups of beta particles. Weighting these by the yields results in an overall efficiency of 82.1% for detection of a beta particle by the plastic scintillator in the Mn^{56} decay. In some preliminary measurements an efficiency of 85.7% was observed, in contrast with a calculated value of 82.1%. Thus, for the purpose of calculating the correction factor $1 + k_{\text{Mn}}$, the individual efficiencies were scaled upward to achieve agreement with the observed overall efficiency. These adjusted values, recorded in Table XVIII, when combined with the individual reported yields result in an effective efficiency identical with that observed in the preliminary measurements.

By graphical integration of the pulse-height distribution from the NaI(Tl) crystal and comparison with an estimate of the Compton electron contribution to the counting rate in the energy window used, it is estimated that 12% of the total counting rate is due to gamma rays other than the 0.845-MeV one on which the window is set. Thus

$$\epsilon_{\gamma_1} = \frac{88}{0.9863} \sum_j \Gamma_j \epsilon_{\gamma_j} = 89.22\% \sum_j \Gamma_j \epsilon_{\gamma_j}$$

The differential cross section for the scattering of a photon of a specific energy into a lower energy interval has been calculated.²⁹ These results allow an evaluation of the response of the gamma detector in the Mn^{56} decay counting to gamma rays of energies other than 0.845 MeV. From such data, the efficiencies for detection of the other gamma rays (γ_2 through γ_7) in the window at 0.845 MeV can be calculated. The results, in terms of the total gamma-ray detection efficiency, are included in Table XVIII.

The data assembled in Table XVIII allow an evaluation of the correction factor required for the manner in which the coincidence counting is performed. The individual and coincidence counting rates are given in terms of the individual yields and efficiencies as follows:

$$\begin{aligned} C_{\beta} &= D \sum_i B_i \epsilon_{\beta_i} \\ &= [(0.47 \times 0.96) + (0.34 \times 0.81) + (0.18 \times 0.70) \\ &\quad + (0.01 \times 0.40)] D = 0.8566 D; \end{aligned} \quad (\text{B-3})$$

$$\begin{aligned}
C_{\gamma} &= D \sum_j \Gamma_j \epsilon_{\gamma_j} \\
&= D \left\{ (\Gamma_1 - \Gamma_2 - \Gamma_3 - \Gamma_4) \epsilon_{\gamma_1} + [\Gamma_2(\epsilon_{\gamma_1} + \epsilon_{\gamma_2}) + \Gamma_5 \epsilon_{\gamma_5}] \right. \\
&\quad \left. + [\Gamma_3(\epsilon_{\gamma_1} + \epsilon_{\gamma_3}) + \Gamma_6 \epsilon_{\gamma_6}] + [\Gamma_4(\epsilon_{\gamma_1} + \epsilon_{\gamma_4}) + \Gamma_7 \epsilon_{\gamma_7}] \right\} \\
&= 0.99997 D \sum_j \Gamma_j \epsilon_{\gamma_j} \quad (B-4)
\end{aligned}$$

This very slight difference from unity results from errors in rounding off the gamma-ray yields and efficiencies. For consistency with the coincidence rate evaluation below, this factor is used.

$$\begin{aligned}
C_{\beta, \gamma} &= D f(B_i, \Gamma_j, \epsilon_{\beta_i}, \epsilon_{\gamma_j}) \\
&= D (\Gamma_1 - \Gamma_2 - \Gamma_3 - \Gamma_4) \epsilon_{\gamma_1} \epsilon_{\beta_1} + [\Gamma_2(\epsilon_{\gamma_1} + \epsilon_{\gamma_2}) + \Gamma_5 \epsilon_{\gamma_5}] \epsilon_{\beta_2} \\
&\quad + [\Gamma_3(\epsilon_{\gamma_1} + \epsilon_{\gamma_3}) + \Gamma_6 \epsilon_{\gamma_6}] \epsilon_{\beta_3} + [\Gamma_4(\epsilon_{\gamma_1} + \epsilon_{\gamma_4}) + \Gamma_7 \epsilon_{\gamma_7}] \epsilon_{\beta_4} \\
&= D(0.4193 \times 0.96 + 0.3833 \times 0.81 + 0.1885 \times 0.70 + 0.0088 \\
&\quad \times 0.40) \sum_j \Gamma_j \epsilon_{\gamma_j} \\
&= 0.84853 D \sum_j \Gamma_j \epsilon_{\gamma_j} \quad (B-5)
\end{aligned}$$

