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A THEORETICAL AND EXPERIMENTAL EVALUATION OF THE
DISTRIBUTION OF THERMALIZED 15 MeV NEUTRONS
IN A FINITE WATER MODERATOR

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A THEORETICAL AND EXPERIMENTAL EVALUATION OF THE DISTRIBUTION OF THERMALIZED 15 MeV NEUTRONS IN A FINITE WATER MODERATOR

*Dennis G. Vasilik and Richard L. Murri
and George P. Fisher, Former Dow Consultant*

Abstract. The energy-dependent diffusion equation was used to describe the spatial distribution of moderated neutrons. One of the most expedient ways of solving this equation is the multigroup method. The theory of this method is discussed and a general solution of the multigroup equation is derived. An explanation of the methods used to determine the values of the parameters needed in the solution of the multigroup equation is also presented.

The multigroup method is then applied to the problem of the thermalization of neutrons from a point source of 15 MeV neutrons in a finite water moderator. In particular, an eight-group approximation is compared to an experimentally measured thermal neutron flux distribution in the water moderator. The experimental data verify the theoretical results. A theoretical slowing-down age of 151.7 cm^2 is calculated for 15 MeV neutrons which have been slowed down below 0.55 eV.

SUMMARY AND CONCLUSIONS

The multigroup method for the solution of the energy-dependent diffusion equation is a good technique to use in problems where neutrons are slowed down in moderators which contain a number of different elements. The method is applied only with laborious calculations, but is greatly simplified by using computer techniques. This method is especially applicable to moderators which contain hydrogen. The more widely used age theory description of the distribution of thermalized neutrons is not applicable for moderators containing hydrogen.

INTRODUCTION

Neutron radiography studies have been conducted at the Rocky Flats Division of Dow Chemical U.S.A. using an accelerator neutron source and a water moderator. A theoretical analysis of the thermal neutron flux distribution in the water moderator was performed. This report discusses the experimental and theoretical analysis of the water moderated system.

The general arrangement of the radiography system is shown in Figure 1. A description of this system has been reported.¹

The yield of the accelerator was 6.18×10^{10} n/sec. The average neutron energy is 15 ± 0.25 MeV and the emission is nearly isotropic. The neutrons are generated by $\text{H}^3(\text{d},\text{n})\text{He}^4$ and $\text{H}^2(\text{H}^3,\text{n})\text{He}^4$ reactions at an accelerating potential of 180 keV.

The energy-dependent diffusion equation, described in the next section, is used to theoretically determine the distribution of moderated neutrons in a water medium.

The boundary conditions to the energy-dependent diffusion equation are chosen from the experimental arrangement shown in Figure 1.

The thermal neutron flux distribution of the bare moderator system was then determined experimentally. The bare moderator system includes only the water moderator and the fast neutron source. The shielding and collimation shown in Figure 1 are not considered to be a part of the bare moderator. This geometry approximates very nearly a point source next to a semi-infinite half-space. The experimental results were then compared to the solution of the energy-dependent diffusion equation.

THE MULTIGROUP METHOD

Multigroup Notation

The multigroup method of calculating the spatial distribution of moderated neutrons was adopted for the problem of an isotropic point source of 15 MeV neutrons in a water moderator. The convention adopted was that of Beckurts and Wirtz² and Goldstein.³ Much work has been done to apply this method to a variety of diffusion problems with fission neutron sources.³ However, very little work has been done using this method in problems with 15 MeV neutron sources.⁴

The starting point for the multigroup method is the diffusion equation:

$$\frac{dq(\bar{r}, u)}{du} + \sum_a(u) \Phi(\bar{r}, u) + \text{div } \bar{J}(\bar{r}, u) = S(\bar{r}, u) \quad (1)$$

Note: (dimensions in centimeters)

Legend

- ① Divergent Beam Collimator
- ② Accelerator Head Position
- ③ Position of Fast Neutron Source

Figure 1. Water Moderated Neutron Radiography System.

where,

$q(\bar{r}, u)$ — is the number of neutrons per $\text{cm}^3\text{-sec}$ which slow down past the lethargy u at $\bar{r}$; the lethargy is defined by:

$$u = \ln \frac{E_Q}{E} \quad (2)$$

where E_Q is an arbitrary reference energy taken to be the energy of the source neutrons.

