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THE EFFECT OF INTERFERENCE INTERACTION
BETWEEN THE RESONANCES OF Pu^{239} AND Th^{232}
ON Pu^{239} FISSION AND CAPTURE CROSS SECTIONS
AND Th^{232} CAPTURE CROSS SECTIONS

by

R. S. Singh

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IN NUCLEAR SCIENCE ABSTRACTS

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R. S. Singh*

Reactor Physics Division

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THE EFFECT OF INTERFERENCE INTERACTION BETWEEN THE RESONANCES OF Pu^{239} AND Th^{232} ON Pu^{239} FISSION AND CAPTURE CROSS SECTIONS AND Th^{232} CAPTURE CROSS SECTIONS

by

R. S. Singh

ABSTRACT

The effect of interference interaction between the resonances of Pu^{239} and Th^{232} on Doppler-effect calculations and sodium-void coefficients has been investigated. The temperature-dependent effective cross sections for Pu^{239} and Th^{232} have been evaluated, taking into account both types of interference interactions, namely, the overlap of neighboring resonances in one isotope and the interaction between resonances of the two isotopes. A simplified method developed by R. N. Hwang¹ has been used to calculate these cross sections over a range of σ_p , the potential-scattering cross section per absorber atom for the two isotopes.

I. INTRODUCTION

The importance of taking into account the interference interactions between the resonances of the fertile and fissile isotopes in the study of the Doppler effect in fast reactors has been emphasized by many persons.^{2,3,4} The two types of interference interactions of interest are the overlap of neighboring resonances of the same isotope and the interference between the resonances of two or more isotopes in the mixture. Extensive studies of this effect have been made for typical Pu^{239} - U^{238} systems. The present study investigates this effect in some Pu^{239} - Th^{232} systems. The evaluation of the temperature-dependent effective group cross sections by direct numerical integration is rather lengthy when more than one type of interaction is involved. R. N. Hwang¹ has used some analytical characteristics of interference interactions to develop a simplified method. In this method, each type of interference interaction is considered separately in the calculation, and the numerical work involved is thereby simplified considerably. Several combinations of the potential-scattering cross sections per thorium atom σ_{p2} and those per plutonium atom σ_{p9} were chosen to represent a range of Th^{232} -to- Pu^{239} atom ratios and core compositions for metal and oxide fuels. The temperature-dependent cross sections were evaluated for these combinations with all the interference interactions included for the standard 22-group set 215.⁵

It has been assumed here that correction factors to the flux due to resonances^{1,6,7,8} can be ignored in computing cross sections. Such factors influence the Doppler effect because of their temperature dependence, but

only to a minor degree for the systems considered. These factors, if applied uniformly to all isotopes, do not affect competition between fission and capture at a given energy, but only the competition between fission and capture on the one hand, and leakage and elastic moderation on the other. Leakage effects in the present case are small in the Doppler energy region (<25 keV), while the variation of k_{∞} with energy is too slow to make a varying ratio of elastic moderation to absorption with temperature a significant source of reactivity change.

Hutchins⁶ and Nicholson⁷ found that the overlap effect can be well-approximated by use of flux correction factors that differ for different isotopes. This would represent an alternate approach to the work presented here.

II. CROSS SECTIONS WITH INTERFERENCE INTERACTIONS INCLUDED

The temperature-dependent effective group cross sections in the resonance region that are influenced by interference and overlap effects occur below group 14 (4 keV) in the 22-group set. These are evaluated separately for the fissile and fertile isotopes as described below.

A. Fissile Isotope: Pu²³⁹

For the fissile isotope, resonances are, in general, closely spaced, and hence the isolated-resonance approximation is not valid. The effective cross section for the reaction x in the overlap-resonance approximation for the self-overlap of neighboring resonances is obtained as

$$\begin{array}{ccc} \tilde{\sigma}_x \text{ overlap} & = & \tilde{\sigma}_x \text{ isolated} + \tilde{0}_x \\ \text{res. approx.} & & \text{res. approx.} \end{array} \quad (1)$$

where $\tilde{0}_x$ is the correction term for the overlap effect, which has been derived and discussed in Ref. 3.

For fast breeder reactors, the system must have at least one additional resonance absorber which is the fertile isotope. This gives rise to an interference effect, in addition to the self-overlap of neighboring resonances in the fissile isotope. The effective cross section, including both effects, is given as

$$\tilde{\sigma}_x = \left(\tilde{\sigma}_x \text{ overlap} \right) \left(1 - \frac{\langle FJ \rangle \text{ fertile isotope}}{\langle S \rangle \text{ fertile isotope}} \right), \quad (2)$$

res. approx.

where Γ is the total width, S the average spacing, and J the usual J -function for an isolated resonance. The second-order term, which is relatively insignificant for the fissile isotope in a dilute system, has been dropped from Eq. (2). The term $\langle \Gamma J \rangle / \langle S \rangle$ for fertile isotope can be easily evaluated from the existing resonance-integral code RE-270 by changing the radiative capture width Γ_γ to the total width Γ . Each effect can be evaluated separately to get the correct value of the effective cross sections.

