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AN IMPROVED MATHEMATICAL MODEL FOR
TRANSIENT SUBCHANNEL ANALYSIS
OF ROD BUNDLE NUCLEAR FUEL ELEMENTS⁽¹⁾

By D. S. Rowe⁽²⁾

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ANALYSIS OF ROD BUNDLE NUCLEAR FUEL ELEMENTS

By:

D. S. Rowe
Battelle-Northwest
Richland, Washington

ABSTRACT

This paper presents a more complete mathematical method for analyzing transient flow and enthalpy transport in rod bundle nuclear fuel elements during both boiling and nonboiling conditions. The mathematical model is formulated much as in the previously published COBRA-III computer program. The bundle flow area is divided into flow subchannels that are assumed to contain one dimensional flow and are coupled to each other by turbulent and diversion crossflow mixing. Diversion crossflow is included to allow for flow redistribution. An improved transverse momentum equation is now used to include both temporal and spatial acceleration of the diversion crossflow. This improvement significantly improves the accuracy and stability of the numerical solutions. The analysis method has also been improved by incorporating a transient conduction model to account for the thermal capacity of the fuel. Boiling and nonboiling conditions are considered by using an equation of state that relates density to enthalpy, flow, heat flux, pressure, position, and time. The mathematical model neglects sonic velocity propagation; therefore, it is limited to transients with time duration greater than the time for a sonic wave to pass through the bundle. The equations of the mathematical model are solved as a

boundary-value problem by using a semiexplicit finite difference scheme that is stable for all time steps. The boundary conditions are the inlet enthalpy, inlet flow rate, and exit pressure. This method of solution is significant since it allows downstream flow disturbances to be felt at upstream locations as governed by the amount of crossflow resistance.

Calculations of flow in a bundle with a partial blockage are presented to demonstrate the relatively weak effect of parameters contained in the transverse momentum equation. The relative effects of the fuel time constant and coolant transit time are discussed in terms of a rapid power transient.

AN IMPROVED MATHEMATICAL MODEL FOR TRANSIENT SUBCHANNEL ANALYSIS

OF ROD BUNDLE NUCLEAR FUEL ELEMENTS

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INTRODUCTION

To establish the safe operating limits of nuclear power reactors, the heat transfer and flow behavior in nuclear fuel bundles must be known for all operating conditions, including transients. In many cases, this information is obtained from laboratory experiments that use electrically heated models to simulate the fuel design. These experiments give thermal-hydraulic performance parameters such as critical heat flux and fuel temperature for selected conditions. Since the experiments can not simulate all possible steady-state and transient operating conditions, analytical techniques must be used to extend the experimental results to the reactor. For simple fuel geometries, this may be done by using a one-dimensional analysis; however, this technique is not completely adequate for establishing the economic operating conditions of today's nuclear power reactors that use rod bundle fuel elements. The enthalpy and mass velocity in some regions of these bundles can be very different from the average conditions during boiling; therefore, more detailed analysis techniques are required.

The purpose of this paper is to present an improved mathematical method for analyzing the steady-state and transient thermal-hydraulic behavior of rod bundle nuclear fuel elements for both boiling and nonboiling conditions.

The method presented in this paper represent an improvement over a transient analysis method presented in an earlier paper.⁽¹⁾ The basic improvements are a more complete transverse momentum equation to describe the radial flow redistribution in rod bundles more accurately and the inclusion of a fuel heat transfer model to couple heat transfer and hydraulics. These improvements allow analysis of a broader range of rod bundle thermal-hydraulic transients.

FLUID TRANSPORT MATHEMATICAL MODEL

The basic approach used to develop the fluid transport mathematical model is shown in Figure 1. The bundle flow cross section is divided into flow subchannels. While the subchannel selection is rather arbitrary the one shown has gained acceptance for steady state analysis of rod bundles.⁽²⁾ This same selection also seems to be suitable for transient analysis. By making suitable assumptions concerning the flow and heat transfer processes within these subchannels a set of fluid transport equations can be derived.

Several steady-state digital computer programs⁽³⁻⁶⁾ have been written that use mathematical models based on this approach; however, there are no publications indicating that these programs have been extended to include transients. This paper presents, a transient subchannel analysis method that is being used in the COBRA-IIIC program being developed at Battelle-Northwest. This program is an extension of the previously published COBRA-III program for transient analysis.⁽⁷⁾

The derivation of the fluid transport equations have been presented elsewhere; therefore, only a summary of the equations and appropriate assumptions are presented in this paper.

BASIC ASSUMPTIONS

Several fundamental assumptions are required to develop the fluid transport model:

1. One-dimensional, two-phase, slip-flow exists in each selected flow subchannel during boiling.
2. Subchannel density is specified in terms of enthalpy, pressure, flow rate, position and time.
3. The subchannels are coupled by the two types of crossflow mixing shown in Figure 1. The first is a turbulent (fluctuating) crossflow rate per unit length that causes no net flow change ($w'_{ij} = w'_{ji}$). The second is a net diversion crossflow per unit length that results from flow redistribution.
4. Sonic velocity propagation is ignored.
5. The diversion crossflow velocity is small compared to the axial velocity within a subchannel.

The first two assumptions are used for one-dimensional analysis of simple channels. By using the third assumption, the model becomes a collection of one-dimensional, parallel flow channels that are coupled by the two types of crossflow mixing. The fourth assumption greatly simplifies the mathematical models and the numerical solution. Sonic velocity propagation can be neglected for transients with times

that are longer than the sonic propagation time through the channel. (8)
 The last assumption allows a transverse momentum equation to be derived by only preserving the vector direction between an adjacent pair of subchannels. In other words, the crossflow loses its sense of direction when it enters a subchannel. This assumption also allows the transverse momentum flux normal to the gap to be neglected. Calculations to be presented later show that even for severe flow diversions this assumption is valid.

