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SONIC DETERMINATION OF ELASTICS CONSTANTS IN GRAPHITE.

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SONIC DETERMINATION OF ELASTIC CONSTANTS IN GRAPHITE

D. E. Cooley

INTRODUCTION

The elastic moduli of a material can be determined from the resonant frequencies of a sample of known shape and weight. The resonance is usually excited by mechanical vibrations from an electromechanical transducer coupled to the sample with a wire. A second transducer receives the vibration induced in the sample. At resonance the sample vibration is subject to negligible impedance. Therefore, at resonant frequencies the received vibration is very much larger than the non-resonant driven vibration. The Young's Modulus, E , can be calculated using either the fundamental flexural or longitudinal resonant frequencies. The modulus of elasticity in shear (shear modulus) can be calculated using the fundamental torsional resonant frequency. The overtones of these modes of vibration also may be used to calculate the same moduli.

The purpose of this report is to present the knowledge that has been gained in the operation of a Magnatest Type FM-500 Elastomat instrument. Information is given on the operation of the machine itself, in the detection and recognition of the resonant frequencies of a sample, the methods and formulae for the calculation of the elastic moduli from these resonant frequencies, and the results obtained from testing several different materials and sample shapes.

OPERATION AND PROCEDURE

The method of specimen excitation used in all of the experiments and experience discussed in this report was by means of a driven Rochelle Salt transducer connected mechanically to the sample with a coupling wire. This is shown in Fig. 1.

Two types of sample supports were tried. The first was with taut transverse support wires. These were spaced so that the sample would rest on wires at the nodal points of the fundamental transverse or flexural mode of vibration. This seemed to be the most satisfactory positioning for the wires. This positioning is ideal for transverse vibration and did not appear to damp the longitudinal or torsional resonances appreciably. The distance between flexural nodal points is equal to 0.55 of the sample length. For metal samples it was found that a 3/16-in. sag in the wires gave good suppression of macroscopic sample vibration due to environmental disturbances, yet the sonic vibrations still were of sufficient amplitude. For the graphite samples the sonic vibrations are of such low amplitude that they were often damped beyond detection by loose support wires. This may be due to lower mechanical inertia of graphite. The environmental vibration did not seem to be as prominent with the taut wires for graphite as for metals; hence with graphite samples better results were obtained with taut support wires than with loose ones.

The best results with graphite were obtained using polyurethane foam supports rather than wires. The foam seemed to have the best macroscopic damping properties yet did not effect the sonic vibration significantly. Results from metal samples would probably be better using foam than wires for the same reason. The foam supports were cut to a right triangular prism shape, notched slightly for the sample, and placed under the nodes of the fundamental flexural vibration.

FIGURE 1

Photograph of Acoustical Bench

The input and receiver wires can be mounted either on the end surfaces or the side of the sample. However, no significant advantage resulted from placing the coupling wires on the side surfaces of the sample, and since the coupling is not conveniently variable from the side, it seemed to be an undesirable positioning. The best position appeared to be on the end surfaces, at opposite corners of the cross section, about one millimeter from the edge of the sample end. It was quite important that the sample have some type of coupling-wire mount; the simplest was an indent in which the coupling wire could rest. In metals a prick-punch dent was adequate, and in graphite a scriber produced a good mounting hole. The coupling was further improved by pressing some beeswax over the indents and inserting the wires into the waxed holes. This greater coupling appeared to have no significant effect on the frequency of resonance, yet did render a high maximum amplitude of vibration and virtually eliminated the inconvenience of the coupling wires becoming detached from the sample in the event of high amplitude vibration. In cases where the application of beeswax is undesirable, an indentation or small hole is adequate.

The transducer mounts are on a bench which provides coarse motion in the direction of the sample axis along a dovetail slide and fine adjustment with a screw. These provide means by which the wire coupling force and direction may be varied. The crystal position may also be varied by rotating the unit and by lateral motion of the mount on the holder. These turning and lateral motions often lead to the detection of a torsional resonance that might not otherwise be observed, especially in graphite.

The desired effect of adjustments is to improve the coupling and thereby to increase the magnitude of the vibration for better definition of the peak position. This change in coupling has no effect on the position of any fundamental high-amplitude resonance peak. Fig. 2 is a plot of the received sample vibration magnitude over two resonance peaks for several coupling wire positions. It can be seen from this figure that the degree of coupling has no significant effect on a strong resonant frequency. In the case of metals for all fundamentals and the first, second, and usually third overtones of the flexural vibrations, the peak position will be unaffected by adjustments of the coupling wire to increase response amplitude. In the case of graphite, coupling wire adjustment will not affect the peak position of the fundamental flexural resonance, and usually not affect the peak positions of the fundamental torsional, fundamental longitudinal, or first flexural overtone resonances.