The potential scattering cross sections per thorium atom σ_{p2} were chosen as 40 and 60 barns, and those per plutonium atom σ_{p9} were selected as 126, 200, and 300 barns, which represents a wide range of core compositions. The effective cross sections for Pu^{239} with the isolated-resonance approximation and with self-overlap corrections were evaluated by R. N. Hwang for studying the $\text{Pu}^{239}\text{-U}^{238}$ systems, for the σ_{p9} values mentioned above. The same cross sections have been used here to study the interference effects of Th^{232} resonances on Pu^{239} resonances. The latter effect, given by the term $\langle \Gamma J \rangle / \langle S \rangle$ for thorium, was evaluated for both the σ_{p2} values at temperatures 300, 750, and 1500°K, by using the RE-270 program. The fission and capture cross sections of Pu^{239} so obtained for the cases (a) $\sigma_{p2} = 40$, $\sigma_{p9} = 126$; (b) $\sigma_{p2} = 40$, $\sigma_{p9} = 200$; (c) $\sigma_{p2} = 40$, $\sigma_{p9} = 300$; (d) $\sigma_{p2} = 60$, $\sigma_{p9} = 126$; and (e) $\sigma_{p2} = 60$, $\sigma_{p9} = 200$ are given in Tables I, II, III, IV, and V respectively.

TABLE I. Interference Interaction of Th^{232} Resonances on Pu^{239} Resonances; Pu^{239} Fission and Capture Cross Sections in Barns; $\sigma_{p2} = 40$ barns; $\sigma_{p9} = 126$ barns

j	Energy E_L , keV	$\tilde{\sigma}_{fB}^a$	$\tilde{\sigma}_{cB}^a$	$\tilde{\sigma}_f$	$\tilde{\sigma}_c$	$1 - \frac{\langle \Gamma_2 J_2 \rangle}{D_2}$	$\tilde{\sigma}_f$	$\tilde{\sigma}_c$	
300°K	14	4.0	1.7162	0.9661	0.0188	0.0111	0.9513	1.6147 (+0.7) ^b	0.9085 (+0.4) ^b
	15	2.8	-	-	-	-	-	2.55 ^c	1.47 ^c
	16	2.0	3.1223	1.8351	0.0293	0.0182	0.9372	2.8987	1.7027
	17	1.4	3.5881	2.1294	0.0343	0.0172	0.9394	3.3385	1.9842
	18	1.0	4.2114	2.5253	0.0351	0.0205	0.9461	3.9516	2.3700
	19	0.55	5.0457	3.0549	0.0373	0.0237	0.9305	4.6605	2.8206
	20	0.30	6.2430	3.7997	-	-	0.9375	5.8530	3.5623
21	0.10	7.9920	4.8093	-	-	0.9540	7.6246	4.5882	
750°K	14	4.0	1.7445	0.9819	0.0262	0.0153	0.9451	1.6239 (+0.7) ^b	0.9135 (+0.4) ^b
	15	2.8	-	-	-	-	-	2.59 ^c	1.49 ^c
	16	2.0	3.2440	1.9099	0.0488	0.0317	0.9264	2.9602	1.7400
	17	1.4	3.7576	2.2361	0.0525	0.0325	0.927	3.4363	2.0437
	18	1.0	4.4577	2.6855	0.0625	0.0389	0.9358	4.1132	2.4768
	19	0.55	5.4143	3.3044	0.0807	0.0497	0.9165	4.8883	2.9829
	20	0.30	6.8073	4.2045	-	-	0.9217	6.2746	3.8754
21	0.10	8.7968	5.4358	-	-	0.9429	8.2946	5.1254	
1500°K	14	4.0	1.7607	0.9908	0.0308	0.0181	0.9407	1.6273 (+0.7) ^b	0.9151 (+0.4) ^b
	15	2.8	-	-	-	-	-	2.61 ^c	1.50 ^c
	16	2.0	3.3200	1.9552	0.0698	0.0430	0.9182	2.9844	1.7558
	17	1.4	3.8670	2.3029	0.0778	0.0483	0.9172	3.4756	2.0681
	18	1.0	4.6242	2.7900	0.0934	0.0584	0.9272	4.2012	2.5329
	19	0.55	5.6799	3.4774	0.1153	0.0721	0.9031	5.0254	3.0753
	20	0.30	7.2554	4.5115	-	-	0.9056	6.5706	4.0857
21	0.10	9.5304	5.9777	-	-	0.9308	8.8715	5.5644	

^aSubscript B refers to Method B calculation, which is the isolated-resonance approximation.

