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AN ANALYSIS OF THE INHERENT STABILITY OF THE EOCR

MASTER

S. R. Gossmann

PHILLIPS
PETROLEUM
COMPANY

ATOMIC ENERGY DIVISION

NATIONAL REACTOR TESTING STATION
US ATOMIC ENERGY COMMISSION

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AN ANALYSIS OF THE INHERENT STABILITY OF THE EOCR*

By

S. R. Gossmann

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ABSTRACT

AN ANALYSIS OF THE INHERENT STABILITY OF THE EOCR*

The inherent stability of the EOCR at full power, exclusive of the heat exchanger is investigated. Space and time dependent differential equations and transfer functions for the nuclear and thermal processes occurring in the reactor core and vessel are derived. Reactor stability is analyzed by application of the Nyquist stability criterion.

The results of the analysis show that the EOCR has an extremely large margin of inherent stability.

*Submitted to the University of Idaho Graduate School on May 15, 1962, in partial fulfillment of the requirements for the degree of Master of Science.

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AN ANALYSIS OF THE INHERENT STABILITY OF THE EOGR

I. INTRODUCTION

The Experimental Organic Cooled Reactor (EOCR) is a 40MW thermal reactor cooled and moderated by an organic fluid. The reactor is specifically designed to investigate the properties of organic materials for use as coolants in high power and high temperature reactors. Two experimental loops have been included in the reactor design to accomplish this objective. The coolant for the first core is Santowax R, a terphenyl mixture ($C_{18}H_{14}$). The Phillips Petroleum Company, Atomic Energy Division, has been selected by the Atomic Energy Commission to operate the reactor. Startup of the EOCR is planned for the fall of 1962.

Necessity for the Analysis

Prior to startup of the reactor, it is the responsibility of the operator to prepare and issue a hazards report. The work reported in this thesis is part of the analysis performed to determine the existence of hazards due to possible instability of the reactor plant. Figure 1 shows the relative location of the various reactor plant components which could conceivably contribute to reactor instability. The nature of the interactions between the components shown in Figure 1 is discussed in the next paragraph.

The organic fluid circulates continuously through the reactor and heat exchanger. The heated fluid leaves the reactor, flows to the heat exchanger where cooling occurs and returns to the reactor. The temperature controller must hold the reactor inlet temperature constant as required by the reactor operating program. It is also required that reactor power be held constant. This is the function of the power level controller.

It is possible that, because of the transport lags and large time constants associated with the heat exchanger and temperature controller, divergent temperature oscillations could arise in that part of the system. In this event, the power controller would attempt to hold the power constant but would be unable simultaneously to control the temperature oscillations. Divergent temperature oscillations could result in damage to the fuel elements, due to overheating, which constitutes a hazard.

The overall stability of a reactor plant can be determined only after the various plant components have been described in terms of their dynamic parameters such as gains, time constants and stability margins. The mathematical analysis performed to assess the inherent stability of the EOCR, exclusive of the heat exchanger, temperature controller and power controller, is the subject of this thesis.

Comparison to other Analyses

A direct comparison of this analysis to other analyses of the EOCR is not possible. The inherent stability of the EOCR had not previously been investigated. Comparison to analyses of other reactors is not possible without detailed information concerning the design and operating conditions of those reactors. Assumptions valid for the EOCR may not be valid for another reactor. The techniques employed in this analysis are well known and extensively used in the analysis of control systems. The representation of power, temperatures and reactivity as continuous functions of time and space within the reactor core is thought to be unique. Within the time allowed for this analysis an extensive

literature search was not possible. A limited search of the literature did not reveal the previous use of this type of representation by other investigators although this author had proposed a very similar representation in an earlier paper (1).

Description of Procedures

The inherent stability margin of the reactor was determined by application of the Nyquist stability criterion to the open-loop reactor transfer function. This transfer function was not obtainable by inspection given the reactor operating conditions. It was derived from a set of linear differential equations describing power generation, heat transfer and reactivity effects in the reactor core. The seven statements listed below summarize the procedures followed in deriving the equations and the open-loop transfer function required for the stability analysis:

1. Operating conditions and the reactor core configuration were studied in order to formulate a set of conditions and assumptions known or thought to apply.
2. The pertinent physical processes and applicable physical laws were identified.
3. The results of statements 1 and 2 were expressed mathematically.
4. Non-linear equations were linearized yielding the mathematical model of the system.
5. The Laplace transformation was applied to all of the linearized equations.
6. Power, temperature and reactivity transfer functions were derived from the transformed equations.
7. From these transfer functions, the open-loop reactor transfer function was derived.

II. THE REACTOR CORE MODEL

The reactor core model used in this analysis was obtained by examining the design of the reactor core, with particular attention given to the fuel element design, and by considering the operating conditions of the reactor.

Description of the Core

The features of the reactor core and fuel element design pertinent to this analysis are shown in Figures 2-a, 2-b, and 2-c. The coolant is that fraction of the organic fluid flowing through the fuel elements and in contact with the fuel plates. The moderator is that fraction of the organic that does not contact the fuel plates. As shown in these figures, the organic fluid enters the reactor inlet plenum at temperature T_{in} and mass flow rate W_i . The flow out of the inlet plenum divides into two streams: coolant and moderator with flow rates W_c and W_m , respectively. The coolant and moderator are constrained to flow in separate channels between the inlet and the outlet plenum. At the outlet plenum, the two streams merge, mix and exit at temperature T_o .

Conditions and Assumptions

In the active core region, the following conditions and assumptions apply:

1. The thermal power and neutron flux vary sinusoidally in the vertical direction.
2. All the power is generated in the active core region.
3. The time and spatial dependence of the power and neutron flux are separable.

4. A fraction, F_f , of the total power is generated in the fuel plates, a fraction, F_c , in the coolant and a fraction, F_m , in the moderator.
5. There is no heat transfer from the fuel plates to the moderator or from the coolant to the moderator.
6. All heat transfer between the fuel plates and the coolant occurs by the process of forced convection or conduction across the coolant film in the direction normal to the fuel plate surfaces.
7. The specific heats and densities of the materials are constant over the temperature and pressure ranges considered.
8. All flow rates are constant and slug or piston flow is assumed through the reactor.
9. The heat transfer coefficient is constant over the range of temperatures considered.
10. The temperature coefficients of reactivity are constant.
11. Reactivities are calculable from the neutron flux-squared weighted averages of the coolant and moderator temperature changes.
12. Thermal energy is accumulated or lost in the inlet and outlet plenums only by the mechanism of physical transport of such energy by fluid motion.
13. Power and temperature distributions are uniform in the plane normal to the reactor vertical axis.

This completes the discussion of the physical model on which the mathematical model of the reactor core is based.

The Model Equations

The linear differential equations describing the nuclear and thermal processes occurring in the reactor core are presented in this section without proof. Derivations and proofs are given in detail in the Appendix. The subscript notation and the symbols used are defined in Tables I and II. The model equations represent the changes or perturbations about the steady state values of the system variables (reactor power, temperatures and reactivity).

Power production. Assuming a small reactivity disturbance, ρ_x , the resulting change in total power, p , is described by the seven equations:

$$\left. \begin{aligned} \dot{p}(t) &= \frac{\bar{\rho}}{\ell} [\rho_x(t) + \rho_r(t)] - \frac{\beta}{\ell} p(t) + \sum_{j=1}^6 \lambda_j \psi_j(t), \\ \dot{\psi}_j(t) &= \frac{\beta_j}{\ell} p(t) - \lambda_j \psi_j(t) ; \quad j=1, 2, \dots, 6. \end{aligned} \right\} \quad (1)$$

Specific power. The change in specific power or power per unit length of the reactor is given by

$$q(z, t) = \frac{\pi}{2L} p(t) \sin \pi z. \quad (2)$$

In contrast to the total power, p , which is a function of time only, the specific power is a function of both time and position. Although the total power is the quantity of primary interest, it is the specific power which must be used for heat balance calculations in a distributed system such as the EOCR core.

