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SC-TM-65-31

FORCE-CONTROLLED VIBRATION TESTING

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February 1965

ABSTRACT

This memorandum introduces the concept of force-controlled vibration testing, where the input force is controlled into a test system which includes not only the test item but a mass which represents the test item's foundation. The fundamental response characteristics for this type of test are presented and compared with those resulting from motion-input controlled tests. In addition, the methods used in determining the force input spectrum and the magnitude of mass, which simulates the foundation, are considered.

The basic intentions of this memorandum are (1) to acquaint test personnel with the type of phenomenon that occurs under this type of test, and (2) to enable the project personnel who request the tests to utilize and understand the data and analyses they receive in the reports.

This memorandum is intended to serve as a forerunner to more specific memorandums concerning the many phases of force-testing and the mechanical impedance concept.

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LIST OF SYMBOLS

g = acceleration/386
 F = force (pounds)
 f = frequency (cps)
 t = time (seconds)
 m = mass (pounds sec^2/inch)
 k = spring constant (pounds/inch)
 D = damping coefficient (pounds sec/in)
 Z = mechanical impedance (pound second/inch)
 M_e = effective mass (pound sec^2/inch)
 W_e = apparent weight (pounds/g)
 ω = circular frequency = $2\pi f$ (radians/sec)
 A = amplification factor
 S_F = force gage sensitivity (millivolts/peak pound)
 S_g = accelerometer sensitivity (millivolts/peak g)
 fn_1 = antiresonant frequency (cps)
 fn_2 = resonant frequency (cps)
 Q = resonance factor

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FORCE-CONTROLLED VIBRATION TESTING

Introduction

Vibration tests, as they are presently conducted throughout the country, rarely (if ever) simulate the environmental conditions for which they are intended. Consequently, the true environmental response characteristics of the test item are seldom determined or even approximated. The test results often force designers to concentrate on unrealistic problem areas and/or overlook those areas that are a potential problem.

The two basic reasons for unrealistic vibration tests are (1) constant motion control of the input to the test item, and (2) the methods used in deriving motion-input specifications. The first requires the false assumption that the foundation has infinite mechanical impedance and infinite effective mass. Consequently, the test item is not allowed to affect its own environment similar to the way it does in the field. Considering the second of the above reasons, it is unfortunate that test specifications are sometimes arbitrarily selected without reference to any applicable field data. It is just as unfortunate that those specifications derived from field data consider only the peak responses of the foundation, and neglect the characteristic valleys. Again, the test item's environmental effects are neglected. When the above practices are combined in a vibration test, overtests as high as 100:1 can result at some frequencies; at other frequencies, the test item may not be tested sufficiently.

A few individuals have crusaded for years to establish the fact that we are basically wrong in utilizing the constant-motion, input-control philosophy. Simultaneously, it has been suggested that the ultimate solution lies in the complete incorporation of the mechanical-impedance concept. Unfortunately, there are factors which prevent its immediate practical application. Some of these factors are the lack of field data from which to derive test specifications, the practical inexperience in applying the mechanical-impedance concept and its complexity.

In the past we have been somewhat analogous to the man who lost his billfold in a dark alley and hunted for it under a streetlight. Most people involved in vibration testing will admit to the fallacies of present test methods; however, the ease with which accelerometers can be used, and the complexity involved with a changeover to force transducers, tend to cause a resistance to change.

The Vibration Division at Sandia is now proposing the utilization of the mechanical-impedance concept in vibration testing. The objective is to insert more meaning into vibration test results.

Discussion

To fully appreciate the contents of this report, the vibration response characteristics of a simple vibration system, consisting of two flexibly connected masses, should be understood. For example, the responses of the system in Figure 1 will be investigated for (1) constant-sinusoidal acceleration (g) input at point one, and (2) constant-sinusoidal force input at point one. It is here assumed that the masses are rigid and of equal magnitude, and the system is lightly damped.

Later in this report this vibration system will be used to represent a test item (m_2) attached to a foundation (m_1). The spring and damper will represent the inherent characteristics of the test item and/or the interconnector between the test item and the foundation.

Figure 1. Simple Vibration System

The above inputs are mathematically described as:

$$(a) \text{ acceleration } (g) - g_1 = g_o \sin 2\pi ft \quad (1)$$

$$(b) \text{ force } - F_1 = F_o \sin 2\pi ft \quad (2)$$

Motion control, as referred to above, includes displacement, velocity, and acceleration control. However, only acceleration (g) control will be used throughout the remainder of this memorandum, although the discussion applies equally well to each type of motion.

