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..... TECHNICAL REPORT

THE TRANSMISSION AND RECEPTION PROPERTIES
OF AN EQUATORIAL SLOT ANTENNA
ON A RE-ENTRY SPHERE

by

Charles W. Harrison, Jr.

and

Clayborne D. Taylor

RELEASED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

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THE TRANSMISSION AND RECEPTION PROPERTIES
OF AN EQUATORIAL SLOT ANTENNA
ON A RE-ENTRY SPHERE

by

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November 1965

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Summary

The re-entry vehicle model employed in this study consists of a plasma clad sphere with a narrow circumferential slot cut at the equator. The slot is excited by a biconical transmission line driven at its center by a coaxial cable. The homogeneous plasma may be in contact with the sphere or an insulating barrier interposed between the sphere and the plasma sheath.

The driving point impedance referred to the input terminals and the radiation field in terms of the input current are determined. The efficiency of transmission is calculated by taking the ratio of the power obtained by integrating the Poynting vector over the surface of a great sphere to the power supplied the linearly tapered radial transmission line by the coaxial cable. The voltage appearing across an impedance connected to the terminals of the biconical transmission line is evaluated in terms of the amplitude of an incident plane wave electromagnetic field.

Extensive numerical results are presented exhibiting the transmission and reception qualities of the equatorial slot antenna on the re-entry sphere, when the plasma is in contact with the vehicle, for a range of frequencies and plasma properties.

THE TRANSMISSION AND RECEPTION PROPERTIES OF AN EQUATORIAL SLOT ANTENNA ON A RE-ENTRY SPHERE

Introduction

This investigation was prompted by the desire to estimate the radar signal loss in both transmission and reception through the plasma sheath surrounding a vehicle during re-entry into the earth's atmosphere. It occurred to the writers that the best physical model of re-entry vehicle to select for analytical simplicity consists of a very wide-angle biconical antenna surrounded by an imperfectly conducting concentric spherical shell representing the plasma sheath. The mathematical formulation provides for either an insulating barrier between the plasma sheath and the re-entry vehicle or for physical contact between the plasma and vehicle.

Thus, the re-entry vehicle model selected for the present study may be regarded as a plasma clad sphere in which is cut a narrow circumferential slot at the equator. The slot is excited by a linearly tapered radial transmission line driven at its center by a coaxial cable. The superiority of this model over those which have been considered by other writers--in which the field in the aperture of an infinite slot is assumed--is obvious. This choice of re-entry vehicle permits reference of all phenomology to a set of terminals. For example, the driving point impedance of the plasma clad structure is obtained referred to these terminals, and the radiation field computed for a specified input current. Also, the voltage developed across an impedance of arbitrary value connected across these terminals is determined in terms of the amplitude of an incident plane wave electromagnetic field.

If the slot consists of a suitably baffled open-ended wave guide it may be possible to obtain the wave impedance in the aperture of the slot. But it is likely to be a difficult task to obtain the driving-point impedance of the electric or magnetic probe exciting the guide in terms of the aforementioned wave impedance.

There are a number of additional applications of the theory set forth in this paper. For example, a warhead consists of an imperfectly conducting spherical shell (containing no slots) encompassing certain electronic circuitry and high explosive. It is desired to calculate the current in a "typical" circuit, in terms of the incident plane-wave electromagnetic field, by representing the circuit as an impedance-loaded wide-angle biconical antenna. Two cases are easily treated: the end caps may be short-circuited to the shield, or insulated from the shield. Evidently this problem is essentially the same as that posed by a wide-angle biconical antenna symmetrically oriented within a spherical radome that is not quite a perfect dielectric. It may be desirable to determine the power lost in the radome both in transmission and reception. As a second example, a spherical cavity is dug in the ground some distance below the earth-air interface. A wide-angle biconical antenna is suspended symmetrically in the cavity and driven by a pulse generator. The signal dispersion in the earth may now be obtained by analytically probing the field, i. e., finding the time history of the field at various locations using Fourier transform techniques. In solving this problem it is necessary to assume that the observation points are sufficiently close to the cavity that interfering reflections from the earth-air boundary cause no difficulty.

In Parts A, B, and C of this paper formulas for the driving-point impedance, radiation field, and effective length are derived for an equatorial slot antenna on a perfectly conducting sphere surrounded by one or two nonstratified concentric spherical shells where the shells may be homogeneous plasma sheaths. This is done by considering the TEM-mode in the biconical transmission line region, and a series of modes in the plasma shell and radiation regions. There are no restrictions in the theory on the sphere diameter and radii of the plasma shells. In Part D of the paper numerical results pertaining specifically to the transmitting and receiving characteristics of an equatorial slot antenna on a

re-entry sphere are presented when the plasma is in contact with the vehicle, for a range of frequencies and plasma properties.

Part A

Transmission from an Equatorial Slot Antenna on a Re-entry Sphere

Figure 1 illustrates one model of re-entry vehicle employed in this study. It consists of a homogeneous plasma-clad perfectly conducting sphere with a narrow circumferential slot cut at the equator. The slot is excited by a biconical transmission line driven at its center by a coaxial cable. Alternatively the model may be thought of as a wide-angle biconical antenna within one concentric non-stratified spherical shell. The vertex of the biconical transmission line is chosen as the origin of a spherical coordinate system (r, θ, ϕ) . The conical surface of the transmission line is given by $\theta = \theta_0$ and $\theta = \pi - \theta_0$, $0 \leq r \leq a_1$, where a_1 is the radius of the sphere; the spherical caps by $r = a_1$, $0 \leq \theta \leq \theta_0$ and $\pi - \theta_0 \leq \theta \leq \pi$, where θ is the flare angle. There are three regions: the biconical transmission line region $0 \leq r < a_1$, $\theta_0 \leq \theta \leq \pi - \theta_0$; the shell or plasma region $a_1 \leq r \leq a_2$, $0 \leq \theta \leq \pi$; and the radiation region $r \geq a_2$, $0 < \theta < \pi$. These regions are labeled I, II, and III in the drawing, respectively.

In the biconical transmission line one considers only an outwardly propagating TEM-wave, and a reflected TEM-wave, the reflection taking place at $r = a$. In the shell region only standing TM-waves are present, and in the radiation region only outwardly propagating TM-waves exist.

The Biconical Transmission Line Region I

Maxwell's equations in spherical coordinates for transverse electromagnetic (TEM) waves are, with suppressed time dependence $\exp(j\omega t)$,¹

¹J. C. Slater, Microwave Transmission, McGraw-Hill Book Co., Inc., (1942) p 202.

FIGURE 1: EQUATORIAL SLOT ANTENNA DRIVEN BY A BICONICAL TRANSMISSION LINE ON A RE-ENTRY SPHERE (THE PLASMA IS IN CONTACT WITH THE SPHERE)

$$\frac{\partial}{\partial r}(rH_\phi) = -j\omega\xi_1(rE_\theta), \quad (1)$$

$$\frac{\partial}{\partial \theta}(\sin \theta H_\phi) = 0, \quad (2)$$

$$\frac{\partial}{\partial r}(rE_\theta) = -j\omega\mu_1(rH_\phi), \quad (3)$$

$$\frac{\partial}{\partial \theta}(\sin \theta E_\theta) = 0, \quad (4)$$

where E_θ and H_ϕ are the θ and ϕ components of the electric and magnetic fields, respectively. Differentiating (1) and using (3) one obtains the wave equation for H_ϕ . It is

$$\frac{\partial^2}{\partial r^2}(rH_\phi) + k_1^2(rH_\phi) = 0. \quad (5)$$

Here

$$k_1^2 = \omega^2 \mu_1 \xi_1, \quad (6)$$

and

$$\xi_1 = \epsilon_1 \left(1 - j \frac{\sigma_1}{\omega \epsilon_1}\right). \quad (7)$$

The subscripts on the constitutive parameters μ , σ , and ϵ , the permeability, conductivity, and dielectric constant, respectively, denote the region. In this case, the biconical transmission line region I is designated.