The quantity $C_{\beta} C_{\gamma} / C_{\beta, \gamma}$ is calculated by the COINC computer program. Inserting the above values in Eq. (B-2) yields the following relation between the computed quantity and the absolute disintegration rate for the manganese sample counted:

$$\begin{aligned}
\frac{C_{\beta} C_{\gamma}}{C_{\beta, \gamma}} &= D(1 + k_{Mn}) \\
&= \frac{0.8566 D \times 0.99997 D \sum_j \Gamma_j \epsilon_{\gamma_j}}{0.84853 D \sum_j \Gamma_j \epsilon_{\gamma_j}} \\
&= 1.00948 D.
\end{aligned}$$

This correction factor of 0.948% for k_{Mn} is in good agreement with the values obtained by Axton.²⁶ His values at 85.7% efficiency for beta detection ranged from 0.9 to 1.2% for various assumptions regarding the mass absorption coefficient and Mn^{56} decay scheme.

The average efficiency for beta detection in the counting of foils from Runs 7, 11, and 12 was 85.36%. Since the above evaluation was for a slightly different value, as obtained in some preliminary counting, a slight modification in the value for k_{Mn} is appropriate. Axton's calculations showed that k_{Mn} is proportional approximately to $1 - \bar{\epsilon}_\beta$. Then the adjusted value applicable to the data in this report is $k_{Mn} = [(1 - 0.8536)/(1 - 0.8566)] \times 0.948\% = 0.968\%$.

2. Correction for Gamma-Gamma Coincidence

Axton has also considered the problem of sensitivity of his beta detector to gamma rays and the attendant probability of a gamma-gamma coincidence occurring when a beta particle is not detected. His evaluation of this effect is largely qualitative, but he concludes that the correction is below 0.1% for 2-mil-thick foils counted with a 4π gas-flow proportional counter. More detailed considerations presented here lead to the same conclusion for the plastic scintillator used in these measurements.

It should be realized that the possibility of a gamma-gamma coincidence is of concern only for a disintegration in which a beta particle is not detected by the plastic scintillator. This happens only 4% of the time for decay through the first excited level of Fe^{56} , 19% through the second, 30% through the third, and 60% through the fourth. Furthermore, these should be weighted by the probabilities for each mode of Mn^{56} decay, namely, 47%, 34%, 18%, and 1%. The probability of a gamma-gamma coincidence is reduced additionally by the very low efficiency of the gamma detector, being only 1.70% for the counting of foils from Runs 7, 11, and 12.

There are several ways in which a gamma-gamma coincidence can occur in the beta-gamma coincidence counting system used here. Radiation produced by the beta particles as they slow down in the foil material (Bremsstrahlung) can escape the foil and be detected by the plastic scintillator. One cascade gamma ray can be detected by the beta counter while the other is detected by the gamma counter. In addition, a single gamma ray may experience Compton scattering in the beta counter, followed by detection of the scattered gamma ray in the gamma counter. Coincidence events arising from pair production are not considered since the cross section for this process in plastic is appreciably lower than that for Compton-type interactions.

Since the correction factor for gamma-gamma coincidences is very small, the cylindrical plastic scintillator will be treated as a sphere of

water of equivalent mass surrounding a point emitter. The equivalent radius is 1.9 cm. This is convenient since the geometrical considerations are simplified and the appropriate gamma interaction probabilities are readily available.³⁰

In regard to Bremsstrahlung (Case I), this is the mechanism contributing the continuous radiation in X-ray tubes. The efficiency of X-ray tubes is known to be quite low, usually in a range from 0.01 to 0.1%. Since the maximum energies of the two beta groups having the highest yields in Mn^{56} decay are appreciably higher than those used in X-ray tubes, a generous upper limit of 10% is assumed for the probability that Bremsstrahlung associated with each undetected beta particle is detected by the beta counter. It is also assumed that the gamma rays associated with the Mn^{56} decay escape the plastic scintillator and are detected by the NaI(Tl) crystal with the efficiencies previously calculated. Then the coincidence rate per Mn^{56} disintegration, $C_{\gamma,\gamma}^I$, is one-tenth of the sum of each term within the brackets of Eq. (B-4) weighted by the probability of not detecting the beta particle which results in the particular excited state of Fe^{56} with which the term is associated. Thus

$$\begin{aligned} C_{\gamma,\gamma}^I &= 0.1[(1 - 0.96) \times 0.4193 + (1 - 0.81) \times 0.3833 + (1 - 0.70) \\ &\quad \times 0.1885 + (1 - 0.40) \times 0.0088] \times 0.0170 \\ &= 2.57 \times 10^{-4}. \end{aligned}$$