Acceleration Control of the Input

Assume a 1.0 g sinusoidal input is applied to m_1 . The responses of m_1 and m_2 , over the frequency range f_1 to f_2 , will be as depicted in Figure 2.

Figure 2. Response for Acceleration Input

The response of m_2 is nothing more than that of a mass supported on a moving foundation (m_1). If the quality factor, Q , is assumed to equal 10, the maximum response is 10 g at fn_1 , where

$$fn_1 = \frac{1}{2\pi} \sqrt{\frac{k}{m_2}} \quad (\text{neglecting damping}) \quad . \quad (3)$$

The two masses are responding equally (1.0 g) at fn_2 , where

$$fn_2 = \frac{1}{2\pi} \sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}} \quad (\text{neglecting damping}) \quad . \quad (4)$$

Force Control of the Input

The constant force, F_0 (Equation 2), will be of the magnitude required to give both m_1 and m_2 a low-frequency ($\approx 0.1 fn_1$) response of 1.0 g. Therefore, the force applied to the base of m_1 will equal $(m_1 + m_2)$ 386 pounds. The resulting responses of m_1 and m_2 are shown in Figure 3.

Figure 3. Response for Force Input

At the frequency fn_1 , which is expressed in Equation 3, m_2 exerts a reaction against m_1 . This results in a nulled response (0.18 g) of m_1 , and an increased response (1.8 g) of m_2 .

At fn_2 , both m_1 and m_2 are responding at a maximum (7.1 g), and are approximately 160° out of phase. As shown in Equation 4, this response frequency is a function of both masses and the interconnecting spring.

Throughout the following discussion, fn_1 and fn_2 will be used to represent the antiresonant and resonant frequencies respectively. Antiresonance and resonance are terms derived from considering the bottom mass as a reference. At fn_1 , the response of mass m_1 has a normal tendency to decrease; at fn_2 , the mass m_1 has a peak response.

Differences Between g and F Control

Through comparison of Figures 2 and 3, it is obvious that the response characteristics are different under the two types of input control. These differences are discussed below.

(1) "g"-Control Requires Large Force at fn_1 -- This can best be exemplified by considering the g response of m_1 during F control (Figure 3). At fn_1 , m_1 is at 0.18 g. Therefore, to maintain m_1 at 1.0 g, an additional force of about $5F_0$ is required. Figure 4 depicts the force requirements for both the g and F controlled tests.

Figure 4. Force Requirements During Force and Acceleration Tests

(2) "g"-Control Requires Small Force at fn_2 -- Using an argument similar to that in (1), consider the response of m_1 during the F control test (Figure 3). At fn_2 , the response of m_1 is 7.1 g; therefore, a much lower force than F_0 ($0.17 F_0$) is required to maintain m_1 at 1.0 g. This is also shown in Figure 4.

(3) "g"-Control Does Not Allow m_2 to Affect the Response of m_1 -- Since m_1 is maintained at a constant g level (Figure 2), its response characteristics are independent of m_2 , D , and k . In other words, the mass, m_2 , is not allowed to affect its environment. This is not the case during force-controlled tests. Comparing the response characteristics during the g-controlled test (Figure 3) to those during the F-controlled test (Figure 4), it is easy to see that the latter type test allows m_2 to affect the response of m_1 . As an elaboration on environmental effects, consider the three vibration systems shown in Figure 4a, where only the lower weight (foundation) is of a different magnitude.

The responses of each system (W_1 and W_2) to a constant, sinusoidal force input below the foundations are depicted in Figure 4b.

Figure 4a. Vibration Systems with Different Foundation Weights

Figure 4b. Responses of the Vibration Systems in 4a

Based upon the above response curves, the following conclusions can be made:

- (a) For equal force input, the response magnitudes of W_2 and W_1 are directly related to the weight of W_1 . For example, increasing the weight of W_1 results in lower responses of W_2 and W_1 across the frequency test range. The depth of the anti-resonant notch and the height of the resonant peaks are also a function of the degree of damping. However, damping was assumed to be the same in each of the above systems.
- (b) The antiresonant frequency (fn_1) does not change since it is a function of W_2 and k . However, the resonant frequency is inversely a function of W_1 . The larger W_1 , the lower the frequency fn_2 . The ratio between fn_1 and fn_2 can be expressed as:

$$\frac{fn_2}{fn_1} = \sqrt{1 + \frac{W_2}{W_1}} \quad (5)$$

Up to this point, the force input has been controlled below the foundation mass (m_1). The response characteristics of the masses m_1 and m_2 , when the force is controlled through k into m_2 (test item), will be discussed later in this report.