$\Sigma_a(u)$ — is the macroscopic absorption cross section (cm^{-1}).

$\Phi(\bar{r}, u)$ — is the flux of neutrons of lethargy u at $\bar{r}$.

$\bar{J}(\bar{r}, u)$ — is the vector current density which is the net number of neutrons of lethargy u which penetrate a 1-cm^2 surface perpendicular to the distribution axis per sec at $\bar{r}$.

$S(\bar{r}, u)$ — is the source density which is the number of neutrons produced at $\bar{r}$ per $\text{cm}^3\text{-sec}$ which have a lethargy greater than u .

The terms on the left of Equation 1 represent neutrons lost per $\text{cm}^3\text{-sec}$ by means of slowing down, absorption, and diffusion, respectively. The term on the right represents all sources.

One now makes the elementary assumption of diffusion theory which relates the current density $\bar{J}$ and the flux gradient, $\nabla\Phi(\bar{r}, u)$:

$$\bar{J}(\bar{r}, u) = -D(u) \nabla\Phi(\bar{r}, u). \quad (3)$$

This equation is called Fick's Law. The diffusion coefficient is given by:

$$D(u) = \frac{1}{3\{\Sigma_{tr}(u) + \Sigma_a(u)\}} \quad (4)$$

where the macroscopic transport cross section is given by:

$$\Sigma_{tr}(u) = \Sigma_s(u)\{1 - \bar{\mu}\} \quad (5)$$

Here, $\Sigma_s(u)$ is the macroscopic scattering cross section and $\bar{\mu}$ is the average cosine of the scattering angle in the laboratory system.

Equation 1 can now be written as:

$$\frac{\partial q(\bar{r}, u)}{\partial u} + \Sigma_a(u)\Phi(\bar{r}, u) - D(u)\nabla^2\Phi(\bar{r}, u) = S(\bar{r}, u) \quad (6)$$

This equation is the energy-dependent diffusion equation and provides a description of the neutron distribution in a moderating medium. The energy-dependent diffusion equation is reduced to a finite set of ordinary differential equations by dividing the lethargy range (0 to u_{max}) into several discrete lethargy groups and integrating Equation 6 over each of these groups. If the total lethargy range is divided into n lethargy groups, the result is:

$$q(\bar{r}, u_\nu) - q(\bar{r}, u_{\nu-1}) + \int_{u_{\nu-1}}^{u_\nu} \Sigma_a(u)\Phi(\bar{r}, u)du - \int_{u_{\nu-1}}^{u_\nu} D(u)\nabla^2\Phi(\bar{r}, u)du = \int_{u_{\nu-1}}^{u_\nu} S(\bar{r}, u)du \quad (7)$$

Then, defining

$$\Phi_\nu(\bar{r}) = \int_{u_{\nu-1}}^{u_\nu} \Phi(\bar{r}, u)du \quad (8)$$

$$S_\nu(\bar{r}) = \int_{u_{\nu-1}}^{u_\nu} S(\bar{r}, u)du \quad (9)$$

$$\Sigma_{a\nu} = \frac{\int_{u_{\nu-1}}^{u_\nu} \Sigma_a(u)\Phi(\bar{r}, u)du}{\int_{u_{\nu-1}}^{u_\nu} \Phi(\bar{r}, u)du} \quad (10)$$

$$D_\nu = \frac{\int_{u_{\nu-1}}^{u_\nu} D(u)\nabla^2\Phi(\bar{r}, u)du}{\int_{u_{\nu-1}}^{u_\nu} \nabla^2\Phi(\bar{r}, u)du} \quad (11)$$

and,

$$q_\nu(\bar{r}) = q(\bar{r}, u_\nu), \quad (12)$$

equation 7 gives the following set of coupled differential equations:

$$-D_1 \nabla^2 \Phi_1 + \Sigma_{a1} \Phi_1 = S_1 - q_1 \quad (13a)$$

$$\text{and,} \quad -D_\nu \nabla^2 \Phi_\nu + \Sigma_{a\nu} \Phi_\nu = S_\nu + q_{\nu-1} - q_\nu \quad (13b)$$

where, $\nu = 2, 3, \dots, n$

The multigroup constants represented in Equations 8-12 are obtained by averaging microscopic cross section data over each energy interval. If a moderator contains strong resonance absorbers then effective cross sections must be introduced. In this case, it is assumed that $\Phi(u)$ is a constant in the individual lethargy intervals. Then, one can write:

$$D_\nu = \frac{\int_{u_{\nu-1}}^{u_\nu} D(u) du}{u_\nu - u_{\nu-1}} \quad (14)$$

$$\Sigma_{a\nu} = \frac{\int_{u_{\nu-1}}^{u_\nu} \Sigma_a(u) du}{u_\nu - u_{\nu-1}} \quad (15)$$

and,

$$\Sigma_{s\nu} = \frac{\int_{u_{\nu-1}}^{u_\nu} \Sigma_s(u) du}{u_\nu - u_{\nu-1}} \quad (16)$$

In the following section, Equations 13a and 13b will be solved for the general Φ_ν .

General Solution of the Multigroup Equations

To solve Equations 13a and 13b, an expression for the slowing-down cross section must be obtained. Since only a few rather wide groups will be used ($\Delta u \geq 1$), the probability that a neutron will jump over a group as a result of a collision with any moderator nucleus is small. One can then set:

$$q_\nu = \Sigma_{b\nu} \Phi_\nu \quad (17)$$

where $\Sigma_{b\nu}$ is defined as the "slowing-down" cross section and q_ν is equal to the number of neutrons per $\text{cm}^3\text{-sec}$ which leave group ν by elastic or inelastic scattering and enter group $\nu + 1$. Also, the number of collisions N in group ν per $\text{cm}^3\text{-sec}$ is $\Sigma_{s\nu} \Phi_\nu$. In addition, a neutron makes $N = (u_\nu - u_{\nu-1})/\xi$ collisions before it is moderated from $u_{\nu-1}$ to u_ν . Here, ξ is the average logarithmic energy decrement per collision:

$$\xi = \frac{1}{N} \ln \left(\frac{E_Q}{E} \right) \quad (18)$$

Hence, from the definition of the slowing-down density:

$$q_\nu = \frac{\Sigma_{s\nu} \Phi_\nu}{N} \quad (19)$$

Thus,

$$\Sigma_{b\nu} = \frac{\Sigma_{s\nu} \xi}{u_\nu - u_{\nu-1}} \quad (20)$$

Since the multigroup constants are now more clearly defined, one can now proceed to solve Equations 13a and 13b. Because of spherical symmetry and isotropic emission of the neutron source, the multigroup equations are a function of only the one spatial variable r .

Consider Equation 13a:

$$-D_1 \nabla^2 \Phi_1 + \Sigma_{a1} \Phi_1 = S_1 - q_1 \quad (13a)$$

From Equation 17,

$$q_1 = \Sigma_{b1} \Phi_1 \quad (21)$$

Also, for a point, isotropic source,

$$S_1 = Q\delta(r) \quad (22)$$

where Q is the source strength in neutrons per sec and $\delta(r)$ is the delta function. If one writes,

$$\Sigma_{c\nu} = \Sigma_{b\nu} + \Sigma_{a\nu}, \quad (23)$$

then Equation 13a can be written:

$$D_1 \nabla^2 \Phi_1 + Q\delta(r) - \Sigma_{c1} \Phi_1 = 0 \quad (24)$$

The general solution of this differential equation can be written as:

$$\Phi_1 = A_{11} \frac{e^{-r/X_1}}{r} + B_{11} \frac{e^{r/X_1}}{r}, \quad (25)$$

where,

$$X_{\nu}^2 = \frac{D_{\nu}}{\Sigma_{c\nu}} \quad (26)$$

X_{ν} is called the diffusion length of the neutron characteristic of the ν -th group.