EQUATIONS OF THE MATHEMATICAL MODEL

By using the previous assumptions and by applying the general equations of continuity, energy and momentum to a segment of an arbitrary subchannel, the equations of the mathematical model may be derived in a manner similar to that shown in Reference 1 and in Appendix A. For simplicity the equations are presented for an arbitrary Subchannel (i) which is connected to another Subchannel (j). The equations are generalized later to account for an arbitrary subchannel layout.

The right side of the continuity equation

$$A_i \frac{\partial \rho_i}{\partial t} + \frac{\partial m_i}{\partial x} = -w_{ij} \quad (1)$$

gives the net rate of change of subchannel flow in terms of the diversion cross flow per unit length. By choice, the diversion cross flow is positive when flow is diverted out of Subchannel (i). The turbulent cross flow does not appear because it does not cause a net flow change. The time

derivative of density gives the component of flow change caused by the fluid expansion or contraction.

The right side of the energy equation

$$\frac{1}{u''} \frac{\partial h_i}{\partial t} + \frac{\partial h_i}{\partial x} = \frac{q_i'}{m_i} - (h_i - h_j) \frac{w_{ij}'}{m_i} + (h_i - h^*) \frac{w_{ij}}{m_i} \quad (2)$$

contains three terms for thermal energy transport in a rod bundle fuel element. The first term is the power-to-flow ratio of a subchannel and gives the rate of enthalpy change if no thermal mixing occurs. The second term accounts for the turbulent enthalpy transport between all interconnected subchannels. The turbulent thermal mixing w' is analogous to eddy diffusion and is defined through empirical correlations.⁽²⁾ The third term accounts for thermal energy carried by the diversion crossflow. This is a convective term that requires a selection of the enthalpy

h^* to be carried by the diversion crossflow. The first term on the left side of Equation (2) gives the transient contribution to the spatial rate of enthalpy change. These are a convective terms with a transport velocity u'' . Since u'' represents the effective velocity for energy transport, the time duration of a transient is related to this velocity. Note that sonic velocity propagation effects are ignored by the absence of a $\partial p / \partial t$ term.

The right side of the axial momentum equation

$$\begin{aligned} \frac{1}{A_i} \frac{\partial m_i}{\partial t} - 2u_i \frac{\partial \rho_i}{\partial t} + \frac{\partial \rho_i}{\partial x} = & - \left(\frac{m_i}{A_i} \right)^2 \left[\frac{v_i f_i \phi_i}{2D_i} + A_i \frac{\partial (v_i' / A_i)}{\partial x} \right] - \rho_i g \cos \theta \\ & - \frac{f_i}{A_i} (u_i - u_j) w_{ij}' + \frac{1}{A_i} (2u_i - u^*) w_{ij} \end{aligned} \quad (3)$$

contains several terms that govern the axial pressure gradient in a subchannel. Without the crossflow terms, these are the frictional, spatial acceleration and elevation components of pressure gradient. The turbulent crossflow term tends to equalize the velocities of adjacent subchannels. This is analogous to turbulent stresses in turbulent flow. The factor f_T is included to help account for the imperfect analogy between the turbulent transport of enthalpy and momentum. The diversion crossflow term accounts for the momentum changes due to changes in subchannel velocity. The first two terms on the left side of Equation (3) are the transient components of the axial pressure gradient.

Appendix A presents a derivation of an improved transverse momentum that more properly accounts for the crossflow momentum coupling. For two adjacent subchannels this equation can be written as

$$\frac{\partial w_{ij}}{\partial t} + u_{ij}^* \frac{\partial w_{ij}}{\partial x} + \left(\frac{s}{l}\right) c_{ij} w_{ij} = \left(\frac{s}{l}\right) (p_i - p_j) \quad (4)$$

The first two terms are new to the momentum equation. They represent the temporal and spatial acceleration of the crossflow. The remaining two terms are the friction and pressure terms used in an earlier transverse momentum equation.^(1,3,9) The new parameter (s/l) represents the importance of friction and pressure terms versus the new inertial terms. These inertial terms also represent the transport of the crossflow at axial velocity u_{ij}^* . These terms cause the crossflows to persist as they move down the channel. This is the "axial inertia" effect often included in crossflow resistance correlations.⁽²⁾

The primary assumption used to derive Equation (4) is that the crossflow through a rod gap loses its sense of direction upon entering or leaving a subchannel. The direction of the crossflow is only considered between the two interconnecting subchannels of interest. This assumption is valid if the crossflow velocities are small compared to the axial velocities. This is usually the case for most computations in nuclear reactor rod bundles.

An equation of state of the form

$$\rho_i = \rho(h_i, P^*, m_i, x, t) \quad (5)$$

is used to define the two-phase fluid density. This equation can be obtained by using an appropriate correlation for void fraction.