Once the above response characteristics are understood, we are in a position to critically evaluate the methods we presently use in (a) vibration testing and (b) the derivation of test specifications.

Objectives of Vibration Tests

Vibration tests are normally requested with one or more of the following objectives in mind: (1) determine the structural integrity of a test item and/or the interconnector between the test item and the foundation, (2) determine the operational integrity of a test item, and (3) determine the acceleration response characteristics (resonant frequencies and amplitudes) of a test item.

Before the above-mentioned objectives (1) and (2) can be realistically achieved, the test setup in the laboratory and the vibration input to the test system should both be similar to those existing in the field. Thus, every item used above the input control point should be the same as in the field setup, and the input spectrum should be accurately determined from field measurements and applied during the test. Objective (3), assuming the system is linear, requires only that the test setup above the control point be completely representative of that in the field.

Few, if any, vibration test inputs are representative of the actual environmental characteristics. An investigation of test procedures, and the resulting fallacies are presented below.

For illustrative purposes, assume that a test item (m_T) is to be vibration-tested. In the field, the test item is attached to the floor (m_F) of an aircraft, which is only one of the many types of structures which could be assumed. This setup is shown in Figure 5.

With further assumptions that m_T and m_F are rigid masses, and the interconnecting structure has negligible mass, the setup can be represented as shown in Figure 6. The spring and damper are here assumed inherent in the interconnector.

Figure 5. Test Item Mounted in the Field

Figure 6. Simple Representation of Test Item Mounted in the Field

Reasons Why Acceleration Input Control Is Basically Wrong

The test item, m_T , is adapted to the vibration exciter by means of a test fixture (Figure 7a). This setup is further simplified in Figure 7b, where k_s is the interconnection between the test item and the fixture, and m_{FE} is the mass of the fixture-exciter armature combination. The interconnector, k_s , is assumed to be the same type used in the field.

(a) Test item on vibration machine

(b) Simple representation of 7a

Figure 7. Laboratory Setup for Vibration Test

The response characteristics of the system in Figure 7 will be essentially the same as those of the system in Figure 1; therefore, this discussion will be referenced to those responses outlined previously.

As is typical in vibration tests, the specified acceleration input to m_T will be controlled at point 1, which is at the base of k_s . What is drastically wrong with controlling the acceleration at this point? The test item is not allowed to affect its own environment. In other words, the dynamic response of m_T is not allowed to influence the motion of the foundation m_{FE} , which it certainly will do in the actual environment. The only possible way the foundation motion can be completely independent of this test item is for the ratio of foundation-mass to test-item-mass to be infinite ($m_F/m_T = \infty$ in Figure 6). Thus, in tests where the acceleration is controlled at the base of the test system, we are assuming that the foundation mass is infinitely large. The important results of not allowing m_T to affect its own environment are discussed below.

At fn_1 , a large amount of force is required to maintain the input motion of m_F at the specified magnitude (Figure 4). The resulting response of m_T is thus extremely high, which leads to overtesting.

In the true environment, and where the system is lightly damped, m_T is more susceptible to a high response at resonance (fn_2) than at any other frequency; i.e., applying the same level of sinusoidal force input (F_0) across the frequency test range, the response of m_T is many times greater at fn_2 than at any other frequency, including fn_1 , which has always been considered the critical frequency under the motion test concept.

The low response of m_T (at fn_2) caused by the severe notching of the force is radically unrealistic compared to the condition that exists in the field; consequently, the test items are vulnerable to undertesting at this frequency.

Unfortunately, a design engineer controls the frequencies where high force requirements occur during testing by his choice of m_T , k , and D when he designs or modifies the test item. Also, these extremes in force will shift in frequency due to test items and/or interconnectors having different properties, even though they are fabricated from the same design.

In summary, acceleration testing of this type causes excessive overdesign due to to overtesting at fn_1 , and in all probability results in a low safety factor or undertest at fn_2 .

Derivation of Test Specifications

The errors resulting from acceleration-control of the input, specifically the overtests, are further magnified through the methods used to derive the test specifications. The input to a test item is derived from field data which is usually obtained under one of the following conditions: (1) the responses on the foundation and/or at some point on the test item are monitored while the test item is mounted, (2) the responses on the foundation are determined without the test item being mounted, (3) the response history from some other test item, which was mounted on the foundation, is used, or (4) an input level is arbitrarily selected without reference to any directly applicable field data.

