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THE RADIATIVE DECAY OF NEUTRON-RESONANT STATES*

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ABSTRACT

Ways in which neutrons are employed to investigate the radiative decay of highly-excited nuclear states are discussed. Neutron capture γ -rays are useful for spin and parity assignments for such states, as well as providing some insight into the capture reaction mechanism. Recent capture γ -ray experiments and threshold photoneutron studies bear on the distribution of radiative strength with excitation energy. Intermediate structure in photon strength functions leads to an interpretation in which doorway states play an important role. Recent experiments relative to these topics are described.

I. INTRODUCTION

Although the first observation of γ -radiation following neutron capture came within a year of the identification of the neutron as an elementary particle,¹ the use of neutrons to probe the electromagnetic properties of nuclei was not really introduced until 15 years ago, with the resonance capture experiments of Landon and Rae,² and of Draper et al.³ At about the same time Bertozzi and Sargent and their collaborators⁴ used time-of-flight methods on the emitted neutrons to examine the behavior of the inverse process, the photoinduced neutron emission. In the intervening years from 1934 to 1957, and particularly since the building of research reactors subsequent to the Second World War, the chief use of neutron capture in nuclear spectroscopy was limited to populating the low-lying levels of the residual nucleus. In my review I will emphasize how neutrons may be used to provide information of the properties of the highly excited states near the neutron separation energy--that is the neutron-resonant states.

The basic tools for such studies are the same today as in the era of the experiments of Landon, Rae, Draper, and Bertozzi--but these tools have become refined in power and resolution in a dramatic manner in the last few years.

On the occasion of this conference on nuclear structure study with neutrons, it is fitting to look backwards in time, to the Antwerp Conference⁵ in 1965, and to reflect on advances in the intervening years. It is striking to observe the rather enormous development in the study of the radiative decay of highly-excited, neutron resonant states, and the clarification such studies have offered to our picture of highly-excited nuclear configurations. This has been due in large part of course to the technical advances in sources, detectors, and techniques in neutron capture γ -ray experiments and also, within the last few years, the increased use of the inverse reaction, as exemplified by the threshold photoneutron technique.

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Fig. 1 The doublet of transitions leading to the ground and first excited state in $^{56}\text{Fe}(n,\gamma)^{57}\text{Fe}$. From right to left, the curves represent the response of a magnetic Compton spectrometer (1965); early Ge(Li) detectors (1965), and recent Ge(Li) performance (1971). The gain for the curve at the left is 0.774 keV/channel. The 7645.6 and 7631.4 keV γ -rays are separated by approximately 14 keV.

(Neg. # 7-151-72)

Some idea of the refinements in the measurements of γ -rays is afforded by Fig. 1. Here I have compared the separation of the doublet of transitions to the ground and 1st excited states of ^{57}Fe , as presently achieved with commercially-available Ge(Li) detectors, with a corresponding picture shown by Vervier at the 1965 Antwerp Conference.⁵ At present an energy resolution of 0.06% is readily achievable, compared to 0.13% for the earliest Ge(Li) detector and 0.24% for the magnetic Compton spectrometer. A parallel improvement in neutron sources has occurred; the presently operating electron accelerators such as those at Oak Ridge and Livermore can produce about 2×10^{10} neutrons per burst as compared to 2×10^9 neutrons/burst available from the best linacs in 1965. Modern refinements in computer-based data acquisition systems, common in most modern laboratories, permit rapid and convenient manipulation of multi-parameter data, which is essential in many applications.

These tools of neutron resonance technology have been applied to the investigation of those states whose excitation energies lie in the range above the neutron separation energy and below the peak of the giant dipole resonance. The surpassingly excellent energy resolution afforded by neutron time-of-flight techniques, the highly intense and versatile neutron sources, and the readily available, highly efficient, and highly resolving solid state detectors combine to provide a microscopically detailed picture of levels at these high excitation energies.

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II. MEASUREMENT TECHNIQUES

Capture γ -ray experiments may be classified into three types, depending on the energy selection of the incoming neutron. In thermal capture, the neutrons are obtained from a Maxwellian distribution with a temperature such that most of the neutrons have energies close to 0.025 eV. Nuclear reactors are copious sources of such neutrons. Very little can be learned about the capturing states in these experiments, since there is no way to vary the neutron energy. They are principally useful in providing information on the final states of the residual nucleus. In the so-called average capture experiment, the sample is exposed to a broad range of neutron energies, from a few electron volts up to several MeV. In this experiment, only average properties of the capturing states can be determined. Spins and parities for final states, however, can be deduced. The most detailed information is obtained, in principle by measuring spectra of γ rays emitted as a function of incoming neutron energy. By using time-of-flight techniques to separate the capturing states, information is obtained, both for the highly excited states near the neutron separation energy and for the low lying nuclear levels. Such a technique is used at the BNL high Flux Reactor and Chalk River NRU experiments, where a neutron chopper is used to produce a pulsed neutron beam. Various other pulsed neutron sources are available, particularly at electron linear accelerators. Most notable examples of the latter are the facilities at Oak Ridge, Livermore, San Diego, Rensselaer Polytechnic Institute, Harwell, Geel, Kurchatov Institute, and Naval Research Laboratory. A significant contribution to resonance capture γ -ray experiments has originated at the IBR pulsed reactor project, Dubna. Most of these installations have been previously thoroughly reviewed.⁶

Fig. 2 A schematic representation of a slow neutron time-of flight experiment yielding γ -ray spectra as a function of neutron energy.