^bp-wave contributions.

^cInterpolated values.

TABLE II. Interference Interaction of Th^{232} Resonances on Pu^{239} Resonances;
 Pu^{239} Fission and Capture Cross Sections in Barns; $\sigma_{p2} = 40$ barns; $\sigma_{p9} = 200$ barns

j	Energy E_L , keV	$\tilde{\sigma}_{fB}^a$	$\tilde{\sigma}_{cB}^a$	$\tilde{\sigma}_f$	$\tilde{\sigma}_c$	$1 - \frac{\langle \Gamma_2 J_2 \rangle}{D_2}$	$\tilde{\sigma}_f$	$\tilde{\sigma}_c$	
300°K	14	4.0	1.7477	0.9833	0.0119	0.0069	0.9513	1.6513 (+0.7) ^b	0.9288 (+0.4) ^b
	15	2.8	-	-	-	-	-	2.65 ^c	1.54 ^c
	16	2.0	3.2641	1.9185	0.0196	0.0121	0.9371	3.0406	1.7864
	17	1.4	3.7893	2.2498	0.0178	0.0103	0.9394	3.5431	2.1039
	18	1.0	4.5119	2.7091	0.0182	0.0113	0.9461	4.2517	2.5525
	19	0.55	5.5157	3.3492	0.0226	0.0139	0.9305	5.1116	3.1036
	20	0.30	7.0269	4.3032	-	-	0.9375	6.5890	4.0344
21	0.10	9.0502	5.5227	-	-	0.9540	8.6342	5.2688	
750°K	14	4.0	1.7665	0.9938	0.0165	0.0096	0.9451	1.6539 (+0.7) ^b	0.9307 (+0.4) ^b
	15	2.8	-	-	-	-	-	2.68 ^c	1.55 ^c
	16	2.0	3.3514	1.9721	0.0328	0.0192	0.9264	3.0745	1.8092
	17	1.4	3.9145	2.3287	0.0347	0.0215	0.9274	3.5983	2.1398
	18	1.0	4.7017	2.8325	0.0398	0.0249	0.9358	4.3629	2.6275
	19	0.55	5.8174	3.5535	0.0450	0.0283	0.9165	5.2904	3.2309
	20	0.30	7.5324	4.6660	-	-	0.9217	6.9429	4.3009
21	0.10	9.8369	6.1291	-	-	0.9429	9.2753	5.7792	
1500°K	14	4.0	1.7771	0.9996	0.0194	0.0114	0.9407	1.6535 (+0.7) ^b	0.9297 (+0.4) ^b
	15	2.8	-	-	-	-	-	2.69 ^c	1.56 ^c
	16	2.0	3.4041	2.0035	0.0455	0.0279	0.9182	3.0839	1.8140
	17	1.4	3.9923	2.3761	0.0515	0.0320	0.9172	3.6146	2.1501
	18	1.0	4.8244	2.9095	0.0616	0.0386	0.9272	4.4164	2.6621
	19	0.55	6.0240	3.6881	0.0725	0.0458	0.9031	5.3748	3.2893
	20	0.30	7.9124	4.9264	-	-	0.9056	7.1656	4.4614
21	0.10	10.5050	6.6179	-	-	0.9308	9.7787	6.1603	

^aSubscript B refers to Method B calculation, which is the isolated-resonance approximation.

^bp-wave contributions.

^cInterpolated values.

TABLE III. Interference Interaction of Th^{232} Resonances on Pu^{239} Resonances;
 Pu^{239} Fission and Capture Cross Sections in Barns; $\sigma_{p2} = 40$ barns; $\sigma_{p9} = 300$ barns