For a large rod bundle the previous equations become unwieldy; therefore, a more compact form is very desirable. Reference 1 presents a vector form of the equations by using a matrix transformation $[S]$ and its transpose $[S]^T$. Using the $[S]$ transformation the previous equations may be written as follows:

Continuity

$$\left\{ A \frac{\partial \rho}{\partial t} \right\} + \left\{ \frac{\partial m}{\partial x} \right\} = - [S]^T \{w\} \quad (6)$$

Energy

$$\left\{ \frac{m}{u} \frac{\partial h}{\partial t} \right\} + \left\{ m \frac{\partial h}{\partial x} \right\} = \{q'\} - [S]^T [\Delta h] \{w'\} + [h] [S]^T \{w\} - [S]^T [h^*] \{w\} \quad (7)$$

Axial Momentum

$$\left\{ \frac{1}{A} \frac{\partial m}{\partial t} \right\} - \left\{ 2u \frac{\partial \rho}{\partial t} \right\} + \left\{ \frac{\partial p}{\partial x} \right\} = - \{a'\} + [A]^{-1} \left[[2u] [S]^T - [S]^T [u^*] \right] \{w\} \quad (8)$$

Transverse Momentum

$$\left\{ \frac{\partial w}{\partial t} \right\} + \left\{ u^* \frac{\partial w}{\partial x} \right\} + \left(\frac{S}{\ell} \right) \{Cw\} = \left(\frac{S}{\ell} \right) [S] \{p\} \quad (9)$$

where

$$\begin{aligned} \{a'\} = & \left\{ \left(\frac{m}{A} \right)^2 \left(\frac{v f \phi}{2D} + A \frac{\partial (v'/A)}{\partial x} \right) + \rho g \cos \theta \right\} \\ & + f_T [A]^{-1} [S]^T [\Delta u] \{w'\} \end{aligned} \quad (10)$$

The elements of the matrix $[S]$ are defined through two column vectors $i(k)$ and $j(k)$ where, for each pair of connected subchannels (i, j) , a unique connection number k is assigned. The connections numbers are assigned in ascending order by considering Subchannel (i) and then assigning a unique connection number for each successively connected Subchannel (j) where j is greater than i . By using this identification procedure the crossflows can be written as w_k where k implies the subchannel pair $i(k), j(k)$. For $w_k > 0$ the crossflow is chosen to be from Subchannel (i) to Subchannel (j) where i is less than j . By examining Equation (9) the elements of the matrix $[S]$ are seen to be:

$$S_{ki} = 0; \text{ except, } S_{ki} = 1, \text{ if } i = i(k); \text{ and } S_{ki} = -1, \text{ if } i = j(k).$$

METHOD OF SOLUTION

The previous equations are solved as a boundary-value problem by using a semiexplicit finite difference scheme. The boundary conditions selected for the problem are the inlet enthalpy, inlet flow, inlet crossflow, and exit pressure. Initial conditions for enthalpy, flow, and crossflow distribution are established from an initial steady-state calculation. No initial condition for pressure is required as it is determined from boundary conditions only. Solving the problem this way is an improvement over the initial-value solutions that are used in many subchannel analysis computer programs. While the use of the initial value solution can be justified as an approximation to the boundary-value solution⁽¹⁰⁾ if the crossflow resistance is small, it restricts the solutions to a limited class of problems.

Equations (6) through (9) are solved by using a finite difference procedure as follows*:

Continuity

$$\left\{ A_n \frac{\rho_{n-1} - \bar{\rho}_{n-1}}{\Delta t} \right\} + \left\{ \frac{m_n - m_{n-1}}{\Delta x} \right\} = -[S]^T \{w_{n-1}\} \quad (11)$$

Energy

$$\left\{ \frac{1}{u_n} \frac{h_n - \bar{h}_n}{\Delta t} \right\} + \left\{ \frac{h_n - h_{n-1}}{\Delta x} \right\} = [m_{n-1}]^{-1} \left\{ q_{n-1} - [S]^T [\Delta h_{n-1}] \{w'_{n-1}\} \right. \\ \left. + \left[[h_{n-1}] [S]^T - [S]^T [h^*_{n-1}] \right] \{w_{n-1}\} \right\} \quad (12)$$

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*The overscore bar ($\bar{}$) indicates previous time.

Axial Momentum

$$\left\{ \frac{1}{A_n} \frac{m_n - \bar{m}_n}{\Delta t} \right\} - \left\{ 2 u_n \frac{\rho_n - \bar{\rho}_n}{\Delta t} \right\} + \left\{ \frac{p_n - p_{n-1}}{\Delta x} \right\} = - \left\{ a'_n \right\} + [A_n]^{-1} \left[[2u_n] [S]^T - [S]^T [u^*_n] \right] \left\{ w_n \right\} \quad (13)$$

Transverse Momentum

$$\left\{ \frac{w_n - \bar{w}_n}{\Delta t} \right\} + \left\{ u^*_n \frac{w_n - w_{n-1}}{\Delta x} \right\} + \left(\frac{S}{\ell} \right) \left\{ C_n w_n \right\} = [S] \left\{ p_{n-1} \right\} \quad (14)$$

Equations (13) and (14) can be combined to eliminate the pressure term

$[S] \left\{ p_{n-1} \right\}$. The result is

$$\begin{aligned} \left\{ \frac{w_n - \bar{w}_n}{\Delta t} \right\} + \left\{ u^*_n \frac{w_n - w_{n-1}}{\Delta x} \right\} + \left(\frac{S}{\ell} \right) \left\{ C_n w_n \right\} &= \left(\frac{S}{\ell} \right) [S] \left\{ p_n \right\} + \left(\frac{S}{\ell} \right) [S] \left\{ a_n \right\} \Delta x \\ &+ \left(\frac{S}{\ell} \right) [S] \left\{ \frac{1}{A_n} \frac{m_n - \bar{m}_n}{\Delta t} \right\} \Delta x - \left(\frac{S}{\ell} \right) [S] \left\{ 2u_n \frac{\rho_n - \bar{\rho}_n}{\Delta t} \right\} \Delta x \\ &- \left(\frac{S}{\ell} \right) [S] [A_n]^{-1} \left[[2u_n] [S]^T - [S]^T [u^*_n] \right] \left\{ w_n \right\} \Delta x \end{aligned} \quad (15)$$