The subscript notation for the A and B coefficients is defined as follows:

1. The first numeral denotes the group number
2. The second numeral denotes the coefficient identification number.

Thus, A_{11} denotes the A_1 coefficient of group one; B_{11} denotes the B_1 coefficient of group one. It is an easy matter to solve for A_{11} and B_{11} by applying the following boundary conditions:

$$\Phi_1 = 0, \text{ at } r = R_1$$

and,

$$\lim_{r \rightarrow 0} 4\pi r^2 J_1 = Q$$

The result is:

$$A_{11} = \frac{Q}{4\pi D_1 (1 - e^{-2R_1/X_1})} \quad (27)$$

and,

$$B_{11} = A_{11} e^{-2R_1/X_1} \quad (28)$$

Here, R_1 is equal to the sum of the radius R of the spherical moderator geometry (at the center of which is located the neutron source) and the extrapolated end point d_1 . The extrapolated end point is explained in detail by Davison⁵ and is assumed to be equal to $0.7104/\Sigma_{tr\nu}$ for the case of a spherical geometry of finite radius. The extrapolated end point d_{ν} is a function of the lethargy group that is being considered.

Now, consider the second group. According to Equation 13b,

$$-D_2 \nabla^2 \Phi_2 + \Sigma_{a2} \Phi_2 = S_2 + q_1 - q_2 \quad (29)$$

Since there are no sources of uncollided group 2 neutrons,

$$S_2 = 0 \quad (30)$$

Also,

$$q_2 = \Sigma_{b2} \Phi_2, \quad (31)$$

and,

$$q_1 = \Sigma_{b1} \Phi_1 \quad (32)$$

Thus,

$$D_2 \nabla^2 \Phi_2 - \Sigma_{c2} \Phi_2 + \Phi_1 \Sigma_{b1} = 0, \quad (33)$$

where the solution of the first group, Φ_1 , is coupled to the second group by Equation 32.

Substituting Equation 25 into Equation 33 produces,

$$D_2 \nabla^2 \Phi_2 + A_{11} \Sigma_{b1} \frac{e^{-r/X_1}}{r} + B_{11} \Sigma_{b1} \frac{e^{r/X_1}}{r} - \Sigma_{c2} \Phi_2 = 0, \quad (34)$$

which can be rearranged to give,

$$\nabla^2 \Phi_2 - \frac{\Phi_2}{X_2^2} = -\frac{\Sigma_{b1}}{D_2} \left\{ A_{11} \frac{e^{-r/X_1}}{r} + B_{11} \frac{e^{r/X_1}}{r} \right\}, \quad (35)$$

where,

$$X_2^2 = \frac{D_2}{\Sigma_{c2}} \quad (36)$$

The particular solution of Equation 35 is,

$$\Phi_{2p} = A_{21} \frac{e^{-r/X_1}}{r} + B_{21} \frac{e^{r/X_1}}{r} \quad (37)$$

where,

$$A_{21} = \frac{-\Sigma_{b1} A_{11} X_1^2 X_2^2}{D_2 (X_2^2 - X_1^2)}, \quad (38)$$

and,

$$B_{21} = \frac{-\Sigma_{b1} B_{11} X_1^2 X_2^2}{D_2 (X_2^2 - X_1^2)} \quad (39)$$

For the homogeneous solution,

$$\nabla^2 \Phi_2 H - \frac{\Phi_2 H}{X_2^2} = 0, \quad (40)$$

and the solution is assumed to be,

$$\Phi_{2H} = A_{22} \frac{e^{-r/X_2}}{r} + B_{22} \frac{e^{+r/X_2}}{r} \quad (41)$$