Since the amplitude of vibration is quite small for graphite samples, the higher frequencies may be poorly defined. This is manifested by a peak which can be moved by a change of wire adjustment. For example, assume a peak had been found at 7800 cps and the coupling wires had been adjusted for maximum response. To test this value, the input is changed to approximately 7750 cps and the wires readjusted to give maximum response at this frequency. If when turning the input back up to 7800 cps the amplitude of the received signal decreases, then a "moving peak" has been encountered. No method was devised for determining the true sample resonant frequency in these cases. This phenomenon was never encountered when testing metal samples.

FIGURE 2

Effect of Coupling on Resonant Response of a Square Prism

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Usually any change in coupling will immediately destroy a forced sample resonance. This is also true in the case of a coupling wire resonance.

The optimum adjustment for the various modes and overtones of vibration for graphite was found unpredictable to a large degree. In a very general sense it might be said that for good flexural resonance the coupling wires were slightly in compression ($3/8$ -in. bow). For torsional resonance, good results were obtained with the coupling wires positioned at non-intersecting angle to the longitudinal axis of the sample. Longitudinal resonance often required more compressive loading than the flexural (bow of $1/2$ to $3/4$ in.). When beeswax was used, the best results were often obtained with the coupling wires very slightly in tension. However, this tends to be unstable since the coupling wires are very apt to pull slowly out of the beeswax.

The coupling wire should be solidly mounted. Should a coupling wire work loose from the transducer and vibrate in the mounting tube, a very poor pattern results on the oscilloscope, and the resonant frequency is difficult to determine. A loose coupling wire can be detected by pinching the wire very lightly between the thumb and forefinger trying gently to move it longitudinally. If it is loose, the coupling wire can be remounted by unscrewing the protective cap from the whisker mount, holding the unit so the wire is horizontal and slowly bring a lighted match down toward the sealing wax until it melts and once again holds the wire. Excess heat will melt the coupling wire and may damage the crystal.

The length of the coupling wire did not seem to have any effect on peak position. In the present work, the coupling-wire length was varied from $1-3/4$ to $2-3/4$ in. with no observable effect.

Much time can be saved if the operator knows roughly at what frequencies the resonances will occur. Accordingly, a set of expected values for the resonance frequencies under 25 kc/sec should be calculated using nominal dimensions for the samples and expected values for the elastic moduli. Fig. 3 shows ratios of the overtone frequencies to the fundamental as a function of the configuration of the sample. The method of calculation is shown in the Appendix. After the first few frequencies are found, identified, and the preliminary calculations established, then the remaining searching is quite rapid, especially in the case of metals. Unfortunately, irregularities of the graphite samples can distort the calculated frequency pattern, sometimes rendering it unrecognizable.

Using the positive-feedback loop enables the experimenter to locate the frequency of the strongest resonances within a few per cent without scanning. This facilitates the confirmation of the pre-calculation, or can be used to obtain fundamental flexural resonance frequencies from which the other resonant frequencies can be calculated. To excite resonant frequencies with the feedback circuit the coupling wires are adjusted to roughly $1/4$ to $3/8$ in. bow, the amplitude set at about 2 and the phase and oscillator dials varied. The line image on the oscilloscope will distort and the counter will begin registering when a resonant frequency is excited and a high amplitude vibration is received.

For the first sample of a group or for a single sample it was found desirable to scan the whole frequency range. This confirmed or refuted the preliminary calculations and established a fairly accurate schedule of the frequencies to be expected for the rest of the group. For the rest of the samples it was found advisable to scan the frequencies where a resonance or group of resonances were expected to appear.

FIGURE 3

Relationship of Overtones

In scanning, the coupling wires were first set with about a $3/8$ in. bow against the sample, and the amplitude knob was set at about 5. The oscillator knob was pulled out and set at $2-1/2$ for metals, and pushed in and set at $2-1/2$ or 5 for graphite. The phase knob is of little importance in the scan, serving only to rearrange the pattern on the oscilloscope. The scan was started at the lowest frequency to insure reception of the fundamental flexural-mode resonance. This is always the lowest and strongest resonance frequency. As this frequency was approached, the scope pattern, which is a straight line where there is no resonance, enlarged and opened into a circle or ellipse. At the same time the amplitude needle deflected up scale. As the oscillator or input frequency continued to approach the resonant frequency of the sample, the receiver amplifier tended to overload and the gain had to be reduced to keep the meter reading on scale. When the resonance was obtained, the coupling wires were adjusted to optimize the coupling. This was accomplished by moving the coupling wire about until a maximum needle deflection was attained. It is preferable to keep the oscillator gain control at a constant setting so that the input amplitude is the same throughout the experiment. By doing this and recording the amplitude knob setting for a given meter deflection a comparative measure of the received amplitude of vibration can be compiled. This procedure was also found best for the other resonance frequencies of graphite, except that the amplitude was not as great and sometimes the only indication that a peak has been passed was a slight opening of the line normally seen on the oscilloscope. This opening was maximized with the input frequency and then adjustment of the coupling wires showed if the frequency was an actual sample resonance or a forced resonance. When testing graphite samples there

sometimes was no indication of an expected peak on the first frequency traverse. In this case, the coupling wires were readjusted with the input frequency at the expected value and the resonance found.