In regard to the simultaneous detection of cascade gamma rays, two possibilities exist. Either the first gamma ray interacts in the plastic scintillator and the succeeding 0.845-MeV gamma is detected by the NaI(Tl) crystal (Case IIa), or the latter is counted by the beta counter and the former interacts with the NaI(Tl) crystal with an energy loss falling between the energy window limits (Case IIb).

The probability for detection of a gamma ray by the plastic scintillator, $P_{\gamma_j}^\beta$, is $(1 - e^{-\mu \rho t})$. Evaluation for a 1.9-cm path length (t), a density of 1 g/cm³ (ρ), and appropriate values for the mass absorption coefficient, $\mu(E)$ cm²/g, from Reference 30 gives the following results:

$$\begin{aligned} P_{\gamma_1}^\beta &= 13.46\%, & P_{\gamma_2}^\beta &= 9.41\%, & P_{\gamma_3}^\beta &= 8.64\%, & P_{\gamma_4}^\beta &= 7.88\%, \\ P_{\gamma_5}^\beta &= 7.65\%, & P_{\gamma_6}^\beta &= 7.24\%, & P_{\gamma_7}^\beta &= 6.79\%. \end{aligned}$$

For Case IIa, the probability of a gamma-gamma coincidence per Mn^{56} decay is the sum of the products of the probability that the 0.845-MeV gamma is detected in the gamma counter, $(1 - P_{\gamma_i}^\beta) \epsilon_{\gamma_i}$, and the probability

that the higher energy gamma ray in cascade with it is detected in the beta counter when the associated beta ray is not. Thus

$$\begin{aligned}
 C_{\gamma,\gamma}^{\text{IIa}} &= (1 - P_{\gamma_1}^{\beta}) \epsilon_{\gamma_1} \sum_{i=j=2}^4 (1 - \epsilon_{\beta_i}) \Gamma_j P_{\gamma_j}^{\beta} \\
 &= 0.8654 \times 0.8922 \times 0.0170(0.19 \times 0.332 \times 0.0941 + 0.30 \\
 &\quad \times 0.1746 \times 0.0864 + 0.60 \times 0.0085 \times 0.0788) \\
 &= 1.43 \times 10^{-4}.
 \end{aligned}$$

In a similar fashion,

$$\begin{aligned}
 C_{\gamma,\gamma}^{\text{IIb}} &= P_{\gamma_1}^{\beta} \sum_{i=j=2}^4 (1 - \epsilon_{\beta_i}) \Gamma_j (1 - P_{\gamma_j}^{\beta}) \epsilon_{\gamma_j} \\
 &= 0.55 \times 10^{-4}.
 \end{aligned}$$

Finally, for the case in which a single gamma ray contributes recorded pulses in both counting channels, more crude approximations are permissible because of the very low probability of such events. It is realistic to assume that the contribution from the 0.845-MeV gamma can be ignored since so few of the scattered gamma rays from interaction of these gammas in the beta counter will have sufficient energy to register in the 0.5-MeV energy window at 0.845 MeV in the gamma counter. Each of the other gamma rays has a probability of interacting in the beta counter, when a beta ray is not detected, of the form $(1 - \epsilon_{\beta_i}) \Gamma_j P_{\gamma_j}^{\beta}$. As a reasonable approximation to the requirement that the only scattered gamma rays detected are those releasing in the gamma detector an amount of energy falling within the limits set by the window width at 0.845 MeV, a relation analogous to the case of neutron-proton scattering is used. The probability for detection of the scattered gamma ray is assumed to be $(0.5 \text{ MeV}/E_{\gamma_j}) \epsilon_{\gamma_j}$. Evaluating the product of these quantities for each of the six gamma rays (excluding γ_1) and adding gives

$$C_{\gamma,\gamma}^{\text{III}} = 0.43 \times 10^{-4}.$$

The total probability of a gamma-gamma coincidence, per Mn^{56} disintegration, is then

$$\begin{aligned}
 C_{\gamma,\gamma} &= (2.57 + 1.43 + 0.55 + 0.43) \times 10^{-4} \\
 &= 4.98 \times 10^{-4}.
 \end{aligned}$$

Thus, even with rather pessimistic assumptions, particularly in regard to a postulated efficiency of 10% for Bremsstrahlung, the estimated correction of $1/20$ of 1% for gamma-gamma coincidences is trivial compared with other uncertainties in the measurement of $\bar{a}$.