The general solution is:

$$\begin{aligned} \Phi_2 = & A_{21} \frac{e^{-r/X_1}}{r} + B_{21} \frac{e^{r/X_1}}{r} \\ & + A_{22} \frac{e^{-r/X_2}}{r} + B_{22} \frac{e^{r/X_2}}{r} \end{aligned} \quad (42)$$

To determine the A_{22} and B_{22} coefficients, two boundary conditions must be applied. First, since there are no sources of group two neutrons at the origin, one can set

$$4\pi r^2 J_2 = 0, \text{ at } r = 0$$

Here,

$$J_2 = -D_2 \frac{\partial \Phi_2}{\partial r} \quad (43)$$

The result is that,

$$B_{22} = -A_{21} - B_{21} - A_{22} \quad (44)$$

Second, at $r = R_2$, $\Phi_2 = 0$.

The result is that,

$$A_{22} = \frac{A_{21} (e^{-R_2/X_1} - e^{R_2/X_2}) + B_{21} (e^{R_2/X_1} - e^{R_2/X_2})}{(e^{R_2/X_2} - e^{-R_2/X_2})} \quad (45)$$

Considering now the third group, it follows that:

$$D_3 \nabla^2 \Phi_3 + q_2 - \Sigma_{c3} \Phi_3 = 0 \quad (46)$$

and,

$$D_3 \nabla^2 \Phi_3 + \Sigma_{b2} \Phi_2 - \Sigma_{c3} \Phi_3 = 0 \quad (47)$$

Thus,

$$\begin{aligned} \nabla^2 \Phi_3 - \frac{\Phi_3}{X_3^2} = & \frac{-\Sigma_{b2}}{D_3} \left\{ A_{21} \frac{e^{-r/X_1}}{r} \right. \\ & \left. + B_{21} \frac{e^{r/X_1}}{r} + A_{22} \frac{e^{-r/X_2}}{r} + B_{22} \frac{e^{r/X_2}}{r} \right\} \end{aligned} \quad (48)$$

In general, the solution of the $(\nu-1)$ -th equation becomes the source for the ν -th equation. Therefore, we can write for the general case of the ν -th group:

$$\begin{aligned} D_\nu \nabla^2 \Phi_\nu - \Sigma_{c\nu} \Phi_\nu = \\ - \Sigma_{b,\nu-1} \left\{ \sum_{i=1}^{\nu-1} \left(A_{\nu-1,i} \frac{e^{-r/X_i}}{r} + B_{\nu-1,i} \frac{e^{r/X_i}}{r} \right) \right\} \end{aligned} \quad (49)$$

The particular solution is assumed to be:

$$\Phi_{\nu p} = \sum_{i=1}^{\nu-1} \left(A_{\nu,i} \frac{e^{-r/X_i}}{r} + B_{\nu,i} \frac{e^{r/X_i}}{r} \right) \quad (50)$$

where,

$$A_{\nu,i} = \frac{\Sigma_{b,\nu-1} A_{\nu-1,i} X_i^2 X_\nu^2}{D_\nu (X_i^2 - X_\nu^2)} \quad (51)$$

and

$$B_{\nu,i} = \frac{\Sigma_{b,\nu-1} B_{\nu-1,i} X_i^2 X_\nu^2}{D_\nu (X_i^2 - X_\nu^2)} \quad (52)$$

For the homogeneous solution,

$$\nabla^2 \Phi_{\nu H} - \frac{\Phi_{\nu H}}{X_\nu^2} = 0. \quad (53)$$

The homogeneous solution is assumed to be:

$$\Phi_{\nu H} = A_{\nu\nu} \frac{e^{-r/X_\nu}}{r} + B_{\nu\nu} \frac{e^{r/X_\nu}}{r} \quad (54)$$

The general solution can thus be written as,

$$\Phi_\nu = \sum_{i=1}^{\nu} \left(A_{\nu,i} \frac{e^{-r/X_i}}{r} + B_{\nu,i} \frac{e^{r/X_i}}{r} \right) \quad (55)$$