(Neg. # 10-233-67)

To clarify further the two-parameter nature of these experiments one may refer to Fig. 2 which is a pictorialization of the way the data are organized. In this figure the top axis represents the neutron capture cross section. A cut along this axis at a particular neutron energy may be taken for the purpose of obtaining a γ -ray spectrum. We are in a position, then, to obtain a spectrum at thermal energies, or in the resonance region, whether it be on resonance or in between resonances. In this fashion, the detailed behavior of each transition can be followed as a function of neutron energy.

The parameters of the ORELA facility are indicative of the present level of time-of-flight technology as applied to resonance capture spectra. As employed in neutron capture γ -ray measurements, the ORELA has been able to deliver some 40 kW of power on a water-cooled tantalum target, at an electron energy of 140 MeV. The 40 ns burst employed in conjunction with a 10 meter flight path, and an 800 sec^{-1} repetition rate allows reasonable spectra to be accumulated to several tens of kilovolts for light to medium-A nuclei, and up to $\sim 1 \text{ keV}$ for heavy nuclei. The neutron current produced at the 10 meter target station is about $42.8 \text{ n/cm}^2/\text{sec/eV}$ at 1 keV, and the flux distribution with energy is somewhat harder ($\sim E^{-0.8}$) than that produced by a reactor. In the low eV range the beam current available from the accelerator is a factor of 2 to 4 lower than the HFBR chopper; however, at higher energies the far superior resolution (by a factor of 125) and harder spectrum allows the accelerators to explore a much higher neutron energy region.

Complementing the time-of-flight method, a neutron monochromating diffraction spectrometer has been employed in resonance capture studies by groups at Idaho and BNL.⁶ The monochromator developed by Walter Kane at BNL provides a 2.54 cm diameter beam at the position of the monochromating crystal with an intensity of $2 \times 10^9 \text{ n/cm}^2/\text{sec}$. The diffracted beam is $2 \times 10^5 \text{ n/sec}$ in the resolution window at 1 eV. An undiffracted beam is also available, through a 38 cm quartz filter with an intensity of $10^8 \text{ n/cm}^2/\text{sec}$ and a cadmium ratio of 20,000. A detailed discussion of all possible types of information that can be obtained from spectral studies of the radiation following neutron capture is not feasible here, but the following list of experiments done at the HFBR for the past six years will give some idea of the possibilities. Some of these will be discussed in detail subsequently.

- 1) Measurements of individual partial radiative widths and their statistical properties, correlations and distributions.
- 2) Determination of the photon strength functions in the 6-8 MeV.
- 3) Resonance spin determinations from high and low-energy capture spectra.
- 4) Directional correlation experiments with unpolarized beams and $J = 1$ resonances. These may be used for initial and final state spin assignments and initial state parity assignments.

- 5) Comparison of off-resonance radiative intensities to resonance intensities. The energy dependence of radiative width amplitudes may be interpreted to infer the presence of a direct reaction component.
- 6) Measurement of isomeric life times through their effect on the time-of-flight dependence of low-energy γ -rays.
- 7) Isotopic analysis by means of neutron capture γ -rays.
- 8) Nuclear structure information by virtue of populating low-lying states of the residual nucleus and assigning their spins and parities.
- 9) Applying capture γ -ray spectral techniques to fissile targets like U-235, Pu-239 for spin determinations of resonances and investigation of the character of the fission γ -ray spectrum.

For experiments in which it proves advisable to average over many neutron resonances, without gaining microscopic detail about each one, some sort of average capture technique is employed. There are two types of reactor-based experiments in which this averaging is done by using a portion of the slowing-down spectrum associated with the moderating material: these are the in-pile target method, and the external-target, filtered beam method.

Fig. 3 The Argonne internal target capture γ -ray facility (ref. 7).

(Neg. # 2-476-70)

Figure 3 shows a schematic view of the in-pile target method as practiced by Bollinger and co-workers at the Argonne National Laboratory's CP-5 reactor.⁷ In this technique the sample is surrounded by boron-10 and placed in a high-flux region near the reactor core. The $1/v$ absorber and $1/E$ reactor spectrum effectively limit the captured neutron spectrum to a moderately narrow region peaked near 1 keV. A beam of capture γ -rays is allowed to emerge from the reactor, filtered to remove neutrons, and viewed by an annihilation pair spectrometer. A high reaction rate is obviously obtainable because of the location of the target. By removing the boron observer, high quality thermal spectra may be obtained, although some contamination by epithermal neutrons is unavoidably present. In an alternate conception, first introduced at the Idaho MTR reactor,⁸ a filtered neutron beam is taken out of the reactor to be captured by an external sample. The filter makes use of the destructive interference between resonance and potential scattering amplitudes below an s-wave resonance to form a cross section window of low attenuation. For a filter made of scandium the "window" is near 2 keV; for iron-56, the window is near 25 keV. The filters are typically 1 meter long. The scandium has a FWHM of 700 eV and provides a flux of 5×10^6 neut/cm²/sec in a 1.9 cm diameter; the iron filter has a FWHM of about 4 keV and provides a flux of 2×10^5 neut/cm²/sec (parameters refer to filters as they were installed at MTR). To remove the effects of secondary windows at higher neutron energies A1 and S components are added to the filters. The filter offers a better defined capture spectrum and one at higher energies than the in-pile technique, at considerable sacrifice in reaction rate. In principle the filtered beam allows γ - γ coincidences to be carried out; although this has not yet been achieved in practice. Such filters are becoming increasingly popular. A scandium filter has been installed at the Karlsruhe research reactor,⁹ while iron filters are being installed at research reactors in Athens,¹⁰ in Mol, Belgium,¹⁰ and at the HFBR at Brookhaven National Laboratory. Filters of scandium and iron have been installed and used at the Physical Power Engineering Institute, Obninsk, USSR.¹¹

Fig. 4 Reduced intensities derived from an average capture experiment using a scandium filter. A Monte Carlo pictorialization of the Porter-Thomas averaging is shown at the left (ref. 6).