Evidently (2) and (5) are satisfied if

$$s_{H_\phi}^I = \frac{s_\alpha e^{-jk_1 r} + s_\beta e^{jk_1 r}}{r \sin \theta}, \quad (8)$$

where s_α and s_β are constants; the superscript s denotes that the single-shell problem is under discussion, Figure 1.

The corresponding electric field may be found by substituting (8) into (1). The result is

$$s_{E_{\theta}}^I = \frac{\zeta_1 \left(s_{\alpha} e^{-jk_1 r} + s_{\beta} e^{jk_1 r} \right)}{r \sin \theta}, \quad (9)$$

where ζ_1 is the characteristic wave impedance of region I.

$$\zeta_1 = \omega \mu_1 / k_1. \quad (10)$$

The voltage between the cones at the vertex ($r = 0$) is

$$V = \int_{\theta_0}^{\pi-\theta_0} r E_{\theta} d\theta = \zeta_1 (s_{\alpha} - s_{\beta}) \int_{\theta_0}^{\pi-\theta_0} \frac{d\theta}{\sin \theta} = 2\zeta_1 (s_{\alpha} - s_{\beta}) \ln \left(\cot \frac{\theta_0}{2} \right). \quad (11)$$

The total current is

$$s_I = \oint \vec{H} \cdot d\vec{\mathcal{L}} = 2\pi r \sin \theta H_{\phi}. \quad (12)$$

Using (8), the current at the vertex ($r = 0$) becomes

$$s_I = 2\pi (s_{\alpha} + s_{\beta}), \quad (13)$$

so that the driving-point impedance $^s Z_o$ of the plasma clad equatorial slot antenna on the sphere referred to the terminals of the biconical transmission line becomes

$$^s Z_o = K_c \left(\frac{1 - \frac{s_{\beta}}{s_{\alpha}}}{1 + \frac{s_{\beta}}{s_{\alpha}}} \right), \quad (14)$$

where

$$K_c = \frac{\zeta_1}{\pi} \ln \left(\cot \frac{\theta_0}{2} \right). \quad (15)$$

The driving-point impedance, as given by (14), is evidently determined when the ratio of the constants s_{β}/s_{α} is found by satisfying boundary conditions on the fields at the interfaces of the shell region.

The Shell Region II

It can be shown² that for transverse magnetic (TM) waves the E_θ and H_ϕ fields may be derived from the scalar potential

$$rv = r^2 E_r, \quad (16)$$

where E_r is the radial electric field. These relations are

$$E_\theta = \frac{1}{n(n+1)} \frac{1}{r} \frac{\partial}{\partial \theta} \frac{\partial}{\partial r} (rv), \quad (17)$$

$$H_\phi = -j \frac{\omega \xi}{n(n+1)} \frac{1}{r} \frac{\partial}{\partial \theta} (rv). \quad (18)$$

If H_ϕ is known, E_θ also may be found from the equation

$$E_\theta = j \frac{1}{\omega \xi} \frac{1}{r} \frac{\partial}{\partial r} (r H_\phi). \quad (19)$$

The scalar potential $v = r E_r$ satisfies the wave equation

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial v}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial v}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v}{\partial \phi^2} + k^2 v = 0. \quad (20)$$

The solution of (20), for rotational symmetry in ϕ , is

$$E_r = \frac{1}{r} \sum_{n=0}^{\infty} \left[A_n h_n^{(2)}(kr) + B_n h_n^{(1)}(kr) \right] P_n(\cos \theta). \quad (21)$$

The magnetic field H_ϕ may now be obtained from (18). It is

$$H_\phi = j\omega \xi \sum_{n=1}^{\infty} \frac{1}{n(n+1)} \left[A_n h_n^{(2)}(kr) + B_n h_n^{(1)}(kr) \right] P_n^1(\cos \theta), \quad (22)$$

where the relation

$$\frac{\partial}{\partial \theta} P_n(\cos \theta) = -P_n^1(\cos \theta), \quad (23)$$

²See Reference 1, p 199.

has been employed. The other electric field component may be obtained by applying (19).

The preliminary discussion of TM waves is now complete.

Returning to a discussion of the fields in the shell (plasma sheath) region, it is convenient to redefine the constants A_n and B_n appearing in (22), and write the magnetic field in region II in the form

$$s_{H\phi}^{II} = \sum_{n=1,3,\dots}^{\infty} \left[s_{A_n}^{II} h_n^{(2)}(k_2 r) + s_{B_n}^{II} h_n^{(1)}(k_2 r) \right] P_n^1(\cos \theta). \quad (24)$$

Note that the indices in (24) are odd integers. This occurs because $s_{H\phi}^{II}$ must be symmetric about $\theta = \pi/2$ and $P_n^1(\cos \theta) = P_n^1[\cos(\pi - \theta)] = P_n^1(-\cos \theta)$ holds only for n odd.

The θ -component of electric field is found from (19) using (24). It is

$$s_{E\theta}^{II} = j\zeta_2 \frac{1}{k_2 r} \sum_{n=1,3,\dots}^{\infty} \left\{ s_{A_n}^{II} \left[k_2 r h_n^{(2)}(k_2 r) \right]' + s_{B_n}^{II} \left[k_2 r h_n^{(1)}(k_2 r) \right]' \right\} P_n^1(\cos \theta), \quad (25)$$

where ζ_2 is given by a formula like (10) and k_2 by a formula like (6), but with subscripts 2 written for 1 designating region II, and

$$\left[k r h_n^{(1)}(kr) \right]' = \left[h_{n-1}^{(1)}(kr) - \frac{n}{kr} h_n^{(1)}(kr) \right] kr. \quad (26)$$

Note that the symbolism $[]'$ means differentiation of the terms within the bracket with respect to the argument.

The Radiation Region III

Remembering that there are only outward traveling waves in the radiation region, and that the time dependence assumed is $\exp(j\omega t)$, it follows that in region III,

$$\left. \begin{aligned}
s_{H\phi}^{III} &= \sum_{n=1,3,\dots}^{\infty} s_{A_n}^{III} h_n^{(2)}(k_3 r) P_n^1(\cos \theta), \\
s_{E\theta}^{III} &= j\zeta_e \frac{1}{k_3 r} \sum_{n=1,3,\dots}^{\infty} s_{A_n}^{III} \left[k_3 r h_n^{(2)}(k_3 r) \right]' P_n^1(\cos \theta).
\end{aligned} \right\} \quad (27)$$

Here ζ_3 and k_3 are the characteristic wave impedance and propagation constants of region III, respectively. It is also of interest to obtain the far-zone fields.