APPENDIX C

Delayed Neutrons "Lost" in Oscillations

The rate of formation of delayed-neutron precursors of disintegration constant λ_i in a neutron flux ϕ is given by

$$N_i(t) = \frac{\phi \Sigma_f V Y_i}{\lambda_i} (1 - e^{-\lambda_i t}). \quad (C-1)$$

Here $\Sigma_f V$ is the total fission cross section of the sample and Y_i is the yield of the i th delayed-neutron precursor. Thus the total number of the i th precursor formed during an irradiation of duration T is $N_{0i}T = \phi \Sigma_f V Y_i T$, and the number present at the end of the irradiation is

$$\frac{N_{0i}}{\lambda_i} (1 - e^{-\lambda_i T}) = N_{Ti}.$$

The neutron flux experienced by the sample varies with time, due to oscillation of the sample between the center of the core and the turret location. In these experiments, the actual time pattern is 2.2-sec transit time from the turret location ("out") to the core position ("in"), 5.3 sec at the "in" position, 2.2 sec in transit to the "out" position, and a storage there for 5.3 sec; then the cycle repeats. For simplicity, this can be approximated by a repetition of 7.5-sec irradiation (T) followed by an equal quiescent period. In practice many cycles occur while equilibrium is being achieved, prior to recording the response to oscillations, so the result of a single irradiation may be considered as representative of the total irradiation and subsequent decay for the sample.

At the end of n successive half-cycles, each of duration T , the precursor population due to production during a single half-cycle at the "in" position is the summation for all delayed-neutron emitters

$\sum_i N_{Ti} e^{-n \lambda_i T}$. The precursor decay during the previous half-cycle is $\sum_i N_{Ti} e^{-(n-1) \lambda_i T} (1 - e^{-\lambda_i T})$. The total precursor decay during the "out" time is then

$$\begin{aligned} P_0 &= \sum_i N_{Ti} (1 - e^{-\lambda_i T}) (1 + e^{-2 \lambda_i T} + e^{-4 \lambda_i T} + e^{-6 \lambda_i T} + \dots) \\ &= \sum_i \frac{N_{Ti} (1 - e^{-\lambda_i T})}{1 - e^{-2 \lambda_i T}}. \end{aligned} \quad (C-2)$$

Similarly, the total precursor decay during the "in" time is

$$\begin{aligned}
 P_I &= \sum_i N_{Ti}(1 - e^{-\lambda_i T})(e^{-\lambda_i T} + e^{-2\lambda_i T} + e^{-3\lambda_i T} + \dots) \\
 &= \sum_i \frac{N_{Ti}(1 - e^{-\lambda_i T}) e^{-\lambda_i T}}{(1 - e^{-2\lambda_i T})} \quad (C-3)
 \end{aligned}$$

The total precursor decay is

$$\begin{aligned}
 P &= P_0 + P_I = \sum_i N_{Ti}(1 - e^{-\lambda_i T})(1 + e^{-\lambda_i T} + e^{-2\lambda_i T} + e^{-3\lambda_i T} + \dots) \\
 &= \sum_i \frac{N_{Ti}(1 - e^{-\lambda_i T})}{(1 - e^{-\lambda_i T})} = \sum_i N_{Ti} \quad (C-4)
 \end{aligned}$$

This is as expected, since in the limit all decay.

