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PROCEEDINGS OF THE TECHNICAL MEETING ON

SUPER-LAMINAR FLOW BEARINGS AND SEALS FOR PROCESS FLUID LUBRICATED TURBOMACHINERY

NOVEMBER 1 AND 2, 1966
ALBANY, NEW YORK

U.S. ATOMIC ENERGY COMMISSION
DIVISION OF REACTOR DEVELOPMENT AND TECHNOLOGY
SPECIAL PROJECTS BRANCH
GERMANTOWN, MARYLAND

RELEASED FOR ANNOUNCEMENT

IN NUCLEAR SCIENCE ABSTRACTS

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Division of Reactor Development and Technology
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Germantown, Maryland

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E. B. Arwas, Editor

December, 1966

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FOREWORD

The Technical Meeting on Development of Super-Laminar Flow Bearings and Seals for Process-Fluid Lubricated Turbomachinery was held in Albany, New York on November 1 and 2, 1966. The publication of the technical presentations made at this meeting is sponsored by the U.S. Atomic Energy Commission and has been prepared by Mechanical Technology Incorporated under U.S. AEC Contract AT(30-1)-3363.

The technology of lubrication in the turbulent-flow regime has practical importance because of the recent and continuing developments of high-speed turbomachinery lubricated with low viscosity fluids such as water, steam, and liquid metals. Examples of such developments include compact, dynamic power-conversion turbomachinery for use in space, undersea, and in mobile, land-based power-plants, as well as large coolant-flow pumps and circulators for use in nuclear reactor installations.

The technical meeting of November 1 and 2, 1966 was held to provide for an exchange of information between the various groups active in research, development and application programs in this area. During the course of three technical sessions, seventeen presentations were made by representatives of U.S. Government Agencies and commercial contractors, on recent progress, current effort and anticipated future requirements in process-fluid lubrication and its applications to nuclear and aerospace turbomachinery.

This report contains all these presentations, reproduced from the manuscripts furnished by the authors to the editor. For continuity and ease of reference, these presentations have been assembled here according to their principal topic into the sections listed below:

- Section 1 - Summary
- Section 2 - Introductory Address
- Section 3 - Background Review
- Section 4 - Technology I - Fundamentals
- Section 5 - Technology II - Seals
- Section 6 - Technology III - Bearings and Materials
- Section 7 - Technology IV - Rotor-Bearing Dynamics
- Section 8 - Application to Large Turbomachines

It should be noted, however, that some of the presentations included in Sections 5 through 7 covered other material, including experience with process-fluid lubrication in high-speed machines.

SUMMARYTECHNICAL MEETING ON DEVELOPMENT OF SUPER-LAMINAR FLOW
BEARINGS AND SEALS FOR PROCESS-FLUID LUBRICATED TURBOMACHINERY

by

Dr. Beno Sternlicht
Technical Director
Mechanical Technology Incorporated
Latham, New York

The use of process-fluid lubrication in turbomachinery presents many advantages. It eliminates oil contamination and permits operation at pyrogenic and cryogenic temperatures, as well as in radioactive and corrosive environments, without lubricant breakdown. The use of process-fluid lubrication, however, requires some fundamental changes in mechanical design. The principal new design considerations include the effects of turbulent flow in bearings and seals, rotor-bearings stability, reduced bearing damping and ultimate load capacity, oxide deposition, selection of compatible materials and surface coatings, and others.

In the early 1960's, as a result of many high-speed machine failures, it was recognized that the application of turbulent lubrication technology to bearing, seal and rotor-bearing dynamics design was very limited. To our surprise, we found that the Rumanians were leaders in this technology. As early as 1957 a book by N. Tipei was published in which turbulent lubrication was treated. In 1963 at the University of Houston, Constantinescu presented a comprehensive lecture on "Theory of Turbulent Lubrication." In 1965 his book, "Theory of Lubrication in Turbulent Regime," was published. In retrospect, this technology should have been vigorously pursued as soon as it was recognized that there was a trend to higher speed machinery and that many of the contemplated cycle fluids (e.g., liquid metals, water) had low kinematic viscosities which would result in turbulent flows at relatively low speeds. In fact, turbulent and compressible lubrication theory and practice should have been the goals of lubrication engineers, recognizing that laminar ($Re < 1000$) and incompressible ($\Lambda \rightarrow 0$) lubrication analyses represent only limiting cases of the more general treatment of this subject.

In recent years a number of hardware programs (e.g., SNAP 2, SNAP 8 and SNAP 50/SPUR) and, to a more limited extent, technology studies have been conducted in the United States by several groups. We have organized this seminar for the purpose of bringing together people actively engaged in this field in order to disseminate information, present the state of knowledge and identify some of the remaining problem areas. The papers were divided into the following categories:

Review

Technology

Applications

The following represents a brief summary of the meeting. More detailed discussions will be found in the other papers which make up this report.

Present State of Knowledge

1. Visual studies have been conducted to gain better understanding of transition from laminar to vortex and turbulent flows in annular gaps. Both concentric and eccentric gaps were studied. The onset and development of film rupture were also investigated visually. Theory has been developed for the boundary conditions at film separation that are to be used with the Reynolds equation.
2. Superlaminar theory neglecting inertia effects has been developed for the calculation of load, frictional losses, stiffness and damping for several bearing types. This theory is in excellent agreement with practice.
3. Several dynamic seal configurations have been investigated experimentally. These include screw seals (visco seals), rotating channels, slingers, and squeeze seals. The experimental studies revealed a fundamental difference in gas ingestion mechanism between grooves on the shaft and grooves on the housing.
4. Stability of several bearing types has been investigated experimentally. Considerable duplication of effort appears in this area. The bearings studied included plain cylindrical, two, three and four axial groove, pressure pad, three lobe, hybrid, herringbone, floating ring, pivoted shoe (tilting pad), etc. Tilting pad bearings exhibited greatest stability; herringbone and floating sleeve bearings were next best.

5. Theory is available for obtaining stiffness and damping coefficients for several bearing types operating in turbulent regime. These coefficients can be coupled in the bearing-rotor dynamics analysis to obtain rotor response. The agreement between theory and practice for critical speeds, amplitudes of vibration and threshold of instability is good.
6. Several machines have been successfully developed using process-fluid lubricated bearings and seals. (Due to the lack of technology information, however, too much trial and error was involved in these developments.)

Remaining Problem Areas

1. Turbulent lubrication theory, incorporating inertia effects, is required.
2. Theoretical analysis of hybrid journal and thrust bearings, also incorporating inertia effects, is required.
3. Theory is required for visco-seals and other potential dynamic seals.
4. Reliable rugged instrumentation for dynamic measurement of bearing and seal film thickness operating in high temperature liquid metal environment is very badly needed for both experimental work and monitoring of machinery.
5. Bearing and seal materials require immediate attention. Cemented carbides, which are good from the standpoints of sliding behavior, corrosion resistance and high-temperature strength, present major design problems due to their low coefficients of expansion, shock resistance, manufacturing difficulties and high cost. Consideration should be given to other materials and to coatings.
6. Theoretical and experimental studies are required on cavitation in bearings and seals operating in liquid metal environment. The complex flow regimes that take place in bearings and seals must be considered in such a study as opposed to simple cavitation in ventury, plate, disk and other experimental apparatus.
7. In the case of hybrid (externally pressurized with hydrodynamic effects) bearings, erosion of restrictors and pockets must be investigated in order to ensure long life, trouble-free operation. This is basically a fluid mechanics and materials problem.

8. Considerable attention must be paid to the design and manufacturing of the large sodium cooled reactor bearings and seals. Because of their size they present major problems of material choice, method of attachment (of sleeves, runners, bearings), flow requirements, power loss, stiffness, damping, etc.
9. Testing in liquid metals is very expensive and time consuming; therefore, it must be used in conjunction with theory in order to minimize the number of tests and establish broadly applicable technology. Accurate, well-instrumented tests are therefore essential (such instrumentation has not been proven out at this time). Theory must be available for guidance and correlation (considerable theory is still missing (refer to items 1, 2, 3 and 6 above)).

The above-mentioned problem areas require immediate attention if reliable process-fluid turbomachinery is to be developed. It is hoped that technical papers on these subjects will be presented in the future and that other similar meetings will be held in order to exchange information. The papers and meetings will be most valuable if the investigators clearly present the data, accuracy of data, and point out problem areas. It is further hoped that such meetings will help to prompt complementary efforts and to minimize duplication.

INTRODUCTORY ADDRESS

X Review of the Role of the *AEC*
Atomic Energy Commission
in Process-Fluid Lubrication

by

Nicholas Grossman
Chief, Special Technology Branch
Division of Reactor Development and Technology
USAEC

I want to thank Mechanical Technology Incorporated for making this engineering get-together possible. I consider this meeting an important milestone in the technology of process-fluid lubrication; it heralds the emergence of process-fluid lubrication from the laboratory and the test bench to practical engineering design. Naturally, we all realize that our analytical knowledge in this field is indeed very small compared with the vast areas yet to be explored -- but the successful demonstration of process-fluid lubrication to rotating machinery is here -- with us today.

Perhaps it will be useful to describe -- in bare outline -- the goals of the Atomic Energy Commission's program in power reactor development; our interest in process-fluid lubrication and the specific needs for process-fluid lubrication.

Many of you are aware of the long standing, well organized and highly successful program in gas lubrication coordinated by the Fluid Mechanics Branch of the Office of Naval Research. Each of us here today is a direct beneficiary of the farsighted and effectively administered ONR program. There are a number of enthusiastic supporters who materially contributed to this undertaking, but I feel we should specifically mention two of our friends, whose unfailing devotion stands out as an example for us all: Mr. Stanley Doroff of ONR, who is directly responsible for procuring the support, defending the program, and dispensing the funds with Solomon-like impartiality and wisdom -- and Professor Dudley Fuller

of Columbia University, who in the role of Technical Secretary of the Coordinating group, has been the program chairman, recording secretary, conciliator and tower of strength.

I recall that during the 1965 summer meeting of the ONR Gas Bearing Coordinating Group, Columbia University was our obliging host. We were fortunate to have Dean Robert Dunning as our keynote speaker. As you know, Dean Dunning was one of the pioneers who helped move atomic energy from the physics laboratory into the realm of engineering.

During his talk to the ONR Gas Bearing Group in 1965, Dean Dunning mentioned how he and his engineering associates working on the design of the gaseous diffusion plant some twenty years earlier, recognized the practical advantages of gas lubrication and made record notes accordingly! Thus if we wish to establish a date to connect gas lubrication and atomic energy, we can go back to the wartime Manhattan Engineering District and state that engineers working with atomic energy on an industrial scale recognized the natural affinity between process-fluid lubrication and atomic energy at the very outset. In my opinion, had rigid security classification not been necessary during the war, the development of process-fluid lubrication would have progressed at a much faster rate, and certainly would have been a practical industry application at a much earlier date.

It is well known that the idea of gas-lubricated machinery was not incorporated in the gaseous diffusion plant built during the war. It is at this junction that we must appreciate the foresight of the Fluid Mechanics Branch of ONR for recognizing the industrial potential of gas lubrication and organizing a sound research program to cover this vast field in an orderly way. Naturally, a clever idea like gas lubrication cannot be kept as a monopoly of any one group, and just as it was recognized in America, similar development efforts were pursued in Europe.

One of the obvious consequences of the war in Europe was the acute shortage of modern central electric power stations. Consequently, there was greater impetus to develop central power stations -- and their construction was pushed much more vigorously in Europe than in the United States. It is understandable,

therefore, that industrial applications of gas bearings in gas blowers appeared in Europe a few years ago.

Coming back to the Atomic Energy Commission -- its role in the promotion of industrial application is specified in Public Law 83-703, popularly known as "The Atomic Energy Act of 1954." Section 3 states in part: "It is the purpose of this Act to effectuate a program of conducting, assisting, and fostering research and development in order to encourage maximum scientific and industrial progress." How successful the Commission has been in carrying out its charter can be readily assessed in terms of nuclear power stations built or being constructed, the use of radioisotopes and other byproducts now in common use, and nuclear propulsion for naval purposes. What is amazing about the success of atomic energy as an established segment of American industry is not that it came into existence, but rather the phenomenal speed of that development which surprised even its most optimistic advocates.

Looking at the typical American commercially built nuclear fueled central power stations, it may appear that -- after stripping away some superficial differences -- it is basically a closed-cycle steam power plant "burning" uranium as its heat source. Looking at the typical British nuclear power station, it appears as a closed gas cycle heat generator, "burning" uranium as its fuel. Both types have proven reliability and economic value.

Important components of these systems are circulators, in many cases using process-fluid lubrication. European gas-cooled reactors -- such as Dragon -- use gas-bearing motor-driven circulators. Just two years ago a significant milestone was achieved when MII, under an AEC- Bureau of Mines contract, demonstrated for the first time a gas bearing turbocirculator operating in a closed-loop Brayton cycle. Since that time a great deal of technical information has been obtained from that program.

Today you will hear about turbine-driven circulators and power conversion systems using water and liquid metal bearings and seals in a turbulent regime. Future developments will lead to the engineering application of two-phase lubricated systems in turbomachinery. Since the AEC is developing two-phase systems, the Commission's interest in process-fluid lubrication is therefore apparent. In fact, most of the advanced process-fluid lubricated machines

have been developed under AEC and Navy contracts.

What then is the goal of the AEC for future development? I will restrict my remarks to the program of the Division of Reactor Development and Technology. Current commercial nuclear power plants use a uranium "burning" reactor operating in the thermal neutron spectrum which converts a portion of the uranium into a new fissionable material: plutonium. There is thus a newly created material that can be used for power generation. However, the amount of new material produced is small compared with the uranium that was used or "burned" in the fission process. Therefore, looking at this process from the point of view of most effective utilization and conservation of our national reserves, thermal reactors are consumers of fissionable material. One of the amazing aspects of nuclear fission -- at least to us engineers -- is that in a suitable fast neutron spectrum more new fissionable material can be "created" than is being "used". This concept is popularly known as "breeder reactor". The orderly development of this system is a high priority goal of civilian nuclear power development programs.

The required combination of nuclear physics and engineering for nuclear reactors operating in the fast neutron spectrum points to the alkaline liquid metals as the heat transport medium. This is the concept that has been carried furthest, and the one that has shown to be technically feasible in actual reactors, both in the United States and Europe. Therefore, it is logical that the Division of Reactor Development and Technology actively pursue an orderly program leading to a reliable alkaline liquid metal bearing and seal system for rotating machinery. I have not seen the reports and papers to be presented during this symposium ahead of time, but I am certain the authors will present the technical details and enumerate the potential advantages and benefits of liquid metal bearings.

Before closing I want to state that the Space Nuclear Systems Division of the AEC, as well as other government agencies, are also pursuing the development of liquid metal bearings and seals, where their application offers attractive benefits to their specific requirements. It is one of the purposes of this symposium to afford us all an opportunity to become better acquainted with the efforts of other groups, and offer the benefit of our experience -- to one another. I am very pleased with the progress to date and I hope you will share this pride.

Thank you.

BACKGROUND REVIEW✓ PROCESS-FLUID LUBRICATION OF TURBOMACHINERY BEARINGS

by

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ABSTRACT

Process-fluid lubrication is being increasingly specified for closed cycle, high speed and high temperature (as well as cryogenic) turbomachinery applications. This is being done to preclude oil contamination of critical system components, to permit bearing operation at the system temperatures, to minimize sealing requirements and for other similar reasons. The use of process-fluid lubrication, however, requires some fundamental changes in mechanical design to cope with the limitations of many cycle fluids when used as lubricants. These design considerations include the effects of turbulent flows in the bearing gap, bearing stability, reduced bearing damping and ultimate load capacity, oxide deposition, selection of compatible bearing materials and surface coatings, potential pivot fretting and others. In this paper, the advantages, problem areas, and design approaches that have been used in current process-fluid lubricated machines are discussed.

(auth)

INTRODUCTION

The theme of this technical meeting is process-fluid lubrication and is attendant phenomenon in high speed applications, fluid-film turbulence. Process-fluid lubrication is not a novel concept - in fact, it predates conventional oil lubrication. The marked difference today, however, lies in its application in the sophisticated turbomachinery that is being developed by the nuclear and aerospace industries, where the bearing and seal requirements are very exacting and the margin for error is small. The development of bearings and seals and the selection of rotor configuration by means of trial and error can be extremely expensive in these advanced applications. It may be minimized, even altogether eliminated in most cases, by rational design based on a thorough understanding of the technology of lubrication with low viscosity fluids.

The papers presented in this technical meeting have ranged over much of the recent work as well as the current and future needs of this technology. Thus, one of the presentations (Section 8.1) discussed the bearing and seal requirements for projected, large, nuclear, liquid sodium facilities. Other presentations (Sections 6.2, 7.1, and 8.3) reviewed recent and current turbomachinery developments involving water or liquid metal lubricated bearings and seals operating in the turbulent flow regime. In still other presentations (Sections 4 through 6), some of the most recent theoretical approaches, empiricisms and experimental analyses of turbulent flow lubrication with low viscosity fluids are described. The important point is that all this effort, comprising research, practical hardware development and definition of specification for future applications, is proceeding simultaneously. This is a healthy condition for a technology. It is the hope behind technical meetings such as this one to provide forums where the hardware development engineers and the research investigators are kept cognizant of one another's progress and changing requirements.

We at M.T.I. have also tried to achieve this balance of theoretical investigation and practical application. Under a number of programs conducted for the Atomic Energy Commission, for the National Aeronautics and Space Administration and for the Agencies of the Department of Defence, we have sought to:

- (a) develop a rational theory of turbulent lubrication, relying only on accepted precepts of turbulent flows generally and without recourse to new constants or "correction factors" to provide agreement with the relatively small amount of turbulent bearing test data available in the open literature.
- (b) prepare numerical procedures and computer programs for calculating the steady-state and dynamic characteristics of various bearing geometries, from the turbulent lubrication theory.
- (c) test the validity of the theory by means of bearing and rotor-bearing dynamics tests with different bearing geometries and rotor arrangements and,
- (d) apply the verified theoretical analysis and the computation procedures in the design of bearings and seals for hardware developments.

The theoretical analysis and its experimental verification have been published in contract reports and in the open literature (References 1 through 5). The results obtained to date and the questions still remaining are noted in Section 4.1, together with our current conclusions regarding the regions of bearing operation where vortex flows, developed turbulence and fluid inertia effects either singly or in combination, will govern performance. The information gathered in this research is being applied to practical bearing designs, such as the ones for HTGR circulator, which are described in Section 8.3.

One aspect of turbulent flow lubrication that appeared to us to be particularly important is the sharp rise in power loss with speed, that occurs immediately after the point of transition from laminar flow. We believe that the attention paid to this problem will increase as experience is gained with lightly loaded, high surface-speed bearings of turbomachines lubricated with water, liquid metals and other low viscosity fluids. We also believe that this problem as well as other factors such as compactness, weight-saving and general design simplifications will lead to a greater acceptance of flexible rotors with small shaft sizes and greatly reduced bearing and seal losses. In high speed bearings design, particularly when flexible rotors are supported, the dynamic characteristics of the bearings and of the coupled rotor-bearings system are of primary importance. Accordingly, we have laid great stress in our work to date on accurate determination of the stiffness and damping characteristics of bearing films. In Section 7.2, the results of an experimental program in which three different rotor arrangements were operated on turbulent flow bearings up to or through the third system critical speed in each case, are described. The lubricant used in this instance was a silicone fluid whose viscosity is intermediate between that of water and those of sodium and potassium.

Currently, under continuing AEC sponsorship this work on bearing and rotor-bearing dynamic performance is being extended to operation with a liquid metal lubricant (NaK). This test program is currently underway, the facility and the proposed test program are briefly described in an Appendix to this paper.

Since the specifics of the effort at M.T.I. are covered here in the presentation of other M.T.I. engineers, the balance of this presentation will be limited to a general review of:

1. some of the advantages of simplification (and, hence, reliability) and improved turbomachinery efficiency that are made inherently possible when process lubrication is used, and
2. some of the important design considerations in lubeication of high speed machines with fluids that have low viscosity and poor lubricity!

ADVANTAGES AND PROBLEMS OF PROCESS FLUID LUBRICATION

It is appropriate here, to briefly review the factors that we take for granted when using hydrocarbon oils as lubricants as well as the problems that weigh against their use in some closed cycle, high temperature systems. These are listed in Table 1 below.

TABLE 1
CHARACTERISTIC PROPERTIES OF HIGH SPEED, OIL LUBRICATED
SLIDER BEARINGS

ADVANTAGES:

- High ultimate load capacity
- Excellent boundary lubrication
- High damping
- High instability threshold
- Non-corrosive
- Low surface tension
- Good wettability

LIMITATIONS IN CLOSED CYCLE, HIGH-TEMPERATURE SYSTEMS:

- Contamination of system components
- Complex cycle-fluid to oil sealing
- Limited operating temperature range
- High friction
- Oil breakdown in radio-active environment
- Complex auxiliary system
- Potential fire hazard

While load capacity is not generally a problem in high speed machines, the relatively high viscosities of many oils, combined with their excellent boundary lubrication properties offer a very desirable margin of safety in event of malfunction or abnormal conditions. In event of a failure, such as the loss of blades in a turbine and the consequent very high dynamic loads, the high ultimate load capacity and the boundary lubrication properties may, in some cases, reduce the severity of the damage to the rotor.

High damping is clearly useful in limiting vibration amplitudes at system resonances. Advantage can also sometimes be taken of the high damping to suppress instabilities or to control the amplitudes of whirl.

Oils are generally non-corrosive, which increases the choice of bearing materials. Finally, the low surface tension and good wettability improve the start up and we believe, on the basis of some fairly preliminary investigations (References 6 and 7), that these properties also tend to reduce the likelihood of cavitation erosion in most applications. This is because they tend to produce a steady striation type of film rupture in the regions of negative static pressure in the film, instead of small, gas or vapor filled bubbles which can then collapse as they move to positive static pressure regions.

Despite these advantages, and they are important ones, oil lubrication is difficult in many of the advanced, closed cycle systems required in nuclear, aerospace and undersea application. Thus, oil fouling and contamination of some of these closed systems cannot be tolerated and it is extremely difficult to prevent if oil lubricated bearings are used. The problem in the way of development of dynamic, zero-leakage seals are, we believe, far greater than those of process-fluid lubrication.

Lubricating oils are also limited in their temperature capability and cannot be used at temperatures exceeding say 600 to 700 F, or at very low temperatures approaching the cryogenic range. Thus, complex bearing temperature control provisions are needed, as well as use of heat dams or other undesirable design compromises.

Because of high viscosity, friction losses in high speed oil lubricated slider bearings are large and represent a sizeable penalty on efficiency.

Hydrocarbon oils cannot sustain radio-activity for long periods without breaking down, which rules them out in some nuclear applications.

The complexity of the auxiliary lube-oil system cannot be minimized. The complex of lube pumps, coolers, filters, controls and other items is complicated and bulky. Moreover, while its component parts are generally well developed they have nonetheless proved to be a major source of unreliability. Finally, of course, the lube-oil system sometimes constitutes a fire hazard.

Turning now to process-fluid lubricated systems, we can make a similar, general assessment of advantages and potential problem areas. The advantages are listed in Table 2 below. Thus, the elimination of the auxiliary lube system removes the problem of oil fouling or contamination of critical system components, as well as the need for cycle-fluid to lube oil seals.

TABLE 2
ADVANTAGES OF FULL-FILM PROCESS-FLUID LUBRICATION
IN HIGH SPEED, HIGH TEMPERATURE,
CLOSED CYCLE SYSTEMS

1. ELIMINATION OF AUXILIARY LUBE SYSTEM:
No oil fouling or contamination
No oil to cycle-fluid seals
Removes oil fire hazard

2. DESIGN SIMPLICITY AND IMPROVED EFFICIENCY
OF TURBOMACHINERY
Reduced machine length
Reduced volume and weight
Bearings operate at system temperatures
Bearings operate at system's level of radio-activity
Low bearing friction (very low with gases and vapors)

Substantial improvements in design simplification and increased efficiency of the turbomachinery are made possible by:

1. reduction of axial length due to the elimination of the cycle fluid to oil seals

2. reduction in volume and weight due to the elimination of the lube-oil system
3. the bearings have at least the potential of operating at the local rotor temperatures, thus eliminating the need for heat shields and the problem of thermal gradients and their associated distortions
4. the bearings can operate at the level of radio activity of the system
5. the bearing friction losses are low, particularly if the lubricant is a gas or vapor.

Conversely, of course, in applying process fluid lubrication, careful attention has to be paid to a number of factors, many of which will depend on the particular process fluid being used as the lubricant. These factors are noted in Table 3 on page 3.8

Thus, in the case of gas or vapor lubrication, the self-generated bearing load capacity is extremely small (of the order of a few PSI), due to the very low absolute viscosities of gases and vapors. Accordingly, externally pressurized bearings have to be used to support large steady state or dynamic loads. The fluid film damping in gas or vapor lubricated bearings is relatively small, and accurate rotor response analysis should be made to insure that system and component resonant frequencies are outside the operating range. The compressibility of gases and vapors complicates the analysis of gas bearings, introducing an additional parameter (the compressibility number Λ) and making the lubrication differential equation non-linear. An important practical consequence of the non-linearity is that the stiffness and damping of gas-bearing films are highly frequency dependent. In particular, gas bearing damping tends to diminish rapidly when the film is subjected to high frequency excitations. Since excitations at multiples of the running speed are not unusual in some turbomachines (e.g. alternator rotors), they can produce large vibrations if they excite a system resonance, due to the reduced damping of the gas film. It is thus particularly important to analyse accurately the dynamic behavior of high speed, gas bearing supported systems.

In the case of externally pressurized bearings, gas compressibility also causes pneumatic hammer. This is prevented by avoiding recessed volumes in the film, by

TABLE 3

SOME DESIGN CONSIDERATIONS WITH PROCESS FLUID LUBRICATION OF HIGH SPEED TURBOMACHINERY

<u>FLUID</u>	<u>SELF-ACTING BEARINGS</u>	<u>EXTERNALLY PRESSURIZED BEARINGS</u>
Gas	Very low load capacity Low damping Compressibility effects: analytical complexity frequency dependent K and B reduced instability threshold Thermal distortions Start-stop wear No boundary lubrication	Provision of pressurized gas Low damping at high speeds Gas compressibility effects: analytical complexity frequency dependent K and B fractional frequency whirl pneumatic hammer No boundary lubrication
Vapor (wet or with low superheat)	As above plus: Condensation Bearing erosion	As above plus: Condensation Bearing erosion Restrictor erosion
Liquid	Low to moderate ultimate load capacity Moderate to poor damping Turbulence: high friction large attitude angle Moderate to poor boundary lubrication Corrosiveness (liquid metals and some other cycle fluids) Oxide deposition Surface reactions High surface tension Poor wettability Cavitation erosion	Provision of pressurized liquid Turbulence Moderate to poor boundary lubrication Corrosiveness (liquid metals and some cycle fluids) Oxide deposition Surface reactions Restrictor and bearing erosion

using inherently compensated restrictors and, where practical, by small external damping provisions.

The low load capacity of self-acting gas bearings makes them relatively intolerant of thermal distortions, which can adversely affect the film geometry or produce misaligning moments and edge loads. This can be aggravated by the fact that the thermal capacity of the gas flow through self-acting bearings is small, so that the heat generated in the bearing is removed by conduction through the bearing elements, resulting in thermal gradients and corresponding distortions. In past applications, the problem of thermal distortions has been controlled by the use of heat dams and heat shunts, by locating the bearings in isothermal regions of the rotor and by external cooling. Currently, effort is being devoted to development of flexure mounts and conformable surfaces to permit bearing operation in the presence of large thermal gradients.

Gases provide little or no boundary lubrication so that in all cases (in particularly for self-acting gas bearings which do not use hydrostatic jacking and where a large number of starts and stops are anticipated over the life of the machine) the materials must be selected to sustain frequent rubs without damage.

Currently, experience with vapor lubrication is much smaller than that with gas lubrication. However, since lubrication of the bearings located at the turbine end of a rotor with the vapor supplied to the turbine (e.g. steam, metal vapors or others) can potentially result in major simplification and increased reliability of the turbomachine, an increased level of effort in this area of lubrication technology is anticipated. Here, the effects of phase change, bearing erosion and, in the case of externally pressurized bearings, restrictor erosion have to be considered (Ref. 6).

With liquid lubricants, the problems, at least from the standpoint of load capacity, fluid film damping and stability, are somewhat less critical than with gas lubrication, due to the higher absolute viscosities of liquids and the absence of compressibility effects.

Many of these liquids, however, have very low kinematic viscosities, leading to onset of superlaminar flows in high speed bearing films. Table 4 on Page 3.10

TABLE 4 - VISCOSITIES OF SOME PROCESS FLUIDS

<u>FLUID</u>	<u>ABSOLUTE VISCOSITY, μ (lb-sec/in.²)</u>	<u>ABSOLUTE VISCOSITY RATIO*</u>	<u>KINEMATIC VISCOSITY, ν (in.²/sec)</u>	<u>KINEMATIC VISCOSITY RATIO*</u>
SAE 10 oil (120F)	35 X 10 ⁻⁷	1	44 X 10 ⁻³	1
Water (120 F)	0.812 X 10 ⁻⁷	1/43	0.890 X 10 ⁻³	1/49
Gasoline (120 F)	0.725 X 10 ⁻⁷	1/48	1.035 X 10 ⁻³	1/33
NaK 78 (400 F)	0.536 X 10 ⁻⁷	1/65	0.695 X 10 ⁻³	1/63
Mercury (400 F)	1.510 X 10 ⁻⁷	1/23	0.123 X 10 ⁻³	1/358
Air (70 F, 15 psia)	0.026 X 10 ⁻⁷	1/1340	22.7 X 10 ⁻³	1/2
Steam (600 psia, sat.)	0.027 X 10 ⁻⁷	1/1290	1.39 X 10 ⁻³	1/32

*Ratio of fluid viscosity to viscosity of SAE 10 oil.

lists the absolute and kinematic viscosities of a number of fluids and compares them with those of an SAE 10 oil. This illustrates the relative load capacities and the turbulence onset speeds, since the absolute viscosity of the lubricant is a measure of the potential bearing load carrying capability (for self-acting bearings), while the speed at onset of turbulence is directly proportional to the kinematic viscosity of the fluid.

The onset of turbulence in a bearing is important, because:

- a) it is accompanied by a very sharp rise in power loss
- b) it results in increased load capacity and changes the dynamic characteristics (i.e. the stiffness and damping) of the bearing
- c) it produces a moderate increase in attitude angle with, potentially, a greater tendency to instability.

All of these changes are important in high speed bearings so that, for rational bearing and rotor-bearing dynamics design, it is necessary to know whether the bearing film is laminar or turbulent and when fluid inertia effects are sizeable. The appropriate theory should then be used to compute the steady-state and dynamic performance characteristics of the bearing.

Some of the other important considerations in designing process-fluid lubricated bearings, where the fluid is a low viscosity liquid such as water or a liquid metal are also noted in Table 3. The bearing surface materials have to be selected to prevent corrosion and surface reactions. Oxide depositions in the bearing and seal regions are prevented by insuring that these elements do not constitute cold traps. Finally, cavitation damage believed to be more likely to occur with fluids that have high surface tension, and poor wettability. This may be prevented, or at least minimized by pressurization to prevent film rupture and by suitable materials selection.

Process Fluid Lubricated Turbomachinery

A recent study conducted jointly by the Army Engineers Reactor Group and MTI (Ref. 7), illustrated some of the potential advantages of process fluids lubrication. In this study, the oil lubrication systems and the bearing power

losses of three closed loop gas turbines developed in connection with the U.S. Army program on compact, mobile nuclear power conversion systems were studied. They were then compared with a gas lubricated system (CSG-1) that has been proposed and for which a dynamic simulator was built and evaluated on externally pressurized gas bearings. Table 5, (Page 3.13) reproduced from Ref. 7, lists the power losses in each case. It is noted that here even for machines in the 2000 shaft H.P. range, there is a significant reduction in power loss associated with process-fluid lubrication. With machines having a smaller power output, the percentage reduction in power loss will be still greater.

The simplification of the lubrication system and controls is seen in Figures 1 and 2 (also reproduced from Ref. 7), which show respectively the oil lubrication system of the CSN2 unit and the process-fluid lubricated system of the CSG1. Note that in this latter case, the pressurized gas for the bearings is bled off from the compressor discharge. Figure 3 is a photograph of the CSN2 lube oil distribution panel and lube oil skid.

The dynamic simulator of the CSG1 was built and tested at MTI. The rotor weight and design point speed of the simulator were 90 lbs. and 28,000 RPM respectively. The unit was operated at up to 30,000 RPM. Figure 4 is a photograph of the simulator parts. It should be noted, however, that full development and operational experience with gas lubricated, closed loop gas turbines in the 2000 HP range has not been undertaken, so that the potential long term operation of such units and their tolerance to off-design conditions are not yet known.

To date, the practical experience with gas bearing turbomachinery has been principally with motor driven gas bearing compressors used mostly in nuclear and chemonuclear installations. There are a number of such units operating in the United States and in Europe (Ref. 8). The several machines operating in the United States have accumulated over 40,000 hours of highly successful field operation, the longest time on any one machine being 28,000 hours.

^a For fewer turbine driven gas bearing machines have been developed to date. Figure 5 shows the first such unit to have been developed and tested at its design speed (24,000 RPM) and its design point turbine inlet temperature (1300 F). This unit was developed and operated for the U.S. Bureau of Mines and the U.S. Atomic Energy

TABLE 5

COMPARISON OF BEARING SYSTEM PARASITIC LOSSES FOR OIL- AND GAS-LUBRICATED TURBOCOMPRESSORS

Machine designation and rated speed	Oil-lubricated Machines			Gas-lubricated Machines	
	TCS 560B 18,000rpm	TCS 670-2 18,338rpm	CSN-2 22,000rpm	CSG-1 28,000rpm	Bu Mines/AEC 24,000rpm
Rated turbine shaft power - KW	1,275.0	2,235.0	2,190.0	2,235.0	84.8
Bearing lubricant viscous shear losses - KW	28.1	61.0*	55.7*	0.91	0.69
Additional heat transfer losses to the lubricant-KW	8.9*			-	-
Compressor bleed losses for buffer-gas flow-KW	13.4*	3.89	5.0*	-	-
Compressor bleed losses for gas-bearing pressuri- zation - KW	-	-	-	0.89*	0.15*
Compressor bleed losses for gas-bearing cooling-KW	-	-	-	-	0.46*
Power for auxiliary pumps, fans, coolers, etc. - KW	2.86*	2.61	11.9*	-	-
Total bearing system losses as a percentage of rated turbine power	4.17%	3.02%	3.25%	0.08%	1.53%

Notes: 1. Numbers marked with an asterisk are measured values. All other numbers are calculated values.

2. Bearing loss data for the CSG-1 machine is based on the CSG-1 rotor-bearing simulator described in Reference 2.

3. The above losses are for the turbocompressor bearing system only. Losses associated with reduction gearing and generator bearings are not included.

Commission by MTI. This unit, which has a rated shaft turbine power of 84.8 HP, is representative (in terms of gas circulation alone) of 100 to 300 HP circulators. In terms of closed Brayton Cycle power generation, the turbocompressor is representative of the gas generator section for a 30 to 100 KW(e) gas-turbine power plant. Over 125 hours of test operation have been accumulated with this machine at turbine inlet temperatures ranging from 1000 to 1400 F.

Gas bearing supported compressors and expanders are also required for cryogenic systems where very long, maintenance free operating life is required. Recently a two stage regenerative helium compressor was operated at MTI at speeds up to 114,000 RPM.

This particular unit, shown in Figure 6, has a rotor weighing 1.5 lbs. which comprises the two regenerative compressor stages, an internal cooling fan and thrust plate and a 3 phase synchronous, hysteresis type drive motor. The rotor is supported in two 3/4" diameter tilting pad bearings, designed for stable operation at up to 150,000 RPM. The unit is cited here, because in the course of developing it, some preliminary experimental investigations were made of the influence of electro-magnetic forces on such small, high speed self-acting gas bearings. When a clean, sinusoidal AC voltage was applied, the unit operated at up to 114,000 RPM without difficulty. When, however, an approximately square wave output was used, the system harmonics resulted in large, unbalanced magnetic forces on the bearings which exceeded their capabilities. Figure 7 illustrates orbits obtained in the two cases. These show the need for careful electrical design, to prevent large, unbalanced magnetic forces in designing motors and alternators which are to be supported in self-acting gas bearings.

Water lubricated bearings have been used extensively and successfully in many applications. Water lubricated pumps are, for example, commonplace in marine and other service. Water lubricated canned-rotor pumps, using graphitar bearings are standard in nuclear submarine service.

Water lubricated bearings are currently being developed for the HTGR helium circulators, because of their advantages in this application, as discussed in Section 8.3 of this report. Figure 8 is a drawing of the test rig for this machine.

Liquid metals have also been used as bearing lubricants. Thus, mercury is the lubricant used in the SNAP 2 rotating unit discussed in Section 7.1 of this report and in other, earlier units (Ref. 9).

Potassium lubricated bearings are contemplated for the potassium-Rankine cycle turbomachines being developed for space power. Much useful technological information on bearing materials for alkali metal lubrication, as well as some testing of alkali metal lubricated bearings has been conducted in connection with SNAP 50/SPUR and other programs (Refs. 10, 11 and 12).

An example of successful application of process-fluid lubrication using the alkali metal process-fluid is in the Sodium-Potassium pump motor assemblies for the SNAP 8 system (Ref. 13). Here, process-fluid lubrication permitted achievement of hermetically sealed units that incorporate on a single shaft the pump impeller, a 400 cycle, three phase induction motor and an internal lubricant coolant circulating pump, supported on NaK lubricated journal and thrust bearings. Both the journal and thrust bearings in this instance were of the tilting pad type for reasons of stability and self-alignment. Several of these units have been operated, accumulating over 4000 hours of test time. One unit has operated for over 3000 hours. Reference 13 notes that the results of the test program of the process fluid lubricated SNAP 8 NaK pump-motor assembly has been most satisfactory

In conclusion then, process-fluid lubrication offers important advantages in simplification, reliability and efficiency which have prompted its acceptance for turbomachinery application in the nuclear and aerospace industries. The use of low viscosity fluids with poor lubricity, however, poses a number of problems some of which were noted above. In order to cope with these problem areas and to minimize the need for trial and error, the bearing and rotor-bearing dynamics design should take account of the results of the technological studies of lubrication with low viscosity fluids. In the case of gas lubrication compressible flow analysis should be used. In the case of liquid lubricated bearings, the Reynolds Number for the fluid film and the clearance ratio of the bearing will establish whether laminar or turbulent flow analysis should be used and whether fluid inertia terms are important. Materials selection is also extremely critical in view of the poor lubricity, the corrosive nature of many of the fluids and the

susceptibility to cavitation damage in some instances. New, electrical aerodynamic and mechanical design approaches are also necessary to reduce bearing loads and otherwise compensate for the lower performance limits of bearings lubricated with low viscosity fluids. Table 6 (Page 3.17), for example, shows some of these design considerations for machinery supported in self-acting gas bearings.

To date, a number of high speed process-fluid lubricated machines using gases, water, liquid metals and other fluids as lubricants have been successfully developed and used. Some of these were also noted above. Vapor lubricated bearings, where phase changes may occur in the bearing or its restrictor system, have not yet been applied, partly because of inadequate technology in this area. However, since vapor lubrication offers important advantages in some instances, such as for rotors supporting vapor turbines, more technological activity in this area is required.

TABLE 6SOME DESIGN CONSIDERATIONS WITH SELF-ACTING GAS BEARING MACHINES

STABILITY	<ul style="list-style-type: none"> - Highly stable bearings, e.g. tilting pad and herringbone grooved bearings <p>Preloading</p> <p>Clearance selection</p>
THERMAL DISTORTIONS	<ul style="list-style-type: none"> - Steady-state and transient thermal analysis: <ul style="list-style-type: none"> Minimize thermal gradients Location of heat shields Coolant flow required <p>Flexible mount of bearings</p> <p>Make bearings support structure independent of housing</p>
LOW LOAD CAPACITY	<ul style="list-style-type: none"> - Aerodynamic balancing: impulse turbines, pressure balancing holes, labyrinths, scalloping compressor end plates, etc. <p>Electrical force balancing: 4 pole machines, rotor to stator concentricity, phase balance, etc.</p> <p>Hollow shafts</p> <p>(External pressurization should be used if above factors are insufficient to provide required load capacity)</p>
LOW DAMPING	<ul style="list-style-type: none"> - In-place balancing <p>Rotor response analysis:</p> <p>Rotor operation away from rigid body critical speeds and below flexural critical speed</p> <p>Avoid locating bearings at nodes</p> <p>Avoid mass shifts:</p> <p>Rigid rotor</p> <p>Single piece construction</p> <p>High shrink fits</p> <p>Pre-compression of motor laminations</p>

APPENDIX

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Under AEC Contract AT(30-1)-3363, MTI is conducting an investigation of lubrication with liquid metals and other low viscosity fluids.

Earlier tasks included:

- a) development of turbulent lubrication theory including the coupled effects of shear and pressure gradient flows (Ref. 2)
- b) exploratory, primarily visual study of film rupture in bearings lubricated with a low viscosity liquid (Ref. 14)
- c) analytical and experimental investigation of flexible rotor operation on tilting pad bearings operating in turbulent flow regime (Ref. 15 and paper 7.2 of this report).

As part of the current task, tables of the steady-state and dynamic characteristics of a number of tilting pad bearings (four-pad bearings with $L/D = 1/4$, $1/2$ and 1 and six-pad bearing with $L/D = 1/4$) have been computed for Reynolds numbers up to 60,000.

In the parallel, experimental work under the task currently in progress, the steady-state and dynamic load capacity, as well as the stiffness and damping coefficients are being accurately determined for a four pad, $L/D = 1$ bearing, for comparison with the theoretical data. The lubricant used is NaK 78 and the measurements made are of the journal location and orbit size and geometry, for a range of values of steady-state loads, dynamic loads and rotational speed. The characteristics of the test apparatus are:

Rotor weight	-	57 lbs.
Speed range	-	3000 to 28,000 RPM, infinitely variable (design speed = 24,000 RPM).
Bearing size	-	2" dia x 2" long
Bearing type	-	four pad, tilting pad with 0.55 pivot position and 0.5 geometrical preload factor.

Machined clearance ratio - 0.004 $\frac{''}{''}$
 Operating clearance ratio - 0.002 $\frac{''}{''}$
 Lubricant - NaK 78 at 100 F
 Range of Reynolds Numbers - 1250 to 11,700 (10,000 at the 24,000 RPM design speed)
 Steady-state load application capability - 0 to 300 lbs. per bearing (i.e.
 0 to 75 psi on projected area).

The steady-state load is applied through a hydrostatic, nitrogen fed bearing located at the center of the rotor. Dynamic loads are applied by introducing known unbalance weights in two planes in the rotor.

The test program calls for determination of the bearing steady-state and dynamic load capacities, as well as of its stiffness and dynamic coefficients, from measurements of journal center displacement and orbits over the above noted load and speed ranges. It is the intent of the test program to obtain this data on the steady-state and dynamic characteristics of the bearings and rotor-bearings system with very high accuracy, in order to establish the degree of confidence with which the existing turbulent lubrication theory may be used to predict alkali metal lubricated bearing performance. Specifically, any deviations between calculated and measured performance due, for example, to fluid inertia effects at pad entrances and exits (velocity lead losses) are to be noted, as functions of Reynolds number and eccentricity ratio. Tilting pad bearings were selected because their high stability makes them prime candidates for high speed, process-fluid lubricated turbomachinery.

The test rig is mounted in a dry box under nitrogen cover gas. The NaK is furnished from a 50 GPM, 40 psi NaK circulation assembly and gas purifier. The assembled dry box and NaK circulation assembly are shown in Figure 9. Figure 10 is a photograph of the test rotor and Figure 11 shows one of the pillow blocks with the bearing pads and seal rings. Figure 12 shows the assembled test rig, but with the upper half of the cylindrical housing and the loader bearing omitted. The hydrostatic loader bearing is shown in Figure 13. The rotor is driven through a splined quill coupling by a variable speed (0 to 36000RPM) motor and MG set. There are six measurement planes, one at each end of the two liquid metal lubricated bearings immediately outside the seal rings, and one at each end of the nitrogen loader bearing. Two capacitance probes installed at 90 degree to one another are located in each measurement plane to measure the journal displacements and orbits.

The tests were started in October 1966 and the unit was operated first in an "easy to use" fluid for check out purposes and later in NaK. To date, three test sequences have been conducted in NaK. The first sequence was at speeds up to 12,000 RPM and was made for the purpose of check out of the system, including the NaK circulation loop, feed and drain systems, instrument performance and others. In the second two sequences, which were conducted at speeds up to 26,400 RPM, test data was obtained first with a balanced shaft and then with an unbalance of 0.29 oz.in., corresponding (at the highest speed of the test) to a dynamic load of 180 lbs. per bearing (i.e. 45 psi on projected area). In both sets of tests the static load was the 57 lb. weight of the rotor, which is equally supported on the two test bearings. Figure 14 shows photographs of some of the orbits at different speeds. Currently, preparations are in progress for conducting other test runs with larger values of steady-state and dynamic loads.

At this early stage of the experimental program, it is premature to draw any final conclusions regarding the performance of the NaK lubricated tilting pad bearings or the degree of confidence with which their characteristics may be predicted from theory. The early tests do, however, indicate generally good correlation between the theoretical and test data. There has been no indication of instability at any speed, although there is some very preliminary evidence that at speeds higher than 15,000 RPM (Reynolds number = 6200), fluid inertia effects may start to have a small, but measurable influence on the eccentricity ratio of the particular bearings used in the tests.

REFERENCES

1. E.B. Arwas, "Topical Report on Turbulent Lubrication," Topical Report under A.I. Subcontract N2-S9-1599 of AEC Contract AT(11-1) GEN 8, MTI Report 64TR67, November 1964.
2. H.G. Elrod, Jr., C.W. Ng and C.H.T. Pan, "A Theory for Turbulent Films and Its Application to Bearings," Topical Report under AEC Contract AT(30-1)-3363, AEC Report No. NYO-3363-2, MTI Report 65TR9, March 1965 (also published under the same title as ASME Paper 66Lub12, June 1966).
3. J.W. Lund et al: "Rotor Bearing Dynamics Technology," Final Reports under USAF Contract No. AF 33(615)-1895, U.S. Air Force Reports AFAPL-TR-65-45 Parts III and V, MTI Report 64TR14 and 65TR15, May 1965.
4. F.K. Orcutt and E.B. Arwas, "Analysis of Turbulent Lubrication, Volume 1 - The State and Dynamic Properties of Journal Bearings in Laminar and Turbulent Regimes," 1st Volume of Final Report under NASA Contract NAS-w-771, MTI Report 64TR19, May 1964. (Condensed version published as ASME paper 66-LUBS-4, (The Steady-State and Dynamic Characteristics of a Full Circular and a Partial Arc Bearing in the Laminar and Turbulent Flow Regimes, June 1966).
5. (a) F.K. Orcutt, "The Steady-State and Dynamic Properties of the Tilting Pad Bearing in the Laminar and Turbulent Flow Regimes," Topical Report under NASA Contract NAS-w-1021, MTI Report 65TR32, June 1965 (condensed version published under the same title, as ASME Paper 66-LUB-19, June 1966).

(b) F.K. Orcutt and C.W. Ng, "Steady-State and Dynamic Properties of the Floating Ring Bearing in the Laminar and Turbulent Flow Regimes," Topical Report under NASA Contract NAS-w-1021, MTI Report 65TR33, June 1965.
6. F.K. Orcutt, "Experimental Investigation of Condensing Vapor Lubricated Thrust Bearing," TRANS. ASLE, Vol. 7, 1964.
7. P. W. Curwen, G.B. Manning, R.A. Harmon, "A Comparison of Oil and Gas Lubrication Systems for Closed-Loop Gas-Turbine Machinery," ASME Paper accepted for presentation at 12th Annual Gas Turbine Conference and Products Show, Houston, Texas, March 1967.
8. B. Sternlicht and E.B. Arwas, "Modern Gas-Bearings Turbomachinery - Part II," Mechanical Engineering, Vol. 88, No. 2, February 1966.
9. G.Y. Ono and O. Decker, "Experience with Liquid-Mercury Lubricated Bearings for Rankine Cycle Space Power Systems," Proceedings of the USAF Aerospace Fluids and Lubricants Conference, San Antonio, Texas, April 16-19, 1963.
10. M.J. Wallace, "Summary Report of Potential Liquid Metal Bearing Materials for SNAP 50/SPUR Pumps," PWAC-468, November 1965 (prepared under Contract AT(30-1)-2789. TID-4500, CAT.UC-25).

11. R.G. Frank (Editor), "Materials for Potassium Lubricated Journal Bearings," NASA CR 72027, General Electric Co. Progress Report under NASA Contract NAS 3-2534.
12. AiResearch Manufacturing Co. of Arizona, Report No. APS-5152-R4, Oct. 1965 (prepared under SNAP 50/SPUR Contract AF 33(615)-2289).
13. H.O. Slone, "SNAP-8 Development Status," Paper No. VI of NASA SP-131, Space Power Systems Advanced Technology Conference.
14. F.K. Orcutt and C.H.T. Pan, "An Experimental Study of Film Rupture in Journal Bearings with Low Kinematic Viscosity Lubricants," Topical Report under AEC Contract AT(30-1)-3363, AEC Report No. NYO-3363-4, MTI Report No. 65TR13, March 1965.
15. F.K. Orcutt and E.B. Arwas, "An Investigation of Rotor-Bearing Dynamics with Flexible Rotors and Turbulent-Flow Journal Bearings," Part 1 - Analysis, Design and Fabrication of the Test Apparatus. Topical Report under AEC Contract AT(30-1)-3363, AEC Report No. NYO-3363-3, MTI Report No. 65TR12, March 1965.

Fig. 1 Schematic Diagram of CSN = 2 Lube-Oil and Seal Gas System

Fig. 2 Schematic Diagram of Lubrication System for the Proposed CSG-1 Gas-Bearing Turbocompressor

Fig. 3 CSN-2 Lube-Oil Distribution Panel and Lube-Oil Skid

Fig. 4 Rotor and Gas Bearing Components for CSG-1 Full Scale Simulator

MTI-1625

Fig. 5 Bu-Mines/AEC, 24,000 RPM Gas-Bearing Turbocompressor

MTI-1623

Fig. 6 Rotor, Housing and Bearing Parts for 150,000 RPM, Two-Stage Regenerative Compressor

COMPRESSOR END BEARING
 SPEED: 18,000 RPM
 DRIVE: S.C.R. POWER PACK
 SQUARE WAVE OUTPUT
 1 CM = 0.00025 IN.

COMPRESSOR END BEARING
 SPEED: 110,000 RPM
 DRIVE: M.G. SET
 SINE WAVE OUTPUT
 1 CM = 0.00025 IN.

Fig. 7 Measured Orbits at Compressor End Bearing of Two-Stage Regenerative Compressor

Fig. 8 Test Rotor for HTGR Water Lubricated Bearings

Fig. 9 Dry Box and NaK Circulation Loop for Liquid Metal Bearings
Test Program (Gas Purifier Not Shown)

Fig. 10 Test Rotor

Fig. 11 Pillow Block, Bearing Pads and Seal Rings

Fig. 12 Assembled Test Rig (Upper Half of Housing and N_2 Loader Bearing Not Shown)

Fig. 13 N_2 Loader Bearing

$N = 5000$ RPM
Rotor Balanced
Scale: 1cm = 0.8 mils

$N = 13,000$
Rotor Balanced
Scale: 1cm = 0.8 mils

$N = 21,000$ RPM
Rotor Balanced
Scale: 1cm = 0.8 mils

$N = 5000$ RPM
Rotor Unbalanced = 0.29 oz.in.
Scale: 1cm = 0.8 mils

$N = 21,000$ RPM
Rotor Unbalanced = 0.29 oz.in.
Scale: 1cm = 0.8 mils

$N = 26,000$ RPM
Rotor Unbalance = 0.29 oz.in.
Scale: 1cm = 0.8 mils

Fig. 14 Measured Orbits

SECTION 4

TECHNOLOGY I - FUNDAMENTALS

SUPER-LAMINAR FLOW IN BEARINGS AND SEALS

by

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ABSTRACT

Possible fluid-dynamic processes in thin films of low kinematic viscosity fluids are discussed and parameters governing the flow regimes are identified. Recent analytical and experimental studies on related subjects are reviewed. Analysis based on the "wall law" of turbulent shear flow has accounted for most experimental data on super laminar thin film flows. The phenomenon of secondary vortices is important only for relatively thick films (according to lubrication practice). Mean inertia effects are the major unresolved items in the analysis of bearings and seals lubricated with low kinematic viscosity fluids.

I. INTRODUCTION

Conventional fluid film bearings operate in a manner that the viscous shear stress predominates in the fluid-dynamical process, which obeys the classical lubrication theory originated by Reynolds (ref. 1). Reynolds' lubrication theory is predicated on the propositions that a state of laminar flow prevails and that the fluid film thickness is considerably smaller than the characteristic dimension of the bearing surface. In recent years, trends toward process fluid lubricated bearings bring to light the need to consider lubrication films of fluids, which, because of a low kinematic viscosity, cannot be adequately described by Reynolds' lubrication theory. In table I, several typical process fluids and their kinematic viscosities are listed for comparison with SAE No. 10 oil. Depending on the hydrodynamic and the geometrical parameters in effect, one or more super-laminar phenomena may dominate the operation of a process fluid lubricated bearing. The purpose of this work is to discuss the relevance of these phenomena to lubrication problems, to review the progresses thus far achieved, and to project future needs of research. The fluid-dynamic point of view will be adopted, therefore this work is as much directed to fluid seals as it is to fluid film bearings.

II. CLASSIFICATION OF SUPER-LAMINAR FLOWS

A. Review of the Classical Lubrication Theory

An appreciation of the significance of a low kinematic viscosity can be gained by reviewing the mathematical foundation of Reynolds' lubrication theory. For the present purpose, validity of the equations of Navier-Stokes will be presumed. Thus, for mass continuity and momentum balance of an incompressible fluid, one finds

$$\frac{\partial U_j}{\partial x_j} = 0 \quad (1)$$

$$\rho \left(\frac{\partial U_i}{\partial t} + U_j \frac{\partial}{\partial x_j} U_i \right) = \frac{\partial}{\partial x_j} (\sigma_{ij}) \quad (2)$$

Here, Cartesian tensor notation is used for brevity, curvature effects as may be related to geometry and kinematics are understood to be implicitly included in above equations. The applicable constitutive law is

$$\sigma_{ij} = -p \delta_{ij} + \mu \left(\frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j} \right) \quad (3)$$

It is sufficient to require that μ be treated as dependent only on T and p , and that $T(x_i, t)$ be obtainable in some consistent manner. The lubricated surfaces are separated by a thin film of fluid, the representative film thickness is designated as C . Let L be a typical linear dimension of the lubricated surfaces, V be a representative velocity magnitude in the fluid film, and ΔT be a representative time period. Make x_2 be the spatial coordinate in a direction normal to the lubricated surfaces, then provided

$$\frac{C}{L} \ll 1 \quad (4)$$

then

$$U_i = O\left\{V\right\} \quad (5)$$

$$\frac{\partial}{\partial x_i} = O\left\{\frac{1}{L}\right\} \quad (6)$$

for $i = 1, 3$

and

$$U_2 = O\left\{\frac{C}{L} V\right\} \quad (7)$$

$$\frac{\partial}{\partial x_2} = 0 \left\{ \frac{1}{C} \right\} \quad (8)$$

Neglecting $O \left(\frac{C}{L} \right)^2$, the righthand side of eq. (2) is reduced to

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_2} \left[\mu \left(\frac{\partial U_i}{\partial x_2} \right) \right] \quad (9)$$

for $i = 1, 3$

In Reynolds' lubrication theory, it is assumed

$$\frac{\rho V C^2}{\mu L} \ll 1 ; \frac{\rho C^2}{\mu \Delta T} \ll 1 \quad (10)$$

so that the left hand side of eq. (2), which accounts for the inertia forces in the fluid film, can be neglected for $i = 1, 3$; and p can be regarded as independent of x_2 . In typical oil bearings

$$\frac{\rho V C}{\mu} \approx 100 \quad (11)$$

$$\frac{C}{L} \approx 10^{-3} \quad (12)$$

and

$$\Delta T \approx \frac{L}{V} \quad (13)$$

Thus the inequalities of eq. (10) are readily justified for an oil film.

It is seen from Table I, that the kinematic viscosity of a process fluid is at least one order of magnitude smaller than that of oil. Thus, eq. (10) would not necessarily be satisfied, and one can be readily convinced that process fluid lubrication films would be distinguished by the relative dominance of the inertia effects.

B. Flow Instabilities

The significance of the inertia effect is more than the simple prevalence of transient and/or convective forces. Two types of breakdown can happen to a laminar flow when the kinematic viscosity is sufficiently small.