j	Energy E_L , keV	$\tilde{\sigma}_{fB}^a$	$\tilde{\sigma}_{cB}^a$	$\tilde{\sigma}_f$	$\tilde{\sigma}_c$	$1 - \frac{\langle \Gamma_2 J_2 \rangle}{D_2}$	$\tilde{\sigma}_f$	$\tilde{\sigma}_c$	
300°K	14	4.0	1.7665	0.9936	0.0079	0.0046	0.9513	1.6729 (+0.7) ^b	0.9408 (+0.4) ^b
	16	2.0	3.3548	1.9719	0.0135	0.0083	0.9371	3.1313	1.8402
	17	1.4	3.9215	2.3291	0.0119	0.0071	0.9394	3.6727	2.1814
	18	1.0	4.7169	2.8348	0.0121	0.0076	0.9461	4.4515	2.6750
	19	0.55	5.8538	3.5620	0.0133	0.0083	0.9305	5.4349	3.3068
	20	0.30	7.6358	4.6974	-	-	0.9375	7.1589	4.4040
	21	0.10	10.1350	6.2400	-	-	0.9540	9.6691	5.9532
750°K	14	4.0	1.7794	1.0009	0.0110	0.0064	-	1.6713 (+0.7) ^b	0.9398 (+0.4) ^b
	16	2.0	3.4179	2.0106	0.0227	0.0132	-	3.1455	1.8504
	17	1.4	4.0138	2.3872	0.0241	0.0149	-	3.7002	2.2001
	18	1.0	4.8611	2.9286	0.0276	0.0173	-	4.5234	2.7246
	19	0.55	6.0937	3.7246	0.0304	0.0192	-	5.5570	3.3960
	20	0.30	8.0685	5.0087	-	-	-	7.4371	4.6167
	21	0.10	10.8800	6.8156	-	-	-	10.2588	6.4265
1500°K	14	4.0	1.7866	1.0048	0.0130	0.0076	-	1.6685 (+0.7) ^b	0.9381 (+0.4) ^b
	16	2.0	3.4551	2.0328	0.0309	0.0189	-	3.1441	1.8491
	17	1.4	4.0696	2.4213	0.0356	0.0221	-	3.7001	2.2006
	18	1.0	4.9514	2.9853	0.0429	0.0268	-	4.5515	2.7433
	19	0.55	6.2518	3.8275	0.0504	0.0320	-	5.6004	3.4277
	20	0.30	8.3794	5.2218	-	-	-	7.5885	4.7289
	21	0.10	11.4830	7.2573	-	-	-	10.6891	6.7555

^aSubscript B refers to Method B calculation, which is the isolated-resonance approximation.

^bp-wave contributions.

TABLE IV. Interference Interaction of Th^{232} Resonances on Pu^{239} Resonances;
 Pu^{239} Fission and Capture Cross Sections in Barns; $\sigma_{p2} = 60$ barns; $\sigma_{p9} = 126$ barns

j	Energy E_L , keV	$\tilde{\sigma}_{fB}^a$	$\tilde{\sigma}_{cB}^a$	$\tilde{\sigma}_f$	$\tilde{\sigma}_c$	$1 - \frac{\langle \Gamma_2 J_2 \rangle}{D_2}$	$\tilde{\sigma}_f$	$\tilde{\sigma}_c$	
300°K	14	4.0	1.7162	0.9661	0.0188	0.0111	0.9633	1.6350 (+0.7) ^b	0.9199 (+0.4) ^b
	15	2.8	-	-	-	-	-	2.60 ^c	1.50 ^c
	16	2.0	3.1223	1.8351	0.0293	0.0182	0.9510	2.9416	1.7279
	17	1.4	3.5881	2.1294	0.0343	0.0172	0.9511	3.3802	2.0090
	18	1.0	4.2114	2.5253	0.0351	0.0205	0.9570	3.9971	2.3973
	19	0.55	5.0457	3.0549	0.0373	0.0237	0.9422	4.7192	2.8562
	20	0.30	6.2430	3.7997	-	-	0.9464	5.9086	3.5962
21	0.10	7.9920	4.8093	-	-	0.9602	7.6744	4.6182	
750°K	14	4.0	1.7445	0.9819	0.0262	0.0153	0.9596	1.6489 (+0.7) ^b	0.9276 (+0.4) ^b
	15	2.8	-	-	-	-	-	2.65 ^c	1.53 ^c
	16	2.0	3.2440	1.9099	0.0488	0.0317	0.9441	3.0166	1.7732
	17	1.4	3.7576	2.2361	0.0525	0.0325	0.9426	3.4926	2.0772
	18	1.0	4.4577	2.6855	0.0625	0.0389	0.9498	4.1747	2.5138
	19	0.55	5.4143	3.3044	0.0807	0.0497	0.9312	4.9667	3.0308
	20	0.30	6.8073	4.2045	-	-	0.9329	6.3510	3.9227
21	0.10	8.7968	5.4358	-	-	0.9502	8.3592	5.1654	
1500°K	14	4.0	1.7607	0.9908	0.0308	0.0181	0.9571	1.6556 (+0.7) ^b	0.9310 (+0.4) ^b
	15	2.8	-	-	-	-	-	2.67 ^c	1.54 ^c
	16	2.0	3.3200	1.9552	0.0698	0.0430	0.9390	3.0521	1.7956
	17	1.4	3.8670	2.3029	0.0778	0.0483	0.9357	3.5456	2.1097
	18	1.0	4.6242	2.7900	0.0934	0.0584	0.9441	4.2774	2.5789
	19	0.55	5.6799	3.4774	0.1153	0.0721	0.9209	5.1249	3.1362
	20	0.30	7.2554	4.5115	-	-	0.9197	6.6734	4.1496
21	0.10	9.5304	5.9777	-	-	0.9398	8.9571	5.6181	

^aSubscript B refers to Method B calculation, which is the isolated-resonance approximation.

^bp-wave contributions.