Reactivity due to temperature changes. Localized changes in temperature of the coolant and moderator produce localized reactivity effects through their respective temperature coefficients. To evaluate the resulting effect on total reactor power, the localized reactivity effects must be averaged over the entire reactor core. For the whole reactor, the reactivity due to temperature changes is given by

$$\rho_T(t) = \frac{\int_0^1 [R_c \theta_c(z, t) + R_m \theta_m(z, t)] \sin^2 \pi z \, dz}{\int_0^1 \sin^2 \pi z \, dz}. \quad (3)$$

Heat balance in the fuel plates. The fuel plates acquire heat by internal generation and lose it by heat transfer to the coolant. Balancing rates of losses and gains results in

$$M_f C_f \frac{\partial \theta_f(z,t)}{\partial t} = -U A [\theta_f(z,t) - \theta_c(z,t)] + L F_f g(z,t) . \quad (4)$$

Heat balance in the coolant. The coolant receives heat by heat transfer from the fuel plates. There is also internal heat generation in the coolant. The net rate of energy accumulation by the coolant is given by

$$M_c C \frac{\partial \theta_c(z,t)}{\partial t} + W_c C \frac{\partial \theta_c(z,t)}{\partial z} = U A [\theta_f(z,t) - \theta_c(z,t)] + L F_c g(z,t) . \quad (5)$$

The second term on the left represents the rate at which heat is transported out of the reactor by the coolant motion.

Heat balance in the moderator. The moderator acquires heat solely by internal generation and loses it by physical transport of the heated fluid out of the reactor. The moderator heat balance equation is simply

$$M_m C \frac{\partial \theta_m(z,t)}{\partial t} + W_m C \frac{\partial \theta_m(z,t)}{\partial z} = L F_m g(z,t) . \quad (6)$$

Heat balance in the inlet plenum. Assuming perfect mixing and neither internal generation nor heat transfer, heat balance in the inlet plenum is attained if

$$M_i C \dot{\theta}_i(t) = W_i C [\theta_{in}(t) - \theta_i(t)] \quad (7)$$

Heat balance in the outlet plenum. Heat is carried into the outlet plenum by the coolant and the moderator. Heat is removed by the exiting fluid. The rate at which the coolant carries in heat depends on its flow rate and on the temperature at which it left the active core region. A similar statement applies to the moderator. As in the case of the inlet plenum, there is no heat transfer or internal generation. Adding the assumption of perfect mixing, the heat balance equation for the outlet plenum is

$$\begin{aligned} M_o C \dot{\theta}_o(t) = & W_c C \theta_{c2}(t - \tau_3) + W_m C \theta_{m2}(t - \tau_4) \\ & - W_o C \theta_o(t) \end{aligned} \quad (8)$$

Conservation of power and flow. The power generated in the reactor core is distributed among the fuel, coolant and moderator in different but constant proportions. Since all the power goes into these materials, the sum of the fractions of the total power generated in each of the three mediums must be unity. Mathematically this means that

$$F_f + F_c + F_m = 1 \quad (9)$$

There are no sources or sinks of fluid in the reactor. This means that the flow rate out of the reactor must be the same as the flow rate into the reactor. Separation of the flow into separate streams while in the core does not alter that fact. Another way of making the same statement is by the equation

$$W_2 = W_0 = W_c + W_m . \quad (10)$$

Initial and boundary conditions. The variables in Equations (1) through (8) are perturbations of power and temperatures from the steady state. As a consequence of being perturbations, their initial conditions are zero. The mathematical representation of the reactor is completed with the specification of the boundary conditions:

$$\left. \begin{array}{ll} \theta_c(0,t) = \theta_{c1}(t) , & (a) \\ \theta_m(0,t) = \theta_{m1}(t) , & \\ \theta_c(1,t) = \theta_{c2}(t) , & (b) \\ \theta_m(1,t) = \theta_{m2}(t) , & \\ \theta_{c1}(t) = \theta_{m1}(t) = \theta_2(t) . & (c) \end{array} \right\} \quad (11)$$

Equations (11-a) and (11-b) simply define the inlet and outlet values of the coolant and moderator temperature with respect to the active core region. The last equation states that fluid enters the coolant and moderator channels at the same temperature that it leaves the inlet plenum. Equations (1) through (11) are the model equations which completely describe the EOCR core within the framework of the stated assumptions. All subsequent equations and results are obtained either directly or indirectly from the model equations.

III. POWER, TEMPERATURE AND REACTIVITY TRANSFER FUNCTIONS

The transfer function representation of the model equations makes the desired information more accessible and amenable to analysis than the differential equation representation given above. The transfer functions presented in this section are derived in the Appendix. Definitions of new symbols may be found in Table II.

Definition of the Transfer Function

The transfer function of any system is a mathematical function relating the response or output of the system to its input (2). The transfer function may also be defined as the Laplace transform of the system output resulting from the application of a unit impulse disturbance. The transfer function is a function of a complex variable. The location of its poles and zeros on the complex plane determine the nature of the system response. Transfer functions are obtained from linear differential equations by taking the Laplace transform of each term in each equation. The transformed equations may be treated as a set of linear algebraic equations and solved for any of the system variables or transfer functions. The description of a system by its transfer function makes it possible to study the system dynamic behavior by investigation of the properties of the transfer function on the complex plane.

The reactor core transfer functions presented below are introduced in the order of their derivation rather than in the order of the model equations. Two independent inputs have been assumed: an arbitrary reactivity, ρ_x , and an arbitrary reactor inlet temperature change, θ_{in} .

Power Transfer Function

The reactor power is only directly affected by changes in the effective reactivity. The transfer function relating power changes to the effective reactivity is

$$\frac{p(s)}{\rho_e(s)} = \bar{P} G_R(s) , \quad (12)$$

where

$$G_R(s) = \left\{ l s \left[1 + \sum_{i=1}^6 \frac{\beta_i / l}{s + \lambda_i} \right] \right\}^{-1} , \quad (13)$$

and

$$\rho_e(s) = \rho_T(s) + \rho_x(s) . \quad (14)$$

As a matter of convenience the same symbol has been used to indicate the transformed variables. The symbol $p(s)$ represents the Laplace transform of $p(t)$ and must not be interpreted as the value of $p(t)$ evaluated at $t = s$.

Coolant Temperature Transform

Coolant temperature changes may result from power changes or from changes in the temperature at the coolant channel inlet. The coolant temperature at any point in the active core region is given by

$$\theta_c(z, s) = e^{-\alpha z} \theta_{c1}(s) + \frac{a_c}{\frac{\alpha^2}{\pi^2} + 1} H_c(z, s) p(s) , \quad (15)$$

where

$$H_c(z, s) = \alpha \sin \pi z - \pi \cos \pi z + \pi e^{-\alpha z} . \quad (16)$$

These two equations may be evaluated at the coolant channel outlet ($z = 1$) resulting in

$$\theta_{c2}(s) = e^{-\alpha} \theta_{c1}(s) + \pi a_c \left[\frac{1 + e^{-\alpha}}{\frac{\alpha^2}{\pi^2} + 1} \right] p(s). \quad (17)$$

Equation (15) is needed for calculating the reactivity transform.

Equation (17) is required for calculating the outlet plenum temperature transform.

Moderator Temperature Transform

Statements similar to those made for the coolant may be made for the moderator. At any point in the active core region, the transform of the moderator temperature change is

$$\theta_m(\gamma, s) = e^{-\tau_m s \gamma} \theta_{m1}(s) + \frac{a_m}{\frac{\tau_m^2}{\pi^2} s^2 + 1} H_m(\gamma, s) p(s), \quad (18)$$

where

$$H_m(\gamma, s) = \tau_m s \sin \pi \gamma - \pi \cos \pi \gamma + \pi e^{-\tau_m s \gamma}, \quad (19)$$

and

$$\theta_{m2}(s) = e^{-\tau_m s} \theta_{m1}(s) + \pi a_m \left[\frac{1 + e^{-\tau_m s}}{\frac{\tau_m^2}{\pi^2} s^2 + 1} \right] p(s). \quad (20)$$

Reactivity Transform

The reactivity perturbations due to coolant and moderator temperature changes are more fundamentally related to power changes and to the changes of coolant and moderator temperature at the

entrance to the active core region. The Laplace transform of this reactivity is

$$\rho_T(s) = G_c(s) \theta_{c_i}(s) + G_m(s) \theta_{m_i}(s) + G_p(s) p(s) . \quad (21)$$