(Neg. # 7-153-72)

Figure 4 shows an intensity plot as obtained by Greenwood⁸ at the MTR reactor, for $^{181}\text{Ta}(n,\gamma)^{182}\text{Ta}$. This illustrates the essential features of an average capture spectrum.

The "average capture" techniques, employing a broad range of incident neutron energies, allow an averaging over many individual transitions. The fluctuations of individual transition probabilities are consequently reduced; assuming Porter-Thomas distributions with comparable mean intensities the relative rms fluctuation is given by the following formula:

$$\frac{\delta I}{I} = \sqrt{\frac{2}{N}}, \text{ where } N = \text{the number of transitions involved.}$$

The resonances with $J_{\lambda}^{\pi} = I \pm \frac{1}{2}$ can decay to final states $J_f^{\pi} = I - 3/2$ to $I + 3/2$ with positive parity by electric dipole emission, or to negative parity states by magnetic dipole radiation. The "inner" spin states $J_f^{\pi} = I \pm \frac{1}{2}$ should be populated by all resonances, while the "outer" spin states $J_f^{\pi} = I \pm 3/2$ by resonances in one spin category or the other. The 2:1 intensity ratio thus divides outer and inner spin states, and the E1/M1 mean intensity ratio divides states of opposite parity. The figure shows the expected Porter-Thomas distribution of intensities for the E-1 transitions to the 2 classes of final states, illustrating the separation expected. There is some evidence in such data for the effect of the giant resonance tail in that the reduced intensities show some energy dependence. A better overall fit to the energy variation is given by an E^5 function, as expected from a Lorentzian dipole absorption peaked near 15 MeV.

Our discussion of resonant neutron spectroscopic methods would be incomplete if we were not to include a description of the threshold photo-neutron technique. Originally suggested by Bertozzi⁴ and his collaborators in 1957, the technique has recently been exploited extensively by Jackson and his collaborators, Bowman et al., and groups at Toronto and Ottawa.¹² The idea of the threshold photoneutron technique is illustrated in Fig. 5.

Fig. 5 An illustration of the threshold photoneutron method (ref.49, 50).

(Neg. # 3-786-72)

A bremsstrahlung beam of precisely determined maximum energy is employed to excite the target nucleus at just above the neutron separation energy. Only a small number of states is thereby excited and these states decay by neutron emission to the ground state of the residual nucleus or by radiative decay to lower states of the target. The resonance yield is given by integrating the Breit-Wigner cross section:

$$A(\gamma) = 2\pi^2 \lambda^2 g \Gamma_{\gamma 0} \Gamma_n / \Gamma$$

The resulting neutron spectrum can be observed with time-of-flight techniques. For all practical cases $\Gamma_n / \Gamma \approx 1$ and the yield is therefore proportional to the ground state radiation width $\Gamma_{\gamma 0}$. The angular distributions can also be measured to identify resonance spins and parities.

As an example of the apparatus used to implement this technique, Fig. 6 shows the Argonne facility. The accelerator is a 2-section L-band electron linear accelerator operating from 2-22 MeV with a pulse width of 6 ns and a repetition rate of 720 sec^{-1} . The peak current is between 10 and 20 amperes and is magnetically analyzed to achieve an energy spread of less than 300 keV. A 20 mil silver foil is used as the bremsstrahlung converter; silver having a desirably high neutron threshold.

Fig. 6 The ANL threshold photoneutron facility (ref. 49,50).

(Neg. # 3-779-72)

These experiments have been directed toward measurement of the electric and magnetic dipole radiation strengths of a variety of nuclei, as well as the distribution of strength with excitation energy. The presence of direct reaction components and intermediate structure has also been reported.

III. RADIATIVE DECAY OF INDIVIDUAL RESONANCES

Since the pioneering experiments of Landon and Rae² and Draper et al.³ in 1957, the measurement of γ -radiation following slow neutron resonances has proved an important technique for the assignment of resonance spin and parities, and the determination of statistical properties of partial radiative widths. The subject of non-statistical effects and the nature of the neutron capture mechanism is of primary interest in motivating studies of the partial decay widths. Since this subject will be treated in detail in the review of Dr. Mughabghab at this conference, I will discuss in this paper only those topics which are interpretable in terms of the conventional theory of compound nuclear states. In the next section of this paper, however, I will refer to certain structural aspects of the distribution of radiative strengths over intermediate ranges of excitation energies.

Neutron resonance spins are determined with great difficulty from total cross section measurements. The transmission effect techniques using polarized neutron beams and polarized targets require elaborate equipment. Capture γ -ray spectral measurements, however, are simple and effective. Both high energy γ -spectra and the low energy end of the spectrum can be used to make spin assignments, and recently these techniques have been extended even to the difficult cases of fissile targets.

Fig. 7 Two resonances of differing spins in ^{171}Yb .

(Neg. # 4-785-71)

A glance at Fig. 7 taken from BNL fast chopper work,¹³ shows how simple and unambiguous spin assignments can be, when they are based on the appearance of dipole transitions to final states of known spin. In this case the spectra of the $J=1$ 46 eV resonance from a Yb-171 target is dominated by transitions to the 0^+ and 2^+ members of the Yb-172 ground state rotational band; these transitions are absent in the 41 eV resonance with $J=0$.