One of these is caused by the centrifugal force of a curved layer of flow. Its classical form exists in the annulus between an inner rotating cylinder and an outer stationary cylinder [ref. 2]. When the rotational speed exceeds a critical value, depending on the kinematic viscosity and the radius ratio, a system of toroidal secondary vortices will appear as illustrated in Fig. 1. When the radii of the inner and outer cylinders are nearly equal, the critical speed for secondary vortices is

$$\frac{\rho V \Delta R}{\mu} \sqrt{\frac{\Delta R}{R}} = 41.1 \quad (14)$$

and the axial spacing of the vortices is approximately same as the annular gap ΔR . At their first occurrence, these vortices are time independent. Until this critical speed is reached the torque required to rotate the inner cylinder is directly proportional to the rotational speed. Above the critical speed, the torque will exceed this condition of being proportional to the speed as shown in Fig. 2. The pertinence of the secondary vortices in the operation of process fluid journal bearings and shaft seals is self-evident; here the annular gap ΔR is the nominal radial clearance C . Secondary vortices do not occur if the outer cylinder instead of the inner one rotates.

The second type of flow instability is due to the tendency of a shear flow to dissipate its kinetic energy in random fluctuations known as turbulence. First discovered by Reynolds in high speed pipe flows [ref. 3], turbulence is now known to exist in all types of shear flows. Without turbulence, the laminar shear stress is linearly proportional to the mean flow rate. With turbulence, the mean shear stress is more or less proportional to the mean kinetic energy or the square of the mean flow rate. The criterion for turbulence to occur is roughly

$$\frac{\rho V C}{\mu} > 10^3 \quad (15)$$

If the above condition already exists, and since the mean state of flow instead of the random flow fluctuations is of interest in lubrication problems, eqs. (1) and (2) should be revised to read

$$\frac{\partial \bar{u}_i}{\partial x_j} = 0 \quad (16)$$

$$\rho \frac{\overline{\partial U_i}}{\partial t} + \overline{U_j} \frac{\partial \overline{U_i}}{\partial x_j} = \frac{\overline{\partial \sigma_{ij}}}{\partial x_j} - \rho \frac{\partial}{\partial x_j} (\overline{U_i' U_j'}) \quad (17)$$

where barred quantities are temporal averages and primed quantities are the fluctuating parts. The last term in eq. (17) represents turbulent momentum transport which causes the mean stress to be higher than the laminar viscous stress. The criteria for neglecting the left hand side of eq. (17) are

$$\frac{1}{C_f} \frac{C}{L} \ll 1 ; \frac{1}{C_f} \frac{C}{V\Delta T} \ll 1 \quad (18)$$

where $C_f = \frac{\text{wall shear}}{\frac{1}{2}\rho V^2}$ is the coefficient of friction. C_f itself is a function of the Reynolds number $\frac{\rho VC}{\mu}$, and in the laminar condition eq. (18) reduces to eq. (10).

C. Flow Regimes Map

In above discussions, super laminar phenomena in lubrication application have been identified as

1. Prevalence of inertia effects when

$$\frac{1}{C_f} \frac{C}{L} \gtrsim 1 ; \frac{1}{C_f} \frac{C}{V\Delta T} \gtrsim 1 ; \quad (19)$$

2. occurrence of secondary vortice for a rotating shaft when

$$\frac{\rho VC}{\mu} \sqrt{\frac{C}{R}} > 41.1 ; \quad (20)$$

and

3. presence of turbulence when

$$\frac{\rho VC}{\mu} > 10^3$$

Since C_f itself is a function of the Reynolds number, above criteria can be expressed in terms of $\frac{\rho VC}{\mu}$, $\frac{C}{L}$, $\frac{C}{V\Delta T}$, and $\frac{C}{R}$. A composite flow regimes map with

$\frac{\rho VC}{\mu}$ as the ordinate, and either $\frac{C}{L}$, or $\frac{C}{V\Delta T}$, or $\frac{C}{R}$ as the abscissa is shown in Fig. 3. The line marking the importance of inertia effects is based on

$$\frac{1}{C_f} \frac{C}{L} = \frac{1}{C_f} \frac{C}{V\Delta T} = 0.25$$

for the Couette flow. Specifically, for $\frac{\rho VC}{\mu} < 2000$,

$$C_f = \frac{8\mu}{\rho VC}$$

and for $\frac{\rho VC}{\mu} > 2000$, C_f is calculated according to [ref. 4]. V is the velocity of the moving surface.

Worthy of special notice is the fact that typical process fluid bearings are designed with $\frac{C}{R}$ between 10^{-3} and 3×10^{-3} . In this range, the regime of flow with secondary vortices but without turbulence occupies a very small portion of the map. Also worthy of notice is the fact that the line marking the importance of inertia effects is almost vertical for $\frac{\rho VC}{\mu} > 2000$. When inertia effects are to be considered, they include transient, convective, centrifugal, and Coriolis accelerations.

III. FLOW WITH SECONDARY VORTICES

A. Vortex Transition in Flow with Pressure Gradients

From the flow regime chart shown in Fig. 3, one would expect that the first transition from laminar flow to occur in bearings with $C/R > 10^{-3}$ would be to a form of vortex flow rather than to turbulence. It is, therefore, of considerable practical importance to know as precisely as possible at what speeds transition to vortex flow will occur. The vortex transition boundary shown in Fig. 3 is for the classical case of concentric cylinders with no pressure gradients in the flow. However, in loaded journal bearings, both axial and circumferential pressure gradients exist. Also, in loaded journal bearings, the radial clearance varies circumferentially. Obviously, then, the problem of the stability of flows in journal bearings to development of Taylor vortices is very much more complex than for the classical case of concentric cylinders.

The effect of axial pressure gradient on the stability of flow to onset of vortices has been shown both theoretically and experimentally to be always a stabilizing effect (Refs. 5, 6, 7 and 8). The effect is illustrated in Fig. 4 for rotating cylinders at different eccentricity ratios. The test cylinders used in these experiments had a clearance ratio of $C/R_1 = .099$ and a length to diameter ratio of $L/D = 6.12$. The axial flow Reynolds number used as the abscissa in Fig. 4 is defined by

$$(N_{Re})_{axial} = \frac{W_m C}{\nu}$$

where W_m is a mean axial flow velocity determined by dividing the axial volume flow rate by the annular cross-section area for the flow.

One can note in Fig. 4 that for $(N_{Re})_{axial} = 0$, flow stability increases with increasing eccentricity ratio. This is due to the net effect of circumferential pressure flows induced by rotation of the eccentric cylinder. This effect is discussed below.

B. Vortex Transition in Eccentric Cylinders

DiPrima (Ref. 9) showed theoretically that for the case of concentric rotating cylinders, a uniform pressure flow in the direction of rotation tends to make the flow less stable to onset of Taylor vortices while a negative pressure flow tends

to make the flow more stable. When a cylinder rotates within a stationary, non-concentric outer cylinder, a negative pressure flow is induced in the region of maximum clearance while a positive pressure flow is induced in the region of minimum clearance. To determine the net effect of these pressure flows on the stability of the flow, DiPrima applied his abovementioned analysis to the local flow at every circumferential point around non-concentric rotating cylinders (Ref. 10). The well known Sommerfeld (Ref. 11) solution for journal bearings was used to calculate the magnitude of the local pressure flows.

The results of DiPrima's analysis are shown in Fig. 5. In the region of maximum clearance ($\theta = 0^\circ$), the increase in clearance with eccentricity tends to make the flow less stable to onset of vortices whereas the negative pressure flow that is developed there tends to stabilize the flow. The resultant effect as ϵ increases from zero is that the flow initially becomes less stable to onset of vortices. However, for $\epsilon > 0.67$, the stabilizing effect of the pressure flow starts to dominate and the flow becomes more stable than it was at $\epsilon = 0$.

In the region of minimum clearance ($\theta = 180^\circ$), the positive pressure flow that is developed acts to destabilize the flow. However, the stabilizing effect of decreasing clearance always dominates here so that flow becomes increasingly more stable to onset of vortices with increase in eccentricity.

DiPrima's analysis of the stability of flow around non-concentric cylinders was developed for the limiting case of $C/R_1 \rightarrow 0$. If, in determining the stability of flow around non-concentric cylinders, we insist that the velocity profiles be stable at every local circumferential position, then the stability curve for $\theta = 0^\circ$ in Fig. 5 will be our minimum stability curve for the entire flow. However, for cylinders with finite radial clearance, we know experimentally (Ref. 12) that, when vortices develop in the flow, they wrap completely around the cylinders extending from regions where the flow is theoretically unstable through regions where it is theoretically stable. Therefore, for vortices to develop, the centrifugal forces acting to produce vortex circulation in any region where flow is unstable must be strong enough to drive not only the local vortex circulation but also the vortex circulation in the stable regions around the cylinders.

Therefore, for cylinders with finite C/R_1 , we expect that at the critical speed at which vortices will first appear in the flow would be greater than that predicted by the line for $\theta = 0^\circ$ in Fig. 5.

In Fig. 6 are shown various experimental data for critical speed for onset of vortices in flow between non-concentric cylinders. For these data, the point of onset of vortices was determined by torque measurements with silicone fluids as the test fluids (Refs. 12 and 13). As can be seen, the measured transition speeds for onset of vortices all are higher than predicted by the minimum transition speed curve of DiPrima. Also, one can note that transition speeds become greater as clearance ratio increases, particularly at large values of ϵ .

The smallest clearance ratio for which experimental measurements of Taylor vortex transition speed have been made is $C/R_1 = 0.0104$. It is not established whether DiPrima's minimum stability curve is valid in the limit as $C/R_1 \rightarrow 0$. However, since DiPrima's curve represents the strictest stability condition that can be applied, one can state with reasonable assurance that Taylor vortices would not develop in journal bearings at speeds below those given by this curve. On the other hand, one would expect vortices to develop in bearing flows at speeds less than those predicted by the experimental curve for $C/R_1 = 0.0104$.

C. Vortex Theory of Lubrication

For the typical clearance ratios found in bearings, the range of operating conditions in which vortex flow will occur without turbulence is fairly narrow (see Fig. 3). As we can see later, once fully developed turbulence sets in, the transport mechanism in the flow will soon be dominated by turbulent fluctuations and the effect of the secondary vortex flow which is present soon becomes negligible. Nonetheless, it is still of practical interest to develop a "vortex theory of lubrication" which can essentially "bridge the gap" between the operating range in which laminar theory applies and the range in which turbulent theory applies. Such a vortex theory of lubrication has been developed (Ref. 13) based on an analysis by DiPrima (Ref. 14). In his analysis, DiPrima developed the following theoretical relationships for flow between concentric cylinders in the vortex regime at speeds just above Taylor critical speed.

$$\frac{V_m}{V_1} = 1/2 + Q/6 + \left(1 - \frac{T_c(Q)}{T}\right) F(Q) G(Q) \quad (21)$$

$$-\frac{\tau'}{\mu V_1/h} = 1 - Q + \left(1 - \frac{T_c(Q)}{T}\right) H(Q) \quad (22)$$

For $T > T_c$

where

- V_m = mean circumferential flow velocity due to sum of Couette flow and pressure flow
 τ' = local shear stress at surface of inner cylinder
 $Q = 3 V_p/V_c = 3 \left[-\frac{h^2}{12\mu R_1} \frac{\partial P}{\partial \theta} \frac{2}{V_1} \right]$
 V_p = mean circumferential velocity due to pressure gradient
 V_c = mean circumferential velocity due to rotation = $V_1/2$
 V_1 = surface velocity of inner cylinder
 R_1 = radius of inner cylinder
 R_2 = radius of outer cylinder
 μ = fluid viscosity
 $T = \text{local Taylor number} = 4 \left(\frac{V_1 h}{\nu} \right)^2 \frac{h}{R_1 + R_2}$
 $h = \text{local radial clearance between inner and outer cylinder}$
 $= C (1 + \epsilon \cos \theta)$
 $T_c(Q) = \text{local transition value of Taylor number for onset of vortices } T_c(Q) \text{ is a function of } Q, \text{ Table II}$
 $F(Q), G(Q), H(Q) \text{ functions of } Q, \text{ Table II}$

Relations (21) and (22) are derived for the case where both clearance and pressure gradient do not vary circumferentially. In the case of non-concentric rotating cylinders, both clearance and pressure gradient do vary around the cylinders. Q , therefore, varies circumferentially. To obtain an approximate calculation of the effect of vortex motion on shear stress and pressure gradient in the flow between non-concentric cylinders, one can apply relations (21) and (22) locally at each circumferential point using the appropriate local values of Q and sum these local contributions. The procedure for doing this is described in Ref. 13. From this procedure, one can calculate the effect of vortex motion on the overall viscous drag associated with rotating non-concentric cylinders.

Typical results for calculated viscous drag are shown in Fig. 7 along with some experimental measurements taken from Ref. 13. The ordinate used in this figure is G/G_L , the ratio of rotational torque in the vortex regime to the torque

that would occur if flow remained laminar. The solid curve shown in Fig. 7 is the theoretical curve calculated using values of critical Taylor number, $T_c(Q)$, which were determined from DiPrima's analysis for $C/R_1 \rightarrow 0$ (see Fig. 5). However, as was shown earlier, the critical speed for first onset of vortices was found experimentally to increase with C/R_1 . To allow for this, the theoretical curve for $C/R_1 = 0$ is shifted horizontally to the right in Fig. 7 so as to align the point of calculated increase in torque with the measured transition point for onset of vortices. Two such shifted curves are shown: one for $C/R_1 = 0.0104$ (dashed curve 1) and one for $C/R_1 = 0.099$ (dashed curve 2). For $C/R_1 = 0.0104$, agreement between measured values of torque and the shifted theoretical curve are fairly good at speeds just above critical speed. For the case of $C/R_1 = 0.099$, measured values of torque increase more rapidly at speed above Taylor transition speed than would be predicted by the corresponding shifted theoretical curve.

To determine the effect of Vortex motion on circumferential pressure profiles around non-concentric cylinders, measurements were made with cylinders having a clearance ratio of $C/R_1 = 0.0104$ for Reynolds numbers up to approximately 2000. Due to this relatively large (by bearing standards) clearance ratio, inertia forces had a significant influence on the measured pressure profiles at the higher Reynolds numbers. However, it was shown (Ref.13) that to first order accuracy, the effect of inertia forces on the pressure profile would be symmetrical about the line of centers of the cylinder while the major portion of the pressure, resulting from viscous forces, would be antisymmetrical about the line of centers. Therefore, it was possible to separate out the inertia effect from the measured pressure profile. Typical separated profiles are shown in Fig. 8. The dashed curve (curve 2) is the measured profile including inertia effects. Curve 1 shows the pressure profile with the symmetrical inertia effects separated out. The inertia correction itself is plotted as the solid line through the points designated by solid triangles. One can note that at the condition shown, the maximum inertia correction amounts to approximately 20 % of the peak pressure.

Once the inertia effects are separated out from pressure profiles measured in the vortex or turbulent flow regimes, one can assess, directly, the influence of vortex motion or turbulence on the magnitude and shape of the profile. In Fig. 9 are plotted three experimental profiles measured, respectively, under conditions of laminar flow, vortex flow, and vortex flow with perhaps the beginning of turbulence. Inertia effects have been separated out from the profiles shown. The ordinate is the dimensionless pressure $PC^2/6\mu V_1 R_1$. If the flow remained laminar, all of the profiles would follow the same curve, since this dimensionless pressure

correctly accounts for the effect of speed on the laminar profile. The fact that the profiles measured at $\sqrt{1/2T^*} = 83$ and $\sqrt{1/2T} = 169$ have larger magnitude than that measured at $\sqrt{1/2T} = 13.6$ is a direct indication of the effect of vortex motion or turbulence. One can note that at $\sqrt{1/2T} = 169$, superlaminar effects have increased the peak magnitude of the experimental profile by approximately 40%.

Theoretically predicted profiles are compared with the measured profiles in Fig. 9. In laminar flow, agreement is excellent. At $\sqrt{1/2T} = 83$ the predicted curve based on vortex theory is seen to be greater than the measured profile. This is due to the fact that the theoretical transition speed is less than the measured transition. At $\sqrt{1/2T} = 169$, the measured curve is greater than the predicted curve. In general, over the range $41 < \sqrt{1/2T} < 170$, vortex theory seems to provide a reasonably accurate prediction of the measured profiles.

D. Interaction between Vortices and Turbulence

The development of secondary vortex flow has a more pronounced effect on the magnitude of pressure profiles than upon their shape. In Fig. 10 is plotted the ratio $P_{\max}/(P_L)_{\max}$ vs Reynolds number for non-concentric cylinders at various eccentricity ratios. Here $P_{\max}$ is the peak magnitude of the measured profile while $(P_L)_{\max}$ is the theoretical peak pressure that would be obtained if flow remained laminar. The point of onset of vortices in the flow is characterized by the ratio of $P_{\max}$ to $(P_L)_{\max}$ becoming greater than unity. Theoretical curves based on both vortex theory and the turbulent theory of Ng and Pan (Ref. 4) are compared with the experimental data in Fig. 10. Data shown are for $C/R_1 = 0.0104$. The significant thing to note in this figure is that for $N_{Re}^{**} > 2400$, the turbulent theory begins to provide a quite accurate calculation of the peak pressure, even though it is known from visual observations that a vortex secondary flow still persists in the flow. Apparently, as soon as turbulence becomes fully developed in the flow, the turbulent transport mechanism dominates over that due to the vortex flow.

$$* T = \left(\frac{V_1 C}{\nu} \right)^2 \left(\frac{4C}{R_1 + R_2} \right) \text{ is known as the Taylor number}$$

$$**N_{Re} = \frac{V_1 C}{\nu}$$

The effect of vortex flow and turbulence on the shape of the pressure profiles around non-concentric cylinders is shown in Fig. 11 where various experimental profiles are plotted normalized to a peak value of 1.0. As noted earlier, flow at the condition $\sqrt{1/2T} = 169$ may have contained some turbulence as well as vortices, since the Reynolds number corresponding to the flow condition was $N_{Re} = 1660$. The turbulent theory of Ng and Pan (Ref. 4) does predict the shape of the profile at $\sqrt{1/2T} = 169$ more accurately than does vortex theory. However, referring back to Fig. 10 we see that turbulent theory does not predict the magnitude of the peak pressure at this condition ($N_{Re} = 1660$) as accurately as does vortex theory. Essentially, at this condition, neither vortex theory alone nor turbulent theory alone can be expected to adequately describe the flow.

For the case in which an outer cylinder rotates about a stationary inner cylinder, vortex flow will not develop. In this situation, the friction factor, λ , for the flow will be a function only of the Reynolds number. λ is defined as $\lambda = 2\tau/\rho V_1^2$ where τ is the shear stress on the surface of the rotating cylinder.

In the case of rotating, inner cylinder, where vortices will develop, the friction factor becomes a function of both Reynolds number and clearance ratio. The dependence of λ on these parameters is expressed quite accurately by the following empirical formulas developed by Wendt* (Ref. 15).

$$\lambda = 0.46 \left(\frac{(R_2 - R_1) R_2}{R_1^2} \right)^{\frac{1}{4}} (N_{Re})^{-0.5} ; 80\sqrt{\frac{R_1}{C}} < N_{Re} < 10^4$$

$$\lambda = 0.073 \left(\frac{(R_2 - R_1) R_2}{R_1^2} \right)^{\frac{1}{4}} (N_{Re})^{-0.3} ; 10^4 < N_{Re} < 10^5$$

A comparison of Wendt's formula with experimental data at $C/R_1 = 0.099$ is shown in Fig. 12. Also shown are data measured at $C/R_1 = 0.0104$ and data measured by Smith and Fuller (Ref. 24) for a journal bearing having $C/R_1 = .00293$. All data shown have been corrected to a clearance ratio of $C/R_1 = 0.099$ by means of Wendt's empirical relationship which states that

$$\lambda \propto \left[\frac{(R_2 - R_1) R_2}{R_1^2} \right]^{\frac{1}{4}}$$

*The range of Reynolds numbers over which these formulae are to be used has been modified slightly by the writer.

The dependence of friction factor on clearance ratio can be seen directly in Fig. 13. Here are plotted curves of λ vs Re for $C/R_1 = 0.01, 0.002, 0.001$ and 0.0005 , calculated from Wendt's empirical formulae. The Taylor critical speeds for predicted onset of vortices at different clearance ratios are indicated along the line for laminar friction factor. Also shown is the friction factor curve based on the turbulent theory of Ng and Pan. This latter theory neglects effects of clearance ratio since it neglects consideration of vortex flow.

It should be noted that Wendt's formulae are based on data taken with test cylinders having clearance ratios greater than 0.095 . Application of Wendt's formulae to clearance ratios on the order of 10^{-3} is therefore questionable. Hence, the discrepancy at high Reynolds numbers between the turbulent theory curve and the curves calculated from Wendt's formulae may not necessarily indicate an inadequacy of the turbulent theory.

IV. THIN FILM TURBULENT LUBRICATION

Analogous to Reynolds' lubrication theory for laminar thin films, turbulent thin film lubrication theory has been formulated recently. While there were earlier works on this topic (refs. 16, 17), the contemporary approach was pioneered by Constantinescu (ref. 18), which invoked in substance Prandtl's "mixing length theory." The most comprehensive treatment available to date (ref. 19) was based on the "wall law" supplemented by Clauser's concept of "core eddy diffusivity" (ref. 20). Although there is no fundamental difference between the "mixing length theory" and the "wall law," ref. 19 treated the buffer zone and the cross pressure gradient effect more thoroughly, furthermore, use of "core eddy viscosity" concept yields more reliable results for the large Reynolds number Poiseuille type flows.

Neglecting inertia effects, the lubrication equation applicable to both laminar and turbulent films in Cartesian coordinates is

$$\frac{\partial}{\partial x_i} \left[\rho \left(-\frac{h^3}{\mu} G_{ij} \frac{\partial p}{\partial x_j} + \frac{1}{2} h V_i \right) \right] + \frac{\partial}{\partial t} (\rho h) = 0 \quad (23)$$

G_{ij} is a diagonal matrix ($G_{ij} = 0$ when $i \neq j$) when one of the coordinates (say, x_1) is chosen to be parallel to the surface velocity ($V_3 = 0$). In particular, $G_{ij} = \frac{1}{12} \delta_{ij}$ for the laminar film, where δ_{ij} is the Kronecker delta. For turbulent films, G_{ij} depends on $\frac{V_1 h}{\nu}$, $\frac{h^3}{\mu \nu} \frac{\partial p}{\partial x_1}$, and $\frac{h^3}{\mu \nu} \frac{\partial p}{\partial x_3}$. Results computed in ref. 19 for G_{ij} are reproduced in Figs. 14 through 17. In Figs. 14 and 15 these are respectively plotted against $\frac{V_1 h}{\nu}$ for various values of $\left| \frac{h^3}{\mu \nu} \frac{\partial p}{\partial x_1} \right|$ with $\frac{\partial p}{\partial x_3} = 0$; in Figs. 16 and 17, $\left| \frac{h^3}{\mu \nu} \frac{\partial p}{\partial x_3} \right|$ is the variable with $\frac{\partial p}{\partial x_1} = 0$. When neither $\frac{\partial p}{\partial x_1}$ nor $\frac{\partial p}{\partial x_3}$ is zero, the following interpolation formula was found to be satisfactory:

$$G_{ij} \left(\frac{V_1 h}{\nu}, \frac{h^3}{\mu \nu} \frac{\partial p}{\partial x_1} = a, \frac{h^3}{\mu \nu} \frac{\partial p}{\partial x_3} = b \right) = \left(1 - \frac{2\theta_p}{\pi} \right) G_{ij} \left(\frac{V_1 h}{\nu}, \frac{h^3}{\mu \nu} \frac{\partial p}{\partial x_1} = \sqrt{a^2 + b^2}, 0 \right) + \frac{2\theta_p}{\pi} G_{ij} \left(\frac{V_1 h}{\nu}, 0, \frac{h^3}{\mu \nu} \frac{\partial p}{\partial x_3} = \sqrt{a^2 + b^2} \right) \quad (24)$$

where

$$\theta_p = \tan^{-1} |b/a|$$

is the angle between the direction of the pressure gradient and the surface velocity.

From Figs. 14 through 17, it is seen the effects of pressure gradients are small so long as

$$\left| \left(\frac{v}{Vh} \right)^2 \frac{h^3}{\mu v} \frac{\partial p}{\partial x} \right| < 0.25 \times 10^{-2}$$

In this range

$$G_{ij} \left(\frac{V_1 h}{v}, \frac{h^3}{\mu v} \frac{\partial p}{\partial x_1}, \frac{h^3}{\mu v} \frac{\partial p}{\partial x_3} \right) \approx G_{ij} \left(\frac{V_1 h}{v}, 0, 0 \right) \quad (25)$$

which was treated in ref. 4 and the results are reproduced in Fig. 18. Using the approximation of eq. (25) in eq. (23), one clearly sees that the equation becomes linear in p , this is referred to as the "linearized turbulent lubrication theory." It is quite evident that the linearized turbulent lubrication theory would be quite accurate for nearly constant film thickness conditions. It is rather surprising that the non-linear effects of $\frac{\partial p}{\partial x_i}$ seem to remain small for a full cylindrical journal bearing even when the eccentricity is very high as shown in Fig. 19. It appears that the linearized turbulent lubrication theory should be adequate for all smooth-film self-acting bearings.

The linearized turbulent lubrication theory has also been used to calculate bearing stiffness and damping which are needed in the prediction of dynamic rotor response. Such a prediction was corroborated by test for a rotor supported by four-shoe tilting-pad journal bearings. Correlation between test and theory for this bearing is shown in Fig. 20 for both static and dynamic conditions. The agreement is quite gratifying.

V. INERTIA EFFECTS

All terms on the lefthand side of the Navier-Stokes equation contribute to inertia effects. According to common mechanics terminology, they include forces due to transient acceleration, convective acceleration, centrifugal acceleration, and Coriolis acceleration. The latter two exist only in fluid films capable of carrying axial loads. Undoubtedly, in the presence of inertia effects, the pressure in the fluid film would not be related to shear stress only. In principle, this situation can exist regardless whether the flow is purely laminar, laminar with secondary vortices, or turbulent (with or without secondary vortices). An examination of the flow regimes map quickly reveals that for a journal bearing with a rotating inner cylinder, the inertia effects by themselves are seldom present without either secondary vortices or turbulence. Of course, if the outer cylinder instead of the inner one is rotating, or if the fluid film is flat, then inertia effects can be important in a thick laminar film.

Experimental evidence of the convective inertia in the vortex flow regime was indicated in Fig. 8 for a full cylinder. The over-all consequence is primarily a shift of the center of pressure such that the attitude angle would be a little larger than 90° and, to a lesser extent, the bearing force is somewhat increased. It should be recalled that the clearance-to-radius ratio is considerably higher than common lubrication practice.

An inertia effect is also manifested along the edge at which flow enters the fluid film. Some data on this phenomenon was reported in ref. 16. Wall pressure and velocity profile at the inlet edge of a cylindrical partial-arc pad were measured for several combinations of inlet film thickness, rotational speed, and inlet flow. This data was correlated by the formula

$$p_o = p_a + C_e \frac{\rho V^2}{2} \quad (26)$$

where

$$p_o = p_e + \frac{\rho U_e^2}{2}$$

p_e = entrance static pressure

U_e = mean entrance velocity

V = surface speed of rotating cylinder.

The entrance coefficient C_e was found to be 0.17, it is quite possible that it is dependent on the geometry of the inlet edge. The major consequence of this inlet effect is a shift in the apparent attitude angle of the bearing force due to the ram pressure. This was observed in a series of tests reported in ref. 23, where the measured bearing force correlated quite well with the linearized turbulent lubrication theory (Fig. 21) but the discrepancy in the attitude angle is considerable (Fig. 22). The ram pressure at the pad inlet can be somewhat relieved by allowing flow bypass around the pad. This was done on a 100 deg. pad test bearing reported in ref. 25 as shown in Fig. 23. The test data of this bearing agree quite well with the linearized turbulent lubrication theory as illustrated in Fig. 24. However, discrepancies suggesting a ram effect are still evident in both the load-deflection and attitude angle curves, for instance, for $C/R = 4 \times 10^{-3}$ $Re = 5820$ and $\epsilon = 0.27$. Indications of increased disparity between the linearized turbulent lubrication theory and test data for a larger C/R are also apparent in the dynamic orbits shown in Fig. 25. It is somewhat surprising that similar discrepancy was not apparent in the data of ref. 21. This may in part be due to a smaller clearance-to-radius ratio, and also in part due to the close spacing between the consecutive pads.

In ref. 22, the inertia effect at an internal edge of a step pad was also studied experimentally. It was found that a steep pressure drop occurs here. This is probably due to the sudden acceleration of flow as it enters the step. For the particular conditions studied, which include a fairly large gap, the land region of the step pad practically does not support any load.

Other inertia effects (transient, centrifugal, and Coriolis accelerations) presumably would become prominent also when the convective inertia is important.

VI. CONCLUSIONS

The experience cited above can be examined in terms of the flow-regime map. The mean film thickness, C_{mean} , is used to calculate the parameters which form the coordinates of the map. C_{mean} is defined as follows:

Bearing Type	C_{mean}
Full Cylindrical	Mean Radial Clearance
Pocket Bearing	
Land Region	Mean Radial Clearance
Pocket Region	Mean Radial Clearance plus Pocket Depth
Partial Arc	Mean Radial Clearance
Pivoted Pad	Concentric Clearance at Pivot

A flow-regime map containing all experimental information discussed above plus several actual and contemplated design points is shown in Fig. 26. The following remarks can be made:

1. Experiments represented by cases 6 through 9 reflect realistic design conditions. In experiments identified by cases 10 through 12, inertia and vortex flow effects are exaggerated on account of the large film thickness.
2. Turbulence is the dominating flow process.
3. The "inertia border" on the map is an adequate basis for neglecting inertia effects. Exception should be made for isolated and lightly loaded partial arcs and sliders. For these bearings the inlet ram effect can be quite substantial at the "inertia border."
4. For $C_{\text{mean}}/L < 2 \times 10^{-3}$, the thin film turbulent lubrication theory is quite satisfactory. In fact, the linearized turbulent lubrication theory is good enough for self-acting smooth films.
5. Inertia effects, in the presence of turbulence, are the important problems which remain to be solved.

NOMENCLATURE

C	representative film thickness, mean radial clearance
C_e	entrance dynamic head coefficient
C_f	coefficient of friction $\tau / \frac{1}{2} \rho V_m^2$
C_{mean}	mean film thickness
D	diameter
$F(Q), G(Q), H(Q)$	functions of Q, Table II
G	rotational torque
G_{ij}	admittance tensor in turbulent lubrication theory
G_L	laminar rotational torque
h	local radial clearance
L	representative linear dimension, pad length, length of cylinder
N	revolutions per second
N_{Re}	$V_1 C / \nu$
$(N_{Re})_{axial}$	$W_m C / \nu$
p	pressure
P	pressure above ambient
p_e	entrance static pressure
$(P_L)_{max}$	maximum P in laminar film
P_{max}	max P in film
p_o	entrance total pressure
Q	$3V_p / V_c$
Re	VC / ν
R_1, R_2	radii of inner and outer cylinders
S	Sommerfeld number $\frac{W(C/R)^2}{N\mu LD}$
t	time

T	temperature, local Taylor number $4 \frac{V_1 h}{\nu}^2 \frac{h}{R_1 + R_2}$
T_c	local transition value of Taylor number for onset of vortices, Table II
U_e	mean entrance velocity
U_i	Cartesian components of flow velocity
V	representative velocity, surface velocity of journal
V_c	mean circumferential velocity due to rotation = $V_1/2$
V_m	mean circumferential flow velocity due to sum of Couette and pressure flow
V_p	mean circumferential velocity due to pressure gradient
V_1	surface velocity of inner cylinder
W	load
W_m	mean axial flow velocity
x_i	Cartesian coordinates
$(\bar{})$	temporal average of ()
$()'$	fluctuating component of ()
β	arc angle
δ_{ij}	Kronecker delta
ΔT	representative time period
ΔR	$R_2 - R_1$
ε	eccentricity ratio
θ_p	angle between directions of pressure gradient and the surface velocity
θ	angular coordinate measured from maximum film thickness position
λ	coefficient of friction, $\tau / \frac{1}{2} \rho V_1^2$
μ	dynamic viscosity coefficient
ν	kinematic viscosity
ρ	fluid density
σ_{ij}	Cartesian stress tensor
ϕ	attitude angle

Subscripts

- 1 direction of surface sliding motion
- 2 normal direction
- 3 direction tangential to surface but perpendicular to sliding motion

REFERENCES

1. Reynolds, O., "On the Theory of Lubrication and Its Application to Mr. Beauchamp Tower's Experiments," Phil. Trans. Roy. Soc., London, vol. 177, part 1, 1886.
2. Taylor, G.I., "Stability of a Viscous Liquid Contained between Two Rotating Cylinders," Phil. Trans. Roy. Soc., London, ser. A. vol. 223, pp. 289-343, 1923
3. Reynolds, O., "An Experimental Investigation of the Circumstances which Determine Whether the Motion of Water Shall be Direct or Sinuous and of the Law of Resistance in Parallel Channels," Trans. Roy. Soc., London, ser. A, vol. 174, 1883, pp. 935 - 982.
4. Ng, C.W. and Pan, C.H.T., "A Linearized Turbulent Lubrication Theory," Journal of Basic Engineering, Trans., A.S.M.E., Series D, Vol. 87, 1965, pp. 675-688.
5. Krueger, E.R., and DiPrima, R.C., "On the Stability of Spiral Flow between Rotating Cylinders," Journal of Fluid Mechanics, vol. 19, part 4, pp. 528-538, 1964.
6. DiPrima, R.C., "The Stability of a Viscous Fluid between Rotating Cylinders with an Axial Flow," Journal of Fluid Mechanics, Vol. 9, Part 4, pp. 621-631, 1960.
7. Donnelly, R.J., and Fultz, D., "Experiments on the Stability of Spiral Flow between Rotating Cylinders," Proc. Nat. Acad., Sci. 46, 1960
8. Snyder, H.A., "Experiments on the Stability of Spiral Flow at Low Axial Reynolds Numbers," Proceedings of the Royal Society of London, Series A, Vol. 265, 1962, pp. 198-214.
9. DiPrima, R.C., "The Stability of a Viscous Flow between Rotating Cylinders with a Pressure Gradient Acting Round the Cylinders," J. Fluid Mech. 6, 462-468 (1959).
10. DiPrima, R.C., "A Note on the Stability of Flow in Loaded Journal Bearings" ASLE Transactions, 6, pp. 249-253 (1963).
11. Sommerfeld, A., "Zur Hydrodynamischen Theorie der Schmiermittelreibung," Z. Math. u. Physik, Vol. 50, page 97, (1904).
12. Vohr, J.H., "Analysis of Turbulent Lubrication, Volume 2, An Experimental Study of Vortex Flow and Turbulence in Concentric and Eccentric Annuli," (Vol 2 of Final Report on NASA Contract NASw-771), May 20, 1964, NASACR-CR-54034, MTI64TR20.
13. Vohr, J.H., "Experimental Study of Superlaminar Flow between Non-Concentric Rotating Cylinders," Mechanical Technology Inc. Report No. MTI-65TR43, Final Report prepared for NASA-Lewis Research Center under contract NASw-1021.
14. DiPrima, R.C., "Viscous Flow between Rotating Concentric Cylinders with a Circumferential Pressure Gradient at Speeds above Critical," ASLE Transactions 7, pp. 333-341, Oct., 1964.
15. Wendt, F., "Turbulente Strömungen Zwischen Zwei Rotierenden Konaxialen Zylindern," Ingenieur-Archiv 4, 1933, pp. 352-353.

16. Tao, L.N., "A Theory of Lubrication in Short Journal Bearings with Turbulent Flow," Trans. A.S.M.E., vol. 80, November, 1958.
17. Chou, Y.T., and Saibel, E.,M "The Effect of Turbulence on Slider Bearing Lubrication," Trans. A.S.M.E., Ser. E, vol. 81, March, 1959.
18. Constantinescu, V.N., "On Turbulent Lubrication," Proceedings of the Institute of Mechanical Engineers, vol. 173, 1959, pp. 881-900d.
19. Elrod, Jr., H.G., and Ng, C.W., "A Theory for Turbulent Fluid Films and Its Application to Bearings," ASME Paper No. 66-Lub-12.
20. Clauser, F.H., "The Turbulent Boundary-Layer," Advances in Applied Mechanics, vol. 4, pp. 1-51, 1956.
21. Orcutt, F.K., "The Steady-State and Dynamic Characteristics of the Tilting-Pad Journal Bearing in Laminar and Turbulent Flow Regimes," ASME Paper No. 66-Lub-19.
22. Burton, R.A., and Carper, H.J., "An Experimental Study of Annular Flows with Applications in Turbulent Film Lubrication," ASME Paper Np. 66-Lub-14.
23. Orcutt, F.K., "Investigation of a Partial Arc Pad Bearing in the Superlaminar Flow Regime," Journal of Basic Engineering, Trans. ASME, Series D, vol. 87, 1965, pp. 145-152.
24. Smith, M.I., and Fuller, D.D., "Journal Bearing Operation at Super-Laminar Speeds," Trans. ASME Vol. 78, 1956, p. 469.
25. Orcutt, F.K., and Arwas, E.B., "The Steady-State and Dynamic Characteristics of a Full Circular Bearing and a Partial Arc Bearing in the Laminar and Turbulent Flow Regimes," ASME Paper No. 66-LubS-4.

TABLE I

Kinematic Viscosity of Various Liquids

Liquid	Kinematic Viscosity $\nu - \text{in}^2/\text{sec}$	$\frac{\nu}{\nu_{\text{SAE 10}}}$
SAE 10 oil (150°F)	2.16×10^{-2}	1.0
Lithium (406°F)	1.70×10^{-3}	0.0788
Water (80°F)	1.33×10^{-3}	0.0616
Silicone oil (0.65 cs)	1.01×10^{-3}	0.0468
NaK (150°F)	0.99×10^{-3}	0.0459
Sodium (1200°F)	3.86×10^{-4}	0.0179
Potassium (1200°F)	3.31×10^{-4}	0.0153
Mercury (400°)	1.20×10^{-4}	0.00556
Cesium (412°F)	0.89×10^{-4}	0.00411

TABLE II
TABLE OF FUNCTIONS OF Q

Q	F(Q) G(Q)	H(Q)	T _c (Q)
-3.665	0.341	0.312	71785.6
-3.65	0.342	0.423	70784.6
-3.60	0.335	0.638	68000.0
-3.50	0.319	0.784	62867.0
-3.25	0.302	0.929	51257.0
-3.00	0.291	0.978	40943.0
-2.6	0.273	1.17	26500.0
-2.2	0.232	1.435	16250.0
-2.0	0.185	1.570	12540.0
-1.8	0.152	1.585	10000.0
-1.6	0.125	1.570	8200.0
-1.2	0.085	1.525	6150.0
-0.8	0.060	1.480	4820.0
-0.4	.039	1.45	3960.0
0	.021	1.433	3390.0
0.4	-0.0055	1.430	2930.0
0.8	-0.01	1.436	2570.0
1.2	-0.026	1.453	2300.0
1.6	-0.042	1.477	2070.0
2.0	-0.058	1.51	1880.0
2.4	-0.0735	1.548	1700.0
2.8	-0.089	1.592	1505.0
21.0	-0.873	4.926	149.0
46.0	-0.873	4.926	149.0

Figure 1 - Velocity Pattern in Taylor Vortex Pair

Figure 2 - Torque vs Speed Showing Effect of Taylor Vortices

Figure 3 - Flow Regimes Map

Figure 4 - Critical Speed vs Axial Flow Reynolds Number

Figure 5 - The Taylor Vortex Transition Value of $\frac{V_1 C}{\nu} \cdot \sqrt{\frac{C}{R_1}}$ as a Function of ϵ for Various Values of θ

Figure 6 - Measurements of Transition Speed for Onset of Vortices in Flow Between Eccentric Cylinders

Figure 7 - Ratio of Superlaminar Torque to Laminar Torque,
Comparison of Experiment with Theory

Figure 8 - Effect of Inertia Forces on Pressure Profile

Figure 9 - Pressure Profiles for Laminar and Superlaminar Flow

Figure 10 - Ratio of Superlaminar Peak Pressure to Laminar Peak Pressure, Comparison of Theory with Experiment

Figure 11 - Effect of Superlaminar Flow on Shape of Pressure Profiles

Figure 12 - Friction Factor for Rotating Cylinders

Figure 13 - Effect of Clearance Ratio on Friction Factor
for Rotating Cylinders

Fig. 14 Coefficient G_{11} for $\partial p / \partial x_3 = 0$

Fig. 15 Coefficient G_{33} for $\partial p / \partial x_3 = 0$

Fig. 16 Coefficient G_{11} for $\partial p / \partial x_1 = 0$

Fig. 18 G_{11} and G_{13} for $\left| \frac{h}{\tau_w} \nabla v \right| \ll 1$

Fig. 19 Circumferential and Axial Pressure Distribution. Compare between Linearized and Non-Linear Turbulent Lubrication Theory for Full Journal Bearing.

Comparison between measured and calculated steady-state load capacity — $m = 0$

Comparison between measured and calculated steady-state load capacity — $m = 0.5$

Comparison between measured and calculated bearing friction torque — $m = 0$

Comparison between measured and calculated bearing friction torque — $m = 0.5$

Schematic of tilting pad bearing

Clearance ratio $(C_p/R) > 10^{-3} \text{ m/in}$
 Slenderness ratio $(l/D) = 1.0$
 Arc length $(\beta) = 80 \text{ deg}$
 Pivot position $(\theta_p/\beta) = 0.57$

Figure 20 - Comparison Between Calculated and Measured Dynamic Load Response Orbit Amplitude

Fig. 21 60-Degree Partial Arc Bearing -
Load Versus Eccentricity

Fig. 22 60-deg. partial arc bearing -
attitude angle versus eccentricity

Figure 23 - 100° Partial Arc Pad

Figure 24 - Measured and Calculated Steady-State Characteristics of 100 Deg. Arc Bearing

Re = 5820
C/R = 4×10^{-3} IN. / IN.

Re = 8314
C/R = 2×10^{-3} IN. / IN.

Figure 25 - Measured and Calculated Dynamic Load Orbits
for 100 Deg. Arc Bearing ($L/D = 1$)

Actual and Contemplated Designs	Bearing Pump	Experimental Bearings	Test Fluid	Remarks	Reference No.
1. 3 KW HG RANKINE	Hydrostatic Pads	6. Partial Arc 60° Pad	Silicone Oil	Data suggests inlet ram effect	23
2. SNAP 8	4 Shoe Tilting Pad 50% pad loading	7. Partial Arc 100° Pad (a) $C/R = 2 \times 10^{-3}$ (b) $C/R = 4 \times 10^{-3}$	Silicone Oil	Good agreement with linearized turbulent lubrication theory	25
3. General Atomics (a) Lower Journal (b) Upper Journal	Hybrid Pocket	8. 4 Shoe Tilting Pad (a) Unloaded (b) 50% Pad Loading	Silicone Oil	Data suggests inlet ram effect	21
4. Liquid Hydrogen Pump	Hybrid Pocket	9. 4 Shoe Tilting Pad	NaK	Test in progress	
5. Liquid Oxygen Pump	Hybrid Pocket	10. Eccentric Cylinders	Silicone Oil	Oversized clearance to obtain exaggerated Vortex flow and inertia effects	8, 9
		11. Partial Arc	Air	Oversized clearance to obtain exaggerated inertia effects	22
		12. Partial Arc	Air	Oversized clearance to obtain exaggerated inertia effects	22

Figure 26 - Flow Regimes of Various Tests and Design

SIMULATION OF TURBULENT LUBRICANT FILMS IN A LARGE SCALE APPARATUS

by

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ABSTRACT

Experiments are reported on turbulent flows in air, in a large-scale bearing model of 6-ft dia, 2-ft length, 0.54-in. film thickness. Simulation of tilted pads, short journal bearings, stepped pads and herringbone grooved bearings is described. Pressure distributions are reported along with velocity and turbulence-intensity profiles. Wall shear stress is computed from the velocity profile measurements. The variations of these factors are compared with available data for plane Couette flow, wall law flow, and pressure flow in pipes, and the relationships among these are discussed. In addition, large pressure jumps at discontinuities are reported, and are shown to provide a major influence on the overall pressure distributions.

INTRODUCTION

Large-scale simulation was decided upon to provide detailed information on turbulent films, for the purpose of guiding and checking analytical techniques. When bearing configurations are operated at turbulent Reynolds number, and have film thicknesses of the order of 0.5 in., it is possible to measure detailed velocity profiles and to relate these to other aspects of the flow. One difficulty with this experimental technique is that pressures produced in the flow vary as some inverse power of film thickness, so the actual measured pressure magnitudes are very small. Means have been devised, however, to make such measurements (Ref. 1). Some preliminary information on both pressure distributions and velocity profiles has been reported elsewhere (Ref. 2); the present report will be restricted principally to interesting features of the pressure distributions, showing some of the earlier data as well as new data.

Apparatus

This apparatus was based upon a 6-ft diameter wheel, 20 in. long. About this surface, which could be turned at speeds above 100 rpm, various stationary bearing configurations could be supported, including tilted pads, stepped pads, spiral grooves and the full bearing. End leakage could be restricted by plastic curtains, thus making possible the simulation of long or short bearings, depending upon the presence or absence of the curtains. Reynolds number in excess of $hu_s/2\nu = R = 2500$ were possible, where h is film thickness, u_s is journal surface speed and ν is kinematic viscosity. This is well within the turbulent regime.

Performance of Tilted Pads

Tilted pads were 3 ft in circumferential length, 20 in. in axial dimension and were pivoted about a film thickness of 0.5 in. at a point 0.75 ft forward of the trailing edge. In these experiments end leakage was (a) free, (b) partially restricted and (c) totally restricted. Pressures were measured at axial midspan. Figure 1 shows two conditions, for a fixed angle of tilt (1.0 in. film thickness at leading edge, 0.26 in. at trailing edge). Note that a significant "ram effect" or "impact pressure" appears at the leading edge. In this figure and others the dimensionless pressure P is defined as $P = p/\rho u_{bc}^2$, where p is pressure, ρ is density and u_{bc} is half the journal surface speed u_s .

Figure 2 shows pressure variations for two tilt angles, and end leakage fully restricted. Again the leading edge pressure plays a dominant role.

Leading Edge Pressures

When entrance region velocity measurements were made for different leading edge pressures in tilted pads and stepped pads, it was found that the correlation presented in Figure 3 prevailed. Here the dimensionless pressure is related to the velocity in such a way that results can be correlated in terms of a dimensionless "impact" pressure according to the equation shown. If the entering flow had been carried at wheel speed (u_s , where $u_s = 2u_{bc}$) then brought to a halt in a frictionless process, as on a pitot impact tube, the following relationship would prevail:

$$p/\rho u_{bc}^2 = u^2/2u_{bc}^2 \quad (1)$$

or

$$\frac{p}{\rho u_{bc}^2} + \frac{u^2}{2u_{bc}^2} = 2 \quad (2)$$

Thus the apparent dimensionless stagnation pressure of the entering flow as described in Figure 3 is made less than if the fluid were carried at wheel speed, is shown in Equation (2). This implies that a boundary layer exists on the rotor surface with velocity dropping off with distance from the surface, and with a mean velocity $< u_{bc}$. Even if a thin film of liquid were transported on the journal, for the fully flooded entry condition a globule of liquid would be expected to accumulate at the entry region, and the slowing of the liquid (movement into an adverse pressure gradient) would be in a process of irreversible mixing; and stagnation pressure would be less than the amount based on the assumption that the flow enters with the velocity u_s . It is believed that the reported data are representative of the fully flooded entry condition and are meaningful beyond the particular experiment. It is also felt that much more is still to be done with the entry-pressure problem.

Stepped Pads

Pressure distributions for stepped pads are shown in Figures 4 and 5. The "unshielded" case represents unrestricted end leakage and the "shielded" one represents zero end leakage. Of special interest is the large pressure drop at the step where film thickness changes from 0.5 in. to 0.25 in. For the pad with leakage the pressure drops to a sizeable negative value at this point. This dimensionless pressure would correspond to -14.7 psig when $(14.7)(144) = \rho u_{bc}^2 (0.1)$ or $\rho u_{bc}^2 = 21,200$. For a fluid with the density of water this would occur at approximately $u_s = 200$ ft/sec. This is not an excessive speed for projected high-performance applications. Thus, it would not be surprising to see cavitation under a pad in the turbulent operating region.

Spiral Grooves

Figure 6 shows pressure tap locations for a spiral groove configuration in the

large-scale apparatus. Figure 7 gives pressure measurements corresponding to the tap locations indicated in Figure 6. The spiral angle is 45° , and the ratio of groove-clearance to land clearance is 2:1. Note that negative dimensionless pressures occur over the land and at one station in the groove. The maximum negative magnitude is very near that measured for the stepped pad.

Short Bearing

Figure 8 shows a polar plot of pressure distribution around the apparatus operated as a full bearing with unrestricted end leakage. Here the radial clearance was 0.5 in., eccentricity was 0.5, and displacement was as shown in the small diagram at the lower right of the figure. Most unexpected was the fact that positive gage-pressures act in the thickest-film regions and negative gage-pressures act on the thinner-film regions. It appears from this that film forces act to augment eccentricity. Preliminary analyses indicate that much of this effect is a consequence of inertial forces in the film.

Conclusion

The above examples serve to illustrate the rather sizeable interial effects in turbulent lubricant film. These are: (1) leading edge "ram effect" which moves center of pressure forward and otherwise significantly affects pressure distribution under pads, (2) convergence effect at a step which causes a strong pressure drop and may lead to cavitation, (3) negative pressure over a land of a spiral groove configuration which can, likewise, lead to cavitation, and (4) tendency of pressure to be high in thick-film regions and low in thin-film regions (as in the Bernoulli effect), thus giving rise to a possible negative-spring effect in turbulent, short, full bearings.

Preliminary analyses, which encompass the experimental thick-film case, predict similar effects in more commonplace thin films. Though less severe in such a case, they nevertheless appear to remain in a dominant role.

References

- (1) Carper, H.J., and Burton, R.A. "A Technique for Micromanometry," AIAA Journal, Vol. 4, No. 3, 1966.
- (2) Burton, R.A., and Carper, H.J., "An Experimental Study of the Annular Flows with Applications in Turbulent Film Lubrication," ASME Paper 66-Lub-14.

Fig. 1 Pressure variation for tilted pads for unrestricted and partially restricted end leakage

Fig. 2 Pressure variation for two tilt angles, for a pad without end leakage

Fig. 3 Variation of pad-entry pressure with inlet velocity

Fig. 4 Pressure variation under a short stepped pad

Fig. 5 Pressure variation under a stepped pad without end leakage

Fig. 6 Layout of experimental groove and land arrangements, showing location of pressure taps

Fig. 7 Dimensionless pressure, P , for concentric operation

Fig. 8 Pressure distribution in a short journal bearing, for $L/D = 0.28$, and eccentricity = 0.5

Motion Picture Visualization of Laminar, Vortex and Turbulent Flows
in the Annular Gap between Concentric and Eccentric
Rotating Cylinders

by

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ABSTRACT

A motion picture was shown of various superlaminar flow patterns occurring in the annular fluid film between a stationary outer cylinder and a rotating inner cylinder for both concentric and eccentric positions of the inner cylinder. The movie contained scenes showing both Taylor vortex flow and turbulent flow. A description of the test apparatus used in the study and descriptions of some of the typical flow patterns seen in the movie are presented below.

DISCUSSION

The test apparatus for the visual study consisted basically of a stationary glass outer cylinder having a 4 inch internal diameter with an aluminum, rotating inner cylinder, 3.64 inches in diameter. The inner cylinder could be positioned at arbitrary eccentricities within the outer cylinder by means of eccentric bushings at the ends of the cylinders. A sketch of the test apparatus is shown in Fig. 1.

The test fluids used in the experiments were silicone fluids of various viscosities. Flow visualization was accomplished by means of fine aluminum particles, suspended in the flow.

The movie presented contained scenes showing both Taylor vortex flow and turbulent flow. Taylor vortices are a form of flow instability which arise due

to centrifugal forces. The instability takes the form of toroidal vortices spaced periodically in the axial direction. The characteristic flow pattern in a pair of Taylor vortices is shown in Fig. 2. In Fig. 3 is shown a photograph of non-concentric rotating cylinders with fully developed vortex flow. Vortices are absent from the ends of the cylinder due to the axial pressure flow there which stabilizes the flow against development of vortices.

When the cylinders are concentric, the vortex activity in the flow is symmetrical around the cylinders. When the cylinders are eccentric, however, the intensity of vortex activity varies considerably around the cylinder although the axial wavelengths of the vortices remain constant. This circumferential variation in vortex intensity is shown in Fig. 4 where a set of 18 photographs taken at 20° intervals around the test rig are assembled into a single "unwrapped" view of the flow around the cylinders. The eccentricity ratio for this figure is 0.475. The widest part of the annulus between the cylinders is at $\theta = 0^\circ$ and the narrowest portion is at $\theta = 180^\circ$. Rotation of the inner cylinder is in the direction of increasing θ . The dark vertical lines at $\theta = 107^\circ$, 227° and 347° in Fig. 4 are the tie rods connecting the upper and lower flanges of the test rig.

As can be seen, the region of strongest vortex activity is in the widest part of the annulus and is centered at approximately $\theta = 50^\circ$. Progressively downstream of this, as the annular clearance decreases, the vortex activity damps out until at $\theta = 180^\circ$ (narrowest part of the annulus) the stream lines appear to be all parallel to the direction of rotation. A close-up high speed motion picture of the flow at $\theta = 180^\circ$ did reveal, however, that the velocity at this point varies in magnitude periodically with Z so that there does appear to be some vortex activity still present at this point. Downstream of $\theta = 180^\circ$ the annulus begins to widen again but vorticity continues to be further damped out and reaches an apparent minimum of strength at $\theta = 230^\circ$. From $\theta = 230^\circ$ onward, vorticity begins to steadily increase again as the widest part of the annulus is approached.

For concentric cylinder, higher modes of vortex wave instability occur as speed is increased above Taylor critical speed. In these higher modes, the vortices develop a circumferential waviness, the number of developed waves being

a characteristic of the particular mode (see Coles, Ref. 1). Ultimately, as speed is further increased, turbulence sets in, and the multiple-wave pattern of Taylor vortices begins to break down with irregular eddies. With further increase in speed, the irregular eddying increases in intensity and appears to become finer in scale. The number of waves in the vortex flow pattern begins to decrease until, at a Reynolds number of 4000, the waviness has disappeared. The flow now has fully developed turbulence but still has a basic vortex structure. This basic vortex structure persists in the flow up to a Reynolds number of 40,000, the maximum obtained in the tests.

The sequence of events described above is illustrated by photos in Fig. 5. Particular things to note are: the beginning of irregular eddying at $N_{RE} = 1080$, the lack of waviness in the vortex pattern of $N_{RE} = 4160$, the feathery appearance of the turbulent vortex flow at Reynolds numbers above 4000, and the persistence of a strong vortex pattern even at $N_{RE} = 39,800$.

When the cylinders of the test rig are positioned at fairly large eccentricity ratios ($\epsilon \geq 0.5$) the sequence of flow patterns that occurs as rotational speed is increased is quite different from that occurring when the cylinders are concentric. When the rotational speed of the inner cylinder is increased above the Taylor critical speed, wave instability occurs first at a position just downstream of the point of maximum annular clearance. Because of the fact that flow conditions and mean flow velocity change rapidly in the circumferential direction, wave instability for non-concentric cylinders never demonstrates the cleanly defined sequence of modes that occurs with concentric cylinders. Instead, the wavy instability in the wide portion of the annulus becomes steadily stronger and more irregular as speed is increased until the flow breaks down into a pattern of large irregular eddies which completely obscure the vortex pattern. This breakdown occurs when the Reynolds number based on the mean clearance is of the order of 500.

The essential features of flow between non-concentric cylinders at Taylor numbers well above critical are shown in the top sequence of pictures in Fig. 6. The right hand picture in this sequence shows the flow near the end of the narrow portion of the annulus. One can see the point at which the laminar flow exiting

from the narrow clearance breaks down suddenly into eddies. The middle picture shows this transition point more directly. The last picture in the sequence shows the eddy pattern that exists further downstream in the wide portion of the annulus. This eddying completely obscures the vortex pattern in the wide portion.

In the bottom three pictures in Fig. 6 is shown the flow at the same locations as in the top pictures but at a much higher Reynolds number ($N_{RE} = 6500$). At this condition the flow in the narrow portion of the channel appears to be turbulent judging from its "grainy" appearance. In the picture to the bottom left in Fig. 6, showing the wide portion of the channel, one can note the reappearance of an axially periodic, vortex structure in the turbulent flow.

With further increase in Reynolds number, flow remains essentially the same in appearance as it does at $N_{RE} = 6500$. The intensity of the turbulence increases, however, and the vortex structure of the flow in the wide portion of the channel becomes more pronounced.

The various flow patterns described above for both concentric and non-concentric cylinders are typical of those which are shown in the motion picture taken of the flow. The motion picture is a 16mm one, approximately 500 feet in length, with no sound track. The film was prepared by MTI for National Aeronautics and Space Administration under contract NASw- 1021. A copy of the film may be obtained from

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A technical report on the experimental project in which the motion film was made is listed as reference 2.

REFERENCES

1. Coles, D., "Transition in Circular Couette Flow" Navy Department Report Division of Engineering and Applied Physics, Harvard University, April, 1964.
2. Vohr, J.H., "Analysis of Turbulent Lubrication, Vol. 2, An Experimental Study of Vortex Flow and Turbulence in Concentric and Eccentric Annuli (Vol. 2 of Final Report on NASA contract NASw-771) May 20, 1964, NASACR-CR-54034, MTL-64TR20.