This equation is a set of simultaneous equations for the diversion crossflow w_n . It can be rearranged into the form

$$[M] \left\{ w_n \right\} = \left\{ b_n \right\} \quad (16)$$

where

$$\begin{aligned} [M] &= \left[\frac{1}{\Delta t} \right] + \left[\frac{u^*_n}{\Delta x} \right] + \left(\frac{S}{\ell} \right) [C_n] \\ &+ \left(\frac{S}{\ell} \right) [S] [A_n]^{-1} \left[[2u_n] [S]^T - [S]^T [u^*_n] \right] \Delta x \end{aligned} \quad (17)$$

and

$$\begin{aligned} \{b\} = & \left\{ \frac{\bar{w}_n}{\Delta t} \right\} + \left\{ \frac{u_n^* w_{n-1}}{\Delta x} \right\} + \left(\frac{S}{\ell} \right) [S] \{p_n\} + \left(\frac{S}{\ell} \right) [S] \{a_n\} \Delta x \\ & + \left(\frac{S}{\ell} \right) [S] \left\{ \frac{1}{A_n} \frac{m_n - \bar{m}_n}{\Delta t} \right\} \Delta x - \left(\frac{S}{\ell} \right) [S] \left\{ 2u_n \frac{\rho_n - \bar{\rho}_n}{\Delta t} \right\} \Delta x \end{aligned} \quad (18)$$

The first three terms on the right side of Equation (17) are diagonal matrices. The first two of these come from the added temporal and spatial acceleration terms in the transverse momentum equation. They are also very important as they provide additional numerical stability. Reducing Δx and Δt adds more diagonal dominance and thus more stability to the numerical solution.

As in earlier difference schemes^(1,3,9) the last term on the right side of Equation (17) is singular for most rod bundle problems. The additional diagonal terms remove the singularity thus allowing a unique solution for the crossflow.

The matrix $[M]$ controls the distribution of crossflow; however, the vector $\{b\}$ contains the crossflow forcing terms. The first two terms on the right side of Equation (18) try to maintain the crossflow that existed at previous time or space. If other forces did not exist crossflow would persist and be convected at velocity u^* . The third term is the pressure difference affecting the crossflow. This is also the term that feeds downstream information into the crossflow solution. The fourth term provides the basic driving force for crossflow. If the pressure gradients

due to friction, acceleration, and gravity are unbalanced, crossflows occur in an attempt to equalize them. The last two terms are transient components of the pressure gradient.

FUEL ROD HEAT TRANSPORT

In a previously published transient subchannel model^(1,7) the fuel heat flux was assumed to be known from input data. This was rather unsatisfactory since it required a priori knowledge of the fuel heat transfer. A more satisfactory approach is to let the heat flux occur naturally by changing the heat generation, heat transfer coefficient or fluid temperature.

The fuel heat transfer model presented here considers only radial conduction within the fuel. Axial and circumferential heat conduction are presently ignored. Axial heat conduction can be ignored because axial temperature gradients are usually small. Circumferential heat conduction is ignored to maintain a reasonable limit on the number of fuel nodes. This assumption can be justified if the heat transfer coefficients and fluid temperatures are rather uniform around a fuel rod. For boiling flow the fluid temperature is usually quite uniform. The validity of assuming a uniform heat transfer coefficient must be evaluated for each problem.

Figure 2 shows the node layout used to construct the conduction model. The fuel is divided into equal radial nodes plus a node for the cladding. For N nodes this gives $N + 1$ temperatures where the temperature

at $i = N + 1$ is at the outer surface of the cladding. An effective gap conductance coefficient is used at the fuel-clad interface to combine the conductance of the cladding and the gap. The surface heat transfer coefficient is arbitrarily specified through heat transfer correlations. An average heat transfer coefficient is determined from a circumferentially weighted average of the heat transfer coefficients in the subchannels surrounding a fuel rod. A similar calculation is performed to obtain an average fluid temperature.

The numerical solution uses an implicit finite difference scheme since it is stable for all time steps.^(11,12) The equations used for the numerical solution are as follows:

$i = 1$, temperature at fuel center

$$\left(\frac{\rho c}{\Delta t} + \frac{4k}{\Delta r^2} \right) T_1 - \frac{4k}{\Delta r^2} T_2 = q_1''' + \frac{\rho c \bar{T}_1}{\Delta t} \quad (19)$$

$1 < i \leq N-1$, temperature in fuel region

$$\left(-\frac{k}{\Delta r^2} + \frac{k}{2(i-1)\Delta r^2} \right) T_{i-1} + \left(\frac{\rho c}{\Delta t} + \frac{2k}{\Delta r^2} \right) T_i + \left(-\frac{k}{\Delta r^2} - \frac{k}{2(i-1)\Delta r^2} \right) T_{i+1} = q_i''' + \frac{\rho c \bar{T}_i}{\Delta t} \quad (20)$$

$i = N$, fuel temperature at fuel-clad interface

$$\left(-\frac{2k}{\Delta r^2} \right) T_{N-1} + \left(\frac{\rho c}{\Delta t} + \frac{2k}{\Delta r^2} + \frac{2h_G}{\Delta r} + \frac{h_G}{(i-1)\Delta r} \right) T_N + \left(-\frac{2h_G}{\Delta r} - \frac{h_G}{(i-1)\Delta r} \right) T_{N+1} = q_N''' + \frac{\rho c \bar{T}_N}{\Delta t} \quad (21)$$

$i = N+1$, cladding temperature

$$-\left(\frac{h_G}{t_c} - \frac{r_N}{r_{N+1}} \right) T_N + \left(\frac{\rho c}{\Delta t} + \frac{h_G}{t_c} \frac{r_N}{r_{N+1}} + \frac{h_s}{t_e} \right) T_{N+1} = q_{N+1}''' + \frac{\rho c \bar{T}_{N+1}}{\Delta t} + \frac{h_s}{t_c} T_c \quad (22)$$

These equations can be arranged as a set of simultaneous equations where the temperature coefficient matrix is tridiagonal. This system of equations is solved by using a compact Gaussian elimination routine.