In some cases parity assignments are also easy to make. Figure 8 shows examples, taken from the work of Wasson et al.¹⁴ on U-238 of transitions to the $5/2^+$ ground state of U-239, as seen in the 10.2 and 263 eV resonances. The strength of these E-1 transitions allow us to unambiguously assign these resonances as $p_{3/2}$, even though the resonances have extremely small neutron widths, e.g. 0.7 microvolts for 10.2 eV resonance.

Porter-Thomas fluctuations sometimes cause problems, since expected transitions may be unobservably small. Many years ago, Huizenga and Vandebosch¹⁵ pointed out that the population of isomeric states by neutron capture should depend on the spin of the capturing state. This idea was later refined by Poenitz.¹⁶ The concept is simple: the state whose spin is closest to that of the compound state is populated most heavily, since there are available more cascades to reach that state. The principle is illustrated in Fig. 9 for Yb-173, taken from BNL fast chopper work.¹⁷ In each case the decay of the state of a given spin is compared to another and the dependence on initial state spin is obvious. In each case the intensity corresponding to the decay of the state whose spin is closest to the spin of the initial capturing state is larger.

Fig. 8 Examples of the transition to the $5/2^+$ ground state of ^{239}U . The 263 eV and 10.2 eV resonances of ^{238}U must have $J, \pi = 3/2^-$.

(Neg. # 1-172-71)

Fig. 9 Low energy γ -rays from $^{173}\text{Yb}(n,\gamma)^{174}\text{Yb}$. These are useful in assigning resonance spins.

(Neg. # 3-252-71)

Many groups have been successfully applying this idea. An example is the experiment of Stolovy et al.¹⁸ at the Naval Research Laboratory's linear accelerator on the spins of resonances in ^{143}Nd and ^{145}Nd . This experiment is especially interesting in that it compares the low energy γ -ray ratio method with another indirect method, which operates under the assumption that the multiplicity of γ -rays is dependent on resonance spin. Figure 10 shows a comparison of the two methods for ^{143}Nd . Clearly the low energy γ -ray ratio method gives a better discrimination between the two spin possibilities.

These methods have been extended to fissile elements, where spin assignments are desired for confirmation of the channel theory of fission and for multilevel analysis of the resonance structure. In spite of the intense competition from fission γ -rays, it is possible to assign spins by these methods. In Fig. 11 we see one of the resonance spectra of Graves et al.¹⁹ obtained at the HFBR fast chopper for U-235. Here there is evidence for a 5588 keV γ -ray populating a doublet of 2^+ states near 957 keV from the 2.04 resonance, suggesting a 3^- assignment for this resonance. Six resonance spins were assigned in this way. More recently Corvi et al.²⁰ have studied the de-excitation of the 6^+ member (160.6 keV) of the ground state rotational band of U-236 compared to the de-excitation of the 2^- level, at 687 keV which emits a 642 keV γ -ray to the first excited state.

Fig. 10 The low-energy γ -ray ratio method compared to a γ -ray multiplicity method for assigning spins in ^{143}Nd . The multiplicity method is shown at the top of the figure; below is the ratio method (ref. 18).

(Neg. # 7-277-72)

Fig. 11 A portion of the γ -ray spectrum following neutron capture in the 2.04 eV resonance of ^{235}U , which has $J=3$. The peak near 5588 keV represents transitions to a doublet of 2^+ levels near 957 keV.

(Neg. # 7-146-72)

Although several fission and background γ -ray peaks lie near the 160.6 keV line, Fig. 12 shows a 2:1 intensity variation for this line, as compared to the 642 keV monitor line, depending on resonance spin. As expected the 6^+ state is more heavily populated from $J=4$ resonances. Corvi et al. have assigned spins to 14 resonances in U-235 by what appears to be a reliable, effective, simple, method. The resonances with large fission widths, however, are still difficult to assign because of the fission background problem.

The ability to determine resonance spin and parity is much enhanced for $l \neq 0$ resonances by the method of particle- γ correlations. This method has found application not only in neutron capture spectra, but also has proved essential in interpreting the results of the threshold photoneutron technique, where resonances of appropriate spin and parity must be classified.

Ferguson²¹ has given the necessary theory for particle- γ correlations and shows that the correlation between neutron and photon directions is given by the following expression:

$$W(\theta) = \sum (-1)^{a-c} (16\pi^2 (2a+1))^{-1} \bar{Z}(\ell_1 b \ell_1' b'; a k) \\ \times \bar{Z}_1(L_2 b L_2' b'; c k) \langle b || \ell_1 | a \rangle \langle b' || \ell_1' a \rangle^* \\ \times \langle c || L_2 || b \rangle \langle c || L_2' || b' \rangle^* P_k(\cos \theta);$$

Fig. 12 Portions of the low energy spectrum following neutron capture in ^{235}U . The ratio of the 160.6 keV to the 642.4 keV lines is an indicator of resonance spin (ref. 20).

(Neg. # 7-142-72)

where a is the channel spin
 b, c are spin of intermediate and compound states
 l the particle orbital momentum
 L the photon multipolarity

The Z coefficients are defined as products of vector coupling coefficients and are tabulated by Ferguson. For dipole emissions, only terms up to $k=2$ need be considered, and since interference between compound states may be ignored on resonance, only even k need be considered: for this case $W(\theta) = a + b \sin^2 \theta$ and only measurements at 2 angles suffice. As an example consider Fig. 13 showing a resonance in Sn-122 at 106.9 eV, taken from BNL fast chopper experiments.²² The ratio of the 5798 and 5924 keV transitions is clearly different at 90° and 135° to the beam, allowing us to identify this resonance as a p -wave resonance with $J=3/2$.