To determine the $A_{\nu,\nu}$ and $B_{\nu,\nu}$ coefficients, two boundary conditions must be applied. First, $S_\nu = 0$ except for the first group where $S_1 = Q\delta(r)$. Thus,

$$4\pi r^2 J_\nu = 0, \text{ at } r = 0$$

$$\nu \neq 1$$

and, from

$$J_\nu = -D_\nu \frac{\partial \Phi_\nu}{\partial r}$$

$$B_{\nu,\nu} = -A_{\nu,\nu} \sum_{i=1}^{\nu-1} (A_{\nu,i} + B_{\nu,i}). \quad (56)$$

Second,

$$\Phi_\nu = 0, \text{ at } r = R.$$

The result is that:

$$A_{\nu,\nu} =$$

$$\sum_{i=1}^{\nu-1} \left\{ \frac{A_{\nu,i} (e^{-R_\nu/X_i} - e^{-R_\nu/X_\nu}) + B_{\nu,i} (e^{R_\nu/X_i} - e^{R_\nu/X_\nu})}{(e^{R_\nu/X_\nu} - e^{-R_\nu/X_\nu})} \right\} \quad (57)$$

Equation 55 is easily solved using computer techniques.

RESULTS

Experimental Thermal Neutron Flux Distribution in a Water Moderator Due to a 15 MeV Point Neutron Source

The experimentally determined thermal neutron flux distribution for the bare water moderator is shown in Figure 2. The values of $r^2 \Phi(r)$ are measured in the water along the axis of the moderator. $\Phi(r)$ is the thermal neutron flux in n/cm^2 -sec. The neutron source is taken as the origin. The experimental geometry is considered to be a point source next to a semi-infinite half space, 38.1-cm thick.

Figure 2. Experimental and Theoretical Distributions of Thermalized 15 MeV-Neutrons in Water.

The thermal neutron flux was obtained by subtracting epithermal contributions from the total neutron flux measured by activating bare gold foils. The gold foils used in this experiment were 0.00254-cm thick. The epithermal contribution was measured by surrounding a gold foil with a 0.051-cm thick cadmium cover. The cadmium-cutoff energy for this thickness is 0.55 eV. Only those neutrons below 0.55 eV contributed to the measured value of the neutron flux. Thermal neutrons are defined in this experiment as those neutrons which are described by a maxwellian distribution which has a most probable energy of 0.0253 eV.

The peak thermal flux was found to be 6.87×10^7 n/cm²-sec at 5.1 cm from the source. A cadmium ratio of 3.4 was measured. The thermalization factor, which is defined as the ratio of the peak thermal neutron flux to the yield of the 15 MeV neutron source, was 0.0011.

Theoretical Description of the Thermal Neutron Flux Distribution in a Water Moderator Due to a 15 MeV Point Neutron Source; The 8-Group Approximation

The theoretical thermal neutron flux distribution in a water moderator due to a 15 MeV point neutron source is also shown in Figure 2. This flux distribution was obtained from an 8-group approximation of the multigroup equations. The theoretical geometry was assumed to be that of an isotropic point source in a spherical water moderator with a radius of 38.1 cm. The source strength Q was measured by a copper-foil-activation-analysis technique to be 6.18×10^{10} n/sec.

In setting up the calculations for an n-group model, one must choose the proper energy intervals in order to be consistent with the physical situation which the theory is to represent. As discussed by Weinberg and Wigner,⁹ the lethargy width of each energy group should be equal to or greater than the highest value of the average logarithmic energy decrement, ξ , in the moderator of interest. In the case of water, the highest value of ξ is 1.0 for hydrogen. Thus, $\Delta u \geq 1.0$.

For the case of water, the groups are not difficult to choose because there are no energy values where significant resonance absorption or some physical discontinuity occurs. Between 0 and 10 MeV, the scattering of neutrons in water is highly isotropic and from 10 to 15 MeV, the scattering is slightly anisotropic. Thus, for the purpose of this report, anisotropic scattering was ignored; it needs to be taken into account only when very accurate calculations are required.