Fig. 1

Schematic of Test Apparatus

Fig. 2 Velocity Pattern in Taylor Vortex Pair

Fig. 3

Taylor Vortex Flow between Non-concentric cylinders, $\epsilon=0.475$

Fig. 4

Circumferential Variation in Vortex Flow around
Non-concentric Cylinders, $\epsilon=0.475$

4.3.10

$N_{RE} = 187$

$N_{RE} = 314$

$N_{RE} = 1080$

$N_{RE} = 1395$

$N_{RE} = 4160$

Fig. 5 Superlaminar Flow Patterns for Concentric Cylinder

$N_{RE} = 1570, \epsilon = 0.707$

$N_{RE} = 6500, \epsilon = 0.707$

Fig. 6 Superlaminar Flow Patterns for Non-concentric Cylinders

CONDITIONS FOR THE RUPTURE OF A LUBRICATION FILM^{*}

by

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ABSTRACT

A theoretical and experimental investigation of the conditions for the rupture of lubricating films is described. The theoretical analysis deals with the boundary conditions at film separation that are to be used with the Reynolds Equation. A simple experiment was performed using a cylindrical reading lens for the slider and phonograph turntable for the moving surface. The experiment was run at very light loads and small values of the dimensionless parameter $\mu U/T$ where μ is the viscosity, U is the turntable speed and T is the surface tension. Under such conditions, the differences between the present theory for film rupture and earlier ones were magnified. The experiment gave excellent agreement with the present theory.

DISCUSSION

The location of the film-cavity interface and the pressure jump occurring across the meniscus of this interface are of interest to lubrication engineers, since such information bears on the proper boundary conditions for Reynolds' pressure equation. Various boundary conditions have been proposed in the literature. The Swift-Stieber condition which is most widely used ends the film where $dp/d\theta$ equals zero. Birkhoff and Hays in a recent paper propose a condition of incipient counterflow for film separation in a diverging clearance. Other older conditions which eliminate the negative pressures or terminate the film at minimum clearance are still in common use.

This earlier work points out the need for determining correct boundary conditions at the edge of a lubricating film, and has provided the motivation for the analysis of free surface separation given in this paper. Here we focus attention on the liquid-vapor interface of a film which is attached to a stationary surface and

^{*} The work described in this presentation and its references was conducted by the author at Columbia University, as part of his doctoral candidacy requirements. The work was carried out under the guidance of Professor H.G.Elrod, Jr, Columbia Univ. and was sponsored by Mechanical Technology Inc.

is being swept along by an opposing moving surface. The physical situation thus described is depicted in Fig. 1. It is one of fundamental interest, and appears in many applications besides cavitation in bearings.

In the theory we consider a smoothly separating film, i.e. fluid velocities only in the x-y plane. The object of the analysis was to find the shape of the liquid-gas interface from the point of attachment out over the lower moving surface. The fluid was assumed to be Newtonian in character with constant density, viscosity, and surface tension. The following conditions are satisfied exactly in the analysis. On the interface:

1. The normal stress is balanced by surface tension and cavity pressure.
2. The tangential stress vanishes
3. The interface is a streamline
4. The streamwise component of the Navier-Stokes equation is satisfied.

The only major assumption is that u can be represented by a quadratic function of y . The constants in this quadratic representation are functions of position along the boundary and must satisfy the no-slip condition at the moving plate and also overall mass continuity. The details of the analysis are given in Reference 1.

Some typical solutions of film height and surface velocity are shown in Fig. 2 for different values of the parameter N . N is defined as:

$$N = (3\mu U/T)^{1/3}$$

where μ is viscosity, U is plate velocity, and T is surface tension. When (N) is large, a single stagnation point is present at $\theta = \pi/2$. As (N) decreases, the ratio of asymptotic thickness h_{∞} to attachment thickness c decreases, and a second stagnation point appears at $h/h_{\infty} = 3.0$, marking the onset of flow recirculation. As (N) further decreases, the second stagnation point moves down the surface and in the limit as (N) approaches zero, the film becomes a meniscus closed at the lower plate. Note that the film attains its asymptotic character at about one clearance thickness.

For any given solution, one can recover the constants in the quadratic velocity

representation, and thereby construct the streamlines of the flow. This has been done for the case of $(N) = 0.5$ in Fig. 3. The separating streamline ($x = 1.0$), which divides the recirculating flow from the flow being swept away on the moving plate, is shown. A further consequence of the assumptions is that the fluid is stagnant along a line normal to the boundary at $\theta = \pi/2$. The solution therefore, corresponds to the situation depicted in Fig. 3. When θ is close to $\pi/2$, the y coordinate samples the flow far upstream. For fully developed upstream flow, the assumed quadratic velocity distribution was proved to be perfectly consistent with Reynolds' equation. This consistence was one of the objects of the analysis and was exploited in developing film edge boundary conditions for Reynolds' equation.

Some comparisons of the theory with bubble-in-tube experiments are shown in Fig. 4. In these experiments a narrow tube was initially filled with a viscous liquid and thereafter partially evacuated by gaseous pressure applied at one end. A long bubble moved along the tube expelling most of the fluid out the far end leaving a layer of h_{∞} on the interior walls (e.g. blowing beverage from a straw). Since h_{∞} was small compared to the tube radius, the differences in geometry between the cylindrical tube and the planar case of the analysis is of minor importance. Figure 3 shows good agreement between the results of the theory and experimental results obtained by Bretherton, Taylor and Cox over a wide range of N .

Boundary conditions for use with Reynolds' equation can now be developed. First the pressure jump across the meniscus due to surface tension establishes the pressure in the film just upstream of separation. Second, the flow carried under the meniscus must equal the upstream Reynolds' flow. This establishes the pressure gradient at film separation. The pressure and pressure gradient boundary conditions for use with Reynolds' equation are shown in Fig. 5, in non-dimensional form, as a function of parameter N . The pressure gradient boundary conditions according to Swift-Stieber and Birkhoff-Hays, which are independent of the parameter N , are also shown. As an example of the application of the boundary condition at film separation of Fig. 5 the pressure profile for an infinitely-wide, parabolically varying clearance was calculated and then plotted in Fig. 6. These pressure profiles are for the case of a fully flooded inlet, that is, the film is assumed to begin infinitely far upstream. The abscissa $\bar{\theta}$ of Fig. 6 is the x -coordinate, distorted such that $\bar{\theta} = 90^\circ$ corresponds to $x = \infty$ and $\bar{\theta} = 0^\circ$ corresponds to the point of minimum film thickness. Note the relatively large sub-ambient pressure

region associated with the pressure profiles for small (N) .

The object of the experimental portion of the investigation was to test the validity of the boundary conditions derived from the theory. The experiment was of simple design requiring only measurements which could be made easily and accurately. The basic set-up consisted of a cylindrical slider bearing riding in a fluid film. Measurements were made of the load on the slider and of the location of the inlet and exit film boundaries. For comparison, Reynolds' equation was solved for the geometry of the slider using the boundary conditions of the theory as well as those of Swift-Stieber and Birkhoff-Hays. The relationship between the load and the location of film separation was computed for nearly flooded inlet conditions. The experiments were run for very light loads and small values of (N) . Under these conditions, surface tension effects become important and the present theory shows the greatest disagreement with others in the literature. However, it should be emphasized that under many conditions of load and N , the various results do not differ substantially.

Figure 7 shows a schematic sketch of the experimental apparatus. The slider was a one inch length cut for a cylindrical glass lens of two inch radius. This section was then cut in half. The moving surface was a glass disc placed over a phonograph turntable. The lubricant was Dow Corning silicone oil. The slider was attached to the end of a phonograph tone arm by means of a dial indicator attachment. The tone arm had a point pivot which allowed free motion of the arm in all planes. The turntable speed was first set to give a desired value of N . The ball and socket joint of the dial indicator attachment was used to set the slider at any arbitrary value of the angle γ . The angle γ was measured optically making use of a mirror attached to the upper surface of the slider. It should be noted that the angle γ which is measured optically bears a geometric relationship to the angular distance between the point of minimum film thickness and the edge of the glass slider. Therefore, when film separation coincides with the edge of the glass slider, the measurement of γ is tantamount to measuring the location of film separation. The sliding weight on the tone arm was used to load the slider to the point where the film was just on the verge of receding from the slider edge. Fine adjustment of load was made by lowering a very light chain onto the slider surface. The location of film inlet was measured by means of small scale attached to the side of the slider. This film inlet location measurement does not require too much accuracy

since small deviations from the fully flooded inlet condition result in small differences in the load carrying capacity of the slider.

As load was applied to the slider, a point was reached where the film began to recede upstream from the slider edge. This recession was always accompanied by some flow in the lateral directions giving the familiar periodic striated flow pattern observed by other investigators. The edge of the lens-slider therefore served two purposes: one to stabilize the separating film and therefore force compliance with the planar-flow of the theoretical model; and two to provide a means of locating the film edge. Figure 8 shows the recession of the film from the slider edge and the start of the striations for the case of $N = .5$. With 1.6 grams on the slider the film is separating from the slider edge. With an additional .2 grams, the film begins to recede upstream and the striated flow pattern is initiated. With 3.6 grams on the slider, the striated flow pattern is fully developed.

The experimental data for $N = 0.5$ are plotted in Fig. 9 along with the predictions of the present theory, the Swift-Stieber and the Birkhoff-Hays theories. Along side of each data point is recorded the observed location of film inlet $\bar{\theta}_1$ ($\bar{\theta}_1$ equals 90° corresponds to a fully flooded inlet). In all theoretical predictions, the pressure at inlet is assumed to be zero, and no correction has been made for the weight of the fluid. The data are in best agreement with the present theory. However, agreement becomes gradually poorer when the load on the slider is reduced. For such lightly loaded conditions, the theory underestimates the load carrying capacity of the slider. In Fig. 10 the same experimental data are compared with the theory where the inlet pressure has been corrected for the hydrostatic head ρgh_i ; a small correction has also been applied to account for the weight of the fluid. The agreement between theory and experiment is excellent. Experimental results for the case $(N) = 1.0$ also gave excellent agreement with the theory. For more detailed description of the experiment, discussion of experimental errors and the calculation of load carrying capacity based on the boundary conditions of the present theory, Ref. 2.

In summary, the main findings of this investigation are as follows:

1. In the absence of inertia or gravity effects, the shape of the liquid-vapor

interface of the separating film depends solely on the parameter N .

2. For (N) smaller than approximately 1.34, a stagnation point appears on the liquid-vapor interface at a height of three asymptotic film thicknesses, marking the onset of flow recirculation just upstream of the separation point.
3. The separating film attains its asymptotic character in approximately one clearance thickness; the flow upstream of separation approaches the conditions of Reynolds' flow also in the order of single clearance.
4. The proper boundary conditions is to be used with Reynolds' equation depend on the parameter N .
5. The pressure gradient in the direction of flow just upstream of separation must be positive; a subambient pressure region must exist just upstream of separation.

References:

1. J.C. Coyne and H.G. Elrod Jr., "Conditions for the Rupture of a Lubricating Film," MTI Report 65TR58, December 1965.
2. J.C. Coyne and H.G. Elrod, Jr., "Slider-Bearing Test of a Theory For Film Rupture or Separation" MTI Report 66TR34, June, 1966.

Fig. 1 Physical Situation

Fig. 2 Typical Curves of Film Height to Attachment Height Ratio (h/c) and Fluid Surface-Velocity to Plate-Velocity Ratio (v/U) for $N = -2.0, -1.0, -0.5$, and -0.2 ; in the Absence of Inertia and Gravitation Forces. Stagnation Points (S.P.) are Shows at $c/h_{\infty} = 3.0$

Fig. 3 Theoretical Streamlines and Fluid Velocity-Profile, Upstream of the Separating Film, for $N = -0.5$, in the Absence of Inertia and Gravitation Forces. a: $X = 1.05$; b: $X = 1.08$; c: $X = 1.18$; d: $X = 1.31$; e: $X = 1.00$; f: $X = 0.95$

Fig. 4 Comparison of Bubble-Experiment Data with the Present Theory and also with the Tape Theory which is Modified by Retaining the Demoninator in the Curvature Term

Fig. 5 Non-Dimensional Pressure and Pressure Gradient to be Used as Boundary Conditions with Reynold's Equation. $N = \left(\frac{3\mu U}{T}\right)^{1/3}$ $R = \frac{\rho U h_{\infty}}{\mu}$

Fig. 6 Pressure Profiles for an Infinitely-Wide, Parabolically-Varying Clearance ($h = h_0 + x^2/2r$) where $2r/h_0 = 900$. The Pressure Profiles were Calculated using Boundary-Conditions in Accordance with the Present Theory.

Fig. 7 Sketch of the Apparatus

(a) 1.6 grams

(b) 1.6 + 0.2 grams

(c) 1.6 + 2.0 grams

Fig. 8 Start of the Film Striations for Medium Load, $N = -0.5$
The force of the unloaded slider on the turntable was
1.6 grams. Loads were increased: (a) 1.6 grams (b) 1.8 grams
(c) 3.6 grams

Fig. 9 Gamma (γ) versus Load Data for $N = -0.5$, $\mu U = -0.76$ dynes/cm, compared with the uncorrected theory (solid lines: $\bar{\theta} = -89$ deg., -85 deg. and -80 deg.) and other theories (dashed lines). The inlet film location $\bar{\theta}_1$ is indicated alongside the corresponding data point.

Fig. 10 Gamma (γ) versus Load Data for $N = -0.5$, $\mu U = -0.76$ dynes/cm, compared with the corrected theory (solid lines: $\theta_i = -89, -88, -87, -85$, and -80 Deg.) The first digit "8" of θ_i alongside the data has been omitted.

SECTION 5

TECHNOLOGY II - SEALS

EXPERIMENTAL AND THEORETICAL STUDY OF THE VISCOSEAL

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SUMMARY

Experimental studies were conducted on pressure patterns, gas ingestion, and sealing capacity of viscoseals. Oil, water, and liquid sodium were used as the sealed fluids. Results indicated that when the groove-land pairs connected directly with the pressurized cavity (without an intermediate circumferential groove), an end effect, or ineffective seal length, was evidenced by a sharp decay of pressure along the land leading edge. Results also disclosed that a helically grooved rotor viscoseal has increasing gas ingestion rates with increasing Reynolds number when the viscoseal liquid interface becomes unstable. However, if the helical grooves are in the housing (smooth rotors), gas ingestion could be eliminated by increasing Reynolds number. Secondary grooves on viscoseal land areas improved sealing capacity and reduced power absorption. Sodium was sealed at 300° to 625° F with negligible liquid loss.

From a theoretical analysis, a set of two-dimensional equations (that includes both convective inertia and viscous forces) were formulated for a quasi-two-dimensional flow field in parallel groove geometries which can be used as a mathematical model for the viscoseal. The mathematical model consisted of an infinite smooth flat plate moving relative to a fixed infinite parallel groove-ridge surface with the clearance filled with an incompressible, homogeneous, Newtonian fluid. A modified Reynolds number evolved that is based on a groove-ridge characteristic length parallel to

the smooth plate velocity vector. Application of the modified Reynolds number to published experimental results reveals that a significant improvement (previously attributed to onset of turbulence) in bearing number or sealing coefficient occurred when the modified Reynolds number was near unity; therefore, it is argued that the strong influence of convective inertia, and not turbulence, is the main cause of this improvement. For this reason a turbulent flow model should include the convective inertia effects. The consideration of convective inertia effects reveals that fluid density ρ , aspect ratio b/h_0 , and number of groove-ridge pairs N are additional parameters of pressure generation dependence that are not revealed by creeping flow analyses. The analysis presented herein provides a unified approach which was previously covered by the creeping flow and the semiempirical turbulent flow analyses. No restriction was placed on the net flow; therefore, the analysis applies equally to hydrodynamic seals, bearings, and pumps. An arbitrary groove-ridge shape is also permissible in solving equations. Consideration of convective inertia effects suggests the principle of viscoseal axial pressure generation and attainment of zero leakage.

INTRODUCTION

The viscoseal is a special case of a general class of parallel groove geometries which have many potential applications for use as pressure generation devices. For example, the parallel groove has been applied in spiral groove thrust bearings (ref. 1), spherical and conical spiral groove bearings (ref. 1), helical groove pumps (ref. 2), as well as helical groove seals (viscoseal, ref. 3).

The viscoseal is potentially useful for sealing liquid metals, such as mercury, potassium, and sodium, in space electric-power generation systems that require 1 to 3 years of unattended operation and near zero leakage (refs. 4 and 5). As pointed out in references 5 and 6, the viscoseal has inherent reliability and long life because of the absence of solid surfaces in rubbing contact.

To date, analyses of parallel groove geometry have been limited to creeping flow solutions where convective inertia effects have been neglected. Convective inertia is the fluid mass acceleration due to a spatial velocity change, for example, a change in velocity due to an obstacle in the flow path. This is contrasted to an inertia effect caused by a temporal velocity change, for example, a variation in velocity at a point in a bearing due to shaft runout (which is a function of time). The creeping flow analysis is restricted to flows where the modified Reynolds number (see DISCUSSION) is much less than 1; that is,

$$\text{Re} = \text{Re}_L \left(\frac{c}{L} \right)^2 \ll 1$$

or

$$\text{Re} \lesssim 0.01$$

where

$$\text{Re}_L = \frac{UL}{\nu} = \frac{\rho UL}{\mu}$$

$$\frac{c}{L} < 1$$

Creeping flow analyses for pumps, screw extruders, seals, and thrust and journal bearings have been published by many authors (e. g.,

refs. 1 to 26). Appendix C contains an outline and discussion of the published creeping flow solutions. The main criticism of the creeping flow solution (generally used in the laminar flow regime) is that its validity is restricted to Reynolds numbers below most engineering applications.

A number of semiempirical and turbulent flow formulations have been devised (refs. 4, 18, 27, and 28) for operation in a regime designated as turbulent, but they neglect the convective inertia terms which are important physically. The main criticism of these semiempirical analyses and turbulent flow model is that the existence of the turbulent regime is assumed. Turbulence, however, may not be necessary to explain reported experimental results and may not even exist. Even if turbulence does exist, convective inertia effects are thought to be far more significant. Kettleborough (ref. 29) numerically analyzed the slider bearing with inertia, turbulent, and viscous terms considered. When inertia only was considered, the results were in qualitative agreement with published (turbulent-attributed) slider bearing experimental results. Kettleborough concluded that the turbulence term did not appear to greatly affect the operation of the slider bearing.

Golubiev (ref. 24) applied centrifugal pump similarity laws to the helical groove seal (including operation where the rotor and housing are both grooved). This case considers an inviscid fluid (viscous forces neglected) whereby the pressure generation is found from Bernoulli's equation modified by a geometric constant, where ΔP varies as the square of the velocity. The viscous force cannot be neglected for two reasons: (1) the viscous force causes fluid motion, and (2) the viscous force contributes to maintaining the generated pressure gradient. Unfortunately, the creeping

flow optimum geometry has been extended to be valid for operation beyond the creeping flow regime. Since the convective change is the main driving force, a complete reevaluation of the optimum geometry including the groove-ridge shape must be investigated before conclusions on visco-seal performance can be made. Experiments show that the optimum geometry changes for flow beyond the creeping flow regime (ref. 27).

Several authors (refs. 20 and 21) reported that the viscoseal had a certain ineffective length at the high-pressure end of the seal. This has been called the end effect, and it has been suggested (ref. 20) that this ineffective rotor length be subtracted from the calculated wetted-seal length in order to compare theory and experiment. Muijderman (ref. 1) derived theoretical pressure patterns for the spiral groove geometry on plane surfaces and developed an analytical method of correcting for the end effect. However, pressure patterns in the viscoseal and end-effect alterations of these patterns have not been determined experimentally. Existing experimental data consists of average pressure measurements from which the existence of an end effect is deduced.

Gas ingestion from the low-pressure to the high-pressure end of the seal is reported to have occurred when sealing potassium (ref. 4) and when sealing water (refs. 5 and 27). Although this gas ingestion characteristic may present no problem with a vacuum on the low-pressure end, it is highly undesirable in other potential applications because of contamination and/or detrimental effect on sealing capacity. Reference 24 reports that in attempting to seal a liquid, the viscoseals (combining helical grooves in rotor and housing) worked on an air-liquid emulsion, and reference 25 reports that viscoseals (helical grooves on shaft)

operate with a mixture of oil and gas. These modes of operation could be due to gas ingestion. Leakage observations reported in reference 21 on "seal breakdown" and in reference 4 on "secondary leakage" could result from gas ingestion into a closed cavity.

The objectives of these studies were:

- (1) Study experimentally the pressure patterns in the viscoseal with particular emphasis on the pattern near the seal end (end effect)
- (2) Investigate experimentally gas ingestion and sealing capacity of the viscoseal
- (3) Develop a physical flow model for a theoretical analysis of the viscoseal

The data and analysis contained herein is covered in detail by the authors' papers (refs. 5, 6, and 30).

SYMBOLS

a	ridge or land width
a'	ridge or land width parallel to viscoseal centerline
$\vec{a}$	acceleration vector
b	groove width
b'	groove width parallel to viscoseal centerline
C	specific heat of fluid
C_p	specific heat of fluid at constant pressure
c	clearance between ridge and smooth flat plate, or radial clearance
$\hat{e}_n$	unit normal directed toward axis
$\hat{e}_t$	unit normal tangent to trajectory
F_x	body force in x-direction
F_y	body force in y-direction

F_z	body force in z-direction
G	dimensionless factor, function of helical geometry
g	acceleration of gravity
h_o	step or edge height (or groove depth)
k	thermal conductivity of fluid
L	characteristic length, length of groove-ridge pair in smooth plate relative velocity vector direction
L'	axial length, normal to plate velocity
N	number of helix starts
n	integer
ΔP	pressure differential
P^*	static pressure at reference state
P_c	static cavity pressure
Pr	Prandtl number
Q	net volume flow rate
Q^*	heat source energy
Q_η	volume flow rate normal to plate relative velocity direction
Q_ξ	volume flow rate in plate relative velocity direction
q	fluid velocity in x-z plane
R	radius of rotor
Re	Reynolds number
Re^*	modified or reduced Reynolds number
T^*	temperature at reference state
t	time
U	smooth flat-plate velocity or rotor surface speed
u	velocity in x-direction

V	characteristic velocity in the y-direction
$\vec{V}$	fluid velocity vector
$V_{p, x}$	pressure flow velocity in x-direction
$V_{p, z}$	pressure flow velocity in z-direction
v	velocity in y-direction
w	velocity in z-direction
x	coordinate along ridge-groove
y	coordinate across film (between plates)
z	coordinate across ridge-groove
α	angle between relative velocity direction and parallel groove- ridges or helix angle
ζ	film height
η	coordinate normal to plate relative velocity vector
θ	temperature ratio = $\frac{T - T_S}{T_M - T_S}$
Λ	bearing number
λG	empirical sealing parameter (defined in ref. 5)
μ	absolute viscosity of fluid
ν	kinematic viscosity of fluid
ν^*	kinematic viscosity of fluid at reference state
ξ	coordinate in direction of plate relative velocity vector
ρ	density
ρ^*	density at reference state
σ	shape parameter
Φ	dissipation function
φ	angle traversed in polar plane

∇ vector differential operator Del or Nabla

∇^2 Laplacian operator

Subscripts:

c based on clearance or film thickness

g groove

L based on characteristic length

M moving surface

n normal direction

r ridge or land

S stationary surface

t tangential direction

z across ridge-groove

ϵ plate relative velocity direction

η normal to plate velocity direction (axial length)

APPARATUS AND PROCEDURE

Experimental Study of Pressure Patterns

Figure 1 shows a schematic drawing of the experimental apparatus and viscoseal assembly used to study end effects and pressure patterns. The rotor of the viscoseal is attached to the power input shaft which is supported by externally pressurized gas journal bearings. Axial thrust due to sealed fluid cavity pressure against the rotor is resisted by the externally pressurized gas thrust bearing. The power input shaft is driven by a variable-speed electric drive and step-up transmission. A magnetic pickup monitors the shaft speed. The viscoseal housing is attached to a support shaft and roller bearing assembly, which permits axial adjustment. A pump pressurizes the seal housing cavity and circu-

lates a cooling fluid flow from the reservoir, through a heat exchanger, to the test cavity and then back to the reservoir. This cooling flow is necessary to control fluid film temperature in the viscoseal. The steel rotors have a centerline average surface finish of 20×10^{-6} inch and are mounted within 0.0002 inch of the total indicator reading. The viscoseal housing is constructed from acrylic plastic and contains thermocouples and pressure taps as shown in figure 2. Pressure taps, in a line parallel to the axis, are located at groove edges and midland and midgroove positions; pressure taps are also located near both edges along one groove. Two sets of four pressure taps (90° apart) were used to align the housing with respect to the rotor within 0.0004 inch as determined by calibration. Thermocouples were located flush with the bore and groove root in a line parallel to the axis and at each midgroove and midland position. Figure 2 also gives the groove dimensions used in this evaluation. The housing had a 2.001-inch bore and a $14\frac{1}{2}^\circ$ helix angle. Groove widths of 0.19 inch and land widths of 0.13 inch were provided by using five helix grooves (five starts). These geometric proportions, which are based on the optimum relations given by reference 19, included a 0.015 inch groove depth and a rotor radial clearance of 0.005 inch.

The kinematic viscosity-temperature relations for the mineral oil used were found to be the same before and after use in the test. These values can be found in table I.

Experimental Study of Gas Ingestion and Sealing Capacity

In studies on sealing water (fig. 3), the water pressure and flow to the pressurized cavity (at the viscoseal high-pressure end) were controlled by throttling valves on the inlet and outlet water lines. This arrangement

provided temperature control of the sealed fluid. A transparent housing permitted visual observations on the rate of gas flow out of the pressurized cavity; and, in studies on grooved rotors, a stroboscope permitted observations of the gas ingestion process in the rotating grooves.

Figure 4 is a diagrammatic sketch of the experimental apparatus used for evaluating viscoseals in sodium. The temperature was maintained by a furnace (resistance heaters) surrounding the viscoseal housing. The temperature was monitored by thermocouples attached to the seal housing outer diameter and by one thermocouple submerged in the sodium in the pressurized cavity.

The enclosure surrounding the viscoseal assembly is pressurized with argon (after prior evacuation) to slightly above ambient to insure exclusion of air. Prior to the introduction of liquid sodium, the input shaft is set at some fixed speed. Sodium is introduced by pressurizing the reservoir and venting the leak detector tank (see sodium supply system in fig. 5). The sodium first fills the pressurized cavity (viscoseal high-pressure end) and then is allowed to fill the leak detector tank to a predetermined level. The leak detector tank float displacement is monitored by recording the differential transformer output that is produced by the transformer core attached to the top of the float. Thermocouples placed at different levels in the tank provide calibration during the filling process.

By holding the reservoir temperature at 220° F, a low degree of oxide solubility was maintained. The 20-micron filter (stainless steel) inside the tank allowed 220° F filtration. A second filter external to the reservoir (5 microns) provides additional filtering at approximately 300° F. Sodium is partially removed from the system by pressurizing the leak detector and

forcing the sodium back into the reservoir. By alternately filling the system and then partially returning the sodium to the reservoir, the sodium can be recirculated and refiltered. This recirculation provides a hot flush or cleaning action on the viscoseal assembly before a run. (All transfer lines and the test section are held at 300° to 500° F during this recirculation process.) For all sodium studies the leak detector tank was held at 300° F, and any seal leakage resulted in a drop of liquid level and float position. Gas ingestion into the pressurized cavity was indicated by a rise in sodium level in the leak detector.

Sealing capacity, when sealing sodium, was obtained by increasing cavity pressure until the leak detector indicated leakage. Line thermocouple readings were also found to be sensitive leak indicators. Viscoseal geometries employed in this evaluation are shown in table II. The geometry near the optimum (derived in ref. 19) was taken as the basis for comparison, and is listed in the first row of table II as geometry 1. Secondary grooves were added to the lands of this basic geometry to form the second geometry evaluated. For the third geometry, a ratio of groove to land width of 1.90 was selected; and for the fourth geometry, secondary grooves were added to geometry 3. Geometry 5 is the internally grooved housing having the same groove dimensions as geometry 1. The rotor outer diameters were 1.993 inches, and the housing bores were 2.001 inches.

EXPERIMENTAL RESULTS AND DISCUSSION

Pressure Patterns

Figure 6 shows an experimentally determined pressure pattern in a viscoseal composed of an internal helically grooved housing and a smooth

rotor arrangement. In a plane orthogonal to the rotor centerline, the pressure increases across the groove (from point A to B) in the direction of rotor rotation and decreases across the land (from point B to C). The increase and decrease in pressure repeat for each groove-land pair and produce a saw-tooth pressure profile in the orthogonal plane. The pressure increases from the low-pressure end (plane 3) to the high-pressure end (plane 1), and the pressure pattern has a helical twist corresponding to the helical grooves. (For a rotating grooved shaft this pressure pattern would be rotating.)

The pressure gradients in the planes orthogonal to the rotor centerline are essentially linear for axial positions not influenced by the end effects. Figure 7 shows typical experimental results. These pressure profiles are the same as those for the orthogonal planes shown in figure 6 except that a single groove-land pair is unwrapped to form a plane figure. (Neglecting curvature is not a significant error since the ratio of rotor radius to groove depth is 65 to 1.)

In these orthogonal planes, the ratio of pressure to cavity pressure P/P_c increases linearly over the groove and then decreases linearly over the land. This linear relation was found to hold to the maximum test speed of 6000 rpm, which produced a modified Reynolds number (Re^*) of 0.25.

End Effect

When the helical grooves connect directly with the pressurized cavity, the pressure developed along the land leading edge decays sharply near the high-pressure end of the seal. Typical results are shown in figure 8 for 1000 and 6000 rpm. The pressure pattern at the end was essentially the same for the speed range investigated (1000 to 6000 rpm) except

that the results for 5000 rpm (not shown) and 6000 rpm start to show a slight shortening of the end-effect length. This shortening is probably due to convective inertia effects. In all cases, the end-effect axial length was approximately equal to one-half the axial groove width. This suggests that end-effect length could be taken to one-half groove width without significant error. The pressure decay within the groove is due to the equalization of the land leading- and trailing-edge pressure, since a common pressure must exist in the cavity.

Gas Ingestion and Sealing Capacity Observations

When Sealing Water

Grooved housing. - The series of photographs in figure 9 shows the gas ingestion process when sealing water with a grooved housing and smooth outer diameter rotor. At a modified Reynolds number (Re^*) of 1.75 (2000 rpm), no gas ingestion is evident, the water film is clear, the interface is stable, and the scavenging length is nonwetted.

At $Re^* = 2.58$ (3000 rpm) the film had large gas pockets, gas was ingested as evidenced by air bubbles passing out of the transparent vent line, and the sealing capacity (parameter, λG) was lower than that at $Re^* = 1.75$ (2000 rpm). Various degrees of gas ingestion and film rupture are evident for $Re^* = 3.50$ (4000 rpm) to 6.10 (7000 rpm). At $Re^* = 7.00$ (8000 rpm), no detectable gas ingestion rate was evident, and gas ingestion was not detectable from $Re^* = 7.00$ (8000 rpm) to the highest speed evaluated, which gave $Re^* = 10.84$ (12 000 rpm). At $Re^* = 7.00$ and above, the seal wetted length had two distinct regions (fig. 10(f)): a gas-liquid region, and a region of homogeneous liquid at the high-pressure end.

Between $Re^* = 3.5$ and 6.10 , it was observed that the bubble size progressively decreased when moving from the low-pressure end to the high-pressure end. It is probably that with longer seal lengths the corresponding higher pressures would reduce the bubble size sufficiently to allow escape over the lands, and thus gas ingestion would be eliminated. It was observed that concentricity of rotor and housing markedly affect gas ingestion occurrence.

Grooved rotor. - In a series of visual observations using a grooved rotor, the gas ingestion started at $Re^* = 1.9$ and increased in rate with increasing Reynolds number (maximum investigated, $Re^* = 10.84$). Below $Re^* = 1.9$ the liquid-to-gas interface was stable and no gas ingestion was observed.

Gas ingestion mechanism. - These observed gas ingestion characteristics of the grooved rotor and the grooved housing are illustrated in figure 10. The probable mechanism producing the difference between the gas ingestion process of the grooved rotor as compared with that of the grooved housing is illustrated in figure 11. For the grooved housing, when the centrifuge action on the liquid becomes great enough, the gas bubbles are displaced to the rotor surface and pass over the lands. Therefore, the bubbles are not pumped to the high-pressure end. For the grooved rotor, the centrifugal action on the liquid forces the gas bubbles into the grooves; therefore, the bubbles are pumped to the high-pressure end of the seal.

When a viscoseal is ingesting gas, the axial pressure gradient is non-linear and increases as the liquid-to-gas volume ratio increases in the direction toward the high-pressure end. For no gas ingestion, the axial pressure gradients are linear as shown in figure 8, which contains data

on sealing oil. As shown in figure 12, gas ingestion results in nonlinear pressure gradients (increasing slope with increasing liquid-gas volume), which are significantly different from the gradients shown in figure 8.

Leak rate. - In both viscoseal combinations (grooved housing and grooved rotor) water was sealed, with no detectable leakage. It was observed that, although gas ingestion reduced the sealing capacity λG , effective sealing of the water was maintained.

Results of Sodium Sealing Studies

Grooved housing. - In sealing sodium, the grooved housing did not ingest gas (fig. 13) for concentric alinement (0.005 in. F.I.R.) over the range investigated ($Re^* = 8.66$ to 50.5) and for pressures ranging from 2 to 100 psig. The sensitivity of the leak detector to measure gas ingestion rate was estimated to be 0.50 cubic centimeter per hour. It was observed that nonconcentricity leads to gas ingestion, but this situation was not investigated.

Grooved rotor. - The grooved rotor started to ingest gas at $Re^* = 10$, and the rate increased with increasing Reynolds number, as shown in figure 13. This ingestion characteristic is similar to that observed when sealing water with the grooved rotor.

Gas ingestion into closed cavity. - In sealing sodium, gas ingestion was readily observed by monitoring the liquid level in the leak detector. As shown in figure 14, a straight line on the oscillograph paper indicated no gas ingestion, no leakage, and thermal equilibrium. When gas ingestion occurred, the liquid level line showed an upward trend, the slope of which is the ingestion rate. As gas ingestion proceeds, the gas bubble in the pressurized cavity increases in size (fig. 14(a)), and when the size

increases to the rotor outer diameter, the gas flows back out the annulus formed by the rotor and housing. This causes a decrease in bubble size and a sharp drop in the leak detector readout trace. If the interface is far enough away from the seal low-pressure end, the liquid will be scavenged back within the nonwetted length, and no loss of liquid will occur. The ingestion process will begin again and the net result is a saw-toothed trace, each sharp drop indicating gas blowback. If the interface is sufficiently close to the seal end, liquid loss will occur, and the leak detector readout will show a saw-toothed trace with a downward trend, which represents a liquid loss at each blowout (fig. 14(c)). This latter action may be the "seal breakdown" phenomenon reported in reference 21 and the "secondary leakage" phenomenon reported in reference 4. (In sealing water, the repeating bubble growth and subsequent seal blowout was visually observed by means of a transparent housing.)

Sealing capacity. - The grooved housing had a higher sealing capacity (higher sealing parameter, λG) than the grooved rotor. The comparison is given in figure 15, which gives the sealing parameter λG as a function of Reynolds number based on c (Re_c). The difference between the value of λG for the grooved housing and the value for the grooved rotor was attributed to the difference in gas ingestion characteristics. The grooved rotor operates with a gas-liquid mixture along the full wetted length similar to that shown in figures 9(b) to (e); the grooved housing operates with a homogeneous liquid at the high-pressure end of the seal and a gas-liquid mixture at the low-pressure end similar to that shown in figure 9(f).

Figures 16(a) and (b) show the sealing parameter λG as a function of Reynolds number (Re_c) for groove configurations of five- and ten-helix

starts (see table II) with and without secondary grooves on the lands. In both cases, a higher sealing coefficient was obtained through the use of secondary grooves. Secondary grooves also operate at lower power absorption levels for equal sealing capacity, as was determined by the amount of heating power required to maintain operating temperature. This lower power absorption is due to less land area as compared with the land area of the seals without secondary grooves.

Sodium was sealed for pressures ranging from 2 to 100 psig with negligible leakage loss. The operating range was within Reynolds numbers (Re^*) of 8.66 to 50.5, and the fluid temperature was in the range 300° to 625° F. The maximum length of any one run was 8 hours and accumulated testing time amounted to 50 hours for 10 runs. The oxygen content of the sodium for the 10 runs varied between 50 to 95 parts per million, as determined by the analytical method described in reference 31.

THEORETICAL ANALYSIS OF THE VISCOSEAL

Basic Model

The rectilinear Cartesian coordinate system was selected for mathematical tractability. A parallel groove-ridge plate geometry with a smooth flat plate moving relative to the parallel groove-ridge plate, both of infinite horizontal extent, was chosen as the basic model (see fig. 17).

Conceptually this model can be thought of as an infinitely long smooth rotor with a very large diameter and a concentric grooved housing. Actually this infinite grooved plate model is the limiting case of a helical grooved cylinder where $R \rightarrow \infty$ or $c \rightarrow 0$. If $c/R \lesssim 0.01$, this model should give a good qualitative picture of the flow field and pressure generation and significant trends for optimum geometry. (See argument in

appendix A.) For example, a practical geometry is a 2-inch-diameter rotor with a 0.005-inch concentric clearance, $c/R = 0.005$.

The clearance between the infinite plates is filled with any incompressible fluid, and it should be noted that for this infinite horizontal extent case there are no end effects (in practical engineering design, of course, end effects must be accounted for). Since experiments at Lewis show that the end effect zone decreases with increasing rotor speed and also that it is less than a groove width, this analysis should be valid in the bulk of the homogeneous fluid (see figs. 18 and 19).

In the flat-plate case, the boundary conditions on a groove-ridge plate in motion are constant; however, it will be advantageous to have the smooth flat plate move relative to the grooved flat plate. This will eliminate the temporal inertia terms and reduce the subsequent independent variables by one.

A word of caution for extension of the results to a case where the groove-ridge pairs on a finite shaft are rotating relative to a fixed smooth housing. The boundary conditions are different when the smooth housing is stationary and the grooved rotor is moving, and when the housing is grooved and the rotating shaft is smooth. When the rotor is grooved, the surface speed is different at each point along the step or edge of the moving boundary. Thus this analysis should be more valid for a smooth rotor and grooved fixed housing (from ref. 5 the grooved housing appears to be the desired mode of operation) with a large radius of curvature and/or small clearance.

When referring to figure 17, it is noted that by the choice of coordinate system the groove-ridge pairs appear only in the y, z -plane. Later it will

be shown that this choice of coordinate system will be necessary to reduce the analysis to a two-dimensional problem for the quasi two-dimensional flow field. It should also be noted that the drag force has been resolved into its components in this coordinate system and there is no equilibrium of forces in the z-direction with inertia (nonlinear) effects neglected.

BASIC EQUATIONS

The Navier-Stokes equations for a homogeneous, incompressible, laminar, Newtonian fluid are the following (see ref. 32):

x-direction:

$$\rho \frac{Du}{Dt} = F_x - \frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (1)$$

y-direction:

$$\rho \frac{Dv}{Dt} = F_y - \frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (2)$$

z-direction:

$$\rho \frac{Dw}{Dt} = F_z - \frac{\partial P}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (3)$$

The Eulerian or spatial derivative is D/Dt , which is composed of the local acceleration (temporal velocity change) and the convective acceleration (spatial velocity change):

$$\frac{D}{Dt} = \underbrace{\frac{\partial}{\partial t}}_{\text{Local, unsteady, or nonstationary acceleration}} + \underbrace{u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}}_{\text{Convective acceleration}}$$

The incompressible continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (4)$$

The incompressible energy equation with constants C and k and neglecting compression work DP/Dt (ref. 32)

$$\rho g C \frac{DT}{Dt} = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \mu \Phi + Q^* \quad (5)$$

where Φ is the dissipation function

$$\begin{aligned} \Phi = 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] &+ \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)^2 \\ &+ \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)^2 \end{aligned}$$

The assumptions for this analysis are as follows:

(1) For steady flow,

$$\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = \frac{\partial w}{\partial t} = 0$$

(2) For no body forces,

$$F_x = F_y = F_z = 0$$

This means there are no electromagnetic fields present and gravitational effects are negligible.

(3) Isothermal and equal temperature plate surface conditions are assumed.

(4) Viscous (frictional) heating is negligible. Therefore, the thermo-physical properties are constant. This appears to be a good assumption for order 10 (water) and smaller (liquid metals) Prandtl number Pr fluids:

$$Pr = \frac{C_p \mu}{k}$$

In experimental setups there is an inherent unsteadiness due to vibrations, shaft runout, and eccentricity of rotor with respect to housing and misalignment. These factors will influence the degree of correlation between the analytical and experimental results.

The conservation of momentum and mass equations were nondimensionalized and a formal ordering procedure was used to determine the relative magnitude of the terms. Consequently, in a formal way the negligible terms were found (see appendix A).

An important parameter, the modified or reduced Reynolds number, was found:

$$Re^* = Re_L \left(\frac{c}{L} \right)^2 = Re_c \left(\frac{c}{L} \right) \quad \text{if } c/L < 1$$

This modified Reynolds number gives the relative magnitude of the inertia forces to the viscous forces; that is,

$$Re^* = \frac{\text{Inertia forces}}{\text{Viscous forces}}$$

Thus, inertia forces can be neglected only if $Re^* \lesssim 0.01$.

After the formal ordering procedure (see appendix A), the three-dimensional flow field equations are as follows:

Conservation of momentum:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \quad (\text{x-direction}) \quad (6)$$

$$\frac{\partial P}{\partial y} \cong 0 \quad (\text{y-direction}) \quad (7)$$

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = - \frac{1}{\rho} \frac{\partial P}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (\text{z-direction}) \quad (8)$$

Conservation of mass:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (4)$$

Fully developed flow is assumed in the x-direction. This means that $\bar{V}$ and $\partial P / \partial x$ are not functions of x and that the pressure gradient in the x-direction (parallel to the groove) is a constant (verified experimentally in ref. 5). In figure 18, it is observed that this assumption is valid only where the average pressure profile at the ridge leading or trailing edges in the axial direction is linear. Gas ingestion results in a nonlinear profile (see figs. 18 and 19).

With the assumption of fully developed flow along the groove-ridge direction (x-direction), the flow field equations become two-dimensional:

$$v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \quad \text{where} \quad \frac{\partial P}{\partial x} = \text{constant} \quad (9)$$

$$\frac{\partial P}{\partial y} \cong 0 \quad (10)$$

$$v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = - \frac{1}{\rho} \frac{\partial P}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (11)$$

Conservation of mass:

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (12)$$

The boundary conditions for an arbitrary shaped groove-ridge geometry (see fig. 20) are:

(1)

$$\left. \begin{array}{l} v = 0 \\ u = U \cos \alpha \\ w = -U \sin \alpha \end{array} \right\} \text{ at } y = 0$$

(2)

$$u = v = w = 0 \quad \text{at } y = h(z)$$

For example, when $h(z)$ is a rectangular groove-ridge pair the boundary condition is:

$$u = v = w = 0 \quad \text{at } c \leq y \leq c + h_0 \quad (\text{on side of groove})$$

$$u = v = w = 0 \quad \text{at } y = c \quad (\text{on ridge})$$

$$u = v = w = 0 \quad \text{at } y = c + h_0 \quad (\text{on groove root})$$

(3) Periodicity of groove-ridge pairs at the clearance interface.

For $0 \leq y < h(z)$

$$v(0, y) = v(L \sin \alpha, y)$$

$$w(0, y) = w(L \sin \alpha, y)$$

$$P(0, y) = P(L \sin \alpha, y) - (\Delta P_{r+g})_z$$

$$\text{where } (\Delta P_{r+g})_z = \text{const}$$

All velocity derivatives must also be the same to satisfy the periodicity boundary conditions

$$\frac{\partial^{n+m} v(0, y)}{\partial y^n \partial z^m} = \frac{\partial^{n+m} v(L \sin \alpha, y)}{\partial y^n \partial z^m}$$

$$\frac{\partial^{n+m} w(0, y)}{\partial y^n \partial z^m} = \frac{\partial^{n+m} w(L \sin \alpha, y)}{\partial y^n \partial z^m}$$

for $0 \leq n, m \leq \infty$

(4) Boundary condition for $\partial P / \partial x$ for a cylindrical geometry (visco-seal).

In appendix B it is shown that for a cylindrical geometry $\partial P / \partial x$ depends on the (ΔP_{r+g}) obtained in the z-direction

$$\frac{\partial P}{\partial x} = \frac{N(\Delta P_{r+g})_z}{2\pi R \cos \alpha}$$

These equations (9), (11), and (12) can be solved for the velocity components. The velocity components can be obtained for arbitrary $(\Delta P_{r+g})_z$. Then the axial pressure gradient $\partial P / \partial L'$ can be found from

$$\frac{\partial P}{\partial L'} = \frac{\Delta P_{\text{axial}}}{L'} = \frac{(N)(\Delta P_{r+g})_z}{L} \cot \alpha$$

where L' is the axial length (see fig. 21). Qualitative pressure distribution from experimental data in the x- and z-directions is shown in figures 22 and 23.

By examining the analysis, boundary conditions, and the derived equations, the axial pressure gradient $\partial P / \partial L'$ is a function of (σ, U, ρ, ν) , where σ is a shape parameter that is a function of $(b/h_o, c/h_o, a/b, \alpha)$.

Since convective inertia effects are important, new geometric parameters appear which are not implied in creeping flow theory. Most notable of these is the aspect ratio b/h_0 , which implies the degree of convective effect. When the b/h_0 and a/b ratios are known, N , the number of helix or groove starts, can be found. (Direct effect of N is seen from boundary condition (4).) Thus the number of groove or helix starts also is an important parameter. Since inertia is important, a high density is desired for pressure generation.

The equations (9) to (12) and boundary conditions are the minimum number of terms that are necessary to mathematically describe the physics of the internal flow. This is the simplified form of the Navier Stokes equations which should be used for an analytical solution; however, a mathematical function to describe the groove-ridge shape may be too complex for a closed-form analytical solution. Thus the equations are solvable by using high speed digital computer numerical methods. Since numerical schemes can accommodate all two independent variable terms with equal ease, it is suggested that the complete set of two-dimensional equations be solved numerically, i.e.,

$$v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = - \frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (13)$$

$$v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = - \frac{1}{\rho} \frac{\partial P}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (11)$$

$$v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad \text{where } \frac{\partial P}{\partial x} = \text{constant} \quad (14)$$

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (12)$$

Of course, the numerical solution will determine under what conditions the previously derived simplified form of the two dimensional Navier Stokes equations are good approximations.

By examining the set of equations (11) to (14) it is seen that v and w can be calculated from equations (11), (12), and (13). The solution is identical to a two-dimensional flow case. Then the u flow field can be found from equation (14) which is linear in u . This is an example of the Independence Principle.

From appendix A the energy equation with no temperature variation along the groove-ridge direction ($\partial T / \partial x = 0$) is

$$v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} = \frac{\nu}{Pr} \left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) + \frac{\mu}{\rho g C} \Phi + \frac{Q^*}{\rho g C} \quad (15)$$

Comparing the above equation with the x-direction momentum equation it is seen that the mathematical form of both equations is identical.

Substituting $\theta \Rightarrow u$

and

$$\frac{\mu}{\rho g C} \Phi \quad \text{or} \quad \frac{Q^*}{\rho g C} = \text{a constant} \Rightarrow -\frac{1}{\rho} \frac{\partial P}{\partial x}$$

Thus with $Pr = 1$ and the above substitutions, the equations (14) and (15) are identical. The thermal boundary conditions are

At $y = 0$

$$\theta = \text{constant} \Rightarrow w = -U \sin \alpha = \text{constant}$$

At $y = h(z)$

$$\theta = 0 \Rightarrow w = 0$$

and the boundary conditions are similar in form.

The above statements mean that once the u velocity field is found

the solution will also give the temperature distribution for the stated restrictions of constant frictional heating or a heat source in the fluid with the appropriate physical parameters equivalent to the constant pressure gradient along the groove-ridge pair direction and $Pr = 1$. Note that this temperature distribution solution violates the initial assumption of constant thermophysical properties; however, for small temperature differences the above solution will give a good qualitative picture of thermal effects.

DISCUSSION

Discussion of Equations

This analysis is valid only for a homogeneous fluid and figures 18 and 19 clarify the region of validity. The analysis is not valid for a scavenging area and a region consisting of a mixture of gas and liquid (see fig. 18) or under conditions of gas ingestion.

It should also be pointed out that the analysis is not valid in the end effect region at the high pressure end of the seal (see fig. 18) where the groove connects with the cavity. Since the appropriate equations are non-linear, the pressure is not a harmonic function; thus, the usual analog methods of finding end effect are not available as found in creeping flow regions (see appendix C).

The equations are unique in many ways. Mathematically, the problem has been reduced to two dimensions (y- and z-directions) but all three velocity components are present. Thus the flow field can be described as quasi-two-dimensional. Also, there is a pressure gradient in the third dimension (x-direction), but fortunately this gradient is a constant and dependent on the y- and z-direction gradients.

The velocity component v is no longer negligible as it was in the creeping flow solution. Physically, v cannot be negligible because of the presence of the edge effect which will result in a component of velocity in the y -direction.

Since there is no restriction on net volume flow, the pressure generation prediction equations should equally apply to hydrodynamic seals, bearings, and pumps.

When further examining the physics of the problem, it is seen that across the groove-ridge pairs the unbalance of the convective acceleration force with the drag force results in a pressure gradient in the z -direction. Since the step or edge causes the convective acceleration, its effects are not negligible unless there is a very large aspect ratio and the mathematical model accounts for the edge effect. Along the groove and ridge (x -direction) the drag force is in equilibrium with the induced pressure and convective inertia force. The drag force not only causes fluid motion but also aids in maintaining equilibrium with the pressure along the groove or ridge.

Both equations of motion illustrate the coupling of u , v , and w in the convective inertia terms. This means that physically a condition such as no end leakage is possible. In creeping flow there is no coupling between the velocity components, which illustrates the shortcoming of that analysis (i. e., end leakage is not zero).

The assumption that the inertia terms are of the same order as the viscous terms results in equations of the boundary layer type; however, the boundary conditions are different.

The equations are of a form that might tempt one to reduce the number of independent variables by one by trying to find the admissible flow field which would result in a similarity transform solution. This technique fails because of the characteristic lengths that are present in this type of confined flow.

The equations are solvable by using high-speed digital computer numerical methods. A mathematical function to describe the groove-ridge shape may be too complex for a closed-form analytical solution. It should be interesting to find out if the step is still the optimum shape with convective inertia effects included. For creeping flow, Lord Rayleigh found in 1912 that the step was the optimum shape by using calculus of variations.

The equations and ordering procedure should be reexamined carefully before extending these results beyond the assumptions used in this analysis.

Discussion of Modified Reynolds Number

A check was made on the significance of the modified or reduced Reynolds number by calculating Re^* from the available published data at the claimed onset of turbulence (breakpoint). In table III it can be noted that the departure in all cases from a constant sealing coefficient occurred when Re^* was in the range of 0.26 to 2.2, which suggests that the so-called beginning of turbulent operation is really the beginning of significant convective inertia effects. This would be in agreement with Kettleborough (ref. 29), who found qualitative agreement between his inertia solution and experimental turbulent results.

For seals 5, 6, and 7 (see ref. 27 and table III), only the groove to ridge ratio has changed. As the ratio increases, the breakpoint decreases.

This is reasonable since a larger ridge area results in a less influential convective inertia effect and delays the beginning of pronounced convective inertia effects. It can also be seen in table III that the calculated Re^* are within an order of magnitude of one another for all of the experimenters. It is difficult, nevertheless, to select the breakpoint (see fig. 24). Even in a range of possible breakpoint numbers, however, the order of magnitude will remain the same. Further experimental results will be necessary, but in general, it can be concluded that if $Re^* > 1$, a constant sealing coefficient or bearing number will not exist.

In figure 25 it is seen that classically the empirical sealing parameter λG was thought to be a constant until some critical Re_c and then to vary exponentially in the turbulent regime. The creeping flow solution was thought to be valid until the critical Re_c and then a semiempirical turbulent theory had to be used. Actually, the onset of significant convective inertia may be gradual, as shown in figure 25. Data (from ref. 27) for seal 7 in figure 24 tends to show this. The derived equations herein pertain to all cases under assumptions stated and will break down only under strong turbulence or an unknown instability phenomena. However, even in the turbulent flow model the convective inertia terms must be considered in addition to the Reynolds' stress terms. This is specifically shown by Kulinski and Ostrach (ref. 33) in their critical evaluation of high speed fluid film lubrication theory.

Principle of Viscoseal Operation

The classical explanation of viscoseal operation is now described. Since the seal is "pumping" the sealed fluid as fast it is leaking out, there is no net end flow (zero leakage). From the basic laws of fluid mechanics,

however, it is seen that the viscoseal axial pressure gradient results from the unbalance of the convective inertia force across the groove-ridge pairs and the drag force. The convective inertia force is much greater than the opposing drag force, and this results in a net pressure force. Along the groove-ridge direction, the drag force maintains equilibrium with the pressure force and convective inertia force. All three velocity components are "coupled," and thus zero net leakage is achieved. This is an excellent example of the physical importance of a nonlinear effect. The inadequacy of the linear theory (creeping flow) has been previously discussed.

SUMMARY OF RESULTS

Pressure Patterns

Pressure patterns were studied for a viscoseal in the laminar-flow regime with a mineral oil and the following results were obtained:

1. For the condition of helical grooves in direct connection with the pressurized cavity, the end effect is evidenced by a sharp decay of land leading-edge pressure.
2. No significant change in end-effect length was noted over the speed range investigated (modified Reynolds numbers (Re^*) of 0.04 to 0.25).
3. Circumferential pressure gradients in the plane of rotation (plane orthogonal to η -axis) increase linearly across the groove and decrease linearly across the land. This pressure pattern repeats for each groove-land pair and provides a saw-tooth pressure profile around the circumference. The pressure gradients along the groove and land were linear.

Gas Ingestion and Sealing Capacity

Visual observations and experimental evaluation of viscoseal assemblies with water (modified Reynolds numbers of 1.75 to 10.84) and sodium

(modified Reynolds numbers of 8.66 to 50.50) as sealed fluids disclosed the following:

1. Gas ingestion characteristics were fundamentally different for the helically grooved rotor-smooth bore housing combination. In the grooved housing-smooth rotor combination, centrifugal action tended to hold the gas bubbles out of the groove; thus the bubbles tended to escape over the lands rather than be pumped into the seal high-pressure end and a condition of no gas ingestion could be achieved. In the grooved rotor-smooth bore housing combination, centrifugal action forced the gas bubbles into the rotor grooves; thus the gas bubbles were pumped into the seal high-pressure end and increasing speeds resulted in increasing gas ingestion.

2. In sealing sodium, the grooved housing-smooth rotor combination had no detectable gas ingestion rate, but the grooved rotor-smooth bore housing combination showed increasing gas ingestion with increasing Reynolds number.

3. No gas ingestion occurred when sealing water with either the grooved rotor-smooth bore housing or the grooved housing-smooth rotor viscoseal when the liquid interface was stable (below Reynolds number (Re^*) of 1.9).

4. In sealing sodium, the grooved housing-smooth rotor combination produced a higher sealing capacity (higher sealing parameter) than the grooved rotor-smooth bore housing combination. This was attributed to the presence of gas ingestion in the case of the grooved rotor.

5. Sealing, with negligible sodium leakage rate, was obtained for short operational periods (maximum length of continuous evaluation was

8 hr) over a pressure range of 20 to 100 lb force/sq in. gage and at sodium temperatures of 300° to 625° F.

Theoretical Analysis of the Viscoseal

A review of the literature on the theoretical analysis of the viscoseal revealed that either the convective inertia forces or the viscous forces were neglected (inviscid fluid assumption). This analysis included both convective inertia and viscous forces. The following two-dimensional set of equations for a quasi-two-dimensional flow field were formulated and are solvable for the velocity and pressure distribution and optimum geometry utilizing numerical methods on a high-speed digital computer:

Conservation of momentum:

$$v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = - \frac{1}{\rho} \frac{\partial P}{\partial z} + \nu \frac{\partial^2 u}{\partial y^2} \quad \text{where} \quad \frac{\partial P}{\partial x} = \text{constant} \quad (9)$$

$$\frac{\partial P}{\partial y} \cong 0 \quad (10)$$

$$v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = - \frac{1}{\rho} \frac{\partial P}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (11)$$

Conservation of mass:

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (12)$$

With the following boundary conditions

(1) At the moving smooth plate surface,

$$u = U \cos \alpha \quad v = 0 \quad w = -v \sin \alpha$$

(2) At the fixed parallel groove geometry surface,

$$u = v = w = 0$$

(3) Periodicity of groove-ridge pairs at the clearance interface.