The fraction of delayed neutrons "lost," that is, those emitted while the sample is in the "out" position and hence make no contribution to the reactivity effect, is given by

$$\begin{aligned}
 F &= \frac{\text{decays during "out" time}}{\text{total precursors produced during } T} \\
 &= \frac{\sum_i [N_{Ti}(1 - e^{-\lambda_i T}) / (1 - e^{-2\lambda_i T})]}{\sum_i N_{oi} T} \quad (C-5)
 \end{aligned}$$

From Eq. (C-1)

$$N_{Ti} = \frac{N_{oi}}{\lambda_i} (1 - e^{-\lambda_i T})$$

and so

$$F = \sum_i a_i \frac{(1 - e^{-\lambda_i T})^2}{\lambda_i T (1 - e^{-2\lambda_i T})} \quad (C-6)$$

where

$$a_i = \frac{N_{0i}}{\sum_i N_{0i}} = \frac{Y_i}{\sum_i Y_i}$$

By use of the accepted values³¹ for the disintegration constants and relative abundance for the delayed neutrons, $F = 0.3755$ for $T = 7.5$ sec. Thus 38% of the delayed neutrons are emitted while the sample is outside the core. Thus the value of $\bar{\nu}$ used in the calculation of $\bar{\alpha}$ requires reduction by 38% of the delayed neutrons, that is, by

$$0.3755 \times 0.0065 \times 2.430 = 0.00593 \approx 0.006.$$

Thus the appropriate value for $\bar{\nu}$ is $2.430 - 0.006 = 2.424$.

APPENDIX D

Cadmium Ratio Measurements at Oscillator Location

Foils, 0.818 cm (0.322 in.) in diameter, were punched from the same material used for the absolute determinations of capture and fission rates, and irradiated in 0.93-cm-diameter, 20-mil-thick boxes of both aluminum and cadmium. These were placed initially at the bottom, and later at both the bottom and top, of otherwise empty aluminum sample cans which were inserted by means of the sample changer and oscillator mechanism to the "in" position used in the oscillator experiments. The aluminum- and cadmium-covered samples of each material were successively irradiated during the same run. During two preliminary runs, the turret of the sample changer was bare. For three subsequent runs it was encased in 0.020-in. cadmium sheet in order to reduce the magnitude of the correction factor for activity induced while the foils were located in the turret. Table XIX summarizes the characteristics of the samples, the observed cadmium ratios, and the correction of the data obtained in these five irradiations.

TABLE XIX. Cadmium Ratio Determinations in Oscillator Tube

Run No.	Material	Average Mass, mg/cm ²	Observed CdR		Correction Factor	Corrected CdR
			No. of Values	Average		
1P	Mn	34.75	11	5.363	0.9698	5.201
2P	U ²³⁵	249.4	15	7.070	0.9735	6.883
14	Mn	41.90	8	5.160	0.9981	5.150
15	U ²³⁵	79.73	7	7.965	0.9986	7.954
21	U ²³⁵	77.25	8	7.963	0.9986	7.952

The foils were positioned between 2-in.-diameter NaI crystals for counting (Channels 3 and 4 of the automated counting facility). Counting of the manganese foils was delayed sufficiently so that the 5.1-min Cu⁶⁶ activity had decayed completely. The bias level was set at the 662-keV Cs¹³⁷ peak, to eliminate the 0.51-MeV annihilation peak from the 12.9-hr Cu⁶⁴ activity. The bare and cadmium-covered manganese foils were counted alternately several times. After suitable correction for background and decay, the cadmium ratio was calculated for successive pairs of values. The averages of the values are recorded in Table XIX. No evidence of a trend among the successive values was apparent.

The U²³⁵ foils were counted alternately for many times over a 6-hr interval after a delay of $3\frac{1}{2}$ hr. before starting the counting. This was intended to make inconsequential the slight differences in the residence times in the sample changer turret. The 411-keV Au¹⁹⁸ peak was used to

set the bias for counting, and, except for the background foil, a minimum of 12,000 counts was obtained each time. The background-corrected activities were plotted as a function of time after the end of the in-core irradiation and the cadmium ratio was calculated from values read from the resulting curves at 20-min intervals. The average values obtained from the successive readings are included in Table XIX. Again, there was no indication of a time dependence for the ratio.