Often when scanning, a resonance will be found for which the display on the oscilloscope will be a Lissajous figure (a figure "8" or a variation thereof with 3, 4, 5, etc. lobes). Adjustment of the phase knob will enable the operator to count the number of lobes in the figure. For an upright figure with two lobes, the sample is vibrating at $1/2$ the input frequency and for a horizontal figure with four lobes the sample is vibrating at 4 times the input frequency. Thus the number of lobes indicate the multiplication factor, and the orientation of the figure indicates whether it is a whole number or a fraction. These were found useful for predicting resonances later in the scan or for finding resonances above the range of the input oscillator. The exact resonance point is found in the same manner as it is when the oscillator and sample are at the same frequency.

Background noise is of but slight concern, being more annoying than detrimental to results. An operating relay in a temperature controller in the room was found to cause the counter to count high when the amplitude knob was set at 4 or over. Since most resonances are of sufficient amplitude to require that the gain be set at 1 or 2, it did not affect many results. Also a desk calculator caused sufficient noise that weak resonances could be missed. This was most annoying when searching for higher frequencies for which the amplitude of resonance vibration was low. Loud sharp noises have a marked effect at high amplitude settings, but are short lived, so are not of importance.

Gross translation or macroscopic motion of the sample appears as a wobbly effect on the scope picture and can be varied by touching the sample with a finger. The foam support eliminates this problem to a very great extent. A loose coupling wire, either loose within its mount or on the sample is manifested by a myriad of fine lines appearing around the scope image, sometimes in the same pattern as the image and sometimes introducing distortions of their own. If they cannot be eliminated by adjustment of the coupling wire compression, then the coupling wire is probably loose in its mount.

A third transducer has been found to be a useful adjunct when connected through a variable amplifier to an ammeter and to a set of earphones. When the third coupling wire is touched to the sample at resonance both an audible and visual response is given. By "listening" to the vibrating sample, nodes can be rapidly identified thereby giving a positive identification of the mode of the sample vibration. The ammeter gives a quantitative indication of sample vibration so that vibration amplitude figures can be plotted. When studying metal samples the amplitude of vibration is great enough that nodes can be identified for first and sometimes second overtones as well as the fundamental resonances. In studying graphite samples it was found that only the fundamental flexural frequencies were of sufficient amplitude to be recognizable with the third transducer. It was useful, however, in identifying these resonances.

CALCULATIONAL METHODS

All of the methods used to calculate the elastic moduli that are discussed in this report utilize the fundamental resonance frequency. The

fundamental can be identified with the greatest accuracy due to its high amplitude compared with its overtones. If it is desirable to use the overtones as a check, then the overtone multiplication factor curves (Fig. 3) should be consulted. The formulae in the Elastomat Instruction Manual give the same results, where applicable, as the theory below.

Young's Modulus. Young's modulus can be calculated from both the flexural and longitudinal resonance frequencies. The flexural resonance is easiest to identify and thus gives the most accurate results. In the Appendix, these formulae are modified for non-uniform cross sections.

A. Flexural Vibration:

From Pickett:¹

$$E_F = \frac{4 \pi^2 L^3}{g I m^4} T_1 W n_F^2 \quad (1)$$

For a sample of any uniform cross section only where

E_F = Young's modulus determined from flexural resonance (psi)

n_F = Fundamental flexural resonance frequency (cps)

$g = 386.09 \text{ in./sec}^2$

I = Area moment of inertia of cross section about neutral axis (in.^4)

L = Sample length (in.)

W = Weight of sample (lb)

$m = 4.730$ as stated in Pickett¹, pp. 851

T_1 = Correction factor determined by the geometry of the sample (R/L), the material (μ), and the direction of bending.

From Fig. 4 and Pickett¹, pp. 852

Where R = Radius of gyration of cross section about neutral axis

μ = Poisson's Ratio ($0 \leq \mu \leq 0.5$)

As can be seen from Table I for any cross sectional configuration for which there are two different principal axes there will appear two different flexural resonance points. The resulting values of E_F have agreed very well, providing an excellent mutual check. Experimental data compiled for graphite ~~specimens~~ are shown in the Appendix as Tables 1A and 2A.

TABLE I - PROPERTIES OF SELECTED CROSS SECTIONS

Cross section	Neutral axis	R	A	I	E_F
<p>Round</p>	Any	$\frac{d}{4}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	$0.0041632 \frac{L^3 W}{d^4} T_1 n_F^2$
<p>Rectangle</p>	L	$\frac{b}{\sqrt{12}}$	bh	$\frac{hb^3}{12}$	$0.0024523 \frac{L^3 W}{b^3 h} T_{1L} n_{FL}^2$
	S	$\frac{h}{\sqrt{12}}$	bh	$\frac{bh^3}{12}$	$0.0024523 \frac{L^3 W}{h^3 b} T_{1S} n_{FS}^2$
<p>Full-length quarter round</p>	L	.30141r	.7854r ²	$7.1350 \times 10^{-2} r^4$	$2.8631 \times 10^{-3} \frac{L^3 W}{r^4} T_{1L} x(n_{FL})^2$
	S	.22117r	.7854r ²	$3.8418 \times 10^{-2} r^4$	$5.3174 \times 10^{-3} \frac{L^3 W}{r^4} T_{1S} x(n_{FS})^2$

B. Longitudinal Vibration:

From sonics for any material

$$V = 2L n_L$$

$$V = \left(\frac{E_L}{\rho_m} \right)^{1/2}$$

Where:

L = Length of sample (in.)