For $0 \leq y < h(z)$

$$v(0, y) = v(L \sin \alpha, y) \quad \text{and} \quad \frac{\partial^{n+m} v(0, y)}{\partial y^n \partial z^m} = \frac{\partial^{n+m} v(L \sin \alpha, y)}{\partial y^n \partial z^m}$$

$$w(0, y) = w(L \sin \alpha, y) \quad \text{and} \quad \frac{\partial^{n+m} w(0, y)}{\partial y^n \partial z^m} = \frac{\partial^{n+m} w(L \sin \alpha, y)}{\partial y^n \partial z^m}$$

$$P(0, y) = P(L \sin \alpha, y) - (\Delta P_{r+g})_z \quad \text{for} \quad 0 \leq n, m \leq \infty$$

(4)

$$\frac{\partial P}{\partial x} = \frac{N(\Delta P_{r+g})_z}{2\pi R \cos \alpha}$$

From the development and analysis of the equations, the following results and conclusions can be made:

1. A modified or reduced Reynolds number evolved which gives the relative magnitude of the convective inertia forces to the viscous forces:

$$Re^* \equiv Re_L \left(\frac{c}{L} \right)^2 \equiv Re_c \left(\frac{c}{L} \right) \quad \text{if} \quad \frac{c}{L} < 1$$

The characteristic length was selected to be the length of the groove-ridge pair in the direction of the smooth plate velocity vector. This is the principal length over which the driving force acts. The flow behavior over each groove-ridge pair is the same. Convective inertia effects can only be neglected if $Re^* \ll 1$, which will result in true creeping flow.

2. A modified Reynolds number equal to 1 will predict a significant change in sealing coefficient or bearing number which heretofore was attributed to the onset of turbulence. This means that the convective inertia effects are the foremost contributing factor to the improvement in sealing

coefficient or bearing number, not the turbulence effects. A calculation of the modified Reynolds number at the onset of turbulence of four different experimenters disclosed that the modified Reynolds number was near 1 ($Re^* = 0.25$ to 2.2). Thus a turbulent flow analysis must also include the convective inertia terms in addition to the appropriate Reynolds stress terms.

3. Several additional parameters of pressure generation dependence were found that are not implied in the creeping flow analysis. These additional parameters, which arise from convective inertia effects, are the fluid density ρ , the groove width to depth or aspect ratio b/h_o , and the number of groove-ridge pairs N .

4. Solving equations (11) to (14) by numerical analysis will permit the investigation of arbitrary groove-ridge shape, for example, sinusoidal, rectangular, triangular, etc. Now a check can be made to see if a step is the optimum shape with convective inertia effects as it is for creeping flow.

5. The analysis provides a unified approach to parallel groove operation previously covered by creeping flow (sometimes called laminar flow) analysis and a semiempirical turbulent flow analysis.

6. No restriction was placed on the net flow situation between the plates. Therefore, the analysis can be used to analyze parallel groove geometries in hydrodynamic seals, bearings, and pumps.

7. The viscoseal axial pressure gradient results from the unbalance of the convective inertia force across the groove-ridge pairs and the drag force. Along the grooves or ridges, the drag force maintains equilibrium with the pressure force and the convective inertia force.

8. Zero leakage in viscoseal analysis can be attained through the consideration of convective acceleration terms (nonlinear effect). It is argued that the creeping flow analysis, which is a linear theory, cannot prove zero leakage.

9. The format of this analysis can be used to evaluate qualitative convective inertia effects in any step or converging wall hydrodynamic seal, pump, or bearing (Rayleigh step, journal bearing, etc.).

10. Solution of the u velocity (along the groove-ridge direction) will give an insight to the heat transfer in the viscoseal for the case when $Pr = 1$.

APPENDIX A

FORMAL ORDERING PROCEDURE FOR SIMPLIFYING

BASIC EQUATIONS

The Navier-Stokes equations for a homogeneous incompressible Newtonian fluid, neglecting body forces and for steady flow, are

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = - \frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (A1)$$

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = - \frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (A2)$$

$$\rho \left(u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = - \frac{\partial P}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (A3)$$

The incompressible continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (A4)$$

To find the important terms in the previous flow field equations, a

formal ordering procedure will be used to determine the relative magnitudes of the terms.

The terms are normalized by scaling them to their appropriate maximum characteristic value as follows: let

$$\begin{aligned}\bar{x} &= \frac{x}{L \cos \alpha} & \bar{y} &= \frac{y}{c} & \bar{z} &= \frac{z}{L \sin \alpha} \\ \bar{u} &= \frac{u}{U \cos \alpha} & \bar{v} &= \frac{v}{V} & \bar{w} &= \frac{w}{U \sin \alpha}\end{aligned}$$

The characteristic dimensions c and L can be seen in figure 17. It is noted that a trigonometric relation exists between x and z characteristic lengths and that $c/L < 1$. The characteristic length was selected to be the length of the groove-ridge pair in the direction of the smooth plate velocity vector. This is the principal length over which the driving force acts. The flow behavior over each groove-ridge pair is the same.

Let ρ^* and ν^* be the fluid density and viscosity corresponding to a reference state (P^*, T^*) . Therefore,

$$\bar{\nu} = \frac{\nu}{\nu^*} = 1$$

and

$$\bar{\rho} = \frac{\rho}{\rho^*} = 1$$

for incompressible fluids with constant thermophysical properties. Now to find the relative magnitude of V so that all terms of the incompressible continuity equation will be of the same order, the incompressible continuity equation is nondimensionalized:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{V}{U} \frac{L}{c} \frac{\partial \bar{v}}{\partial \bar{y}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \quad (\text{A5})$$

To have all terms of the equation the same order of magnitude, $\frac{V}{U} \frac{L}{c}$ must be of the order of 1. Therefore, $V = U \frac{c}{L}$, and thus $\bar{v} = \frac{v}{U} \frac{L}{c}$.

Next the equations of motion are nondimensionalized. The pressure term must first be nondimensionalized; even though the pressure is not the driving force, it is the function of primary importance. Thus the characteristic pressure is not extremely important. Since the pressure gradient is related to the rotor speed, it appears to be logical to nondimensionalize the pressure with respect to the dynamic pressure ρU^2 (also called the dynamic head):

$$\bar{P} = \frac{P}{\rho U^2}$$

It can be seen that the characteristic dynamic pressure ρU^2 has reduced the number of nondimensional groups by one (i. e., if P^* (a reference pressure) had been used, another nondimensional group would be present in the equations). Also, since the incompressible case is being analyzed, the pressure appears in the dynamic role only, not in the thermodynamic role.

Nondimensionalization of x-Direction Momentum Equation

When the x-direction momentum equation is nondimensionalized, equation (1) results in

$$\begin{aligned} u \frac{\partial \bar{u}}{\partial \bar{x}} + v \frac{\partial \bar{u}}{\partial \bar{y}} + w \frac{\partial \bar{u}}{\partial \bar{z}} = & - \frac{1}{\cos^2 \alpha} \frac{\partial \bar{P}}{\partial \bar{x}} \\ & + \frac{\nu^*}{UL} \left[\frac{1}{\cos^2 \alpha} \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \left(\frac{L}{c} \right)^2 \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{1}{\sin^2 \alpha} \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right] \end{aligned}$$

Next a Reynolds number is defined based on the characteristic length L :

$$Re_L = \frac{UL}{\nu^*}$$

where

$$\nu = \nu^* \bar{\nu} = \nu^*$$

and thus

$$\begin{aligned} \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{u}}{\partial \bar{z}} = & - \frac{1}{\cos^2 \alpha} \frac{\partial \bar{P}}{\partial \bar{x}} \\ & + \frac{1}{Re_L} \left[\frac{1}{\cos^2 \alpha} \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \left(\frac{L}{c} \right)^2 \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{1}{\sin^2 \alpha} \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right] \quad (A6) \end{aligned}$$

The convective acceleration and pressure terms are of unit order; thus,

the viscous term must be of unit order also. The largest term of the viscous forces is $\left(\frac{L}{c} \right)^2 \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}$, since $\left(\frac{c}{L} \right) < 1$; therefore, the $\frac{1}{Re_L} \left(\frac{L}{c} \right)^2$

should be of unit order. The modified Reynolds number is defined by

$Re^* = Re_L \left(\frac{c}{L} \right)^2$. This is the identical "reduced" or "effective" Reynolds

number that is used in the bearing theory to show the true relative magnitude of the inertia and viscous forces; therefore,

$$Re^* = \frac{\text{Inertia forces}}{\text{Viscous forces}}$$

(e.g., see ref. 32). Now for curved geometries, a Reynolds number based on clearance has appeared. This is especially true in viscoseal analysis where

$$Re_c = \frac{Uc}{\nu}$$

Writing the previous x-direction momentum equation in terms of Re_c gives

$$\bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{u}}{\partial \bar{z}} = - \frac{1}{\cos^2 \alpha} \frac{\partial \bar{P}}{\partial \bar{x}} + \frac{\nu^*}{Uc} \left(\frac{c}{L} \frac{1}{\cos^2 \alpha} \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{L}{c} \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{c}{L} \frac{1}{\sin^2 \alpha} \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right)$$

or

$$\bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{u}}{\partial \bar{z}} = - \frac{1}{\cos^2 \alpha} \frac{\partial \bar{P}}{\partial \bar{x}} + \frac{1}{Re_c} \left(\frac{c}{L} \frac{1}{\cos^2 \alpha} \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{L}{c} \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{c}{L} \frac{1}{\sin^2 \alpha} \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right)$$

Again it is seen that the $\partial^2 \bar{u} / \partial \bar{y}^2$ term is the largest.

Thus $1/Re_c(L/c)$ should be of unit order. Now

$$Re^* \equiv Re_c \left(\frac{c}{L} \right)$$

which leads to

$$Re^* \equiv Re_L \left(\frac{c}{L} \right)^2 \equiv Re_c \left(\frac{c}{L} \right)$$

It does not matter which Reynolds number is used to find the relative inertia force to viscous force ratio provided that $c/L < 1$, which corresponds to the conditions

$$\frac{\partial^2}{\partial y^2} \gg \frac{\partial^2}{\partial z^2}$$

and

$$\frac{\partial^2}{\partial y^2} \gg \frac{\partial^2}{\partial x^2}$$

The proper x-direction momentum equation then becomes

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} \quad (A7)$$

It is assumed that α does not go to 0° or 90° for the formal ordering procedure to be valid.

Example

A numerical example will be used to illustrate the validity of neglecting the $\partial^2/\partial z^2$ and $\partial^2/\partial x^2$ viscous terms. A typical viscoseal has the following construction and operating conditions: smooth 2-inch-diameter rotor, 0.0035-inch concentric radial clearance between the ridge (housing) and rotor surfaces, 1000 rpm, and sealed fluid (water) at 60° F. Therefore,

$$\nu = 17.5 \times 10^{-4} \text{ sq in./sec}$$

$$c = 0.0035 \text{ in.}$$

$$U = 2\pi R(1000 \text{ rpm})/60 = 100 \text{ in./sec (plate speed)}$$

$$c/R = 0.0035 \text{ (therefore, curvature effects should be slight)}$$

$$\begin{aligned} L &= \text{Ridge width} + \text{Groove width (in circumferential direction)} \\ &= 0.2 \text{ in.} + 0.2 \text{ in.} = 0.4 \text{ in.} \end{aligned}$$

$$\alpha = 15^\circ$$

$$\text{Re}_L = UL/\nu = 23,000$$

$$\text{Re}_c = Uc/\nu = 200$$

Using the Reynolds number based on the characteristic length L yields

$$\begin{aligned} \frac{1}{\text{Re}_L} \left[\frac{1}{\cos^2 \alpha} \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \left(\frac{L}{c} \right)^2 \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{1}{\sin^2 \alpha} \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right] \\ = \frac{1}{\text{Re}_L} \left(1.1 \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + 13,000 \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + 15 \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right) \end{aligned}$$

Using the Reynolds number based on the clearance c gives

$$\begin{aligned} \frac{1}{\text{Re}_c} \left[\left(\frac{c}{L} \right) \frac{1}{\cos^2 \alpha} \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \left(\frac{L}{c} \right) \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \left(\frac{c}{L} \right) \frac{1}{\sin^2 \alpha} \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right] \\ = \frac{1}{\text{Re}_c} \left(0.01 \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + 114 \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + 0.13 \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right) \end{aligned}$$

In both equations it is seen that $\partial^2 \bar{u} / \partial \bar{y}^2$ is the largest term. Now the modified or reduced Reynolds number is

$$\text{Re}^* = \text{Re}_L \left(\frac{c}{L} \right)^2 = \text{Re}_c \left(\frac{c}{L} \right) = 1.75$$

Nondimensionalization of y-Direction Momentum Equation

Nondimensionalizing the y-direction momentum equation (eq. (2))

yields

$$\bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{v}}{\partial \bar{z}} = - \left(\frac{L}{c} \right)^2 \frac{\partial \bar{P}}{\partial \bar{y}} + \frac{1}{\text{Re}_L} \left[\frac{1}{\cos^2 \alpha} \frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \left(\frac{L}{c} \right)^2 \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \frac{1}{\sin^2 \alpha} \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} \right] \quad (\text{A8})$$

Again using the definition of Re^* and examining the terms of unit order

$$\bar{u} \frac{\partial \bar{v}}{\partial \bar{x}}, \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}}, \bar{w} \frac{\partial \bar{v}}{\partial \bar{z}}, \frac{1}{\text{Re}^*} \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} \sim \mathcal{O}(1)$$

thus

$$\left(\frac{L}{c} \right)^2 \frac{\partial \bar{P}}{\partial \bar{y}} \cong \mathcal{O}(1)$$

Since

$$\frac{\partial \bar{P}}{\partial \bar{y}} \sim \mathcal{O} \left[\left(\frac{c}{L} \right)^2 \right]$$

therefore

$$\frac{\partial P}{\partial y} \cong 0 \quad (\text{A9})$$

This result is expected for a flat plate where there is no mechanism to generate a radial pressure gradient (e.g., a centrifugal force). For small curvatures, $c/R \leq 0.1$ however, the radial pressure gradient $\partial P/\partial y$ should also be a small quantity and have little physical influence on the axial pressure gradient. This can be shown by considering the following simplified model. Consider the flat plate model accelerating steadily about an origin. From elementary dynamics

$$\vec{a} = a_t \hat{e}_t + a_n \hat{e}_n$$

where: $a_t = \dot{q} = 0$ for steady acceleration

$$a_n = \frac{q^2}{R} = \frac{(u \cos \alpha - w \sin \alpha)^2}{R}$$

q is the fluid velocity in the tangential plane (x-z plane) and

$q = u \cos \alpha - w \sin \alpha$ (see fig. 17). $\hat{e}_n$ is the unit normal that is directed towards the axis of rotation. This is the y-coordinate direction in the flat plate model. Thus the centrifugal inertia force effects the y-momentum equation only.

Add ρa_n to the inertia force in equation (A2) and nondimensionalize as before and let $U = U\bar{U}$ and $R = R\bar{R}$. Thus equation (A8) becomes

$$\begin{aligned} \bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{v}}{\partial \bar{z}} + \frac{1}{\bar{R}} \left(\frac{\bar{L}}{\bar{c}} \right)^2 \left(\frac{\bar{c}}{\bar{R}} \right) (-\sin \alpha \bar{w} + \cos \alpha \bar{u})^2 = - \left(\frac{\bar{L}}{\bar{c}} \right)^2 \frac{\partial \bar{P}}{\partial \bar{y}} \\ + \frac{1}{\text{Re}_L} \left[\frac{1}{\cos^2 \alpha} \frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \left(\frac{\bar{L}}{\bar{c}} \right)^2 \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \frac{1}{\sin^2 \alpha} \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} \right] \quad (\text{A8a}) \end{aligned}$$

Now

$$\frac{1}{\bar{R}} \left(\frac{L}{c} \right)^2 \left(\frac{c}{R} \right) (-\sin \alpha \bar{w} + \cos \alpha \bar{u})^2 \sim \mathcal{O}(1)$$

Comparing the largest viscous term, pressure and centrifugal inertia term

$$\frac{\partial \bar{P}}{\partial \bar{y}} = \frac{1}{\text{Re}_L} \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \frac{1}{\bar{R}} \left(\frac{c}{R} \right) (-\sin \alpha \bar{w} + \cos \alpha \bar{u})^2$$

Now use a typical Re_L (as in example) $\cong 10^3$ and apply restriction

$$\frac{c}{R} < 0.01$$

$$\frac{\partial \bar{P}}{\partial \bar{y}} \cong (10^{-3}) \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \frac{(10^{-2})}{\bar{R}} (-\sin \alpha \bar{w} + \cos \alpha \bar{u})^2$$

Again

$$\frac{\partial \bar{P}}{\partial \bar{y}} \cong 0$$

if

$$\frac{c}{R} < 0.01$$

which substantiates the claim that the centrifugal effects should be small.

In this model Coriolis forces were also thought to be negligible. The above centrifugal effect is for the case when the grooves are on the stationary housing. A slight modification is required for the groove-ridge pattern rotating. Note that as $c/R \rightarrow 1$ the centrifugal force effects are no longer negligible.

Nondimensionalization of z-Direction Momentum Equation

In a like manner the z-direction momentum equation is nondimensionalized and appears in a similar form as the nondimensional x-direction momentum equation (A6)

$$\bar{u} \frac{\partial \bar{w}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{w}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{w}}{\partial \bar{z}} = -\frac{1}{\sin^2 \alpha} \frac{\partial \bar{P}}{\partial \bar{z}} + \frac{\nu^*}{UL} \left[\frac{1}{\cos^2 \alpha} \frac{\partial^2 \bar{w}}{\partial \bar{x}^2} + \left(\frac{L}{c}\right)^2 \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + \frac{1}{\sin^2 \alpha} \frac{\partial^2 \bar{w}}{\partial \bar{z}^2} \right] \quad (\text{A10})$$

Examining the viscous terms

$$\frac{1}{\text{Re}_L} \left[\frac{1}{\cos^2 \alpha} \frac{\partial^2 \bar{w}}{\partial \bar{x}^2} + \left(\frac{L}{c}\right)^2 \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + \frac{1}{\sin^2 \alpha} \frac{\partial^2 \bar{w}}{\partial \bar{z}^2} \right]$$

The largest term is $(L/c)^2 (\partial^2 \bar{w} / \partial \bar{y}^2)$ and it would appear that the $\partial^2 \bar{w} / \partial \bar{z}^2$ term could be neglected; however, this term must be retained due to a physical argument. Because of the "edge" there is a sharp gradient in the z-direction and the nature of the flow is such that the $\partial^2 \bar{w} / \partial \bar{z}^2$ term must be retained. Thus the z-direction momentum equation in dimensional form is

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (\text{A11})$$

This was pointed out by Dr. A. Mager of Aerospace Corporation, Los Angeles, California (personal communication, October 5, 1966).

Nondimensionalization of the Energy Equation

The energy equation for steady temperature variation for an incompressible fluid with constant thermophysical properties and negligible compression work is

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{k}{\rho g C} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{\mu}{\rho g C} \Phi + \frac{Q^*}{\rho g C} \quad (\text{A12})$$

Nondimensionalize as before and nondimensionalize the temperature by letting

$$\theta = \frac{T - T_s}{T_M - T_s} \quad \text{and} \quad \Phi' = \frac{\Phi}{\Phi^*}$$

Thus

$$\bar{u} \frac{\partial \theta}{\partial \bar{x}} + \bar{v} \frac{\partial \theta}{\partial \bar{y}} + \bar{w} \frac{\partial \theta}{\partial \bar{z}} = \left(\frac{k}{\rho g C} \frac{L}{U c^2} \right) \left[\left(\frac{c}{L} \right)^2 \frac{1}{\cos^2 \alpha} \frac{\partial^2 \theta}{\partial \bar{x}^2} + \frac{\partial^2 \theta}{\partial \bar{y}^2} + \left(\frac{c}{L} \right)^2 \frac{1}{\sin^2 \alpha} \frac{\partial^2 \theta}{\partial \bar{z}^2} \right] \\ + \frac{\Phi'}{L \rho g C (T_M - T_S)} + \frac{Q^*}{\rho g C (T_M - T_S)}$$

now

$$\left(\frac{k}{\rho g C} \frac{\nu}{U c} \frac{L}{c} \right) = \frac{1}{Pr} \frac{1}{Re_c} \left(\frac{L}{c} \right) = \frac{1}{Pr Re^*} = \frac{1}{P_e^*}$$

where P_e^* is the modified Péclet number

$$P_e^* = P_e \left(\frac{c}{L} \right) = Pr Re_c \left(\frac{c}{L} \right)$$

or

$$\bar{u} \frac{\partial \theta}{\partial \bar{x}} + \bar{v} \frac{\partial \theta}{\partial \bar{y}} + \bar{w} \frac{\partial \theta}{\partial \bar{z}} = \frac{1}{P_e^*} \left[\left(\frac{c}{L} \right)^2 \frac{1}{\cos^2 \alpha} \frac{\partial^2 \theta}{\partial \bar{x}^2} + \frac{\partial^2 \theta}{\partial \bar{y}^2} + \left(\frac{c}{L} \right)^2 \frac{1}{\sin^2 \alpha} \frac{\partial^2 \theta}{\partial \bar{z}^2} \right] \\ + \frac{\Phi'}{L \rho g C (T_M - T_S)} + \frac{Q^*}{\rho g C (T_M - T_S)}$$

Now assume constant temperature in the x-direction (along the groove ridge pairs) then

$$\bar{v} \frac{\partial \theta}{\partial \bar{y}} + \bar{w} \frac{\partial \theta}{\partial \bar{z}} = \frac{1}{P_e^*} \left[\frac{\partial^2 \theta}{\partial \bar{y}^2} + \frac{1}{\sin^2 \alpha} \left(\frac{c}{L} \right)^2 \frac{\partial^2 \theta}{\partial \bar{z}^2} \right] + \frac{\Phi'}{L \rho g C (T_M - T_S)} + \frac{Q^*}{\rho g C (T_M - T_S)}$$

Now dimensionalize all terms but the temperature

$$v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} = \frac{k}{\rho g C} \left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) + \frac{\mu}{\rho g C} \Phi + \frac{Q^*}{\rho g C}$$

$$\text{Pr} = \frac{C\mu}{k}$$

therefore

$$v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} = \frac{\nu}{\text{Pr}} \left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) + \frac{\mu}{\rho g C} \Phi + \frac{Q^*}{\rho g C} \quad (\text{A13})$$

APPENDIX B

FORMULATION OF ALONG GROOVE-RIDGE BOUNDARY CONDITION FOR A CYLINDRICAL GEOMETRY (VISCOSEAL)

A special case of parallel groove geometry is a cylindrical geometry which is the helically grooved fluid film seal (viscoseal). For this cylindrical geometry a special boundary condition must be placed on the pressure gradient along the groove-ridge direction. This can best be illustrated by referring to figure 26. It is seen that the point o to point a can be traversed along both coordinate axes on the surface (x and z axes).

From figure 26 and figure 21, it is seen that going from point o to point a along the z-axis involves the traversing of an angle φ .

$$\tan \alpha = \frac{R\varphi}{L'} \quad (\text{B1})$$

Now going the same distance along the x-axis

$$\cot \alpha = \frac{2\pi R - R\varphi}{L'} \quad (\text{B2})$$

Equating (B1) and (B2)

$$\frac{2\pi - \varphi}{\tan \alpha} = \frac{\varphi}{\cot \alpha}$$

or

$$\varphi = 2\pi \cos^2 \alpha \quad (\text{B3})$$

The arc distance (path length) in the x-direction is:

$$S = \sqrt{(R\varphi)^2 + (L')^2}$$

Upon substitution for L' and φ

$$S = 2\pi R \cos \alpha \quad (\text{B4})$$

The pressure differential at a from traversing the z-direction from o is

$$(\Delta P_{o-a} = (\Delta P_{r+g})_z(N)$$

then

$$\frac{\partial P}{\partial x} = \frac{N(\Delta P_{r+g})_z}{S} = \frac{N(\Delta P_{r+g})_z}{2\pi R \cos \alpha} \quad (\text{B5})$$

It is seen that $\partial P / \partial x = \text{constant}$ which follows from fully developed flow along the groove-ridge direction and verified by experiment in reference 5.

APPENDIX C

SUMMARY OF CREEPING FLOW ANALYSIS

The optimization from creeping flow analysis has been formulated by considering a flat-plate model (unwrapped cylinders, see fig. 27) and the following:

(1) General incompressible plane Couette flow is assumed in the ξ -direction (Poiseuille flow and simple Couette flow). The ξ -direction momentum equation is

$$\frac{\partial^2 u_\epsilon}{\partial \xi^2} = \frac{1}{\mu} \frac{\partial P}{\partial \xi}$$

with the boundary conditions

$$u_{\epsilon} = U \quad \text{at } \zeta = 0$$

$$u_{\epsilon} = 0 \quad \text{at } \zeta = h$$

where

$$h = c \quad \text{at ridge surface}$$

$$h = c + h_0 \quad \text{at groove root surface}$$

The well known solution (e. g., see ref. 32) is

$$u_{\epsilon} = \frac{\zeta}{h} U - \frac{h}{2\mu} \frac{dP}{d\xi} \zeta \left(1 - \frac{\zeta}{h}\right)$$

(2) Poiseuille flow is assumed in the η -direction. The η -direction momentum equation is

$$\frac{\partial^2 w_{\eta}}{\partial \zeta^2} = \frac{1}{\mu} \frac{\partial P}{\partial \eta}$$

with the boundary conditions

$$w_{\eta} = 0 \quad \text{at } \zeta = 0$$

$$w_{\eta} = 0 \quad \text{at } \zeta = h$$

The well-known solution (see ref. 32) is

$$w_{\eta} = \frac{1}{2\mu} \left(\frac{\partial P}{\partial \eta} \right) \zeta (\zeta - h)$$

Now the condition $Q = 0$ is imposed in the control volume; that is,

$$Q = Q_{\xi} + Q_{\eta} = \iint u_{\epsilon} d\eta d\zeta + \iint w_{\eta} d\zeta d\xi = 0$$

which means that there is no net end leakage or closed channel flow. This is a restriction that limits the optimization and pressure gradient prediction to a seal only.

Solving for pressure gradient that satisfies the Reynolds equation (ref. 34) results in a sealing parameter

$$\frac{\Delta P}{L'} = \frac{6\mu UG}{c^2}$$

or

$$G = \frac{c^2}{6\mu U} \left(\frac{\Delta P}{L'} \right) = \frac{1}{\Lambda}$$

where Λ is the bearing number (ref. 34) and G is a geometry factor. See references 1 to 26 for various expressions and values for G .

The creeping flow analysis has some paradoxes. It can best be illustrated by referring to figures 17 and 28 and by observing the resolution of the drag force into components along the groove (x-direction) and normal to the groove (z-direction). In creeping flow, the drag force is in equilibrium with the pressure force. Thus along the groove (x-direction) the drag force is indeed in equilibrium with the pressure force. In the normal groove direction (z-direction), however, the pressure and drag forces are in the same direction, which is also the direction of a very large pressure gradient (see figs. 22 and 23). Therefore, an unbalance of forces exists in this direction. Physically, the $Q = 0$ restriction is not possible. Also, this point can be illustrated from ξ - and η -direction momentum equations, respectively:

$$\frac{\partial^2 u_\epsilon}{\partial \xi^2} = \frac{1}{\mu} \frac{\partial P}{\partial \xi}$$

$$\frac{\partial^2 w_\eta}{\partial \xi^2} = \frac{1}{\mu} \frac{\partial P}{\partial \eta}$$

Notice that there is no velocity "coupling" between the velocity components; that is, u_ϵ and w_η are independent of one another. It is obvious that for zero leakage these velocity components are not independent of one

another. Thus, physically as well as mathematically it is not possible under the restrictions of the classical creeping flow analysis to get zero leakage.

Also in the creeping flow analysis the edge effects are assumed negligible. The full meaning of this assumption has not really been clarified; it is only close to physical reality when c is very small and/or b/h_0 is very large. For example, the creeping flow analysis is valid for spiral groove thrust bearings where c is on the order of 500 microinches and the aspect ratio is on the order of 1000 ($c/L \ll 1$, see fig. 29). As $b \rightarrow h_0$, the edge effect becomes more pronounced as the convective forces becomes more and more important. To date only reference 35 has explicitly stated this restriction.

To resolve the paradox of having the drag and pressure forces unbalanced in the z -direction, a convective inertia force is in equilibrium with the pressure and drag forces (a nonlinear effect). The step is causing a convective change that results in generation of an axial pressure gradient. The role of the step is now described.

If a concentric rotor is rotating about a stationary smooth sleeve, no axial pressure gradient is generated. This is the classic axisymmetric Couette flow (see ref. 36). Furthermore, if an axial pressure gradient is imposed (e.g., by having the rotor translate in the axial direction) an "uncoupled" helical flow will result. Subsequently, of course, there will be net flow out.

In the helical groove seal (viscoseal), the step or edge h_0 is the pressure generation mechanism. Without the step, that is, the limiting case when $h_0 \rightarrow 0$ (rotor and housing smooth), there is no axial pressure

gradient generated. Eccentricity will generate a radial pressure gradient. Therefore, the effect of the groove wall is a convective acceleration or inertia effect (nonlinear effect). To neglect the convective force terms means that the primary axial pressure generating mechanism is neglected.

The creeping flow solution equations are linear and pressure is a potential function. The creeping motion or slow viscous flow momentum equation in vector notation is

$$\nabla P = \mu \nabla^2 \vec{V}$$

The incompressible continuity equation says

$$\nabla \cdot \vec{V} = 0$$

Taking the divergence of both sides of the equation yields

$$\nabla \cdot \nabla P = \mu \nabla^2 (\nabla \cdot \vec{V}) = 0$$

or

$$\nabla^2 P = 0$$

which is Laplace's equation, since pressure is a scalar quantity. Consequently, the multitude of mathematical analog methods of solving linear equations and specifically Laplace's equation can be used. Thus, solutions for end effect can readily be found (refs. 1 and 34).

Muijderman (ref. 1) used conformal mapping to analyze the spiral grooved bearing. Again, this technique can be readily applied because pressure is a potential function.

REFERENCES

1. Muijderman, E. A.: Spiral Groove Bearings. Philips Res. Rept. Suppl. 1964, no. 2.: See also Scientific Lubr., vol. 17, no. 1, Jan. 1965, pp. 12-17.

2. Rowell, H. S.; and Finlayson, D.: Screw Viscosity Pumps. Engineering, vol. 114, Nov. 1922, pp. 606-607.
3. Boon, E. F.; and Tal, S. E.: Hydrodynamic Seal for Rotating Shafts. DEG. Inf. Ser. 13, United Kingdom Atomic Energy Authority, 1961.
4. King, Alan E.: Screw Type Shaft Seals for Potassium Lubricated Generators. IEEE Trans. on Aerospace, vol. AS-3, Supplement, June 1965, pp. 471-479.
5. Ludwig, Lawrence P.; Strom, Thomas N.; and Allen, Gordon P.: Gas Ingestion and Sealing Capacity of Helical Groove Fluid Film Seal (Viscoseal) Using Sodium and Water as Sealed Fluids. NASA TN D-3348, 1966.
6. Ludwig, Lawrence P.; Strom, Thomas N.; and Allen, Gordon P.: Experimental Study of End Effect and Pressure Patterns in Helical Groove Fluid Film Seal (Viscoseal). NASA TN D-3096, 1965.
7. Vohr, J. H.; and Chow, C. Y.: Characteristics of Herringbone-Grooved Gas-Lubricated Journal Bearings. J. Basic Eng., vol. 87, no. 3, Sept. 1965, pp. 568-578.
8. Billet, A. B.: Hydraulic Sealing in Space Environments. Proceedings of the Second International Conference on Fluid Sealing, B. S. Nau, H. S. Stephens, and D. E. Turnbull, eds., British Hydromechanics Research Association, Harlow, Essex, England, 1964, pp. C2-17 - C2-36.
9. Rowell, H. S.; and Finlayson, D.: Screw Viscosity Pumps. Engineering, vol. 126, Aug. 1928, pp. 249-250.

10. Rowell, H. S.; and Finlayson, D.: Screw Viscosity Pumps. Engineering, vol. 126, Sept. 1928, pp. 385-387.
11. Rogowsky, Z.: Mechanical Principles of the Screw Extrusion Machine. Engineering, vol. 162, no. 4213, Oct. 11, 1946, pp. 358-360.
12. Strub, R. A.: Spindle Drag Pump. Machine Design, vol. 25, July 1953, pp. 149-151.
13. Pigott, W. T.: Pressures Developed by Viscous Materials in the Screw Extrusion Machine. ASME Trans., vol. 73, Oct. 1951, pp. 947-955.
14. Anon.: Theory of Extrusion. Ind. Eng. Chem., vol. 45, no. 5, May 1953, pp. 969-993.
15. Eccher, Silvio; and Valentinotti, Aldo: Experimental Determination of Velocity Profiles in an Extruder Screw. Ind. Eng. Chem., vol. 50, no. 5, May 1958, pp. 829-836.
16. Griffith, R. M.: Fully Developed Flow in Screw Extruders. Ind. Eng. Chem. Fundamentals, vol. 1, no. 3, Aug. 1962, pp. 180-187.
17. Squires, P. H.: Screw Extrusion - Flow Patterns and Recent Theoretical Developments. SPE Trans., vol. 4, no. 1, Jan. 1964, pp. 7-16.
18. McGrew, J. M.; and McHugh, J. D.: Analysis and Test of the Screw Seal in Laminar and Turbulent Operation. J. Basic Eng., vol. 87, no. 1, Mar. 1965, pp. 153-162.
19. Zotov, V. A.: Research on Helical Groove Seals. Russ. Eng. J., vol. 10, Oct. 1959, pp. 3-7.

20. Asanuma, T.: Studies on the Sealing Action of Viscous Fluids.
Paper No. A3 presented at the First International Conference on Fluid Sealing. Cranfield, England. British Hydromechanics Research Association, Harlow, Essex, England, Apr. 1961.
21. Lessley, R. L.; and Hodgson, J. N.: Low-Leakage Dynamic Seal-to-Space. Paper 65-GTP-14, ASME, Feb. 1965.
22. Whipple, R. T. P.: The Inclined Groove Bearing. Rep. AERE-T/R-622 (rev.), Research Group, Atomic Energy Research Establishment, United Kingdom Atomic Energy Authority, 1958.
23. Gruntz, Robert D.; and Rackley, Ray A.: Snap 50/Spur Power Conversion System-Objectives, Current Status and Lunar Applications. Paper No. 650321, SAE, May 1965.
24. Golubiev, A. I.: Studies on Seals for Rotating Shafts of High-Pressure Pumps. Wear, vol. 8, no. 4, July/Aug. 1965, pp. 270-288.
25. Holan, Karel: Sealing in Engineering. Proceedings of the Second International Conference on Fluid Sealings, B. S. Nau, H. S. Stephens, and D. E. Turnbull, eds., British Hydromechanics Research Association, Harlow, Essex, England, 1964, pp. E5-73 - E5-88.
26. Stair, William K.: Analysis of the Visco Seal. Rep. No. ME 65-587-2, University of Tennessee, Jan. 18, 1965.
27. Stair, William K.: Theoretical and Experimental Studies of Visco-Type Shaft Seals. Rep. No. ME 66-587-5, University of Tennessee, Apr. 28, 1966.

28. Lessley, R. L.: Snap-8 Seals-to-Space Development Test Program.
Vol. 1 - Visco Pump. Rep. No. 2808 (Topical) (NASA CR-54234),
Aerojet-General Corp., May 1964.
29. Kettleborough, C. F.: Turbulent and Inertia Flow in Slider Bearings,
ASLE Trans., vol. 8, no. 3, July 1965, pp. 286-295.
30. Zuk, John; Ludwig, Lawrence P.; and Johnson, Robert L.: Flow and
Pressure Field Analysis of Parallel Groove Geometry for an Incom-
pressible Fluid With Convective Inertia Effects. NASA TN D-3635,
1966.
31. Kuivinen, David E.: Determination of Oxygen in Liquid Alkali Metals
by the Mercury Amalgamation Method. Paper presented at the
Nineteenth Meeting, Chemical Rocket Propulsion Group (St. Paul,
Minn.), July-Aug., 1963.
32. Schlichting, Hermann (J. Kestin, Trans.): Boundary Layer Theory.
Fourth ed., McGraw-Hill Book Co., Inc., 1960.
33. Kulinski, E. S.; and Ostrach, S.: A Critical Evaluation of the Status
and Trends in High Speed Fluid Film Lubrication (to be published).
34. Bisson, Edmond E.; and Anderson, William J.: Advanced Bearing
Technology. NASA SP-38, 1964.
35. Booy, M. L.: Influence of Oblique Channel Ends on Screw-Pump
Performance. J. Basic Eng., vol. 88, no. 1, Mar. 1966, pp.
121-131.
36. Langlois, W. E.: Slow Viscous Flow. Macmillan Co., 1964, pp.
105-107.

TABLE I. - VISCOSITY
OF MINERAL OIL

Temperature		Kinematic viscosity, cs
$^{\circ}\text{F}$	$^{\circ}\text{C}$	
100	37.8	71.2
130	54.4	32.4
210	98.9	8.4

TABLE II - VISCOSEAL GEOMETRIES

Geometry	Configuration	Primary grooves							Secondary grooves		
		Groove width, z_p , cm	Land width, z_l , cm	Ratio of groove to land width, z_p/z_l	Groove depth, h_c	Radial clearance, c , cm	Primary helix angle, α , deg	Number of helix starts	Number of helix starts per land	Secondary helix angle, α_s , deg	Cross section through land
2		0.430 (0.170 in)	0.30 (0.118 in)	1	0.0 (0.0)	()	1		1		
Geometric visco seal profiles		0.40 (0.157 in)	0.35 (0.138 in)	1	()	()	14		1	14
3		0.26 (0.102 in)	0.14 (0.055 in)	1.9	0.0 (0.01 in)	0.010 (0.004 in)	4	1	0		
Geometric visco seal profiles		0.40 (0.157 in)	0.014 (0.0005 in)	1.9	0.0 ()	0.1 ()				14

0.40 (0.157 in)	0.00 (0.130 in)	1	()	0.00 (0.000)			0		

a. Use as base for

TABLE III. - CRITICAL MODIFIED REYNOLDS NUMBER FOR ONSET OF TURBULENCE OF VARIOUS EXPERIMENTERS

Experimenter	Reference and fluid	Seal	Diameter, in.	Radial clearance, c, in.	Ratio of radial clearance to rotor radius, c/R	Ridge width, a, in.	Groove width, b, in.	Helix angle, α , deg	Characteristic length, L, in.	Clearance Reynolds number, Re_c	Modified Reynolds number, Re^*
Stair	27 Water	a1	1.2430	0.0042	0.0068	0.0934	0.1596	14.5	1.01	400	1.7
		2	1.2465	.00235	.0038	.1176	.0494	9.67	.98	(b)	(b)
		2B	1.2420	.0047	.0076	.1176	.0494	9.67	.98	(b)	(b)
		3	1.2461	.00265	.0043	.0828	.0842	9.67	.98	350	.95
		3B	1.2408	.0053	.0085	.0828	.0842	9.67	.98	(b)	(b)
		4	1.2461	.00265	.0043	.0514	.1156	9.67	.98	200	.95
		4B	1.2408	.0053	.0085	.0514	.1156	9.67	.98	400	2.2
		5	1.2455	.00295	.0047	.1585	.0607	5.81	1.83	600	.97
		6	1.2455	.00295	.0047	.1070	.1083	5.81	1.83	200	.32
		7	1.2455	.00295	.0047	.0609	.1600	5.81	1.83	160	.26
King	4 and 27 Oil, water, potassium ^d	3C	2	0.0029	0.0029	0.1063	0.1063	3.86	3.2	550	0.50
		1B	2	.0020	.0020	.1063	.1063	3.86	3.2	400	.25
		2E	2	.0025	.0020	.1000	.1000	7.26	1.54	650	1.1
		5D	2	.0015	.0015	.1000	.1000	7.26	1.54	650	.63
McGrew and McHugh	18 Oil	SF96-5 oil	1	0.00325	0.0065	(c)	(c)	14.5	3.14	400	0.41
Lessley	28 Oil	Configuration B	2	0.0032	0.0032	0.075	0.122	14.5	0.79	350	1.4
	Water Mercury ^d	Configuration C	2	.0032	.0032	.083	.136	4	3.13	500	.51

^aEccentricity of rotor to housing, $\epsilon = 0.1$.^bBreak point not well defined.^cFour thread starts.^dNot explicitly found at breakpoint but falls on data line.

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Figure 1. - Viscoseal and hydraulic system.

Figure 2. - Pressure tap and thermocouple locations.

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Figure 3. - Internal grooved housing viscoseal used with water.

Figure 4. - Grooved rotor viscoseal used to seal sodium.

Figure 5. - Sodium supply system.

Figure 6. - Isometric representation of viscoseal and pressure patterns.

Figure 7. - Pressure gradients on planes orthogonal to viscoseal axis centerline.

(a) Rotational speed, 1000 rpm; average temperature of oil film, 97°F ; temperature change along oil film, 11°F ; cavity pressure, 54.1 lb/sq in. absolute.

(b) Rotational speed, 6000 rpm; average temperature of oil film, 132°F ; temperature change along oil film, 19°F ; cavity pressure, 80.6 lb/sq in. absolute.

Figure 8. - Axial pressure gradients near helical groove edges with grooves in direct communication with pressurized cavity.

Figure 9. - Grooved housing viscoseal operation when sealing water.

High-
pressure
end

(c) Modified Reynolds number, 3.50 (4000 rpm).

High-
pressure
end

(d) Modified Reynolds number, 4.42 (5000 rpm).

Figure 9. - Continued.

High-
pressure
end

(e) Modified Reynolds number, 5.25 (6000 rpm).

High-
pressure
end

(f) Modified Reynolds number, 7.00 (8000 rpm).

Figure 9. - Concluded.

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Figure 10. - Comparison of observed gas ingestion when sealing water for grooved housing and grooved rotor viscoseals.

Figure 11. - Gas ingestion mechanisms of grooved housing compared with grooved rotor viscoseal.

High-
pressure
end

65 102

← Gas ingestion direction and increasing liquid-gas density

- No leakage at low-pressure end

Figure 12 - Pressure as function of axial length with gas ingestion for grooved rotor Sealed fluid, water

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Figure 13. - Gas ingestion rate when sealing liquid sodium at 400° F.

(a) Mechanism involved in gas ingestion into pressurized cavity.

(b) Schematic liquid level oscillograph trace for repeated cycles of gas ingestion, subsequent gas blowout, and no liquid leakage at blowout.

(c) Schematic liquid level oscillograph trace for repeated cycles of gas ingestion, subsequent gas blowout, and liquid leakage at blowout.

Figure 14. - Gas ingestion into closed cavity.

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Figure 15. - Comparison of sealing parameters for grooved rotor and housing operating in sodium. Sodium temperature, 165° to 335° C (329° to 635° F).

Figure 16. - Comparison of sealing parameter obtained with helically grooved rotor with and without secondary land grooves operating in sodium. Sodium temperature, 165° to 335° C (329° to 635° F).

(a) Plan view.

(b) Cross-sectional view along z -axis illustrating relative motion of smooth flat plate with respect to fixed parallel groove-ridge plate.

Figure 17. - Mathematical model of parallel groove-ridge geometry of infinite horizontal extent

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Figure 18. - Region of analysis validity (homogeneous fluid) of a grooved housing viscoseal. Reynolds number based on clearance or film thickness, 1700; modified or reduced Reynolds number, 19; 400-microinch shaft rotational movement during photographic exposure; shaft speed, 8000 rpm; sealed fluid water

Figure 19. - Viscoseal with grooved housing operating in a regime of water-air mixture (gas ingestion). Analyses does not apply for this discontinuous fluid film case; 250-microinch shaft rotational movement during photographic exposure; shaft speed, 5000 rpm, sealed fluid, water

Figure 20 - Cross-sectional view along z -axis illustrating arbitrary groove-ridge pair shape

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Figure 21 - Resolution of across and along groove-ridge pressure gradients into axial pressure gradient "Unwrapped" cylinder is shown

Figure 22 Pressure distribution over parallel groove geometry (Qualitative graphical representation from experimental data)

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(a) Along ridges or grooves (no end effects are shown).

(b) Across ridges and grooves (no restriction that profile must be linear).

Figure 23. - Pressure profiles (qualitative graphical representation from experimental data)

Figure 24. - Experimental sealing parameter for seals 5, 6, and 7 from reference 27.

Figure 25 Empirical sealing coefficient as function of clearance Reynolds number

Figure 26 Illustration that a point on the viscoseal surface can be reached from the origin axis along both the x constant and z constant surface coordinate paths (for example paths from o to a)

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Figure 27 Model used in creeping flow solution

Figure 28 - Resolution of plate velocity (or drag force) into components along and across groove or ridge

Figure 29 - Example where the creeping flow mathematical model can be considered a valid physical model. Modified Reynolds number (Re) much less than 1 since clearance c is much less than characteristic length L .

THE EFFECT OF TURBULENCE ON VISCO SEAL PERFORMANCE

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ABSTRACT

Experimental study of the visco seal reveals that the laminar seal performance is in excellent agreement with the analysis of Boon and Tal. In the turbulent region, however, the sealing performance exceeds the theoretical laminar prediction and the optimum laminar seal geometry is not optimum for turbulent operation. A theoretical equation has been obtained for the sealing coefficient of a visco seal which is suitable for both laminar and turbulent flow. Experimental results from 13 different seal geometries confirm the utility of the analysis.

Air ingestion has been observed during turbulent operation of each experimental seal. This phenomenon, however is not fully understood, may become a major problem in the application of the visco seal.

I. INTRODUCTION

The viscoseal, viscosity pump, screw seal, spiral groove seal or bearing are various names given the device whose working principle is based on the pressure generation in a viscous fluid, enclosed in a narrow annulus or slit, by means of grooves on a rotating shaft or plate. The viscoseal, shown basically in Figure 1, is an old but little used device which functions without sliding contact and, unlike other clearance-type seals, holds promise of zero leakage. The lack of contact, long reliable life, and the possibility of zero leakage has recently created a renewed interest in the viscoseal as a sealing element for use in the critical applications associated with the nuclear energy and space programs (1).¹

II. BACKGROUND

The performance of the viscoseal may be expressed in terms of a sealing coefficient defined as:

$$\Lambda = \frac{6\mu UL}{c^2 \Delta P}$$

A number of theoretical analyses have been presented for laminar operation of the viscoseal with homogeneous fluids in which the theoretical laminar sealing coefficient is presented as a function of groove geometry (2). The grooves shown on the shaft in Figure 1, may be located on the shaft or housing insofar as laminar theory is concerned. These analyses lead to

¹Numbers in parenthesis designate references at end of paper.

expressions for the sealing coefficient which are, or can be modified to be, in the form:

$$\Lambda = \frac{6\mu UL}{c^2_{\Delta P}} = f(\text{seal geometry}). \quad (2)$$

Sealing coefficients determined experimentally for a number of sealing geometries may be characterized as follows: (2,3,4,5,6)

1. At low Reynolds numbers the sealing coefficient is constant (independent of Reynolds number) and is a function of the groove geometry. The experimental laminar sealing coefficients are in excellent agreement with the equation,

$$\Lambda_T = \frac{\beta^3(1+t^2) + t^2\gamma(1-\gamma)(\beta^3-1)^2}{t\gamma(1-\gamma)(\beta^3-1)}, \quad (3)$$

which is based on the work of Boon and Tal (7). Figure 2 presents a comparison of several laminar analyses with data for test seal No. 1.

2. At high Reynolds numbers the sealing coefficient improves (Λ decreases) and is a function of Reynolds number as well as seal geometry.

3. There is a smooth systematic transition from laminar to turbulent operation and the Reynolds number at which the transition begins is a function of seal geometry. As γ is increased in a seal test series, having the same α and β , the onset of turbulence begins at lower Reynolds numbers. These observations suggest that whatever the theoretical expression for the turbulent Λ might be, it should become equivalent to equation (3) at low values of Reynolds number.

The analyses of the turbulent viscoseal are very limited and are largely empirical. McGrew and McHugh (5) made use of the Prandtl mixing length concept and deduced the form of the turbulent sealing coefficient as:

$$\Lambda_o = (C_1 + C_2 \text{Re}_h^m)^{-1} \quad (4)$$

Since the experimental constants were determined for a single seal only, equation 4 is of limited utility in design.

III. TURBULENT SEALING COEFFICIENT

Making use of the same physical model employed by Boon and Tal (7) for the laminar analysis, a theoretical sealing coefficient for the turbulent viscoseal has been derived (2). The procedure utilized in the turbulent analysis was to modify the equations of motion and continuity used in the laminar analysis by replacing instantaneous velocity components and pressure with mean components and turbulent fluctuating components in order to represent turbulent flow. From these basic equations the Reynolds equations for turbulent flow were formed. Following a method similar to that employed by Pai, and representing the velocity profiles for Poiseuille and Couette flow as power series, the equations for the Poiseuille and Couette velocity components were obtained. The flow components in the seal were the same as those identified in the laminar analysis (9). Setting the sum of the flow components to zero, the turbulent sealing coefficient was obtained as:

$$\Lambda_T = K_4 \left[\frac{I_1 + I_2}{I_4} \right] + K_5 \left[\frac{I_3}{I_4} \right], \quad (5)$$

where

$$I_1 = (1 - \gamma) t^2, \quad (6)$$

$$I_2 = \beta^3 \gamma t^2, \quad (7)$$

$$I_3 = \beta^3 / \left[\gamma + \beta^3 (1 - \gamma) \right], \quad (8)$$

and

$$I_4 = t \left[1 - \gamma + \gamma \beta - \frac{\gamma + \beta^3 (1 - \gamma) + \gamma (\beta - 1)}{\gamma + \beta^3 (1 - \gamma)} \right]. \quad (9)$$

The terms I_1 , I_2 , I_3 , and I_4 are functions of the seal geometric parameters α , β , and γ and are independent of Reynolds number. K_4 and K_5 , however, are functions of seal geometry and Reynolds number and are given by:

$$K_4 = \frac{3}{2F_\xi} \left[1 - \frac{F_\xi}{10.5F_\xi - 7.5} + \frac{1 - F_\xi}{3.92F_\xi^2 - 1.4F_\xi - 1} \right], \quad (10)$$

and

$$K_5 = \frac{3}{2F_n} \left[1 - \frac{F_n}{10.5F_n - 7.5} + \frac{1 - F_n}{3.92F_n^2 - 1.4F_n - 1} \right]. \quad (11)$$

K_4 , which is a function of F_ξ , relates to flow in the direction parallel to the grooves, while K_5 , a function of F_n , is related to flow in the direction normal to the grooves. The factors F_ξ and F_n are expressed as the ratio of the wall shearing stress in turbulent flow to the wall shearing stress for laminar flow with the same maximum channel velocity.

Thus,

$$F_\xi = (\tau_o / \tau_l)_\xi = (f_o / f_l)_\xi, \quad (12)$$

and

$$F_n = (\tau_o / \tau_l)_n = (f_o / f_l)_n. \quad (13)$$

The formulation of equation 5 considers that the onset of turbulence in the grooves and over the lands does not occur at the same Reynolds number. The Taylor criterion for the onset of turbulence in concentric cylinders with the inner cylinder rotating is

$$Re \leq 41.1 \sqrt{\frac{D}{2c}}. \quad (14)$$

In this analysis, an average critical Reynolds number for the visco seal was defined as:

$$Re_{crit} \leq 41.1 \left[\frac{D/2}{(1 - \gamma)c + \gamma \beta c} \right]^{1/2}. \quad (15)$$

At this juncture the coefficients K_4 and K_5 may be evaluated in terms of the experimental data which have been obtained. However, it was observed early in this program that the plots of the friction parameter versus

Reynolds number for the visco seal were similar in general appearance to plots of the friction factor versus Reynolds number for ordinary pipe flow. In an effort to utilize readily available, through indirect, data to predict the turbulent visco seal performance, K_4 and K_5 were estimated by the use of Figure 3. In this figure the commonly used friction factor for laminar flow in pipes is expressed by the equation:

$$f = \frac{64}{Re_p} \quad (16)$$

In the turbulent range the coefficient of resistance for flow normal to the grooves was expected to be different from the coefficient for flow parallel to the grooves. In Figure 3 two lines were selected to represent estimates of the resistance coefficients for these two flow directions. The lower line, defined by the equation

$$f = 0.326/Re_p^{0.253}, \quad (17)$$

was selected to represent the friction factor for flow parallel to the grooves. This curve was approximated by extending the conventional "smooth pipe" curve to the intersection with equation 16. The upper curve, defined by the equation

$$f = 0.646/Re_p^{0.246}, \quad (18)$$

was selected to represent the friction factor for flow normal to the grooves. This curve was estimated by extending the curve for a roughness ratio of approximately 0.03, which corresponds to that for corrugated pipe, to its intersection with the laminar equation 16. This procedure, of course, represents the transition region as straight lines.

Assuming the critical Reynolds number for pipe flow to be 2,000, the term F_ξ is evaluated, according to equation (12), by determining the ratio of the friction factor for turbulent flow to the friction factor for laminar flow, both at the pipe Reynolds number of:

$$Re_p = (2000 \cos \alpha) Re_c / Re_{crit}. \quad (19)$$

Re_c refers to the seal Reynolds number based on the clearance and $Re_{crit.}$ is defined by equation (15). For evaluation of F_ξ the lower turbulent line on Figure 3 is used. F_η is determined in a similar manner except at a pipe Reynolds number of:

$$Re_p = (2000 \sin \alpha) Re_c / Re_{crit.}, \quad (20)$$

and using the upper turbulent line on Figure 3. The coefficients K_4 and K_5 can now be evaluated by equation (10) and (11) respectively.

When the seal is operating in the laminar range K_4 and K_5 are both unity and equation (5) becomes equal to equation (3). As the Reynolds number increases F_ξ increases, causing K_4 to decrease, and the sealing coefficient begins to decrease. With further increases in Reynolds number F_η also begins to increase thus causing K_5 to decrease which produces a further decrease in the sealing coefficient.

IV. GENERALIZING THE THEORETICAL SEALING COEFFICIENT

Equation (5) was evaluated for a series of seals having helix angles of 5.81° , 9.67° , 14.5° , and 20.15° for Reynolds numbers of 100, 500, 1000, and 5000. In this general case the seal diameter and clearance were fixed at 1.250" and 0.003" respectively. Typical results of these calculations are plotted as Figures 4 through 8. Several observations may be made regarding these figures:

1. The optimum screw geometry in the laminar range is not the optimum geometry for turbulent operation. As shown in Figure 4, a seal having an α of 14.5° has a minimum sealing coefficient for laminar operation when $\gamma = 0.5$ and $\beta \approx 4$. However, when operation becomes turbulent the minimum value of the sealing coefficient is found at higher values of γ and β . At higher α values the same observation can be made. However, the shift in the optimum β becomes smaller.

2. The β producing the minimum sealing coefficient in the turbulent range increases with increasing Reynolds number. This effect is less significant for large α than for small α .

3. In both laminar and turbulent flow, a given change in β produces a greater change in Λ_T with screws having large α than with small α . Thus, from the standpoint of pressure stability the lower values of α are to be preferred.

4. In laminar operation the optimum γ for any screw is 0.5. However, in turbulent flow the sealing coefficient is improved slightly as γ is increased from 0.5 to 0.7.

5. For seals which must operate in both the laminar and turbulent regions, the smaller helix angles will give the best values of the sealing coefficient for both regions.

6. The transition from laminar to turbulent operation is a function of γ . As γ increases the onset of turbulence occurs at lower Reynolds numbers.

7. As the visco seal operation begins to be turbulent the effect of γ on Λ_T at the larger β values is rather pronounced. However, as the degree of turbulence increases this effect becomes less pronounced.

8. At a Reynolds number of 100, Λ_T is the same as the one computed in equation (3), which is independent of Reynolds number.

V. EXPERIMENTAL FACILITIES AND TEST PROCEDURE

The visco seal test section and drive is shown in Figure 9, and a schematic diagram of the test section is shown in Figure 10. The sealant fluid, which was distilled water in all tests reported, was introduced under regulated pressure through the sealant inlet. Pressure taps and thermocouples are located along the axis of the test sleeve, the dimensional location of which are shown in Figure 11. Inductance probes located in two planes and 90° apart are used to determine the eccentricity between the test sleeve and spindle. A torque arm connected to a strain gage bridge is used to measure the frictional torque. Since the torque measured under some operational conditions is very low, the test sleeve is mounted on eight hydrostatic bearing pads to minimize static friction. The dimensions of the test spindles are presented in Table I. Test spindles 1, 2, 3, 4, 2B, 3B, and 4B, shown in

Figure 12, were threaded the total seal length. Spindles 5, 6, and 7, shown in Figure 12, were constructed with end dams in order to study the end effects of the seal and the phenomenon of air ingestion.

During a test run the data recorded included spindle speed, torque, eccentricity, pressure distribution, and temperature distribution. Using the information in Table I, the recorded experimental data, and the physical properties of the test fluid, the experimental sealing coefficient (hereafter referred to as Λ_E) and the friction parameter were calculated. Lambda and friction parameter were plotted versus Reynolds number based on the clearance.

The quantity $\Delta P/L$ in equation (2) for Λ_E may be interpreted in two ways. In a practical application this value should be $(P_{\text{supply}} - P_{\text{atmosphere}})/L$. However, in this study the effect of screw geometry on the sealing coefficient was a factor of major concern. Therefore, in order to minimize the end effects, the $\Delta P/L$ was interpreted as dP/dL and evaluated as shown in Figure 11. The viscosity of the test fluid was evaluated at an average temperature determined from the distribution as shown in Figure 11. The temperature variation in the seal was quite small being on the order of 2 degrees F.

VI. EXPERIMENTAL RESULTS

Sealing Coefficient:

The geometry of spindle number 1 was made similar to the one employed by McGrew and McHugh for the purpose of comparison of experimental results. The data compared favorably in the turbulent range, as shown in Figure 13, to the experimental curve of McGrew and McHugh. In the laminar range, however, the value of McGrew and McHugh's experimental curve is higher than the present data.

A summary of test results is presented in Table II. Using the theory presented in section III, the theoretical sealing coefficient is compared with the experimental values in Figures 13 through 18.