Applying the Laplace transformation to the boundary conditions of Equation (11-c) yields

$$\theta_{c_i}(s) = \theta_{m_i}(s) = \theta_i(s) . \quad (22)$$

Combining the last two equations results in

$$\rho_T(s) = [G_c(s) + G_m(s)] \theta_i(s) + G_p(s) p(s) . \quad (23)$$

This last equation relates the reactivity, ρ_T , to power changes and to inlet plenum temperature changes. This reactivity may further be related to the reactor inlet temperature changes θ_{in} through the inlet plenum transfer function which is

$$\frac{\theta_i(s)}{\theta_{in}(s)} = \frac{1}{\tau_i s + 1} . \quad (24)$$

Combining Equations (23) and (24) yields

$$\rho_T(s) = G_T(s) \theta_{in}(s) + G_p(s) p(s) , \quad (25)$$

where

$$G_T(s) = \frac{G_c(s) + G_m(s)}{\tau_i s + 1} . \quad (26)$$

Outlet Plenum Temperature Transform

Outlet plenum temperature changes result primarily from power changes and from reactor inlet temperature changes. The Laplace transform of the outlet temperature change is

$$\theta_o(s) = H_T(s) \theta_{in}(s) + H_p(s) p(s) , \quad (27)$$

where

$$H_T(s) = \frac{B_3 e^{-(\tau_3 s + \alpha)} + B_4 e^{-(\tau_4 + \tau_m)s}}{(\tau_3 s + 1)(\tau_o s + 1)} , \quad (28)$$

and

$$\begin{aligned} H_p(s) = & \pi a_c B_c e^{-\tau_3 s} \left[\frac{1 + \bar{e}^{-\alpha}}{\frac{\alpha^2}{\pi^2} + 1} \right] (\tau_o s + 1)^{-1} \\ & + \pi a_m B_m e^{-\tau_4 s} \left[\frac{1 + \bar{e}^{-\tau_m s}}{\frac{\tau_m^2}{\pi^2} s^2 + 1} \right] (\tau_o s + 1)^{-1} . \end{aligned} \quad (29)$$

Discussion of Transforms and Transfer Functions

The transforms of all the system variables have been presented above for the sake of completeness. Of the above set of transformed equations, four may be regarded as the key system equations because these four completely describe the reactor core. The key system equations are

$$\frac{p(s)}{p_e(s)} = \bar{P} G_R(s) , \quad (12)$$

$$\rho_e(s) = \rho_T(s) + \rho_x(s) \quad , \quad (14)$$

$$\rho_T(s) = G_T(s) \theta_{in}(s) + G_P(s) p(s) \quad , \quad (15)$$

and

$$\theta_o(s) = H_T(s) \theta_{in}(s) + H_P(s) p(s) \quad . \quad (16)$$

The reactor open-loop transfer function may be derived from these equations but the effects of the various physical processes occurring in the reactor core are more easily discussed with the aid of a block diagram. The block diagram of the reactor core suggested by the above four equations is shown in Figure 3. Examination of this figure confirms that it is a pictorial representation of the key equations. The utility of the block diagram is due to the ease with which the various functions appearing in the key equations may be identified and their effect on the reactor variables understood.

Considering constant reactor inlet temperature ($\theta_{in} = 0$), it can be seen with the aid of Figure 3 that the only contribution to the reactivity, ρ_T , is due to changes in reactor power, p . Power changes affect ρ_T and consequently the effective reactivity, ρ_e , through the function G_P . Thus it is seen that the function, G_P , is a reactivity feedback transfer function. The function G_T is also a reactivity transfer function but it is not a feedback transfer function unless the reactor inlet temperature, θ_{in} , is related to the outlet temperature, θ_o . By similar reasoning it is seen that the functions H_T and H_P are not feedback transfer functions. By excluding the heat exchanger from this analysis, the temperature θ_{in} is made independent of the temperature

θ_o and the above statements are valid. It has thus been shown that without the heat exchanger only one feedback loop exists and the feedback transfer function is G_P .

The transfer function H_P relates the reactor outlet temperature changes to reactor power changes. The transfer function H_T relates the reactor outlet temperature changes to reactor inlet temperature changes.

Open-loop Reactor Transfer Function

As discussed above, a study of Figure 3 revealed only one feedback loop. The stability of a feedback system such as this depends only upon the transfer functions in the closed loop. In this case the transfer functions are $\bar{P} G_R$ and G_P . The other transfer functions G_T , H_T and H_P are not feedback functions and, hence, have no affect on the reactor stability. Having identified the feedback loop of the reactor and the associated transfer functions, it remains to obtain the open-loop reactor transfer function from which the stability of the reactor can be ascertained. Application of elementary control system theory to the analysis of the $\bar{P} G_R - G_P$ closed loop reveals that the open-loop reactor transfer function is

$$Y_o(s) = -\bar{P} G_R(s) G_P(s) \quad (30)$$

The negative sign in this equation falsely indicates a regenerative or unstable system. The negative sign has resulted from not having specified the signs of the temperature coefficients of reactivity earlier in the analysis. Examination of the transfer function G_P reveals that it is inherently a negative function if the temperature

coefficients are negative which is true for the EOCR. Thus the negative value of G_p cancels the negative sign of the open-loop transfer function. The fact that Y_o is basically a negative function does not assure a stable system. The complexity of Y_o prohibits the determination of stability by inspection. In this case, stability must be determined by the techniques discussed in the next section.

IV. STABILITY ANALYSIS

The inherent stability margin of the EOCR has been investigated by application of the Nyquist stability criterion in its most general form. The brief discussion of the criterion given below is adequate for the purposes of this analysis. A more detailed discussion of the theory and applications of this criterion is given by Chestnut and Mayer (3).

General Discussion of the Nyquist Stability Criterion

The Nyquist stability criterion essentially predicts the stability of a closed-loop or feed-back system from the behavior of the system open-loop transfer function in the complex plane. In applying this criterion, the open-loop transfer function, $Y(s)$, is considered as a mapping function which maps a contour or region in the S - complex plane into a contour or region in the Y - complex plane. It can be shown that, if the mapping function satisfies certain conditions, the entire right half of the S - plane may be mapped simply by mapping the entire imaginary axis of the S - plane. The right half of the S - plane is chosen because a linear feedback system is stable only if the closed-loop transfer function has no poles in the right half of the S - plane. Thus, mapping this region of the S - plane constitutes a search for poles of the closed-loop transfer function in this region. The Nyquist stability criterion allows the open-loop transfer function, which is invariably a simpler function, to be used for this search rather than the more cumbersome closed-loop transfer function. The mapping of the

imaginary axis from minus to plus infinity is accomplished by plotting $Y(S)$ as S varies over this range.

The general Nyquist stability criterion states that, if the system is to be stable, the net number or counter-clockwise encirclements of the $(-1 + j0)$ point by the locus of Y as S traverses the imaginary axis from minus to plus infinity must be equal to the number of poles of Y located in the right half of the S - plane.

The equation for the imaginary axis of the S - plane is simply

$$S = j\omega .$$

Chestnut and Mayer (11) have shown that, when S is the Laplace transform variable associated with the real variable, t , the symbol ω represents the angular frequency in radians per second of a sinusoidal forcing function. Moreover, the function $Y(j\omega)$ is the frequency response of the open-loop system. Therefore, the first step in applying the Nyquist stability criterion is to plot on the complex plane the open-loop frequency response for all values of ω , both positive and negative.

Poles of $Y(S)$ at the origin or on the imaginary axis result in an open-ended locus of $Y(j\omega)$. In this case the open ends of the locus must be connected to form a closed curve on the Y plane so that the encirclements of the $(-1+j0)$ point by the $Y(j\omega)$ locus can be counted. The Nyquist stability criterion as stated above requires a knowledge of the number of poles of $Y(S)$ located in the right half of the S - plane. The location of the poles of Y is usually determined by visual inspection of Y or by Routh's criterion (12) if the

denominator of Y is a high order polynomial.

Open-loop Frequency Response

The EOCR open-loop transfer function has been shown to be

$$Y_o(s) = -\bar{P}G_R(s)G_P(s) \quad (30)$$

The open-loop frequency response is obtained by plotting $Y_o(j\omega)$ as ω varies from zero to infinity. This plot yields half of the Y_o curve required for application of the stability criterion. The other half of the curve $Y_o(-j\omega)$ is obtained by reflection of $Y_o(j\omega)$ about the real axis of the complex plane. The complete open-loop frequency response is shown in Figure 4. This is the polar or Nyquist diagram of the function $Y_o(j\omega)$ for all values of ω , both positive and negative. The solid curve is $Y_o(j\omega)$. The short-dash curve is $Y_o(-j\omega)$. Frequency values are indicated at various points along the $Y_o(j\omega)$ curve. The amplitude or absolute value of $Y_o(j\omega)$ is plotted radially in decibels (db) as a matter of convenience. Use of a decibel scale converts the $(-1+j0)$ point to zero db amplitude at 180 degrees phase angle.