Table 1 shows the energy groups and constants that were used for the results of Figure 2. The upper limit of the 8th group was chosen to conform to the cadmium-cutoff energy of 0.55 eV used in the gold activation analysis measurements of the thermal neutron flux. The values of the constants for group 8 in Table 1 are for thermal neutrons. They are very accurately known and are tabulated in Beckurts and Wirtz.² The 8th group represents the thermal neutron flux that was measured experimentally.

The procedure for determining the values of D_ν , R_ν , $\Sigma_{c\nu}$, and $\Sigma_{b\nu}$ for each energy group is now discussed. All of these constants can be evaluated from published cross section data. The values of the cross sections used to represent a given group are those which are characteristic of the high-energy limit of the group.

The diffusion constant, D_ν , is determined from Equation 4. Equation 4 is rewritten:

$$D_\nu = \frac{1}{3(\Sigma_{tr\nu} + \Sigma_{a\nu})} \quad (58)$$

The macroscopic absorption cross section can be represented by:

$$\Sigma_{a\nu} = \Sigma_{a\nu O} + \Sigma_{a\nu H} \quad (59)$$

and,

$$\Sigma_{a\nu} = N_O \sigma_{a\nu O} + N_H \sigma_{a\nu H}, \quad (60)$$

Table 1. Constants Used for the 8-Group Calculations of the Thermal Neutron Flux Distribution in a Spherical Water Moderator of Radius 38.1 cm and a 15-MeV Point Fast Neutron Source. (Source Strength = 6.18×10^{10} n/sec.)

ν	D_ν (cm)	R_ν (cm)	$\Sigma_{c\nu}$ (cm ⁻¹)	$\Sigma_{b\nu}$ (cm ⁻¹)	Energy Group	Δu
1	3.0146	45.672	0.0288	0.0205	15.0000 MeV - 3.6680 MeV	1.41
2	1.5040	41.405	0.0881	0.0847	3.6680 MeV - 1.2700 MeV	1.06
3	1.1180	40.483	0.0800	0.0800	1.2700 MeV - 0.4450 MeV	1.05
4	0.5669	39.308	0.2693	0.2693	0.4450 MeV - 0.1500 MeV	1.09
5	0.9885	40.207	0.1741	0.1741	0.1500 MeV - 0.0407 MeV	1.30
6	0.8263	40.264	0.0349	0.0349	0.0407 MeV - 1.4000 eV	10.1
7	0.7049	39.602	0.4460	0.4460	1.4000 eV - 0.5500 eV	1.1
8	0.1440	38.406	0.0189	0.0000	0.5500 eV - 0.0000 eV	

where, oxygen is represented by O and hydrogen by H. The microscopic absorption cross section is σ_a , N_O is the number of oxygen atoms per cm^3 , and N_H is the number of hydrogen atoms per cm^3 . In water N_O is $6.7 \times 10^{22} \text{ cm}^{-3}$ and N_H is $3.35 \times 10^{22} \text{ cm}^{-3}$.

The macroscopic transport cross section can be represented by:

$$\Sigma_{tr\nu} = \Sigma_{tr\nu O} + \Sigma_{tr\nu H} \quad (61)$$

and,

$$\Sigma_{tr\nu} = N_O \sigma_{s\nu O} (1 - \bar{\mu}_O) + N_H \sigma_{s\nu H} (1 - \bar{\mu}_H) \quad (62)$$

where σ_a is the total microscopic scattering cross section. The values of $\bar{\mu}$ for oxygen and hydrogen can be calculated from the relationship:

$$\bar{\mu} = \frac{2}{3A} \quad (63)$$

where A is the atomic weight. The use of this relationship is justified in light of the slight anisotropy in scattering between 10 and 15 MeV. For oxygen $\bar{\mu} = 0.0417$ and for hydrogen $\bar{\mu} = 0.667$. These values are used for each lethargy interval.

From a knowledge of $\Sigma_{tr\nu}$ and $\Sigma_{a\nu}$, one can calculate D_ν using Equation 58.