For all spindles tested Λ_E remained essentially constant

during laminar operation. This result is in agreement with equation (3) which indicates that $\Lambda_T \neq f(\text{Re}_c)$ in the laminar region. The average ratio of Λ_E to Λ_T for laminar operation is 0.968. During the turbulent operation of each of the seals 2, 3, 4, 2B, 3B, and 4B, the ratio of Λ_E to Λ_T remained essentially constant. However, because of the variation of slopes in the turbulent region for test series 5, 6, and 7, the ratio of Λ_E to Λ_T was not constant.

With the onset of turbulence Λ_E begins to decrease with increasing Reynolds numbers. The Reynolds number at which transition from laminar to turbulent operation occurs is a function of γ . In test series 2, 3, and 4, the three test spindles have essentially the same α , β , c , and D with variations in γ only. Spindle 2, having the lowest γ , resulted in the longest laminar operational region, while spindle 4, with the largest γ , resulted in the shortest operational laminar region, as may be observed in Figure 14. Thus, as γ increases the transitional Reynolds number decreases. The decrease in the transitional Reynolds number, defined in equation (15), is to be expected since the average clearance in the seal is increasing. This direct relationship between the onset of turbulence and γ is in agreement with the theoretical projection in section III.

The relationship between α & Λ_E was demonstrated in test series 5, 6, and 7. The geometric parameters of spindles 5, 6, and 7 are essentially the same as for spindles 2, 3, and 4 with the exception of α which was decreased from 9.67° to 5.81° . In tests of seals having α 's of 9.67 and 14.5° , the slope $d\ln \Lambda_E / d\ln \text{Re}_c$, in the turbulent zone was approximately -0.70 , which agrees with the theoretical prediction of -0.709 . However, for an α of 5.81° the value of $d\ln \Lambda_E / d\ln \text{Re}_c$ was found to be approximately -0.43 for the range of experimental data recorded. The relationship between Λ_T and α is demonstrated in Figure 8. In this figure it is observed that Λ_T , for α equal to 5.81° , remains laminar to a Reynolds number of 300, while for α equal to 20.15° ,

Λ_T begins to decrease at a Reynolds number of approximately 100. Therefore, as α increases the transition point from laminar to turbulent operation for Λ_T decreases. However, it is noted from Figure 8 that, for either large or small α 's, the predicted slope, $d\ln \Lambda_E / d\ln Re_c$, for Reynolds numbers greater than 1000, is the same. In this regard Λ_E and Λ_T do not agree.

The effect of β on Λ_E may be observed in Figures 16, 17, and 18. In series 2B, 3B, and 4B α and γ remain unchanged from series 2, 3, and 4, while β was decreased from approximately 7 to 3. Λ_E for 2B, 3B, and 4B is, on the average, 1.29 times the values for spindles 2, 3, and 4 in both laminar and turbulent operation. This ratio compares favorably with theoretical predictions which indicate that the ratio should be 1.28. In test series 2B there is a sudden large increase in Λ_E at a Reynolds number of approximately 1600. This discontinuity in Λ_E is believed to be the result of air ingestion which produces a decrease in the value of β . A similar observation was made for spindles 3B and 4B, but the perturbation was much lower in magnitude.

Friction Parameter and Power Loss:

The power loss in the visco seal during laminar operation may be described in terms of a dissipation function (8) defined by the equation:

$$\phi = 1 - \gamma + \frac{\gamma}{\beta} + 3 \frac{t^2 \gamma (1 - \gamma) (\beta - 1)^2 (1 - \gamma + \gamma \beta^3)}{\beta^3 (1 + t^2) + t^2 \gamma (1 - \gamma) (\beta^3 - 1)^2}, \quad (21)$$

$$\phi = \frac{qc}{\pi \mu D L U^2}, \quad (22)$$

where q is the power loss. The dissipation function depends upon α , β , and γ and is independent of rotational speed. The friction parameter may be defined in terms of the dissipation function as:

$$F.P. = \frac{4\pi \phi}{Re_c}. \quad (23)$$

Thus, F.P. is a function of the seal parameters and speed. The laminar experimental data for the friction parameter are in agreement with the equation (23) as shown in Figures 19, 20, and 21. The experimental data in the turbulent range follows the trend noted by Smith and Fuller (10) whose data for the friction parameter of a plain journal bearing operating under laminar and turbulent conditions can be expressed as:

$$\text{F.P.}_l = 4\pi/\text{Re}_c, \quad (24)$$

and

$$\text{F.P.}_o = 0.156 \pi / \text{Re}_c^{0.43}. \quad (25)$$

As the visco seal more nearly approaches a journal bearing, γ becoming smaller, the experimental data for the friction parameter approaches the laminar experimental curve presented by Smith and Fuller. In the turbulent region, however, the experimental data for the seal are very slightly higher than that of Smith and Fuller. Thus, in laminar operation the friction parameter is described by equation (23) and during turbulent operation Smith and Fuller's turbulent journal bearing results can be utilized to estimate the friction parameter.

Effect of Eccentricity on Sealing Coefficient:

McGrew and McHugh (5) predicted that eccentric operation of the visco seal could cause as much as a fifty percent increase in the sealing coefficient obtained under concentric conditions. For laminar operation in the absence of gas ingestion it was found that Λ_E was essentially independent of the seal eccentricity in the range of eccentricity ratios up to 0.6. In the turbulent range Λ_E was observed to decrease slightly with increasing eccentricity. This effect is shown in Figure 13.

Gas Ingestion:

Extensive observations of a phenomenon which has been termed "gas ingestion" have been made by Ludwig and others (4). Other

investigators have also observed seal leaks or instabilities which may be related to gas ingestion. McGrew and McHugh (5) observed small leaks past the seal interface which were called "seal breaks". These leaks occurred at different Reynolds numbers for the various test fluids used, and the fluid surface tension was believed to be a significant property controlling the seal break point. King (11) referred to a similar observation as a "secondary leak," and indicated that the leak point was a function of speed and seal clearance and independent of seal pressure. An actual leak was not encountered in any of the tests represented by Figures 13 through 18. However, these tests were limited to Reynolds numbers below which the pressure gradient remained stable for the 15 to 20-minute period required to complete a single test point. During these apparently stable runs gas ingestion was usually observed when in the turbulent zone.

While the exact cause of gas ingestion is not known, the experimental observations made in the study support the hypothesis first stated by Ludwig (4), that the portion of the grooved shaft which is exposed to gas tends to pump the gas through the liquid-gas interface. The gas and liquid become mixed, thus having lower density, and the effective length of the seal increases slightly and the pressure gradient at the low pressure end of the seal is reduced. The centrifugal force in the fluid element causes the gas bubbles to migrate to the root of the groove.

The changes in seal performance caused by gas ingestion appear to depend upon the seal geometry. For example, seal 2B experienced a sudden rise in the sealing coefficient as shown in Figure 11. This rise was quite reproducible since a number of excursions were made through this region. Similar discontinuities were experienced with other test seals but to a smaller degree. Test seal 2B had a $\beta = 2.96$ and a γ of 0.296. Note in Figure 6 that the slope of the Λ versus β curve is very steep at low values of β . Thus, the accumulation of a small amount of gas at the root of the grooves could effectively reduce β and thus cause

Λ to suddenly rise. Seals 3B and 4B had β which were almost identical to that for 2B, yet these seals exhibited only slight increases in Λ during turbulent operation with gas ingestion. Seals 3B and 4B, however, had γ values of 0.504 and 0.692 respectively suggesting that the wider grooves may be less susceptible to gas accumulation at the thread root.

The question naturally arises as to whether the threaded shaft visco seal can operate for extended periods in the turbulent region when gas ingestion is occurring. Beginning with test seal 2B, a series of long term tests was initiated to gain insight into the problem of turbulent operation with gas ingestion. After a sufficient number of normal test runs are made to construct a performance curve, the seal is then subjected to one or more extended runs during which the effect of gas ingestion is observed. The seal test facility is not presently instrumented to make a quantitative measurement of gas ingestion. Rather, the pressure gradient in the seal is monitored to observe gradual changes which occur due to gas ingestion and to note the occurrence of pressure pulsations. Also changes in the sealing coefficient at the start and end of the extended test run are noted. A long term test is continued at the same operation conditions for a period of one to two hours or until severe pressure fluctuations are encountered. The long term test is run under two system conditions. First, the seal is operated with the pressure-regulated sealant supply system connected to the seal supply inlet. This condition produces a regulated pressure cavity having a volume of 239 in³. Then the seal is bled and operated with the supply system valved off which produces a trapped volume of 4.9 in³ and the pressure in the trapped volume is determined by the seal. The location of typical test points for long term tests are shown in Figures 16, 17, and 18.

Test seals 5, 6, and 7 were machined with small clearance dams on each end of the grooved section of the shaft as shown in Figure 12. These dams were expected to have two effects (12) : The inner restriction is expected to reduce the end effect and the outer restriction

is used to reduce gas ingestion. Following the tests with seals 5, 6, and 7 the clearance dams were removed and the test series repeated as 5D, 6D, and 7D. The results of the latter test series indicate that the end dams do not adversely affect the sealing coefficient, the end effect is reduced but not eliminated, and the severity of gas ingestion is reduced.

The study of gas ingestion continues. Though not complete, the observations which have been made may be summarized as follows:

1. Gas ingestion under laminar conditions is so low as to be undetectable in the present test facility in either normal or long term tests. The pressure gradient in the seal remains essentially constant and the sealing coefficient shows no significant change during extended runs. Operation of the seal is not affected by changing from a large regulated pressure supply to a small non-regulated trapped volume at the high pressure end of the seal.
2. As speed of operation is increased a point is reached at which gas ingestion can be observed. The Reynolds number at which gas ingestion is first encountered is not the same for each seal. The beginning and intensity of gas ingestion appears to be a function of seal geometry. After gas ingestion has started the operation of the seal may be affected by the volume and pressure regulation of the sealed cavity. Increasing the seal eccentricity ratio increases the intensity of gas ingestion.
3. The seal may operate in a stable manner with gas ingestion in the turbulent region. The maximum Reynolds number for stable operation is different for each seal. At low turbulent Reynolds numbers the seal performance changes during a long term test and may reach a stable operating point at a slightly higher sealing coefficient than is obtained in a normal short term run. See Figures 16 and 17. Under these conditions the pressure profile changes slightly and the effective seal length increases.
4. As the Reynolds number increases a point is reached at which the

Seal pressure gradient becomes unstable. When this occurs the seal performance may show one of perhaps several characteristics. The following have been identified:

- (a) The seal pressure profile experiences small fluctuations followed by a substantial drop in the sealed head after which a new stable operating curve is reached which permits operation at even higher Reynolds numbers. This is the characteristic exhibited by seal 2B (See Figure 16).
- (b) The seal pressure profile experiences small low frequency fluctuations accompanied by an increased effective seal length. The seal will then continue in a steady low frequency pressure fluctuation. If the sealed head is increased the effective seal length required exceeds the threaded length and a small more or less steady leak occurs and continues until the sealant supply is exhausted. If both speed and the sealed pressure are increased the small steady pressure fluctuations will increase in amplitude and frequency and the leak becomes pulsating.
- (c) The seal pressure profile exhibits a high frequency fluctuation with an increasing amplitude and a pulsating or spurting type of leak occurs. This condition is more characteristic when operating with a small trapped sealed cavity than when using the large regulated cavity.

Other Experimental Results:

In Figure 22 the experimental data taken from a series of tests conducted by King (11), along with Λ_T computed by equation (5) for the thread geometry used by King, are shown. The predicted sealing coefficient is in good agreement with the 5D and 2E series of data in the turbulent range. In the laminar range the predicted value for the sealing coefficient is smaller than King's extrapolated data curves. It is difficult to draw conclusions about the laminar zone since King obtained limited data in this region. In series 3C and 1B, the slopes

of King's experimental sealing coefficients are -0.527 and -0.466 , agreeing with the slopes of Λ_E in test series 5, 6, and 7. Data curves 5D and 2E have slopes in the turbulent region of -0.706 and -0.686 , which are similar to the values obtained in test with spindles 1 through 4B. From the experimental evidence reported in this work, along with data from King, the indication is that for small helix angles, 3° to 6° , the slope of the experimental sealing coefficient in the turbulent zone is approximately -0.46 . For helix angles of 7° to 14° corresponding to the range of optimum α recommendations of various investigators, the slope of the experimental sealing coefficient is approximately -0.70 .

VII. CONCLUSIONS AND RECOMMENDATIONS

Based on the analytical laminar analysis of the visco seal developed by Boon and Tal, equations were derived to predict the operation of the visco seal in the laminar and turbulent regions. The theoretical sealing coefficient developed by this analysis was compared with the experimental sealing coefficient obtained for ten experimental seals. In view of the observations and comparisons presented in this work, the following conclusions are indicated:

1. The laminar analysis leading to equation (3), upon which the present analysis is based, agrees more closely with the experimental results in laminar flow than any of the other analytical approaches considered.
2. The optimum screw geometry for laminar operation was not optimum for operation in the turbulent range.
3. For screws which must operate in both the laminar and turbulent range, smaller helix angles will provide more flexible operation in both regions.
4. In turbulent operation the visco seals having small helix angles operate with a greater degree of pressure stability.
5. Experimental Λ is essentially constant in laminar operation and in agreement with equation (3).

6. The transition Reynolds number at which Λ_E begins to decrease, decreases with increasing γ and is in agreement with equation (15).

7. The slope $d\ln \Lambda_E / d\ln Re_c$ in the turbulent range for helix angles of 9.67° and 14.5° was approximately -0.70 , while this value decreased to -0.43 for helix angles of 5.81° .

8. The phenomenon of gas ingestion will be encountered in turbulent operation. This condition was observed to be a function of shaft wetability, Reynolds number, seal geometry, and eccentricity ratio, but until more experimental data are obtained this relationship cannot be precisely defined. However, in order to minimize the effects of gas ingestion on Λ_E , large β and γ along with small α should be employed.

9. There exists a maximum Reynolds number beyond which long term operation is no longer feasible. However, more experimental data are needed before this value can be precisely defined.

10. The frictional losses in the visco seal may be described with existing equations. In the laminar range the theoretical predictions by equation (24) closely approximate the experimental data. During turbulent operation the experimental data presented by Smith and Fuller, represented by equation (25), are 0.951 times the value of the friction parameter observed in these tests. Therefore, the frictional losses in the seal may be calculated either for laminar or turbulent operation by:

$$F.P._l = 4\pi \phi / Re_c ,$$

and

$$F.P._o = 0.164\pi / Re_c^{0.43} .$$

11. For laminar operation Λ_E is essentially independent of the seal eccentricity ratio and under turbulent conditions the effect of seal eccentricity is slight.

A number of aspects of the visco seal operation in the turbulent range require further and more intensive study. Specific

problem areas requiring additional attention are as follows:

1. More experimental data in the turbulent range for Reynolds numbers higher than 3000 are needed in order to determine the maximum Reynolds number for long term seal operation.

2. The construction of test spindles with helix angles of approximately 5° and β 's in the range from 10 to 20 are needed in order to verify the observations concerning the relatively small effect of β on Λ_T for high Reynolds numbers and small helix angles.

3. A refinement of equation (5) for the calculation of Λ_T is needed in order to better describe the sealing coefficient in the turbulent region for small helix angles. It is realized that the two lines in Figure 3 may not be the best estimate for all geometric parameters.

4. An investigation of the effects of grooves located in the housing rather than along the shaft would be desirable. This is based on the observation by Ludwig (4) that a smooth shaft located in the threaded housing would lessen the effects of gas ingestion on Λ_E .

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NOMENCLATURE

a	Axial land width, in.
b	Axial groove width, in.
c	Radial clearance, in.
D	Seal diameter, in.
$F.P. = \frac{4\pi\phi}{Re_c}$	Friction parameter
f	Resistance coefficient, Figure 3
h	Groove depth, in.
L	Active seal length, in.
l	Axial threaded length of seal, in.
n	Number of thread starts
P	pressure, lbf./in. ²
q	Power loss, in. lbf./sec.
$Re_c = \frac{Uc\rho}{\mu}$	Reynolds number based on clearance
$t = \tan \alpha$	Tangent of the helix angle
U	Surface velocity of the seal, in./sec.
x	x coordinate in the direction of motion
y	y coordinate along shaft axis
z	z coordinate in the radial direction
α	Helix angle, degrees
$\beta = \frac{h+c}{c}$	Dimensionless parameter
$\gamma = \frac{b}{a+b}$	Dimensionless parameter
η	η coordinate
Λ	Sealing coefficient as presented in this work
μ	Absolute viscosity, lbf. sec./in. ²
ξ	ξ coordinate
ρ	Density, lbf. sec. ² /in. ⁴
τ	Wall shearing stress, lbf./in. ²
ϕ	Dissipation function

Subscripts

c	Denoting evaluation on clearance
E	Denoting experimental
h	Denoting film thickness
l	Denoting laminar flow
o	Denoting turbulent flow
p	Denoting pipe Reynolds number
T	Denoting theoretical
η	Denoting η direction
ξ	Denoting ξ direction

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REFERENCES

1. W. K. Stair, "The Visco Seal - A Survey, " Report ME 5-62-2, The University of Tennessee, March 1962, USAEC Report TID - 15987.
2. W. K. Stair and R. H. Hale, "Analysis of the Visco Seal, Part II - The Concentric Turbulent Case," University of Tennessee, Mechanical and Aerospace Engineering Research Report ME 66-587-7, June 28, 1966.
3. W. K. Stair, "Theoretical and Experimental Studies of Visco-Type Shaft Seals," Semi-Annual Progress Report, University of Tennessee, Mechanical and Aerospace Engineering Research Report ME 66-587-5, April 28, 1966.
4. L. P. Ludwig, T. N. Strom and G. P. Allen, "Gas Ingestion and Sealing Capacity of Helical Groove Fluid Film Seal (Viscoseal) Using Sodium and Water As Sealed Fluids," NASA TN D-3348, March, 1966.
5. J. M. McGrew and J. D. McHugh, "Analysis and Test of the Screw Seal in Laminar and Turbulent Operation," Journal of Basic Engineering, Trans. ASME, Series D, Vol. 87, 1965, pp. 153-162.
6. W. K. Stair, "Effect of Groove Geometry on Viscoseal Performance," to be presented at the ASME Winter Annual Meeting, Nov. 27 - Dec. 1, 1966, New York, N. Y. ASME Preprint 66-WA/FE-28.
7. E. F. Boon and S. E. Tal, "Hydrodynamische Dichtung fur rotierende Wellen," Chemie-Ing.-Technik., vol. 31, no. 3, January 31, 1959, pp. 202-212. Translation by R. Presser published by UKAEA, 1961, as DEG, Information Series 13 (CA).
8. S. I. Pai, "On Turbulent Flow Between Parallel Plates," Journal of Applied Mechanics, vol. 20, Trans. ASME, vol. 75, 1953, pp. 109-114.
9. W. K. Stair, "Analysis of the Visco Seal, Part I - The Concentric Laminar Case," Report ME 65-587-2, The University of Tennessee, January 18, 1965, NASA Report CR-285.
10. M. I. Smith and D. D. Fuller, "Journal-Bearing Operation at Superlaminar Speeds," Trans. ASME, vol. 78, 1956, pp. 469-474.

11. A. E. King, "Engineering Information Report on Tests Made With a Hydrodynamic Seal (Viscoseal) in Oil, Water, and Potassium," Westinghouse Electric Corporation, Aerospace Electrical Division, Report No. WAED 63.3 (Revision A), September, 1964.
12. Robert L. Johnson, NASA Lewis Research Center, Personal Communication, August 13, 1965.

TABLE I
DIMENSIONS AND GEOMETRIC PARAMETERS FOR TEST SEALS

Test Seal No.	D Diameter Inches	c Clearance Inches	h Groove Depth Inches	a Land Width Inches	b Groove Width Inches	α Angle Angle Degrees	β $\frac{(h+c)}{c}$	$\frac{b}{(a+b)}$
1	1.2430	0.0042	0.0101	0.0934	0.1596	14.5	3.38	0.631
2	1.2465	0.00235	0.0116	0.1176	0.0494	9.67	5.94	0.296
3	1.2461	0.00265	0.137	0.0828	0.0842	9.67	6.16	0.504
4	1.2461	0.00265	0.128	0.0514	0.1156	9.67	5.84	0.692
2B	1.2420	0.0047	0.00925	0.1176	0.0494	9.67	2.96	0.296
3B	1.2408	0.0053	0.01165	0.0828	0.0842	9.67	3.09	0.504
4B	1.2408	0.0053	0.01015	0.0514	0.1156	9.67	2.96	0.692
5	1.2455	0.00295	0.0179	0.1585	0.0607	5.81	6.96	0.277
6	1.2455	0.00295	0.0180	0.1070	0.1083	5.81	7.00	0.502
7	1.2455	0.00295	0.0180	0.0609	0.1600	5.81	7.00	0.724

TABLE II
COMPARISON OF TEST RESULTS

Test Seal Number	1	2	3	4	2B	3B	4B	5	6	7
Λ_T (Laminar)	11.73	13.09	12.39	12.86	17.54	14.29	17.20	14.08	12.44	14.10
Λ_E (Laminar)	10.98	12.86	12.25	11.62	17.90	16.02	14.40	14.92	11.33	11.75
$\frac{\Lambda_E}{\Lambda_T}$ (Laminar)	0.93	0.98	0.99	0.90	1.02	1.12	0.84	1.06	0.91	0.83
$-\frac{d \ln \Lambda_E}{d \ln Re_c}$ (Turb.)	0.698	0.731	0.695	0.676	0.709	0.709	0.709	0.414	0.430	0.438

P_a = ambient pressure

P_s = system pressure

Fig. 1 Basic Elements of a Visco Seal

Fig. 2 Comparison of Theories and Experiment for the Laminar Visco Seal

Fig. 3 Resistance Coefficient versus Reynolds Number for Pipe Flow

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Fig. 4 Theoretical Sealing Coefficient as a Function of β , Re_c , and γ for $\alpha = 14.5^\circ$, $c = 0.003$ in., and $D = 1.25$ in.

Fig. 5 Theoretical Sealing Coefficient as a Function of α and β for $c = 0.003$ in., $D = 1.25$ in., $\gamma = 0.5$, and $Re_c = 100$.

Fig. 6 Theoretical Sealing Coefficient as a Function of α and β for $c = 0.003$ in., $D = 1.25$ in., $\gamma = 0.5$, and $Re_c = 1000$.

Fig. 7 Theoretical Sealing Coefficient as a Function of α and β for $c = 0.003$ in., $D = 1.25$ in., $\gamma = 0.5$, and $Re_c = 5000$.

Fig. 8 Theoretical Sealing Coefficient versus Re_c for $\alpha = 5.81^\circ$ and 20.15° with γ of 0.3 and 0.7

Fig. 9 Visco Seal Test Facilities

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Fig. 10 Schematic Diagram of the Visco Seal Test Section.

Fig. 11 Typical Pressure and Temperature Gradients in the Visco Seal.

Fig. 12 Visco Seal Test Spindles.

Fig. 13 Theoretical and Experimental Sealing Coefficients for Seal 1.

Fig. 14 Theoretical and Experimental Sealing Coefficients for Seals 2, 3, and 4.

Fig. 15 Theoretical and Experimental Sealing Coefficients for Seals 5, 6, and 7.

Fig. 16 Theoretical and Experimental Sealing Coefficients for Seals 2 and 2B.

Fig. 17 Theoretical and Experimental Sealing Coefficients for Seals 3 and 3B.

Fig. 18 Theoretical and Experimental Sealing Coefficients for Seals 4 and 4B.

Fig. 19 Theoretical and Experimental Friction Parameters for Seals 1, 2, 3, and 4.

Fig. 20 Theoretical and Experimental Friction Parameters for Seals 2B, 3B, and 4B.

Fig. 21 Theoretical and Experimental Friction Parameters for Seals 5, 6, and 7.

Fig. 22 Comparison of King's Experimental Data with the Theoretical Sealing Coefficient

SECTION 6

TECHNOLOGY III - BEARINGS AND MATERIALS

EXPERIMENTS WITH HYDRODYNAMIC JOURNAL BEARINGS
 OF VARIOUS MATERIALS AND DESIGNS IN SODIUM N_a
 AT TEMPERATURES TO 800° F

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ABSTRACT

Experiments were conducted with 1.5 inch diameter hydrodynamic journal bearings in liquid sodium at 500° and 800° F, speeds to 12 000 rpm and unit loads to 31 psi. The stability characteristics of four different geometries and the wear and seizure properties of several material combinations were investigated. Tilting pad bearings were most stable. Combinations of a cobalt alloy with nickel alloys or with a titanium carbide cermet showed the best wear and seizure properties.

INTRODUCTION

Extended space exploration missions of the future will necessitate long periods of continuous, reliable operation of the power generation system incorporated in the space vehicle. Power levels on the order of 30 000 watts to the million-watt range are anticipated (ref. 1). At the present time it appears that a turbogenerator system employing a liquid metal as the working fluid is the most favorable system for the high power levels desired. Requirements for light weight, high reliability, and minimum complexity dictate the use of a hermetically sealed pump with process fluid lubricated bearings to circulate the fluid through the entire system, including the space radiator.

Fluid film bearings have been selected over rolling element bearings because the former bearing type maintains a full fluid film more easily. This eliminates or minimizes the rubbing contact problem which is usually present in rolling element bearings. Also, the materials problem in fluid film bearings is much simpler because of less stringent hardness requirements.

Fluid film bearings do, however, have some disadvantages over rolling contact bearings, namely, increased power losses, greater breakaway torques, and the need for a separate bearing to carry thrust loads. In addition, the disadvantage of principal interest here is the tendency of journal bearings to exhibit instability under light or zero load conditions that will exist in a space vehicle in a zero gravity environment. Instability here refers to half-frequency whirl or the tendency of the journal center to orbit about the bearing center at an angular velocity about half that of the journal around its own center.

Materials chosen as bearing and journal pairs in an alkali metal system must have good resistance to corrosion in the liquid, good wear and seizure resistance, and a low coefficient of friction to insure reasonable breakaway torque. A survey of existing literature on usefulness of materials for bearings in liquid alkali metals (refs. 2 to 4) and the need for using materials of particular expansion coefficients led to the selection of the bearing and journal materials reported herein.

The object of this investigation, the data of which are reported fully in reference 5, was to determine the stability characteristics of four bearing configurations and which bearing and journal material combinations had the best compatibility and wear and seizure resistance in liquid

sodium at temperatures to 800° F.

The bearings were submerged in the liquid sodium and operated hydrodynamically. Bearings with $1\frac{1}{2}$ -inch bore by $1\frac{1}{2}$ -inch length were tested in liquid sodium at radial loads from 0 to 70 pounds and journal speeds to 12 000 rpm at 500° and 800° F. Bearing friction torque at varying speeds and loads was recorded and compared with theoretical values under both laminar and turbulent flow conditions.

APPARATUS

Bearing Test Rig

A cutaway view of the bearing rig, shown in figure 1(a), illustrates the configuration of the rig and its loading mechanism. The test shaft was positioned vertically so that normal gravity forces would not act on the journal. A 15 horsepower d. c. motor powered the test shaft through a 7.5 to 1 ratio gear box. The sodium test vessel is located immediately below the main support bearing housing and floats between the upper and lower gas bearings as shown schematically in figure 1(b). Two semicircular wheels connected by a cable belt comprise the radial loading system. Radial load was applied by means of an air cylinder between the two semicircular wheels, one of which pivots on a knife edge. Bearing torque is measured by a force transducer. Vertical positioning of the test vessel was achieved by means of a vertically mounted air cylinder located below the vessel.

The test shaft was mounted on two support ball bearings which were preloaded to about 40 pounds by means of a wave spring shown in figure 1(a). This preload was necessary to insure a minimum amount of test shaft run-out. Cooling fins were mounted on the test shaft immediately below the

bottom support bearing to dissipate heat and prevent excessive lower support bearing temperatures due to soak back from the high temperature sodium in the test vessel. The test journal was mounted and keyed to the bottom end of the test shaft. The test bearing was mounted in a housing in the test vessel as shown schematically in figure 1(b).

Liquid sodium at 400⁰ F was introduced to the test vessel and heated to the desired test temperature by means of an induction heater. The induction heater coil around the test vessel (fig. 1(b)) does not come into physical contact with the outer surface of the test vessel and therefore does not inhibit its free swinging motion.

A drain was provided at the bottom of the test vessel to facilitate draining of contaminated sodium.

Sodium Supply System

The sodium supply system was a noncirculating, once-through system composed of the following major components: the supply tank, the supply line, the sodium filter, the filter by-pass line, and the control valve.

The supply tank was sized to contain 20 gallons of sodium and was equipped with a fill valve, dual thermowells, diffusional cold trap, pressure transmitter, vapor trap, and a sodium supply line. The liquid metal was supplied to the bearing test rig by means of a differential pressure between the tank and the test rig. A diffusional cold trap was provided to control the sodium oxide content during operation of the sodium system. The cover gas for pressurizing and venting the supply tank was passed through a wire-mesh demister (vapor trap) which prevented any sodium vapors from getting into the cover gas system.

A micrometallic filter was provided in the supply line to the bearing rig permitting continuous filtering of all supply sodium. The porous micrometallic filter unit was so designed as to permit replacing the filtering element.

Bearings and Journals

Hydrodynamic bearings of four configurations were tested fully immersed in liquid sodium. Bearings with two axial grooves, three axial grooves, a herringbone groove journal with a plain bearing, and tilting pad bearings with three pads were evaluated. One of the three groove bearings was run with an axial flow pump attached to the test shaft, pressure feeding the test bearing through a hole in the test journal.

The bore and length of the bearings in all cases were nominally $1\frac{1}{2}$ inches. The journal outside diameter and bearing inside diameter were machined to a 4 to 8 microinch finish, rms.

A three pad configuration was chosen for the tilting pad bearings because it affords greater load capacity than a configuration of more than three pads. Also, it is much easier to accurately align all pivot points on the common pivot circle center with a three pad configuration than one with more than three pads. The load was applied symmetrically between two support points (pivots).

The bearing and journal materials were: Stellite Star J, Mo-0.5Ti, Hastelloy X, Inconel, and titanium carbide (K184B). The composition and hardnesses of these materials are given in table I.

Instrumentation

Dual chromel alumel thermocouples were attached to the test bearing back and into the liquid sodium in the test vessel. The induction heater

was controlled by one of the dual thermocouples in the sodium bath.

Two capacitance probes, which measured the movement of the test vessel during a test run, were mounted outside of the test vessel on the vessel cover, 90° from each other. The signal from the probes was fed through displacement meters to an x-y display in an oscilloscope where the actual pattern of motion of the test vessel could be observed. The orbital frequency of the test vessel was measured by means of a frequency counter.

Shorting probes were used in the test vessel to indicate the level of the sodium. These probes would short out when sodium came into contact with them, thereby either lighting a level indicator light or closing the main sodium supply valve automatically.

Test shaft speed was measured with a magnetic pickup head mounted in close proximity to a six-toothed gear on the test shaft. The signal from the pickup was displayed on a four-channel frequency counter.

PROCEDURE

Pretest Preparation for Two and Three Axial Groove Bearings

Prior to each test run, the test bearing was pressed into its housing with a slight interference fit. The bearing was then machined in place to a predetermined inside diameter at room temperature that would result in the desired bore size at test temperature. Nine bore gage readings, each accurate to within 0.001 inch, were averaged and used as a measure of the bearing bore. The outside diameter of the mating journal was then ground to a size that would result in the desired clearance for the test bearing. To insure a minimum amount of runout, the journal outside diameter was machined to within 0.0002 inch concentricity with its inside diameter.

The test bearing housing was then assembled into the test vessel which was carefully raised into position around the test journal on the shaft by means of the lower air cylinder.

Pretest Preparation for the Tilting Pad Bearings

The radii of the three pads of a bearing assembly were checked after delivery from the vendor to insure an accurate geometry. The pads were assembled into an annular housing by means of a threaded pivot and nut arrangement. Desired preload was obtained by adjustment of the threaded pivots until the bearing surfaces of the pads made intimate contact around a presized set-up plug. The plug was then removed and the tilting pad housing was assembled in a manner like that for the axial groove bearings.

General Pretest Preparation

After the bearing and journal were assembled and the test vessel raised to its run position with the upper and lower gas bearings turned on, the test vessel was filled with alcohol and drained as a final cold cleaning procedure. The test vessel was then purged with argon and a cover gas of argon was supplied to the test vessel throughout the test. The test vessel was preheated to 500⁰ F and liquid sodium at about 400⁰ F was introduced into the test vessel through the inlet port under about 5 pounds per square inch pressure from the 20-gallon supply system. Sodium flowed into the test vessel until it made contact with the liquid level probe which automatically closed the main sodium supply valve preventing overfilling.

With the bearing and journal completely submerged in sodium, the test vessel was heated to 500⁰ F and allowed to soak for approximately

4 hours to remove remaining contaminants. The vessel was then drained and refilled with clean sodium and the bearing test was started after equilibrium conditions had been attained.

Test Procedure

Two types of tests were conducted, one with load and the other at zero load conditions. Under zero load conditions the semicircular wheel that pivots on a knife edge was removed along with its loading cable. The force transducer for torque measurement was attached directly to the semicircular wheel on the test vessel.

Shaft speed was increased in 1000 rpm increments from 3000 rpm to a maximum of 12 000 rpm. In the loaded bearing tests, the loads were changed as required to fulfill the purpose of the particular test run. In some tests the load had to be changed to maintain bearing stability and in others the load was left unchanged to observe the effect of prolonged instability in a bearing. The time interval between speed and load changes varied, but was of sufficient duration so as to allow the friction torque to stabilize. Speed, load, bearing temperature, and bearing friction torque were recorded at each speed and load condition.

Test vessel movement was noted by observation of the oscilloscope screen in an attempt to identify bearing instability at each test interval.

A test was terminated for the following reasons: (a) if the bearing seized, (b) if the bearing showed indications of imminent failure by erratic or excessively high torque, (c) if the test vessel no longer floated freely on its gas bearings, or (d) if the desired objective of the test had been attained.

RESULTS AND DISCUSSION

Two and Three Axial Groove Cylindrical Bearings

The results of tests on 14 two and three axial groove bearings are summarized in table II. The results of these tests, which were conducted over a speed range to 11 000 rpm at unit loads to 26.7 pounds per square inch, were generally poor. Some rig problems were encountered in the early phase of the test program. These problems included test vessel cocking, which caused excessive wear at the ends of one bearing and two failures and contamination of the sodium which may have caused four failures due to particle ingestion into the test bearing clearance area. In three instances, sodium migrated into the upper gas bearing causing the test vessel to cock, producing erratic torque readings.

A typical failure caused by contaminant particle ingestion is shown in figure 2. This type of failure resulted in a rather pronounced gouge in the bearing and journal. It was after this type of failure that a cleaning procedure was adopted in which the test bearing and journal were soaked for 4 hours in the sodium bath at 500⁰ F in the test vessel. The contaminated sodium was then drained and a clean supply of filtered sodium was introduced into the test vessel before a test was begun.

In some instances, at the higher speeds, excessive sloshing of sodium in the test vessel caused some sodium to migrate up into the top gas bearing of the test vessel. These sodium droplets would solidify and test vessel cocking would result causing bearing failure.

The generally poor results obtained with the two and three axial groove bearings was principally due to their inherent instability which will be discussed later.

Herringbone Journal Bearing

Table III summarizes the results of experiments with two herringbone groove journals running in plain cylindrical bearings. These bearings, one of which is illustrated in figure 3, were tested in an effort to determine their stability characteristics.

The two herringbone journal bearings were run over a speed range of 4000 to 12 000 rpm at loads of 4 to 20 psi at sodium temperatures of 500⁰ and 800⁰ F. Higher bearing torques were observed than for the two and three axial groove bearings due to the pumping action of the herringbone grooves.

Bearing MP-3 (table III) ran for 490 minutes and showed very little wear (fig. 3). The slight scoring of the journal was probably due to a small contaminant particle. Bearing MP-4 seized after 90 minutes due to a line heater expansion pushing the test vessel and overloading or cocking the bearing.

Tilting Pad Bearings

Five tilting pad bearings having a three-pad configuration were tested in sodium at 500⁰ and 800⁰ F at speeds to 12 000 rpm and loads to 70 pounds (31.1 psi). These results are summarized in table IV. Of the five bearings tested, one seized due to overloading (bearing number T-2, table IV), and one seized due to a combination of insufficient clearance and incompatible bearing and journal material (bearing number T-1A, table IV). The wear on the unloaded pad of bearings T-1 and T-2A1, recorded in table IV, was probably due to insufficient preload which resulted in pad flutter.

Table IV lists the various preload coefficients that were investigated. Although inconclusive, because of the limited number of tests, it appeared

that tilting pad bearings with large machined in radial clearances (0.0028 to 0.0036 in.) and high preload coefficients (0.50 to 0.72) yielded the best test results, as far as wear to pads is concerned.

Figure 4 shows a tilting pad bearing that failed due to overloading. Pads B and C were the loaded pads, with most of the wear shown on the leading edge of pad C. Analysis of the wear pattern on the outer edges of all three pads would indicate that the pads had rolled. The surface damage to the unloaded pad A was attributed to the failure of this bearing when wear debris was transferred completely around the bearing just prior to seizure.

Bearing torque data obtained from tilting pad bearings at zero load and at 10-pound radial load are shown in figures 5(a) and (b), respectively. These data are shown compared to the theoretical laminar torque calculated from the Petroff equation,

$$T_l = \frac{\mu \pi^2 D^3 L N}{120 C_r}$$

and the theoretical turbulent torque calculated from the equation given by Smith and Fuller (ref. 6) where

$$T_t = T_l (0.039 \text{ Re}^{0.57})$$

The theoretical torques are for a full circular bearing with zero eccentricity, so they are only rough approximations for a tilting pad bearing, and only for zero preload. Bearing torque values indicated that turbulent flow conditions prevailed over the greater part of the speed range, due primarily to the low sodium viscosity. The experimental transitional speed, where the bearing passes from the laminar to the turbulent regime, occurred at a higher speed than that predicted by theory (critical transi-

tional speed, N_T). This may be due to the fact that this transition occurs over a range of speeds rather than at one definite speed. Similar results with the transition speed were reported in reference 7.

No abrupt increase in torque readings was noticed when turbulent flow conditions were approached. The rather gradual increase in torque with speed made it difficult to specify exactly the speed at which full turbulence was attained.

Bearing Instability

The bearing instability of principal concern here is half-frequency whirl. Figure 6 shows oscilloscope traces of bearing motion obtained with a two axial groove bearing in sodium at 500° F with a 10-pound radial load. At 4100 rpm the trace indicated stable bearing operation (fig. 6(a)). When the speed was increased to 5000 rpm, however, the increase in attitude angle was sufficient to sustain half-frequency whirl. The whirl pattern observed on the oscilloscope screen is shown in figure 6(b). If the bearing is allowed to operate unstably, the supporting film between the bearing and journal soon breaks down, and the bearing eventually fails.

One of the most undesirable characteristics of the two and three groove bearings was their instability. The result of such instability is graphically shown in figure 7. Test bearing J-2 is shown in this figure after 257 minutes of operation at 11 psi unit bearing load in 500° F sodium under half-frequency whirl conditions. The excessive wear shown is the result of unstable bearing operation.

Of the 14 two and three groove cylindrical bearings tested, five showed excessive wear due to half-frequency whirl and one seized because of this instability.

One of the three groove bearings (bearing number M-9, table II) was loaded sufficiently at each speed throughout its evaluation to keep it running stably. After 290 minutes of running time in 500⁰ F sodium, the bearing was removed and no measurable wear was present on either the journal or the bearing. The maximum load on this bearing was 26.7 psi at a maximum speed of 10 000 rpm indicating that a three groove cylindrical bearing will run successfully in sodium if properly loaded to suppress half-frequency whirl.

Theory indicates that herringbone groove bearings operate at considerably lower attitude angles than do smooth bearings resulting in more favorable stability characteristics. This type of bearing assembly was indeed more stable than the two and three groove bearings and plain journal assemblies. However, the herringbone groove bearing assembly did show evidence of half-frequency whirl at low load conditions. A more judicious design of the herringbone groove journal might lead to a bearing that would be stable even at zero load conditions. Such a bearing has been run at zero load in air up to 60 000 rpm without any evidence of half-frequency whirl (ref. 8).

Figure 8 shows the relative stability of the four different cylindrical bearing configurations tested. The two and three axial groove bearing configurations were the least stable of the four since they required the highest load at any specific speed to maintain stable operation. The plain bearing with a herringbone groove journal was the most stable of the four since it required the lowest load at a given speed to keep it running stably. Axial grooved bearings appeared to require a linear increase in load with

speed to maintain stable operation whereas the herringbone groove bearing was stable at 25-pound load at speeds of 7000 to 10 000 rpm.

Experimental data on the threshold of instability of two and three axial groove bearings correlated well with the theoretical curves reported in reference 9. Figure 9 shows theoretical curves of Sommerfeld number plotted against the dimensionless critical rotor mass for a 100° partial bearing and full circular journal bearing. The data points for the two axial groove and three axial groove bearings generally fall between the curves indicating good correlation.

The procedure for determining the threshold speed is to calculate the dimensionless rotor mass

$$\frac{\sqrt{C_r MW}}{\mu DL \left(\frac{R}{C_r}\right)^2}$$

Enter figure 9 with this value and determine the corresponding Sommerfeld number using the appropriate bearing curve. The speed corresponding to this Sommerfeld number is the threshold speed, that is, the rotor speed at onset of instability.

The tilting pad bearings were the most stable of the four configurations tested. However, half-frequency whirl was observed and measured with a frequency counter on bearing number T-3 at 12 000 rpm and zero load. Tilting pad bearings T-3 and T-2A1 were run up to 11 000 rpm at zero load without exhibiting any half-frequency whirl instability.

Although not tested at zero load, the remaining three bearings of the tilting pad group showed good stability at light loads of 4.5 psi to speeds of 11 000 rpm.

Material Compatibility

Table V lists the bearing and journal material combinations that had good wear and seizure properties in sodium to 800° F.

Stellite Star J material mated with Hastelloy X, titanium carbide (K184B), or Inconel showed the best wear and seizure properties. Also titanium carbide (K184B) mated with Mo-0.5Ti, showed excellent promise. Materials such as Hastelloy X and Inconel having high nickel content, were prone to catastrophic seizure when paired in a bearing and journal combination.

Figure 10 shows the results of a seizure of a three axial groove bearing due to an incompatible bearing and journal material combination. The bearing material was Mo-0.5Ti and the journal, Hastelloy X. The surface of the journal shows a galling typical of this type of failure.

Another example of the results of pairing incompatible materials is shown in figure 11. The bearing material was Inconel and the journal material was Hastelloy X, both high in nickel content. Seizure resulted in galling of the pads and journal. The bearing radial clearance at the pivots of 0.0003 inch would not allow many particles to pass through the bearing without initiating surface damage. With a poor combination of materials, galling, severe surface damage, and possible seizure quickly follow the initial surface damage.

Figure 7 shows an example of mating materials with good compatibility. The combination of Stellite Star J and Hastelloy X material showed excellent seizure resistance since this bearing and journal combination did not seize even after prolonged operation with whirl present which produced the excessive wear shown in the figure. Figure 12 shows a photo-

graph of the pivot arrangement used on the tilting pad bearings: a sphere against a flat. Figure 13 shows a typical example of surface damage to the pivot that was observed in some tests, even after very short runs at light load. This damage occurred with both the titanium carbide (K162B) and the Stellite Star J pivot materials which were always mated against themselves. As shown in table IV, bearing number T-1 ran for 175 minutes with a maximum load of 15 pounds on the pivot producing a Hertz stress of only 52 600 psi which was sufficient, however, to cause slight pivot surface damage.

SUMMARY OF RESULTS

A series of hydrodynamic journal bearing experiments was run in sodium at 500° and 800° F at speeds to 12 000 rpm and loads to 70 pounds. Four different configurations were tested; cylindrical bearings with two and three axial grooves, plain cylindrical bearings with herringbone groove journals, and tilting pad bearings with three pads. The bearing bore in all cases was 1.5 inches and all bearings had a length to diameter ratio of 1. The following results were obtained:

1. The tilting pad bearings were the most stable. Following in order were: (a) plain cylindrical bearing with a herringbone groove journal, (b) three axial groove bearing, pressure fed from an axial shaft pump through a hole in the journal, and (c) three and two axial groove bearings.

2. Stellite Star J material mated with Hastelloy X titanium carbide (K184B), or Inconel showed the best wear and seizure properties. Also titanium carbide (K184B) mated with Mo-0.5Ti showed excellent promise. Materials having high nickel content, such as Hastelloy X and Inconel,

were prone to catastrophic seizure when paired in a bearing and journal combination.

3. Data on the threshold of instability of circular, grooved bearings obtained from this investigation compared favorably with predicted theoretical values published in reference 9.

4. Actual bearing torque values (for the tilting pad bearings) agreed favorably with turbulent flow theory, indicating that turbulent flow conditions prevailed over the greater portion of the tested speed range, due primarily to the low sodium viscosity. Critical transitional speed from laminar to turbulent flow occurred at a higher speed than the theoretical Taylor criterion predicts. Similar results were reported in reference 7.

5. No abrupt increase in torque readings was noticed when turbulent flow conditions were approached, indicating that the transition from laminar to turbulent flow occurs gradually over a range of Reynolds numbers rather than suddenly. The rather gradual buildup of torque made it difficult to specify exactly at what speed turbulence was initiated.

6. Surface damage of the tilting pad bearing pivots was observed in some tests even after very short runs at light pivot loads. The pivot configuration used in this investigation was a spherical surface against a flat and the materials were titanium carbide (K162B) against itself and Stellite Star J against itself.

7. Although inconclusive, because of the limited number of tests, it appeared as though the tilting pad bearings with large machined-in clearances (0.0028 to 0.0036 in. radial) and high preload coefficients (0.50 to 0.72) yielded the most favorable test results. (These bearings had radial clearances at the pivots of 0.0003 to 0.0017 in.) The tilting pad

bearings were the most stable of the four configurations tested, however, bearing instability in the form of half-frequency whirl was observed in one bearing at 12 000 rpm at zero load conditions.

APPENDIX - SYMBOLS

C_r radial bearing clearance, in.

D bearing diameter, in.

g acceleration due to gravity, in./sec²

L bearing length, in.

M rotor mass per bearing, W_r/g , (lb)(sec²)/in.

N journal speed, rpm

N' journal speed, rps

N_T transitional journal speed from laminar to turbulent regime,

$$\frac{41.1 \nu}{\pi D C_r} \sqrt{\frac{D}{2 C_r}}, \text{ rps}$$

R bearing radius, in.

Re Reynolds number, $\pi D N \rho C_r / 60 \mu$, dimensionless

S Sommerfeld number, $\frac{\mu N' D L}{W} \left(\frac{R}{C_r} \right)^2$

T_l Petroff's torque for laminar flow, zero load: $T_l = \mu \pi^2 D^3 L N / 120 C_r$

T_t turbulent torque: $T_t = T_l (0.039 Re^{0.57})$ (Smith and Fuller, ref. 6)

W bearing load, lb

W_r load due to rotor mass, lb ($W_r = Mg$)

$$\frac{\sqrt{C_r MW}}{\mu DL \left(\frac{R}{C_r}\right)^2}$$

dimensionless critical rotor mass

μ	absolute lubricant viscosity, lb sec/in. ² (reyns)
ν	kinematic viscosity, centistokes (in. ² /sec)
ρ	lubricant mass density, lb-sec ² /in. ⁴

REFERENCES

1. Slone, Henry O.; and Lieblein, Seymour: Electric Power Generation Systems for Use in Space. Paper presented at Second Int. Cong., Int. Council of Aero Sci., Zurich, Switzerland, Sept. 12-16, 1960.
2. Basham, S. J.; Stang, J. H.; and Simons, E. M.: Corrosion Screening of Component Materials for NaK Heat Exchange Systems. Battelle Memorial Inst., Columbus, Ohio. Preprint 24 Session XXII. Contributed by American Institute of Chemical Engineers, 1958.
3. Apkarian, H.: Investigation of Liquid Metal Lubricated Bearings. Rep. 50GL231, General Electric Co., Nov. 27, 1950.
4. Cook, W. H.: Corrosion Resistance of Various Ceramics and Cermets to Liquid Metals. Rep. ORNL - 2391, Oak Ridge Nat. Lab., June 15, 1960.
5. Schuller, Fredrick T.; Anderson, William J.; and Nemeth, Zolton N.: The Operation of Hydrodynamic Journal Bearings of Various Materials and Designs in Sodium at Temperatures to 800⁰ F. (NASA TN to be published.)

6. Smith, M. I.; and Fuller, D. D.: Journal Bearing Operation at Super-Laminar Speeds, Trans. ASME, vol. 78, 1956.
7. Stahlhugh, P. H.; and Trippett, R. J.: Liquid Metal Bearing Performance in Laminar and Turbulent Regimes. Trans. ASLE, vol. 5, no. 2, Nov. 1962.
8. Malanoski, S. B.: Experiments on an Ultra-Stable Gas Journal Bearing. MTI-65TR37. Mechanical Technology, Inc., Nov. 1965.
9. Churchill, A. V.: Rotor Bearings Dynamics Design Technology Part III - Design Handbook for Fluid Film Type Bearings. Tech Rept. AFAPL TR-65-45, Part III, May 1965, Wright-Patterson Air Force Base, Ohio.

TABLE I. - NOMINAL COMPOSITION AND HARDNESS OF BEARING AND JOURNAL MATERIALS

Materials	Rockwell hardness, room temperature	Composition												
		Al	C	Cr	Co	Fe	Mn	Ni	Si	W	Ti	TiC	Mo	Others
Stellite Star J	C-62	--	2.5	32	40.5	3	----	2.5	----	17	---	--	----	2.5
Hastelloy X	B-87	--	0.2	22	1.5	18	----	47.2	----	0.6	---	--	9	1.5
Mo-0.5Ti	B-87	--	---	--	----	--	----	----	----	----	0.5	--	99.5	---
Inconel	B-75 to B-95	--	0.04	15	----	7	0.35	78	0.20	----	---	--	----	---
Titanium Carbide (K-184B) (nickel bonded)	C-67	3	----	3	----	--	----	40	----	----	---	50	4	---

TABLE II - RESULTS OF 2 AND 3 GROOVE BEARING TESTS IN SODIUM

Bearing number	Bearing material	Journal number	Journal material	Bearing type	Test temperature, °F	Measured ^a radial clearance at test temperature, in	Journal speed range tested, rpm	Unit load range tested, psi	Total test time, min	Observed instability	Remarks
M-1	Mo-0 5Ti	K-6	TiC	2 Groove	500	0.0010	3000 to 8000	4.5 to 20.0	470	At various speed and load conditions	Bearing and journal showed moderate wear due to half frequency whirl. Increasing load stabilized bearing. Bearing did not seize. Ran deliberate half-frequency-whirl operation at momentary intervals.
M-7	Mo-0 5Ti	K-B	TiC	2 Groove	800	0.0010	4000 to 11 000	4.5 to 20.0	703	None present	Bearing showed moderate wear at both ends indicating the test vessel had cocked. Bearing did not seize. Sodium had migrated to top gas bearing of test vessel.
J-1	Stellite Star J	HX-B	Hastelloy X	3 Groove	500	0.0013	5000	22.0	5	Not set up to observe	Contaminant particle score bearing and caused seizure almost immediately after start of test.
J-2	Stellite Star J	HX-C	Hastelloy X	3 Groove	500	0.0012	5000 to 7000	11.0	257	At 7000 rpm and 11.0 psi load	Bearing and journal showed severe wear but no seizure resulted. Held bearing at 7000 rpm and 11.0 psi deliberately for 90 minutes to observe results of half-frequency-whirl operation.
J-3	Stellite Star J	HX-D	Hastelloy X	3 Groove	500	0.0011	5000 to 7000	9.0	260	Not set up to observe	Bearing and journal showed heavy wear probably due to unstable operation at light load. Bearing did not seize.
HX-B	Hastelloy X	J-5	Stellite Star J	3 Groove	500	0.0011	5000	11.0	30	Not set up to observe	Bearing seized after 30 minutes apparently due to contaminant particle.
HX-D	Hastelloy X	J-6	Stellite Star J	3 Groove	500	0.0015	3000 to 7000	4.5 to 8.9	460	Not set up to observe	Bearing and journal showed heavy wear but did not seize. Light loads probably caused the bearing to run unstably.
HX-5	Hastelloy X	J-3	Stellite Star J	3 Groove	800	0.0008	5000	Not recorded	Not recorded	Too short a run to observe	Bearing seized immediately, probably due to contaminant particle. Bearing showed a score mark completely around on one end.

^aBy observation of oscilloscope pattern as the test vessel was moved by hand, to the limits of its outward motion, in a circular orbit about the stationary journal.

TABLE II - Concluded. RESULTS OF 2 AND 3 GROOVE BEARING TESTS IN SODIUM

Bearing number	Bearing material	Journal number	Journal material	Bearing type	Test temperature, °F	Measured ^a radial clearance at test temperature, in.	Journal speed range tested, rpm	Unit load range tested, psi	Total test time, min	Observed instability	Remarks
K-3	TiC	J-7	Stellite Star J	3 Groove	500	0.0017	5000	4.5	315	Not set up to observe	Bearing and journal showed heavy wear but did not seize. Bearing apparently was operating unstably because of light load.
M-9	Mo-0.5Ti	J-11	Stellite Star J	3 Groove	500	0.0010	5000 to 10 000	11.0 to 26.7	290	At various speed and load conditions	Bearing and journal showed discoloration but no measurable wear. Bearing was purposely loaded sufficient to keep it running stably. Also ran deliberate half-frequency-whirl operation at momentary intervals.
M-12	Mo-0.5Ti	J-1	Stellite Star J	3 Groove	800	0.0011	5000 to 10 000	13.3 to 22.2	355	None present	Bearing was loaded sufficient to keep it running stably, however, bearing seized at 8000 rpm during shutdown 1/8 inch gall mark around bearing possibly caused by contaminant particle, otherwise light wear.
M-10	Mo-0.5Ti	J-2	Stellite Star J	3 Groove	800	0.0010	4000 to 7000	15.5 to 17.8	242	None present	Bearing seized at 7000 rpm and 17.8 psi load. Two small wear areas on each end of bearing indicating the test vessel had cocked.
M-11	Mo-0.5Ti	HX-7	Hastelloy X	3 Groove	800	0.0013	Not recorded	0	Not recorded	Present immediately	Bearing seized immediately at 0 load with definite indication of half-frequency whirl. Wear on both ends of bearing was observed.
HX-7	Hastelloy X	J-8	Stellite Star J	3 Groove	500	0.0012	4000 to 9000	13.3 to 22.2	647	At various speed and load conditions	A shaft screw pump forced sodium into the bearing through a hole in the journal. Bearing showed small wear area in loaded zone probably due to deliberate half-frequency-whirl operation at momentary intervals.

^aBy observation of oscilloscope pattern as the test vessel was moved by hand, to the limits of its outward motion, in a circular orbit about the stationary journal.

TABLE III. - RESULTS OF PLAIN CYLINDRICAL BEARING AND HERRINGBONE GROOVE JOURNAL TESTS IN SODIUM

Bearing number	Bearing material	Journal number	Journal material	Bearing type	Test temperature, °F	Measured ^a radial clearance at test temperature, °F	Journal speed range tested, rpm	Unit load range tested, psi	Total test time, min	Groove angle (measured from a perpendicular to the journal axis) (β)	Number of grooves and lands	Width of grooves and lands, in.	Depth of grooves, in.	Observed instability	Remarks
MP-3	Mo-0.5Ti	K-14	TiC	Plain bearing. Herringbone groove journal	500	0.0013 (on lands)	4000 to 12 000	^b 0 to 17.8	490	33°	20	0.064	0.0014	At various speed and load conditions	Light wear on bearing and journal, due to deliberate half-frequency-whirl operation at momentary intervals.
MP-4	Mo-0.5Ti	K-A	TiC	Plain bearing. Herringbone groove journal	800	0.0008 (on lands)	5000 to 7000	^b 0 to 20.0	90	33°	20	0.064	0.0014	At 5000 rpm and 0 load	Bearing seized due to overload caused by heater expansion pushing test vessel increasing radial load.

^aBy observation of oscilloscope pattern as the test vessel was moved by hand, to the limits of its outward motion, in a circular orbit about the stationary journal.

^bZero load only momentary since half-frequency-whirl occurred.

TABLE IV. - RESULTS OF TILTING PAD BEARING TESTS IN SODIUM

Bearing number	Bearing material	Journal number	Journal material	Bearing type	Test temperature, °F	Measured ^a radial clearance at test temperature, °F	Preload coefficient	Journal speed range tested, rpm	Unit load range tested, psi	Total test time, min	Observed instability	Remarks
T-1	Inconel	J-10	Stellite Star J	3 Tilting pad	500	$bC_p = 0.0021$	0.19	5000 to 11 000	4.5 to 6.7	175	None present	Very little wear observed with most wear on unloaded pad. Pivot surfaces showed slight surface damage.
						$cC_p = 0.0017$						
T-2	Inconel	J-12	Stellite Star J	3 Tilting pad	500	$C_p = 0.0011$	0.36	5000 to 9000	4.5 to 31.1	311	None present	Bearing seized at 9000 rpm at 31 psi because of overloading. Loaded pad showed most wear.
						$C_p = 0.0007$						
T-2A1	Inconel	J-13	Stellite Star J	3 Tilting pad	500	$C_p = 0.0028$	0.50	3000 to 11 000	0 to 8.9	590	None present	Unloaded pad showed most wear indicating possibly insufficient preload. Journal showed light wear. Pivots showed mating surface damage.
						$C_p = 0.0014$						
T-3	Hastelloy X	J-14	Stellite Star J	3 Tilting pad	500 and 800	$C_p = 0.0036$	0.72	2000 to 12 000	0 to 17.8	1013	At 12 000 rpm and 0 load	Very little wear on all shoes. Pivots showed mating surface damage. Clearance at 800° F was not appreciably different from 500° F value.
						$C_p = 0.0010$						
T-1A	Inconel	HX-8	Hastelloy X	3 Tilting pad	800	$C_p = 0.0010$	0.70	5000 to 8000	4.5	175	None present	Bearing seized when higher load than 4.5 psi was attempted, probably due to tight clearance and incompatible materials. Wear evenly distributed between pads.
						$C_p = 0.0003$						

^aBy observation of oscilloscope pattern as the test vessel was moved by hand, to the limits of its outward motion, in a circular orbit about the stationary journal.