For convenience, the manganese counting data were treated as if all of the activation occurred while the foil was in the core. Thus, the calculated activities are too high. The bare foil spent the equivalent of 4 min in the sample changer (turret), followed by 15 min in the core, and then an additional 16 min in the turret. Thus the calculated disintegration rate at the end of the core irradiation results from three terms, as follows:

$$\lambda N_0 = \lambda N_{T_1} + \lambda N_C + \lambda N_{T_2}.$$

From this, the desired disintegration rate due to core activation is given by

$$\lambda N_C = \frac{\lambda N_0}{\left[1 + \frac{\lambda N_{T_1} + \lambda N_{T_2}}{\lambda N_C} \right]}.$$

Hence, the computed bare foil activities should be divided by a factor

$$\begin{aligned} 1 + \frac{\lambda N_{T_1} + \lambda N_{T_2}}{\lambda N_C} &= 1 + \frac{(1 - e^{-4\lambda}) e^{-15\lambda} + (1 - e^{-16\lambda}) e^{+16\lambda}}{1 + e^{-15\lambda}} \frac{\phi_T}{\phi_C} \\ &= 1 + 1.399 \frac{\phi_T}{\phi_C}. \end{aligned} \quad (D-1)$$

Similarly, since the cadmium-covered manganese foil spent an equivalent of 20 min at full power in the turret followed by a 15-min irradiation in the core, the activities of the cadmium-covered foils should be divided by a factor

$$1 + \frac{(1 - e^{-20\lambda}) e^{-15\lambda}}{1 - e^{-15\lambda}} \left(\frac{\phi_T}{\phi_C} \right)_{\text{epi Cd}} = 1 + 1.233 \left(\frac{\phi_T}{\phi_C} \right)_{\text{epi Cd}}. \quad (D-2)$$

In Table XIII and Fig. 8 are presented data regarding the relative fluxes in the core and at various locations in the turret when the latter is enclosed in 0.020-in. cadmium sheet. Similar measurements for the bare turret indicated a value of $\phi_T/\phi_C = 3.68 \times 10^{-2}$ at the bottom of the turret. From these data and the cadmium ratio for manganese at the oscillated

sample location in the core it is calculated that $(\phi_T/\phi_C)_{\text{epi Cd}} = 0.434 \phi_T/\phi_C$. Thus, for the preliminary irradiation of manganese, the factor by which the observed cadmium ratio must be multiplied to correct for activity induced while the foils were in the turret is, from Eqs. (D-1) and (D-2):

$$F = \frac{1 + 1.233 (\phi_T/\phi_C)_{\text{epi Cd}}}{1 + 1.399 (\phi_T/\phi_C)}$$

$$= \frac{1 + 1.233 \times 0.0368 \times 0.434}{1 + 1.399 \times 0.0368} = 0.9698.$$

Since the U^{235} foils were counted from $3\frac{1}{2}$ to $9\frac{1}{2}$ hr after their irradiation, it is realistic to assume that the slight difference in the times (not duration) that the bare and cadmium-covered foils were located in the turret is negligible. Then, for correcting the observed average ratio of 7.070 in the preliminary run for the activation received in the turret, consider that each foil was activated for 20 min in the turret simultaneously with the 15 min in the core. Then the multiplying factor is

$$F = \frac{1 + \frac{20}{15} (\phi_T/\phi_C)_{\text{epi Cd}}}{1 + \frac{20}{15} (\phi_T/\phi_C)}$$

$$= \frac{1 + 0.0491 \times 0.434}{1 + 0.0491} = 0.9735.$$

The factors for the subsequent irradiations were obtained in a similar manner. Since the addition of cadmium around the turret resulted in a decrease in ϕ_T/ϕ_C by more than a factor of 10, the correction factors were appreciably smaller. Thus it was practical to work with average values for the foils at the top and bottom of the sample cans during these subsequent irradiations.

The corrected results given in Table XIX exhibit good agreement for the manganese foils, while the results for U^{235} are reasonable in view of the appreciable difference in thickness, and hence self-shielding, for the preliminary and subsequent measurements.

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J. Armstrong and F. Kodrick operated the ZPR-7 facility during most of these measurements and assisted in the data taking. W. Robinson performed the gamma counting of the various manganese and U^{235} foils as well as providing advice and assistance for some of the data processing. F. Ozer performed the coincidence counting required for the determination of the manganese absorption rates, and F. Lyon handled the radiochemical determination of the U^{235} fission rate. The Cf^{252} source was prepared by R. Sjöblom. K. Plumlee provided expert guidance and trouble-shooting for the recording of the oscillation data. Truly, without this skilled assistance the accuracy achieved in this experiment would not have been possible. We are most appreciative of the valuable contributions made by these individuals to the success of this measurement.

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