^cInterpolated values.

TABLE V. Interference Interaction of Th^{232} Resonances on Pu^{239} Resonances;
 Pu^{239} Fission and Capture Cross Sections in Barns; $\sigma_{p2} = 60$ barns; $\sigma_{p9} = 200$ barns

j	Energy E_L , keV	$\tilde{\sigma}_{fB}^a$	$\tilde{\sigma}_{cB}^a$	$\tilde{\sigma}_f$	$\tilde{\sigma}_c$	$1 - \frac{\langle \Gamma_2 J_2 \rangle}{D_2}$	$\tilde{\sigma}_f$	$\tilde{\sigma}_c$	
300°K	14	4.0	1.7477	0.9833	0.0119	0.0069	0.9633	1.6721 (+0.7) ^b	0.9405 (+0.4) ^b
	15	2.8	-	-	-	-	-	2.68 ^c	1.57 ^c
	16	2.0	3.2641	1.9185	0.0196	0.0121	0.9510	3.0856	1.8128
	17	1.4	3.7893	2.2498	0.0178	0.0103	0.9511	3.5874	2.1302
	18	1.0	4.5119	2.7091	0.0182	0.0113	0.9570	4.3008	2.5819
	19	0.55	5.5157	3.3492	0.0226	0.0139	0.9422	5.1760	3.1427
	20	0.30	7.0269	4.3032	-	-	0.9464	6.6516	4.0727
21	0.10	9.0502	5.5227	-	-	0.9602	8.6906	5.3032	
750°K	14	4.0	1.7665	0.9938	0.0165	0.0096	0.9596	1.6793 (+0.7) ^b	0.9445 (+0.4) ^b
	15	2.8	-	-	-	-	-	2.71 ^c	1.585 ^c
	16	2.0	3.3514	1.9721	0.0328	0.0192	0.9441	3.1331	1.8436
	17	1.4	3.9145	2.3287	0.0347	0.0215	0.9426	3.6573	2.1748
	18	1.0	4.7017	2.8325	0.0398	0.0249	0.9498	4.4281	2.6668
	19	0.55	5.8174	3.5535	0.0450	0.0283	0.9312	5.3752	3.2827
	20	0.30	7.5324	4.6660	-	-	0.9329	7.0275	4.3533
21	0.10	9.8369	6.1291	-	-	0.9502	9.3476	5.8242	
1500°K	14	4.0	1.7771	0.9996	0.0194	0.0114	0.9571	1.6823 (+0.7) ^b	0.9459 (+0.4) ^b
	15	2.8	-	-	-	-	-	2.72 ^c	1.59 ^c
	16	2.0	3.4041	2.0035	0.0455	0.0279	0.9390	3.1539	1.8551
	17	1.4	3.9923	2.3761	0.0515	0.0320	0.9357	3.6874	2.1935
	18	1.0	4.8244	2.9095	0.0616	0.0386	0.9440	4.4964	2.7104
	19	0.55	6.0240	3.6881	0.0725	0.0458	0.9209	5.4813	3.3544
	20	0.30	7.9124	4.9264	-	-	0.9197	7.2778	4.5312
21	0.10	10.5050	6.6179	-	-	0.9398	9.8731	6.2198	

^aSubscript B refers to Method B calculation, which is the isolated-resonance approximation.

^bp-wave contributions.

^cInterpolated values.

Parameters for Pu^{239} used in these calculations are as follows:

$$\bar{\Gamma}_f = 0.099 \text{ eV}, \quad \Gamma_\gamma = 0.039 \text{ eV}, \quad \frac{\langle \Gamma_n^0 \rangle}{D} = 1.0 \times 10^{-4},$$

$$D_{J=0} = 11.6 \text{ eV}, \text{ and } D_{J=1} = 3.86 \text{ eV}.$$

A χ^2 distribution of level widths with $\nu = 2$ were used for fission widths (represented by five discrete widths) and $\nu = 1$ was used for s-wave neutron widths (represented by 10 discrete widths).

The percentage reduction in the fission cross-section change $\delta\sigma_f$ due to temperature (750-1500°K), in going from the cross sections evaluated with the isolated-resonance approximation to those obtained with overlap and interference effects included can be calculated for the energy groups of interest from these tables, and these percentage reductions are plotted as a function of Th/Pu ratio in Fig. 1. The percentage reduction increases with the Th/Pu ratio and is higher for the higher-energy groups.