V = Velocity of sound (in./sec)

n_L = Fundamental longitudinal resonant frequency (cps)

ρ_m = Mass density (lb sec²/in.⁴)

E_L = Young's modulus as determined from longitudinal resonance (psi)

Equating these and solving for E_L

$$E_L = 4L^2 \rho_m n_L^2 \quad (2)$$

For a sample of uniform cross section

$$\rho_m = \frac{W}{gAL}$$

A = Cross sectional area (in.²)

g = 386.09 in./sec²

Substituting

$$E_L = \frac{4LW}{gA} n_L^2 \quad (3)$$

Shear Modulus. The shear modulus, G , can be calculated from the torsional resonance frequency of a sample.

For samples of uniform cross section only¹

$$G = \frac{4LW}{gA} R n_T^2 \quad (4)$$

Where:

L = Length (in.)

g = 386.09 in./sec

A = Cross sectional area (in.²)

W = Sample weight (lb)

n_T = Fundamental torsional resonance frequency (cps)

R = Form factor for cross section (Pickett, pp. 857)

For a circle: $R = 1$

Square: $R = 1.183$

Rectangle: $R = \frac{a/b + b/a}{4 a/b - 2.52 (a/b)^2 + 0.21 (a/b)^6}$ (side $a \leq$ side b)

Quarter Round: $R = 1.3305$ (derived in Appendix)

Poisson's Ratio. Poisson's ratio, μ , can readily be calculated for isotropic materials from the above results since²

$$\mu = \frac{E}{2G} - 1$$

For anisotropic materials such as graphite this relation no longer holds. The above expression yields values less than zero and greater than 1 for different

graphites even though theoretical limits of Poisson's ratio are $0 \leq \mu \leq 0.5$. No satisfactory method of calculating μ from the elastic moduli was found, and it is expected that the only legitimate method of determination of this quantity for graphite is from tensile tests.

For the purpose of the determination of T_1 , the correction factor shown as Fig. 4, the Poisson's ratios listed in the ASTM Metals Handbook were used for metals. For graphite the value of 0.33 was used for lack of better information. In the range of R/L under consideration the total variation for T_1 for $0.1 \leq \mu \leq 0.3$ is 3 per cent, so the possible error introduced by this arbitrary choice is not very large.

Relation of Harmonics to Fundamental. It is advisable when possible to check the assumed fundamental flexural frequency by comparing it to its overtones. Or, if a scan of a sample has been run and there is a question as to the identity of some high-frequency resonances, the overtones of the flexural resonance can be identified if the relation of these overtones to the fundamental is known. This results in isolation of the torsional and longitudinal resonances, rendering them easily identifiable. Often the first overtone of the flexural resonance is close to the fundamental torsional resonance, and this enables the operator to distinguish between them. Pickett¹ has calculated and drawn curves showing the ratio of the flexural resonance overtones to their fundamental as a function of the geometry of the sample. These curves were amply substantiated for any direction of flexural vibration for the samples of uniform cross section including full length quarter rounds. (See again Fig. 3). The relation of the overtones to their fundamentals is readily found by using these curves and a computed value of the radius of gyration about the neutral axis divided by the length.

FIGURE 4

Correction Factor, T_1 , as a Function of Geometry and Poisson's Ratio

Non-Uniform Cross Section. The only non-uniform cross section studied was a stepped quarter-round 3-7/8 in. long with a major radius of 1/2 in., a minor radius of 3/8 in. on both of the 7/16 in. long steps. It was decided that the best way to establish a means of calculating the elastic moduli from the resonant frequencies for this configuration was to impose factors by which the resonance frequencies for the stepped quarter-round could be made equal to that of the calculable full-length quarter round. To do this, four stainless steel full-length quarter rounds were first made 3-7/8 in. long with a radius of 1/2 in. They were tested on the Elastomat and the resulting resonant frequencies were used to determine the elastic moduli with the formulae listed above for uniform quarter rounds. The same full-length quarter rounds were then stepped to 3/8 in. radius for 7/16 in. on each end, and again run on the Elastomat. The resulting resonant frequencies were then divided by the corresponding resonant frequencies of the full-length quarter rounds to obtain a non-uniformity factor, ϕ , for each mode of resonance. The same procedure was followed for four different types and orientations of graphite. In order to render the formulae more manageable for the non-uniform cross section, they were altered to be a function of density of the material rather than of the total weight.

