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STRUCTURAL DESIGN OF NUCLEAR ROCKET FUEL ELEMENTS

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ABSTRACT

This paper presents a status report on NERVA fuel element analytic techniques, including stress calculations in a complicated stress field by finite element methods, the problem of defining a satisfactory criterion of failure, and the application of state-of-the-art probabilistic techniques in predicting the fuel reliability.

The energy source in the NERVA nuclear rocket engine is a high power density reactor that operates at an impressive temperature level (nozzle chamber temperature of 4250°R) and uses hydrogen as the propellant. The fuel elements are graphite, fueled by enriched uranium carbide which is dispersed through the volume in bead form or in solid solution with another carbide. The elements are hexagonal, about three-quarters of an inch across flats, about 52 inches long, and contain 19 circular coolant channels. At the NERVA operating temperatures, the carbon in the fuel elements is highly reactive with the hydrogen propellant, with hydrocarbons like methane and acetylene formed and exhausted at the expense of fueled material. As a corrosion reduction measure, the fuel is coated with a refractory carbide.

The principal causes for stresses in the elements are: (1) the high power density, (2) the interaction between the fuel matrix and the corrosion resistant carbide coating, and (3) the interaction between adjacent elements due to inevitable element-to-element temperature differences. The method of calculating the combined stress field due to these mechanisms and others of less importance is discussed.

NOTE: The Nuclear Engine for Rocket Vehicle Application Program (NERVA) is administered by the Space Nuclear Propulsion Office (SNPO), a joint office of the U. S. Atomic Energy Commission and the National Aeronautics and Space Administration. Aerojet Nuclear Systems Company as prime contractor for the engine system and Westinghouse Electric Corporation as subcontractor for the nuclear subsystem, are developing a nuclear propulsion system for space applications.

The conversion of the calculated stresses to a safety factor or a reliability by means of an adequate criterion of failure is discussed. The possible criteria of failure for the brittle graphite elements are explored with comments on the feasibility of each criterion. Possible approaches are: (1) fracture mechanics, (2) Weibull (weakest link) statistics, (3) maximum principal stress, (4) maximum principal strains, and (5) a strain energy criterion.

The final topic is a description of the probabilistic techniques incorporated into the NERVA fuel stress analysis. Both the mean values and variances of the individual stress components are calculated and are then combined statistically to define the failure-governing stress. This stress and the component strength, both defined probabilistically, are subsequently compared by the "interference theory" or "stress-strength overlap" method to predict the fuel reliability.

I. INTRODUCTION

This paper presents a status report on NERVA fuel element analytic techniques, including stress calculations in a complicated stress field by finite element methods, the problem of defining a satisfactory criterion of failure, and the application of state-of-the-art probabilistic techniques in predicting the reliability of the fuel.

The energy source of the NERVA nuclear rocket engine is a high-power-density reactor that operates at an impressive temperature level (nozzle chamber temperature of 4250°R) and uses hydrogen as the propellant. The NERVA nuclear subsystem is shown in cutaway form in Figure 1. chamber temperature of 4250°R) and uses hydrogen as the propellant. The NERVA nuclear subsystem is shown in cutaway form in Figure 1.

The fuel configuration is shown in Figure 2. The fuel elements are graphite, fueled by enriched uranium carbide which is dispersed through the element volume in bead form or in solid solution with another carbide. The elements are hexagonal, about three-quarters of an inch across flats, about 52 inches long, and contain 19 circular coolant channels. At the NERVA operating temperatures, the carbon in the fuel elements is highly reactive with the hydrogen propellant, with hydrocarbons like methane and acetylene formed and exhausted at the expense of fueled material. As a corrosion reduction measure, the fuel is coated with a refractory carbide. The presence of the carbide coating, as a material differing in stiffness and thermal expansion from the fuel but intimately attached to the fuel, naturally complicates the fuel stress analysis.

The principal causes for stresses in the elements are: (1) the high power density, (2) the interaction between the fuel matrix and the corrosion-resistant carbide coating, and (3) the interaction between adjacent elements due to inevitable element-to-element temperature differences.