These are

$$\left. \begin{aligned}
s_{H\phi}^{III} &\simeq \frac{e^{-jk_3 r}}{k_3 r} \sum_{n=1,3,\dots}^{\infty} s_{A_n}^{III} (-1)^{(n+1)/2} P_n^1(\cos \theta), \\
s_{E\theta}^{III} &\simeq \zeta_3 s_{H\phi}^{III}.
\end{aligned} \right\} \quad (28)$$

Boundary Equations

The tangential component of the electric field is continuous at the boundary $r = a_1$, $\theta_0 \leq \theta \leq \pi - \theta_0$, and is zero on the surface of the perfectly conducting sphere $0 \leq \theta \leq \theta_0$, $\pi - \theta_0 \leq \theta \leq \pi$. E_{θ}^I is given by (9) and E_{θ}^{II} by (25). Hence,

$$\begin{aligned}
&\zeta_1 \left(s_{\alpha} e^{-jk_1 a_1} - s_{\beta} e^{jk_1 a_1} \right) \\
&= j \frac{\zeta_2}{k_2} \sum_{n=1,3,\dots}^{\infty} \left\{ s_{A_n}^{II} \left[k_2 a_1 h_n^{(2)}(k_2 a_1) \right]' + s_{B_n}^{II} \left[k_2 a_1 h_n^{(1)}(k_2 a_1) \right]' \right\} \\
&\times P_n^1(\cos \theta) \sin \theta.
\end{aligned} \quad (29)$$

One now multiplies both sides of (29) by $P_m^1(\cos \theta)$ and integrates the left side over the range $\theta_0 \leq \theta \leq \pi - \theta_0$ and the right side over the range $0 \leq \theta \leq \pi$. (Evidently this choice of limits in the integration is permissible because the tangential component of electric field must vanish over the surface of the sphere.) The result is

$$\begin{aligned} & -jk_2 \sqrt{\frac{\epsilon_2 \mu_1}{\epsilon_1 \mu_2}} \left[s_\alpha e^{-jk_1 a_1} - s_\beta e^{jk_1 a_1} \right] P_n(\cos \theta_0) \\ & = \frac{n(n+1)}{2n+1} \left\{ s_A^{II} \left[k_2 a_1 h_n^{(2)}(k_2 a_1) \right]' + s_B^{II} \left[k_2 a_1 h_n^{(1)}(k_2 a_1) \right]' \right\}. \end{aligned} \quad (30)$$

In deriving (30) use has been made of the relations

$$\int_{-1}^1 P_m^1(u) P_n^1(u) du = \frac{2n(n+1)}{2n+1} \delta_{mn}, \quad (31)$$

$$\int P_m^1(\cos \theta) d\theta = -P_m(\cos \theta), \quad (32)$$

$$\text{and} \quad P_m(\cos \theta) = -P_m[\cos(\pi - \theta)], \quad (33)$$

for m odd.

The tangential component of the magnetic field is continuous at the boundary $r = a_1$, $\theta_0 \leq \theta \leq \pi - \theta_0$; i.e., in the aperture region of the slot antenna. (The tangential component of the magnetic field is not continuous across the surface of the perfectly conducting sphere.) Equating (8) and (24) and integrating over the range $\theta_0 \leq \theta \leq \pi - \theta_0$, the result is

$$\begin{aligned} & \frac{1}{a_1} \epsilon n \cot \frac{\theta_0}{2} \left[s_\alpha e^{-jk_1 a_1} + s_\beta e^{jk_1 a_1} \right] \\ & = \sum_{n=1,3,\dots}^{\infty} \left[s_A^{II} h_n^{(2)}(k_2 a_1) + s_B^{II} h_n^{(1)}(k_2 a_1) \right] P_n(\cos \theta_0). \end{aligned} \quad (34)$$

The boundary equations at $r = a_2$ are obtained in a similar manner. They are:

$$\begin{aligned} & \zeta_2 \left\{ s_{A_n}^{II} \left[k_2 a_2 h_n^{(2)}(k_2 a_2) \right]' + s_{B_n}^{II} \left[k_2 a_2 h_n^{(1)}(k_2 a_2) \right]' \right\} \\ &= \frac{k_2}{k_3} \zeta_3 s_{A_n}^{III} \left[k_3 a_2 h_n^{(2)}(k_3 a_2) \right]', \end{aligned} \quad (35)$$

$$s_{A_n}^{II} h_n^{(2)}(k_2 a_2) + s_{B_n}^{II} h_n^{(1)}(k_2 a_2) = s_{A_n}^{III} h_n^{(2)}(k_3 a_2). \quad (36)$$

Remembering that the radiation field of the plasma clad re-entry sphere, and the input impedance of the slot antenna referred to the terminals at the center of the biconical transmission line are to be found, one finds $s_{B_n}^{II}$ in terms of $s_{A_n}^{II}$ from (35) and (36). The result is

$$s_{B_n}^{II} = s_{\rho_n} s_{A_n}^{II}, \quad (37)$$

where

$$s_{\rho_n} = \frac{C_{32} h_n^{(2)}(k_3 a_2) \left[k_2 a_2 h_n^{(2)}(k_2 a_2) \right]' - h_n^{(2)}(k_2 a_2) \left[k_3 a_2 h_n^{(2)}(k_3 a_2) \right]'}{h_n^{(1)}(k_2 a_2) \left[k_3 a_2 h_n^{(2)}(k_3 a_2) \right]' - C_{32} h_n^{(2)}(k_3 a_2) \left[k_2 a_2 h_n^{(1)}(k_2 a_2) \right]'}, \quad (38)$$

and

$$C_{32} = \frac{k_3 \sqrt{\mu_2 \xi_3}}{k_2 \sqrt{\mu_3 \xi_2}} = \frac{k_3 \zeta_2}{k_2 \zeta_3} = \frac{\xi_3}{\xi_2}. \quad (39)$$

Substituting (37) into (30) and solving for $s_{A_n}^{II}$:

$$s_{A_n}^{II} = -jk_1 C_{21} \frac{2n+1}{n(n+1)} \left[\frac{P_n(\cos \theta_o) \left(s_{\alpha} e^{-jk_1 a_1} - s_{\beta} e^{jk_1 a_1} \right)}{\left[k_2 a_1 h_n^{(2)}(k_2 a_1) \right]' + s_{\rho_n} \left[k_2 a_1 h_n^{(1)}(k_2 a_1) \right]'} \right]. \quad (40)$$

Here

$$C_{21} = \frac{k_2 \sqrt{\mu_1 \xi_2}}{k_1 \sqrt{\mu_2 \xi_1}} = \frac{k_2 \zeta_1}{k_1 \zeta_2} = \frac{\xi_2}{\xi_1}. \quad (41)$$

The values of $s_{A_n}^{II}$ and $s_{B_n}^{II}$, as given by (40) and (37), respectively, are now substituted into (34). The result is

$$\begin{aligned}
& \frac{s_{\alpha} e^{-jk_1 a_1} + s_{\beta} e^{jk_1 a_1}}{s_{\alpha} e^{-jk_1 a_1} - s_{\beta} e^{jk_1 a_1}} \\
&= -jk_1 a_1 \frac{C_{21}}{\ln \cot \frac{\theta_o}{2}} \sum_{n=1, 3 \dots}^{\infty} \frac{2n+1}{n(n+1)} \left[\frac{h_n^{(2)}(k_2 a_1) + s_{\rho} h_n^{(1)}(k_2 a_1)}{\left[k_2 a_1 h_n^{(2)}(k_2 a_1) \right]' + s_{\rho} \left[k_2 a_1 h_n^{(1)}(k_2 a_1) \right]'} \right] \\
&\times \left[P_n(\cos \theta_o) \right]^2. \tag{42}
\end{aligned}$$

$$\text{Then, } \frac{s_{\beta}}{s_{\alpha}} = e^{-j2k_1 a_1} \left(\frac{s_y - 1}{s_y + 1} \right). \tag{43}$$

where

$$\begin{aligned}
s_y = -j \frac{k_1 a_1 C_{21}}{\ln \cot \frac{\theta_o}{2}} \sum_{n=1, 3 \dots}^{\infty} \left\{ \frac{2n+1}{n(n+1)} \right. \\
\left. \times \frac{\left[h_n^{(2)}(k_2 a_1) + s_{\rho} h_n^{(1)}(k_2 a_1) \right] \left[P_n(\cos \theta_o) \right]^2}{\left[k_2 a_1 h_n^{(2)}(k_2 a_1) \right]' + s_{\rho} \left[k_2 a_1 h_n^{(1)}(k_2 a_1) \right]'} \right\}. \tag{44}
\end{aligned}$$

The Driving-Point Impedance of the Plasma Clad Slot Antenna
Referred to the Terminals of the Biconical Transmission Line

Since s_{Z_o} is given by (14), and s_{β}/s_{α} by (43), it is a simple matter to show that

$$s_{Z_o} = K_c \left(\frac{\cos k_1 a_1 + j s_y \sin k_1 a_1}{s_y \cos k_1 a_1 + j \sin k_1 a_1} \right), \quad (45)$$

where K_c is given by (15) and s_y by (44).