Table II presents the average of the frequency ratio factors (ϕ) found for the materials listed. This factor is the ratio of a given resonance frequency of the stepped quarter round to the corresponding resonant frequency of the full length quarter round. These were determined with a stepped quarter round 3-7/8 in. long with a major radius of 1/2 in., and a minor radius of

3/8 in. for 7/16 in. at each end. These were cut from full length quarter rounds 3-7/8 in. long with a radius of 1/2 in. Each value represents an average of four values with the exception of ϕ_{S2} , ϕ_{L2} (2 each) and ϕ_{S3} (one value). For a complete listing of the values obtained, see Appendix Tables 3A and 5A. Subscript 1 refers to first overtone, 2 to second overtone, etc.

TABLE II - FREQUENCY FACTORS FOR QUARTER-ROUND SAMPLES

<u>Material</u>	<u>ϕ_S</u>	<u>ϕ_{S1}</u>	<u>ϕ_{S2}</u>	<u>ϕ_L</u>	<u>ϕ_{L1}</u>	<u>ϕ_{L2}</u>	<u>ϕ_T</u>
CSF (II)	1.146			1.150			
CSF (I)	1.145			1.154			
TSX (II)	1.147			1.154			
TSX (I)	1.155			1.162			
Stainless Steel	1.147	1.076	1.07	1.154	1.149	1.107	1.138

CONCLUSIONS AND RECOMMENDATIONS

The accuracy and reproducibility of the sonic modulus method for graphite is very good. This accuracy is dependent on the amplitude of sample vibration, which is larger for fundamental flexural resonance than any of the other fundamentals or overtones. For metal samples in flexural resonance the reproducibility is about 0.05 per cent. For metals the reproducibility of the higher fundamentals and the first two flexural overtones is about 0.1 per cent. For graphite samples, the fundamental flexural resonance is reproducible to within 0.2 per cent, with the reproducibility of the high resonances decreasing to about 0.5 per cent.

Sample temperature has a significant effect on the resonant frequencies. For an aluminum round sample the fundamental flexural resonance changed 50 cps (1 per cent) for a temperature change of about 75° F. The fundamental flexural resonance (n_{FL}) for a full-length quarter-round sample of TSX graphite dropped 14 cps (0.5 per cent) for a temperature rise of about 35° F. Inasmuch as these changes are several times larger than the precision for which the machine is capable, it seems advisable to run all tests at room temperature with samples that have had time to equilibrate.

The compatibility of the resulting moduli of elasticity as calculated from the resonant frequencies is quite good. In general, Young's Modulus as determined from the longitudinal resonance is slightly (0.7 per cent) higher than that as determined by the flexural resonance. In the case of quarter rounds the Young's modulus as determined by n_{FL} is slightly (0.5 per cent) larger than that determined by n_{FS} . This effect is more marked for graphite, due largely to the anisotropy. This can be appreciated by studying the tables of results in the Appendix.

In one case a sample of aluminum that was originally quarter round was machined down to a round and the resulting elastic moduli compared. The Young's modulus was 1 per cent lower for the round shape, whereas the shear modulus was the same. In cases where the same material but not the same sample have been tested in different shapes, there are no significant differences in the elastic moduli. During the course of the graphite experiments uniform square prisms were tested, and these were later machined to full-length quarter rounds. In the latter case the results were essentially the same, the item of

major importance being that the uniform square prism is not a good sample configuration. Resonances were difficult to detect and identify and overall accuracy was poorer than with the quarter-round samples.

The overtone frequency determination factor as determined by Pickett¹ agree to within 1 per cent with those found experimentally for samples of uniform cross section. This can be seen by studying Fig. 3 which shows Pickett's lines along with the experimental points. For non-uniform samples, in this case stepped quarter rounds, Pickett's values no longer apply.

Since the present interest is the determination of elastic constants in graphite and their changes as a result of irradiation, recognition was made of certain engineering necessities of irradiation technique. The samples are scraped to obtain samples for X-ray measurements. A large hole for a thermocouple insulator is sometimes drilled in the sample, and the smaller holes are always drilled for optical-length measurement and thermal conductivity tests.

The effect on the flexural resonance frequencies of scraping the sample for X-ray tests was found to be negligible. After three scrapings had been taken the flexural resonances were still within the range of reproducibility of the machine.

The 1/8 in. hole drilled in the sample for the purpose of thermocouple insulator insertion is quite detrimental to the results of the test. Samples with such holes should not be considered suitable for use in the sonic determination of Young's modulus. Drilling the hole at a nodal point of flexural vibration made it extremely difficult to excite any resonance at all and did not appear

to be a solution of the problem. It might be worthwhile to note that two CSF parallel samples were drilled with the hole and the flexural resonances determined. The n_{FS} was badly distorted by the hole while both of the n_{FL} were increased by 0.35 per cent. Additional tests should be run to determine the reliability of this result, however.

The other smaller holes in the samples appear to have no significant effect on the resonant frequencies of the samples. This includes holes for length measurement by the optical comparator and holes for thermal conductivity tests.