FUEL STRESS MECHANISMS

First discussing the stress mechanisms contributing to transverse stress in the element, there is the internal heat generation stress, represented by the expression

$$\sigma_{q'''} = \frac{E\alpha}{k} \cdot q''' \cdot \psi ,$$

where

$$\begin{aligned}
 \psi &= F(L^2), \text{ geometric shape factor,} \\
 \sigma_q''' &= \text{maximum thermal stress,} \\
 E &= \text{modulus of elasticity,} \\
 \alpha &= \text{coefficient of thermal expansion,} \\
 k &= \text{thermal conductivity,} \\
 q''' &= \text{heat generation rate,} \\
 L &= \text{characteristic length.}
 \end{aligned}$$

The other large contributor to element transverse stresses is the coating-matrix interaction stress. The expression for this stress component is

$$\sigma_{c \cdot m} = E_m \left[-\alpha_m \Delta T + \Delta \alpha T_c - \Delta \alpha_d T_d \right] \cdot \phi,$$

where

$$\begin{aligned}
 \sigma_{c \cdot m} &= \text{coating-matrix interaction stress in the matrix,} \\
 E_m &= \text{matrix modulus of elasticity at the matrix average temperature, } T_m, \\
 \alpha_m &= \text{matrix coefficient of thermal expansion at temperature } T_m, \\
 \Delta T &= \text{temperature difference between the matrix and the coating} = T_m - T_c, \\
 T_m &= \text{average matrix temperature,} \\
 T_c &= \text{average coating temperature,} \\
 \Delta \alpha &= \text{difference in coefficient of thermal expansion between matrix and coating} = \alpha_c - \alpha_m, \\
 \alpha_c &= \text{coating coefficient of thermal expansion at temperature } T_c, \\
 \Delta \alpha_d &= \text{difference in coefficient of thermal expansion between matrix and coating at coating deposition temperature } T_d = \alpha_{cd} - \alpha_{md}, \\
 T_d &= \text{coating deposition temperature,} \\
 \phi &= \text{geometry and relative stiffness factor between coating and matrix.}
 \end{aligned}$$

Our present best model of this complicated stress component is based on three assumptions:

- a. The coating and substrate interact elastically with one another.
- b. The coating is perfectly adherent.
- c. The coating-matrix assembly is in a stress-free condition at the temperatures which prevailed during the final coating deposition process (a chemical vapor deposition operation).

Since, as has been mentioned, the fuel in the graphite elements is in the form of beads, there is a fuel bead-matrix interaction stress, calculated by the expression

$$\sigma_{fb} = E_m \left[\Delta\alpha (T_m - T_g) + \alpha_b \cdot \Delta T \right] \cdot \theta \text{ for } T_m \geq T_g,$$

where

- σ_{fb} = fuel bead-matrix interaction stress in the matrix,
- E_m = matrix modulus of elasticity at the matrix temperature, T_m ,
- $\Delta\alpha$ = difference in coefficient of expansion between bead and matrix at temperature $T_g = \alpha_b - \alpha_m$,
- T_g = graphitization temperature,
- α_b = bead coefficient of thermal expansion,
- ΔT = difference between bead temperature and matrix temperature = $T_b - T_m$,
- θ = geometry and stiffness factor.

It is assumed for the purposes of this calculation that the stress-free condition is the fuel graphitization temperature.

A final transverse stress mechanism is the transverse compressive bundling load placed on the fuel elements for the simple purpose of maintaining the core assembly as a more or less integral cylinder. This stress is so small compared with the other stress mechanisms, and the element compressive strength is high enough, that it is neglected.

Turning now to the mechanisms which induce axial stresses in the element, there are internal heat generation, coating-matrix interaction, and bead-matrix interaction, all present in the transverse stress picture.

A major additional contributor is the axial friction stress. Under the tightest controls of flow distribution, fuel loading, etc., there are bound to be element-to-element temperature differences. The calculation of maximum tensile stress in the coolest element in the core is performed according to

$$\sigma_f = E \alpha \Delta T,$$

where

- E = modulus of elasticity,
 α = coefficient of thermal expansion,
 ΔT = local interelement temperature difference,

or (whichever gives the lower stress)

$$\sigma_f = \int_x^{52} \mu P_b \cdot \frac{P}{A} dx ,$$

where

- x = element station,
 μ = coefficient of friction,
 P_b = bundling pressure,
 $\frac{P}{A}$ = perimeter to cross-sectioned area ratio of fuel element,
 σ_f = axial friction stress.