The Radiation Field of the Slot Antenna on a Re-Entry Sphere

It is desirable to express the radiation field in terms of the input current.

Equation (13) may be written in the form

$$s_\alpha = \frac{s_{I_o}}{2\pi \left(1 + \frac{s_\beta}{s_\alpha} \right)}. \quad (46)$$

Also, from (43),

$$1 + \frac{s_\beta}{s_\alpha} = 2 e^{-jk_1 a_1} \left(\frac{s_y \cos k_1 a_1 + j \sin k_1 a_1}{1 + s_y} \right), \quad (47)$$

so that (46) becomes

$$s_\alpha = \frac{s_{I_o} e^{jk_1 a_1}}{4\pi} \left(\frac{1 + s_y}{s_y \cos k_1 a_1 + j \sin k_1 a_1} \right). \quad (48)$$

Substituting (48) into (43),

$$s_\beta = s_{I_o} \frac{e^{-jk_1 a_1}}{4\pi} \left(\frac{s_y - 1}{s_y \cos k_1 a_1 + j \sin k_1 a_1} \right). \quad (49)$$

It follows that

$$s_{\alpha} e^{-jk_1 a_1} - s_{\beta} e^{jk_1 a_1} = \frac{s_{I_o}}{2\pi} \left(\frac{1}{s_y \cos k_1 a_1 + j \sin k_1 a_1} \right). \quad (50)$$

Using this result in (40),

$$s_{A_n}^{II} = -jk_1 C_{21} \frac{s_{I_o}}{2\pi} \left[\frac{1}{s_y \cos k_1 a_1 + j \sin k_1 a_1} \right] \\ \times \left[\frac{\left(\frac{2n+1}{n(n+1)} \right) P_n(\cos \theta_o)}{\left[k_2 a_1 h_n^{(2)}(k_2 a_1) \right]' + s_{\rho_n} \left[k_2 a_1 h_n^{(1)}(k_2 a_1) \right]'} \right]. \quad (51)$$

$s_{A_n}^{III}$ may now be determined from (36), (37), and (51). It has the value

$$s_{A_n}^{III} = -jk_1 C_{21} \frac{s_{I_o}}{2\pi} \frac{P_n(\cos \theta_o)}{s_y \cos k_1 a_1 + j \sin k_1 a_1} \frac{2n+1}{n(n+1)} \\ \times \left[\frac{h_n^{(2)}(k_2 a_2) + s_{\rho_n} h_n^{(1)}(k_2 a_2)}{\left[h_n^{(2)}(k_3 a_2) \left\{ \left[k_2 a_1 h_n^{(2)}(k_2 a_1) \right]' + s_{\rho_n} \left[k_2 a_1 h_n^{(1)}(k_2 a_1) \right]' \right\} \right]} \right]. \quad (52)$$

Having determined $s_{A_n}^{III}$ one may proceed to calculate $s_{H_{\phi}}^{III}$ and $s_{E_{\theta}}^{III}$ with (27) and (28).

The Power Radiated by the Equatorial Slot Antenna on the Plasma Clad Sphere-Poynting Vector Method

The power radiated is

$$s_{P_{\text{rad}}} = \lim_{r \rightarrow \infty} 2\pi \int_0^\pi E_\theta^{\text{III}} H_\phi^{\text{III}*} r^2 \sin \theta d\theta. \quad (53)$$

But according to (28)

$$\zeta_3^s H_\phi^{\text{III}} = \lim_{r \rightarrow \infty} s_{E_\theta}^{\text{III}}. \quad (54)$$

Hence

$$\begin{aligned} s_{P_{\text{rad}}} &= \lim_{r \rightarrow \infty} 2\pi \zeta_3 \int_0^\pi |s_{H_\phi}^{\text{III}}|^2 r^2 \sin \theta d\theta \\ &= \lim_{r \rightarrow \infty} 2\pi \zeta_3 \sum_{m, n}^{\infty} s_{A_n}^{\text{III}} s_{A_m}^{\text{III}*} \frac{r^2}{|k_3 r|^2} e^{-j[k_3 r - \frac{\pi}{2}(n+1)]} e^{j[k_3 r - \frac{\pi}{2}(m+1)]} \\ &\quad \times \int_0^\pi P_n^1(\cos \theta) P_m^1(\cos \theta) \sin \theta d\theta, \end{aligned} \quad (55)$$

where use has been made of (27) and the asymptotic formula

$$h_n^{(2)}(z) \xrightarrow{z \rightarrow \infty} \frac{e^{-j[z - \frac{\pi}{2}(n+1)]}}{z}. \quad (56)$$

Applying (31) and considering k_3 to be real, it follows that

$$s_{P_{\text{rad}}} = \frac{4\pi \zeta_3}{k_3^2} \sum_{n=1, 3, \dots}^{\infty} \frac{n(n+1)}{2n+1} |s_{A_n}^{\text{III}}|^2. \quad (57)$$

The input power is

$$s_{P_o} = |s_{I_o}|^2 R_e(s_{Z_o}), \quad (58)$$

so that the power lost in the antenna region I and the shell region II is $sP_{\text{lost}} = sP_o - sP_{\text{rad}}$. Considering the re-entry sphere to be perfectly conducting, and the biconical transmission line lossless, the efficiency of power transmission through the plasma sheath is

$$\eta = \frac{sP_{\text{rad}}}{sP_o} \times 100 \text{ percent.} \quad (59)$$

In the foregoing relations it is assumed that all voltages and currents are rms values so that the power lost in the plasma, power radiated, and power supplied to the biconical line is in watts.

Part B

Transmission from an Equatorial Slot Antenna When an Insulating Barrier Exists Between the Plasma and the Re-Entry Sphere

Figure 2 represents a homogeneous plasma sheath insulated from but encompassing a re-entry sphere. As before, the equatorial slot antenna is excited by a biconical transmission line driven at its center by a coaxial cable.

The formal solution of this problem is the same as that of Part A; there is simply one more shell to consider. In view of this, only an outline of the solution of this problem will be given. The notation is the same as that used before, except that the superscript d means that the double shell problem is under discussion, Figure 2.

The fields in the biconical line, shell, and radiation regions may be written down by analogy with the work in Part A, as follows:

Biconical line region I, $r \leq a_1$.

$$d_{H\phi}^I = \frac{d_{\alpha} e^{-jk_1 r} + d_{\beta} e^{jk_1 r}}{r \sin \theta}, \quad (60)$$

FIGURE 2: EQUATORIAL SLOT ANTENNA DRIVEN BY A BICONICAL TRANSMISSION LINE ON A RE-ENTRY SPHERE (THE PLASMA SHEATH IS SEPARATED FROM THE SURFACE OF THE SPHERE)

$$d_{E\theta}^I = \zeta_1 \frac{\left(d_{\alpha} e^{-jk_1 r} - d_{\beta} e^{jk_1 r} \right)}{r \sin \theta}. \quad (61)$$

Shell region II, $a_1 \leq r \leq a_2$.

$$d_{H\phi}^{II} = \sum_{n=1,3,\dots}^{\infty} \left[d_{A_n}^{II} h_n^{(2)}(k_2 r) + d_{B_n}^{II} h_n^{(1)}(k_2 r) \right] P_n^1(\cos \theta), \quad (62)$$

$$d_{E\theta}^{II} = j\zeta_2 \frac{1}{k_2 r} \sum_{n=1,3,\dots}^{\infty} \left\{ d_{A_n}^{II} \left[k_2 r h_n^{(2)}(k_2 r) \right]' + d_{B_n}^{II} \left[k_2 r h_n^{(1)}(k_2 r) \right]' \right\} \\ \times P_n^1(\cos \theta). \quad (63)$$

Shell region III (plasma sheath), $a_2 \leq r \leq a_3$.