In selecting coupling-wire mounting points for stepped quarter-round samples, the opposite corners of the shoulder appear adequate. This positioning rather than at the end of the sample may account for the difficulty of exciting the torsional and longitudinal resonant frequencies. It is quite easy to excite both flexural modes of vibration with this mounting.

Inasmuch as there is a great deal of difficulty involved in recognizing the higher modes and overtones it seems desirable in the interest of accuracy and time to confine the testing to the flexural vibration. The quarter-round configuration has the desirable quality of furnishing two high-amplitude fundamental flexural vibrations which serve as a check for the value of Young's modulus. The longitudinal resonance is virtually unexcitable in the stepped quarter-round configuration, and serves only to give a third value of Young's modulus. The torsional resonance is difficult to excite or recognize and probably is not worth the time involved in obtaining it. Therefore it would seem expedient to record the two flexural resonances only and determine

Young's modulus and the radiation effects thereon from these. Table III shows the ranges of flexural resonances obtained on samples of unirradiated graphite. This should prove useful for the resonance-determination process, and can be supplemented as more graphite types are measured.

TABLE III - FREQUENCIES OF PRINCIPAL FLEXURAL RESONANCES FOR STANDARD-SIZE QUARTER-ROUND SAMPLES

	<u>n_{FS}</u>	<u>n_{FL}</u>
TSX ()	3700-3800	4980-5060
(⊥)	2100-2240	2830-3030
CSF ()	2960-3040	3980-4060
(⊥)	2060-2080	2760-2810

APPENDIX

MODIFICATION OF FORMULAE FOR YOUNG'S MODULUS FOR A NON-UNIFORM CROSS SECTION

The quarter-round sample shape is routinely used in graphite irradiation programs. Much of the effort of this study was expended to determine the effect on resonant frequencies of this non-uniform cross section. The derivation of appropriate modification to the general formulae for elastic moduli is as follows:

A. Flexural Vibration

From above:

$$E_F = \frac{4 \pi^2 L^3}{g I m^4} T_1 W n_F^2 \quad (1)$$

$$\rho = \frac{W}{AL}$$

so

For uniform quarter round:

$$\rho = \frac{W}{0.7854 r^2 L}$$

$$I_S = 3.8418 \times 10^{-2} r^4$$

$$I_L = 7.1350 \times 10^{-2} r^4$$

Substituting into (1) and simplifying for vibration about the S axis:

$$E_{FS} = 4.1763 \times 10^{-3} \frac{L^4 \rho}{r^2} T_{IS} (n_{FS})^2, \quad \text{for full length quarter rounds.}$$

Likewise for vibration about the L axis:

$$E_{FL} = 2.2487 \times 10^{-3} \frac{L^4 \rho}{r^2} T_{IL} (n_{FL})^2, \quad \text{for full length quarter rounds.}$$

But since $\phi = \frac{\text{Frequency of stepped quarter round } (n_F)}{\text{Frequency of full quarter round } (n_F)}$

$$n_{FS} = \frac{n_{FS}}{\phi_S}$$

$$n_{FL} = \frac{n_{FL}}{\phi_L}$$

Then for stepped quarter rounds:

$$E_{FS} = 4.1763 \times 10^{-3} \frac{L^4 \rho}{r^2} T_{IS} \left(\frac{n_{FS}}{\phi_S} \right)^2 \quad (5)$$

and

$$E_{FL} = 2.2487 \times 10^{-3} \frac{L^4 \rho}{r^2} T_{IL} \left(\frac{n_{FL}}{\phi_L} \right)^2 \quad (6)$$

where E_{FS} = Young's modulus as determined from flexural resonance about the S axis (psi)

E_{FL} = Young's modulus as determined from flexural resonance about the L axis (psi)

L = Length of sample (in.) (approx. 3-7/8 in.)

r = Major radius of sample (in.) (approx. 1/2 in.)

ρ = Density of material (lb/in.³)

n_{FS} = Fundamental flexural resonance frequency about S axis of stepped quarter round (cps)

n_{FL} = Fundamental flexural resonance frequency about L axis of stepped quarter round (cps)

ϕ_S, ϕ_L = Frequency ratio factors determined experimentally.

T_{IS}, T_{IL} = Pickett's¹ factor depending on configuration, bending direction, and material.

B. Longitudinal Vibration

From above:

$$E_L = 4 L^2 \rho_m (n_L)^2 \quad (2)$$

$$\rho_m = \frac{\rho}{g}$$

so for convenience:

$$E_L = \frac{4 L^2}{g} \rho (n_L)^2 \quad \text{for uniform quarter rounds}$$

$$\text{letting } \phi_{Lg} = \frac{\text{Longitudinal frequency of stepped quarter round } (n'_L)}{\text{Longitudinal frequency of full quarter round } (n_L)}$$

$$n_L = \frac{n'_L}{\phi_{Lg}}$$

so

$$E_L = \frac{4 L^2 \rho}{g} \left(\frac{n'_L}{\phi_{Lg}} \right)^2 \quad \text{for stepped quarter rounds} \quad (7)$$

E_L = Young's modulus as determined from longitudinal resonance (psi)

L = Length of sample (in.) (approx. 3-7/8 in.)