COMPUTATION PROCEDURE

Steady-state computations are performed first to obtain initial conditions for the transient. Since the previously presented finite difference equations are stable for large time steps, those same equations are used for the steady state calculations by setting Δt equal to some arbitrarily large value. An iteration is performed until convergence of the crossflow solution is obtained.

For each iteration the computation sweeps from the inlet to the exit of the channel. All values normally considered to exist at previous time are set equal to zero to start the iteration. With inlet boundary information on flow and enthalpy known, the flow and enthalpy can be advanced one space step by using Equations (11) and (12). The crossflow solution is advanced in a similar manner by using the previous iterate value of the subchannel pressure difference $[S]\{P_n\}$. After the crossflow $\{w_n\}$ is calculated, $[S]\{P_{n-1}\}$ is calculated for use during the next iteration. From Equation (13) this is calculated as

$$\begin{aligned}
 [S] \{P_{n-1}\} = [S] \{P_n\} &+ \{a'_n\} \Delta x - [A_n]^{-1} [2u_n] [S]^T \{w_n\} \Delta x \\
 &+ [A]^{-1} [S]^T [u^*_n] \{w_n\} \Delta x + \left\{ \frac{1}{A_n} \frac{m_n - \bar{m}_n}{\Delta t} \right\} \Delta x \\
 &- \left\{ 2 u_n \frac{\rho_n - \bar{\rho}_n}{\Delta t} \right\} \Delta x
 \end{aligned} \tag{23}$$

Note that $[S] \{P_n\}$ is downstream of $[S] \{P_{n-1}\}$; therefore, iteration allows the downstream pressure differences to be felt at upstream locations.

At the end of the channel the boundary condition is $[S] \{P\} = 0$. In this way a boundary value solution is obtained. Only a few iterations are sufficient for convergence in most problems.

The above equations do not require actual pressures since pressure difference is only used in the combined momentum equation. The calculation of pressure is, therefore, only a back calculation. It is calculated from Equation (13) in a forward direction. When the exit is reached the pressures are corrected to agree with the exit pressure boundary condition.

Transient calculations are performed in the same way but for a selected time step Δt . Boundary conditions and other forcing functions are set to their desired values at new time; then, the calculation sweeps through the channel for the number of iterations required to achieve convergence on the crossflow. The procedure then repeats for the next time step.

DISCUSSION

Solutions to the previous set of equations require a certain amount of empirical information. Because of the incomplete knowledge of steady state and transient two-phase flow in bundles some of this information must be supplied by assumption. For example, h^* and u^* are commonly assumed to be their respective values from the donor subchannels. Other selections of u^* and h^* may be made⁽⁷⁾ to account for the nonuniform enthalpy distribution in a subchannel. This usually requires information about the liquid-and vapor-phase distribution. The factor f_T is also unknown but its effect is weak. For many problems f_T can be safely set equal to

zero. The correlations required for calculating the pressure gradient are of major importance. This includes the correlations for friction factor, subcooled void fraction, bulk void fraction and two-phase friction multiplier. Fortunately, correlations of these effects developed for simple channels may be applied to rod bundle subchannels for steady state with reasonably good results. Since their accuracy is questionable for high speed transients, additional experimental work is needed in this area. Turbulent mixing must also be specified from empirical correlations or data. At the present time a definitive correlation for mixing does not exist for all bundle geometries and all flow conditions. Some progress has been made to describe mixing for single⁽²⁾ and two-phase flow;^(9,13,14) however, very little is known about two-phase mixing processes during transients.

The transverse momentum equation and fuel heat transfer model add still more parameters to the subchannel analysis method. These parameters and their effect on typical rod bundle calculations are discussed in the succeeding sections.

Transverse Momentum Equation Parameters

The improved transverse momentum introduces a new parameter (s/ℓ) and also suggests possible new importance for the terms u^* and C . Some insight concerning the importance of these parameters can be found by inspecting the finite difference form of the transverse momentum equation (17). For discussion purposes, consider the matrix $[M]$ for two

interconnected subchannels of equal area. The matrix has only one element given by

$$m = \frac{1}{\Delta t} + \frac{u^*}{\Delta x} + \left(\frac{s}{\ell}\right) C + \left(\frac{s}{\ell}\right) \frac{2u^*}{A} \Delta x \quad (24)$$

Where u^* is the average of the two subchannel velocities. The relative importance of the terms can be controlled by arbitrarily selecting Δx .