Fig. 1
Effect of Interference on $\delta\sigma_x$
(750-1500°K) for Pu^{239} and
 Th^{232} at Various Th/Pu Ratios

B. Fertile Isotope: Th^{232}

The effective group-capture cross sections for Th^{232} have been evaluated at different temperatures by using the isolated-resonance approximation, with the latest resonance data at different temperatures and σ_{p2} values of 40 and 60 barns in our earlier studies.⁵ The effect of interference interactions on the fertile isotope is relatively small because the resonances of

the fertile isotopes are much stronger than those of fissile isotopes. For energies below about 25 keV, the resonances of the fertile isotopes can be considered as isolated, since the average separation between them is greater than their Doppler width. Therefore, the self-overlap effect is negligible in the fertile isotopes, and only the interference effects of Pu^{239} resonances on those of Th^{232} have to be considered. The expression for the effective cross section of the fertile isotope under these conditions is given by¹

$$\begin{aligned} \tilde{\sigma}_x = \tilde{\sigma}_{x \text{ isolated}} & \left(1 - \sum_{\text{spin states}} \frac{\langle \Gamma(J+H) \rangle_{\text{fissile}}}{\langle S \rangle_{\text{fissile}}} \right) \\ & + \frac{N_{\text{fissile}}}{N_{\text{fertile}}} \cdot \tilde{\sigma}_{t \text{ fissile}} \cdot \frac{1}{\langle S \rangle_{\text{fertile}}} \left\langle \frac{\Gamma_x^i}{2} \int_{-\infty}^{\infty} \left(\frac{\psi_i}{\beta_i + \psi_i} \right)^2 dx_i \right\rangle, \quad (3) \end{aligned}$$

where $\tilde{\sigma}_{t \text{ fissile}}$ is the total resonance cross section for the fissile isotope. The term, $\langle \Gamma(J+H) \rangle_{\text{fissile}} / \langle S \rangle_{\text{fissile}}$, accounts for the interference effect of Pu^{239} resonances whose overlap has also been considered and has been evaluated in earlier studies. The integral in the second term was evaluated by a separate program for the fertile isotope Th^{232} by feeding the resonance parameters in the energy groups of interest. The capture cross sections so obtained for Th^{232} for all the five cases as for Pu^{239} at the three temperatures are tabulated in Tables VI-X.

TABLE VI. Interference Effect of Pu^{239} Resonances on Th^{232} Resonances; Th^{232} Capture Cross Sections in Barns; $\sigma_{p2} = 40$ barns; $\sigma_{p9} = 126$ barns

j	Energy E_L , keV	Integral Term**	$\tilde{\sigma}_{t \text{ fissile}}$	Second Term*	$\tilde{\sigma}_{c \text{ Th}^{232}}^{\text{iso. res.}}$	$1 - \frac{\langle \Gamma(J+H) \rangle}{\langle S \rangle}$	First Term*	$\tilde{\sigma}_c^{\text{Th}^{232}}$, barns
300°K	17 1.4	0.0101	8.1273	0.0275	1.348	0.9354	1.2606	1.2882
	18 1.0	0.0112	9.1465	0.0342	1.070	0.9274	0.9923	1.0264
	19 0.55	0.0146	10.4386	0.0508	1.088	0.9171	0.9978	1.0486
	20 0.30	0.0137	12.2533	0.0561	1.030	0.9027	0.9298	0.9859
	21 0.10	0.0188	14.3120	0.0900	0.895	0.8864	0.7933	0.8834
750°K	17 1.4	0.0105	8.5274	0.0298	1.577	0.9323	1.4702	1.5001
	18 1.0	0.0114	9.7036	0.0369	1.254	0.9229	1.1574	1.1943
	19 0.55	0.0163	11.2335	0.0612	1.323	0.9108	1.2050	1.2663
	20 0.30	0.0184	13.5193	0.0831	1.288	0.8927	1.1498	1.2329
	21 0.10	0.0244	15.9760	0.1300	1.119	0.8732	0.9771	1.1071
1500°K	17 1.4	0.0103	8.7583	0.0302	1.759	0.9304	1.6367	1.6670
	18 1.0	0.0101	10.0540	0.0337	1.395	0.9202	1.2836	1.3174
	19 0.55	0.0173	11.7936	0.0681	1.528	0.9064	1.3849	1.4531
	20 0.30	0.0213	14.5151	0.1034	1.540	0.8848	1.3625	1.4660
	21 0.10	0.0294	17.4470	0.1715	1.359	0.8615	1.1622	1.3337

*The terms are from Eq. (3).