ρ = Density of material (lb/in.³)

$g = 386.09 \text{ in./sec}^2$

n'_L = Fundamental longitudinal resonance frequency of stepped quarter round

ϕ_{Lg} = Frequency ratio factor

Inasmuch as the longitudinal resonance frequency was never recognized in the stepped quarter-round configuration, there are no values yet known for ϕ_{Lg} .

C. Torsional Vibration

From above:

$$G = \frac{4LW}{\pi A} R (n_T)^2 \quad (4)$$

For a uniform quarter round:

$$A = 0.7854 r^2$$

$$W = 0.7854 r^2 L \rho$$

$$R = 1.3305$$

Substituting and simplifying for uniform quarter rounds

$$G = 1.3784 \times 10^{-2} L^2 \rho (n_T)^2$$

since

$$\phi_T = \frac{\text{Frequency of stepped quarter round } (n_T')}{\text{Frequency of full quarter round } (n_T)}$$

$$n_T = \frac{n_T'}{\phi_T}$$

and for stepped quarter rounds

$$G = 1.3784 \times 10^{-2} L^2 \rho \left(\frac{n_T'}{\phi_T} \right)^2 \quad (8)$$

where G = Shear modulus

L = Length of sample (in.) (approx. 3-7/8 in.)

ρ = Density (lb/in.³)

n_T = Fundamental torsional resonance frequency of stepped quarter round

ϕ_T = Frequency ratio factor

DERIVATION OF THE FORM FACTOR, R, FOR A QUARTER ROUND

For free end torsional resonance³

$$n_T = 1/2 \left(\frac{C_0}{J_0} \right)^{1/2}$$

Where $J_0 = M R_0^2$

$M = \text{Mass (lb sec}^2/\text{in.)}$

$R_0 = \text{Radius of gyration about centroid (in.)}$

$$C_0 = \frac{G I_p}{L} = \frac{M t}{L \theta}$$

$G = \text{Shear modulus}$

$I_p = \text{Polar moment of inertia (in.}^4\text{)}$

$L = \text{Length (in.)}$

$M t = \text{Torque about axis (in. lb)}$

$\theta = \text{Twist per unit length (radians/in.)}$

Substituting:

$$n_T = 1/2 \left[\left(\frac{M t}{L \theta} \right) \left(\frac{1}{M R_0^2} \right) \right]^{1/2}$$

For a quarter round²

$$\frac{M t}{\theta L} = 0.0825 \frac{G r^4}{L}$$

so

$$n_T = 1/2 \left(\frac{0.0825 G r^4}{L M R_0^2} \right)^{1/2}$$

or

$$M = \frac{W}{g}$$

Solving for G

$$G = \frac{4 L W R_0^2}{0.0825 r^4 g} n_T^2$$

Setting this equal to Pickett's expression

$$\frac{4LW R_o^2}{0.0825 Gr^4 g} n_T^2 = \frac{4LW R}{g A} n_T^2$$

$$R = \frac{AR_o^2}{0.0825 r^4}$$

Since for a quarter round

$$A = 0.7854 r^2$$

and

$$R_o^2 = \frac{r^2}{2} - (0.6002 r)^2 = 0.13976 r^2$$

then

$$R = \frac{(0.7854 r^2) (0.13976 r^2)}{0.0825 r^4}$$

$$\underline{\underline{R = 1.3305}}$$

RESULTS OF SONIC TESTS ON GRAPHITE

In the following tables, results of this work are shown. Tables 1A and 2A give elastic parameters for graphite quarter rounds and square prisms. Table 3A shows the ratio of the resonant frequency of stepped to straight quarter rounds of graphite. Table 4A and 5A give elastic moduli for stainless steel and frequency ratios, respectively.

TABLE IA - MECHANICAL MODULI FOR GRAPHITE FULL LENGTH QUARTER ROUNDS

Sample	All Elastic Moduli x 10 ⁻⁶							
	Flexural		Longitudinal	Torsional	f_{21S}	f_{31S}	f_{21L}	f_{31L}
	E_{FS}	E_{FL}	E_L	G				
CSF ()								
142-301	1.71	1.71	1.79	0.568	2.59		2.50	4.38
-302	1.60	1.63	1.76	0.513	2.59	4.69	2.54	4.55
-304	1.62	1.65	1.71	0.524	2.59	4.82	2.51	
-305	1.69	1.72	1.74	0.572	2.60		2.49	4.34
AVE.	1.66	1.68	1.75	0.544	2.59	4.75	2.51	4.42
CSF (⊥)								
142-101	0.786	0.807	0.855	0.494			2.54	4.54
-103	0.795	0.816	0.804				2.54	4.53
-106	0.778	0.783	0.791	0.492			2.54	4.56
-108	0.773	0.786	0.782				2.53	4.58
AVE.	0.783	0.798	0.808	0.493			2.54	4.55
TSX ()								
153-201	2.83	2.84	2.96	0.737	2.57		2.46	
-202	2.70	2.70	2.85	0.719			2.48	
-211	2.81	2.83	2.93	0.736			2.47	
-214	2.77	2.83	2.87	0.715		4.84		4.34
AVE.	2.78	2.80	2.90	0.727	2.57	4.84	2.47	4.34
TSX (⊥)								
153-C14	0.845	0.861	0.855	0.212	2.64	4.87	2.56	4.59
-C15	0.852	0.869	0.862		2.64	4.84	2.56	4.59
-M17	0.896	0.915			2.63	4.83		
-E18	0.958	0.982	0.979	0.209			2.55	
AVE.	0.888	0.907	0.899	0.211	2.64	4.85	2.56	4.59