The numerical values of the steady state variables and associated constants are listed in Table III. The operating condition of the EOCR considered for this analysis is summarized by the following steady state variables:

1. Total power - 40MW.
2. Inlet reactor temperature - 500°F.
3. Outlet reactor temperature - 523°F.
4. Total flow through the core - 25,000 gpm.
5. Coolant flow rate - 23,650 gpm.
6. Moderator flow rate - 1,350 gpm.

The numerical values of the amplitude and phase angle of $Y_o(j\omega)$ were obtained by digital computation using the IBM-650 digital computer.

Determination of Stability

Examination of Figure 4 reveals that the open-loop frequency response is an open-ended curve on the complex plane. The amplitude approaches infinity at -90 degrees phase angle as the frequency approaches zero. This is due to the pole at the origin of the transfer function $G_R(s)$. The feedback transfer function $G_P(s)$ also appears to have a pole at the origin. Such is not the case. In fact, it is easily shown that

$$\lim_{s \rightarrow 0} G_P(s) = \frac{R_c(F_c + F_f)}{2W_c C} + \frac{R_m F_m}{2W_m C}$$

A cursory inspection of G_P also indicates the presence of four poles on the imaginary axis at the points:

$$s = \pm j\pi/\tau_m,$$

and

$$s = \pm j2\pi/\tau_m.$$

These poles are canceled by zeroes of G_P located precisely at the same points. It can be shown that

$$\lim_{s \rightarrow \pm j\pi/\tau_m} G_P(s) = \text{complex constants.}$$

and

$$\lim_{S \rightarrow \pm j2\pi/\tau_m} G_p(S) = \text{complex constants.}$$

Therefore, the open-endedness of the Y_o locus is due solely to the pole at the origin of the power transfer function $G_R(S)$. The open ends of the $Y_o(j\omega)$ and $Y_o(-j\omega)$ plots are connected simply by a clockwise circular arc of infinite radius starting at the open end of the $Y_o(-j\omega)$ and terminating at the open end of the $Y_o(j\omega)$ curve (3). The connecting arc is shown by the long-dash curve in Figure 4. The $Y_o(j\omega)$ and $Y_o(-j\omega)$ curves together with the connecting circular arc make up a closed curve on the Y_o - plane. The point at zero db and 180 degrees phase is not encircled by the closed curve, therefore the EOOR is inherently stable if the open-loop transfer function $Y_o(S)$ has no poles in the right half of the S - plane. The transfer function $G_R(S)$ is known to have no poles in the right half of the S - plane (10). The function G_p , however, will have poles in the right half of the S - plane if the factors

$$X(S) = \frac{\alpha^2(S)}{\pi^2} + 1$$

and

$$Z(S) = \frac{\alpha^2(S)}{4\pi^2} + 1$$

have zeroes in the right half-plane. From Table II, the function $\alpha^2(S)$ is seen to be a fourth order polynomial in S . Therefore the zeroes of X and Z are equal to the roots of the equations:

$$X(s)=0=\tau_c^2\tau_f^2s^4+2\tau_c\tau_f\tau s^3+(\tau^2+\pi^2\tau_f^2)s^2+2\pi^2\tau_f s+\pi^2, \quad (31)$$

and

$$Z(s)=0=\tau_c^2\tau_f^2s^4+2\tau_c\tau_f\tau s^3+(\tau^2+4\pi^2\tau_f^2)s^2+8\pi^2\tau_f s+4\pi^2, \quad (32)$$

where

$$\tau = \tau_c + \tau_f.$$

Application of Routh's criterion to Equations (31) and (32) reveals that neither X nor Z have zeroes in the right half-plane. Therefore, the transfer function G_p has no poles in the right half-plane. It then follows that the open-loop reactor transfer function has no poles in the right half-plane.

At very large values of S (high frequencies) the open-loop transfer function $Y_o(S)$ can be approximated by

$$Y_o(s) \simeq \frac{4\bar{P}}{3\ell C} \left[\frac{|R_c|F_c}{M_c} + \frac{|R_m|F_m}{M_m} \right] \frac{1}{s^2},$$

or

$$Y_o(j\omega) \simeq -\frac{4\bar{P}}{3\ell C} \left[\frac{|R_c|F_c}{M_c} + \frac{|R_m|F_m}{M_m} \right] \frac{1}{\omega^2}.$$

This shows that as ω approaches infinity, the open-loop amplitude approaches zero along the negative real axis. This axis is approached from the third quadrant of the complex plane and is never crossed by

the Y_0 locus.

The conclusion may now be drawn, based on the entire results of this section, that the EOCR inherent stability is absolute. The results predict an infinite gain margin and a phase margin of approximately 120 degrees. This high degree of stability is not surprising since reactors with large negative temperature coefficients of reactivity are known to be inherently stable devices. The infinite gain margin may, in reality, be only extremely large. Additional phase shift at frequencies above 40 radians per second would force the Y_0 locus to cross the negative real axis, thereby resulting in a finite but very large gain margin. A more sophisticated model considering heat diffusion in both the vertical and transverse directions, more realistic flow profiles in the coolant and moderator channels and considering the fuel cladding as a separate heat transfer medium, might yield additional high frequency phase shift. The analysis of such a model would, of course, be extremely more difficult than the present analysis. The results of the present analysis, although, admittedly based on many simplifying assumptions, are realistic and of sufficient accuracy to permit predictions of reactor performance.

Examination of the form of the open-loop transfer function reveals that the gain is directly proportional to the steady state value of the power. A decrease in the power results mainly in a decrease of phase margin. The gain margin tends to increase but it is already so large that this effect is of secondary importance.

Closed-loop Frequency Response

The closed-loop reactor transfer function may be readily obtained by inspection from the block diagram in Figure 3 assuming constant inlet temperature. This transfer function relates reactor power changes to the reactivity forcing function, ρ_x . If the power changes are normalized to the steady state power, $\bar{P}$, the result is

$$Y_c(s) = \frac{\rho(s)}{\bar{P} \rho_x(s)} = \frac{G_R(s)}{1 - \bar{P} G_R(s) G_p(s)}$$

or

$$Y_c(s) = \frac{G_R(s)}{1 + Y_o(s)}$$

This transfer function is of interest because it more clearly shows the effect on the closed-loop system of the temperature feedback loop. The closed-loop frequency response of the EOCR is shown in Figure 5. On this figure is also shown the frequency response of the reactor assuming no temperature feedback or zero temperature coefficients of reactivity. This response is calculated from the function G_R . The transfer function G_R has a pole at the origin. This causes the amplitude of its frequency response to become infinite as the frequency approaches zero. The temperature feedback removes this pole at the origin thereby resulting in a stable reactor having finite low frequency gain. The amplitude and phase of G_R are mostly affected by the feedback loop at low frequencies.

Above 10 radians per second the effect of the G_p loop vanishes. The transient response of the reactor following a reactivity disturbance, while stable, would be sluggish, requiring long periods of time to achieve a new steady state condition. The sluggishness of the response can be directly attributed to the extremely large gain and phase margins.

It is easily shown that the zero frequency closed-loop gain is given by

$$Y_c(0) = -1/\bar{P}G_p(0) \quad .$$

This equation shows that at lower power the zero frequency gain of the reactor is larger than at higher power levels. This result agrees with intuition. At sufficiently low power the temperature feedback becomes ineffective and the closed-loop transfer function, Y_c , reduces to G_R .

V. SUMMARY AND CONCLUSIONS

The inherent stability of the EOCR core at full power, exclusive of the heat exchanger has been investigated. In pursuing this objective, mathematical models of the nuclear and thermal processes occurring in the reactor were derived. Transfer functions characteristic of the reactor core dynamics were derived from the model equations. The stability of the reactor was analyzed by application of the Nyquist stability criterion to the open-loop reactor transfer function. Transfer functions and a block diagram of the reactor core, not necessary for this analysis but necessary for more comprehensive studies considering the heat exchanger, were derived.

The results of this analysis show that the EOCR has an extremely large margin of inherent stability. Attendant with the large margin of stability is sluggish dynamic performance of the temperature feedback loop. At lower power the reactor remains stable but the response approaches that of a reactor with no temperature feedback. The results of this analysis do not apply at other flow rates or reactor inlet temperatures.