The method can be applied to non-spin-zero target nuclei to fix the relative proportions of channel spin admixtures for each resonance.²³ For example in ^{93}Nb , p -wave capture can take place in the resonance at 35.8 with $J=5$ either via channel spin 4 or 5. Figure 14 shows the various possible γ -ray distributions for decays to final states of spin 4, 5 or 6^+ . For any given resonance a mixture of channel spins is present, so that for an unpolarized beam:

Fig. 13 A comparison of the 5798 keV transition ($p_{3/2} \rightarrow s_{1/2}$) and the 5924 keV transition ($p_{3/2} \rightarrow d_{3/2}$) in $^{122}\text{Sn}(n, \gamma)^{123}\text{Sn}$. Angles of emission of 90° and 135° to the incident neutron beam are shown.

(Neg. # 1-361-72)

Fig. 14 Angular distributions for $5^- \rightarrow 6^+$, $5^- \rightarrow 5^+$, and $5^- \rightarrow 4^+$ transitions for channel spins $s = 5$ and 4 . Also shown is the transition $4^- \rightarrow 5^+$.

(Neg. # 3-657-72)

$$\bar{W}(\theta) = g(S=4)W_4(\theta) + g(S=5)W_5(\theta)$$

$$\text{where } g = \frac{(2S+1)}{(2s+1)(2I+1)}.$$

Figure 15 shows the experimentally measured ratios as a function of $\alpha \equiv \Gamma_n(S=5)/(\Gamma_n(S=4) + \Gamma_n(S=5))$ at 35.8 eV. These results are all consistent with the statement that the formation of the 5^- resonance takes place through the $S=5$ channel with a probability of 70%. This is a good example of the kind of measurement not possible with conventional neutron cross section techniques.

IV. STRUCTURE OF THE PHOTON STRENGTH FUNCTIONS

I will now discuss some problems in which the average radiative intensities over some defined region of excitation are of interest. Much interest has been focused on the E-1, M-1 and E-2 photon strength functions, which are conventionally defined as follows.²⁴

Fig. 15 The ratio of channel spin admixture ($\alpha = \Gamma_n(s=5)/(\Gamma_n(s=5) + \Gamma_n(s=4))$) for the 35.8 resonance of ^{93}Nb .

(Neg. # 3-932-72)

$$k_{E_1} = \langle \Gamma_Y(E_1) \rangle / DE_A^{3/2} \quad \dagger$$

$$k_{M_1} = \langle \Gamma_Y(M_1) \rangle / DE^3$$

$$k_{E_2} = \langle \Gamma_Y(E_2) \rangle / DE_A^{5/4}$$

Γ_Y in eV

D, E in MeV

† occasionally an alternate definition is used:

e.g. $f(E_1) = \langle \Gamma_Y \rangle / DE^3$

Γ_Y, D, E in MeV

The distribution of the radiative strength with excitation energy has been investigated fairly thoroughly since the late 40's when photoabsorption experiments were first employed by Baldwin and Klaiber²⁵ to investigate nuclear structure. The great advantage of the use of electromagnetic interaction as a nuclear probe lies in the fact that is a well-understood interaction with a known Hamiltonian. The observed giant resonance peak seen in betatron experiments was easily explained by the induced electric dipole moment of interpenetrating neutron and proton fluids in a hydrodynamical nuclear model. The explanation in terms of a shell model was given in terms of a coherent superposition of particle-hole states. The residual interaction between particle and hole is responsible for the shift in resonance energy in photoabsorption far above the value characteristic of major shell energy spacing. The existence of the various dipole sum rules and the measured photon absorption cross sections lead naturally to the conclusion that there is only about 1-2% of the total dipole strength left in the energy range near neutron emission threshold.

Danos and Greiner²⁶ suggested that because of time reversal invariance and the zero photon rest mass, the form of the absorption cross section is Lorentzian:

$$\sigma_f = \sigma_0 \frac{E^2 \Gamma^2}{(E_0^2 - E^2)^2 + E^2 \Gamma^2}$$

if a constant damping with Γ is assumed. For deformed nuclei a superposition of two Lorentz cross sections is suggested.

We can consider E-1 photon absorption as forming a doorway state whose strength is subsequently distributed through a series of interactions among fine structure compound states occurring at high excitation energy. Thus the strength function Γ_Y/D for these fine structure states can be related to the photon absorption cross section. Indeed for most of the energy range from the neutron separation energy to just beyond the giant resonance peak, single neutron emission is the dominant decay process.

Hence using (γ, n) with the well developed neutron time-of-flight measurement techniques, a powerful tool is available. Furthermore as Axel²⁷ showed, the Brink hypothesis of a giant resonance built on each excited state allows us to interpret the distribution of radiation strength among the exit channels of an (n, γ) reaction in terms of the giant resonance phenomenon.

Neutron capture reactions can thus shed considerable light on the distribution of radiation strength at and below the neutron binding energy, while photoneutron reactions are useful at higher excitations. The exceedingly high resolution available with neutrons allows us to microscopically examine the radiation strength distribution among the fine structure resonances.

There are two kinds of questions to be answered here:

- 1) Is the overall distribution (averaged over the fine structure), well described by the tail of the Lorentzian?
- 2) Is the distribution over fine structure states statistical, or are these nuclear structure effects? In particular is there evidence for particle-hole annihilations characteristic of the shell model transitions? Are there other doorway states characteristic of more complicated excitations?