The stress in the element length over which element-to-element slippage occurs is based on a fuel element without positive axial constraint at the hot end. The stress over the non-slipping, or "locked-up", element length is a simple function of local interelement temperature difference and appropriate properties. Schematically, the axial friction stress varies as shown in Figure 3.

Another axial stress component derives from the temperature difference transversely across the element. Superimposed on the complex thermal pattern within an element (channel walls coolest, element flats hottest) there can be a transverse temperature gradient due to a gross core radial temperature distribution inevitable especially at the core periphery. Assuming a linear gradient across the given element, this stress equals $1/2E\alpha\Delta T$, where ΔT = temperature difference across element flats.

The core pressure drop is the final axial component, but is compressive and is presently neglected to add conservatism to the analysis.

Most of the stresses just described are combined in a two-dimensional (plane strain) finite element structural model (currently 1500 nodes to represent a 1/12 model of the 0.75 inch across flats element) which calculates its own internal temperature distribution and includes such refinements as taking into account the graphite orthotropic properties and the stress-free reference temperature in the coating-matrix interaction computation.

CRITERION OF FAILURE

Having developed, with the complicated model just described, the fuel stress distribution, the next task is prediction of a safety factor or a reliability. Such a prediction depends on defining an adequate criterion of failure. Pos-

sible approaches are: (1) fracture mechanics, (2) Weibull (weakest link) statistics, (3) maximum principal stress, (4) maximum principal strain, and (5) a strain energy criterion.

The fracture mechanics approach¹, with comments on salient advantages and disadvantages, is depicted below.

Definition

failure occurs when

$$\sigma > \sigma_c ,$$

where

$$K_{IC} = Q \sigma_c \sqrt{\pi a} = \text{fracture toughness,}$$

$$Q = \text{geometry factor,}$$

$$a = \text{anticipated flaw size.}$$

Advantages

1. Reasonably well developed technique for selected geometries and stress distributions.
2. Moderate level of testing required.

Disadvantages

1. Of doubtful applicability to material with distributed internal "defects".
2. As-extruded geometry precludes testing of standard geometry fracture mechanics specimens.
3. No obvious method of accounting for volume under load in reliability prediction.

Weibull analysis² is a statistical approach to calculating brittle material failure. In simple terms, the probability of fracture of a component is calculated as a weighted function of the volume under tensile stress and the size of the tensile stresses which are present.

Definition

$$P = 1 - \exp \left[- \int_V \left(\frac{\sigma - \sigma_u}{\sigma_o} \right)^m dV \right] ,$$

where

$$P = \text{probability of fracture,}$$

$$V = \text{volume under tension,}$$

$$\sigma = \text{local tensile stress,}$$

σ_u = minimum material strength,
 σ_o = volume-dependent scale parameter,
 m = scatter parameter;
 (σ_u , σ_o , and m are Weibull constants).

Advantages

1. Directly predicts failure probability including effect of volume under load.
2. Computational method reasonably direct and small extrapolation from current finite element methods.

Disadvantages

1. Requires very extensive testing program.
2. Applicability to graphite not established.
3. Risk of extraneous surface finish effects in small volume specimens.

Maximum principal stress theory is a classic approach to predicting brittle failure when the ultimate tensile strength is locally exceeded. Its potential is summarized as follows.

Definition

Failure occurs when
 maximum tensile stress > ultimate tensile strength.

Advantages

1. Moderate level of testing required.
2. Directly compatible with current finite element methods.

Disadvantages

1. Definition of maximum stress problematic in vicinity of sharp-edged flaws.
2. Probably overconservative with zero allowance for local yielding.
3. No obvious method of accounting for volume under load in reliability prediction.

Maximum principal strain theory has only recently received serious attention as an appropriate failure criterion. Progress to date with this idea does not warrant further discussion here.

A final alternate theory of failure is a strain energy criterion proposed by Andrae³. A prediction of failure based on crack initiation, not crack propagation, is suggested.