The field data obtained under condition (1) is the only data that should be used in deriving input specifications. Condition (2) could be used if this data were supplemented with impedance measurements of the test item and foundation. This would allow the modification of the measured response data to account for the test item load effects. To validate data obtained under condition (3), impedance data of the two different test items and the foundation would have to be known. None of these modifications (through impedance usage) has ever been made up to the present time. Condition (4) needs no explanation as to why it is wrong.

Finally, assuming that the above data is completely representative of the test item's motion in the field, this data is analyzed into acceleration amplitude vs frequency spectra. Many methods are used to derive input specifications from this data. Two of the most common methods used are discussed below. Unfortunately, the highest peaks are usually encompassed, and this acceleration level is called out as the test input. This extremism is sometimes reduced by calling out a step function for the input where the highest response is encompassed over particular frequency ranges. What is wrong with these practices? They disregard the important frequency fn_1 , where the motion of the foundation tends to be low, and consider only the high responses at other frequencies. Thus, the input at fn_1 is required to be high during a test. The test item's influence on its environments, where it causes antiresonance of the support, is ignored, and this is equivalent to assuming that the foundation is infinitely more massive than the test item.

What Can or Should Be Done

The main fault in present-day vibration tests, where the input is acceleration-controlled, is that we cannot predict or control the degree of overtest on any one test item, or cannot guarantee that we have a sufficient test. As the demand for higher performance and reliability of systems increases, with the simultaneous requirement that the size and weight of systems be reduced, test accuracy becomes progressively more important.

Test accuracy is definitely a function of being realistic in both the test setup and the input spectrum. No doubt better simulation between the field and test conditions can result through utilization of the mechanical-impedance concept. However, the complexity of mechanical impedance will probably result in a compromise type of test that is short of the limit suggested by theory. The Vibration Division at Sandia Corporation is presently pursuing this problem in an attempt to determine ways of running future tests that have more meaning.

Limiting Acceleration Response

To hold the acceleration response of a test item, or one of its components, to a specified level is called g-limiting. As an example, consider the system in Figure 8a. The response of m_T , for a constant acceleration input on m_F , is shown in 8b. Limiting m_T to a 20 g response requires that the input be reduced between f_3 and f_4 . This can be accomplished best by switching over to a control of the acceleration on m_T at f_3 , holding this at 20 g from f_3 to f_4 , and then switching the control back to the base at f_4 . The resulting input and response curves are shown in Figures 9a and 9b.

Figure 8. Simple Resonant System and the Response of m_T Prior to g-Limiting

Figure 9. Response of m_T and Input to m_T During g-Limiting

What does this accomplish? First, the test item is allowed to affect its own environment to some degree between f_3 and f_4 . Consequently, the ridiculously high force normally required between f_3 and f_4 is reduced to a lower, more sensible level. Indirectly, the assumption can be made that the mass of the foundation is reduced to some finite value between f_3 and f_4 . Note, however, that this method in no way corrects the undertest potential at resonant frequencies, such as fn_2 .

Caution should be used in selecting the reference point for acceleration limiting. The point should not be extremely easy to drive in comparison to the rest of the test system. This means it should not be nearly massless (low-mechanical impedance) across the whole test range.

Mechanical-Impedance Testing

The second step in the program has been the application of force-control of the input at a point below a mass which is intended to represent the test item's support structure in the field. This is a form of mechanical-impedance testing.

Mechanical Impedance

Mechanical impedance (Z) is defined as the complex ratio of the sinusoidal force applied at a point to the resulting velocity at that point. Since force and velocity are vector quantities, the phase angle between the two vectors must be considered.

If the applied force is $F = F_0 \cos (\omega t + \theta)$ and the resulting velocity is $v = v_0 \cos \omega t$, then mechanical impedance is mathematically expressed as:

$$Z = \frac{F_0 \cos (\omega t + \theta)}{v_0 \cos \omega t} \quad (6)$$

where θ is the phase angle between the two vectors. Mechanical impedance is simply a measure of the dynamic characteristics at the point of force application. Z is a measure of resistance to motion; high Z is analogous to a massive or rigid structure, and low Z is analogous to a light or flexible structure.

Effective Mass

Effective mass (M_e) is the complex ratio of sinusoidal force to the resulting sinusoidal acceleration. Thus, the relationship between Z and M_e is as follows:

$$Z = \omega M_e \quad (7)$$

where

Z = mechanical impedance

M_e = effective mass

ω = circular frequency = $2\pi f$.