^b C_p = bearing radial clearance before preload.

^c C_p = bearing radial clearance at pivot location after preload.

TABLE V. - BEARING AND JOURNAL MATERIAL
COMBINATIONS THAT HAVE GOOD RESISTANCE
WEAR AND SEIZURE IN SODIUM TO 800⁰ F

(a) Cutaway view.

Figure 1. - Liquid metal bearing rig.

(b) Detailed view of test-bearing installation.

Figure 1. - Concluded.

CS-41286

Figure 2. - Seizure caused by contaminant particle. Bearing (J-1) versus journal (HX-B).

Figure 3. - Plain bearing (MP-3) versus Herringbone groove journal (K-14) after test.

Figure 4. - Failure due to overloading. Tilting pad bearing (T-2) versus journal (J-12).

Figure 5. - Comparison of experimental and theoretical friction torques for a 3-pad tilting pad bearing. Lubricant, 500° F sodium; bearing material, Hastelloy X; Journal material, Stellite Star J; nominal diameter, 1.5 inches; nominal length, 1.5 inches; radial clearance, 0.0036 inch.

(b) 10 pound load.

Figure 5. - Concluded.

1 CM = 0.5 MILS

(A) STABLE OPERATION
AT 4100 RPM; 10-LB
RADIAL LOAD.

1 CM = 0.5 MILS

(B) UNSTABLE OPERATION
(HALF FREQUENCY
WHIRL) AT 5000 RPM;
10-LB RADIAL LOAD.

CS-41290

Figure 6. - Oscilloscope traces of bearing motion obtained with a two-axial groove bearing in sodium at 500° F. Diametral clearance, 0.0020 inches. Bearing (M-1) against journal (K-6).

Figure 7. - Wear due to unstable bearing operation (one-half frequency whirl). Stellite star "J" Bearing (J-2) versus Hastelloy "X" journal (HX-C).

Figure 8. - Speed versus load at which bearing half frequency whirl is initiated. Comparison of 4 different bearing and journal configurations.

Figure 9. - Dimensionless critical rotor mass as a function of Sommerfeld number at the onset of half-frequency whirl for the plain cylindrical and the 100° partial arc bearings operating with a turbulent film.

Figure 10. - Seizure due to incompatible material combination. Mo -0.5Ti bearing. (M-11) versus Hastelloy "X" journal (HX-7).

Figure 11. - Tilting pad bearing seizure due to incompatible material combination. Inconel bearing (T-1A) versus Hastelloy "X" journal (HX-8).

Figure 12. - Spherical pivot and flat mating surface in pad. (Titanium carbide (K162B)) versus (Titanium carbide (K162B)).

SURFACE DAMAGE TO SPHERICAL PIVOT. (TITANIUM CARBIDE (K162B))

Figure 13. - Tilting pad bearing pivot after operation in sodium.

ALKALI METAL BEARING & SEAL DEVELOPMENT AT SPACE POWER AND PROPULSION SECTION

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ABSTRACT

SPPS engaged in three major programs associated with the development of Rankine cycle power conversion technology for space application: The low viscosity bearing stability investigation under NASA contract, the development of dynamic seals under Air Force contract and the preparations for a liquid metal bearing test, simulating the conditions of a space generator, on CIRP funding. All programs were supported by a corresponding analytical effort, presently geared towards predicting bearing/rotor response. Experimental results are expected in 1966, and 1967.

Screw seals, rotating channel and slinger squeeze seals were investigated in the dynamic seal program. Interface instability was identified as one of the major problems. It was resolved for the case of the slinger squeeze seal, which was successfully tested in potassium.

DISCUSSION

Alkali metal bearings and seals comprise two of the basic components necessary for reliable operation of Rankine cycle turbomachinery for space. As a result of the high temperatures and radiation fields in nuclear space power systems, and the general complexity of multi-fluid systems, present turbogenerator bearings and seals employ the primary thermodynamic fluid of the system in their operation. SPPS is engaged in the development of alkali metal bearings and seals for incorporation in a full turbogenerator system.

A typical 350 KWe turbogenerator for space power conversion - General Electric concept - is shown in Figure 1. It consists of a six-stage turbine driving an axial gap generator. Each component is supported on two radial bearings and a thrust bearing. The two components are connected by an elastic coupling. The design has been optimized for minimum rotor weight and distribution of this weight over four equal size pivoted pad bearings. Each component, turbine as well as generator, has its own thrust bearing primarily to fix the rotor position accurately, in relation to its stator. Thrust loads are minimized in the turbine by proper fluid design, and in the generator by carefully positioning the rotor in the middle of the stator. The generator cavity is

open to space and is based on the availability of zero leakage seals. The generator is uncanned and its temperature distribution is so that no potassium vapor condensation or accumulation of condensate within the generator housing will take place.

Within SPPS, bearing and seal development is being carried out under four different programs, three of which are basic, while the last one will combine the knowledge gained in the three previous ones. Of the three basic programs, the first one is devoted to the study of bearing stability which implies the establishment of proper bearing geometries and the verification of bearing rotor response behavior. This program is carried out under NASA contract and uses water at 120°F as a lubricant (Appendix A and B).

The second program is devoted to the establishment and selection of suitable bearing materials. Under this program (Contract No. NAS 3-2534, Appendix C), accurate physical properties have been established for these materials, such as hot hardness, compressive strength, modulus of elasticity, thermal expansion, potassium wetting and dimensional stability at 1600°F. Also, corrosion resistance and compatibility in potassium. Friction testing is being carried out in two highly sophisticated test rigs, one operating in a vacuum and the other one in a potassium atmosphere. This program favors extremely hard and stable refractory materials primarily Tungsten and Titanium Carbides using special binders. This program will not be further discussed here because of the limitation in time. It covered three years and is close to its completion, so its results should be available in a reasonable time.

The third program was carried out under Air Force contract and was devoted to the development of zero leakage seals employing potassium as a sealant (Appendix D). Its main application will be in the electric generator for space.

The fourth program, which is based on the knowledge accumulated in the three aforementioned programs, is carried out under Company Research Funds and has as its objective the simulation of the bearing and seal arrangement in a space generator. This presents the more difficult problem than the space turbine due to rotor and containment materials involved. This program will comprise performance and endurance testing of a 70-pound rotor operating on large carboloy potassium bearings and a zero leakage seal exposed to vacuum (Appendix E).

Bearing Stability Program

The first portion of the Bearing Stability Program was carried out in 1961 through 1963 under NASA Contract No. NAS 3-2111. The test rig in this time was designed and manufactured by SPPS. Testing and test evaluation, however, was carried out in the Bearing and Lubricant Center in Schenectady by Gerry Fox and his staff. The program was continued in 1965 under a new contract, NAS 3-6479 (Appendix A and B).

Under the first contract, six different bearing configurations were compared in regard to their stability behavior, especially their capability of avoiding or suppressing partial frequency whirl in the turbulent operating regime. Out of this program, the pivoted pad and the three-lobe bearing

emerged as the outstanding candidates for stable operation up to high speeds. These two bearing types are being further investigated under the present contract under aligned as well as purposely misaligned conditions. The intent is also to study the effects of seizure of pivoting points on the four-pad pivoted pad bearing by running tests with one or two pads fixed.

The test rig for this purpose had to be drastically modified. In addition, better and more instrumentation has been introduced. Shaft motions are monitored by four Bently gages per plane, arranged in four separate planes.

Opposed gages are used combining their signals in summing amplifiers to avoid errors due to differential thermal expansion. Gages are arranged on both sides of each bearing to be able to measure shaft bending under aligned as well as misaligned conditions. As a new venture, provisions have been made to measure bearing loads and to establish the load vector by size and angle during testing up to 500 cycles per second. The Bearing and Lubricant Center has had good success in making such measurements by using strain gaged buttons for shaft speeds up to 7,000 rpm. For high speed testing up to 30,000 rpm, originally similar strain gaged buttons had been considered; however, piezoelectric force gages were finally selected due to their high spring constant and their extreme capability of picking up fractions of a pound under preloads of several thousand pounds. Each bearing support is suspended on Kistler force gages in two planes, adequate to register the forces transmitted to the bearings.

Another innovation is introduced in this test program by using a digital data handling system for data taking. In high frequency testing, the normal tools are oscilloscopes and photography, as well as tape recorders. When it comes to the evaluation of test results, both of these methods are inadequate and it is usually an extremely time-consuming and difficult process to reduce tape recordings to printed language. In using a digital data handling system, special precautions have to be taken in measuring high frequency responses. This is done by using peak-to-peak detectors for forces and displacement and average level detectors for equilibrium position of the shaft. The other inputs, such as speed, torque, flows, temperatures and pressures, are straight forward. Accurate measurement of the phase angle constitutes a problem in itself and has been accomplished to within $\pm 2^{\circ}\text{F}$.

The test rig is shown in Figure 2. It comprises a 1.25-inch diameter shaft supported by two test bearings approximately 12 inches apart. The shaft is driven by an electric motor receiving its power from a variable frequency power supply through a quill shaft to speeds of 30,000 rpm. Side loads are imposed on the test shaft by two partial-arc loader bearings pneumatically operated. The lower bearing assembly can be adjusted both transversely and angularly to allow variation of bearing alignment. Measurements of shaft motion and force transmitted to the bearings are made in four planes, located on both sides of the test bearings as shown in detail in Figure 3. Shaft motion is monitored by Bently inductance gages in an assemblage shown in Figure 4. The gages have Teflon tips to protect them against water, as suggested by J. McHugh. To eliminate the effect of shaft inhomogeneities on gage readings, the shaft is silver plated in the zones of gage measurement

(Figure 5). A measuring accuracy within ± 50 micro-inch is an objective hard to meet. The alignment of the bearings prior to testing is accomplished optically by a high-precision telescope in conjunction with optical targets inserted in the bearings, Figure 6.

Figures 7 and 8 show the test rig under testing conditions at room temperature (open) as well as temperature controlled for 120°F operation (closed).

Figure 9 shows the instrumentation and control panel, subdivided into five compartments, one handling the lube flow, pressure and temperature control, another for power and frequency remote control of a variable frequency power supply arranged in another building. The third panel holds a speed counter, the proximity gage oscilloscopes, a temperature recorder and electronic equipment producing the signals for a digital recording system, also arranged in a separate building. The fourth panel is devoted to phase angle and force measurements. The fifth panel holds the equipment required for vibration and torque readout.

Figure 10 shows the pivoted pad bearing design incorporated in the present test program. It has four pads with gimbaling points at 55 percent partial arc. Figure 11 shows the calculated load carrying capacity of this bearing with oil and with liquid metal as lubricants. This graph brings out the weakness of the low viscosity bearing resulting in a load carrying capability of roughly 1/1000 of the oil lubricated bearing.

Computer programs are available which allow prediction of the dynamic response of a given bearing-rotor configuration. One such program was

generated by G.E., Bearing and Lubricant Center. Another one was published by MTI under Air Force contract. SPPS has used both of these programs and has generated an additional program which combines VAST and LSE^{*}, offering higher accuracy and flexibility. In the near future, a comparison of actual test results with those predicted will be possible. Figure 12 shows a typical predicted curve of bearing stiffness as a function of rotative speed, which in turn is used to calculate fluid film forces as shown in Figure 13. Figures 14 and 15 show typical amplitudes of shaft motion as a function of speed with two different levels of shaft unbalance in the easy-fluid test rig, calculated by using the MTI computer programs. In these curves, the occurrence of critical speeds can be identified with the high amplitude zones, and it can be seen that for the larger value of unbalance, the rotor amplitude actually exceeds the bearing clearance, predicting failure of the bearing. As a major test objective, generalized dynamic coefficients will be obtained for bearing rotor system operating conditions involving the practical problems of misalignment and varying support structure stiffness, both of which are difficult to predict analytically.

Dynamic Zero Leakage Seal

A program was conducted by SPPS on the development of such a dynamic seal, Figure 16. Considerable testing in water was done to identify the proper seal concept, and to prove its hydrodynamic capabilities. Ultimately, the seal was successfully tested in potassium (Appendix D).

* VAST - Vibration Analysis Systems
LSE - Linear Simultaneous Differential Equations

Figure 17 shows some of the seal concepts investigated. All of these share the common characteristic of being non-contacting hydrodynamic seals, which is a necessary requirement for long-time operation in space turbo-machinery. Figure 18 shows the physical properties of potassium which assist the successful functioning of such a seal, where, because of low vapor pressure, the molecular diffusion from a free liquid surface to the vacuum of space is small.

A detailed configurational drawing of the dynamic zero leakage seal ultimately tested is shown in Figure 19. Liquid is injected into the rotating disc cavity, and sealing is accomplished by centrifugal force of the liquid. Any migration of liquid along stationary walls toward the vacuum side results in re-ingestion of the liquid into the seal before it escapes. Figure 20 is a photograph of the disassembled seal parts. Testing was accomplished in the liquid metal bearing and seal facility in Figure 21, which will be used in the forthcoming liquid metal bearing development program. Figures 22 and 23 show typical performance curves for seals of diameters of interest.

Figure 24 shows the liquid potassium which was accumulated in a vacuum tank during 173 hours of testing the dynamic seal. Most of this was accumulated during two startups and shutdowns.

Liquid Metal Bearing Test Program (Appendix E)

The liquid metal bearing test rig, shown in Figure 25, is similar to the previously discussed water test rig, in that two test bearings support a

vertical shaft driven by an electric motor. The rotor weighs 70 pounds and simulates the mass and dynamic characteristics of an axial gap generator. Along with the very accurate manufacturing requirements of the bearing, the design of a bearing for the generator shaft constitutes a severe mechanical design requirement, since the rotor of the generator must be made from magnetic steel, the casing from stainless steel and the bearing is fabricated from refractory metal carbides having thermal coefficients of expansion of roughly one-third those of steels, Figure 26. A number of geometric solutions to the differential thermal expansion problem have been investigated, involving radial springs, radial faced teeth, and conical faces as shown in Figure 27, the latter approach having been selected as the most practical. The bearing detail is shown in Figure 28, and comprises four segmented pads, retained by spherical pins, allowing self-alignment of the bearing during operation. The pads are weight balanced so that their centers of gravity coincide with the pivot contact points to minimize fretting motions. The entire bearing region is enclosed by close-running shaft screw seals, and is flooded with potassium. A side load of 200 to 300 pounds can be applied to the rotor by generating a magnetic field with an induction coil. The test rig has other new features such as a high temperature induction gage measuring the potassium film thickness through an aluminum oxide window against an aluminum oxide ring which is part of the rotor. The lower shaft end is exposed to vacuum and sealed off by a zero leakage seal modified for a low power requirement. Dynamic sealing, in this case, is accomplished by a screw seal. An additional lift-off face seal provides sealing at standstill and during startup. Figures 29 and 30 show rotor and stator hardware. The test rig is

presently being assembled and will undergo its easy-fluid checkout in November 1966. Liquid metal testing is scheduled for the first quarter of 1967.

Conclusions

The engineering know-how required to resolve the complex problems of a space turbogenerator operating on liquid metal bearings and employing zero leakage seals is progressing rapidly. As compared to eight years ago, when even the problems involved were poorly identified, much knowledge and much practical experience has been accumulated today. Computer programs are available and will eventually reach close agreement with actual test conditions. SPPS is prepared to contribute to this general realm of knowledge by coordinating computer predictions with actual test results, by establishing best qualified pairs of materials for long-time operation of liquid metal bearings and by developing seals which will combine the zero leakage feature with a minimum power consumption.

As a bearing configuration, SPPS presently favors the pivoted pad bearing not because of its resistance to partial frequency whirl, but also because of its self-adjustment capability to misalignments or structural distortions. It is realized that the pivoted pad bearing is complex and has an inherent weakness of fretting or seizing of pivoting points. A less complex stabilized sleeve bearing with equal resistance to partial frequency whirl and supported in a way providing equal self-adjustment to misalignments would be desirable.

The differential thermal expansion problem of the Tungsten carbide bearing in a steel environment is being actively attacked in our liquid metal bearing program. Answers and test results should be available in 1967.

APPENDIX A

TITLE OF CONTRACT:	JOURNAL BEARING INVESTIGATIONS
NASA CONTRACT NUMBER:	NAS 3-2111
DATE CONTRACT WAS ISSUED:	OCTOBER 10, 1961
DATE OF FINAL REPORT:	DECEMBER 31, 1963
NASA CONTRACTING OFFICE:	LEWIS RESEARCH CENTER
NASA PROGRAM MANAGER:	JOSEPH P. JOYCE

EXCERPT OF WORK STATEMENT

The Contractor shall conduct an investigation of hydrodynamically lubricated journal bearings having the capability to suppress shaft whirl while operating at high journal speeds and with zero bearing load.

The Contractor shall fabricate a test bearing rig with a vertically mounted shaft (1.25-inch diameter) and speed capability up to 35,000 rpm.

The test rig shall be designed with the facility to alter and to control accurately the following input variables:

1. Shaft speed
2. Bearing loads
 - Unit directional
 - Rotating
3. Inlet lubricant pressure
4. Inlet lubricant temperature
5. Lubricant flow rate

The test rig shall be so designed and instrumented to facilitate the determination of impending whirl speed and accurate readout of the following data:

1. Bearing load
2. Inlet lubricant pressure
3. Inlet and outlet lubricant temperature
4. Lubricant flow rate
5. Shaft speed
6. Friction torque
7. Shaft position in the bearing with respect to time
8. Eccentricity

APPENDIX A - (Cont'd)

The Contractor shall study analytically approximately eleven (11) different bearing configurations and shall then select approximately six (6) of these for experimental testing.

The Contractor shall determine experimentally the antiwhirl characteristics for each bearing configuration selected for testing over ranges of the following variables as specified.

1. Shaft speed: 0 - 35,000 rpm
2. Bearing load: Unidirectional - 0 - 200 lbs.
Rotating - 0 - Maximum level as governed by
maximum attainable speed or by
bearing load capacity
3. Inlet lubricant pressure: 3 - 30 psi
4. Inlet lubricant temperature: 100 - 160°F
5. Clearance: 0.0016" - 0.004"
6. Bearing length to diameter ratio (L/D): 0.75 - 1.5

The test lubricant for this test phase shall be distilled water.

APPENDIX B

TITLE OF CONTRACT:	INVESTIGATION OF STABILITY OF HYDRODYNAMIC JOURNAL BEARINGS
NASA CONTRACT NUMBER:	NAS 3-6479
DATE CONTRACT WAS ISSUED:	APRIL 29, 1965
DATE OF CONTRACT COMPLETION:	END OF 1966
NASA CONTRACTING OFFICE:	LEWIS RESEARCH CENTER
NASA PROGRAM MANAGER:	JOSEPH P. JOYCE

EXCERPT OF WORK STATEMENTSpecific Requirements

The Contractor shall extend the investigation of the stability of the two hydrodynamically lubricated bearings that have demonstrated under Contract NAS 3-2111 an ability to suppress half-frequency shaft whirl while operating at rotational speeds up to 21,000 rpm and with near zero radial loads. This study shall extend to rotational speeds of 30,000 rpm and shall include effects of bearing misalignment. The two bearing designs to be studied are the three lobe and the four pad pivot pad configurations. The test bearing dimensions are:

Inside Diameter	1.25 inches (nominal)
($\frac{\text{Length}}{\text{Diameter}}$)	1.0
Diametrical Clearance	0.005 inch

The bearing test facility, its auxiliary equipment and instrumentation used on Contract NAS 3-2111 shall be modified and improved to demonstrate the capability of testing through the range of variables stated below:

<u>Variables</u>	<u>Range</u>
1. Shaft Rotational Speed	3,600 to 30,000 rpm
2. Bearing Angular Misalignment	0 to 400 sec.

APPENDIX B - (Cont'd)

3.	Bearing Linear Misalignment	0 to 0.004 inches
4.	Combined Bearing Angular and Linear Misalignment	Same as Items 2 & 3
5.	Lubricant Supply Pressure	15 to 150 psia
6.	Bearing Mount Rigidity	Rigid & 50,000 pounds/inch
7.	Bearing Unidirectional Radial Load (Each Test Bearing)	0 - 60 pounds
8.	Bearing Synchronous Rotating Load (Each Test Bearing)	0 - 40 pounds at 30,000 rpm
9.	Bearing Reaction Force Vector	0 to 360 ^o

APPENDIX C

TITLE OF CONTRACT:	RESEARCH ON MATERIALS SUITABLE FOR POTASSIUM-LUBRICATED JOURNAL BEARING AND SHAFT COMBINATIONS
NASA CONTRACT NUMBER:	NAS 3-2534
DATE CONTRACT WAS ISSUED:	APRIL 22, 1963
DATE OF CONTRACT COMPLETION:	END OF 1966
NASA CONTRACTING OFFICE:	LEWIS RESEARCH CENTER
NASA PROGRAM MANAGER:	ROBERT L. DAVIES

EXCERPT OF WORK STATEMENT

The Contractor shall conduct an evaluation of materials suitable for potassium-lubricated journal bearings and shaft combinations destined for use in space system turbogenerators.

The Contractor shall provide capsule corrosion test facilities capable of subjecting single and/or multiple bearing material specimens to potassium at 400° F to 1600° F. Test capsules shall be contained in a vacuum better than 1×10^{-7} torr pressure after initial startup and inception of stable operation of the tests. During the startup time, the vacuum shall be 1×10^{-6} torr or better. The capsule facility shall alter, control and give accurate readout of potassium temperature. Similar facilities shall be provided to determine dimensional stability characteristics of the bearing materials.

The Contractor shall provide a physical and mechanical properties test facilities capable of maintaining a vacuum in the range of 10^{-6} torr pressure, except for thermal expansion measurement apparatus which shall maintain an inert gas atmosphere. These facilities shall be fitted to determine material physical mechanical properties.

The Contractor shall provide a friction and wear test facilities capable of maintaining a vacuum in the range of 10^{-9} torr pressure or better. This facility shall be designed to alter, control, and read out accurately the following variables:

1. Test sample sliding speed from 0 to 1000 ft/min.
2. Test sample normal load to produce yield stress in contact materials.

APPENDIX C - (Cont'd)

3. Induced torque.
4. Test environment
 pressure, torr
 potassium level.
5. Test temperature 400° to 1200°F.

The Contractor shall provide facilities to evaluate the wetting behavior of the bearing materials with liquid potassium by the sessile drop method. The facilities shall be capable of being evacuated to pressures in the 10^{-9} torr range.

It is anticipated that about 14 materials and about 7 material combinations shall be chosen for test.

Corrosion capsule tests shall be conducted as outlined below:

1. At temperatures of 800°, 1200° and 1600°F.
2. Continuous test duration of 1,000 hours at each temperature listed in 1. above.
3. Capsules shall be fabricated from Cb-1Zr Alloy.
4. Test specimen combinations shall be tested with equal surface areas exposed to potassium.

Dimensional stability - the amount of permanent dimensional change at constant temperatures shall be conducted at 800°, 1200°, and 1600°F for 1000 hours.

The following physical and mechanical properties shall be determined:

1. Coefficient of thermal expansion at 400°, 800°, 1200° and 1600°F.
2. Compressive yield stress at room temperature, 400°, 800°, 1200° and 1600°F.
3. Ultimate compressive stress at room temperature, 400°, 800°, 1200° and 1600°F.
4. Modulus of elasticity at room temperature, 400°, 800°, 1200° and 1600°F.

APPENDIX C - (Cont'd)

5. Hardness at 400°, 800°, 1200° and 1600°F.

Coefficients of friction and wear rates shall be determined under the following conditions:

1. At temperatures of 400°, 800° and 1200°F.
2. At speeds to 1000 SFM.
3. At loads to 90 percent of the 0.2% yield strength or ultimate strength of the test materials.
4. At pressures in the 10^{-9} torr range and in liquid potassium.

APPENDIX D

TITLE OF CONTRACT:	DESIGN CRITERIA FOR ROTARY SEALS FOR A SPACE ENVIRONMENT
AIR FORCE CONTRACT NUMBER:	AF 33(657)-8469
DATE CONTRACT WAS ISSUED:	MARCH 1, 1962
DATE OF FINAL REPORT:	SEPTEMBER, 1965
AIR FORCE CONTRACTING OFFICE:	AERONAUTICAL SYSTEMS DIVISION
AIR FORCE PROGRAM MANAGER:	ROBERT J. SMITH

EXCERPT OF WORK STATEMENTObjectives

The objective of this program is to acquire the techniques for completely sealing high speed rotating shafts under the operating conditions of high temperature liquid metals and vapors, the near-vacuum environments of space, and long life.

Requirements

Concepts of seals to meet the objective and the following conditions shall be designed, fabricated and experimented upon.

1. Fluid - The fluid to be sealed shall be potassium.
2. Temperature - The seals shall be operative at fluid temperatures from the melting point of the fluid selected to 800°F.
3. Pressure - The pressure on the fluid side of the seal shall be 15 psi and the external pressure shall be 10^{-6} mm Hg.
4. Speed - The speed of the rotating shaft shall be 24,000 rpm.
5. Life - The seal, or seal combinations, shall be designed for 10,000 hours of maintenance-free life.
6. Seal Lubricant - The working fluid, potassium, shall be used as the seal lubricant.

APPENDIX D - (Cont'd)

7. Leakage - The seal, or seal combinations, shall be capable of maintaining absolute zero leakage under all conditions of operation.
8. Size - The seals shall be designed for a 1.0-inch diameter shaft.
9. Zero G - The seals shall be capable of operation in a zero "g" environment.

Evaluation

The seal evaluation shall consist of the following:

1. Preliminary experiments
2. 100-hour operational screening test
3. Thermal-cycling test
4. 1000-hour life test

Test Conditions

The test conditions are potassium at 1400⁰F and 15 psi on one side of the seal, a pressure of 10⁻⁶ mm Hg on the other side, and a shaft speed of 24,000 rpm.

4

6

APPENDIX E

TITLE: RESEARCH ON LIQUID METAL BEARINGS
AND DYNAMIC SEAL SYSTEMS

FUNDING: GENERAL ELECTRIC COMPANY
INDEPENDENT RESEARCH FUNDING
(CIRP) DA # L06-09F

PROGRAM INITIATED: 1965

PROGRAM TO BE COMPLETED: MIDDLE OF 1967

Objectives

1. Establish feasibility of potassium lubricated bearings and seals for application in electric generator.
2. Employ optimum bearing materials as identified by Bearing Materials Program (NAS 3-2534).
3. Determine bearing stiffness.
4. Establish load carrying capability, dynamic as well as static.
5. Determine power requirements.
6. Determine rotor/bearing response.
7. Establish mechanical integrity
 - a) During thermal cycling
 - b) During long-time operation.
8. Establish low power DZL seal.

Specifications

Lubricant	Liquid Potassium
Temperature Level in Bearings and Seals	600°F
Bearing Diameter	2.625 inches

APPENDIX E - (Cont'd)

Bearing L/D Ratio	1.0
Speed	18,000 rpm
Bearing Type	Four-Pad Pivoted Pad Bearing
Fixed Pivoting Points Preload Coefficient	0.5
Rotor Weight	70 pounds
Rotor Material	H-11 Tool Steel
Residual Unbalance	0.15 gr-in
Provisions for Rotational Loads up to 1000 pounds	By Unbalance
Electromagnetic Side Load	Up to 200 pounds
Containment Material	316 Stainless Steel
Bearing Material	Carboloy 907

Fig. 1. Bearings and Seals in a Potassium Turbogenerator for Space.

Fig. 2. Bearing Stability Test Rig.

Fig. 3. Force Button and Proximity Gage Installation.

Fig. 4. Proximity Gage Holder.

Fig. 5. Test Shaft.

MTI-2133

Fig. 6. Optical Alignment Measurement.

MTI-2134

Fig. 7. Test Rig in Room Temperature Test.

MTI-2135

Fig. 8. Test Rig Temperature Controlled 120°F.

MTI-2136

Fig. 9. Instrumentation Panel.

1.875"

Fig. 10. Self-Aligning Pivoted Pad Bearing.

Fig. 11. Load Carrying Capacity (Pivoted Pad Bearing).

Fig. 12. Bearing Stiffness Vs. Speed and Side Load.

Fig. 13. Force Vs. Speed.

Fig. 14. Displacement Vs. Speed, 0.25 in-gr Unbalance.

MTI-2142

Fig. 15. Displacement Vs. Speed, 0.50 in-gr Unbalance.

MTI-2143

- OBJECTIVE

ESTABLISH "ZERO LEAKAGE" SEAL FOR SPACE GENERATOR

- SPECIFICATIONS

SEAL FLUID POTASSIUM AT 600⁰F

SHAFT DIAMETER 1"

SHAFT SPEED 24,000 RPM

EXTERNAL: LOW VACUUM (10^{-6} TORR)

LEAKAGE: 1 TO 10 LBS. IN 10,000 HOURS

Fig. 16. Dynamic Seal Development.

Fig. 17. The Seal Concept.

Fig. 18. Vapor Pressure of Liquid Metals.

Fig. 19. Dynamic Zero Leakage Seal.

Fig. 20. Seal Hardware and Test Spindle.

Fig. 21. Liquid Metal Bearing and Seal Facility.

Fig. 22. Sealing Capacity of DZL Seal.

MTI-2150

Fig. 23. Power Consumption of DZL Seal.

MTI-2151

Fig. 24. Seal Leakage in 173 Hours of Testing.

Fig. 25. Liquid Metal Bearing Test Rig.

Fig. 26. Thermal Expansion Coefficients.

Fig. 27. Approach to Differential Thermal Expansion.

Fig. 28. Carboloy Tilting Pad Bearing.

Fig. 29. Liquid Metal Bearing Test Rig Rotor.

Fig. 30. Liquid Metal Bearing Test Rig Stator.

✓ Bearing and Seal Materials for Liquid Metal Lubrication

S. Frank Murray

Mechanical Technology Incorporated

ABSTRACT

The obvious problems of selecting materials for high temperature corrosion resistance and strength have masked the fact that many of these liquid-metal lubricated bearing and seal surfaces may fail by seizure long before corrosion becomes a significant factor. Some of the hard, refractory materials, such as the cemented carbides, appear to offer a means for achieving good sliding behavior, corrosion resistance and high temperature strength all in one package. Actually, these materials are pacifiers. Design and fabrication problems generally outweigh the advantages that the cermets offer. In addition, the use of these materials may limit the size of machine components.

One promising approach is to use coatings or surface treatments to protect surfaces in relative motion. The feasibility of this approach has been demonstrated; however, there is a reluctance to use coatings because of quality control problems. Unless this problem is resolved, the design and fabrication of liquid metal lubricated components will continue to be unduly complex and in some instances, will be a source of built-in unreliability.

INTRODUCTION

There is certainly no controversy over the importance of bearing and seal materials in process fluid-lubricated systems. In many cases, materials have been the limiting item in the successful operation of machinery, especially when liquid metals were used as the lubricating fluids. Their poor viscosity characteristics, coupled with the fact that liquid metals have little or no boundary lubricating ability, impose very stringent limitations on the reliability of the bearing systems. Even hydrostatically-supported bearings are in constant jeopardy of failure because of an accidental rub.

There are materials available with good sliding behavior and corrosion resistance in NaK, but their physical characteristics impose many limitations on design and fabrication. The purpose of this discussion is to review briefly the status of the work on bearing materials selection and to recommend an approach to the problem of achieving good sliding characteristics without penalizing the bearing or seal design.

BACKGROUND ON BEARING MATERIALS FOR LIQUID METALS

In the early nineteen fifties, Vail, at the Knolls Atomic Power Laboratory and Basham et al, at Battelle Memorial Institute, both published results of material compatibility tests with NaK (Refs. 1 - 3). Their results are summarized in Table 1. The work done at Battelle was concerned with static tests on valve and seat material combinations in 1500F NaK. The purpose was to determine if welding and transfer would take place. Vail's work at KAPL consisted of basic sliding tests on a large number of material combinations in NaK, at temperatures up to 850F. In both studies, it was found that certain metal-bonded carbides were very satisfactory. Basham also showed that tungsten and molybdenum were promising materials for the valve and seat materials.

Since that time, there have been a number of other investigations made to select bearing material combinations for various liquid metal environments, e.g., Refs. 4 - 9. In general, the conclusions have been much the same. The cemented carbides and tungsten or molybdenum are the most compatible sliding combinations

for liquid metal-lubricated bearings. Stellite Star J has also been added to the list of promising materials. Certainly, these studies were useful since they broadened our knowledge of the behavior of these materials in various liquid metals. However, from the standpoint of the design engineer, who is concerned with building practical machinery, this approach of trying to use solid carbides or refractory metals appears to be the wrong way to attack this problem.

That last statement is backed up by practical experience. Some of the early work on journal bearings operating in NaK was done by Apkarian (Ref. 4) using the cemented carbides which were recommended by Vail as the bearing materials. The same background is still being used to select materials, e.g., Figure 1 shows a cemented carbide thrust bearing which was built by MTI for Aerojet-General under NASA Contract No. NAS 5-417. That bearing ran successfully in the Snap 8 NaK pump for 3000 hours at temperatures from 300-600F. Pivot damage studies have also been run in 600F NaK using various material combinations, including the cemented carbides.

Granted that these cemented carbides have excellent sliding compatibility in liquid metals, there are still many problems involved in the design and fabrication of parts from these materials which discourages their use in machinery. For example, Table 2 compares some of the characteristics of candidate liquid metal lubricated bearing materials with common structural alloys. All of the bearing materials have very low coefficients of thermal expansion and relatively poor shock resistance. In contrast, most of the structural alloys, with the exception of the columbium alloy, have much higher coefficients of thermal expansion. These mismatches in material properties have a strong tendency to force the design engineer to ignore sliding compatibility as a bearing material requirement and to select materials based on other attributes.

For these reasons, MTI has leaned heavily on the use of coatings to provide good bearing surfaces for unconventional fluid-lubricated systems.

EXPERIENCE ON COATINGS FOR BEARING APPLICATIONS

Most of the work which has been done at MTI on using coatings for bearings has been directed toward gas-lubricated bearing systems. However, this work does apply to the problem of liquid metal lubrication. In most of the gas bearing work, the gases were inert and no contaminating oxide films were present to provide surface protection. In addition, these non-reactive gases provided no boundary lubrication. These two conditions apply equally well to lubrication by NaK or by inert gases, although NaK is certainly a more severe case.

Basically, our approach has been to select one base alloy for the shaft, bearings, and any other critical structural members. This alloy is chosen for strength, density, corrosion resistance and any other essential attributes. Then, plasma or flame-sprayed oxide or cermet coatings are used on the journal and bearing surfaces to provide the necessary resistance to sliding damage. These coatings are applied, then ground back to a standard thickness of .003 inches. In this way, problems of matching thermal expansion are minimized. Experience has also shown that these thin coatings are much more capable of withstanding thermal or mechanical shock than are the solid cermets or ceramics.

Even among these so-called wear resistant coatings, there are wide gradations in effectiveness. Under NASA contract, MTI has evaluated some promising gas bearing material combinations for use in an inert environment (Ref. 10). In those tests, a single hydrodynamic tilting pad test bearing was dead weight loaded down against a 1.5 inch diameter test shaft. A photograph of the partially assembled test rig is shown as Figure 2. Capacitance probes, mounted on each side of the pad, were used to monitor lift-off speed, running film thickness and wear. The purpose of those tests was to measure changes in bearing performance as a result of sliding contacts.

In Figure 3, photographs of three different tilting pad bearings are shown after these bearings had been evaluated in start-stop tests. Two of these bearings ran very effectively for 1000 starts and stops at a stress level of 4 psi, based on the projected area of the shaft. These were a nickel-bonded tungsten carbide coated bearing against an Al_2O_3 coated shaft and a chrome

oxide coated bearing against a chrome oxide coated shaft. The contacting surfaces were lightly polished and the capacitance probes indicated less than forty microinches of wear. The third bearing, a hardened M-50 tool steel (60Rc) running against a cobalt-bonded tungsten carbide coated shaft was effective for about 350 starts and stops. At this point, the capacitance probe signals indicated that the bearing performance was beginning to degrade. The test was continued for the full thousand starts and stops, but there the bearing was not lifting effectively at the end of the test. Examination of the bearing surfaces after the test showed that the tool steel had transferred and welded to the carbide coated shaft and that this transferred material was scoring the pad surface in one area.

These same bearing combinations were also subjected to a series of high speed rubs by bringing the shaft up to a stable speed of 60,000 rpm and then impacting the pad against the shaft under various shock loads. Time in contact was on the order of .02 - .05 seconds. The appearance of the bearings after those high speed rub tests is shown in Figure 4. The chrome oxide coated pad running against the chrome oxide coated shaft was still in excellent condition after ninety rubs. The capacitance probe signals indicated that the bearing was operating on a larger film thickness at the end of the test than it was in the beginning. The contacting surfaces were highly polished and smooth.

As was the case in the start-stop tests, the M-50 tool steel pad running against the cobalt bonded tungsten carbide coated shaft was a very poor combination. This test had to be stopped before the full sequence of shock loads could be run because the pad was no longer lifting. The pad was found to be severely cold-worked and the shaft was covered with transferred metal.

The tungsten carbide coated pad running against an Al_2O_3 coated shaft showed a gradual deterioration in running performance throughout the test. After ninety rubs, the probe signals indicated that intermittent contact was taking place between the pad and the shaft. Examination of the test specimens showed that a smooth film of the binder metal was smeared on the Al_2O_3 coated shaft. The carbide-coated pad was severely abraded, either by the transferred metal or by

carbide grains being torn out of the pad surface.

Similar results have also been obtained with nickel and cobalt bonded tungsten carbide coated pads running against the same coatings on the shafts.

From the results of these tests the self-mated chrome oxide coating was selected as being the most promising material for good sliding behavior. This coating has since been used successfully in a number of gas and water-lubricated machinery applications.

MTI EXPERIENCE ON COATINGS FOR LIQUID METAL LUBRICATED BEARINGS

When a decision was required on the bearing materials for the NaK lubricated test rig, which was being built by MTI under AEC Contract No. AT(30-1)3363, three coatings were selected for preliminary evaluations of their corrosion resistance to NaK. The selection was based on our previous experience of the sliding compatibility of these materials in inert environments. These coatings were:

- a) Chrome oxide
- b) Chrome oxide, undercoated with molybdenum to improve the adherence of the oxide.
- c) Cobalt-bonded tungsten carbide.

The coatings were sprayed on one end of a hardened, cylindrical, 410 stainless steel test specimen which was part of a dumbbell tensile test bar. The geometry and dimensions of the specimens are shown in Figure 5. The coatings were then ground back to a standard thickness of .003 inches. This specimen configuration was used so that some measure of the bond strengths of the coatings could be obtained before and after exposure to NaK. To determine the coating bond strength, the coated face of the specimen was cemented to the other half of the tensile test bar with an epoxy resin. The bar was then pulled on a tensile test machine. Normally, the break will occur at the interface between the coating and the substrate since this bond is rarely as strong as the epoxy joint. Figure 6 is a

photograph of some typical test specimens showing the type of coating failure which should normally occur. By comparing the bond strength of control specimens with the bond strength of specimens which have been exposed to NaK, a measure of the resistance of the coatings to attack by the liquid metal can thus be obtained. This bond strength test, coupled with visual observation of the condition of the coating on the surface, is a reasonable measure of the resistance of the coating to corrosive attack.

The first test was a 300 hour static test with the specimens immersed in NaK at 200F. The specimens were then removed, examined and subjected to the tensile test. Table 3 gives the bond strength values for the control specimens (which had not been exposed to NaK), and for the specimens which had been exposed to 200F NaK. Two discrepancies were noted. First, the bond strength of the chrome oxide coating, which was undercoated with molybdenum, actually increased by a significant amount after exposure to the 200F NaK. At this time, we have no explanation for this behavior.

The second discrepancy was the fact that the joints broke at the epoxy when the tungsten carbide coated specimens, which had been exposed to NaK, were tested. At that time, it was felt that this was simply the result of a poor epoxy joint. However, repeat tests gave essentially the same results and in no case were we able to get good adhesion between the epoxy and any of the carbide coated specimens after the carbide coatings had been exposed to NaK. It is possible that the NaK had reacted with the carbide surface or the metal binder to form a film which did not make a good bond with epoxy. This introduced some uncertainty into the test results but time did not permit us to resolve this.

There was no visible indication of corrosion on any of the test specimens.

The second part of the screening study consisted of a series of fifty hour tests in 600F NaK. After each fifty hour cycle, the specimens were removed, washed and examined. The reason for removing the specimens every fifty hours was to see if the formation of caustic, as the result of exposure to air, would cause more corrosion than the continuous immersion in NaK.

After the first fifty hour test at 600F, the chrome oxide coatings were found to be completely disintegrated. Not only had the coating separated from the surface, the particles of chrome oxide had also separated from each other and had ended up in the bottom of the basket as fine debris. The molybdenum undercoat was still intact on the specimen which had been undercoated. A photograph of one of the chrome oxide coated specimens after this test is shown in Figure 7.

The cobalt-bonded tungsten carbide specimens had a dark film on the surface, but the coatings were still intact. These carbide coated specimens were then put back for another fifty hours. After this second run, there was a faint suspicion of pitting starting to appear on the surfaces. This could have been due to the fact that the NaK had washed out some grinding debris which had been embedded in the surface. In retrospect, this explanation seems reasonable since the pitting never became any worse.

The tests were suspended temporarily at this point while a new set of specimens was obtained. This time, 410 stainless specimens with a nickel-bonded carbide coating (25% WC, 7% Ni + mixed W-Cr carbides) were prepared. This nickel-bonded carbide is known to be more corrosion resistant than the cobalt-bonded material. In addition, a tilting pad bearing with this nickel-bonded carbide coating on a 416 stainless substrate was also included. This pad bearing had been ground and lapped to a 1-3 rms finish for some previous tests. It was included here because the highly polished surface would make it easier to see if the coating was actually pitting.

Finally, a type 316 stainless flat with an Al_2O_3 coating was also evaluated just to see if NaK would attack the Al_2O_3 . The Al_2O_3 coating was as-sprayed.

The specimens described above, plus the two cobalt-bonded tungsten carbide specimens which had already been subjected to the two fifty hour cycles in 600F NaK, were then run through three more fifty hour cycles at 600F in NaK.

At the end, the specimens were washed and examined. The appearance of the surfaces is summarized in Table 4. Figure 8 is a photograph of the polished carbide

coated specimen and the Al_2O_3 coated flat after the test. The carbide coated pad appeared to be in very good condition, perhaps a little duller than before the test but certainly not pitted. The Al_2O_3 coated flat had discolored badly and looked much rougher. However, the coating was still intact. This test should be repeated with an Al_2O_3 coating which has been ground back to a smooth finish. The result does not prove that Al_2O_3 is unsuitable, in fact, Al_2O_3 has been reported to be resistant to NaK at much higher temperatures but it does indicate that there maybe some questions as to the ability of the coating to withstand corrosion by NaK.

The shiny cylindrical specimen, with the threaded end, which is also shown in Figure 8, is a test sample of 6061 T-6 aluminum alloy that had been plated with .0002 - .0003" of electroless nickel. This aluminum specimen had gone through the five 50 hour cycle tests at 600F with no apparent corrosion taking place.

Table 5 summarizes the results of the bond strength tests on the carbide-coated specimens before and after exposure to 600F NaK. Once again, poor epoxy bonds were obtained on the specimens which had been exposed to NaK, particularly with the cobalt-bonded carbide. However, the results do indicate that the losses in bond strength, if any, were not large.

Based on the results of these static corrosion tests, the nickel-bonded tungsten (W-Cr) carbide coating was selected as being the best choice for the bearing and journal surfaces in the MTI NaK loop. As far as sliding behavior was concerned, this coating was known to be satisfactory for start-stop operation. Its ability to withstand high speed rubs was not outstanding, but seemed to be adequate for several lightly-loaded momentary contacts. The corrosion resistance of the carbide coating appeared to be suitable at temperatures up to at least 600F.

This conclusion of the suitability of the carbide coatings for use in 600F NaK was also shown by Wallace (Ref. 6) who had evaluated cobalt-bonded tungsten carbide coatings for their sliding behavior in NaK at 600F, and had found that these coatings had good wear resistance.

Figure 9 is a photograph of the shaft and pads from the MTI NaK test rig. The journals and pad surfaces have the nickel-bonded carbide coating. Although the number of operating hours on this facility is still low, these bearings have been run in water, alcohol and NaK. As far as sliding behavior is concerned, the coatings appear to be very satisfactory.

One point which should be emphasized here is the capability of these coatings for protecting larger areas such as face seal and thrust bearing surfaces. There is no limit to the diameter of a part as far as the application of coatings is concerned.

PROBLEM AREAS IN THE USE OF COATINGS FOR PROCESS FLUID LUBRICATION

Up to this point, all of the emphasis has been placed on the attributes of plasma or flame-sprayed coatings. They do have certain disadvantages, chiefly in the area of quality control. By varying the techniques used to apply the coatings, it is possible to get wide variations in coating properties and adherence. This point was also discussed by Wallace (Ref. 6), who concluded that a vigorous development effort was required to define a standard coating procedure which would insure reliable bearing surfaces. Most of the techniques available at the present time for evaluating the bond strength and integrity of the coatings are destructive in nature. It is necessary to coat test specimens or coupons at the same time that the machine elements are being coated and then to use these specimens for the coating evaluations. This introduces considerable uncertainty as to the condition of the coating on the actual machine parts. Non-destructive techniques have been proposed to determine coating adherence. These include ultrasonics, thermal or infra-red evaluations and eddy current flux field applications. Each of these is a development problem, but the feasibility of using each of these techniques has been demonstrated.

There are other problems in selecting the best coating for a given application and in establishing reasonable temperature limitations for the coatings but this information can be developed in a straightforward manner.

CONCLUSIONS

Flame or plasma-sprayed hard coatings offer many significant advantages in simplifying the design and fabrication of process fluid lubricated bearing and seal components. This is particularly true for large diameter parts where mismatches in the thermal expansion coefficients of materials may otherwise pose insurmountable difficulties. The effectiveness of certain of these coatings in protecting the surfaces against sliding damage in inert environments has been demonstrated in a number of practical applications. All of the test results which have been obtained to date show that these coatings can also be used effectively for liquid metal-lubricated systems.

The major drawback to the use of coatings is a lack of adequate quality control.

Ultimately, the choices boil down to these two:

Either we live with the solid cermets and refractory metals and accept the design problems and limitations which these materials impose, particularly for larger bearings and seals, or we learn to use coatings and develop the necessary quality control procedures which can ensure a reliable product.

REFERENCES

1. Vail, D. B. "Compatibility of Materials in Liquid Metal". USAEC Report KAPL-589. August 18, 1951.
2. Vail, D. B. "Compatibility of Materials in Liquid Metal; Second Report". USAEC Report KAPL-1021. January 5, 1954.
3. Reactor Handbook. Vol. I - Materials Edited by C. R. Tipton, Jr. Interscience Publishers, Inc., New York 1960. pp. 762-774.
4. Apkarian, H. "Investigation of Liquid Metal Lubricated Bearings". G.E. Report R50GL231. November, 1950.
5. Kissel, J. W., Glaeser, W. A., and Allen, C. M. "Frictional Behavior of Sodium-Lubricated Materials in a Controlled High-Temperature Environment". Wear. Vol. 5 (1962) p 446.
6. Wallace, M. J. "Summary of Potential Liquid Metal Bearing Materials for SNAP-50/SPUR Pumps". PWAC-468. November 1965. Pratt & Whitney Aircraft-CANEL.
7. Kumpitsch, R. C., Granan, J. R., and Kroon, P. J. "Study of a Liquid Metal, NaK 77, For Application in Flight Control Systems". ASD-TR-62-597. Vol. 1 May, 1962.
8. Quarterly Technical Progress Report No. APS-5152-R3 SNAP-50/SPUR Contract by AiResearch Mfg. Co. July 18, 1965. Contract No. AF33(615)2289.
9. Schuller, F. T., Anderson, W. J. and Nemeth, Z. N. "The Operation of Hydrodynamic Journal Bearings of Various Materials and Designs in Sodium at Temperatures to 800F". (NASA TN, to be published).
10. Murray, S. F. "Evaluation of Material Combinations for Hydrodynamic Gas-Lubricated Bearings". Presented at ASME Spring Lubrication Symposium. New Orleans. June 5-9, 1966.

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3. Reactor Handbook. Vol. I - Materials Edited by C. R. Tipton, Jr. Interscience Publishers, Inc., New York 1960. pp. 762-774.
4. Apkarian, H. "Investigation of Liquid Metal Lubricated Bearings". G.E. Report R50GL231. November, 1950.
5. Kissel, J. W., Glaeser, W. A., and Allen, C. M. "Frictional Behavior of Sodium-Lubricated Materials in a Controlled High-Temperature Environment". Wear. Vol. 5 (1962) p 446.
6. Wallace, M. J. "Summary of Potential Liquid Metal Bearing Materials for SNAP-50/SPUR Pumps". PWAC-468. November 1965. Pratt & Whitney Aircraft-CANEL.
7. Kumpitsch, R. C., Granan, J. R., and Kroon, P. J. "Study of a Liquid Metal, NaK 77, For Application in Flight Control Systems". ASD-TR-62-597. Vol. 1 May, 1962.
8. Quarterly Technical Progress Report No. APS-5152-R3 SNAP-50/SPUR Contract by AiResearch Mfg. Co. July 18, 1965. Contract No. AF33(615)2289.
9. Schuller, F. T., Anderson, W. J. and Nemeth, Z. N. "The Operation of Hydrodynamic Journal Bearings of Various Materials and Designs in Sodium at Temperatures to 800F". (NASA TN, to be published).
10. Murray, S. F. "Evaluation of Material Combinations for Hydrodynamic Gas-Lubricated Bearings". Presented at ASME Spring Lubrication Symposium. New Orleans. June 5-9, 1966.

TABLE 1
PROMISING COMBINATIONS FOR NaK

S. J. Basham - Battelle
 Valve-Seat Tests in 1500F NaK

Molybdenum vs. Molybdenum

Molybdenum vs. Cobalt-Bonded
 Tungsten Carbide

Tungsten vs. Tungsten

Tungsten vs. Cobalt-Bonded
 Tungsten Carbide

D. B. Vail - KAPL
 Sliding Tests in 850F NaK

Cobalt-Bonded Tungsten
 Carbide vs. Itself

Nickel-Bonded Titanium
 Carbide vs. Cobalt-Bonded
 Tungsten Carbide

TABLE 2
TYPICAL CORROSION RESISTANT BEARING AND STRUCTURAL
MATERIALS FOR LIQUID METAL SERVICE

	Coefficient of Thermal Expansion in./in./°F x 10 ⁻⁶	Density - Pounds Per Cu.In.	Mechanical Shock Resistance
<u>BEARING MATERIALS</u>			
Cobalt-Bonded Tungsten Carbide	2.38	0.54	Poor
Nickel-Molybdenum Bonded Titanium Carbide	3.7	0.22	Poor
Tungsten	2.5	0.70	Fair
TZM (Mo-Base)	2.7	0.37	Fair
Stellite Star J	6.8	0.316	Fair-Poor
<u>STRUCTURAL MATERIALS</u>			
347 Stainless	9.3	0.29	Good
410 Stainless	5.7	0.28	Good
Columbium - 1Zr	3.82	0.31	Good
Haynes 25	9.4	0.33	Good

TABLE 3

RESULTS OF BOND STRENGTH TESTS BEFORE AND AFTER EXPOSURE TO 200 F NAK FOR 300 HOURSALL TESTS IN DUPLICATEA. Control Specimens Which Were Not Exposed to Nak

Type of Coating	Tensile Stress at Break	Average Tensile Stress	Percent of Area Broken Away
Cobalt Bonded Tungsten Carbide on 410 Stainless	3080 psi 3900	3490 psi	About 40%
Chrome Oxide on 410 Stainless	1650 psi 1470	1560 psi	About 20% Less than 10%
Chrome Oxide with Molybdenum Undercoat on 410 Stainless	1920 psi 2000	1960 psi	Almost complete separation

B. Specimens After 300 Hour Exposure to Nak at 200F

Cobalt Bonded Tungsten Carbide on 410 Stainless	2175 psi 2560	2367 psi	Surfaces did not fail, break occurred at epoxy
Chrome Oxide on 410 Stainless	1625 psi 1700	1662 psi	About 10% Most of coating
Chrome Oxide with Molybdenum Undercoat on 410 Stainless	2975 psi 3150	3062 psi	Almost complete separation

TABLE 4
APPEARANCE OF TEST SPECIMENS AFTER STATIC CORROSION TESTS IN NAK AT 600F

Coating	After One 50 Hour Test	After Two 50 Hour Tests	After Three More 50 Hour Tests
Chrome Oxide on 410 Stainless Steel	Coating Disintegrated	Not Run	Not Run
Chrome Oxide plus Molybdenum Undercoat on 410 Stainless Steel	Chrome Oxide Disintegrated, Molybdenum intact	Not Run	Not Run
Cobalt-Bonded Tungsten Carbide on 410 Stainless Steel	Dark film, easily washed off	Faint suspicion of pitting	Dark film, easily washed off. Same as after first two 50 hour tests
Nickel-Bonded Tungsten (W-Cr) Carbide on 410 Stainless Steel	Not Run	Not Run	Covered by dark film, easily washed off. No damage visible
Nickel-Bonded Tungsten (W-Cr) Carbide on 416 Stainless Steel, Lapped to High Polish	Not Run	Not Run	Covered by dark film, easily washed off. Still shows high polish
Al ₂ O ₃ Coating on 316 Stainless steel	Not Run	Not Run	Coating still intact but it is black and appears to be rougher

TABLE 5
RESULTS OF BOND STRENGTH TESTS BEFORE AND AFTER EXPOSURE TO 600F NAK

A. Control Specimens Which Were Not Exposed To Nak

Type of Coating	Average Tensile Stress	Percent of Area Broken Away
Cobalt-Bonded Tungsten Carbide on 410 Stainless	3490 psi	About 40%
Nickel-Bonded Tungsten (W-Cr) Carbide on 410 Stainless	>4500 psi	Epoxy bond failed before coating

B. Specimens After Exposure To 600F Nak

Cobalt-Bonded Tungsten Carbide on 410 Stainless	>2180 psi ^(a)	Surfaces did not fail, break occurred at epoxy
Nickel-Bonded Tungsten (W-Cr) Carbide on 410 Stainless	>3800 psi ^(b)	Surfaces did not fail, break occurred at epoxy

(a) After five 50 hour tests

(b) After three 50 hour tests

Fig. 1 Cemented Carbide Thrust Bearing

Fig. 2 Disassembled View of Test Rig Showing
Test Shafts and Tilting-Pad Bearing

M-50 STEEL

CHROME OXIDE

TUNGSTEN CARBIDE

Fig. 3 Tilting Pad Bearings After 1000 Starts and Stops in Argon Atmosphere at 4 psi

M-50 STEEL

CHROME OXIDE

TUNGSTEN CARBIDE

Fig. 4 Tilting Pad Bearings After High Speed Rubs at 60,000 rpm in Argon Atmosphere

MTI-1963

UNLESS OTHERWISE SPECIFIED USE	STANDARD PRACTICES	SURFACES ✓	TOLERANCES		
			DIMENSIONS	FRACTIONS	ANGLES
				1/64	+
APPROVED <i>H. JONES</i> 3/16/66			<i>SKA 2253</i>		
CHECKED					
DRAWN <i>W. Shuford</i> 16 MAR 66		SCALE FULL	SHEET OF		

MTI-2119

Fig. 6 Tensile Test Specimens Showing Typical Coating Failures

Fig. 7 Chrome Oxide Coating After Exposure to 600F NaK

Fig. 8 Appearance of Test Specimens After Exposure to NaK at 600F

Fig. 9 Shaft and Tilting Pad Bearings from MTI NaK Test
Showing Coatings on Journal and Pad Surfaces

SECTION 7TECHNOLOGY IV - ROTOR-BEARING DYNAMICS

BEARING SHAFT SYSTEM DYNAMICS :
MERCURY RANKINE EXPERIENCE AT TRW

by

R. Kasuba
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SUMMARY

The CRU V's were designed following a combined experimental and analytical approach. With the past and present general scarcity of rigorous, analytical methods in deriving a successful mercury lubricated rotor-bearing system operating beyond the laminar regime, the initially selected analytical techniques were progressively extended or empirically adjusted as needed.

Satisfactory correlations between the predicted and measured responses of the CRU V rotor-bearing system were obtained.

Deviations between the predicted and measured CRU V rotor-bearing responses could be further decreased as in most similar complex design cases by:

- a) Development of required instrumentation
- b) More accurate description of rotor-bearing parameters
- c) Improvements of analytical techniques

These further refinements would somewhat improve the accuracy of correlations between the predicted and experimental rotor-bearing performance; however, this was not necessarily required for the CRU V development objectives.