To illustrate these ideas let us examine Fig. 16 taken from the work of Veyssiere et al.²⁸ for photoabsorption in ^{208}Pb . We can see the giant resonance well represented by a Lorentzian, but on the lower energy end, structure appears, from 7-11 MeV, seen with a resolution of approximately 140 keV. Just above threshold this structure has further been resolved by Bowman et al.,²⁹ and Bertozzi et al.,³⁰ (Fig. 17) using the threshold photonuclear technique; resolution broadening of this fine structure

Fig. 16 The total photo-excitation cross section of ^{208}Pb as measured by Veyssiere et al. (ref. 28).

(Neg. # 7-147-72)

Fig. 17 A portion of the ^{208}Pb photoexcitation cross section showing the high resolution photo-neutron results of (1) Bowman et al. (ref. 29) and (2) Bertozzi et al. (ref. 30). These are broadened to compare to the total cross sections of Veyssiere (ref. 28).

(Neg. # 7-148-72)

yields good agreement with the results of Veyssiere. This is demonstrated in Fig. 17. The interpretation of the gross structure peaks by Veyssiere is that they correspond to $1p-1$ hole excitations, several of which have unperturbed energies in this region, as shown in the following table.

Particle-Hole States in ^{208}Pb

<u>Proton</u>		<u>Neutron</u>	
7.69	$p_{3/2} d_{3/2}^{-1}$	7.7	$i_{11/2} h_{9/2}^{-1}$
7.70	$h_{9/2} g_{9/2}^{-1}$	8.25	$g_{7/2} f_{7/2}^{-1}$
8.18	$p_{1/2} s_{1/2}^{-1}$	9.4	$g_{7/2} h_{9/2}^{-1}$
8.53	$p_{1/2} d_{3/2}^{-1}$		

Additional evidence for a local concentration of photon strength function corresponding to single-particle resonance is that reported by K. H. Lokan and co-workers³¹ at Ottawa. They find that the photo-absorption cross section on ^{40}Ar , leading by neutron emission to the ground state of ^{39}Ar shows a peak (Fig. 18) near 10-11 MeV which contains about 2% of the exchange-augmented dipole sum rule. The peak appears to have a width appropriate to a single

Fig. 18 The photoneutron results of Lokan et al. (ref. 31) on ^{40}Ar . The derived photoexcitation cross section in the inset suggests a single particle peak near 11 MeV.

(Neg. # 7-150-72)

particle resonance, and neutron emission to the ground state is emphatically favored. The authors interpret this behavior as resulting from the photon interaction raising one of the two valence $1f_{7/2}$ neutrons in ^{40}Ar to the $1g_{9/2}$ shell. The 27 resonances excited in the 2 MeV interval from 10.4 to 12.4 MeV possess a $\Gamma_{\gamma g}/D$ value of 8×10^{-5} which is in reasonable agreement with values observed in neutron capture. Taken with evidence from p-wave neutron capture in the molybdenum region, as well as from the investigations of Bartholomew and co-workers³² from (d,p), and (n, γ) reactions in heavier nuclei, it seems now quite clear that some remnants of single particle strengths remain at the unperturbed shell-model energies, and also that some intermediate structure may appear as well. These simple excitations will result in width correlations and dependences on the character of the final states.

Of course a more definitive experiment would require the spin and parity identification for each fine structure resonance. Furthermore the interpretation would call for a common doorway in both photon and neutron channels.

Lane³³ and Beer³⁴ have shown that the presence of doorway states is manifested by the appearance of correlations between partial widths. If there exist two reaction channels c,c' which behave the same way over some range of the fine structure resonances λ , then:

$$\gamma_{\lambda c} = \langle \lambda | d | \gamma \rangle \gamma_{dc}$$

$$\gamma_{\lambda c'} = \langle \lambda | d | \gamma \rangle \gamma_{dc'}$$

Such a doorway is termed "common" to channels c, c' . A correlation coefficient of unity is implied for the above case: a smaller coefficient would imply that the structure of the fine resonances is such that more complicated excitations also play a role in the decay processes. Lane has also shown that the presence of correlations implies the presence of a background component between resonances; the size of this component can be estimated from the relation:

$$\sigma_{cc'}(\text{background}) = \rho(\Gamma_{\lambda c}^{1/2}, \Gamma_{\lambda c'}^{1/2}) \sigma_c(\text{abs}) \frac{\Gamma_{\lambda c'}}{D} \frac{\pi}{2} \left(\frac{\text{Re} R_{cc'}}{\text{Im} R_{cc'}} \right)^2$$

where
$$\sigma_{\text{abs}} = g \frac{2\pi^2}{k_c^2} \frac{\langle \Gamma_{\lambda c} \rangle}{D}$$

One recent photoneutron experiment can be interpreted as showing considerable support for this picture, indicating that the radiative strength is non-uniformly distributed over resonances, and that this strength is concentrated into definite energy regions giving rise to intermediate structure. I refer here to the work of Jackson and Toohey in $^{29}\text{Si}(\gamma, n)$ carried out at the ANL photo-neutron facility.³⁵

Jackson observes a concentration of radiative strength at the same excitation as that proposed by Newson³⁶ for a neutron doorway in the reaction $^{28}\text{Si}+n$. This concentration is illustrated in Fig. 19. From angular correlation measurements on the emitted neutrons, the resonances are shown to be mostly $3/2$ in character. The data suggest the presence of an isolated doorway corresponding to a $2p_{3/2}$ neutron coupled to a ^{28}Si core. A calculation of the radiative width corresponding to a valence neutron strength of such a state comes to about 10% of the expected value.

Fig. 19 A concentration of E-1 Strength in $^{29}\text{Si}(\gamma, n)^{28}\text{Si}$ from the work of Jackson and Toohey (ref. 35). The inset shows the effect of a background cross section on the shape of the resonance at 761 keV.