The phase angle between Z and M_e is 90° . High M_e is analogous to high Z and low M_e is analogous to low Z .

Apparent Weight

Apparent weight (W_e) is the complex ratio between the sinusoidal force and the resulting acceleration in terms of g . Thus,

$$W_e = \frac{F}{g} = \frac{F}{ax386} \quad (8)$$

Apparent weight and effective mass are in phase.

Classification of Test Methods

(A) Acceleration (Motion) Control of the Input -- Maintaining the acceleration level of the foundation independent of the test item is assuming that the foundation has infinite Z and M_e . Thus, the test is referred to as an infinite-impedance test. (Figure 10).

(B) Force Control of the Input into The Test Item -- Maintaining the force input to the test item (Figure 11) at a level which is independent of the foundation mass is assuming that the foundation has zero (or negligible) Z and M_e . This type of test is referred to as a zero-impedance test.

Figure 10. Infinite Impedance Test
on Simple System

Figure 11. Zero Impedance Test
on Simple System

Differences Between Infinite- and Zero-Impedance Test Results

Consider the test item (m_T) mounted on the foundation mass (m_F), as shown in Figure 12. The spring and damper are considered to be inherent in the test item and/or the inter-connector between the test item and the foundation.

Figure 12. Test Item m_T Mounted On the Foundation Mass m_F .

The sinusoidal responses of m_T , m_F , and the sinusoidal force F_1 are shown in Figures 13 and 14 for motion control (infinite-impedance) and force control (zero-impedance), respectively at point 2.

Figure 13. Responses for Infinite Impedance Test

Figure 14. Responses for Zero Impedance Test

As previously discussed, an infinite-impedance test makes the following assumptions about force availability below the support structure, (Figure 13):

- (a) A high force available at fn_1
- (b) A low force available at fn_2
- (c) A moderate force available at high frequencies ($> fn_2$)

A zero-impedance test makes the following assumptions about the available force (Figure 14):

- (a) A moderate force at fn_1
- (b) A low force at fn_2
- (c) A high force at high frequencies considerably greater than fn_2 . In most vibration tests, this would be a limitation at the higher frequencies since the shaker output would be exceeded.

The similarity between either of the above force spectrums and that in the true environment is quite improbable. Until a better understanding of the actual force characteristics in a test item's environment can be attained, there appears to be a great deal of logic in testing with a force level that is both controllable and at a reasonable level across the frequency test range.

The assumption of infinite or zero impedance and effective mass of the foundation across the complete test range is also ambiguous. Antiresonance of a support structure results in a peak of the effective mass, and resonance results in a valley (low effective mass). However, these extremes vary radically with frequency and hence a constant M_e magnitude across the frequency test range is unrealistic. Attempts are now being made to determine the effective mass characteristics of several support structures. However, until this can be done for each situation, or until experience allows a fairly close approximation prior to each test, a reasonable approach is to assume an effective mass based upon the static weight or low-frequency dynamic properties of the structure.

A method has been suggested¹ whereby the infinite-impedance concept is used at the lower test frequencies and a transition to the zero-impedance concept is made at the higher frequencies. This reasoning was based on evidence that indicated (1) that at low frequencies the foundation tends to have a high impedance by comparison with the equipment, and (2) that the foundation (in this case a missile airframe) tends to have a low impedance by comparison with the equipment at the middle and high frequencies. Regardless of the advantage of using these concepts separately or in combination, the fact still remains that zero or infinite impedance and effective mass are far from descriptive of the foundation's true dynamic characteristics.

¹Morrow, C. T., Shock and Vibration Engineering, Volume I, John Wiley & Sons, 1963, pp. 136-137.

Under our present pursuit, both of these extreme concepts are ignored. We are proposing testing under, and trying to improve, the concept of controlled effective mass testing. This method is discussed below.

Controlled Effective Mass Testing

Controlling the force input into a mass which represents the test item's foundation is termed a controlled effective mass (M_e) test. Depending upon the dynamic characteristics of the mass (m_F), a test is either a constant- or variable- M_e test. Unless m_F remains rigid across the test range, a variable- M_e test results. Considering the test system in Figure 12, the acceleration response characteristics and force (F_1) below the mass m_F are shown in Figure 15. It is here assumed that the force input is constant and that the effective mass of m_F is constant across the test range.

Figure 15. Responses for Controlled Effective Mass Test

The advantages of controlled- M_e testing over other methods previously discussed are listed below.