Definition

Failure occurs when

$$\frac{1}{A_F} \int_{A_F} \frac{1}{2E} \sigma^2 dA > \frac{1}{2E} \sigma_{ult}^2 ,$$

where

A_F = initial fracture area,

E = modulus of elasticity,

σ = local stress,

σ_{ult} = ultimate tensile strength.

Advantages

1. Moderate level of testing required.
2. Computational method reasonably direct and small extrapolation from current finite element methods.

Disadvantages

1. Lacks adequate experimental verification.
2. No obvious method of accounting for volume under load in reliability prediction.

These theories continue to get our analytic and experimental attention, but a confident choice of a standard failure criterion for the NERVA fuel analysis still seems elusive. In this connection, a notable program to define the graphite failure criterion has been underway in the Oak Ridge National Laboratory Graphite Mechanics group, with much of the work completed under this program being discussed elsewhere during this conference.

RELIABILITY PREDICTIONS

The final comments are a brief discussion of the probabilistic thinking which pervades the NERVA fuel analysis and the interpretation of the analytic results. Excluding the Weibull analysis with its built-in probabilistic failure prediction, the essence of our prediction of probability of failure lies in the so-called "interference theory" or "stress-strength overlap" method⁴. The approach is sketched in Figure 4. The strength curve is generated simply from a probabilistic description of the appropriate material property, ultimate strength, ultimate strain, or fracture toughness, for example. A probabilistic description of the stress is, however, more difficult. The present approach is to use "engineering judgment" on each stress component at the point of maximum combined stress in a manner similar to that shown below. The example below is the calculation of uncertainty in the component of stress due to heat generation:

1. $\sigma_q''' \propto \frac{E\alpha_q'''}{k}$
2. Calculate standard deviation of $(E\alpha_q'''/k)$ from known standard deviations of all parameters by algebra of normal functions.
3. Calculate standard deviation of σ_q''' at point of maximum combined stress by assuming:

$$\frac{\text{standard deviation } (\sigma_q''')}{\text{mean } (\sigma_q''')} = \frac{\text{standard deviation } (E\alpha_q'''/k)}{\text{mean } (E\alpha_q'''/k)}$$

4. Similarly treat all components of stress and combine standard deviations statistically to determine standard deviation of maximum combined stress.

All stress components can be comparably operated upon and then combined statistically to give the failure-governing stress and its standard deviation. The statistical description of the failure-governing stress and of the selected measure of "strength" necessarily involves a statistical description of numerous input parameters. These parameters include fuel geometry, operating environment and fuel material properties. To meet the need of statistically determined properties an extensive material properties test program has been mounted within the NERVA project. The program is unique in covering some exotic materials, in requiring large sample sizes, and in having to take into account extreme radiation and thermal environments as well as exposure to hydrogen and carbon. With the assumption of a normal distribution on both stress and strength, a probability of failure in the component is routinely calculated.

SUMMARY

In summary, the points discussed have been the NERVA fuel element stress mechanisms, the various alternatives available as a criterion of failure, and, finally, the probabilistic schemes present in the analysis. The NERVA fuel element is a highly developed piece of hardware with an impressive history of performance success. However, the complex loads on the element combined with the two-edged demand of even greater performance with increased reliability keep the stress analytic methods stretching current state-of-the-art techniques.

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Fig. 1. NERVA Nuclear Subsystem Cutaway.

Fuel Elements

- SUSTAIN CONTROLLED NUCLEAR HEAT GENERATION
 - PYROCARBON COATED UC₂ FUEL BEADS DISPERSED THROUGH AXM GRAPHITE MATRIX (630 MG/CC MAXIMUM FUEL LOADING)
 - UC IN COMPOSITE WITH ZrC AND GRAPHITE (700 MG/CC MAXIMUM FUEL LOADING)
- LIMIT TOTAL REACTIVITY LOSS TO \$1.00 AT END OF LIFE
 - CARBIDE COATING OF FLOW CHANNELS
- AFFORD HEAT TRANSFER FROM FUEL ELEMENT TO H₂ PROPELLANT
 - 19 FLOW CHANNELS IN EACH 3/4 INCH HEX, x 52 INCH LONG FUEL ELEMENT

Fig. 2. NERVA Fuel Element Description.

Fig. 3. Axial Friction Stress Schematic.

Fig. 4. Schematic of Overlap Method of Failure Probability Prediction.