$$d_{H\phi}^{III} = \sum_{n=1,3,\dots}^{\infty} \left[d_{A_n}^{III} h_n^{(2)}(k_3 r) + d_{B_n}^{III} h_n^{(1)}(k_3 r) \right] P_n^1(\cos \theta), \quad (64)$$

$$d_{E\theta}^{III} = j\zeta_3 \frac{1}{k_3 r} \sum_{n=1,3,\dots}^{\infty} \left\{ d_{A_n}^{III} \left[k_3 r h_n^{(2)}(k_3 r) \right]' + d_{B_n}^{III} \left[k_3 r h_n^{(1)}(k_3 r) \right]' \right\} \\ \times P_n^1(\cos \theta). \quad (65)$$

Radiation region IV, $a_3 \leq r$.

$$d_{H\phi}^{IV} = \sum_{n=1,3,\dots}^{\infty} d_{A_n}^{IV} h_n^{(2)}(k_4 r) P_n^1(\cos \theta), \quad (66)$$

$$d_{E\theta}^{IV} = j\zeta_4 \frac{1}{k_4 r} \sum_{n=1,3,\dots}^{\infty} d_{A_n}^{IV} \left[k_4 r h_n^{(2)}(k_4 r) \right]' P_n^1(\cos \theta). \quad (67)$$

The boundary equations are:

$$r = a_1 \text{ [see Equations (30), (34), and (41)]}$$

$$\begin{aligned} & -jk_1 C_{21} \left(d_\alpha e^{-jk_1 a_1} - d_\beta e^{jk_1 a_1} \right) P_n(\cos \theta_o) \\ & = \frac{n(n+1)}{2n+1} \left\{ d_{A_n}^{II} \left[k_2 a_1 h_n^{(2)}(k_2 a_1) \right]' + d_{B_n}^{II} \left[k_2 a_1 h_n^{(1)}(k_2 a_1) \right]' \right\}, \end{aligned} \quad (68)$$

$$\begin{aligned} & \frac{1}{a_1} \ln \cot \frac{\theta_o}{2} \left(d_\alpha e^{-jk_1 a_1} + d_\beta e^{jk_1 a_1} \right) \\ & = \sum_{n=1,3,\dots}^{\infty} \left[d_{A_n}^{II} h_n^{(2)}(k_2 a_1) + d_{B_n}^{II} h_n^{(1)}(k_2 a_1) \right] P_n(\cos \theta_o). \end{aligned} \quad (69)$$

$$r = a_2 \text{ [see Equations (35), (36), and (39)]}$$

$$\begin{aligned} & C_{32} \left\{ d_{A_n}^{II} \left[k_2 a_2 h_n^{(2)}(k_2 a_2) \right]' + d_{B_n}^{II} \left[k_2 a_2 h_n^{(1)}(k_2 a_2) \right]' \right\} \\ & = d_{A_n}^{III} \left[k_3 a_2 h_n^{(2)}(k_3 a_2) \right]' + d_{B_n}^{III} \left[k_3 a_2 h_n^{(1)}(k_3 a_2) \right]', \end{aligned} \quad (70)$$

$$d_{A_n}^{II} h_n^{(2)}(k_2 a_2) + d_{B_n}^{II} h_n^{(1)}(k_2 a_2) = d_{A_n}^{III} h_n^{(2)}(k_3 a_2) + d_{B_n}^{III} h_n^{(1)}(k_3 a_2). \quad (71)$$

$$r = a_3.$$

$$\begin{aligned} & C_{43} \left\{ d_{A_n}^{III} \left[k_3 a_3 h_n^{(2)}(k_3 a_3) \right]' + d_{B_n}^{III} \left[k_3 a_3 h_n^{(1)}(k_3 a_3) \right]' \right\} \\ & = d_{A_n}^{IV} \left[k_4 a_3 h_n^{(2)}(k_4 a_3) \right]', \end{aligned} \quad (72)$$

$$d_{A_n}^{III} h_n^{(2)}(k_3 a_3) + d_{B_n}^{III} h_n^{(1)}(k_3 a_3) = d_{A_n}^{IV} h_n^{(2)}(k_4 a_3). \quad (73)$$

In (72),

$$C_{43} = \frac{k_4}{k_3} \sqrt{\frac{\mu_3 \xi_4}{\mu_4 \xi_3}} = \frac{k_4 \xi_3}{k_3 \xi_4} = \frac{\xi_4}{\xi_3}. \quad (74)$$

In each pair of boundary equations the θ -component of the electric field in two regions is equated at the interfaces first; then the ϕ -component of the magnetic field.

Eliminating $d_{A_n}^{IV}$ from (72) and (73) leads to the result

$$d_{B_n}^{III} = d_{\sigma_n} d_{A_n}^{III}, \quad (75)$$

where

$$d_{\sigma_n} = \frac{C_{43} \left[k_3 a_3 h_n^{(2)}(k_3 a_3) \right]' h_n^{(2)}(k_4 a_3) - h_n^{(2)}(k_3 a_3) \left[k_4 a_3 h_n^{(2)}(k_4 a_3) \right]'}{h_n^{(1)}(k_3 a_3) \left[k_4 a_3 h_n^{(2)}(k_4 a_3) \right]' - C_{43} \left[k_3 a_3 h_n^{(1)}(k_3 a_3) \right]' h_n^{(2)}(k_4 a_3)}. \quad (76)$$

Substituting (75) into (70) and (71) the relation between $d_{A_n}^{II}$ and $d_{B_n}^{II}$ may be established. It is

$$d_{B_n}^{II} = d_{\rho_n} d_{A_n}^{II}, \quad (77)$$

where

$$d_{\rho_n} = \frac{C_{32} \Delta_n \left[k_2 a_2 h_n^{(2)}(k_2 a_2) \right]' - h_n^{(2)}(k_2 a_2) \gamma_n}{h_n^{(1)}(k_2 a_2) \gamma_n - C_{32} \Delta_n \left[k_2 a_2 h_n^{(1)}(k_2 a_2) \right]'}, \quad (78)$$

$$\Delta_n = h_n^{(2)}(k_3 a_2) + d_{\sigma_n} h_n^{(1)}(k_3 a_2), \quad (79)$$

$$\gamma_n = \left[k_3 a_2 h_n^{(2)}(k_3 a_2) \right]' + d_{\sigma_n} \left[k_3 a_2 h_n^{(1)}(k_3 a_2) \right]'. \quad (80)$$

Evidently, the input impedance of the slot antenna referred to the center of the biconical transmission line is

$$d_{Z_o} = K_c \frac{(\cos k_1 a_1 + j^d y \sin k_1 a_1)}{d_y \cos k_1 a_1 + j \sin k_1 a_1}, \quad (81)$$

where

$$d_y = -j \frac{k_1 a_1 C_{21}}{\theta_o} \sum_{n=1,3,\dots}^{\infty} \left\{ \frac{2n+1}{n(n+1)} \right. \\ \left. \times \frac{\left[h_n^{(2)}(k_2 a_1) + d_{\rho} h_n^{(1)}(k_2 a_1) \right] \left[P_n(\cos \theta_o) \right]^2}{\left[k_2 a_1 h_n^{(2)}(k_2 a_1) \right]' + d_{\rho} \left[k_2 a_1 h_n^{(1)}(k_2 a_1) \right]'} \right\}. \quad (82)$$

[See Equations (44) and (45).] Also,

$$d_{A_n}^{II} = -jk_1 C_{21} \frac{d_{I_o}}{2\pi} \left[\frac{1}{d_y \cos k_1 a_1 + j \sin k_1 a_1} \right] \\ \times \left[\frac{\left(\frac{2n+1}{n(n+1)} \right) P_n(\cos \theta_o)}{\left[k_2 a_1 h_n^{(2)}(k_2 a_1) \right]' + d_{\rho} \left[k_2 a_1 h_n^{(1)}(k_2 a_1) \right]'} \right]. \quad (83)$$

[See Equation (51).]