**Integral term = $\frac{1}{\Delta E} \sum_i \frac{\Gamma_x^i}{2} \int \frac{\psi_i^2}{(\beta_i + \psi_i)^2} dx_i$.

TABLE VII. Interference Effect of Pu^{239} Resonances on Th^{232} Resonances; Th^{232} Capture Cross Sections in Barns; $\sigma_{p2} = 40$ barns; $\sigma_{p9} = 200$ barns

j	Energy E_L , keV	Integral Term**	$\tilde{\sigma}_t$ fissile	Second Term*	$\tilde{\sigma}_c^{\text{Th}^{232}}$ iso. res.	$1 - \frac{\langle \Gamma(J+H) \rangle}{\langle S \rangle}$	First Term*	$\tilde{\sigma}_c^{\text{Th}^{232}}$, barns
300°K	17 1.4	0.0101	8.6860	0.0176	1.348	0.9565	1.2894	1.3071
	18 1.0	0.0112	9.9284	0.0222	1.070	0.9503	1.0168	1.0391
	19 0.55	0.0146	11.5832	0.0338	1.088	0.9420	1.0249	1.0588
	20 0.30	0.0137	13.9299	0.0382	1.030	0.9303	0.9582	1.9965
	21 0.10	0.0188	16.8350	0.0635	0.895	0.9158	0.8196	0.8832
750°K	17 1.4	0.0105	8.9782	0.0188	1.577	0.9551	1.5062	1.5250
	18 1.0	0.0114	10.3640	0.0236	1.254	0.9481	1.1890	1.2126
	19 0.55	0.0163	12.2666	0.0401	1.323	0.9386	1.2418	1.2819
	20 0.30	0.0184	15.0812	0.0556	1.288	0.9245	1.1908	1.2465
	21 0.10	0.0244	18.5310	0.0904	1.119	0.9073	1.0153	1.1058
1500°K	17 1.4	0.0103	9.1449	0.0189	1.759	0.9542	1.6785	1.6975
	18 1.0	0.0100	10.6243	0.0213	1.395	0.9468	1.3208	1.3422
	19 0.55	0.0173	12.7007	0.0440	1.528	0.9364	1.4309	1.4750
	20 0.30	0.0213	15.9364	0.0681	1.540	0.9203	1.4172	1.4854
	21 0.10	0.0294	19.9490	0.1176	1.359	0.9002	1.2234	1.3411

*The terms are from Eq. (3).

$$\text{**Integral term} = \frac{1}{\Delta E} \sum_i \frac{\Gamma_x^i}{2} \int \frac{\psi_i^2}{(\beta_i + \psi_i)^2} dx_i.$$

TABLE VIII. Interference Effect of Pu^{239} Resonances on Th^{232} Resonances; Th^{232} Capture Cross Sections in Barns; $\sigma_{p2} = 40$ barns; $\sigma_{p9} = 300$ barns

j	Energy E_L , keV	Integral Term**	$\tilde{\sigma}_{tq}$	Second Term*	$\tilde{\sigma}_c^{\text{Th}^{232}}$ iso. res.	$1 - \frac{\langle \Gamma(J+H) \rangle}{\langle S \rangle}$	First Term*	$\tilde{\sigma}_c^{\text{Th}^{232}}$, barns
300°K	17 1.4	0.0101	9.0350	0.0122	1.348	0.9698	1.3074	1.3196
	18 1.0	0.0112	10.4376	0.0156	1.070	0.9652	1.0327	1.0483
	19 0.55	0.0146	12.3625	0.0240	1.088	0.9587	1.0431	1.0672
	20 0.30	0.0137	15.2536	0.0279	1.030	0.9491	0.9776	1.0055
	21 0.10	0.0188	19.0180	0.0478	0.895	0.9366	0.8382	0.8861
750°K	17 1.4	0.0105	9.2551	0.0129	1.577	0.9691	1.5283	1.5413
	18 1.0	0.0114	10.7737	0.0164	1.254	0.9640	1.2089	1.2253
	19 0.55	0.0163	12.9127	0.0281	1.323	0.9569	1.2660	1.2942
	20 0.30	0.0184	16.2510	0.0399	1.288	0.9458	1.2182	1.2582
	21 0.10	0.0244	20.6440	0.0672	1.119	0.9311	1.0419	1.1092
1500°K	17 1.4	0.0103	9.3775	0.0129	1.759	0.9687	1.7040	1.7169
	18 1.0	0.0101	10.9697	0.0147	1.395	0.9634	1.3439	1.3587
	19 0.55	0.0173	13.2525	0.0306	1.528	0.9558	1.4605	1.4911
	20 0.30	0.0213	16.9568	0.0483	1.540	0.9434	1.4529	1.5012
	21 0.10	0.0294	21.9370	0.0862	1.359	0.9268	1.2596	1.3458

*The terms are from Eq. (3).

$$\text{**Integral term} = \frac{1}{\Delta E} \sum_i \frac{\Gamma_x^i}{2} \int \frac{\psi_i^2}{(\beta_i + \psi_i)^2} dx_i.$$