(Neg. # 7-155-72)

In Fig. 20 is a comparison of the (γ, n) and (n, n) widths for the resonances. A striking correlation is apparent, and although the sample is a small one, the authors believe that there is a chance of 5% or less that the observed correlation of ~ 0.9 could be ascribed to a statistical fluctuation.

The observed asymmetry of the 761 keV level in Fig. 19 is explicable in terms of a background amplitude corresponding to a total photoneutron cross section of 0.32 mb. The estimate from the above equation is about ~ 0.1 mb, sufficiently close to the observed value to establish, at least qualitatively, the relationship between correlations and background cross section. There are, in summary, 3 effects which support the above interpretation:

- 1) localized neutron and photon strengths
- 2) correlations between reaction channels
- 3) background component present

Structure in the photon absorption cross section would imply a redistribution in photon strength functions as measured in the inverse reaction. Departures from the smooth trend expected from the Lorentzian tail of the giant resonance were noticed many years ago in the thermal neutron capture γ -ray spectra by Groshev and Bartholomew.³⁷ In the region between tantalum and thallium--that is just below the $N=126$ shell closure--an anomalous bunching of strength was noticed. This bunching is indicated in Fig. 21 in which the experimental data are compared to a prediction from a smoothed giant resonance and a level-density formula of the Gilbert and Cameron type, normalized to experimental densities at and below neutron threshold.

This bunching has been confirmed by studying various reaction mechanisms including (d, py) and photo-excitation experiments. As pointed out by Lundberg and Starfelt,³⁸ and confirmed recently by the Chalk River

Fig. 20 Evidence for a common doorway in ^{29}Si (ref. 35).

(Neg. # 3-783-72)

Fig. 21 Evidence for a non-statistical distribution in E-1 radiative strength in the mass region from thallium to tantalum (ref. 39).

(Neg. # 7-143-72)

group,³⁹ the position of the anomaly does not change with excitation energy. This is illustrated in Fig. 22, taken from recent CRNL work,³⁹ showing unfolded NaI spectra obtained from fast neutron capture. The anomaly seen in thermal capture persists up to 2.6 MeV of neutron energy.

High resolution experiments on Tl in the region $E_n = 0$ to 100 keV, done at the Oak Ridge Electron Linear Accelerator,⁴⁰ have established that the anomalous bump is composed of primary radiation of the electrical dipole type. The multipolarity had been confirmed earlier in Chalk River experiments on Hg²⁰⁰, using coincidence and polarization correlation techniques.⁴¹ Presumably, then, we may assume the primary and E-1 natures of the anomalous radiation throughout this region of mass number.

The simple interpretation of the anomaly has been given in terms of the formation of two particle-1 hole doorway states by neutron capture. The subsequent particle-hole annihilation would produce radiation corresponding to the energy difference of the major closures of the shell model near $N=126$.

Bartholomew has recently advanced a slightly modified view of the nature of this anomalous radiation.³⁹ The redistribution of radiation strength is conceived to be a sudden diminution in strength function below what is expected from the dipole resonance extrapolation below 5 MeV, rather than an excess over what is predicted above that energy. This re-interpretation is partly required by the recent measurements of the

Fig. 22 Capture spectra from thallium at various neutron energies compared to the predictions of the statistical model (ref. 39).

(Neg. # 7-141-72)

Fig. 23 Gamma ray intensities for ^{182}Ta and ^{198}Au as determined by the average capture method (ref. 42).

(Neg. # 7-368-72)

Bollinger group on thermal and average capture, in which discrete lines are measured with high resolution Ge(Li) detectors. The results of the average capture measurements of Loper, Bollinger, and Thomas⁴² are presented in Fig. 23. Here the results for gold are compared to tantalum, which is known to present a "bumpless" spectrum. As the figure shows, gold behaves just like tantalum. Moreover it has been established that the absolute value of the E-1 strength function in this region is well-predicted by the giant resonance extrapolation. Thus the departure from the tail must occur for energies below 5 MeV. This interpretation would remove a long-standing controversy between the Argonne and Chalk River groups on this question.

Bartholomew has recently estimated that the area of strength redistribution represented by the effect represents anywhere from about 25% at Tl down to near zero of the predicted E-1 strength below the neutron separation energy for Ta (Fig. 24). It is worth bearing in mind that we are discussing no more than about 1 to 2% of the total giant dipole strength when we integrate the strength function up to the neutron threshold.

Although confirmation of the anomaly has been achieved from (d, p_γ) , (γ, γ_0) , (n_{fast}, γ) experiments, it is remarkable that the anomaly is not seen in $(n, n'\gamma)$, $(p, p'\gamma)$, and u^- capture.³⁹ This fact may be a manifestation of a breakdown of the statistical model of the compound nucleus in that the photon strength function may be structure dependent. In this event different reactions pick out states whose differing structures would lead to differing radiative decay patterns.

It has been known for some time that the energy variation of the strength function, as observed in average capture measurements, is not always consistent with the giant resonance extrapolation. This is illustrated in measurements by Smither⁴³ on capture in the isotopes of samarium. Although he finds that the photon intensities follow a "normal" E^5 variation for the even-even compound nuclei ^{148}Sm and ^{150}Sm , the variation

Fig. 24 The size of the anomaly in the E-1 strength function for $A=180$ to 208 (ref. 39).

(Neg. # 7-152-72)

Fig. 25 Non-Lorentzian behavior of the E-1 strength function for odd A samarium isotopes. From the work of R. K. Smither, ref. 43.