1. There is a definite control of the force input below the foundation (Figure 15). Thus, the force will not have sharp peak or notch characteristics across the frequency test range (particularly at fn_1 and fn_2). The magnitude of overtesting, and the possibility of undertesting, can both be lessened.

2. As shown in Figure 15, the test item has influenced the response of the foundation and, therefore, affected its own environment in a manner similar to that in the field. As a result, the test item response at fn_1 is lower than it would be for infinite- M_e testing, and the higher test item response at fn_2 is allowed to occur.
3. Every test item from a particular model does not have the same dynamic characteristics. Also, test items are sometimes modified between vibration tests. Because of the above, infinite- M_e testing results in a different force spectrum. This should not occur and it doesn't when this force spectrum is controlled. During controlled- M_e testing, the response magnitudes and frequencies change as they should, but not the force spectrum.
4. Another advantage of controlled- M_e testing is that the axial-fundamental response of the test item is not a function of the test equipment. The test equipment below the force-control point (armature and fixtures) has no fundamental influence on the system above the control point. However, the test equipment's effects upon rotation, distortion, etc., are still not eliminated.

The degree of test accuracy resulting from controlled- M_e testing obviously depends upon the force spectrum used and the M_e used to simulate that of the foundation. At present, the selection of these test values is somewhat arbitrary; however, the basic principle is aimed in the right direction, since the test item is being allowed to affect its environment in the manner it should. As the available field data increases and/or our ability to predict test conditions improves, we will come closer and closer to a truly simulative vibration test.

It should be remembered that one of the most accurate ways to test would be to measure the acceleration of the foundation (with test item mounted), or measure the force input to the test item, and then exactly reproduce one of these spectrums (peaks, valleys, etc.) in the laboratory test. This would eliminate the need to be concerned about M_e of the foundation being simulated in the test, since only the equipment above the control point has to be the same in the test as it was in the field. Overlooking inaccuracies that would result because each test item and foundation does not have the same characteristics, and motion or force in the field is not exactly repetitive, this method is basically sound. To lessen these inaccuracies, enough motion and/or force data would have to be accumulated to allow a good statistical description of the input spectrum. In addition, one could compensate for the differences between individual test items by measuring the Z or M_e properties of each item to be tested, and then using this data to alter the input spectrum for each case. Since this field data is not available, we have proposed the method of controlled- M_e testing, discussed above, to insert more realism into our vibration tests. The methods used in this type of test are discussed below.

Force Input Magnitude

Until a better understanding of the environmental forcing function can be obtained through field measurements, a constant-sinusoidal force input or a flat random-force

spectrum is recommended. Normally, the specified g input level is used as a guide in determining the force input. For example, assume that a test item of weight W_1 pounds was to be tested with a simulated foundation of weight W_2 pounds. If the specified acceleration input were to be 2.0g from 20-2000 cps, the constant-force input below the foundation would be set at 2.0 ($W_1 + W_2$) pounds from 20-2000 cps. It should be clear by now that the force 2 ($W_1 + W_2$) will not give the specified 2.0g constant input across the frequency range unless everything above the force control point is as one rigid body.

Another method which has been used is that of applying 10 percent of the maximum rocket thrust as a constant-force input into the payload and simulated rocket-motor mass.

Representing the Foundation Effective Mass

As long as a constant-force input spectrum is used in vibration tests, it is necessary to insert a mass (representative of the foundation) above the control point in order to allow the test item to affect its environment in a manner similar to that in the field. Without this mass above the force control point, a zero-impedance test would result. The disadvantages of this type test have already been discussed.

The dynamic characteristics (M_e) of typical foundations are definitely not constant; i.e., a plot of M_e vs frequency for most any foundation will be variable over the frequency range of interest. Therefore, to truly represent the foundation's characteristics during a test, the effective mass (as seen by the test item) should match the variations of the real foundation over the test frequency range. As yet, this variable mass control has not been attempted during a test. The inserted mass has always been designed to remain rigid across as much of the frequency range as possible. Thus, in tests to date, the M_e of the foundation has been nonvariable at the lower frequencies, and is a noncontrollable function above the frequency where rigidity is not the case. Future work includes the experimental determination of the foundation's M_e as a function of frequency, and also finding ways to duplicate this function in the laboratory.

Currently, the magnitude of the foundation M_e to be used in a test is selected in one of the following ways. In tests where there is no reference available, a constant M_e equal to the static mass of the test item is being considered for use. For tests on rocket payloads, an M_e equal to that of the empty rocket motor is held constant across the test range. Motor-empty M_e is used rather than motor-full, since the former results in higher test item responses during testing.