TABLE I

EXPLANATION OF NOTATION

1. A dot over a variable represents the total derivative of that variable with respect to time.
2. A bar over a variable represents the steady state value of that variable. Barred quantities may, in general, be functions of the position coordinate.
3. Subscripts on symbols carry the following connotation:
 - c refers to fluid in the coolant channel.
 - i refers to fluid in the inlet plenum.
 - f refers to the fuel plates.
 - m refers to fluid in the moderator channel.
 - o refers to fluid in the outlet plenum.
 - 1 refers to the inlet value.
 - 2 refers to the outlet value.
4. Subscripts used in the text and not listed above pertain to lumped constants or auxiliary functions and are given in Table II.

TABLE II
DEFINITIONS OF SYMBOLS

Variables

n	= Laplace transform variable associated with z .
P	= total reactor power.
p	= change in P .
Q	= specific power or power per unit length.
q	= change in Q .
S	= Laplace transform variable associated with t .
T	= temperature.
T_{in}	= reactor inlet temperature.
t	= time.
y	= position measured with respect to fuel plate surface.
z	= y/L = non-dimensional position coordinate.
θ	= change in T .
θ_{in}	= change in T_{in} .
ρ_e	= effective reactivity.
ρ_T	= reactivity due to temperature changes.
ρ_x	= reactivity forcing function.
$\bar{\Psi}_j$	= variable proportional to the concentration of delayed neutron emitters of type j .
ψ_j	= change in $\bar{\Psi}_j$.
ω	= angular frequency of sinusoidal forcing function.

Basic Constants

A	= total heat transfer surface area.
C	= specific heat.
F	= fraction of total power generated in the medium.

TABLE II (continued)

L	= vertical length of fuel plates.
ℓ	= prompt neutron lifetime.
M	= mass.
R	= temperature coefficient of reactivity.
U	= heat transfer coefficient.
W	= mass flow rate.
β	= total effective delayed neutron fraction.
β_j	= effective delayed neutron fraction of group j .
λ_j	= decay constant of delayed neutron emitters of group j .
τ_3	= transport lag from coolant channel outlet to outlet plenum.
τ_4	= transport lag from moderator channel outlet to outlet plenum.

Lumped Constants

a_m	= $K_m/2\pi L$
B_c	= W_c/W_o
B_m	= W_m/W_o
K	= $UA/W_c C$
K_c	= $LF_c/W_c C$
K_f	= LF_f/UA
K_m	= $LF_m/W_m C$
τ_c	= M_c/W_c
τ_f	= $M_f C_f/UA$
τ_i	= M_i/W_i
τ_m	= M_m/W_m
τ_o	= M_o/W_o

TABLE II (continued)

Auxiliary Functions

$$a_c = [K_c g_f + K K_f] / 2\pi L g_f$$

$$g_c = \tau_c s + K$$

$$g_f = \tau_f s + 1$$

$$G_c = \frac{R_c (1 - e^{-\alpha})}{\frac{\alpha}{4} \left(\frac{\alpha^2}{\pi^2} + 4 \right)}$$

$$G_m = \frac{R_m (1 - e^{-\tau_m s})}{\tau_m s \left(\frac{\tau_m^2}{4\pi^2} s^2 + 1 \right)}$$

$$G_p = \frac{R_c a_c}{\frac{\alpha^2}{\pi^2} + 1} \left[\frac{8\alpha}{3\pi} + \frac{\pi (1 - e^{-\alpha})}{\alpha \left(\frac{\alpha^2}{4\pi^2} + 1 \right)} \right] \\ + \frac{R_m a_m}{\frac{\tau_m^2}{\pi^2} s^2 + 1} \left[\frac{8\tau_m s}{3\pi} + \frac{\pi (1 - e^{-\tau_m s})}{\tau_m s \left(\frac{\tau_m^2}{4\pi^2} s^2 + 1 \right)} \right]$$

$$G_R = \left[\lambda s \left(1 + \sum_{j=1}^6 \frac{\beta_j / \lambda}{s + \lambda_j} \right) \right]^{-1}$$

$$G_T = (G_c + G_m) / (\tau_c s + 1)$$

$$H_p = \pi a_c B_c e^{-\tau_c s} \left(\frac{1 + e^{-\alpha}}{\frac{\alpha^2}{\pi^2} + 1} \right) / (\tau_c s + 1)$$

$$+ \pi a_m B_m e^{-\tau_m s} \left(\frac{1 + e^{-\tau_m s}}{\frac{\tau_m^2}{\pi^2} s^2 + 1} \right) / (\tau_c s + 1)$$

TABLE II (continued)

$$H_T = \frac{B_c e^{-(\alpha + \tau_c s)} + B_m e^{-(\tau_m + \tau_4) s}}{(\tau_i s + 1)(\tau_o s + 1)}$$

$$Y_o = -\bar{P} G_R G_P$$

$$\alpha = (g_f g_c - k) / g_f = \frac{(\tau_c + \tau_f) s}{\tau_f s + 1} \left(\frac{\tau_c \tau_f}{\tau_c + \tau_f} s + 1 \right)$$

TABLE III
NUMERICAL VALUES OF CONSTANTS

Steady State Variables

$$\bar{P} = 40 \text{ MW} = 3.792 \times 10^4 \text{ BTU/sec}$$

$$\bar{T}_{\text{in}} = 500.0^\circ\text{F}$$

$$\bar{T}_O = 523.0^\circ\text{F}$$

Basic Constants

$$A = 2.146 \times 10^3 \text{ ft}^2$$

$$C = 5.073^{-1} \text{ BTU/lb/}^\circ\text{F}$$

$$C_f = 1.1 \times 10^{-1} \text{ BTU/lb/}^\circ\text{F}$$

$$F_c = 5.0 \times 10^{-2}$$

$$F_f = 9.3 \times 10^{-1}$$

$$F_m = 2.0 \times 10^{-2}$$

$$L = 3.0 \text{ ft}$$

$$l = 1.56 \times 10^{-5} \text{ sec}$$

$$M_c = 6.67 \times 10^2 \text{ lb}$$

$$M_f = 1.364 \times 10^3 \text{ lb}$$

$$M_i = 3.92 \times 10^4 \text{ lb}$$

$$M_m = 2.840 \times 10^3 \text{ lb}$$

$$M_o = 2.37 \times 10^4 \text{ lb}$$

$$R_c = -1.12 \times 10^{-4} \text{ reactivity units /}^\circ\text{F}$$

$$R_m = -1.0 \times 10^{-5} \text{ reactivity units /}^\circ\text{F}$$

$$U = 2.314 \times 10^{-3} \text{ BTU/ft}^2\text{/sec/}^\circ\text{F}$$

$$W_c = 3.093 \times 10^3 \text{ lb/sec}$$

$$W_i = 3.269 \times 10^3 \text{ lb/sec} = 2.5 \times 10^4 \text{ gpm}$$

$$W_m = 1.76 \times 10^2 \text{ lb/sec}$$

TABLE III (continued)

$$W_0 = 3.269 \times 10^3 \text{ lb/sec}$$

$$\beta = 6.738 \times 10^{-3}$$

$$\beta_1 = 2.24 \times 10^{-4}$$

$$\beta_2 = 1.477 \times 10^{-3}$$

$$\beta_3 = 1.326 \times 10^{-3}$$

$$\beta_4 = 2.651 \times 10^{-3}$$

$$\beta_5 = 7.78 \times 10^{-4}$$

$$\beta_6 = 2.82 \times 10^{-4}$$

$$\lambda_1 = 1.244 \times 10^{-2} \text{ sec}^{-1}$$

$$\lambda_2 = 3.051 \times 10^{-2} \text{ sec}^{-1}$$

$$\lambda_3 = 1.114 \times 10^{-1} \text{ sec}^{-1}$$

$$\lambda_4 = 3.014 \times 10^{-1} \text{ sec}^{-1}$$

$$\lambda_5 = 1.136 \text{ sec}^{-1}$$

$$\lambda_6 = 3.014 \text{ sec}^{-1}$$

$$\tau_3 = 2.1 \times 10^{-1} \text{ sec}$$

$$\tau_4 = 4.12 \times 10^1 \text{ sec}$$

FIG. 1

EOCR PLANT DIAGRAM

(a)

FUEL PLATE ARRANGEMENT
 IN FUEL ELEMENT

(b)

FUEL ELEMENT
 CROSS-SECTION

(c)

FIG. 2
 REACTOR THERMAL MODEL

FIG. 3

BASIC BLOCK DIAGRAM OF THE REACTOR CORE

FIG. 4
EOCR NYQUIST DIAGRAM

FIG.- 5
EOCC CLOSED-LOOP FREQUENCY RESPONSE

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APPENDIX

Derivation of model equations. The differential equations describing the time behavior of the EOCR are derived in this appendix. The notation is explained in Table I and symbols are defined in Table II. The equations are derived in the order used in section II to allow easier cross-reference between that section and this one. Following the derivation of the equation or equations describing a process, linearization of same will be performed where required.