From the use of (71), (75), and (77) it is easily demonstrated that

$$d_{A_n}^{III} = \frac{h_n^{(2)}(k_2 a_2) + d_{\rho} h_n^{(1)}(k_2 a_2)}{h_n^{(2)}(k_3 a_2) + d_{\sigma} h_n^{(1)}(k_3 a_2)} d_{A_n}^{II}. \quad (84)$$

The values of $d_{B_n}^{III}$ and $d_{A_n}^{III}$ given by (75) and (84) are now substituted in (73) to obtain the value of $d_{A_n}^{IV}$. It is

$$d_{A_n}^{IV} = \left[\frac{h_n^{(2)}(k_3 a_3) + d_{\sigma n n}^{(1)}(k_3 a_3)}{h_n^{(2)}(k_4 a_3)} \right] \left[\frac{h_n^{(2)}(k_2 a_2) + d_{\sigma n n}^{(1)}(k_2 a_2)}{h_n^{(2)}(k_3 a_2) + d_{\sigma n n}^{(1)}(k_3 a_2)} \right] d_{A_n}^{II}, \quad (85)$$

where $d_{A_n}^{II}$ is given by (83). The radiation magnetic and electric fields are now determined using (66) and (67), respectively.

Evidently, the power radiated is

$$d_{P_{\text{rad}}} = \frac{4\pi\zeta_4}{k_4^2} \sum_{n=1,3,\dots}^{\infty} \frac{n(n+1)}{2n+1} |d_{A_n}^{IV}|^2, \quad (86)$$

where ζ_4 and k_4 are real for this calculation to be meaningful. The power lost in the antenna and shell regions and the efficiency of transmission may be obtained as in the foregoing development.

Part C

Reception by an Equatorial Slot Antenna Cut in a Perfectly Conducting Sphere Encompassed by a One- or Two-Layer, Imperfectly Conducting, Concentric Shell

The effective half-length of the slot antenna is defined in this paper as that complex quantity (dimensionally in meters) which when multiplied by the incident vertically polarized electric field (in volts per meter) gives one-half the open-circuit voltage at the terminals of the biconical transmission line. King³ has shown that the complex field function $F_o(\theta, k_o h)$ for any two-terminal antenna, that is located in free space and used for transmission, is equal to the complex effective half-length $k_o h_e$ in radians when the same antenna is used for reception. Thus

³R. W. P. King, Theory of Linear Antennas, Chap. V, Section 14, pp 568-571.

$$F_o(\theta, k_o h) = k_o h_e, \quad (87)$$

where $k_o = \omega/c$, $c = 3 \times 10^8$ m/sec. For an isolated dipole $F_o(\theta, k_o h)$ is, by definition,

$$F_o(\theta, k_o h) = -j \frac{2\pi r e^{jk_o r} E(\theta)}{\zeta_o I_o}, \quad (88)$$

where $\zeta_o = 120\pi$ ohms.

It follows by analogy with (87) and (88) that

$$s_F(\theta) = k_3 s_{a_e} = -j \frac{2\pi r e^{jk_3 r} s_{E_{\theta}^{III}}}{\zeta_3 s_{I_o}}, \quad (89)$$

$$d_F(\theta) = k_4 d_{a_e} = -j \frac{2\pi r e^{jk_4 r} d_{E_{\theta}^{IV}}}{\zeta_4 d_{I_o}}, \quad (90)$$

where E_{θ}^{III} and E_{θ}^{IV} are given by (28) and (67), respectively. s, d_{a_e} is the effective half-length of the equatorial slot antenna on the perfectly conducting sphere within a single or dual shell. Note that in the treatment of the reception of a plane wave by the slot antenna it is mandatory that the radiation region be a perfect dielectric because equations (89) and (90) are valid only when k and ζ are real quantities.

The equivalent circuit of the receiving antenna may be treated as an idealized voltage generator, whose open-circuit voltage is

$$s, d_{V_{oc}} = -2 s, d_{a_e} E^{inc}, \quad (91)$$

driving a circuit consisting of the load impedance Z in series with the driving-point impedance s, d_{Z_o} . Ordinary circuit theory may then be used to find the current in the load, or voltage across the load in terms of the incident field E^{inc} . Usually, one would be interested in the response of the impedance-loaded slot antenna

on a sphere within one or two concentric spherical shells to the incident field, when $\theta = \pi/2$. In this case $E^{\text{inc}} \rightarrow E_z^{\text{inc}}$.

Part D

Numerical Determination of the Transmission and Reception Properties of an Equatorial Slot Antenna on a Re-Entry Sphere

An extensive numerical analysis was made to determine the transmitting and receiving properties of an equatorial slot antenna on a re-entry sphere. The Dikewood Corporation IBM-7044 computer was used in the program development. At every step in the program evolution several points were computed manually, using a desk calculator. The final computation was carried out on the Sandia Laboratory CDC-3600 computer. The only numerical case considered is that of a homogeneous plasma sheath possessing various properties in contact with the perfectly conducting sphere. The biconical transmission line feeding the slot is assumed to be lossless and have a relative dielectric constant of unity.

Specifically, the following properties of the re-entry vehicle are determined.

- (a) The driving point impedance of the structure referred to the input terminals of the biconical transmission line feeding the circumferential slot.
- (b) The efficiency of power transmission.
- (c) The voltage appearing across the terminals of the linearly tapered radial transmission line in terms of the amplitude of the incident plane wave electromagnetic field.
- (d) The radiation field of the plasma clad vehicle referred to the input current.

In addition to the above investigations, for checking purposes, the transfer functions for several infinite plasma sheets were computed, and the percentage of power transmission per unit area evaluated.

The effective conductivity and effective dielectric constant of the plasma with collisions considered in this study are given by

$$\sigma = \frac{\epsilon_0 \omega_{\text{eff}}^2 \omega_p^2}{\omega^2 + \omega_{\text{eff}}^2}$$

and

$$\epsilon = \epsilon_0 \left(1 - \frac{\omega_p^2}{\omega^2 + \omega_{\text{eff}}^2} \right),$$

respectively, where $\omega_p^2 = Nq^2/m\epsilon_0$. The notation used follows: $\omega = 2\pi f$ where f is the radar frequency; $\mu = \mu_0 = 4\pi \times 10^{-7}$ henry/m is the fundamental magnetic constant of space. $\epsilon_0 = 8.85 \times 10^{-12}$ farads/m is the fundamental dielectric constant of space. ω_{eff} is the collision frequency in collisions/sec. ω_p is the plasma frequency in cycles/sec. N is the electron density in electrons per cu/m. q is the charge on an electron; $q = 1.60210 \times 10^{-19}$ coulomb. m is the mass of an electron; $m = 9.1091 \times 10^{-31}$ kg. The relations for σ and ϵ apply when the motion of the ions can be neglected, i.e., ω is sufficiently high. Static fields that may be present are neglected.

In all of the numerical work reported here (with the exception of results presented in connection with Figure 9) $a_1 = 12$ cm and $\omega_{\text{eff}} = 10^{12}$ collisions/sec. Plasma shell thicknesses $a = a_2 - a_1$ of 0.2, 1.0, and 1.5 cm were considered, and the electron densities employed in most of the work were $N = 10^{17}$ and 10^{19} electrons/cu. m. Biconical transmission lines of two characteristic impedances were treated: $K_c = 10.48$ ohms and 42.8 ohms. It follows that the angles at the vertices of the lines are 10° and 40° , respectively.

Figures 3 to 8, inclusive, present the input impedance $Z_o = R_o + jX_o$ in ohms of the plasma clad equatorial slot antenna on a sphere over the frequency range 0.5 to 4.0 gcps. In each figure θ_o and $a = a_2 - a_1$ are specified. When no plasma sheath is present the curves for Z_o coincide with those labeled $N = 10^{17} \text{ m}^{-3}$ provided $a = 0.2$ cm. This is true for $\theta_o = 70^\circ$ as well as for $\theta_o = 85^\circ$.