(Neg. # 7-280-72)

is more like E^7 for ^{151}Sm and ^{153}Sm . The effect is illustrated in Fig. 25. It is perhaps significant that the redistribution of radiative strength seen by Bartholomew in the tantalum-to-thallium region would result in a similarly strong variation in the energy dependence of the transition probabilities.

The previous remarks have been confined to E-1 radiation. Somewhat less is known about M-1 strengths, although for many cases they are comparable in strength. The suggestion of an M-1 giant resonance has been made on several occasions to explain the anomalously large M-1 widths reported, for example in $^{136}\text{Ba}^{44}$ and in the tin isotopes.⁴⁵ A spin-flip mechanism was first suggested by Mottelson⁴⁶ in terms of the shell model. Later Shapiro and Emery⁴⁷ calculated M-1 strengths for deformed nuclei, on the basis of two quasiparticle excitations. They concluded that the energy of the two-quasiparticle excitations should be raised up by an MeV or so by the inclusion of a long-range spin-spin interaction between the neutron and proton pairs. The result is a resonance-like shape for k_{M1} , peaking near 6 MeV for the heavier nuclei. Results from recent experiments are at least in qualitative agreement with the predictions, although the detailed energy dependence has not been verified. For example Shapiro and Emery predict $k_{M1} \approx 20 \times 10^{-3}$ near 4.8 MeV in ^{239}U . Wasson et al.¹⁴ report a value of 24.5×10^{-3} from measurements of partial widths of discrete resonances in ^{238}U , while Bollinger and Thomas⁴⁸ get 15.9×10^{-3} from thermal and average capture measurements. For the neighboring nuclide ^{240}Pu , Ottmar et al.⁹ have done an average capture experiment with a scandium filtered

Fig. 26 Anomalous concentrations of M-1 strength in ^{57}Fe . From photoneutron experiments of Jackson and Strait (ref. 49, 50).

(Neg. # 3-933-72)

beam at Karlsruhe, obtaining a value of $\approx 30 \times 10^{-3}$ near 5.5 MeV. This value fits in with the expected energy trend for an M-1 peak near 6 MeV.

For spherical nuclides, and nuclides in the light mass region, considerably more complex structure in the M-1 strength function may be expected. Jackson and Strait^{49,50} report the presence of very sharp concentrations of M-1 excitation for p-wave levels in ^{57}Fe . These are shown in Fig. 26 where the region near 600 keV is covered with a proton-recoil type detector, and the region near 200 keV has been measured with a Li-6 glass scintillation detector. The unusual concentrations of p-wave neutron emission are visible near 230 and 600 keV. Such concentrations are also noted in the neighboring nuclides ^{53}Cr and ^{61}Ni . Spin and parity assignments were made through angular correlations and comparison with neutron total cross section data on ^{56}Fe . Although the integrated M-1 strength in ^{57}Fe is not unusually large, $k_M = 10^{+10}_{-3} \times 10^{-3}$, it is found to be concentrated in a doublet of $3/2^-$ levels at 224 and 235 keV, and a group near 606 keV. This grouping of strength may be analyzed in terms of statistical fluctuations, and in both instances cannot be reconciled with normal Porter-Thomas fluctuations. The structure is interpreted by Jackson as due to a 2 quasiparticle doorway consisting of a $(f_{5/2})^{-1}$ excitation coupled to the ^{57}Fe ground state. Both the $1/2^-$ and $3/2^-$ resonances can be formed from this particle-hole coupling. This situation suggests that the spreading width of this doorway in ^{57}Fe is small enough to invest most of the doorway strength one or two levels.

Fig. 27 Anomalous M-1 strength concentrations in ^{208}Pb , from the work of Berman et al. (ref.51).

(Neg. # 7-260-72)

A much higher concentration of strength has been reported by Berman et al.⁵¹ for the case of $^{208}\text{Pb}(\gamma, n)$, at an excitation of about 8.3 MeV, or 900 keV above the neutron separation energy for ^{207}Pb . This is shown in Fig. 27. The total γ -ray strength in this region is deduced to be 50.8 eV (5 Weisskopf units). This strong enhancement represents at least half the total M-1 strength for lead and strongly suggests a collective doorway state excited by spin flip transitions from the $i_{13/2}$ neutron shell and $h_{11/2}$ proton shell. A similar concentration of M-1 strength near 8 MeV has been postulated by Winhold et al.⁵² to account for broad peaks in the photoneutron yields of ^{117}Sn and ^{119}Sn . The peaks were not identified as to spin and parity, but if they were $1/2^+$ in character they could be ascribable in terms of $g_{9/2} \rightarrow g_{7/2}$ proton transitions, as might be expected in the tin region.

Only a few values for E-2 strengths have so far been reported.⁷ Values of k_{E-2} obtained by the average capture method for ^{106}Pd , ^{166}Ho and ^{168}Er are in the range 30 to 60 $\times 10^{-9}$. However, recent measurements of E-2 widths for ^{239}U are considerably larger. From measured partial widths from microscopic resonances, Wasson et al.¹⁴ deduce a value of $(300 \pm 100) \times 10^{-9}$, while from the average capture method, Bollinger and Thomas⁴⁸ get $(280 \pm 100) \times 10^{-9}$. The significance of the relatively large value of k_{E-2} for ^{239}U is not known; there exists no model with which to compare the data.

This survey of recent experiments involving the radiative decay of highly excited states produces the inescapable conclusion that radiation strengths are not distributed in a statistical manner. Anomalous concentrations of E-1 and M-1 strengths appear to be attributable to transitions between simple nuclear configurations which play the part of doorways in the formation of the compound nuclear states.

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