Power production. The power production equations for a nuclear reactor are well known and their derivation may be found in any standard text on reactor theory having a section on the time behavior of nuclear reactors. The seven equations used in this analysis are due to Schultz (4) and are of the form:

$$\left. \begin{aligned} \dot{P} &= \frac{\rho_e - \beta}{\lambda} P + \sum_{j=1}^6 \lambda_j \Psi_j, \\ \dot{\Psi}_j &= \frac{\beta_j}{\lambda} P - \lambda_j \Psi_j \quad j = 1, 2, \dots, 6. \end{aligned} \right\} \quad (A-1)$$

The effective reactivity, ρ_e , is defined as the relative change in the effective multiplication factor of the reactor. The effective multiplication factor can be changed by motion of control rods, temperature changes of the materials in the reactor and by other physical processes occurring in the reactor. For this analysis, only two methods of changing the effective multiplication are assumed. An arbitrary change can be produced by rod motion and will result in a reactivity ρ_x . Temperature changes produce a

reactivity ρ_T . These reactivities are additive and result in an effective reactivity given by

$$\rho_e = \rho_T + \rho_x \quad (A-2)$$

The reactivity ρ_x is considered as an independent variable or forcing function while ρ_T is a dependent variable depending on temperature changes of materials in the core. The temperature changes are in turn related to power and reactor inlet temperature changes.

The first equation in the set of Equations (A-1) describes the reactor power at anytime in response to variations in ρ_e . The other six equations are related to the production of delayed neutrons. The power equation is non-linear because of the term containing the product of the variables ρ_e and P . The subsequent techniques to be employed in the analysis are applicable only to linear differential equations. This requires linearization of the power equation.

Assuming that the reactor is in steady state operation at power $\bar{P}$, the equilibrium is disturbed by a change in reactivity. The power at any instant after the disturbance is given by

$$P(t) = \bar{P} + p(t) \quad (A-3)$$

Similarly the other variables are given by

$$\bar{\Psi}_j(t) = \bar{\Psi}_j + \psi_j(t) \quad (A-4)$$

The variables p and ψ represent perturbations from the steady state of the respective variables P and $\bar{\psi}$. Combining Equations (A-1), (A-3) and (A-4) results in

$$\dot{p} = \frac{\bar{P}}{\ell} \rho_e - \frac{\beta}{\ell} p + \sum_{j=1}^6 \lambda_j \psi_j + \left[\sum_{j=1}^6 \lambda_j \bar{\psi}_j - \frac{\beta}{\ell} \bar{P} \right] + \frac{\rho_e}{\ell} p, \quad (A-5)$$

$$\dot{\psi}_j = \frac{\beta_j}{\ell} p - \lambda_j \psi_j - \left[\lambda_j \bar{\psi}_j - \frac{\beta_j}{\ell} \bar{P} \right] ; \quad j = 1, 2, \dots, 6. \quad (A-6)$$

The last term in Equation (A-5) is a second order term and may be neglected if both p and ρ_e are small. The terms in the brackets are steady state terms adding up to zero. In fact, it is these terms that define the steady state condition for the reactor power. Therefore, the linear reactor power perturbation equations are

$$\left. \begin{aligned} \dot{p} &= \frac{\bar{P}}{\ell} \rho_e - \frac{\beta}{\ell} p + \sum_{j=1}^6 \lambda_j \psi_j \\ \dot{\psi}_j &= \frac{\beta_j}{\ell} p - \lambda_j \psi_j \quad ; \quad j = 1, 2, \dots, 6. \end{aligned} \right\} \quad (A-7)$$

Specific power. The symbol P in Equation (A-1) represents the total reactor power as a function of time only. Since the power varies sinusoidally over the length of the reactor core, a more meaningful quantity for this analysis is the specific power or power per unit length. The specific power is defined by the equation

$$P(t) = \int_0^L Q(y, t) dy, \quad (A-8)$$

where Q is the specific power. In addition to being a defining equation, Equation (A-8) imposes the condition that all of the power be generated in the active core region. Since the power distribution has been assumed sinusoidal, Q must be of the form

$$Q(y, t) = N(t) \sin \pi y / L, \quad (A-9)$$

where N is a normalizing factor. Combining Equations (A-8) and (A-9) yields

$$P(t) = N(t) \int_0^L \sin \pi y / L \, dy = \frac{2L}{\pi} N(t). \quad (A-10)$$

The last equation defines the normalizing factor. Finally, the expression for specific power is

$$Q(y, t) = \frac{\pi}{2L} P(t) \sin \pi y / L. \quad (A-11)$$

Assuming steady state conditions at zero time,

$$\bar{Q} = Q(y, 0) = \frac{\pi}{2L} \bar{P} \sin \pi y / L. \quad (A-12)$$

Following a disturbance let Q be given by

$$Q(y, t) = \bar{Q} + q(y, t). \quad (A-13)$$

Combining Equations (A-3), (A-11) and (A-13) and imposing the steady state condition of Equation (A-12) results in the specific

power perturbation equation

$$\phi(y,t) = \frac{\pi}{2L} p(t) \sin \pi y/L \quad . \quad (A-14)$$

In terms of the non-dimensional coordinate $z = y/L$, Equation (A-14) may be written as

$$\phi(z,t) = \frac{\pi}{2L} p(t) \sin \pi z \quad . \quad (A-15)$$

Reactivity due to temperature changes. Murray (5) has shown that temperature changes in a reactor result in a reactivity proportional to the temperature change over a limited temperature range. The constant of proportionality is defined as the temperature coefficient of reactivity. The EOCR may be characterized by three such coefficients corresponding to coolant, moderator and fuel plate temperature changes. It has been shown that the temperature coefficient of the fuel plates is negligible (6); therefore, only those of the coolant and moderator need be considered. Since the coolant and moderator temperatures vary over the length of the reactor core, the overall reactivity effect due to temperature changes must be calculated from some spatial average of these changes over the length of the core. Murray (7) has shown that the correct average is obtained by use of the square of the neutron flux as a weighting function. Reducing these statements to mathematical form results in the expression

$$\rho_T(t) = \frac{\int_0^L [R_c \theta_c(y,t) + R_m \theta_m(y,t)] \phi^2(y,t) dy}{\int_0^L \phi^2(y,t) dy} \quad (A-16)$$

The constants R_c and R_m are, respectively, the coolant and moderator temperature coefficients of reactivity while θ_c and θ_m represent the changes in temperature of these fluids from the steady state. The neutron flux is represented by the symbol ϕ . Recalling the assumption that the time and spatial dependence of the flux are separable and that the flux varies sinusoidally along the length of the reactor, Equation (A-16) reduces to

$$\rho_T(t) = \frac{\int_0^1 [R_c \theta_c(z,t) + R_m \theta_m(z,t)] \sin^2 \pi z dz}{\int_0^1 \sin^2 \pi z dz} \quad , \quad (A-17)$$

where, as before, $z = y/L$.