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TABLE 2A - MECHANICAL MODULI FOR GRAPHITE SQUARE PRISMS

Sample	All Elastic Moduli x 10 ⁻⁶					
	<u>E_F</u>	<u>E_L</u>	<u>E_{AVE}</u>	<u>G</u>	<u>f₂₁</u>	<u>μ</u>
CSF (H)						
142-301	1.67	1.75	1.71	0.548	2.43	0.561
-302	1.64	1.69	1.67	0.535	2.43	0.560
-304	1.63	1.69	1.66	0.547	2.44	0.515
-305	1.62	1.68	1.65	0.588		0.403
AVE.	1.64	1.70	1.67	0.555	2.43	0.510
CSF (L)						
142-101	0.780	0.797	0.789	0.520	2.43	-0.242
-103	0.799	0.791	0.795	0.515	2.44	-0.235
-106	0.781	0.780	0.781	0.505	2.44	-0.227
-108	0.772	0.776	0.774	0.504	2.44	-0.232
AVE.	0.783	0.786	0.785	0.511	2.44	-0.234
TSX (H)						
153-201	2.76	2.91	2.84	0.656	2.40	1.17
-202	2.67	2.85	2.76	0.672	2.39	1.06
-211	2.58	2.94	2.76	0.652		1.12
-214	2.69	2.87	2.78			
AVE.	2.68	2.89	2.79	0.660	2.40	1.12
TSX (L)						
153-C14	0.830	0.846	0.838	0.205	2.47	1.04
-C15	0.839	0.843	0.841	0.207	2.46	1.03
-M17	0.872	0.996	0.934			
-E18	0.952	1.010	0.982	0.186		1.64
AVE.	0.873	0.924	0.899	0.199	2.47	1.24

TABLE 3A - RATIO, ϕ , OF FREQUENCY OF STEPPED TO STRAIGHT GRAPHITE QUARTER ROUNDS

<u>Sample</u>	<u>ϕ_S</u>	<u>ϕ_L</u>
CSF (11)		
142-301	1.144	1.148
-302	1.146	1.150
-304	1.145	1.148
-305	1.147	1.152
AVE.	1.146	1.150
CSF (1)		
142-101	1.149	1.155
-103	1.136	1.150
-106	1.149	1.156
-108	1.146	1.153
AVE.	1.145	1.154
TSX (11)		
153-201	1.150	1.154
-202	1.147	1.162
-211	1.143	1.151
-214	1.148	1.149
AVE.	1.147	1.154
TSX (1)		
153-C14	1.154	1.160
-C15	1.156	1.164
-M17	1.154	1.159
-E18	1.154	1.163
AVE.	1.155	1.162
AVE.	1.147	1.154

TABLE 4A - MECHANICAL MODULI FOR STAINLESS STEEL

FULL LENGTH QUARTER ROUNDS

All Elastic Moduli $\times 10^{-6}$

<u>Sample No.</u>	<u>#1</u>	<u>#2</u>	<u>#3</u>	<u>#4</u>	<u>Ave.</u>	<u>Round Std.</u>
E_{FS}	27.87	28.18	28.32	27.94	28.09	28.74
E_{FL}	28.01	28.43	28.31	28.15	28.23	
E_L	28.21	28.40	28.42	28.26	28.32	28.93
E_{AVE}	28.03	28.34	28.35	28.12	28.21	28.84
G	11.01	11.07	11.13	11.03	11.06	11.10
μ	0.273	0.280	0.274	0.275	0.275	0.299
f_{2LS}	2.62	2.60				
f_{3LS}	4.74	4.75				
f_{2LL}	2.52	2.52				

TABLE 5A - RATIO, ϕ , OF FREQUENCY OF STEPPED TO STRAIGHT STAINLESS STEEL QUARTER ROUNDS

<u>Sample No.</u>	<u>#1</u>	<u>#2</u>	<u>#3</u>	<u>#4</u>	<u>Ave.</u>
ϕ_S	1.144	1.143	1.144	1.143	1.144
ϕ_{S1}	1.072	1.080			1.076
ϕ_{S2}	1.074	1.062			1.068
ϕ_L	1.150	1.148	1.150	1.149	1.149
ϕ_{L1}	1.107	1.106			1.107
Tors	1.137	1.138	1.138	1.137	1.138

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