TABLE IX. Interference Effect of Pu²³⁹ Resonances on Th²³² Resonances;
Th²³² Capture Cross Sections in Barns; $\sigma_{p2} = 60$ barns; $\sigma_{p9} = 126$ barns

j	Energy E _L , keV	Integral Term**	$\tilde{\sigma}_{tq}$	Second Term*	$\tilde{\sigma}_{c}^{Th^{232}}$ iso. res.	$1 - \frac{\langle \Gamma(J+H) \rangle}{\langle S \rangle}$	First Term*	$\tilde{\sigma}_c^{Th^{232}}$ barns
300°K	17 1.4	0.0069	8.1273	0.0270	1.608	0.9354	1.5040	1.5310
	18 1.0	0.0072	9.1465	0.0316	1.251	0.9274	1.1601	1.1918
	19 0.55	0.0108	10.4386	0.0541	1.334	0.9171	1.2234	1.2775
	20 0.30	0.0121	12.2533	0.0710	1.310	0.9027	1.1826	1.2536
	21 0.10	0.0161	14.3120	0.1101	1.156	0.8864	1.0246	1.1347
750°K	17 1.4	0.0068	8.5274	0.0277	1.838	0.9323	1.7131	1.7408
	18 1.0	0.0070	9.7036	0.0324	1.431	0.9229	1.3207	1.3532
	19 0.55	0.0116	11.2335	0.0624	1.594	0.9108	1.4518	1.5143
	20 0.30	0.0147	13.5193	0.0944	1.628	0.8927	1.4533	1.5477
	21 0.10	0.0204	15.9760	0.1558	1.455	0.8732	1.2705	1.4263
1500°K	17 1.4	0.0061	8.7583	0.0255	2.014	0.9304	1.8736	1.8992
	18 1.0	0.0066	10.0540	0.0316	1.565	0.9202	1.4401	1.4717
	19 0.55	0.0119	11.7936	0.0670	1.812	0.9064	1.6423	1.7094
	20 0.30	0.0165	14.5151	0.1142	1.929	0.8848	1.7067	1.8210
	21 0.10	0.0241	17.4470	0.2007	1.760	0.8615	1.5162	1.7169

*The terms are from Eq. (3).

**Integral term = $\frac{1}{\Delta E} \sum_i \frac{\Gamma_x^i}{2} \int \frac{\psi_i^2}{(\beta_i + \psi_i)^2} dx_i$.

TABLE X. Interference Effect of Pu²³⁹ Resonances on Th²³² Resonances;
Th²³² Capture Cross Sections in Barns; $\sigma_{p2} = 60$ barns; $\sigma_{p9} = 200$ barns

j	Energy E _L , keV	Integral Term**	$\tilde{\sigma}_{tq}$	Second Term*	$\tilde{\sigma}_c^{Th^{232}}$ iso. res.	$1 - \frac{\langle \Gamma(J+H) \rangle}{\langle S \rangle}$	First Term*	$\tilde{\sigma}_c^{Th^{232}}$ barns
300°K	17 1.4	0.0069	8.6860	0.0183	1.608	0.9565	1.5378	1.5562
	18 1.0	0.0072	9.9284	0.0218	1.251	0.9503	1.1888	1.2107
	19 0.55	0.0108	11.5832	0.0382	1.334	0.9420	1.2567	1.2949
	20 0.30	0.0121	13.9299	0.0514	1.310	0.9303	1.2187	1.2701
	21 0.10	0.0161	16.8350	0.0824	1.156	0.9158	1.0586	1.1411
750°K	17 1.4	0.0068	8.9782	0.0185	1.838	0.9551	1.7550	1.7735
	18 1.0	0.0070	10.3640	0.0221	1.431	0.9481	1.3568	1.3789
	19 0.55	0.0117	12.2666	0.0434	1.594	0.9386	1.4962	1.5396
	20 0.30	0.0146	15.0812	0.0670	1.628	0.9245	1.5052	1.5722
	21 0.10	0.0204	18.5310	0.1150	1.455	0.9073	1.3201	1.4351
1500°K	17 1.4	0.0061	9.1449	0.0170	2.014	0.9542	1.9215	1.9385
	18 1.0	0.0066	10.6243	0.0212	1.565	0.9468	1.4818	1.5031
	19 0.55	0.0119	12.7007	0.0459	1.812	0.9364	1.6969	1.7429
	20 0.30	0.0165	15.9364	0.0798	1.929	0.9203	1.7752	1.8550
	21 0.10	0.0241	19.9490	0.1461	1.760	0.9002	1.5844	1.7305

*The terms are from Eq. (3).

**Integral term = $\frac{1}{\Delta E} \sum_i \frac{\Gamma_x^i}{2} \int \frac{\psi_i^2}{(\beta_i + \psi_i)^2} dx_i$.

The percentage reduction in the change of capture cross section ($\delta\sigma_c$) due to temperature (750-1500°K) when the interference effects are considered, is plotted for Th^{232} also in Fig. 1 as a function of Th/Pu ratio. The percentage reduction for Th^{232} is small compared to that for Pu^{239} isotope, indicating that the interference effects of the fissile isotope on the fertile isotope are relatively small.

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