Heat balance in the coolant, moderator and fuel plates. The heat balance equation for the coolant is more general in form than that of either the moderator or fuel plates and for that reason it is derived first. The coolant acquires energy by heat transfer from the fuel plates and by absorption of energy from neutrons and gamma radiation. The time rate of energy transfer from a metal surface to a moving fluid in contact with the surface is given by the product of the film coefficient of heat transfer, the area of the contact surface and the temperature difference across the film. This

thermal process is called forced convection or conduction across the fluid film. If E_c represents the thermal energy in the coolant, then, at any time t and position y , the net rate of energy accumulation per unit length is

$$\frac{1}{L} \frac{dE_c(y,t)}{dt} = \frac{UA}{L} [T_f(y,t) - T_c(y,t)] + F_c Q(y,t) , \quad (A-18)$$

where F_c is the fraction of the total power (due to neutrons and gamma radiation) generated in the coolant. Based on the assumptions made for this analysis, the heat capacity of the coolant is given by

$$M_c C_c = \frac{dE_c}{dT_c} . \quad (A-19)$$

In calculus it is shown that

$$\frac{dE_c}{dt} = \frac{dE_c}{dT_c} \frac{dT_c}{dt} = M_c C_c \frac{dT_c}{dt} . \quad (A-20)$$

Equations (A-18) and (A-20) combine to give

$$M_c C_c \frac{dT_c(y,t)}{dt} = UA [T_f(y,t) - T_c(y,t)] + L F_c Q(y,t) . \quad (A-21)$$

In advanced calculus, it is shown that

$$\frac{dT_c(y,t)}{dt} = \frac{\partial T_c(y,t)}{\partial t} + \frac{dy}{dt} \frac{\partial T_c(y,t)}{\partial y} . \quad (A-22)$$

If the position of a coolant particle is measured with respect to the stationary fuel plates, then the derivative dy/dt in Equation (A-22) is the coolant velocity, v_c , relative to the fuel plate surface. Combining the last two equations results in

$$M_c C_c \frac{\partial T_c(y,t)}{\partial t} + M_c C_c v_c \frac{\partial T_c(y,t)}{\partial y} = U A [T_f(y,t) - T_c(y,t)] + L F_c Q(y,t) \quad (A-23)$$

If the coolant density is d_c and the coolant channel cross-section A_c , the mass flow rate of the coolant is

$$W_c = d_c A_c v_c \quad (A-24)$$

The volume of the coolant channel is $A_c L$ and the mass of coolant in the channel is $d_c A_c L$. The mass flow rate can then be calculated from the equation,

$$W_c = \frac{M_c}{L} v_c \quad (A-25)$$

Utilizing this relationship in Equation (A-23) results in

$$M_c C_c \frac{\partial T_c(y,t)}{\partial t} + W_c C_c L \frac{\partial T_c(y,t)}{\partial y} = U A [T_f(y,t) - T_c(y,t)] + L F_c Q(y,t) \quad (A-26)$$

The heat balance equation for the moderator is obtained by identical procedures. It differs from the coolant equation in that

it has no heat transfer term as stated in the assumptions. Thus the moderator heat balance equation is

$$M_m C_m \frac{\partial T_m(y,t)}{\partial t} + W_m C_m L \frac{\partial T_m(y,t)}{\partial y} = L F_m Q(y,t) . \quad (A-27)$$

The heat balance equation for the fuel plates is of the form

$$M_f C_f \frac{\partial T_f(y,t)}{\partial t} = -2A [T_f(y,t) - T_c(y,t)] + L F_f Q(y,t) . \quad (A-28)$$

The heat transfer term in this equation is negative because heat is lost from the fuel plates to the coolant. This equation also does not have the derivative of the temperature with respect to position. It was seen in the derivation of the coolant equation that the derivative with respect to position was associated with the velocity of the fluid or medium relative to a frame of reference. The frame of reference being the fuel plates, such a derivative does not exist in the fuel plate heat balance equation. The presence of that derivative in the equation would represent motion of the fuel plates with respect to the fuel plates which is a meaningless concept.

To obtain perturbation equations for the three temperatures, it is necessary to use the relationships:

$$\left. \begin{aligned} T_c(y,t) &= \bar{T}_c + \theta_c(y,t) , \\ T_f(y,t) &= \bar{T}_f + \theta_f(y,t) , \\ T_m(y,t) &= \bar{T}_m + \theta_m(y,t) . \end{aligned} \right\} \quad (A-29)$$

and

Considering Equations (A-13), (A-26), (A-27) and (A-29) the perturbation equations are easily shown to be

$$M_c C \frac{\partial \theta_c(z,t)}{\partial t} + W_c C \frac{\partial \theta_c(z,t)}{\partial z} = U A [\theta_f(z,t) - \theta_c(z,t)] + L F_c g(z,t) , \quad (A-30)$$

$$M_f C_f \frac{\partial \theta_f(z,t)}{\partial t} = -U A [\theta_f(z,t) - \theta_c(z,t)] + L F_f g(z,t) , \quad (A-31)$$

and

$$M_m C \frac{\partial \theta_m(z,t)}{\partial t} + W_m C \frac{\partial \theta_m(z,t)}{\partial z} = L F_m g(z,t) . \quad (A-32)$$

The specific heats of all the organic fluid in the various parts of the reactor are equal making it possible to omit the subscripts in the above equations. The coordinate transformation $z = y/L$ has also been performed.

The corresponding set of steady state equations for coolant, fuel plates and moderator is

$$M_c C \frac{d\bar{T}_c}{dz} = U A [\bar{T}_f - \bar{T}_c] + L F_c \bar{Q} , \quad (A-33)$$

$$0 = -U A [\bar{T}_f - \bar{T}_c] + L F_f \bar{Q} , \quad (A-34)$$

and

$$M_m C \frac{d\bar{T}_m}{dz} = L F_m \bar{Q} . \quad (A-35)$$

The last three equations give the steady state temperature distributions.

Heat balance in the inlet and outlet plenum. Heat balance equations for mixed fluids applicable to the inlet and outlet plenum are derived by Campbell (8). The inlet plenum heat balance equation is

$$M_i C \dot{T}_i = W_i C [T_{in} - T_i] \quad . \quad (A-36)$$

Derivation of this equation assumes no gain or loss of heat by heat transfer or by internal generation. Perfect mixing requires that the bulk temperature of the fluid in the plenum be equal to the outlet temperature. To derive the perturbation equation for the inlet plenum temperature it is necessary to introduce the relationships:

$$T_i(t) = \bar{T}_i + \theta_i(t) \quad , \quad (A-37)$$

and

$$T_{in}(t) = \bar{T}_{in} + \theta_{in}(t) \quad . \quad (A-38)$$

A perturbation equation and a steady state equation is obtained by combining Equations (A-36), (A-37) and (A-38). The perturbation equation is

$$M_i C \dot{\theta}_i(t) = W_i C [\theta_{in}(t) - \theta_i(t)] \quad . \quad (A-39)$$

The steady state equation is

$$0 = W_i C [\bar{T}_{in} - \bar{T}_i] \quad . \quad (A-40)$$

Heat balance in the outlet plenum may be expressed by an equation of the form

$$M_o C \dot{T}_o(t) = W_m C T'_m(t) + W_c C T'_c(t) - W_o C T_o(t) \quad (A-41)$$

The temperatures T'_m and T'_c are the moderator and coolant temperatures, respectively, at the entrance to the outlet plenum. These two temperatures can be calculated from their values at the end of the active core region by considering the transport time for both fluids to flow from the active core region to the outlet plenum. Since no heat is lost or gained in transit, it follows that

$$\left. \begin{aligned} T'_m(t) &= T_{m2}(t - \tau_4) \\ \text{and} \\ T'_c(t) &= T_{c2}(t - \tau_3) \end{aligned} \right\} \quad (A-42)$$

Substituting for T'_m and T'_c in Equation (A-41) results in

$$M_o C \dot{T}_o(t) = W_m C T_{m2}(t - \tau_4) + W_c C T_{c2}(t - \tau_3) - W_o C T_o(t) \quad (A-43)$$

The perturbation equation is derived with the aid of the equations:

$$\left. \begin{aligned} T_o(t) &= \bar{T}_o + \theta_o(t) \\ T_{m2}(t - \tau_4) &= \bar{T}_{m2} + \theta_{m2}(t - \tau_4) \\ \text{and} \\ T_{c2}(t - \tau_3) &= \bar{T}_{c2} + \theta_{c2}(t - \tau_3) \end{aligned} \right\} \quad (A-44)$$

Using these three equations it is possible to write:

$$M_o C \dot{\theta}_o(t) = W_m C \theta_{m_2}(t - \tau_4) + W_c C \theta_{c_2}(t - \tau_3) - W_o C \theta_o(t) . \quad (A-45)$$

The outlet plenum steady state equation is

$$0 = W_m C \overline{T}_{m_2} + W_c C \overline{T}_{c_2} - W_o C \overline{T}_o . \quad (A-46)$$

This completes the derivation of the linear model equations.