The second and fourth terms can be made of comparable magnitude by selecting $\Delta x^2 = A$ which usually requires $\Delta x < 0.5$ inch for most nuclear reactor rod bundle problems. A larger value of Δx is normally used as it reduces computation time for long bundles. In those cases the last term is the largest one. The smallest term is usually the transverse friction term, C . It contains the friction coefficient K_{ij} which could be expected to be on the order of unity or less. This friction coefficient does not include an "axial inertia" term contained in some previous crossflow resistance correlations.^(2,15) It is improper to use those correlations here because the "axial inertia" effect is contained in the transverse momentum term $u^*/\Delta x$. This can be seen by writing the pressure difference $p_i - p_j = u^* dw/dx$. If dw is approximated by w and if dx is approximated by an appropriate length, then, $p_i - p_j \propto (u^*/v) v^2$ and the resistance coefficient due to axial inertia is proportional to u^*/v . This ratio is large for small crossflow velocity. The same result is reported in a combined momentum and friction crossflow correlation by Khan⁽¹⁵⁾ at large values of u^*/v . At small values of u^*/v Khan's correlation, et. al., reduces to $K_{ij} \sim 0.25$ which would be the value of the friction coefficient for pure crossflow. This discussion points out the usual dominance of $u^*/\Delta x$ over $(s/\ell)C$. The values of C are therefore

relatively unimportant for most problems. This result may not be true, however, for very closely spaced rod bundles since C is inversely proportional to the square of the gap spacing.

The (s/ℓ) parameter appears in all but the transverse momentum terms; therefore, as long as $u^*/\Delta x$ and $1/\Delta t$ are small components of $[M]$, (s/ℓ) is a weak parameter. Since this is usually the case for the value of Δx used in rod bundle analysis (s/ℓ) need not be specified to high accuracy. For core-wide analysis of pressurized water reactors this conclusion may not be valid because Δx^2 may be small compared to A .

The velocity u^* appears in the transverse and axial momentum terms. Since these are usually the largest components of $[M]$, u^* could have a stronger effect on crossflow solutions than the previously discussed parameters. The choice of u^* must presently be made by assumption because of insufficient data to define it accurately. Fortunately, a variety of assumptions can be made that give comparable values of u^* . This is because most bundles have rather uniform subchannel velocity distributions. The most serious errors in u^* could occur where large subchannel velocity differences occur. In those cases the effect of the u^* assumption should be checked.

The previous comments concerning the usually small magnitude of $u^*/\Delta x$ and C does not mean that they can be dropped from the calculations. They play a vital role in the numerical stability. The last term of the matrix $[M]$ is a singular matrix for any problem where one or more transverse flow loops exist such as around a fuel rod. In those cases the crossflow around the loop is not unique. Including $u^*/\Delta x$ and C remove the singularity by imposing the friction and momentum constraint

that the sum of the pressure drop around any transverse loop is equal to zero. Mathematically these terms add diagonal dominance to $[M]$, thus, improve the condition of $[M]$. The diagonal dominance can be improved by decreasing Δx .

To evaluate the influence of these parameters, calculation results are presented for a partial subchannel flow blockage in a rod bundle of typical boiling water reactor design. The subchannel layout is shown in Figure 3 and the input parameters are in Table I. The blockage is introduced into Subchannel 8 at the third grid spacer by increasing the grid pressure loss coefficient from 1.0 to 2.0. The result of the calculation with and without the blockage is shown in Figure 4. The subchannel flow decreases abruptly at blockage and gradually recovers downstream where a grid spacer aids the flow recovery at $x/L = 0.8$. The influence of the blockage is slightly evident at the first node before the blockage. The enthalpy increase downstream of the blockage is less than 10 Btu/lb for the chosen value of crossflow mixing. The diversion crossflow mass flux is very small compared to the axial mass flux except at the blockage. There it is still less than 10% of the average. This shows that the assumption of small transverse velocity compared to axial velocity is well satisfied for this problem even in the presence of a large subchannel flow diversion. Although this result would apply to most rod bundle problems the assumption should be checked, especially for problems where gross flow redistributions occur under low flow conditions.

TABLE I. Input Parameters for Sample Problems

Water Thermodynamic Properties	1967 ASME Tables
Subchannel Friction Factor	$f = 0.186 R_e^{-0.2}$
Subcooled Voids	Not Included
Void Fraction*	$\alpha = x v_g / [(1-x) v_f + x v_g]$
Two-phase Multiplier*	$\phi = \rho_f / \rho$
Two-phase Density*	$\rho = \rho'' = \alpha \rho_g + (1-\alpha) \rho_f$
Two-phase Specific Volume*	$v' = 1/\rho$
Enthalpy Carried by Crossflow (h^*)	$h^* = h$ from donor subchannel
Axial Velocity of Crossflow (u^*)	Average velocity of adjacent subchannels
Axial Heat Flux	Chopped Cosine, peak/avg = 1.26
Momentum Turbulent Factor	0
Transverse Momentum Parameter (s/ℓ)	0.5
Crossflow Resistance (K_{ij})	0.5
Number of Axial Nodes	60
Turbulent Mixing Parameter, β	0.02
Fuel Thermal Properties:	
k	2.0 Btu/hr ft °F
ρ	640 lb/ft ³
C	0.08 Btu/lb °F
Diameter	0.502 inches
Cladding Thermal Properties:	
k	8.8 Btu/hr ft °F
ρ	405 lb/ft ³
C	0.076 Btu/lb °F
Thickness	0.030 inches
Gap Conductance	1000 Btu/hr ft ² °F
Surface Heat Transfer Coefficient	5000 Btu/hr ft ² °F
Grid Spacer Location	$x/L = 0.2, 0.4, 0.6, 0.8$
Spacer Pressure Loss Coefficient	$k = 1.0$

*Homogeneous two-phase flow correlations.