R_ν is determined next. As previously defined,

$$R_\nu = R + d_\nu,$$

so,

$$R_\nu = 38.1 + \frac{0.7104}{\Sigma_{tr\nu}} \quad (64)$$

The next step in the procedure is to calculate $\Sigma_{b\nu}$. From Beckurts and Wirtz²

$$\Sigma_{s\nu} = N \sigma_{s\nu} \bar{\mu} \quad (65)$$

Thus, Equation 20 can be written:

$$\Sigma_{b\nu} = \frac{\xi_O N_O \sigma_{s\nu O} \bar{\mu}_O + \xi_H N_H \sigma_{s\nu H} \bar{\mu}_H}{\Delta u} \quad (66)$$

where,

$$\Delta u = u_{\nu-1} - u_\nu$$

$$= \ln \frac{E_Q}{E_{\nu-1}} - \ln \frac{E_Q}{E_\nu}$$

$$= \ln \frac{E_\nu}{E_{\nu-1}} \quad (67)$$

The values of ξ for oxygen and hydrogen as tabulated by Beckurts and Wirtz² are:

$$\xi_O = 0.12$$

and,

$$\xi_H = 1.00$$

$\Sigma_{c\nu}$ can then be calculated from Equation 23:

$$\Sigma_{c\nu} = \Sigma_{b\nu} + \Sigma_{a\nu} \quad (23)$$

The values of the microscopic scattering and absorption cross sections used to determine the values of the parameters in Table 1 are shown in Table 2 for the pertinent energy groups.

Table 2. Values of the Microscopic Scattering and Absorption Cross Sections Used to Calculate the Values of the Input Parameters which Are Shown in Table 1.

ν	Energy Group	σ_{aO} (barns)	σ_{aH} (barns)	σ_{sO} (barns)	σ_{sH} (barns)
1	15.0000 MeV - 3.6680 MeV	0.25 ^a	0.0	1.35 ^a	0.64 ^b
2	3.6680 MeV - 1.2700 MeV	0.10 ^a		3.00 ^a	2.00 ^b
3	1.2700 MeV - 0.4450 MeV	0.00		4.00 ^b	3.70 ^b
4	0.4450 MeV - 0.1500 MeV			13.80 ^b	6.50 ^b
5	0.1500 MeV - 1.0407 MeV			3.50 ^b	10.075 ^b
6	0.0407 MeV - 1.4000 eV			3.50 ^b	16.00 ^b
7	1.4000 eV - 0.5500 eV			3.80 ^b	20.50 ^b
8	0.5500 eV - 0.00005 eV			-	-

^a J. Stehn, et al., "Neutron Cross Sections," V. 1, BNL 325, 1964.

^b USAEC Selected Reference Material on Atomic Energy, Volume 5, "Neutron Cross Sections," Geneva, August 1955.

In Figure 2, the discrepancy between theory and experiment for distances less than approximately 20 cm arises because the theoretical distribution was calculated for a point source in a semi-infinite full-space whereas the experimental model was for a point source next to a semi-infinite half-space.

From the theoretical results, one can calculate the slowing-down age to 0.55 eV, τ_E , of 15 MeV neutrons in water. The slowing-down age is defined by:²

$$\tau_E = \frac{\bar{r}^2}{6} = \sum_{\nu} X_{\nu}^2$$

where X_{ν} is the diffusion length of the neutron which characterizes the ν th group, and $\bar{r}^2$ is the mean-squared slowing-down distance.

From the data presented in Table 1 for the 8-group calculations, one calculates,

$$\sum_{\nu} X_{\nu}^2 = \tau_E = 151.7 \text{ cm}^2,$$

This can be compared to the experimental results of Caswell⁴ where a flux age to indium resonance (1.46 eV) was determined to be $\tau_E = 150 \pm 6 \text{ cm}^2$. This result was for 14.1 MeV neutrons from the $\text{H}^3(\text{d},\text{n})\text{He}^4$ reaction at a deuteron energy of 150 keV.

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