Derivation of transforms and transfer functions. The Laplace transform is an integral transform defined by the equation

$$F(s) = \int_0^{\infty} F(t) e^{-st} dt .$$

The function $F(s)$ is defined as the Laplace transform of $F(t)$ and is not to be interpreted as the value of $F(t)$ at $t = s$. A function of two independent variables can be transformed by successive application of the transform integral. For example,

$$F(n, s) = \int_0^{\infty} e^{-nz} \left[\int_0^{\infty} F(z, t) e^{-st} dt \right] dz .$$

As shown above, the transform variable s is associated with the real variable t and the transform variable n is associated with the real variable z . A more detailed discussion of the Laplace transform and the companion inverse Laplace transform is beyond the scope of this thesis. Full details may be found in Churchill (9).

Power transfer function. Derivation of the reactor power transfer function may be found in Schultz (10). The results of Schultz may be written in the form

$$\frac{p(s)}{\rho_e(s)} = \bar{P} G_R(s) , \quad (A-47)$$

where

$$G_R(s) = \left[l s \left(1 + \sum_{j=1}^6 \frac{\beta_j / l}{s + \lambda_j} \right) \right]^{-1} .$$

Temperature transforms. The temperature transforms are obtained from the heat balance equations. Successive transformations are required because the temperatures and the specific power are functions of position and time. The procedure will be demonstrated by detailed treatment of the moderator heat balance equation. For notational simplicity the moderator equation is written in the form

$$\tau_m \frac{\partial \theta_m(z,t)}{\partial t} + \frac{\partial \theta_m(z,t)}{\partial z} = K_m q(z,t) . \quad (A-48)$$

All new symbols introduced in what follows are defined in Table II.

Transforming Equation (A-48) with respect to t results in

$$\tau_m s \theta_m(z,s) + \frac{d \theta_m(z,s)}{dz} = K_m q(z,s) . \quad (A-49)$$

Transforming with respect to z yields

$$\tau_m s \theta_m(n,s) + n \theta_m(n,s) = \theta_{m1}(s) + K_m q(n,s) , \quad (A-50)$$

where

$$\theta_{m1}(s) = \theta_m(0, s)$$

Solving Equation (A-50) for the moderator temperature,

$$\theta_m(n, s) = \frac{\theta_{m1}(s)}{n + \tau_m s} + \frac{K_m q(n, s)}{n + \tau_m s} \quad (A-51)$$

The desired result is the moderator temperature transform with respect to z expressed as a function of position. This quantity is obtained by calculating the inverse transform of Equation (A-51) with respect to z . It can be shown that the resulting transform is

$$\theta_m(z, s) = e^{-\tau_m s z} \theta_{m1}(s) + K_m \int_0^z q(x, s) e^{-\tau_m s(z-x)} dx,$$

where x is a dummy variable and

$$q(x, s) = \frac{\pi}{2L} p(s) \sin \pi x$$

Performing the indicated integration results in

$$\theta_m(z, s) = e^{-\tau_m s z} \theta_{m1}(s) + \frac{a_m}{\frac{\tau_m^2}{\pi^2} s^2 + 1} H_m(z, s) p(s), \quad (A-52)$$

where

$$H_m(z, s) = \tau_m s \sin \pi z - \pi \cos \pi z + \pi e^{-\tau_m s z}.$$

The coolant and fuel plate equations must be treated as a set of simultaneous differential equations but the coolant temperature transform is derived in the manner demonstrated above for the moderator temperature. The resulting transform of the coolant temperature is

$$\left. \begin{aligned} \theta_c(z, s) &= e^{-\alpha z} \theta_c(s) + \frac{a_c}{\frac{\alpha^2}{\pi^2} + 1} H_c(z, s) p(s), \\ H_c(z, s) &= \alpha \sin \pi z - \pi \cos \pi z + \pi e^{-\alpha z}. \end{aligned} \right\} \quad (A-53)$$

where

The fuel plate temperature transform is not needed but for the sake of completeness it is presented below

$$\begin{aligned} \theta_f(z, s) &= \frac{e^{-\alpha z}}{g_f} \theta_c(s) \\ &+ \frac{1}{g_f} \left[\frac{\pi K_f}{2L} \sin \pi z + \frac{a_c}{\frac{\alpha^2}{\pi^2} + 1} H_c(z, s) \right] p(s). \end{aligned}$$

Reactivity transform. The transform of the reactivity due to temperature changes is obtained by calculating the transform with respect to t of Equation (A-17). The result is simply

$$\rho_T(s) = \frac{\int_0^1 [R_c \theta_c(z, s) + R_m \theta_m(z, s)] \sin^2 \pi z dz}{\int_0^1 \sin^2 \pi z dz}.$$

The temperatures specified in the above equation are given in Equations (A-52) and (A-53). After performing the indicated integration, the result may be written in the form

$$\rho_T(s) = G_c(s)\theta_{c_i}(s) + G_m(s)\theta_{m_i}(s) + G_p(s)p(s). \quad (A-54)$$

This transform equation relates the reactivity due to coolant and moderator temperature changes to the changes of the coolant and moderator temperature at the active core region entrance and to the change in total reactor power.

The transforms of the boundary conditions specified by Equation (11-c) are simply

$$\theta_{c_i}(s) = \theta_{m_i}(s) = \theta_i(s).$$

Considering this relationship, Equation (A-54) may be rewritten in terms of the inlet plenum temperature change.

$$\rho_T(s) = [G_c(s) + G_m(s)]\theta_i(s) + G_p(s)p(s). \quad (A-55)$$

The transfer function for the inlet plenum is easily shown to be

$$\frac{\theta_i(s)}{\theta_{in}(s)} = \frac{1}{\tau_i s + 1}. \quad (A-56)$$

Combining Equations (A-55) and (A-56) leads to the result

$$\rho_T(s) = G_T(s)\theta_{in}(s) + G_p(s)p(s). \quad (A-57)$$

where

$$G_T(s) = \frac{G_c(s) + G_m(s)}{\tau_i s + 1}$$

Equation (A-57) states that the reactivity due to coolant and moderator temperature changes actually results from power and reactor inlet temperature changes. Since this result is intuitively obvious from physical considerations, it may be argued that Equation (A-57) could have been written by inspection. It is this investigator's opinion that the form of the equation may be deduced strictly from physical intuition but that the transfer functions G_T and G_P cannot be defined by intuition. To determine these two transfer functions it is necessary to follow, very closely, the procedures employed in arriving at Equation (A-57). Thus the mathematical results agree with intuition in this case.

Outlet plenum temperature transform. The transform of the outlet plenum temperature is obtained by transforming Equation (A-45). The result is

$$M_o \theta_o(s) = W_c e^{-\tau_3 s} \theta_{c2}(s) + W_m e^{-\tau_4 s} \theta_{m2}(s) - W_o \theta_o(s)$$

Collecting terms and introducing new constants leads to

$$(\tau_o s + 1) \theta_o(s) = B_c e^{-\tau_3 s} \theta_{c2}(s) + B_m e^{-\tau_4 s} \theta_{m2}(s) \quad (A-58)$$

The temperatures θ_{c2} and θ_{m2} are the coolant and moderator temperature changes at the active core region exit. They are obtained from Equations (A-52) and (A-53) by evaluating these equations at $z = 1$. Thus, it follows that

$$\left. \begin{aligned} \theta_{c2}(s) = \theta_c(1, s) &= e^{-\alpha} \theta_{c1}(s) + \frac{a_c}{\frac{\alpha^2}{\pi^2} + 1} H_c(1, s) p(s), \\ \text{and} \\ \theta_{m2}(s) = \theta_m(1, s) &= e^{-\tau_m s} \theta_{m1}(s) + \frac{a_m}{\frac{\tau_m^2}{\pi^2} s^2 + 1} H_m(1, s) p(s), \end{aligned} \right\} \quad (\text{A-59})$$

where

$$H_c(1, s) = \pi(1 + e^{-\alpha}),$$

and

$$H_m(1, s) = \pi(1 + e^{-\tau_m s}).$$

Combining Equations (A-58) with (A-59) and recalling that

$$\theta_{c1}(s) = \theta_{m1}(s) = \theta_i(s) = \frac{\theta_{in}(s)}{\tau_i s + 1},$$

leads to the result

$$\theta_o(s) = H_T(s) \theta_{in}(s) + H_P(s) p(s). \quad (\text{A-60})$$

This equation states that perturbations of the outlet plenum temperature are basically due to power and reactor inlet temperature changes. This, too, is a result that is intuitively obvious but

the transfer functions H_T and H_P cannot be obtained by intuition.

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