The dynamic quantities of the selected three-pad bearing may be characterized by high damping capacity and wide regions of stable operation.

The CRU V bearing system exhibited the critical damping factors of about .25 to .5 for the considered geometric and operational parameters. As the consequence the shaft orbits and bearing reactions were effectively limited for the entire speed range and the predicted critical speed could not be experimentally observed.

The three-pad bearing provided sufficiently large stability regions by selecting appropriate pressures and geometric parameters. For example, at the CRU V operating speed of 36,000 rpm the lubricant supply pressures of 225 psia were sufficient to prevent fractional whirl instabilities for all variations of bearing clearances and unidirectional and rotating load combinations.

Finally, the results described here were not only dependent upon development of improved analytical techniques, but also reflect advanced design and dimensional control of critical system elements through fabrication and assembly procedures.

INTRODUCTION

This presentation sets as its primary task to discuss briefly some of the rotor-bearing dynamical quantities and responses of the Mercury Rankine Power Systems (SNAP 2) turbo-alternator units which are being developed for the United States Atomic Energy Commission by TRW. The above turbo-alternator units, more popularly named as the Combined Rotating Units (CRU's) use a liquid mercury lubricated bearing system. A degree of success in developing the CRU's may be illustrated by the accumulated operational times. For example, the CRU V type rotors have accumulated almost 22,000 hours. Individual CRU V units show operational times up to 4700 hours. An additional 20,000 hours were accrued by Mercury Rankine turbo-machinery units of earlier designs.

Design and development of the CRU's was not an easy task. The rotor-bearing system to be considered was in the little-known and investigated nonlaminar flow regimes involving a flexible rotor. Stringent bearing requirements in addition to the very limited state-of-the-art involving fluids with high inertial effects complicated the analytical design and development of mercury lubricated bearings. The usual budgetary and engineering schedules for developing the system were also present. Consequently, within all these constraints the basic CRU rotor bearing system's design was pursued along the parallel and combined paths of analytical and experimental procedures.

Experimentation and testing was conducted at two levels:

- a) Component level, which was used for basic development of individual CRU elements, such as the bearings
- b) System level involving the entire turbo-alternator system with the preselected bearings.

7.1.4

In deriving the analytical design procedures of the rotor-bearing system for operation in the turbulent regime theoretical methods were similarly employed in two stages. For example, the rotor responses and bearing characteristics were initially predicted with the available analytical methods which during bearing testing at the component level were progressively extended or empirically adjusted as needed.

With the analytical and experimental techniques used in developing the required rotor-bearing system well defined in relevant references, this presentation will attempt to convey only some of the more typical results or observations derived from designs of these turbo-alternator units. The presented results will primarily include the bearing stability regions, distributions of rotating loads, bearing film stiffness and damping coefficients, and the CRU V rotor-three-pad bearing system responses.

DEVELOPMENT OF THE CRU V ROTOR-BEARING SYSTEM

In the Mercury Rankine Space Power Systems the size and the main mass of the rotor and to some extent the bearing span are considerably predetermined by the electrical power requirements, thus leaving the responsibility for optimization of the dynamical rotor-bearing performance to the proper design of the extending shaft sections and the bearings. On the basis of force response analyses, structural and thermoelastic studies, some adjustments or trade-offs were made in designs to cover three entirely different mechanical and thermal regimes of startup, steady-state and spindown. A large number of considerations involved in the system's analyses may be represented by the factors considered in selecting the suitable bearings for this specific application. These factors, not necessarily in the order of importance, may be listed as follows:

- a) Load carrying capacity
- b) Power loss
- c) Bearing film coefficients to provide satisfactory range of bearing-rotor responses
- d) Stability
- e) Resistance to cavitation-erosion
- f) Compatibility of required materials
- g) Mechanical and thermoelastic deformations (system and bearings)
- h) Satisfactory performance within systems pressure and flow capabilities.
- i) Spinup and spindown transient conditions with no metal to metal contact
- j) Operation exceeding Reynolds numbers of 8,000
- k) Long-term operation $\geq$ 10,000 hours
- l) Easy CRU integration

7.1.6

The three-pad, the three-sector, the tilting-pad journal bearings, and the spiral groove and hydrostatic thrust bearings were analyzed for the CRU application. Employing a system's approach which also included testing of the above mentioned bearings, the three-pad journal bearing and the hydrostatic thrust bearing were selected for the CRU V system.

The selected hybrid three-pad bearing shown in Figure 4 has three supply pads equally spaced around the circumference and three equally sized compensating orifices. The three-pad bearing can be also described as geometrically identical to the three-sector bearing with the exception of the axial drain grooves.

The main experimental portion of the previously mentioned combined CRU design program was conducted at the component test level. The obtained bearing data on endurance, stability and bearing film coefficients provided additional information for selecting the bearings for testing at the system's level.

For assuring the dynamical compatibility between these two test levels the actual CRU and the counterpart bearing test rotors were designed to be dynamically similar units. The main features of both systems will be reviewed by a few general comments.

CRU V Unit

The CRU V and the associated test setup at the system's test level are illustrated in Figures 1 and 2. The compactness, electrical, hermetical sealing and structural requirements limited direct measurements of the shaft orbits at a single location far outboard of the turbine bearing. Limited direct instrumentation and structural inaccessibility made the actual CRU V somewhat less suitable for the fundamental and convenient research of the bearings. For

implementation of the shaft orbit data taken at a single axial location, several accelerometers were installed on the CRU V housing which was in turn supported by the test rig structure. Acceleration levels were subsequently used as a cross-checking element of the calculated shaft orbits for the entire rotor length. The obtained accuracy of correlations of measured orbits and acceleration levels with the predicted counterpart values was quite satisfactory and will be commented upon in the later stages of this presentation.

The basic instrumentation and display systems used in monitoring the rotor responses at both levels of testing included redesigned capacitance probes for high temperature application (600°F), panoramic analyzers, dual-beam oscilloscopes, and magnetic tape recording and playback systems for studies of startups and shutdowns. The capacitance probe signals were displayed both as individual probe signals and as Lissajous patterns. All rotating loads and dynamic phase angles were referenced to the scribe marks in the shaft at the capacitance probe locations.

Bearing Test Rig

The basic test rig shown in Figure 3 has capabilities for operation at elevated temperatures to 500°F. The test rotor which is dynamically similar to the CRU rotor can be subjected either separately or in combination to both rotating and unidirectional loads. The unidirectional loads can be introduced in either upward or downward direction by means of the hydrostatic gas type load pads and the desired initial mode of unbalance can be selected by inserting appropriate unbalancing screws.

The basic instrumentation was similar to that used in the actual CRU's; however, the structural accessibility to instrumentation permitted the use of four

capacitance probes in sets of two located 90 degrees apart at two axial locations. Techniques used in displaying the capacitance probe data at the system's level were also employed at this testing level with the sample data presented in Figure 5.

LOADS, BEARING STABILITY, AND BEARING FILM COEFFICIENTS OF CRU ROTOR-BEARING SYSTEM

Several basic dynamical quantities are needed for predicting the responses for a given rotor-bearing system. These relevant quantities generally include definition of loads, areas of stable (non-fractional frequency whirl) operation and bearing film coefficients. The presented quantities, it should be mentioned, will refer to the CRU V units and specific three-pad bearing geometries and thus may not be directly applicable to other systems. It is hoped, however, with this presentation to indicate the general magnitudes of loads, stability fields and the range of bearing film coefficients for similar power systems involving mercury lubricated three-pad bearings operating in a non-laminar regime.

Loads and Balancing Techniques

As in any complex long-duration rotating electro-mechanical systems the CRU rotors are subjected to both rotating and unidirectional loads of various distribution and sources. The unidirectional CRU loads consist of the rotor weight, some magnetic loads, turbine nozzle and mercury pump loads. These loads are of similar magnitudes as the operational rotating loads and for some CRU units they may be self-balancing with the rotating loads becoming as the primary loading mode.

The CRU rotating loads are comprised of the rotating magnetic unbalance, and the inherent and developed mass unbalance loads. The magnitudes of these loads were predicted and wherever it was practically possible they were measured. For example, the magnitudes, distributions and angular orientations of the rotating magnetic loads were experimentally established while the distribution of the inherent mass eccentricities were derived from the geometrical

7.1.10

tolerances. The exact distributions of the inherent mass unbalances were not fully known; however, this situation was theoretically analyzed by assessing the responses for a wide range of practical and theoretical distributions of inherent mass unbalances.

The approximate range of the CRU V load magnitudes can be given as:

Rotating magnetic unbalance loads - 2.0 to 7.0 pounds

Direct mass unbalance - mass eccentricity vectors of about .0002 in.
All rotors were balanced in two planes within .0003 in-oz at 5600 rpm

Indirect mass unbalance - rotor loads induced due to shaft bending by various distributions of mass eccentricities of about 3.0 to 5.0 pounds at 36,000 rpm

Thermally and time induced mass unbalance loads which cannot be practically counterbalanced at the unit's assembly were calculated to be approximately 3.0 pounds at 36,000 rpm

Complex load distributions in high speed applications always tend to present a problem in selecting a proper balancing method. Another complication may arise in that the off-site balancing and actual operational conditions may be entirely different with structural inaccessibility permitting no operational or on-site balancing.

The multi-plane and two-plane balancing methods were considered and used at one or another stage of the rotor development program. The type of balancing and the number of balancing planes can be selected by examining the computed responses of the flexible rotor for various distributions of the previously discussed direct and indirect unbalances. Using this theoretical approach it was established that the two-plane balancing at 5600 rpm will provide for various load distributions satisfactorily low operational bearing reactions at 36,000 rpm for the present generations of CRU's.

A characteristic computed response pattern is presented in Figure 12.

Bearing Film Coefficients

The force transmitted into the bearing film or bearing reactions can be expressed as:

$$FTR_X = \bar{K}_X \ddot{X} + \bar{C}_X \dot{X} \quad - \text{ in the X direction}$$

$$FTR_Y = \bar{K}_Y \ddot{Y} + \bar{C}_Y \dot{Y} \quad - \text{ in the Y direction}$$

where $\bar{K}$ and $\bar{C}$ are the effective film stiffness and damping coefficients in the given directions. Following this representation the quantities $\bar{K}_X$, $\bar{C}_X$, $\bar{K}_Y$ and $\bar{C}_Y$ can be experimentally established. As such they are the effective coefficients which include all dynamic and geometric effects of a given bearing. Theoretically, these effective coefficients can be expressed as complex functions of eccentricities, direct and cross-coupled film coefficients which in this particular case must include the significant liquid mercury inertia effects.

The above effective film coefficients were obtained following Hagg and Sankey from the steady-state solutions of the following basic set of equations of motion:

$$M \ddot{X} + C_X \dot{X} + K_X X = F \cos (wt + \phi)$$

$$M \ddot{Y} + C_Y \dot{Y} + K_Y Y = F \sin (wt + \phi)$$

F and M are the applied unbalance force and the applicable rotor mass per bearing, respectively, and the mutually perpendicular X and Y axes correspond to the major and minor axes of the journal locus. Employing the measured values of displacements and phase angles of:

$$X = \delta_X, \phi = \phi_X \quad \text{for } Y = 0$$

and

$$Y = \delta_Y; \phi = \phi_Y \quad \text{for } X = 0$$

the K and C quantities become:

$$\bar{K}_X = \frac{F \cos \phi_X}{\delta_X} + m \omega^2$$

$$\bar{K}_Y = \frac{F \cos \phi_Y}{\delta_Y} + m \omega^2$$

$$\bar{C}_X = \frac{F \sin \phi_X}{\delta_X \omega}$$

$$\bar{C}_Y = \frac{F \sin \phi_Y}{\delta_Y \omega}$$

The required displacements and phase angles for solving the above relationships were obtained from the component level tests and are illustrated in Figure 5. Circularity of the Lissajous patterns were maintained until the unidirectional loads exceeded the applied rotational loads by about 1.5 to 2.0 times.

For the case where the predominant loads are rotational loads such as is the case in CRU's the measured orbits are circular and thus the effective coefficients may be given as:

$$\bar{K}_X = \bar{K}_Y \text{ and } \bar{C}_X = \bar{C}_Y$$

which then arbitrarily could be referred to as the radial bearing film stiffness and damping coefficients K and C, respectively.

Some of the characteristic trends of bearing film coefficients are depicted graphically in Figures 6 and 7, which show the effects of supply pressures and two magnitudes of impressed unbalance loads at ambient temperatures for two clearances which will be designated here as the standard and large (1.3 x standard) clearances. The basic trends of the hot bearing test results (400°F) were similar as predicted to the cold test data.

- A. For the considered range of loads which represent the eccentricity ratios below 0.5 the bearing film coefficients of a three-pad bearing can be taken to be almost independent of applied loads. Most of the spread between the two load curves can be assigned to a scatter of experimental data points.
- B. The bearing film stiffness curves suggest again two distinctive regions of operation of these hybrid bearings. In the low speed region the hydrostatic action appears to be the predominant mode which is identified by somewhat flattened stiffness curves. In this region the bearing film stiffness is strongly dependent on the magnitude of supply pressure. At the inception of the high speed region somewhere above 25,000 rpm range the hydrodynamic action begins to predominate and the stiffness becomes more dependent on speed than on the initial supply pressure.

The general trends of the obtained bearing film coefficients of a three-pad bearing may be identified as:

Film stiffness - increasing slope with speed

Film damping - somewhat decreasing slope with speed to flattened curves for some geometric bearing properties

- C. The thermal tests suggested the following adjustments:

- 1) The bearing film stiffness is generally reduced from 0 to 15% depending on the speed by increasing the lubricant supply temperature from 70° to 400°F. Within the same temperature range the

bearing film damping is reduced by approximately 35% which is equivalent to the percentage reduction in mercury viscosity.

Stability Threshold

Two stability threshold patterns of bearing supply pressure versus shaft speed with no load, and unidirectional load versus bearing flow at a given speed are given in Figures 8 and 9. These threshold stability patterns were derived experimentally for the vertical and horizontal shaft operation and can be considered as the limiting patterns for the system since they represent the largest expected bearing clearances.

The instabilities observed with the three-pad bearings for several orifice sizes did not exhibit an abrupt transition between stable and unstable operation. The whirl orbit of these bearings gradually increased at the onset of instability for each incremental parameter change until reaching a stabilized orbit plateau when the bearing was operated well into the unstable region. This transitional area may be arbitrarily defined as the region of hysteresis. The orbits of these bearings in the instability region remained several times below the available bearing clearance. In characteristic unstable gas or oil lubricated bearings the whirl orbits typically increase from a stable small orbit quite rapidly to a whirl orbit approximating the bearing clearance orbit. The observed fractional frequency whirl was at .48 to .49 of the shaft whirling speed.

The fractional whirl instabilities were easily identifiable by harmonic analyzers and also by four notches in the Lissajous pattern or two notches in individual traces of the capacitance probe data. Otherwise the recorded data was similar to one shown in Figure 5.

In summary, at the CRU V operating speed of 36,000 rpm the lubricant supply pressures of 225 psia were sufficient to prevent fractional whirl instabilities for all operational variations of bearing clearances and unidirectional and rotating load combinations.

RESPONSES OF THE CRU V ROTOR-BEARING SYSTEM

The main objectives during the system level or CRU V testing were directed toward correlation of the measured and predicted shaft orbits. This was an imposing task since only one axial location was accessible in the CRU for installation of the capacitance probes.

Referring to the previously reviewed dynamic quantities, some of the typical CRU V rotor-bearing system responses will be discussed. The initial basis of discussion will be set by establishing the theoretical relationship between the operational and critical speeds of the system. As shown in Figure 10, the intersections of the stiffness curves with the critical speed curves indicate the CRU V operation between the first and second critical speeds with the passage through the first critical speed between 17,000 to 27,000 rpm depending on bearing clearances and operational conditions.

The CRU V rotor-bearing system is a heavily damped (critical damping factor of .25 to .5) and comparatively lightly loaded system and, consequently, the dynamical rotor orbits and bearing reactions were effectively limited for the entire operational speed range. Experimental confirmation of critical speeds based on acceleration levels and capacitance probe measurements, including spindowns and startups, could not be firmly established, although three types of indirect checks were available to establish the CRU V critical speed ranges:

- A. Transient state - startups and spindowns
- B. Steady-state operation with a fixed set of parameters at various speed plateaus
- C. Steady-state speed operation with varying parametric inputs to affect bearing characteristics.

With orbit measurements taken at a single axial location far outboard of the turbine bearing, the maximum orbit at that location for some load configurations may not determine the actual critical speed. The phase angles between the rotating forces and displacements at critical speeds easily visualized for the equivalent rigid mass type members - displacement relationships are not directly transferable to the flexible type members under various load distribution. Finally, with a small change in transmissibility ratios of about 20% or lower for the entire CRU speed range, the small changes in acceleration at low acceleration levels are hardly detectable.

Closest experimental confirmation of the CRU V critical speeds can be based on the component level test data which indicates the first critical speed for the dynamically equivalent CRU V shaft in the predicted critical speed ranges defined by a change in phase angle of 90 degrees. Test data at the component level test were characteristic of a heavily damped system with no dramatic changes of orbits and phase angles between the applied shaft forces and journal displacement vectors over the entire speed range from 0 to 45,000 rpm or above.

The behavior of rotor whirl orbits in the entire operational CRU V speed range is shown in Figure 11. This presentation contains two types of data:

- a) Envelope of measured and calculated startup and spindown shaft extension orbits for a large number of CRU V units.
- b) Calculated journal orbits representing the more heavily loaded CRU V units at average operating conditions.

These two quantities are presented in relationship to the standard bearing clearance to indicate an approximate degree of available load capacity.

In subsequent paragraphs the developed bearing reactions will be correlated with the predicted and measured acceleration levels.

The approximate distributions of the previously discussed loads are shown in Figure 12 with the theoretical shaft orbits calculated at the design speed of 36,000 rpm for a typical set of bearing operational conditions (approximately 400 psia, 400°F). The loads considered for this illustration are:

Eccentricities:

$$e_1 = .0001 \text{ in.}$$

$$e_2 = .0002 \text{ in.}$$

$$e_3 = .0001 \text{ in.}$$

$(me)_1$ and $(me)_2$ - balancing weights needed for a close counterbalance of the
above inherent mass eccentricities

F_m - rotating magnetic unbalance load of 6.0 pounds

$(me)_6$ - thermally induced load of about 3 pounds at 36,000 rpm

$(me)_7$ - localized assembly and thermally induced unbalance of about 3 pounds
at 36,000 rpm

The presented curves show the corresponding shaft orbits and phase angles between the total rotor rotating force vector and the maximum displacement vectors at the journal and capacitance probe locations. Curve I represents the effects of the vectorial load combination as shown in Figure 12, with the Curve II representing the response when only the rotating magnetic load was rotated by 90 degrees from the original position. Curve III represents the load condition as Curve I with the load $(me)_7$ removed. This is the load condition which may be identified with some of the later CRU V units. As it may be expected, the change in localized loads at the flexible rotor section affects to a significant degree the shaft extension and to a much lesser

degree the closest bearing. The illustrated technique was used to predict the coupled rotor-housing responses for a variety of load distributions, balancing effects and later for correlations of measured and calculated responses.

The CRU V housing and its own suspension system within the test rigs form an equivalent mass-spring system upon which the measured housing acceleration levels depend. Some of the earlier CRU housing-housing suspension systems and the entire test rig were subjected to vibrolator excitations which yielded satisfactory checks with predicted responses. Later with changing types of the test rigs, it was resorted to analytical models for predicting the combined housing-rotor responses.

The obtained correlations between the predicted and measured orbits and accelerations are illustrated in a single presentation (Figure 13) for several CRU V units identified as -3D, -1C, etc. An immediate observation can be made from this presentation that various units were subjected to different loads and load distributions. At this time, assuming no problems in instrumentation, which will be discussed later, the iterations with slightly varied load magnitudes and distributions to provide closer correlations between the measured and predicted accelerations, measured and predicted, shaft extension orbits and the accompanying phase angle were used to establish a better assessment of loads. The responses shown in Figure 13 were based on the first selection of loads to show the agreement of correlations for typical operating conditions.

Operational or test times for various CRU V units identified in Figure 13 can be considered as another factor in assessing the obtained correlations of the orbits and accelerations. These operational times are shown to indicate the times at the test completions of the identified CRU V units and should

not be construed as the failing times of the bearings or the system.

Some problems in instrumentation may be suggested by the varying degree of satisfactory correlations of responses of some units. For example, a degree of poor correlation of the shaft extension orbits in the case of CRU unit ID related to the obtained satisfactory correlations between the measured and predicted acceleration levels can be taken as an indication of some problems in capacitance probes and associated instrumentation. Continuing along a similar thought, deviations between the predicted and experimental acceleration levels most probably can be assigned to inaccurately defined or selected load distributions.

At the beginning of the CRU V testing program the desired or required instrumentation was not available and some elements had to be improvised and designed, such as the capacitance probe extension stems, probe cooling systems, seals, etc. With time the CRU V test instrumentation was progressively improved to more reliable and effective measuring systems.

Some of the problems experienced in instrumentation related to the measurements of shaft orbits can be assigned to the off-site calibration techniques, shaft runouts, temporary capacitance probe circuit shortings due to flooding of the gap, long duration and thermal drifts, etc. Accelerometers on the other hand were not affected by most of the listed factors and correlations between the measured and calculated responses may be considered as reliable indicators of the bearing reactions.

The directly measured rotor orbits were comprised by the actual shaft extension orbit and the physical shaft runout at the probe location. The shaft extensions were ground to less than 100 micro-inches (TIR) referred simultaneously

to both journal surfaces. However, in most cases even such a small runout constituted a considerable portion of the directly measured shaft extension orbits. Thus, for obtaining a true shaft extension orbit from which the journal orbits were partly derived, it was mandatory to consider a vectorial relationship between the recorded runout and directly measured orbit at the capacitance probe locations.

At the beginning of testing it was quite desirable to use a dual monitoring system such as the accelerometers and capacitance probes to establish confidence and accuracy levels of the analytical methods and instrumentation in hostile environment.

In the later CRU V units utilizing the established confidence levels in predicting the coupled rotor-housing responses and experience with previous units, the capacitance probes were removed leaving the housing accelerometers as the only monitoring instrumentation of the rotor responses.

FIGURE 1 TURBOALTERNATOR ASSEMBLY - CRU V

FIGURE 2 CRU V TEST SET-UP

FIGURE 3 BEARING TEST RIG

FIGURE 4 THREE-PAD BEARING

FIGURE 5 TYPICAL COMPONENT LEVEL
BEARING TEST CAPACITANCE PROBE DATA

LUBRICANT : LIQUID MERCURY
LUBRICANT INLET TEMPERATURE : ROOM TEMP.
SUPPLY PRESSURE : 400 PSIA
ROTATING LOAD : .00725 IN-OZ/BRG
UNIDIRECTIONAL LOAD : ZERO
SPEED AS SHOWN

15,000 RPM

25,000 RPM

36,000 RPM

45,000 RPM

FIGURE 6 THREE-PAD BEARING FILM STIFFNESS AND DAMPING COEFFICIENTS (STANDARD BEARING CLEARANCE)

LUBRICANT - MERCURY

TEST CONDITIONS:

SUPPLY PRESSURE 250 AND 400 PSI

ROTATING LOADS:

.00335 AND .00725 IN-OZ/BRG

TEMPERATURE: ROOM

FIGURE 7 THREE-PAD BEARING FILM STIFFNESS AND DAMPING COEFFICIENTS (1.3 X STANDARD BEARING CLEARANCE)

1.3 X STD. BEARING CLEARANCE

LUBRICANT - MERCURY

TEST CONDITIONS:

ROTATING LOAD:

.00725 IN - OZ / BRG

TEMPERATURE - ROOM

----- 206 PSIA

-.-.-.- 256 PSIA

———— 406 PSIA

-.-.-.- 456 PSIA

FIGURE 9 STABILITY THRESHOLD WITH UNIDIRECTIONAL LOAD

FIGURE 10 CRU V CRITICAL SPEEDS VERSUS BEARING FILM STIFFNESS CURVES

FIGURE 11 SHAFT ORBITS DURING TRANSIENT STATE OPERATION (SPINUPS AND SPINDOWNS)

FIGURE 12 TYPICAL LOAD DISTRIBUTION AND
THEORETICAL SHAFT ORBITS AT 36,000 RPM

LOCATIONS
1 ALTERNATOR BEARING
2 BALANCING PLANE
3 BALANCING PLANE
4 TURBINE WHEEL
5 TURBINE WHEEL
6 TURBINE BEARING
7 SHAFT EXTENSION
8 CAPACITANCE PROBE LOCATION

LOAD DISTRIBUTIONS
(me) ₁ BALANCING PLANE EFFECTS
(me) ₃ BALANCING PLANE EFFECTS
F _m ROTATING MAGNETIC LOAD
(me) ₆ THERMALLY INDUCED UNBALANCE
(me) ₇ LOCALIZED ASSEMBLY OR THERMALLY INDUCED UNBALANCE
e ₁ , e ₂ , e ₃ DISTRIBUTED UNBALANCES

FIGURE 13 COMPARISON OF MEASURED AND PREDICTED ROTOR RESPONSES AT 36,000 RPM

CALCULATION AND EXPERIMENTS ON THE UNBALANCE
RESPONSE OF FLEXIBLE ROTORS SUPPORTED BY TILTING
PAD BEARINGS OPERATING IN THE TURBULENT FLOW REGIME

by

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ABSTRACT

The results of a combined analytical and experimental investigation of the unbalance vibrations of a rotor are presented. The analysis applies to a general rotor-bearing system in which the dynamic bearing forces are represented by four spring coefficients and four damping coefficients. The rotor can be represented either as a lumped or a distributed parameter system, and gyroscopic moments are included. In general, the unbalance whirl motion of the rotor will be elliptical. The analysis has been programmed for a digital computer to obtain results for comparison with the experimental data.

The test rotor is a uniform, flexible shaft with heavy wheels mounted at the ends and in the middle. The rotor is supported in two silicone fluid lubricated, tilting-pad bearings. The rotor amplitude caused by an induced unbalance has been measured over a speed range of 3000 to 24,000 RPM for three different rotor configurations, obtained by removing one or both end wheels. This speed range extends to or through the third critical speed for each of the rotor configurations. The results are compared with the theoretical values and, in general, the agreement is found to be good.

INTRODUCTION

To date, the shaft diameters of high-speed, process-fluid lubricated machines have generally been selected with the intent of maintaining high shaft rigidity, so that all flexural critical speeds are well above the highest operating speed. Application of this design approach, however, becomes increasingly difficult as operating speeds are raised, in addition to introducing needlessly high parasitic losses in the bearings and seals. Accordingly, it is anticipated that as the level of experience with high speed, process-fluid lubricated turbomachinery increases, there will be greater acceptance of the flexible rotor design approach.

Parasitic bearing power losses may be minimized by adopting the flexible rotor approach to the design of turbomachinery. This is particularly the case with units that operate at high speeds and are lubricated with low viscosity cycle fluids, such as water or liquid metals, so that the bearings operate beyond the transition from laminar flow. In the superlaminar flow regimes, but where the Reynolds Number is less than 10^4 , the bearing and seal losses vary as the 3.5 power of the diameter. At higher values of Reynolds Number, the exponent is still larger. Moreover, the use of flexible rotors permits greater machinery compactness and it may also allow design simplification. For these reasons, it has long since been generally adopted for high speed, oil lubricated machines such as steam and gas turbines.

In units that operate at high speeds and where the steady-state load on the radial bearings is small, reductions in bearing diameter are generally permissible from the standpoint of load capacity. The effect of increased flexibility of the rotor will, however, cause highly significant changes in the dynamic response of the rotor-bearings system. Accordingly, an accurate and detailed investigation of rotor response to unbalanced loads is necessary during the design phase, in order to arrive at arrangements that are not subject to high vibration amplitude within operating speed ranges. Such an analysis must couple the dynamic characteristics of the bearings and those of the rotor, as well as the pedestals or other flexible supports.

Analytical procedures for computing the dynamic properties of fluid film bearings in turbulent as well as in laminar flow regimes are currently available, from earlier studies of turbulent fluid films 1,2 . These dynamic properties of fluid film bearings, expressed in the form of stiffness and damping coefficients, are then coupled with the dynamic properties of the rotor and those of the pedestals or other flexible supports, to analyze the dynamic response of the overall rotor-bearings-supports system, using procedures which are also currently available 3 .

In this presentation, the dynamic response analysis of the general, rotor-bearings system is described, indicating how the rotor may be treated either by a lumped or distributed parameter system. In addition, an experimental investigation that was conducted under AEC Contract AT(30-1)-3363 is described, which was performed for the purposes of:

- (a) evaluating the effectiveness of the combination of theoretical bearing dynamic properties in turbulent flow regime and the rotor dynamic response analysis, as a design tool for high-speed rotor-bearing system design, and
- (b) demonstrating the feasibility of operating systems with flexible rotors, supported on bearings lubricated with low viscosity fluids and operating in the turbulent flow regime.

The rotor vibration amplitudes caused by induced mechanical unbalances were measured in five radial planes, over a speed range of 3000 to 24,000 RPM, with three different rotor configurations. This speed range extended to or through the third critical speed, for each of the rotor configurations. The results were in good agreement with the calculated values and it was concluded that the currently existing theoretical analyses of bearing and rotor-dynamics are effective design tools.

Before proceeding with the review of the analysis and the test data, it is appropriate to emphasize here the need for investigating the response of high-speed, rotor-bearing systems over the entire speed range of interest. It is not generally sufficient to merely compute the critical speeds, considering the rotor and bear-

ing flexibilities and then adjust the rotor-bearings design so that these critical speeds lie outside the operating speed range. While critical speed calculations are obviously valuable, they provide only a part of the information which the designer needs. In particular, the following information is not provided by critical speed calculations, but may be obtained from calculation of the unbalance response of the rotor-bearings system, including the effects of bearing damping:

1. There can be significant effects of bearing film damping on the critical speeds. While bearing flexibility lowers the critical speeds, the bearing damping usually has the opposite effect. Hence, a conventional critical speed calculation which includes the bearing flexibility, but not the bearing damping, will tend to give values for the critical speeds which are lower than the ones actually observed. The actual critical speeds can be determined from a series of unbalance response calculations made at intervals over the speed range to locate the response peak.
2. Depending on the rotor and bearing designs, there will be great differences in the amplitudes of vibration which are encountered at the critical speeds. In most cases, bearing film damping is the dominant controlling influence on rotor vibration at the critical speeds. Bearing damping can be so effective that, in many machines, it is very difficult to identify the critical speeds without special instrumentation. On the other hand, if the bearings are not effective in controlling the rotor vibration either because of their location or because they have insufficient damping, there may be very large amplitudes at the critical speeds. Thus, unbalance response calculations can be used to determine the importance of the critical speeds and to evaluate the adequacy of the proposed rotor and bearing designs.
3. The response of a particular rotor-bearing system will depend on the amount and distribution of rotor unbalance. By placing the unbalance at different positions on the rotor when performing unbalance response calculations, the sensitivity of the rotor to unbalance can be evaluated. Also, guidelines can be established for specifying the maximum tolerable unbalance of individual rotor components. In the same way, response calculations may be helpful in choosing the best locations for balancing planes.

ANALYSIS *

A typical rotor consists of a shaft on which is mounted a number of wheels, impellers or similar components. The shaft is seldom uniform but its diameter changes in steps along the length of the rotor. The rotor is supported in two or more fluid film journal bearings.

At any given speed and in the absence of rotor mass unbalance, the axis of the shaft is in a stationary equilibrium with a shape determined by the static deflection of the rotor and the eccentricity of the journals in the bearings. However, when the rotor contains an unbalance, the centrifugal forces cause the rotor to whirl and deform such that each point of the shaft axis whirls in a closed orbit around the corresponding equilibrium position. The whirl orbits are closely elliptical and their size, orientation and phase angle vary along the length of the rotor. It is the purpose of the analysis to set up a method for calculating these whirl orbits for an arbitrary rotor subjected to specified mass unbalances.

In order not to complicate the analysis unnecessarily, the hydrodynamic forces of the bearing fluid film are linearized. If the journal center amplitudes are x and y , measured from the static equilibrium position, and the corresponding dynamic forces are F_x and F_y , a first order Taylor series expansion of the bearing forces can be written:

$$\begin{aligned} F_x &= -K_{xx} x - B_{xx} \frac{dx}{dt} - K_{xy} y - B_{xy} \frac{dy}{dt} \\ F_y &= -K_{yx} x - B_{yx} \frac{dx}{dt} - K_{yy} y - B_{yy} \frac{dy}{dt} \end{aligned} \quad (1)$$

The four spring coefficients K_{xx}, K_{xy}, K_{yx} and K_{yy} , and the four damping coefficients B_{xx}, B_{xy}, B_{yx} and B_{yy} are determined directly from the partial derivatives of F_x and F_y , evaluated at the journal center equilibrium position corresponding to the particular rotor speed. They may be obtained by perturbing and solving the lubrication equation (Reynolds Equation) [3,6,8,9,10]. Thus, the coefficients depend on the bearing geometry, the lubricant viscosity, the static bearing reaction and the rotor speed. They can be expressed in dimensionless form as functions of the

* See Nomenclature on Page 7.2.21.

bearing Sommerfeld number and have been measured and computed for several bearing types 4 through 12. For most bearing types, the coefficients do not possess any symmetry characteristics and it is not possible to select a set of principal axes such that the cross-coupling terms are eliminated [11]. Hence, the whirl orbit becomes elliptical.

Although Eq. (1), strictly speaking, is valid only for small amplitudes, the expressions prove to represent the actual fluid film force also for large amplitudes with surprisingly good accuracy as long as the minimum film thickness during the whirling motion does not become less than approximately 25 per cent of the radial clearance [13]. In an actual application the rotor amplitude is rarely allowed to be that large and the chosen linearized representation of the fluid film forces is, therefore, adequate for most practical purposes. This seems also confirmed by the present experimental investigation as discussed later.

For analysis purposes the rotor itself can be represented by a series of stations connected by shaft sections of uniform cross-section as indicated in Fig. 1. The stations are chosen to coincide with any concentrated rotor masses such as wheels, impellers, etc. and, furthermore, stations are introduced at each location where the shaft diameter changes, at each bearing location and at both rotor ends. Thus, each station can be assigned a mass, a transverse mass moment of inertia, a polar mass moment of inertia, an unbalance and a set of 8 bearing spring and damping coefficients. For any particular station, several or all of these quantities may be equal to zero depending on which station is considered. Specifically, it should be noted that the mass of the shaft is treated separately and is not included at the rotor stations. Then applying a force balance and a moment balance, the jump in shear force and bending moment across a rotor station can be expressed in terms of the rotor amplitude and slope at the station. The equations are derived in Appendix A where they are given as Eqs. (9) to (12).

For each shaft section the length, the cross-sectional area, the cross-sectional moment of inertia and the shape factor for shear deformation are known. The shaft material properties: Young's modulus, shear modulus and mass density, are also given. Then, the relationships for the amplitude, the slope, the bending moment and the shear force between the two ends of the shaft section can be established

as shown by Eqs. (40) to (43) in Appendix B. The coefficients in these equations are speed dependent and include the effect of shaft inertia and shear deformation.

The equations for the rotor stations together with the equations for the shaft sections establish a set of recurrence formulas by which a step-by-step calculation of the rotor can be performed. Assuming the rotor ends to be free, a successive application of the recurrence formulas allows computing the amplitude, the slope, the bending moment and the shear force along the rotor in terms of the amplitude and the slope at the first station. Adding the contribution from the unbalance forces and equating the bending moment and the shear force at the last station to zero, makes it possible to determine the unknown amplitude and slope at the first station. By back-substitution, the amplitude at each station can finally be calculated. The procedure is given in more detail in Appendix A and is readily programmed for a digital computer. Such a program has been used to obtain the results for comparison with the experimental data as discussed later.

The analysis is seen to be an extension of the Prohl Method [14] for calculating the critical speeds of a rotor. The present method differs from the original one by including the anisotropic stiffness and damping characteristics of the fluid film bearings whereby the rotor motions in the horizontal and vertical directions become coupled and the whirl orbits become elliptical rather than circular. This feature also necessitates a different treatment of the gyroscopic moments as shown in Appendix A. Furthermore, in the original method the mass of the shaft is lumped at the rotor stations and the shaft is represented by its static influence coefficients, whereas the present method includes the effect of the distributed shaft mass and employs dynamic influence coefficients. Thus, within the assumptions inherent in conventional beam theory, the present method gives an exact solution to the problem where the original method involves approximations. It must be noted, however, that the more accurate treatment is only necessary if the number of rotor stations is less than approximately four to five times the number of that critical speed which follows after the upper limit of the operating speed range of the rotor. In this connection it may be observed that the solution by the present method, and also by the Prohl Method, depends directly on evaluating a determinant which is essentially the characteristic determinant of the system. For illustration, consider a simply supported, uniform shaft of length ℓ . The characteristic determinant as computed from the analysis is:

$$\frac{1}{4}(\sinh(\beta l) + \sin(\beta l))^2 - \frac{1}{4}(\sinh(\beta l) - \sin(\beta l))^2 = \sinh(\beta l) \sin(\beta l) \quad (2)$$

where β is proportional to the square root of the speed and is defined by Eq. (31) in Appendix B. The determinant is zero for $(\beta l) = m\pi$ where m then gives the number of the critical speed. The characteristic determinants for other cases are analogous. It is seen, however, that the actual value of the determinant is the remainder of a subtraction involving:

$$\frac{1}{4} \sinh^2(\beta l) - \frac{1}{4} \sin^2(\beta l)$$

Thus, if the computer works to an accuracy of k significant figures the remainder will be lost when $\frac{1}{4} \sinh^2(\beta l) \geq 10^k$ or when $(\beta l) \geq 2 + 2.3 \cdot k$, i.e. when $m \geq 0.6 + 0.7 \cdot k$. Most computers carry eight significant figures and in that case the present method suffers an increasing loss of accuracy beyond the fourth critical speed until it finally breaks down completely around the sixth critical speed. Going to double precision ($k=16$) raises the ultimate limit to around the eleventh critical speed.

For the calculations of the present test rotor, these considerations are of little importance. Furthermore, the test bearings are tilting pad bearings with four shoes arranged symmetrical with respect to both a vertical line and a horizontal line. Thereby the spring and damping coefficients become the same in all directions and the rotor whirl orbits are circular. This greatly simplifies the analysis and the calculations.

EXPERIMENTAL APPARATUS

The test rotor is a cylindrical steel shaft with overall length of 41 inches and a diameter of 2.5 inches except for a centrally located integral disk of 6 inches diameter and 6 inches length. Detachable disks of 6 inches diameter, 3 inches length can be mounted on the ends of the rotor by heavy interference fit and lock nuts which clamp them against shoulders on the shaft. The total weight of the rotor, exclusive of the detachable end masses, is 88 lbs. The center disk, not counting the inner 2.5 inch diameter section, weighs 36 lbs. and the end disks each weigh 18 lbs.

The rotor is supported by two tilting-pad journal bearings whose centerlines are 12.5 inches on either side of the rotor center plane. These are four-pad bearings with slenderness ratio (L/D) of 1.0 and bearing diameter of 2.5 inches. Other design characteristics include:

Clearance ratio (C/R) - 3×10^{-3} in/in (based on machined clearance)

Pad arc length - 80 degrees

Pivot position - 0.55 of the arc length (44 degrees) measured from the leading edge.

Geometrical preload coefficient - 0.5

Geometrical preloading is used to give improved bearing film stiffness at high speeds with low steady-state loads. It is achieved by moving the pads radially inward so that the actual clearance at the pivot location is less than the machined clearance (based on pad and shaft radii of curvature). In this case, the clearance at the pivots is just half the machined clearance.

The pivot configuration is a sphere seated in an internal cylindrical surface machined into the back surface of the pad. The bearings are oriented in the housing so the steady-state, gravity, load line bisects the pivot positions. That is, the pivots are at 45 degrees from a vertical line drawn through the bearing center.

Axial positioning of the shaft is accomplished by externally-pressurized, gas-lubricated thrust bearings on either side of the center disk.

The bearings are rigidly mounted in housings machined from a single block of aluminum jig plate. The housings are bolted and keyed to a massive structural steel base whose moment of inertia about the horizontal axis exceeds $9,000 \text{ in}^4$. There were no detectable traces of flexure or vibration of the base during the experiments.

The rotor is driven by an electric motor through a crowned spline coupling. The coupling can accommodate misalignment, between the motor and rotor axes, of up to 0.030 inch without restraint to the shaft. The motor is bolted and keyed to the same base surface which supports the bearing housings. Motor and test rotor shafts were aligned with less than 0.002 inch total runout measured radially and 0.0015 inch total runout measured axially on a 2 inch radius. A variable-frequency motor-generator set supplying power to the motor provides for variable speed operation within a range from about 3000 to 24,000 rpm.

The journal bearings are lubricated by a very low viscosity silicone fluid, 0.65 cs at 77F and 0.51 cs at 130F. Lubricant at controlled temperature is pumped into the bearing housing so the bearings are operated submerged in the lubricant. There are clearance seals and scavenging rings on either side of the bearing housings.

The motion of the rotor is measured in both horizontal and vertical directions at each of five measurement planes along its length. There are measurements planes at each end, at the center, and at the inboard sealing ring of each bearing housing (8.65 in. from the rotor center plane). These measurements are obtained with eddy-current, non-contacting proximity sensors. Readout is by oscilloscope with the signals connected to the horizontal and vertical deflection plates of the cathode ray tube. With this arrangement, the rotor center axis at the measurement plane is represented by a spot on the oscilloscope screen and motions of the rotor axis are reproduced by the spot on the screen.

Accurate measurement of the response of the rotor to imposed unbalance load requires care in minimizing other causes of real or apparent shaft axis motion. The principal such causes are shaft surface out-of-roundness or non-concentricity and residual rotor unbalance. Rigid specifications were imposed on rotor surface roundness and concentricity with the result that subsequent gaging showed all surfaces to be

round and concentric with the bearing journals within less than 0.00012 inch total runout. The rotor was balanced first in a balancing machine and then in place in the apparatus. In-place balancing was done by a trial and error procedure. The single, centered, disk rotor was balanced so that the maximum orbit amplitude at any measurement plane over the entire speed range was 0.00014 inch radius. For the two-disk system, which is more sensitive to unbalance, the maximum orbit radius was 0.00021 inch at the light end and all other positions were always below 0.00015 inch radius. The three-disk system was balanced at speeds up to 10,800 rpm. At this speed, the maximum whirl orbit radius was 0.00035 inch. At all speeds below 10,000 rpm, the maximum orbit radius was below 0.00013 inch.

In order to minimize the effects of runouts and residual unbalance on the test results, it is desirable to impose large deliberate unbalances to obtain sizeable response orbits. On the other hand, the assumption of linear bearing stiffness and damping about the steady-state rotor position will be strained if the orbits are too large. For most operating conditions, a good balance between these considerations was considered to be: the larger of the orbit radii at the two stations adjacent to the bearings should be between 0.20 and 0.40 times the bearing radial clearance measured at the pivots, or, between about 0.0004 and 0.0008 inch orbit radius. This limit was not applied to the three-disk rotor for reasons which will become clear. During the experiments, the amount of unbalance weight was adjusted as the speed was changed in order to remain within the limits given above. Since the orbit amplitudes at the center and end positions were usually larger than those at the bearing positions, the orbits with imposed unbalances were usually at least 4 times larger than the corresponding orbits with no imposed unbalance.

Bearing temperatures were measured by thermocouples welded to the trailing edge of each of two pads, flush with the bearing surface. These measured bearing temperatures were used to establish the lubricant film viscosity which, in turn, is needed to specify bearing stiffness and damping from the bearing design data. Bearing temperatures ranged up to about 130F at full speed in the experiments. Within limits, bearing temperature could be controlled by varying lubricant flow and inlet temperature.

CALCULATION OF TEST ROTOR UNBALANCE RESPONSE

A computer program based on the preceeding analysis was used to calculate the unbalance response of each of the three rotor configurations, that is, with one, two or three disks. Dynamic stiffness and damping values for the test bearings were obtained from the data given in Ref. 5. The stiffness and damping vary with the shaft speed and steady-state load and, since the test bearings operate in the turbulent flow regime, with the mean film Reynolds Number. In the case of the four-pad bearing with steady-state load line midway between the pivots, symmetry results in elimination of the cross-coupling terms and equality of stiffness and damping in x and y directions. Sample dimensional stiffness and damping values are given in Table 1 to illustrate the values obtained.

The computer program output included the calculated vibration amplitude at each of the planes where experimental measurements were made. Because of test bearing symmetry, the calculated vibration is a circular orbit whose radius is given in mils/oz-in. total unbalance.

Flexibility of the pad pivots was introduced into the calculations as a stiffness in series with the bearing. The calculated Hertzian contact stiffness of the combined pivots at 100 lb. bearing load is 5×10^5 lb/in. and this is the value used in the calculations at all speeds.

The undamped critical speeds of the three rotor-bearing systems were calculated using the Prohl method (Ref. 11) together with the theoretical bearing stiffness data. This was done for comparison with the calculated and measured unbalance response data. The comparisons are significant since it is common practice to use calculated critical speeds in rotor-bearing design analysis, but much less common to perform unbalance response calculations. The results are tabulated below for each of the three rotor configurations.

<u>Rotor Configurations</u>	<u>Calculated Critical Speeds by Prohl Method</u>		
	<u>1</u>	<u>2</u>	<u>3</u>
One-Disk Rotor	7200	17,800	24,500
Two-Disk Rotor	6100	8,700	21,000
Three-Disk Rotor	5000	5,800	11,000

TABLE 1THEORETICAL TEST BEARING STIFFNESS AND DAMPING

Stiffness, lb/in.

Speed,rpm	(1) One-Disk Rotor	Two-Disk Light-End	Rotor Heavy-End	Three-Disk Rotor
6050	8.1×10^4	8.1×10^4	8.4×10^4	8.5×10^4
8100	10.6	10.8	11.8	12.3
11,000	15.3	15.4	15.6	15.8
16,000	26.2	26.8	27.9	29.0
24,000	50.3	50.6	51.2	51.4

Damping, lb.sec/in.

6050	148	156	168	170
8100	156	161	170	174
11,000	173	178	183	186
16,000	218	223	228	231
24,000	266	269	273	274

- (1) The bearing film viscosity is assumed to be 0.65 cs at speeds up to 9000 rpm, 0.60 cs from 9000 to 17,000 rpm and 0.53 cs up to 24,000 rpm. Bearing temperature was controlled during the experiments to match these values.

COMPARISON OF EXPERIMENTAL AND CALCULATED ROTOR RESPONSE TO UNBALANCE

Calculated curves and experimental data points of rotor response as a function of speed for the single-disk rotor are given in Figs. 2 through 4. The location and relative magnitudes of the imposed unbalance weights are indicated schematically on the figures. Because of symmetry, the response at corresponding measurement planes on either side of the centerline should be the same and so the results at both ends are given together. At each measurement plane there are two sets of data. These are from two separate series of runs in which two different angular positions of unbalance weights were used. That is, after the first series was obtained, all of the unbalance weights were moved to a different angular position on the rotor, usually opposite the first location. Differences between these sets of data points can be taken as an indication of the influence of runout and residual unbalance effects on the experimental results.

The theoretical curves show the one-disk rotor system to be well behaved through the operating speed range. The orbit amplitudes level off about midway between the first and second calculated critical speeds and remain about the same or gradually decline through the remainder of the operating range. The experimental data show the same features and the absolute levels of vibration amplitude agree rather well with the calculated levels. While there is some experimental scatter, the measurements show no marked differences between the coupling and free ends or between the two angular positions of the unbalance weights.

According to theory, the response orbits should always be circular. The actual measured orbits were not always so, although their ellipticity was seldom marked. There were no recorded instances when the ratio of major to minor axes was greater than 3 and very few when it exceeded 2. When the orbits were elliptical, the major and minor axes were measured and the mean between them was taken as the experimental data point. There are a number of possible reasons for non-circular orbits of which the most significant is believed to be asymmetry of the bearings, because of slightly different preload between the two opposing pairs of pads.

Calculated curves and experimental data for the two-disk rotor with the unbalance weights in-line are given in Figs. 5 through 9. This configuration experiences

significantly higher vibration amplitudes for an equivalent total unbalance weight than the one-disk rotor. Also, there are well defined vibration peaks of a translation whirl mode at about 10,000 rpm and a conical whirl mode at about 12,000 rpm. The measured behavior of the rotor is consistent with the calculated results and, with one exception, the measured and calculated vibration amplitudes agree quite well. At the light end, one of the two sets of data shows orbits which are substantially larger than either the calculated results or those for the other set of data. There was no obvious explanation for this.

Results for the two-disk rotor when the unbalance weights at the two disks are 180 degrees out-of-line are given in Figs. 10 through 14. This results in a sharply defined conical mode vibration peak at about 11,000 rpm with noticeably larger amplitude than occurred when the unbalance weights were in-line. Again, agreement between experimental and theoretical results is good with respect to both the trend and the whirl orbit amplitudes.

At the third, free-free mode, critical speed of the three-disks rotor system, the bearings are very close to the rotor nodal points. The result is a sharp peak with very large whirl orbit amplitudes at the calculated third critical speed. Whirl orbit amplitudes for the out-of-line arrangement of unbalance weights are shown in Figs. 15 through 17.

The total unbalance weight used when passing through the response peak was about 0.18 in-oz which results in very small orbits and comparatively poor agreement at higher speeds when the response amplitudes are much lower. When passing through the third critical speed with this small unbalance, the orbits at the stations adjacent to the bearings were appreciably larger than the bearing clearance. Because of concern over the well-being of the apparatus and its operators, the rotor was accelerated through the response peak and there was no steady-state operation at that point. Peak amplitude data were obtained by using a camera attachment for the oscilloscope with the shutter held open during the passage through the peak. The largest recorded orbit amplitude is shown as the data point at the peak in Figs. 15 through 17.

The response of the three-disk rotor with the unbalance weights in-line is

similar in trend to the results when the weights are out-of-line. The amplitudes are considerably lower though they are still very large. Agreement between calculated results is good for both cases and there was precise agreement on the speed at which the response peak occurred.

The modes of rotor vibration during the experiments were determined from observations of the phase angles between the direction of the unbalance force and the response at the different measurement planes. The measurement was accomplished by using a sensor which responds with a sharp voltage peak as a mark on the shaft passes. The amplified voltage peak is used to momentarily dim the beam of the oscilloscope so that there is a short interruption of the orbit on the screen which identifies the location of the rotor axis on its orbit at the time the mark on the shaft passes the sensor. The phase angle is measured on the oscilloscope as the angle between the break in the orbit and the known angular position of the unbalance weights at the time the mark passes the sensor. The accuracy of the measurement is adversely affected if the orbit is not truly circular. For this reason and because calculated phase angles are of comparatively little significance in design analysis, the emphasis has been placed on the response amplitude results. However, a typical sample of the agreement between measured and calculated phase angles is given in Table 2 for the two-disk rotor with the unbalance weights in line.

TABLE 2

MEASURED AND CALCULATED PHASE ANGLES
FOR TWO-DISK SYSTEM, UNBALANCES IN-LINE

Calculated

Speed, rpm	Light End	Light End Bearing	Center	Heavy End Bearing	Heavy End
3000	30	32	32	34	195
6000	80	89	90	90	220
9000	124	136	146	170	280
10,000	131	146	158	195	288
11,000	137	154	163	260	318
12,000	143	158	163	340	340
15,000	156	168	158	126	33
18,000	163	174	164	134	60

Measured

3000	40	310	20	30	200
6000	63	65	65	58	175
9000	65	80	120	123	200
10,000	100	95	107	100	240
11,000	110	113	168	220	300
12,000	145	100	160	355	30
15,000	168	160	135	132	35
18,000	165	185	160	160	110

DISCUSSION

Comparison of the measured response of the test rotor-bearing system with the results calculated from the combination of the rotor response analysis described in this paper and theoretical bearing stiffness and damping has shown good agreement. In particular, there is agreement in the following respects

- (1) For each of three rotor configurations, the trend of the response to unbalance as the speed was varied over a range extending to or through the third undamped critical speed was substantially as calculated.
- (2) With few exceptions, the measured and calculated amplitudes of vibration are comparable. Examination of either measured or calculated results for the purpose of determining the relative sensitivity to unbalance of the three different rotor configurations clearly leads to the same conclusions.
- (3) One application of rotor response calculations is to determine the sensitivity of the rotor to unbalance at different locations. Substantial differences in the response of the experimental rotor were predicted depending on whether the unbalance weights were in-line or out-of-line. The experiments showed differences which were in good agreement with these theoretical predictions.

Comparison of unbalance response characteristics with the calculated, undamped critical speeds is interesting, especially since critical speed calculations are usually the only type of rotor dynamics analysis performed in design. The three-disk rotor results show the large amplitude vibration peak occurring precisely at the calculated third critical speed. At this vibration mode, the bearing properties have little effect on the rotor vibration because the bearings are very close to the nodal points. For this reason, the agreement between calculated critical speed and rotor response in this instance is principally a verification of the accuracy of the rotor response program in calculating the characteristics of the rotor. When the bearings exert considerable control over the rotor vibration, as with the one and two disk rotors, the response results, both calculated and measured, show much less correlation with critical speed calculations. The peak response with the one-disk system occurs about midway between the first two

critical speeds. Results for the two-disk system with unbalance weights in-line are especially interesting since they show two well defined peaks whose mode shapes correspond to those of the first two critical speeds. These vibration peaks occur at 1.7 and 1.4 times the first and second critical speeds respectively. These differences between critical speed and rotor response results are caused by the effects of bearing damping which is not included in the critical speed analysis.

A principal assumption in the theoretical analysis is that the bearing stiffness and damping are linearized about the steady-state rotor position. This assumption was examined during the experiments by operating with different values of unbalance weight under otherwise identical conditions. Proportionality between unbalance weight and response held up within 20 percent or less for weight changes of as much as 3 to 1 giving orbits as large as 0.6 times the bearing clearance.

These results are regarded as an encouraging demonstration of the feasibility of using the rotor response analysis and theoretical bearing properties for design analysis of proposed high-speed rotor-bearing systems. Moreover, they have shown that rotor response calculations are a valuable supplement to critical speed calculations. Undamped critical speed calculations may not accurately identify the speeds at which maximum vibration amplitudes will occur and they do not give any indication of the relative amplitudes of vibration at the different critical speeds or between different rotor configurations. Rotor response calculations provide a direct indication of the speeds at which peak vibration amplitudes will occur and they can be used for comparison of alternate rotor-bearing configurations and for determination of the relative effects of unbalance at different locations on the rotor.

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NOMENCLATURE

A	Cross-sectional area of shaft section, in^2
$a, \text{ to } a_7$	Influence coefficients, see Eqs. (44) to (50)
a, b	Major and minor semiaxes of elliptical whirl orbit, inch
$B_{xx}, B_{xy}, B_{yx}, B_{yy}$	Bearing damping coefficients, lbs. sec/in.
E	Young's modulus for shaft section, lbs/in^2 .
G	Shear modulus for shaft section, lbs/in^2 .
I	Cross-sectional transverse moment of inertia of shaft section, in^4 .
I_P	Polar mass moment of inertia at rotor station, lbs.in. sec^2 .
I_T	Transverse mass moment of inertia at rotor station, lbs.in. sec^2 .
i	$= \sqrt{-1}$
J_P	Polar mass moment of inertia per unit length of shaft section, lbs. sec^2 .
J_T	Transverse mass moment of inertia per unit length of shaft section, lbs. sec^2 .
$K_{xx}, K_{xy}, K_{yx}, K_{yy}$	Bearing spring coefficients, lbs/in.
k_1, k_2, k_3	Static influence coefficients, see Eqs. (51) to (53)
l	Length of shaft section, inch
M	Bending moment, lbs.in.
m	Mass at rotor station, $\text{lbs. sec}^2/\text{in.}$
n	Index giving the rotor station number
q	Index for last rotor station
t	Time, sec.
U	$= U_f + i U_g$. Mass unbalance at rotor station, lbs. sec^2
U_f, U_g	Components of unbalance along directions fixed in rotor, lbs. sec^2 .
u	Mass unbalance of shaft section per unit length, $\text{lbs. sec}^2/\text{in.}$

Nomenclature (cont'd)

V	Shear force, lbs.
x, y	Rotor amplitudes, inch
z	Axial coordinate, inch
$Z_{xx}, Z_{xy}, Z_{yx}, Z_{yy}$	$Z_{xx} = K_{xx} + i\omega B_{xx}$, etc. Bearing impedance, lbs/in.
α	Cross-sectional shape factor for shear deformation
β	See Eq. (31), inch^{-1}
β_1, β_2	See Eqs. (34) and (35), inch^{-1}
γ	Angle between x-axis and major semiaxis of elliptical orbit
δ	See Eq. (32), inch
Θ	x-component of rotor slope, radians
λ_1, λ_2	See Eq. (54)
μ	Inertia of shaft section, Eq. (55), lbs/in.
ρ	Mass density of shaft material, $\text{lbs}\cdot\text{sec}^2/\text{in}^4$
ϕ	y-component of rotor slope, radians
ψ	Phase angle for whirl orbit, see Eqs. (19) and (20)
ω	Angular speed of rotor, radians/sec.