FIGURE 3: INPUT IMPEDANCE OF A PLASMA CLAD EQUATORIAL SLOT ANTENNA ON A SPHERE REFERRED TO THE TERMINALS OF THE BICONICAL TRANSMISSION LINE. $\theta_0 = 70^\circ$, $a = 0.2 \text{ cm}$.

FIGURE 4: SAME AS FIGURE 3, EXCEPT $a=1.0$ cm.

FIGURE 5: SAME AS FIGURE 3, EXCEPT $a = 1.5$ cm.

FIGURE 6: SAME AS FIGURE 3, EXCEPT $\theta_0 = 85^\circ$ AND $a = 0.2$ cm.

FIGURE 7: SAME AS FIGURE 3, EXCEPT $\theta_0 = 85^\circ$ AND $a = 1.0$ cm.

FIGURE 8: SAME AS FIGURE 3, EXCEPT $\theta_0 = 85^\circ$ AND $a = 1.5$ cm.

Figure 9 (upper curves, right-hand scale) present the efficiency of power transmission of a thin biconical antenna, $\theta_o = 1^\circ$, surrounded by an isotropic plasma shell of thickness a . These curves were taken from a report by Chen and Tai⁴. Figure 9 (lower curves, left-hand scale) present the efficiency of power transmission of a plasma clad equatorial slot antenna, $\theta_o = 85^\circ$, cut in a sphere over the frequency range 0.2 to 2 gcps. In all data appearing in Figure 9, $\omega_p = 1.26 \times 10^{10}$ radians/sec; $\omega_{eff} = 5 \times 10^9 \text{ sec}^{-1}$; $N = 4.986 \times 10^{16}$ electrons/cu. m., and $a_1 = 10$ cm. Evidently, for the same plasma properties, the efficiency of power transmission of the thin biconical antenna surrounded by an isotropic plasma shell is somewhat higher than that of the equatorial slot antenna cut in a re-entering sphere. But agreement is sufficiently close, considering the different geometries of the antennas, to establish credence in the accuracy of all computations represented by the set of curves appearing in Figure 9.

Figures 10 and 11 show the efficiency of power transmission of a plasma clad equatorial slot antenna cut in a sphere when $\theta_o = 70^\circ$ and 85° , respectively, over the frequency range 0.5 to 4.0 gcps. Evidently, as the conductivity and thickness of the plasma sheath increase the efficiency of power transmission decreases. An interesting feature of these results is that the efficiency of power transmission is essentially frequency independent over a wide frequency range.

Figures 12 to 17, inclusive, illustrate the receiving characteristics of a plasma clad equatorial slot antenna cut in a sphere. The magnitude and phase of the voltage $V_L \big|_{\theta_p}$ developed across the load impedance $Z_L = R_L + j0.0$ connected to the biconical transmission line terminals is given. The incident electric field is directed parallel to the z -axis shown in Figure 1, and has an amplitude of 1 volt/m. The frequency range covered is 0.5 to 4.0 gcps. When $\theta_o = 70^\circ$ the load resistance is 42.8 ohms; and when $\theta_o = 85^\circ$, the load resistance is 10.5 ohms.

⁴Sinclair N. C. Chen and Chen-To Tai, "An Analytical Study of a Thin Biconical Antenna Surrounded by an Isotropic Plasma Shell," Department of Electrical Engineering, The Ohio State University Research Foundation, Columbus Ohio, Report 1573-4 of July 31, 1963, Figure 3.

FIGURE 9: UPPER CURVES: EFFICIENCY OF POWER TRANSMISSION OF A THIN BICONICAL ANTENNA SURROUNDED BY AN ISOTROPIC PLASMA SHELL; $\theta_0 = 1.0^\circ$. LOWER CURVES: EFFICIENCY OF POWER TRANSMISSION OF A PLASMA CLAD EQUATORIAL SLOT ANTENNA CUT IN A SPHERE; $\theta_0 = 85^\circ$. IN ALL DATA $\omega_p \approx 1.26 \times 10^{10}$ rad/sec, $\omega_{eff} = 5 \times 10^9$ sec⁻¹; $N = 4.986 \times 10^{16}$ e/cu. m; $a_1 = 10$ cm. THE RIGHT HAND SCALE APPLIES TO THE UPPER SET OF CURVES; THE LEFT HAND SCALE TO THE LOWER SET.

FIGURE 10: EFFICIENCY OF POWER TRANSMISSION OF A PLASMA CLAD EQUATORIAL SLOT ANTENNA CUT IN A SPHERE. $\theta_0 = 70^\circ$.

FIGURE 11: SAME AS FIGURE 10, EXCEPT $\theta_0 = 85^\circ$.

FIGURE 12: PLASMA CLAD EQUATORIAL SLOT ANTENNA ON A SPHERE. MAGNITUDE AND PHASE OF THE VOLTAGE DEVELOPED ACROSS A LOAD RESISTANCE OF 42.8 OHMS CONNECTED TO THE BICONICAL TRANSMISSION LINE TERMINALS. $\theta_0 = 70^\circ$; $a = 0.2$ cm. $E^{inc} = 1$ VOLT/M DIRECTED PARALLEL TO THE z -AXIS. $|V_L|$ IS REPRESENTED BY THE SOLID CURVES; θ_p BY THE DOTTED CURVES.

FIGURE 13: SAME AS FIGURE 12, EXCEPT $a = 1.0$ cm.

FIGURE 14: SAME AS FIGURE 12, EXCEPT $a = 1.5$ cm.

FIGURE 15: SAME AS FIGURE 12, EXCEPT $\theta_0 = 85^\circ$, $a = 0.2$ cm. AND $R_L = 10.5$ ohms.

FIGURE 16: SAME AS FIGURE 12, EXCEPT $\theta_0 = 85^\circ$, $a = 1.0$ cm. AND $R_L = 10.5$ ohms.

FIGURE 17: SAME AS FIGURE 12, EXCEPT $\theta_0 = 85^\circ$, $a = 1.5 \text{ cm}$. AND $R_L = 10.5 \text{ ohms}$.

The previously used range of plasma shell thicknesses and electron densities are considered, i.e., $a = 0.2, 1.0, \text{ and } 1.5 \text{ cm}$; $N = 10^{17}$ and 10^{19} electrons cu/m . When no plasma sheath is present the curves for $|V_L|$ and θ_p coincide with those marked $N = 10^{17} \text{ m}^{-3}$, provided $a = 0.2 \text{ cm}$. This statement is true for $\theta_o = 70^\circ$ as well as for $\theta_o = 85^\circ$.

In Figures 18 and 19 the electric field (volts/meter) evaluated at $\theta = 90^\circ$ in the radiation zone of a plasma clad equatorial slot antenna on a sphere, multiplied by the distance from the vertex of the biconical transmission line to the point of observation of the field (meters) is presented for several plasma shell thicknesses, electron densities, and θ_o values. The input current to the line terminals is 1.0 ampere. Hence the power supplied by the generator in transmission is not constant because R_o is frequency dependent as reference to Figures 3 to 8 show. However, by careful study the writers have concluded that Figures 3 to 8, inclusive, Figures 10 and 11, and Figures 18 and 19 are self-consistent. When no plasma sheath is present the curves for $r|E_\theta^{III}|$ coincide with those marked 10^{17} m^{-3} , provided $a = 0.2 \text{ cm}$. This statement is true for $\theta_o = 70^\circ$ as well as for $\theta_o = 85^\circ$.