Changing the value of K_{ij} from .5 to 0.1 and 2.5 had no significant effect on the flow solution. This could be expected based on the previous discussion concerning the magnitude of C . Increasing K_{ij} had the larger effect by increasing the subchannel pressure difference near the blockage and by moving the start of flow diversion slightly upstream. The values of crossflow resistance used in this example probably bracket those that could be expected for rod bundles. These calculations indicate that crossflow resistance does not have to be known precisely as its effect on the computed flow is small. This should be checked for particular problems to determine the importance of crossflow resistance.

Changing the value of (s/l) from 0.5 to 0.25 and 1.0 also had no significant effect on the flow solution. Increasing it increases the importance of the friction and pressure terms. This has the effect of allowing more rapid flow diversions by reducing the importance of the "axial inertia". This range of (s/l) would seem to be reasonable from geometric considerations.

Figure 5 presents the results of calculations using various assumptions for u^* . The reference assumption is that u^* is the average velocity of the adjacent subchannels. Another assumption is that u^* is the velocity of the donor subchannel. This gives a discontinuity in u^* when the crossflow changes direction. The other assumption is that u^* is the smaller of the two adjacent subchannel velocities. While this may not be realistic, it can permit stable flow solutions near regions of severe flow maldistribution. All of these assumptions are seen to cause only moderate changes in the flow solution. This is fortunate because it would be difficult to provide a meaningful generalized correlation for u^* .

The previous parameter evaluation indicates that rather large changes in the parameters C , (s/λ) and u^* cause only small-to-moderate changes in the subchannel flow solution. Similar results have been found for calculations with large inlet flow imbalances for core-wide flow analysis in pressurized water reactors. These types of calculations tend to amplify the effects of the above parameters. For more typical design analysis problems, where flow imbalances are not large, changes in the above parameters are weak. For most problems the reference values of the three parameters provide reasonable results; however, computational experiments should be performed to check this. For cases where the parameters have a meaningful effect on flow solutions laboratory experiments may be necessary to define them more accurately.

Fuel Temperature and Flow Transients

The use of the fuel temperature model introduces some additional considerations for transient analysis of rod bundles. The fuel heat transfer and flow are coupled by their respective time response. The time response of the fuel is its time constant and the time response for the flow is the transit time through the bundle. Depending upon the nature of the transient being considered the time responses may or may not be comparable.

If the time response of the fuel is much longer than the transit time through the bundle, the flow solution can be treated as a series of quasi steady state solutions during the duration of the fuel temperature transient. If, for these conditions a flow related transient occurs over a short period of time the fuel may not have time to respond. Conversely, if the time response of the fuel is much faster than the transit time

through the bundle, then the flow is significantly affected by fuel temperature transient.

As an example, consider the power transient shown in Figure 6 which might occur after a turbine trip for a boiling water reactor. The flow and enthalpy solutions are shown in Figure 7 as a function of time. Note that the flow is only slightly perturbed by the power transient and that the enthalpy decays rather slowly when the fuel heat transfer model is used. The transit time through the bundle is approximately 1.5 seconds which is about a factor of 10 less than the fuel time constant. For the .05 second time step used for this problem, each time step solution is nearly a quasi-steady flow solution that is following a fuel temperature transient. As a contrast to this, the same calculation was performed but without the fuel heat transfer model where the heat flux has an infinitely fast response and decays with the power. In this case the transient has a very dramatic affect on the flow and enthalpy solution. Initially the exit flow increases due to the void growth in the bundle. Then as the power and enthalpy decay, the flow decreases rapidly as the voids begin to collapse. At the end of 10 seconds the entire bundle has all liquid flow. While this is a fictitious case it does show how the fuel thermal response can have a varied effect on the flow depending upon the relative time response of the fuel and channel.

CONCLUSION

The inclusion of the temporal and spatial acceleration of the diversion crossflow adds important physical phenomena to the subchannel analysis method with only a small increase in the complexity of the numerical solution. The importance of these additional phenomena are governed by the parameters u^* , c , and (s/l) . These have only a small-to-moderate affect on subchannel flow solutions for most rod bundle analyses. The effect of the parameters should be evaluated and justified experimentally if they have important influence on the flow solutions. The applicability of the basic assumptions for the low crossflow velocity and the suitability of the two-phase flow model should always be justified. This is especially important for geometric and operating conditions that are far from those where the validity of the subchannel analysis method has been checked. (2, 9, 13, 14)

The use of coupled fuel rod heat transfer and subchannel analysis method provides a more complete way of performing transient thermal-hydraulic analysis of rod bundle nuclear fuel elements. By using appropriate heat transfer correlations the fuel temperature response to selected transients can now be analyzed in much greater detail. There still is the problem, however, of obtaining meaningful general heat transfer correlations for rod bundle. This includes critical heat flux correlations and heat transfer coefficient correlations that are applicable to the full range of heat transfer nodes that could occur during transients.

While the use of an inlet flow boundary condition may be entirely satisfactory for a wide range of problems, there are many analyses where the pressure at each end of the bundle could be defined with greater ease.

This type of boundary condition would allow the inlet flow rate to change as governed by the boundary pressure and the internal driving forces. Work is underway at Battelle-Northwest to provide this type of boundary condition. Numerical methods are also being developed to consider transient flow reversal, coolant expulsion and countercurrent subchannel flow in the presence of a pressure boundary condition.