Indices

n	Refers to rotor station n or to shaft section between station n and $(n+1)$
x	x-component
y	y-component
c	Real part ($\cos(\omega t)$ -component)
s	Imaginary part (negative $\sin(\omega t)$ -component)

Nomenclature (cont'd)Superscripts

- ' Refers to bending moment or shear force, just to the right
 of a rotor station, see Fig. 1.
- Used in few places to denote complex quantity. Note, that
 the bar notation is implied but dropped in many equations,
 notably Eqs. (9) to (15).

REFERENCES

1. E.B. Arwas, "Topical Report on Turbulent Lubrication," Topical Report under A.I. Subcontract N2-S9-1599 of AEC Contract AT(11-1) GEN 8, MTI Report 64TR67, November 1964.
2. H.G. Elrod, Jr., C.W. Ng and C.H.T. Pan, "A Theory of Turbulent Films and Its Application to Bearings," Topical Report under AEC Contract AT(30-1)-3363, AEC Report No. NYO-3363-2, MTI Report 65TR9, March 1965 (Also published under the same title as ASME Paper 66Lub12, June 1966).
3. J.W. Lund et al: "Rotor Bearing Dynamics Technology," Final Reports under USAF Contract No. AF 33(615)-1895, U.S. Air Force Reports AFAPL-TR-65-45 Parts III and V, MTI Report 65TR14 and 65TR15, May 1965.
4. F.K. Orcutt and E.B. Arwas, "Analysis of Turbulent Lubrication, Volume 1 - The State and Dynamic Properties of Journal Bearings in Laminar and Turbulent Regimes," 1st Volume of Final Report under NASA Contract NAS-w-771, MTI Report 64TR19, May 1964. (Condensed version published as ASME paper 66-LUBS-4, "The steady state and dynamic characteristics of a full circular and a partial arc bearing in the laminar and turbulent flow regimes, June 1966).
5. (a) F.K. Orcutt, "The Steady State and Dynamic Properties of the Tilting Pad Bearing in the Laminar and Turbulent Flow Regimes," Topical Report under NASA Contract NAS-w-1021, MTI Report 65TR32, June 1965 (condensed version published under the same title, as ASME Paper 66 Lub 19, June 1966).
- (b) F.K. Orcutt and C.W. Ng, "Steady State and Dynamic Properties of the Floating Ring Bearing in the Laminar and Turbulent Flow Regimes," Topical Report under NASA Contract NAS-w-1021, MTI Report 65TR33, June 1965,
6. B. Sternlicht, "Elastic and Damping Properties of Cylindrical Journal Bearings," JOURNAL OF BASIC ENGINEERING, TRANS. ASME, Series D, Vol. 81, 1959, pp. 101-108.
7. A.C. Hagg and G.O. Sankey, "Some Dynamic Properties of Oil-Film Journal Bearings with Reference to the Unbalance Vibration of Rotors," JOURNAL OF APPLIED MECHANICS, Vol. 23, TRANS. ASME. Vol. 78, 1956, pp. 302-306.
8. A.C. Hagg and G.O. Sankey, "Elastic and Damping Properties of Oil-Film Journal Bearings for Application to Unbalance Vibration Calculations," JOURNAL OF APPLIED MECHANICS, Vol. 25, TRANS. ASME. Vol. 80, 1958, p. 141.
9. P.C. Warner, "Static and Dynamic Properties of Partial Journal Bearings," JOURNAL OF BASIC ENGINEERING, TRANS. ASME, Series D, Vol. 85, 1963, pp. 247-257.
10. P.C. Warner and R.J. Thoman, "The Effect of the 150-Degree Partial Bearing on Rotor-Unbalance Vibration," JOURNAL OF BASIC ENGINEERING, TRANS. ASME, Series, D, Vol. 86, 1964, pp. 337-347.

11. J.W. Lund and B. Sternlicht, "Rotor-Bearing Dynamics with Emphasis on Attenuation," JOURNAL OF BASIC ENGINEERING, TRANS. ASME, Vol. 84, Series D, 1962, pp. 491-502.
12. J.W. Lund, "Spring and Damping Coefficients for the Tilting Pad Journal Bearing," ASLE Transactions, Vol. 7, 1964, pp. 342-352.
13. J.W. Lund, "Self-Excited, Stationary Whirl Orbits of a Journal in a Sleeve Bearing," PhD Thesis, Rensselaer Polytechnic Institute, Department of Mechanics, Troy, N.Y. 1966.
14. M.A. Prohl, "A General Method for Calculating Critical Speeds of Flexible Rotors," JOURNAL OF APPLIED MECHANICS, Vol. 67, 1945, pp. A-142 to A-148.

APPENDIX A

This appendix describes the analysis for calculating the whirl amplitudes of an unbalanced rotor. The analysis takes into account the anisotropic stiffness and damping characteristics of the bearings which cause the rotor whirl orbit to be elliptical. Furthermore, the gyroscopic moments of the rotor wheels are included. The analysis is an extension of the Prohl Method [14] and is readily programmed for a digital computer.

To define the rotor motion, introduce a cartesian coordinate system with the x-axis vertical downwards, the y-axis horizontal and the z-axis along the rotor axis. The origin of the x-y-system coincides at each axial location with the static deflection of the rotor, corrected at any given speed for the eccentricity of the journals in the bearings. Thus, the rotor amplitudes are x and y, the corresponding slope components are Θ and Φ , and the bending moment and the shear force in the rotor are denoted as M and V, respectively. They have the components M_x and M_y , and V_x and V_y .

The rotor is represented as a series of stations connected by shaft sections of uniform cross section. Each station is assigned a mass m_n , a transverse mass moment of inertia I_{Tn} , a polar mass moment of inertia I_{pn} , a bearing reaction (expressed by Eq. (1)) and an unbalance force. The latter force can be determined by introducing two mutually perpendicular, fixed directions in the rotor, denoted as the ξ -axis and the η -axis. The instantaneous angle between the ξ -axis and the x-axis is ωt , where ω is the angular speed of the rotor and t denotes time. The rotor unbalance can then be defined by its components U_ξ and U_η , and the corresponding forces, measured in the x-y-system, become:

$$\begin{aligned}\omega^2 U_x &= \omega^2 U_\xi \cos(\omega t) - \omega^2 U_\eta \sin(\omega t) \\ \omega^2 U_y &= \omega^2 U_\xi \sin(\omega t) + \omega^2 U_\eta \cos(\omega t)\end{aligned}\tag{3}$$

Considering the n'th rotor station as shown in Fig. 1, a force equation and a moment equation yields:

$$m_n \frac{d^2 x_n}{dt^2} = V_{x_n} - V'_{x_n} + \omega^2 U_{x_n} - K_{xxn} x_n - B_{xxn} \frac{dx_n}{dt} - K_{xy_n} y_n - B_{xy_n} \frac{dy_n}{dt} \quad (4)$$

$$m_n \frac{d^2 y_n}{dt^2} = V_{y_n} - V'_{y_n} + \omega^2 U_{y_n} - K_{yx_n} x_n - B_{yx_n} \frac{dx_n}{dt} - K_{yy_n} y_n - B_{yy_n} \frac{dy_n}{dt} \quad (5)$$

$$I_{Tn} \frac{d^2 \theta_n}{dt^2} + \omega I_{pn} \frac{d\theta_n}{dt} = M'_{x_n} - M_{x_n} \quad (6)$$

$$I_{Tn} \frac{d^2 \phi_n}{dt^2} - \omega I_{pn} \frac{d\phi_n}{dt} = M'_{y_n} - M_{y_n} \quad (7)$$

The rotor is forced to vibrate by the unbalance forces with the frequency ω . The vibration is harmonic whereby the amplitude, the slope, the bending moment, the shear force and the unbalance can be expressed in complex notation:

$$x = \bar{x} e^{i\omega t}$$

$$\theta = \bar{\theta} e^{i\omega t}$$

$$M = \bar{M} e^{i\omega t}$$

$$V = \bar{V} e^{i\omega t}$$

$$\omega^2 U_x = \omega^2 U e^{i\omega t}$$

$$\omega^2 U_y = -i\omega^2 U e^{i\omega t}$$

where: $U = U_x + iU_y$, $\bar{x} = x_c + ix_s$, etc., and only the real parts apply.

Similar definitions hold for the y-components. Further, define:

$$Z_{xx} = K_{xx} + i\omega B_{xx} \quad (8)$$

and analogously for Z_{xy} , Z_{yx} and Z_{yy} . With these definitions, and dropping the bar notation, Eqs. (4) to (7) become:

$$V'_{xn} = V_{xn} + (\omega^2 m_n - Z_{xxn})x_n - Z_{xy_n} y_n + \omega^2 U_n \quad (9)$$

$$V'_{yn} = V_{yn} - Z_{yx_n} x_n + (\omega^2 m_n - Z_{yy_n})y_n - i\omega^2 U_n \quad (10)$$

$$M'_{xn} = M_{xn} - \omega^2 I_{Tn} \theta_n + i\omega^2 I_{pn} \phi_n \quad (11)$$

$$M'_{yn} = M_{yn} - i\omega^2 I_{pn} \theta_n - \omega^2 I_{Tn} \phi_n \quad (12)$$

These equations express the jump in shear force and bending moment across a rotor station.

The n 'th station is connected with the $(n+1)$ 'th station by a shaft section of length ℓ_n with a cross-sectional area A_n , a cross-sectional moment of inertia I_n and a cross-section shape factor α_n for shear deformation. The shaft material has a mass density ρ_n , Young's modulus E_n and shear modulus G_n . Then, as shown in Appendix B, it is possible to express $x_{n+1}, \theta_{n+1}, M_{x,n+1}$ and $V_{x,n+1}$ as linear combinations of x_n, θ_n, M_{xn} and V_{xn} (the relationships for the y -components are identical). The equations are given as Eqs. (40) to (43) in Appendix B. The coefficients in the equations are functions of the shaft properties and the speed of the rotor. For simplicity, it is assumed that unbalance, rotary inertia and gyroscopic moments in the shaft can be ignored, and that these effects instead are included at the rotor stations.

Equations (9) to (12) together with Eqs. (40) to (43) are a set of recurrence relationships from which the rotor amplitudes can be computed. Assuming the rotor to have free ends, the bending moments and shear forces at the ends are zero:

$$M_{x1} = M_{y1} = V_{x1} = V_{y1} = 0 \quad (13)$$

$$M'_{xq} = M'_{yq} = V'_{xq} = V'_{yq} = 0$$

where station q is the last rotor station. Selecting x_1, y_1, θ_1 and ϕ_1 as unknowns, repeated application of Eqs. (9) to (12) and Eqs. (40) to (43) results in:

$$\begin{Bmatrix} x_n \\ y_n \end{Bmatrix} = \begin{Bmatrix} c_n \end{Bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ \theta_1 \\ \phi_1 \end{Bmatrix} + \begin{Bmatrix} c_{n15} \\ c_{n25} \end{Bmatrix} \quad (14)$$

$$\begin{Bmatrix} M'_{xq} \\ M'_{yq} \\ V'_{xq} \\ V'_{yq} \end{Bmatrix} = \begin{Bmatrix} d \end{Bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ \theta_1 \\ \phi_1 \end{Bmatrix} + \begin{Bmatrix} d_{15} \\ d_{25} \\ d_{35} \\ d_{45} \end{Bmatrix} = 0 \quad (15)$$

where $\{c_n\}$ is a 2 by 4 matrix and $\{d\}$ is a 4 by 4 matrix. The matrix elements c_{ni} and d_{ij} are complex. They are found simplest by performing a total of 5 rotor calculations. In the first calculation, set $x_1=1$ and $y_1=\theta_1=\phi_1=U_n=0$ whereby $c_{n11}, c_{n21}, d_{11}, d_{21}, d_{31}$ and d_{41} are obtained. Next, set $y_1=1$ and $x_1=\theta_1=\phi_1=U_n=0$ and determine the second columns of the matrices. Proceeding in this way, all the coefficients are obtained. Solving Eq. (15) for x_1, y_1, θ_1 and ϕ_1 allows computing x_n and y_n from Eq. (14) for all the rotor stations. Noting that:

$$\bar{x}_n = x_{cn} + i x_{sn}$$

$$x_n = \bar{x}_n e^{i\omega t} = x_{cn} \cos(\omega t) - x_{sn} \sin(\omega t)$$

and similarly for y_n , the semiaxes and the orientation of the elliptical whirl orbit are calculated from

$$a_n = \left[\frac{1}{2}(x_{cn}^2 + x_{sn}^2 + y_{cn}^2 + y_{sn}^2) + \sqrt{\frac{1}{4}(x_{cn}^2 + x_{sn}^2 - y_{cn}^2 - y_{sn}^2)^2 + (x_{cn}y_{cn} + x_{sn}y_{sn})^2} \right]^{1/2} \quad (16)$$

$$b_n = \frac{x_{sn}y_{cn} - x_{cn}y_{sn}}{a_n} \quad (17)$$

$$\gamma_n = \frac{1}{2} \tan^{-1} \left[\frac{2(x_{cn}y_{cn} + x_{sn}y_{sn})}{x_{cn}^2 + x_{sn}^2 - y_{cn}^2 - y_{sn}^2} \right] \quad (18)$$

$$\psi_n = \frac{1}{2} \tan^{-1} \left[\frac{2(x_{cn}x_{sn} + y_{cn}y_{sn})}{x_{cn}^2 - x_{sn}^2 + y_{cn}^2 - y_{sn}^2} \right] \quad (19)$$

where a_n is the major semiaxis, b_n the minor semiaxis, γ_n the angle from the x-axis to the major semiaxis in the direction of rotor rotation, and ψ_n is the phase angle. The definition of the phase angle is such that if the x-y-coordinate system is rotated the angle γ_n into an x'-y'-system (i.e. x' is along the major semiaxis), then the rotor motion can be expressed as:

$$\begin{aligned}x'_n &= a_n \cos(\omega t + \psi_n) \\ y'_n &= b_n \sin(\omega t + \psi_n)\end{aligned}\tag{20}$$

If the value for the minor semiaxis is negative, the rotor precesses backwards.

For those bearing types where $Z_{xx} = Z_{yy}$ and $Z_{xy} = -Z_{yx}$ it is seen from Eqs. (9) to (12) that:

$$y = -ix \qquad \varphi = -i\theta \tag{21}$$

Hence, the whirl orbit becomes circular and the outlined calculation procedure can be considerably simplified. This condition applies to the four shoe tilting pad bearing used in the present tests.

APPENDIX B

In this appendix a uniform shaft section is considered and the relationships are established for the amplitude, slope, bending moment and shear force at one end of the section in terms of the corresponding quantities at the other end.

Including shear deformation, the rotation Θ of a shaft element becomes:

$$\Theta = \frac{V}{\alpha GA} + \frac{\partial x}{\partial z} \quad (22)$$

and similarly for the y-component, where G is the modulus of shear and α is a cross-sectional shape factor ($\alpha \cong 0.75$ for a circular cross section).

The bending equation is given by:

$$M_x = EI \frac{\partial \Theta}{\partial z} \quad (23)$$

and similarly for the y-component. When the shaft is subjected to unbalance forces per unit length of $\omega^2 u_x$ and $\omega^2 u_y$ a force equation yields:

$$\rho A \frac{\partial^2 x}{\partial t^2} = -\frac{\partial V}{\partial z} + \omega^2 u_x \quad (24)$$

and analogously for the y-direction. Finally, if per unit length the shaft has a transverse mass moment of inertia J_T and a polar mass moment of inertia J_P a moment equation results in:

$$J_T \frac{\partial^2 \Theta}{\partial t^2} + \omega J_P \frac{\partial \Theta}{\partial t} = \frac{\partial M_x}{\partial z} - V_x \quad (25)$$

$$J_T \frac{\partial^2 \phi}{\partial t^2} - \omega J_P \frac{\partial \Theta}{\partial t} = \frac{\partial M_y}{\partial z} - V_y \quad (26)$$

Assuming the shaft section to have constant cross-sectional properties, Θ, ϕ, M and V can be eliminated from the equations. In this way the equations governing the shaft motion become:

$$\begin{aligned}
EI \frac{\partial^4 x}{\partial z^4} - \rho I \left(\frac{E}{\alpha G} + \frac{J_T}{\rho I} \right) \frac{\partial^4 x}{\partial z^2 \partial t^2} + \rho A \frac{\partial^2 x}{\partial t^2} + \frac{\rho J_T}{\alpha G} \frac{\partial^4 x}{\partial t^4} - \omega J_P \frac{\partial^3 y}{\partial z^2 \partial t} + \frac{\omega \rho J_P}{\alpha G} \frac{\partial^3 y}{\partial t^3} \\
= \omega^2 \left[u_x + \frac{J_T}{\alpha GA} \frac{\partial^2 u_x}{\partial t^2} + \frac{\omega J_P}{\alpha GA} \frac{\partial u_y}{\partial t} - \frac{EI}{\alpha GA} \frac{\partial^2 u_x}{\partial z^2} \right]
\end{aligned} \quad (27)$$

$$\begin{aligned}
EI \frac{\partial^4 y}{\partial z^4} - \rho I \left(\frac{E}{\alpha G} + \frac{J_T}{\rho I} \right) \frac{\partial^4 y}{\partial z^2 \partial t^2} + \rho A \frac{\partial^2 y}{\partial t^2} + \frac{\rho J_T}{\alpha G} \frac{\partial^4 y}{\partial t^4} + \omega J_P \frac{\partial^3 x}{\partial z^2 \partial t} - \frac{\omega \rho J_P}{\alpha G} \frac{\partial^3 x}{\partial t^3} \\
= \omega^2 \left[u_y + \frac{J_T}{\alpha GA} \frac{\partial^2 u_y}{\partial t^2} - \frac{\omega J_P}{\alpha GA} \frac{\partial u_x}{\partial t} - \frac{EI}{\alpha GA} \frac{\partial^2 u_y}{\partial z^2} \right]
\end{aligned} \quad (28)$$

Since the analysis is restricted to forced vibrations with a frequency equal to the angular speed of the shaft, ω , a complex notation can be used:

$$x = \bar{x} e^{i\omega t}$$

$$y = \bar{y} e^{i\omega t}$$

and similarly for the slope, the bending moment and the shear force. Only the real part applies. Furthermore, setting:

$$u_x = u e^{i\omega t}$$

results in: $u_y = -i u e^{i\omega t}$. Thus, Equations (27) and (28) can be written:

$$\begin{aligned}
EI \frac{d^4 \bar{x}}{dz^4} + \omega^2 \rho I \left(\frac{E}{\alpha G} + \frac{J_T}{\rho I} \right) \frac{d^2 \bar{x}}{dz^2} - \omega^2 \rho A \left(1 - \frac{\omega^2 J_T}{\alpha GA} \right) \bar{x} - i \omega^3 J_P \frac{d^2 \bar{y}}{dz^2} - i \frac{\omega^4 \rho J_P}{\alpha G} \bar{y} \\
= \omega^3 u \left[1 + \frac{\omega^2 (J_P - J_T)}{\alpha GA} \right] - \frac{\omega^2 EI}{\alpha GA} \frac{d^2 u}{dz^2}
\end{aligned} \quad (29)$$

$$\begin{aligned}
EI \frac{d^4 \bar{y}}{dz^4} + \omega^2 \rho I \left(\frac{E}{\alpha G} + \frac{J_T}{\rho I} \right) \frac{d^2 \bar{y}}{dz^2} - \omega^2 \rho A \left(1 - \frac{\omega^2 J_T}{\alpha GA} \right) \bar{y} + i \omega^3 J_P \frac{d^2 \bar{x}}{dz^2} + i \frac{\omega^4 \rho J_P}{\alpha G} \bar{x} \\
= -i \omega^3 u \left[1 + \frac{\omega^2 (J_P - J_T)}{\alpha GA} \right] + i \frac{\omega^2 EI}{\alpha GA} \frac{d^2 u}{dz^2}
\end{aligned} \quad (30)$$

Although these equations may be solved, the resulting solution is impractical. Instead, it shall be assumed that the shaft section is free of unbalance forces ($u=0$), and that the effect of rotary inertia and gyroscopic moment can be neglected ($J_T = J_P = 0$). If these effects are significant they can be accounted for with good accuracy by lumping them at the ends of the shaft section (i.e. at the rotor stations). It should be noted that even though it is simple to keep the rotary inertia terms in the above equations, this is not permissible without also including the gyroscopic moments since they are of the same magnitude.

With these assumptions, Eqs. (29) and (30) become identical and it is only necessary to consider the first equation. The following definitions are introduced:

$$\beta = \left(\frac{\omega^2 \rho A}{EI} \right)^{1/4} \quad (31)$$

$$\delta^2 = \frac{EI}{2\alpha GA} \quad (32)$$

Dropping the bar notation and setting $u = J_T = J_P = 0$, Eq(29) becomes:

$$\frac{d^4 x}{dz^4} + 2\delta^2 \beta^4 \frac{d^2 x}{dz^2} - \beta^4 x = 0 \quad (33)$$

The characteristic equation has the roots: $\pm \beta_1$ and $\pm i\beta_2$ where:

$$\beta_1 = \beta \left[\sqrt{1 + (\delta\beta)^4} - (\delta\beta)^2 \right]^{1/2} \quad (34)$$

$$\beta_2 = \beta \left[\sqrt{1 + (\delta\beta)^4} + (\delta\beta)^2 \right]^{1/2} \quad (35)$$

Hence, the general solution can be written:

$$x = C_1 \cosh(\beta_1 z) + C_2 \sinh(\beta_1 z) + C_3 \cos(\beta_2 z) + C_4 \sin(\beta_2 z) \quad (36)$$

where C_1 to C_4 are constants to be determined from the boundary conditions.

Combining Eqs. (22) to (25), setting $u = J_T = J_P = 0$ gives:

$$\frac{1}{EI} M_x = (\beta_2^2 - \beta_1^2) x + \frac{d^2 x}{dz^2} \quad (37)$$

$$V_x = \frac{dM_x}{dz} \quad (38)$$

$$\Theta = \frac{(\beta_2^2 - \beta_1^2)}{\beta^4} \frac{V_x}{EI} + \frac{dx}{dz} \quad (39)$$

Substitution from Eq. (36) allows determining M_x , V_x and Θ . Next, the four constants can be evaluated by setting:

$$\text{At } z=0: \quad x=x_n \quad \Theta=\Theta_n \quad M_x=M'_{xn} \quad V_x=V'_{xn}$$

At the other end of the shaft section, set:

$$\text{At } z=l_n: \quad x=x_{n+1} \quad \Theta=\Theta_{n+1} \quad M_x=M_{x,n+1} \quad V_x=V_{x,n+1}$$

Thus, the desired relationships become:

$$x_{n+1} = a_{1n} x_n + l_n a_{3n} \Theta_n + k_{2n} a_{4n} M'_{xn} + k_{3n} a_{7n} V'_{xn} \quad (40)$$

$$\Theta_{n+1} = \frac{1}{3} \mu_n k_{2n} a_{5n} x_n + a_{2n} \Theta_n + k_{1n} a_{6n} M'_{xn} + k_{2n} a_{4n} V'_{xn} \quad (41)$$

$$M_{x,n+1} = \frac{1}{2} \mu_n l_n a_{4n} x_n + \frac{1}{6} \mu_n l_n^2 a_{5n} \Theta_n + a_{2n} M'_{xn} + l_n a_{3n} V'_{xn} \quad (42)$$

$$V_{x,n+1} = \mu_n a_{3n} x_n + \frac{1}{2} \mu_n l_n a_{4n} \Theta_n + \frac{1}{3} \mu_n k_{2n} a_{5n} M'_{xn} + a_{1n} V'_{xn} \quad (43)$$

where:

$$a_{1n} = (\beta_1^2 \cosh \lambda_1 + \beta_2^2 \cos \lambda_2) / (\beta_1^2 + \beta_2^2) \quad (44)$$

$$a_{2n} = (\beta_2^2 \cosh \lambda_1 + \beta_1^2 \cos \lambda_2) / (\beta_1^2 + \beta_2^2) \quad (45)$$

$$a_{3n} = (\beta_1 \sinh \lambda_1 + \beta_2 \sin \lambda_2) / (\beta_1^2 + \beta_2^2) l_n \quad (46)$$

$$a_{4n} = 2(\cosh \lambda_1 - \cos \lambda_2) / (\beta_1^2 + \beta_2^2) l_n^2 \quad (47)$$

$$a_{5n} = 6(\beta_2 \sinh \lambda_1 - \beta_1 \sin \lambda_2) / (\beta_1^2 + \beta_2^2) \beta^2 l_n^3 \quad (48)$$

$$a_{6n} = (\beta_2^3 \sinh \lambda_1 + \beta_1^3 \sin \lambda_2) / (\beta_1^2 + \beta_2^2) \beta^2 l_n \quad (49)$$

$$a_{7n} = 6(\beta_1^3 \sinh \lambda_1 - \beta_2^3 \sin \lambda_2) / (\beta_1^2 + \beta_2^2) \beta^4 l_n^3 \quad (50)$$

$$k_{1n} = \frac{l_n}{EI} \quad (51)$$

$$k_{2n} = \frac{l_n^2}{2EI} \quad (52)$$

$$k_{3n} = \frac{l_n^3}{6EI} \quad (53)$$

$$\lambda_1 = \beta_1 l_n \quad \lambda_2 = \beta_2 l_n \quad (54)$$

$$\mu_n = \omega^2 g A l_n \quad (55)$$

It should be noted that k_{1n} , k_{2n} and k_{3n} are the static influence coefficients neglecting shear deformation. Furthermore, for sufficiently low speeds and sufficiently many mass stations, the shear effect can be ignored and $\beta \approx 0$. Then, the coefficients a_{1n} to a_{7n} become equal to 1 and the shaft mass can be lumped with good approximation at the ends of the shaft section, setting $\mu_n = 0$ in the above equations. In most practical cases, this will be accurate enough.

Outline of Rotor with Location of Rotor Stations

Sign Convention for Amplitude, Slope, Bending Moment and Shear Force

Fig. 1. Location of Rotor Stations and Sign Convention for Analysis

Fig. 2. Unbalance Response of the One-Disk Rotor, End Positions
 (There are two sets of data for each measurement plane by using
 two different angular positions of the unbalance weights on the rotor).

Fig. 3. Unbalance Response of the One-Disk Rotor, Bearing Positions

Fig. 4. Unbalance Response of the One-Disk Rotor, Center Position

Fig. 5. Unbalance Response of the Two-Disk Rotor, Weights In-Line, Heavy End Positions

Fig. 6. Unbalance Response of the Two-Disk Rotor, Weights In-Line, Heavy End Bearing Position

Fig. 7. Unbalance Response of the Two-Disk Rotor, Weights In-Line, Center Position

Fig. 8. Unbalance Response of the Two-Disk Rotor, Weights In-Line, Light End Bearing Position

Fig. 9. Unbalance Response of the Two-Disk Rotor, Weights-In-Line, Light End Position

Fig. 10. Unbalance Response of the Two-Disk Rotor, Weights Out-of-Line, Heavy End Position

Fig. 11. Unbalance Response of the Two-Disk Rotor, Weights Out-of-Line, Heavy End Bearing Position

Fig. 12. Unbalance Response of the Two-Disk Rotor, Weights Out-of-Line, Center Position

Fig. 13. Unbalance Response of the Two-Disk Rotor, Weights Out-of-Line, Light End Bearing Position

Fig. 14. Unbalance Response of the Two-Disk Rotor, Weights Out-of-Line, Light End Position

Fig. 15. Unbalance Response of the Three-Disk Rotor, End Positions

Fig. 16. Unbalance Response of the Three-Disk Rotor, Bearing Positions

Fig. 17. Unbalance Response of the Three-Disk Rotor, Center Position

SECTION 8APPLICATIONS IN LARGE TURBOMACHINES

BEARING AND SEAL REQUIREMENTS FOR LARGE
LIQUID-METAL COOLED REACTOR SYSTEMS*

by

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SUMMARY

Bearing and seal technology is well advanced in the components to which it has been applied. So far, it has been concentrated primarily in the mechanical sodium pumps; an order of magnitude extrapolation in pumping capacity will be required for prospective plants, which in turn requires technology extrapolation and extensive tests to assure satisfactory performance. Some additional applications of bearings and seals are possible if they are economically justified, although at least partially satisfactory substitutes are currently available.

INTRODUCTION

Fast breeder reactors of power levels of 1000 Mwe or greater are presently considered to be necessary for the continued conservation of nuclear resources in an expanding nuclear power economy. Component requirements in the large future systems transcend any experience now available; the largest sodium cooled reactor built to date has a design power of 300 Mwt, with components sized to meet this power generation capability. Therefore, an order of magnitude extrapolation in component size for power generation at about 2500 Mwt is a near term objective.

Sodium, because of its high chemical reactivity, and its rather poor lubricating qualities is not suitable for anti-friction bearings. Consequently, sodium systems have demanded bearings lubricated by the

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working fluid and rotating seals to retain inert gas over the sodium wherever mechanical shafting had to be introduced into the heat transfer systems, almost since the inception of sodium cooled reactors. Sodium also becomes highly radioactive in the primary circuit. While this does not affect the bearing performance, it does introduce stringent requirements for reliability, accessibility and repairability. In small plants, the requirements could sometimes be dodged by devices such as frozen sodium shaft seals which permitted conventional bearings to be used outside the sodium, with varying degrees of success. However, with the advent of even the moderately large plants constructed to date, it has become apparent that at least for pumps the impeller and shaft must be stabilized by sodium lubricated bearings. This automatically requires the use of a rotating shaft seal to retain inert gas over the sodium in the pump casing; we are faced again with a large extrapolation from current practice.

CURRENT APPLICATIONS

Bearings

The largest sodium pumps built to date have been installed in the Enrico Fermi Fast Breeder Reactor, with a capacity of approximately 11,000 gpm. These have been operated quite satisfactorily, indicating that the principle of sodium lubricated bearings is a sound one. The bearing dimensions for these pumps are 12-in. diameter and 14-in. length. The six pumps installed at the Hallam Nuclear Power Facility, of 7500 gpm capacity and identical bearing size, have operated equally satisfactorily for a total of 70,000 hours on all pumps.

The bearings in sodium main circulating pumps to date have been hydrostatic, supporting only the radial loads imposed on the shaft. High pressure sodium supplied to the "pads" is furnished from the discharge of the pump, with 1 to 4% of the total pump flow being bypassed through the bearing to the suction side of the pump. Bearing sodium is relieved into the free surface volume of the sodium above the pump impeller and flows back to the suction side of the pump through an overflow line and

pressure balancing "weep holes" in the impeller. Radial loads of up to 120 lb force have been sustained by these bearings, which operate in the turbulent regime. No attempt has been made as yet to sustain thrust loads by such bearings; these are typically absorbed by roller or duplex ball bearings located above the rotating inert gas seal, where they are available for inspection and receive ordinary oil lubrication. Journal surfaces are hard faced to prevent galling during startup and shutdown with evident success; in the case of the Hallam pumps, inspection after 250 starts and stops showed no measurable wear and only slight polishing of the surfaces. Experimental evidence has been adduced which indicates that bearing stabilization and "lift off" from the rubbing surfaces takes place after less than one revolution of the shaft. Because of the success of this design on the pumps constructed to date, this type of bearing will undoubtedly be at least one of the reference designs for the 60,000 to 120,000 gpm pumps now under consideration. There are, however, obvious unknowns in extrapolating this bearing performance to the rather larger shaft sizes envisioned. The design parameters of these larger pumps have not yet been firmly fixed. It is reasonable to expect, however, that bearing journal diameters of at least 20-in. will be required. Shaft rotating speeds will probably not be substantially different from the 500-1500 rpm commonly utilized now, but it is obvious that the lineal velocity at the bearing surface will be substantially higher due to the increased bearing diameter. These pumps will be used in commercial power plants, where any loss of efficiency means an increase in parasitic plant load, which in turn means internal consumption of power which cannot be sold at a profit. Efficiency is thus a matter of great interest, particularly when one is considering pump drivers of 5-10 Mw in power. Bearing design thus assumes one of the critical roles in plant design and pump selection, which must be done on the basis of economics and reliability. A high bypass flow and/or a high bearing drag will contribute a significant efficiency loss to the plant with a measurable economic penalty. There are other types of pumps which can be utilized for moving sodium which are currently less efficient than mechanical pumps, but whose other potential advantages may prevail should the mechanical pump efficiency fall undesirably low. Therefore, it should not be assumed that

free surface mechanical pumps with sodium lubricated bearings are the only possible choice regardless of their mechanical design and efficiency. For instance, small canned motor pumps have been used in sodium to a limited degree; these utilize hydrodynamic bearings and have apparently been satisfactory although their cost has been quite high compared to shaft sealed overhung impeller pumps which can perform the same service. Bearings are probably not the key to reducing cost in this case; electrical insulation is a more likely candidate for cost reduction.

Seals

An inert gas blanket must be maintained over the sodium pool within the free surface pumps. This gas is normally argon, in fast reactors, which is maintained at 1 to 5 psi over atmospheric pressure, to permit outleakage of argon rather than inleakage of air in case of seal failure. In addition to minimum inert gas leakage, the rotating seal must not permit any contaminants to enter the sodium stream. Oil lubricated face type seals must be carefully designed so that the lubricant is trapped before it can leak down the pump shaft, and the oil volume should be arranged such that the traps can contain any conceivable amount of leakage. Carbon is quite deleterious to sodium systems, carburizing and embrittling thin stainless steel sections such as fuel cladding, and its introduction into a sodium system must therefore be avoided. It is possible to find non-carbon bearing lubricants; the effects of their decomposition products in sodium has not, to my knowledge, been completely explored. The simplest course is therefore to assure that potential contaminants cannot escape from the seal into the sodium.

Rotating seals utilized in large mechanical sodium pumps to date have been carbon faced, oil lubricated, operating on shafts up to 6-in. in diameter. Again, a substantial extension of technology is required when considering this type of seal for shafts up to 10-in. in diameter. Present seal technology is demonstrated at lineal speeds up to about 30,000 inches per minute; whether this experience, technology and manufacturing practice for flatness, finish, etc., can be extended by factors of 2 to 3 satisfactorily has not been established. Complete reliability for at least

10,000 hours of operation at speeds of 1500 rpm is required, as the seal is essential to pump operation, and the reactor system cannot operate without pumps. It will be necessary to perform both analysis and extensive tests to ensure that this critical component meets the requirements of economy, reliability, and tightness to gas and lubricant leakage which are required for performance in the central station fast breeder reactors of the future.

POTENTIAL APPLICATIONS OF BEARINGS AND SEALS

Bearings

Some possible additional applications of sodium lubricated bearings come to mind, providing the technology permits. It should be understood that these are not necessarily recommended applications, and that reasonably suitable equipment now exists. The examples cited must offer significantly improved reliability and/or cost advantages to be seriously considered. The first of these is in turbine type flowmeters, completely immersed in the sodium stream. These are not now practical primarily because of the bearings, which cannot be lubricated by anything but sodium. There is no source of high pressure sodium for these small bearings as in the pump case; a hydrodynamic bearing might possibly be suitable if the cost and complexity can be reduced to where such a flowmeter would be competitive with the electromagnetic type (which suffers from lack of accuracy in large pipes) and the differential pressure type (which suffers from lack of accuracy due to imperfect pressure measurement). A considerable amount of ingenuity will also be required in the electrical insulation and connections to a turbine flowmeter in sodium, even if the bearing problem is solved.

It is also possible to conceive control and safety mechanisms which could operate at reasonably high speeds in sodium, providing adequate bearings were available. As it is, all bearing surfaces are normally kept out of sodium and sodium vapor atmosphere, with the possible exception of surfaces which slide or roll quite slowly over one another. Sliding and rubbing under sodium, or in sodium vapor, has proven to be most unrewarding to date, although development along this field is

continuing. A "canned motor" drive with sodium lubricated bearings is conceivable, although many other problems such as electrical insulation would face the designer.

Seals

Any time a sodium system is penetrated, it is necessary to utilize a seal. Normally, the seal must retain inert gases containing sodium vapor. The main sodium coolant pumps are the prime examples. Control rod drive mechanisms, which usually penetrate the top of the shield, are excellent candidates for improved seals. Currently, the problem is sometimes dodged by isolating the operating mechanisms from the reactor atmosphere by a bellows which operates only in (vertical) linear motion. This is not always desirable, due to the questionable integrity and lifetime of thin-walled bellows with large extension ratios. This problem has also been dodged by containing the whole control mechanism, including the absorber column, in a "thimble" thus isolating the complete unit from sodium and cover gas. This is wasteful of neutrons, and is often rather clumsy in a small, high power density, reactor core. Therefore, a reliable rotating seal, operating at modest speeds up to perhaps 100 rpm, which can be depended upon to permit the escape of cover gas containing radioactive sodium vapor and maintaining close alignment throughout its lifetime, may well find application in drive and safety mechanisms for sodium cooled reactors.

So far, sealing against sodium directly has been most successfully accomplished by a frozen sodium annulus, such as in valve stems. Cooling is either by air or a liquid jacket. This method of sealing does introduce a necessity for periodically exercising the valve stems to break up sodium oxide deposits which may form. Also, of course, some small amount of sodium oxide is deposited on the shaft each time the valve is operated, thus making operating force erratic. So far, this has not proven difficult to handle, requiring only a sufficiently energetic valve operator. These are normally electro-mechanical or pneumatic. An economical seal, with constant force characteristics and which can be depended upon to reliably seal high temperature sodium from the atmosphere in translation and/or rotation would be highly desirable.

DESCRIPTION OF A LARGE SODIUM PUMP CONCEPT
FOR FUTURE SODIUM COOLED POWER REACTORS

by

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SUMMARY

A review of sodium pump and sodium cooled reactor technology was made by Westinghouse in 1963 under U.S. AEC Contract No. AT(30-1)-3123*. A purpose of this review was to establish criteria and limits for the sodium pumps which will be required for large (1000-1500 MWe) sodium cooled power reactors. This study indicated possible pumping requirements of 5000-10,000 hydraulic horsepower per pump.

DISCUSSION

The conceptual design shown Figure 1 depicts a 6000 HP electric motor coupled to a vertical pump shaft. The shaft extends down through a five foot thick shielding plug into the pump tank. A free surface of sodium is maintained in the pump tank by gravity or cover gas pressure balancing. By providing an inert gas cover above the sodium level in the tank, ingress of air is prevented and the requirements of the pump shaft seal (located just above the floor level) are reduced to that of sealing 5-50 psig gas pressure and sodium vapor. Radial baffling within the pump tank is provided to prevent vortexing of sodium around the shaft. A labyrinth or series of conical baffles is shown above the free surface to limit the passage of sodium vapor and to reduce convective heat transfer in the cover gas area. A sodium lubricated radial bearing is shown just above the pump impeller at the lower end of the shaft. The upper radial and thrust bearings are located above floor level and above the gas shaft seal. They are shown as conventional oil lubricated bearings.

The hydraulic or pump end shown is a double suction pump in which the sodium is drawn into the top and bottom of the impeller and discharged through a diffuser into the pump casing or volute. The pump internals (impeller, diffuser, lower

* see bottom of Page 8.2.3 for report titles and numbers.

bearing assembly, and baffling) can be withdrawn vertically from the pump tank without requiring personnel to work below the shield floor level.

The leading particulars for the pump hydraulic design shown are:

Fluid	-	800-1200 ^o F Sodium
Flow	-	80,600 GPM
Total Head	-	300 Ft.
RPM	-	890
Specific Speed	-	3500
Hydraulic Efficiency	-	88%
NPSH Req'd.	-	62 Ft.
Impeller O.D.	-	42 Inches
Suction Nozzle, I.D.	-	40 Inches
Discharge Nozzle, I.D.	-	36 Inches

The overall length of this unit is approximately 45 feet and the diameter of the pump is five feet.

This design is strictly conceptual and major development areas include:

1. Shaft Seals - Development of large diameter seals capable of sealing 5-50 psi cover gas and sodium vapor with little or no maintenance.
2. Bearings - Development of large diameter sodium lubricated radial bearings for 800-1200^oF sodium service.
3. Pumps - Development of high capacity pumps with low NPSH requirements.

Other areas which will require detailed analysis are:

1. Thermal Studies - To provide information necessary to design for thermal shock and expansion.
2. Dynamic Vibrations Analysis - To provide information to assure satisfactory shaft and rotor design.

3. Fluid Stability Studies - To provide information to assure sodium level control and prevention of gas entrapment in the sodium stream.
4. Mechanical Design Considerations - Obtain information for proper layout with respect to the reactor and other nuclear considerations, and to insure ease of installation, assembly, and disassembly.
5. Materials Selection - To assure structural integrity and long life for motor and pump components.
6. Electrical - To assure proper motor sizing and speed regulation with associated nuclear considerations for motor lubricating and cooling systems.
7. Instruments and Control - Selection and location of the various instruments required for monitoring temperatures, pressures, vibration, liquid level, etc. of pump and pump auxiliary systems.

Westinghouse is presently performing parametric and design studies under U.S. AEC Contract No. AT(30-1)-3762 directed towards development of Engineering data for large mechanical centrifugal sodium pumps.

Due to the size and speed requirements envisioned for the large sodium pumps of the future, development of reliable turbulent flow sodium bearings and gas seals will be necessary.

Reports published in the study under U.S. AEC Contract AT(30-1)-3123:

WCAP 2347 - Sodium Pump Development and Test Facility Design

WCAP 2255 - Survey of Sodium Pump Technology

Fig. 1 Large Sodium Pump Concept

✓ PROCESS FLUID LUBRICATED BEARINGS FOR HIGH-
TEMPERATURE GAS-COOLED REACTOR CIRCULATORS

by

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ABSTRACT

The development of a new gas cooled power reactor, has required the engineering of a new steam-driven, high-pressure, helium circulator. Because of the simplifications possible in seals and lubricant systems, water lubricated bearings were selected.

A shrouded step design was chosen for both journal and thrust bearings. These designs offer the advantages of a large tolerance to wear, no moving parts, and the flexibility of running as hybrid or even hydrostatic bearings if desired.

An approximate solution, accounting for turbulence effects (the Reynolds Number may exceed 25,000) has guided initial design. Some results from a computer solution for the thrust bearing taking into account inertial effects as well as turbulence are described.

DISCUSSION

The 330 MWe Power Plant being constructed by General Atomic for the Public Service Corporation of Colorado has the entire primary circuit consisting of the reactor core, steam generators and circulators, contained entirely within a prestressed concrete pressure vessel. The output power of the plant is produced by a conventional 2400 psi 1000°F/1000°F reheat steam turbogenerator which is located together with the remainder of the associated power plant equipment in a separate turbine building. The only penetrations of the PCRV required are those for

power production; feed water in, high pressure steam out, cold reheat steam in, and hot reheat steam out. The concept of the series steam turbine driven helium circulator was conceived by General Atomic to best provide the means for primary helium flow in the reactor power plant. Each of the circulators consist of a single stage axial flow steam turbine that is in series or tandem with the power producing steam turbogenerator. These driver turbines are located in the cold reheat line between the discharge from the high pressure element of the turbogenerator and the reheater. Figure 1 shows the series steam turbine driven circulator installed in the bottom head of the PCRV. The machine is oriented vertically and its housing forms the closure to the primary gas circuit. The arrangement of the elements comprising the circulator is shown in Figure 2. The compressor and turbine rotors are overhung from a center bearing housing on opposite ends of the shaft. The bearing housing which contains the bearings and seals is supported by the conical section that forms the closure to the primary system.

The choice of the bearing type and lubrication system is strongly influenced by the operational and system requirements. The loads imposed on the radial bearings are those resulting from unbalance since the orientation of the machine is vertical while the thrust bearing loads result from the pressures acting at each end of the machine and the axial reactions of the turbo machine elements. It should be borne in mind that although nuclear plants are, in general, base loaded, they require starting and stopping and operation under emergency conditions. The thrust loads for all of the required conditions are identified in Figure 3, where the speed (plant load) is the abscissa and the thrust loads and pressure are the ordinates.

Furthermore, these large variations of pressure of the two working mediums, steam and helium, the mechanical requirements of clearances of at least one thousandths of an inch per inch of shaft and the variable speed operational requirement determine the need for hydrostatic lubrication for at least the low speed part of the operating range and emphasize the problem of insuring effective sealing of the fluids in the machine. The fluids in the machine must be kept separate and the only proven and effective method of sealing for long periods of operation is to allow leakage in a controlled fashion. One, therefore, resorts to a buffer gas leaking into the process and into a scavenge chamber into which

the lubricant also leaks. The lubricant and buffer gas are then separated in the accessory system and returned to their respective systems. Moreover, since the pressures in the two process fluids vary considerably from each other at part load, large flows of buffer gas would be required or each of the bearings would have to be pressure referenced to the process it is contiguous to, with resulting leakage flows in the lubrication system between the two pressure systems. If, however, one of the process fluids is used in the lubricant, a single pressure system referenced to the other process fluid is feasible with a considerable savings of equipment and complication as compared to the two-pressure lubricant system and a two buffer gas system. The choice of water as the lubricant allows for this simplification.

As was pointed out earlier, hydrostatic bearings are required by the operating conditions so that the boundary lubrication properties of oil are not necessary for startup or shutdown, and it is possible to always have large clearances separating the possible rubbing surfaces. Another contributing factor to the choice of water as the lubricant, is that redundancy of the lubrication supply is easily achieved by the use of the feed water system as an emergency lubricant supply.

Three types of thrust bearing were considered, a tilting-pad bearing, a pocketed hydrostatic bearing, and a shrouded step bearing. Because of the difficulties associated with providing hydrostatic starting for a tilting-pad bearing, as well as its complexity, the tilting-pad design was not considered further for the thrust application. Furthermore, since the shrouded step design could provide a large measure of hydrodynamic lift performance, and in addition could operate as a hydrostatic bearing at low speeds, it was selected as the design to be investigated further.

This paper describes the preliminary design for the shrouded step thrust bearing. It is discussed here because it is the critical bearing in the design because of power loss and thrust capacity considerations.

BEARING DESCRIPTION

The shrouded step bearing is shown diagrammatically in Figure 4. A number of pads of the general shape shown are arranged on the annular thrust bearing surface. The step is depressed an amount $(h_2 - h_1)$ below the otherwise uniform top surface of the thrust bearing. Note that a feed is provided at the entrance to the step and that a drain is provided between each pad and its neighbor. In operation, fluid pressure is developed in the step to support load.

Approximate Analysis

Several assumptions were made which permitted a closed form solution to be obtained. In addition to the usual assumptions involved in establishing Reynold's equation, the bearing was assumed to be a linear strip, thus ignoring the curvature of the boundaries and the difference in length between inner and outer edges. Furthermore, pressure across the step (in the radial direction) was assumed to be constant, so that a one dimensional solution could be obtained. Finally, the corner areas beyond the step (Bxe) were ignored both for their contribution to flow and their contribution to load capacity.

The basic differential equations were the same by summing flows into and out of the strip of length dx running across the bearing. Both shear flow and pressure flow were considered in the direction of motion, and end flow out of the ends of the strip was assumed to be due to pressure flow only.

Because these bearings were to operate in the turbulent regime, with Reynolds' numbers as high as 50,000, provision was made to introduce an effective viscosity, corrected for the influence of turbulent on viscosity.

The resulting differential equation was

$$\frac{d^2 p}{dx^2} - \frac{2p}{bL} \frac{\mu_{x2}}{\mu_z} \frac{h}{h_2}^3 = 0 \quad (1)$$

where

μ_{x2} is the effective viscosity in the x direction over the step and μ_z is the effective viscosity in the z direction over the lands. μ_{x1} is the effective viscosity in the x direction over the lands.

Equation (1) applies along the step. Assuming a linear pressure drop over the lands, Equation (1) may be integrated and with the appropriate boundary conditions, giving the following relationship for the load:

$$W = \Lambda_S P_i E (L + b) \left\{ \frac{\cosh \Lambda_S}{\Lambda_S} + \frac{1}{2} \frac{e}{E} \sinh \Lambda_S - \frac{1}{\Lambda_S} + \frac{1}{\Lambda_S} \sinh \Lambda_S + \frac{e}{2\pi E} \cosh \Lambda_S \right\} \quad (2)$$

where

$$\Lambda_S = \left[\frac{2E^2}{bL} \left(\frac{\mu_{z1}}{\mu_{x2}} \right) \left(\frac{h_1}{h_2} \right)^3 \right]^{1/2} \quad (3)$$

$$H = \left(\frac{h_2}{h_1} \right)^3 \left(\frac{\mu_{x1}}{\mu_{x2}} \right) \frac{e}{E} \quad (4)$$

$$\pi = \frac{\left[6V(h_2 - h_1) e \mu_{x1} / p_i h_1^3 \right] - \cosh \Lambda_S - H \Lambda_S \sinh \Lambda_S}{\sinh \Lambda_S + H \Lambda_S \cosh \Lambda_S} \quad (5)$$

Equation (2) is conservative in that it does not include the load carried by pressure in the feed groove itself or over the land between the drain groove and the feed groove.

Calculated Performance

It was desired to have this bearing fully operable over the full speed range from 30 percent to 100 percent of full speed. Bearing proportions were analyzed therefore at the worst load condition at 30 percent speed.

The effect of the width of the side shroud is shown in Figure 5, holding the total radial width of the bearing constant. While the maximum load is obtained at a land width of about 0.3 inches, the fluid flow through the bearing increases rapidly as the land width is decreased. The dotted curve in Figure 5 shows the load per unit flow, perhaps a better measure of overall bearing performance. Therefore, a value of land width of 0.475 was selected as a compromise between load capacity and flow.

The step depth is also a critical design parameter. If the film thickness, h_1 , is

held constant the step depth is gradually increased, the load carried by the bearing increases rapidly at first then reaches a optimum value at approximately 5 times the film thickness. The optimum point is a function of feed pressure, film thickness and the other design variables. A value of 5 was selected for this design, corresponding to the optimum at the 30 percent speed point with the desired minimum film thickness of .002 inches.

The length of the land beyond the step, e , cannot be optimized with these approximate equations. These equations indicate an increasing load capacity as small e is increased. However, due to the approximations made in eliminating the corner flows, it is very doubtful whether this is the case. For this reason, small e was chosen at an arbitrary value that would provide reasonable flow rates, and was roughly comparable with the side land width.

The pressure generated at the step is shown in Figure 6 for two supply pressure. Here the step depth is kept constant at 0.012 inches and the h_2/h_1 ratio is varied by varying the film thickness h_1 . It is seen that the pressure at the step increases rapidly with h_2/h_1 for the low supply pressure. This calculation, made at 110 percent speed, shows one evidence of the hybrid or partially hydrostatically nature of this bearing. At small values of h_2/h_1 the step pressure is very little greater with a high supply pressure than with a low supply pressure, suggesting that pressure drop along the step is a strong factor when a high supply pressure is used in the feed groove.

In Figure 7 the pressure at the step is shown as a function of speed, with both the step depth and the bearing film thickness held constant. With a low supply pressure the pressure at the step increases regularly with speed at a rate faster than linear. For a high supply pressure, 250 psi, the pressure at the step is nearly equal to the feed pressure at 0 speed drops to a minimum of about 1/2 feed pressure at 4000 rpm and then increases again. This again is evidence of the quasi-restrictor action of the step towards lubricant flow in the bearing.

The overall performance of a 5 x 9-3/4 6-pad thrust bearing is shown in Figure 8, as a function of speed. The load curve is that imposed by the machine performance, and it should be noted that this may differ from the final load curve reported in

the first part of the paper, because in the course of design the load curve has shifted. The solid curve for film thickness at an inlet pressure of 250 psi shows that at no time does the bearing go below the desired minimum film thickness of .002 inches. The solid curve marked flow in gallons per minute gives the flow through a single pad. Note that the flow is very high at full speed and again is very high at very low speeds.

Because it was noted that high flows occurred where the film thickness was much greater than the minimum required, the bearing was redesigned to include a flow controlled restrictor in series with the bearing. Using a higher, but available, supply pressure of 600 psi plus a restrictor, the film thickness and flow rate curves shown by the dash lines were obtained. By this means, the flow rate was kept nearly constant over the entire speed range, and the film thickness was maintained above the desired minimum value from 30 percent speed to full speed. A single restrictor in series with the six bearing pads was used rather than an individual restrictor per pad. Thus the restrictor served primarily as a constant flow control device, reducing the speed pressure at the bearing in the speed ranges where large flows would otherwise be obtained.

More Exact Analysis

While the approximate solution discussed above takes account of turbulence effects in the bearing, it ignores the influence of other inertia effects. These inertia effects can be expected to manifest themselves in three ways. These are Bernoulli pressure drops at steps or entrance areas, ram effects at entrance, and centrifugal effects. The scale of these effects can be large in bearings of this size. For example, the Bernoulli pressure drop from an entrance groove into an area with an average velocity equal to half the runner velocity, is more than 100 psi. For these reasons, a more exact analysis is underway in which these inertia effects are accounted for in every point in the film. The solution is realized on a digital computer in two dimensions (R, β) using an iterative technique.

The inlet, exit and transition boundary conditions have been carefully considered. At a transition such as the step, conservation of total pressure has been maintained. At the exit boundaries, the static pressure in the film is made equal to the external ambient pressure, on the assumption that there is no sensible

recovery of the velocity head possible. At the inlet, no velocity head correction is made on the assumption that the fluid exiting from the land between the drain and the feed groove into the feed groove forms a boundary layer which increases in thickness due to energy imparted by the runner as it traverses the feed groove. In fact, one can hypothesize everything from a Bernoulli pressure to a full ram pressure rise at the entrance to the step. Until experimental information is available, the compromise assumption of no pressure change is made.

A few results from this program are now available. Figure 9 shows a pressure profile along the circumferential mid plane of the bearing for a Reynold's number of 2500 and for a step depth equal to the film thickness. In this representation, the feed groove is at Station 3 and the step at Station 8. The drain groove is at Station 0, 10. The lower, solid, curve is the solution with no inertia terms but including the turbulent effects. The upper curves show the solution with the inertia correction, including the sharp drop at the step. It can also be that the values obtained on the first iteration are quite close to the final converged solution. The centrifugal affect is shown in Figure 10. Here under the same conditions as in Figure 9, one can compare the inner half profile with the outer half profile. Perhaps surprisingly, the outer half profile is lower than the inner half profile, indicating that the influence of the longer leakage path overrides the centrifugal effect. The influence of Reynold's numbers is shown in Figure 8, in which the dimensionless load is plotted as a function of Reynold's number. Note that while the dimensionless load appears to decrease as Reynold's number increases, the true bearing load will increase with Reynold's number since it is proportional to the square of the angular velocity. Figure 11 shows the curve with no inertia as the lower solid line, and the first term correction as the upper dash line. The middle curve showing the conversion solution has been obtained up to a Reynold's number of 2500, and appears to be reasonably extrapolatable beyond that point. Thus the first term correction may serve as a guide at Reynold's numbers for which full conversion cannot be obtained.

PLANNED PROGRAM

An experimental program for the evaluation of this bearing is planned to begin shortly. Figure 12 is a photograph showing the thrust and journal bearing. The displacement probes for the measurement of axial and radial clearance are shown as are the static pressure tapings in and around the bearing pocket. The same type of probes are used to measure speed and are seen mounted in brackets which protrude in front of the thrust bearing and over the outer diameter of the thrust runner which is not shown. The inlet ports in the pockets of both the thrust and journal bearings are observable in the photograph.

It is planned to carry out the experimental program on a test rig in which the inertia of the circulator rotor is simulated. In fact, the actual shaft and discs will be used. This experimental work will provide information on the bearings, seals and rotor dynamics. Figure 13 shows the contemplated measurements. There will be five planes in which the radial displacement of the shaft relative to the stator and the axial displacement of the thrust runner relative to the thrust bearing will be measured. In addition, the static pressure profiles in one of the pockets of both the thrust and journal bearing will be measured. In addition, the flows, temperatures and pressures in the seal and lubrication system are to be observed.

CONCLUSIONS

The step bearing appears to have particular advantages for this installation which requires a bearing capable of long life with no attention for 10 years or more while operating in a high pressure water environment. The step bearing offers the decided advantage of excellent hydrodynamic performance in addition to capability as a hydrostatic bearing. Because of the relatively large step depth, and the small rate of variation in load capacity with step depth when operating near the optimum point on the step depth in the load versus step depth curve, the step bearing can be expected to be very insensitive to wear. Thus, in the bearing under discussion here, wear of one to two mils can be tolerated without seriously affecting the bearing performance. Although inertia effects can play a major role in the load capacity of this bearing when operating at high Reynold's numbers and the nature of the boundary condition at the inlet of the step is an unknown area which can benefit from experimental investigation, it is possible to meet most any exigency that might occur through a modification of the hydrostatic component of this type of hybrid bearing.

SERIES STEAM TURBINE CIRCULATOR
PCRV INSTALLATION

Figure 1: Circulator Thrust Load, Helium Pressure and Steam Pressure as Functions of Speed

SERIES STEAM TURBINE CIRCULATOR

Figure 2: Series Steam Turbine Circulator,
PCR Installation

CIRCULATOR THRUST LOAD, HELIUM PRESSURE, AND STEAM PRESSURE AS A FUNCTION OF SPEED

Figure 3: Series Steam Turbine Circulator

Figure 4: Schematic of Shrouded Step Bearing

Figure 5: Load Versus Land Width

Figure 6: Pressure at Step Versus Step Depth Ratio

Figure 7: Pressure at Step Versus Speed

Figure 8: Film Thickness and Flow Versus Speed

Figure 9: Computed, Normalized Pressure Profile in Direction of Motion

Figure 10: Computed, Normalized Pressure Profile in Radial Direction

Figure 11: Normalized Load Versus Reynolds Number

Figure 12: Journal and Thrust Bearing

INSTRUMENTATION BEARING TEST RIG

Figure 13: Instrumentation for Bearing Test Rig

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