M_e Simulation

Two methods are used to insert the desired foundation M_e into the test setup. The first is physical addition of a mass. In this case it is desirable to design a mass that remains rigid across the frequency test range if at all possible. As long as the mass is

rigid, M_e is controlled at a constant, known value. The second method, which is now being used almost exclusively, is electronic simulation of the desired M_e value. This concept utilizes a simple analog computer circuit which determines what the force input to the test item would be if the desired M_e were actually inserted in the test setup. A complete description of this procedure will be presented at a later date.

Two advantages of electronic representation over physical representation are the economics and the potential ability to electronically vary M_e as a function of frequency and/or time. As previously mentioned, it is known that a foundation's M_e is not constant over the frequency test range. When the variable function is determined from field measurements, it is hoped that it will be possible to reproduce or approximate this function in the laboratory.

A brief discussion of electronic mass simulation will now be given. Consider a rigid weight (W), mounted on a vibration machine with the force gage (FG) intermediate. This setup is shown in Figure 16a. In Figure 16, the millivolt output of the accelerometer is represented by g , and the millivolt output of the force gage is represented by F . E_c in Figure 16b represents the instantaneous sum (vector sum) of g and F , while E_c in Figure 16c represents the instantaneous difference (vector difference) of g and F .

The variable amplifier is used to amplify or attenuate the g signal, the degree of which depends upon the weight to be electronically added or subtracted. The 180-degree inversion of the g signal in Figure 16c is necessary in this setup in order to get the instantaneous difference.

Once the desired amount of weight has been electronically inserted in the system, the desired force-input spectrum is controlled through the E_c signal rather than the immediate output from the force gage. For example, the amount of amplification set into the variable amplifier is determined by

$$A = \frac{S_F}{S_g} W_A \quad (9)$$

where

- A = amplification factor
- S_F = force gage sensitivity (RMS millivolts/peak pound)
- S_g = accelerometer sensitivity (RMS millivolts/peak g)
- W_A = weight electronically added (pounds).

Figure 16. Setup and Instrumentation for Electronic Mass Addition

The signal E_c would be controlled during the test at a level which is determined from the force input spectrum and the sensitivity of the force gage (E_c millivolt level = force input (peak pound) x force gage sensitivity (millivolts/peak pound)).

As mentioned previously, controlling E_c rather than the direct output of the force gage (F) provides the analog computer with information that determines what the force input to the test item would be if the desired mass were physically, rather than electronically, inserted. If a variable effective mass (M_e) function is to be used during testing, the degree of amplification (A) can be varied during testing to give the desired M_e versus frequency function. However, one disadvantage to electronic addition does still exist since, as yet, there is no proven way to electronically control the phase angle existing between the effective masses of the test item and foundation. (This is also another disadvantage to physical mass addition.) To be completely realistic in simulating M_e of the foundation, the phase relationship with the test item should be preserved across the frequency test range. The phase problem is being investigated to determine the degree of severity if it is not controlled in a test. Methods of phase control will also be investigated in the near future.

As mentioned previously, it is quite probable that field data will eventually eliminate the constant-force input and the insertion of a simulated foundation M_e . If the motion at the base of the test item, or the force input to the test item is determined from field measurements and reproduced in the laboratory, there is no need for either constant-force input or duplication of M_e .

In summary, the above discussion concerning the assumptions as to force input and effective mass levels used in tests is quite indicative of the need for accurate field data and impedance measurements of vehicles, payloads, and foundations.

Constant-Force Input and Controlled- M_e Test on the RFD II Re-Entry Vehicle

The first constant-force input and controlled- M_e test run at Sandia Corporation was on the RFD II Re-Entry Vehicle (RV). The RV was designed for a SNAP 10A nuclear reactor. The RV and reactor are shown mounted on a vibration fixture in Figure 17.

At the time the test was requested, the 450-pound system was to be powered by a four-stage NASA Scout rocket, which is connected to the RV by means of the interconnector shown in Figure 18. With the exception of momentary pulses at ignition of each stage, field data indicated that the only significant vibration occurred during fourth-stage burnout; thus, only this vibration was requested to be simulated.

Figure 17. RFD II Re-entry Vehicle Mounted on Vibration Machine

Figure 18. RFD II Test Setup (Internal)

Prior to testing, decisions had to be made concerning the input force level, the motor M_e representation, and the type of force transducer to be used. Vibration energy during flight was mainly attributed to burning of the rocket. Since there was no reference from which to determine the proper forcing function, the value of 500 pounds constant-force (recommended by NASA), was used over the frequency range of 20-2000 cps.