Figures 20 and 21 present the real and imaginary parts of the transfer function for an infinite plasma sheet when the incident plane wave electric field is directed parallel to the sheet. Note that in Figure 20, $10^{13} \text{ m}^{-3} \leq N \leq 10^{17} \text{ m}^{-3}$ so that over the range of electron densities the actual density selected is immaterial. But when $N = 10^{19} \text{ m}^{-3}$, Figure 21 is needed to present the facts. The transfer function from which these data were computed appears in the literature⁵. As in all of the numerical results reported previously in this paper, $\omega_{\text{eff}} = 10^{12}$ collisions/sec.

⁵Charles W. Harrison, Jr., "Transient Electromagnetic Field Propagation through Infinite Sheets, into Spherical Shells, and into Hollow Cylinders," IEEE Transactions on Antennas and Propagation, Vol. AP-12, No. 3, pp 319-334, May 1964, Eq. (8). Note that Equations (10) and (11) appearing in this referenced paper have no application to plasmas, only to highly conducting metals.

FIGURE 18: THE RADIATION ZONE FIELDS OF A PLASMA CLAD EQUATORIAL SLOT ANTENNA ON A SPHERE FOR VARIOUS PLASMA SHELL THICKNESSES AND ELECTRON DENSITIES, $\theta_0 = 70^\circ$ AND $\theta = 90^\circ$. INPUT CURRENT TO THE TERMINALS OF THE BICONICAL TRANSMISSION LINE IS 1 AMPERE.

FIGURE 19: SAME AS FIGURE 18, EXCEPT $\theta_0 = 85^\circ$ AND $\theta = 90^\circ$

FIGURE 20: TRANSFER FUNCTION FOR AN INFINITE PLASMA SHEET. $10^{13} \text{ m}^{-3} \leq N \leq 10^{17} \text{ m}^{-3}$

FIGURE 21: TRANSFER FUNCTION FOR AN INFINITE PLASMA SHEET. $N=10^{19}\text{m}^{-3}$.

Figure 22 presents the efficiency of power transmission through an infinite plasma sheet for a number of sheet thicknesses and electron densities. Here e_i is the tangential electric field on the "far" side of the plasma sheet; e_o is the incident tangential electric field on the "near" side of the sheet. Comparing Figures 10, 11, and 22 reveals an interesting result. It appears to be possible, under present circumstances, to estimate the efficiency of power transmission from a plasma clad equatorial slot antenna cut in a sphere using the efficiency of power transmission of a plane wave per unit area through an infinite plasma sheet. This result is surprising when it is remembered that in the case of the slot antenna the field in the plasma just beyond the aperture of the slot is elliptically polarized. Generally speaking, one would expect the power loss in a conducting dielectric to differ for linearly polarized waves as compared to elliptically polarized waves.

Conclusions

A comprehensive theory of transmission from and reception by an equatorial slot antenna on a re-entry sphere under steady-state conditions has been presented. The plasma sheath may be in contact with or insulated from the sphere. An important feature of the present analysis is that all electrical characteristics of the re-entry vehicle have been referred to a set of terminals at the center of the biconical transmission line. The radar transmitter and receiver would be normally connected to these terminals by use of a coaxial cable. Sufficient information is supplied in this paper (both theoretical and numerical) to permit an engineer to proceed with the design of the radar transmitting and receiving circuitry and the selection of frequency of operation.

Performance data on the transmission and reception properties of the re-entry vehicle model selected for this study, when the plasma sheath is absent, has been included.

FIGURE 22: POWER TRANSMISSION PER UNIT AREA THROUGH AN INFINITE PLASMA SHEET.

It should be emphasized that a homogeneous plasma, without ablation products present, has been assumed in the analysis. This plasma is equivalent to a conducting dielectric.

Thus, a static plasma is employed in a dynamic problem. No account is taken in the theoretical development of the change in plasma properties occasioned by the application of a gcps frequency voltage to the terminals of the biconical transmission line, or by the excitation of the plasma clad re-entry vehicle by the incidence of an electromagnetic field in the gcps frequency range. The presence of static fields is ignored.

Figures 10, 11, and 22 show that the efficiency of power transmission for an equatorial slot on a plasma clad sphere compares favorably with the power transmission per unit area through an infinite plasma sheet provided that the plasmas have the same properties and the frequency is sufficiently high. But it cannot be inferred that the efficiency of power transmission from a partially circumferential slot in a plasma clad sphere may be obtained from Figure 22. This is because the electrical properties of an unending slot are necessarily completely different from those of a slot of finite length, i. e., one having end boundaries.

The solution of the boundary value problem is essentially exact provided the equatorial slot aperture cut in the sphere is sufficiently narrow. The reason for this is that only TEM waves are considered in the biconical line region. Even if both TEM and TM waves are assumed to be present, it is still possible to recast the theory so that the boundary condition is satisfied, for certain values of θ_0 requiring the radial component of the electric field in the line region to vanish on the biconical surfaces.

There are many applications for the theory developed in this paper. Some of these have been mentioned in the Introduction.

The response of the antenna-transmission line configurations discussed here to transient voltages or fields is easily obtained by use of Fourier transform methods. The procedure to follow has been set forth in the literature.^{5, 6, 7}

⁶Charles W. Harrison, Jr., Margaret L. Houston, Ronold W. P. King, and Tai T. Wu, "The Propagation of Transient Electromagnetic Fields into a Cavity Formed by Two Imperfectly Conducting Sheets," IEEE Transactions on Antennas and Propagation, Vol. AP-13, No. 1, pp 149-158, January 1965.

⁷Charles W. Harrison, Jr., and Charles S. Williams, Jr., "Transients in Wide-Angle Conical Antennas," IEEE Transactions on Antennas and Propagation, Vol. AP-13, No. 2, pp 236-246, March 1965.

APPENDIX

Procedures and Numerical Techniques Relating to the Determination of the Transmission and Reception Properties of an Equatorial Slot Antenna on a Re-Entry Sphere

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The majority of the numerical computations reported in this paper are straightforward and do not warrant discussion.

Whenever a series summation was required in the calculations a convergence criterion was met such that the last term taken in the series before truncation contributed less than 0.01 percent to the summation to that order.

Both forward and backward recursion techniques were used in the computation of the spherical Hankel functions of both kinds. For real argument only (regions I and III, Figure 1) backward recursion was used to calculate the real parts of the functions (spherical Bessel functions of the first kind) and forward recursion was used to calculate the imaginary parts (spherical Bessel functions of the second kind). In backward recursion, the real part of the Nth function is set equal to zero ($N \gg 1$) and the real part of the (N-1)st function is set equal to a very small number. The recursion relation,

$$f_{n-1}(z) = \frac{2n+1}{z} f_n(z) - f_{n+1}(z),$$

is then used to determine the remaining values of the real parts to $n = 0$. The ratio of the exact value of the real part to the calculated value for zero order is then taken and each of the N calculated values are multiplied by this ratio. This

technique avoided any loss in machine accuracy since the magnitude of the real parts of the spherical Hankel functions of both kinds decreases as the order increases. Accuracy to eight significant figures was obtained over the range of the arguments of interest and confirmed by comparison with existing tables. Forward recursion, used for the imaginary parts of the functions, produced the same degree of accuracy.

When the argument of the spherical Hankel functions was complex (region II, Figure 1) only forward recursion was used to calculate complete functions. This method was found to be the most reliable by comparing it to two other methods. These methods were: first, the calculation of the functions from the standard complex series representation

$$h_n^{1,2}(z) = 2 e^{\pm iz} \sum_{r=0}^n (\pm 1)^{n-r-1} (n+1)! / r! (n-r)! (2z)^{r+1},$$

and second, the calculation of the functions from the spherical Bessel Functions of the first and second kinds for which standard routines exist. For the range of arguments of interest, it was found that the forward recursion technique produced values that agreed with either one or both of the two test methods to eight significant figures although the test methods themselves rarely agreed with one another over all orders. This type of verification of the calculations was necessary since tabulated values of spherical Hankel functions with complex arguments are virtually nonexistent.

Supplementary References

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