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NOMENCLATURE*

Equations

A	Cross-sectional area, (L^2)
{b}	Column vector defined by Equation (18)
c	Specific heat, ($H/M\theta$)
C	Crossflow resistance, function, $C_{ij} = (K_{ij} w_{ij}) / (2g_c s_{ij}^2)$, (T/L)
D	Hydraulic diameter, $4A/P_w$, (L)
f	Friction factor based on all-liquid flow, (Dimensionless)
f_T	Turbulent momentum factor, (Dimensionless)
F	Force, (ML/T^2)
h	Enthalpy, $Xh_g + (1-X)h_f$, (H/M)
h^*	Enthalpy carried by diversion crossflow, (H/M)
h_g, h_f	Saturated vapor and liquid enthalpy, (H/M)
k	Thermal conductivity, ($H/T L\theta$)
m	Flow rate, (M/T)
[M]	Matrix of coefficients defined by Equation (17)
p	Pressure, (F/L^2)
P_w	Wetted perimeter, (L)
q'	Heat addition per unit length, (H/L)

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*Dimensions are denoted by:

L = Length, T = time, M = mass, θ = temperature,

$F = ML/T^2$ = force and $H = ML^2/T^2$ = energy

Equations

q'''	Power density, (H/L^3)
r	Radius, (L)
s	Rod spacing, (L)
$[S]$	Matrix transformation defining adjacent subchannels, (Dimensionless)
T	Temperature, (θ)
u	Effective momentum velocity, $mv'/A, (L/T)$
u''	Effective energy transport velocity, (L/T)
u^*	Effective velocity carried by diversion crossflow, (L/T)
v	Liquid specific volume, (L^3/M)
v'	Effective specific volume for momentum, $(1-X)^2/\rho_f(1-\alpha) + X^2/\rho_g\alpha, (L^3/M)$
w	Diversion crossflow between adjacent subchannels, (M/TL)
w'	Turbulent (fluctuating) crossflow between adjacent subchannels, (M/TL)
x	Distance, (L)
X	Quality, $m_g/(m_g+m_f), (Dimensionless)$
α	Void fraction, $A_g/(A_g+A_f), (Dimensionless)$
θ	Orientation of channel with respect to vertical, $(Radians)$
ρ	Density, (M/L^3)
ρ	Two-phase Density, $\rho_g\alpha + \rho_f(1-\alpha), (M/L^3)$
ρ_g, ρ	Saturated vapor and liquid density, (M/L^3)
ρ''	Effective density for enthalpy transport, (M/L^3)
ρ^*	Effective density of the diversion crossflow, (M/L^3)
ϕ	Two-phase friction multiplier, $(Dimensionless)$

Subscripts

f,g	Saturated conditions for liquid and vapor, respectively
i	Fuel node number
i,j	Subchannel identification number
ij, ji	Double subscripts imply subchannel connection i to j and j to i, respectively
i(k), j(k)	Subchannel pair for connection number (k)
k	Subchannel connection number
n	Axial node number

APPENDIX A

DERIVATION OF CROSSFLOW MOMENTUM EQUATION

Consider a rectangular control volume placed in the gap between two subchannels.

Assume that the crossflow momentum flux entering and leaving the control volume through the transverse surfaces is negligibly small. This assumption is valid if the crossflow velocity is small compared to the axial velocity. Applying the conservation of momentum equation to this control volume in the transverse direction gives

$$\begin{aligned}
 -F_{ij} \Delta x l - p_j s \Delta x + p_i s \Delta x &= \frac{\partial \rho^*}{\partial t} s l \Delta x v - (\rho^* s l u^* v)_x \\
 &\quad + (\rho^* s l u^* v)_{x+\Delta x}
 \end{aligned}
 \tag{A-1}$$

By rearranging this equation and taking the limit at $\Delta x \rightarrow 0$, the following equation is obtained

$$\frac{\partial w_{ij}}{\partial t} + \frac{\partial (u^* w_{ij})}{\partial x} = \frac{s}{l} (p_i - p_j) - F_{ij}
 \tag{A-2}$$

where $w_{ij} = \rho^* s v$. F_{ij} represents the friction and form pressure loss due to crossflow. For steady flow let

$$p_i - p_j = K \frac{\rho^* v^2}{2} \quad (A-3)$$

Therefore,

$$F = \frac{K|w|w}{2s^2\rho^*} \left(\frac{s}{\ell}\right) \quad (A-4)$$

To put this in the form used previously in COBRA-III, let

$$F = Cw \left(\frac{s}{\ell}\right) \quad (A-5)$$

where $C = K|w|/2s^2\rho^*$.

Equation (A-2) can be simplified further to be consistent with the assumption on the transverse momentum flux. The second term can be written as

$$\frac{\partial(u^*w_{ij})}{\partial x} = u^* \frac{\partial w_{ij}}{\partial x} + w_{ij} \frac{\partial u^*}{\partial x} \quad (A-6)$$

Neglecting density changes, the term $\partial u^*/\partial x$ is proportional to the difference in crossflow entering and leaving the control volume through the area $s\Delta x$. Since the difference in the momentum flux terms ($\rho s v^2$) through the same area have been neglected, it is appropriate to neglect $\partial u^*/\partial x$. The result is that the primary transverse momentum term is that due to changing the magnitude of the crossflow.

LIST OF FIGURES

<u>Figure</u>	<u>Title</u>
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7	Computed Flow and Enthalpy in Subchannel (8) With and Without Fuel Heat Transfer Model

FIGURE 1. Method of Subchannel Selection

FIGURE 2. Fuel Model Node Designation

FIGURE 3. Subchannel Layout for a 1/8 Section
 of Symmetry from a 49-Rod Bundle

FIGURE 4. Computed Flow in Subchannel (8)
With and Without Blockage

FIGURE 5. Effect of u^* Assumption on Subchannel (8) Flow Solution

FIGURE 6. Comparison Between the Computed Heat Flux and Power Transient

FIGURE 7. Computed Flow and Enthalpy in Subchannel (8) With and Without Fuel Heat Transfer Model

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