The test setup and field setup were identical for the system above the motor. Thus, the only decision remaining was the value of M_e to be used in representing that of the motor. Since field data indicated that the maximum vibration response (30 g) occurred just prior to fourth-stage burnout, when the motor weight was 60 pounds, it was decided to represent the motor-empty condition. Another strong factor in the decision to represent the motor-empty M_e was the results obtained in an electrical analog study, where the response of the RV was investigated for both motor-empty and motor-full conditions. For the same force input below the motor, it was determined that the empty condition resulted in RV responses about seven times higher at fn_2 than for the motor-full condition. Thus, a 60-pound cylinder was used to represent the motor (Figure 18). The attachment between the cylinder and interconnector simulated the actual connection between the fourth-stage motor and the interconnector.

The force transducer used in the first series of tests is shown in Figure 18. Forty strain gages were mounted around the periphery, with every fourth gage connected in series to form the four legs of the bridge. Two of these legs were used for temperature compensation. Due to low sensitivity (< 0.1 microvolts per pound), the signal-to-noise ratio was low enough to cause accuracy problems during input control. The final test series utilized ten commercial transducers mounted under the 60-pound cylinder. The transducer outputs were instantaneously summed. The resulting force sensitivity was about one thousand times that of the first transducer.

Typical response characteristics of the RV and motor are shown in Figures 19a and 19b. As predicted, the response characteristics during testing simulated those of the simple system shown in Figure 20. In this sketch, m_t represents the weight of the RV, the reactor, and all internal components of the RV. The spring, k , represents the relatively light adapter's spring characteristics, and m_m represents the 60-pound motor. As can be seen by the antiresonance (fn_1) and resonance (fn_2) of the motor, the system above the adapter was allowed to affect its own environment. As a result, the highest response on the RV occurred at fn_2 rather than fn_1 .

Had a constant acceleration of 1.0 g (infinite- M_e test) been maintained on the motor, the force required at fn_1 would have been about 3000 pounds, six times the force estimate actually used for an input. The resulting RV response at fn_1 would have been about 10 g rather than 1.6 g. Also, the force required at fn_2 would have been about 50 pounds, and the resulting RV response approximately 1.0 g.

Figure 19. Vibration Response of RFD Re-Entry Vehicle and Simulated Motor

Figure 20. Simple Representation of RFD II Test Setup.

A report by Belsham and Harris² includes apparent weight (W_e) analyses of the full and empty fourth-stage motor (X - 248). These plots are reproduced in Figures 21a and 21b. The dashed lines superimposed on these figures represent the motor-full and motor-empty apparent weights that were considered for use in the RFD II tests. The antiresonant frequency (fn_1) and resonant frequency (fn_2) which were found during vibration tests are also shown on these plots.

Up to about 120 cps, the 60-pound apparent weight used in the test was fairly accurate in its representation of the actual empty motor. There was considerable error involved at the frequency (≈ 130 cps) where there was a peak in the apparent weight (antiresonance). Above 200 cps, 60 pounds was higher than either motor-empty or motor-full apparent weights, with the exception of two frequencies on the motor-empty plot.

A better representation of the system's response to motor-full and motor-empty M_e , above 200 cps, would have been obtained if the apparent weight of the motor had been rolled off as some function of frequency after being maintained constant up to 200 cps. However, at the time of this test sequence we were not aware of the method whereby apparent weight could be electronically varied.

The above discussion applies to random-force inputs as well as sinusoidal. Several random tests have been run where the force spectrum was equalized, and electronic M_e representation was used. No difficulties or discrepancies in the data were experienced.

Summary

The basic intention of this memorandum was (1) to illustrate the fact that motion-controlled vibration tests are unrealistic, since the test item is not allowed to affect its environment as it does in the field, and (2) to propose the utilization of force-controlled vibration tests, where the force is controlled below a mass which represents the test item's foundation.

Force testing (controlled- M_e test) is one avenue to realistic testing, since the test item is allowed to affect its environment. Although the force-control concept, in its most primitive stage, offers a more realistic test than the motion-control concept, the validity of the results is strongly dependent upon the force spectrum used and the ability to accurately simulate the dynamic characteristics of the test item's foundation.

²Belsham, R. O., and Harris, J. J., Apparent Weight Measurements of Rocket Payload and Test Structures, NRL Memorandum No. 1099, U. S. Naval Research Laboratory, 1960.

Figure 21. Apparent Weights of Fourth-Stage Motor