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EXPLICIT FORMULAE FOR THE MAG-
NETIC INDUCTION IN UNIFORMLY
MAGNETIZED ELLIPSOIDS OF
REVOLUTION

by

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EXPLICIT FORMULAE FOR THE MAGNETIC INDUCTION IN UNIFORMLY MAGNETIZED ELLIPSOIDS OF REVOLUTION

Candace A. Schau and Dennis R. Stone

ABSTRACT

The Stoner-Wohlfarth theory has been extended to give implicit formulae for the magnitude and direction of the magnetic induction in a single-domain ferromagnetic ellipsoid of revolution. Explicit formulae for the magnetic induction have been suggested by Gold; quantitative comparison shows that these approximate expressions compare quite well with the exact results obtained from the Stoner-Wohlfarth theory.

INTRODUCTION

In recent years it has become clear that the effective magnetic field acting on conduction electrons in a single domain of a ferromagnetic metal should be regarded as the total induction $\underline{B}$. In particular, the de Haas-van Alphen effect in long "whisker" crystals oriented parallel to an external field $\underline{H}_0$ has been demonstrated (Anderson and Gold, 1963) to be periodic in B^{-1} , where $\underline{B} = \underline{H}_0 + 4\pi \underline{M}_S$ and M_S is the saturation magnetization. However, in most practical cases, the sample cannot be regarded as an infinite cylinder, and it need not, of course, be oriented with its long axis parallel to the field so that it

is usually necessary to take demagnetizing effects into account. Stoner and Wohlfarth (1948) have calculated numerically the direction of the magnetization vector $\underline{M}_S$ for prolate ellipsoids of revolution in relatively weak fields, but no attempt was made to calculate the magnetic induction $\underline{B}$. In general, $\underline{B}$ is a function of both the magnitude and direction of the applied field $\underline{H}_0$, and explicit formulae for the magnetic induction would be useful to account for these effects in interpreting experimental data. Such formulae have been suggested heuristically by Gold (1964), and some spot checks indicated that the magnitude and direction of $\underline{B}$ calculated from these explicit formulae were in quite close agreement with the exact results calculated by extending the implicit Stoner-Wohlfarth theory. The purpose of this report is to make a more quantitative comparison of the approximate explicit formulae with the exact results.

THE FORMULAE FOR B

If only the effects of shape anisotropy are considered, the energy per unit volume U of a uniformly magnetized specimen can be written as (cf. Brown, 1962)

$$U = - \underline{M} \cdot \underline{H}_0 - (1/2) (4\pi) \underline{M}_S \cdot \underline{D} \cdot \underline{M}_S, \quad (1)$$

where $\underline{D}$ is the demagnetizing tensor

$$D_{ij} = \frac{1}{4\pi\tau} \int \frac{\partial^2 \psi}{\partial x_i \partial x_j} d\tau$$

and ψ is the scalar potential of unit pole density distributed throughout the volume τ of the specimen.

For both prolate and oblate ellipsoids of revolution, the magnetization vector $\underline{M}_S$ lies in the plane determined by the external field $\underline{H}_O$ (which we assume to be uniform) and the polar axis of the ellipsoid. With reference to Fig. 1, we let θ be the angle between the polar axis and $\underline{H}_O$, ϕ that between $\underline{M}_S$ and $\underline{H}_O$, and χ the angle between the polar axis and the total induction $\underline{B}$ (all angles are expressed in radians). For ellipsoids of revolution, the principal demagnetizing factors (D_{11} , D_{22} , D_{33}) are constants which satisfy the relations

$$\begin{aligned} D_{22} &= D_{33} \\ D_{11} + D_{22} + D_{33} &= 1. \end{aligned} \quad (2)$$

If we let $D_{11} = D$, then $D_{22} = D_{33} = (1-D)/2$. Using the coordinates of Fig. 1, the variable part of the energy per unit volume is found from (1) and (2) to be

$$U_v = -M_S H_O \cos \phi - (\pi/2) M_S^2 (1 - 3D) \cos 2(\theta - \phi). \quad (3)$$

If $\underline{H}_O$ and θ are regarded as independent variables, the extremal values for U_v occur when

$$H_O \sin \phi + \pi M_S (1 - 3D) \sin 2(\phi - \theta) = 0, \quad (4)$$

and it can be shown that this expression will always correspond to an energy minimum, i. e. to stable equilibrium, when $H_O > 4\pi M_S$ (Stoner and Wohlfarth, 1948). For convenience we introduce the reduced field $\underline{H}$ defined by

$$\underline{H} = \underline{H}_O / 4\pi M_S$$

Fig. 1. Definitions of angles used in formulae for $\underline{B}$ (or $\underline{\mathcal{B}}$).

so that (4) becomes

$$4H \sin \phi + (1-3D) \sin 2(\phi-\theta) = 0. \quad (5)$$

The magnetic induction $\underline{B}$ is given by (cf. Brown, 1962)

$$\underline{B} = \underline{H}_0 + 4\pi \underline{M}_S - 4\pi \underline{D} \cdot \underline{M}_S$$

or in terms of a reduced induction $\underline{\mathcal{B}}$ by

$$\underline{\mathcal{B}} = \underline{B}/4\pi M_S = \underline{H} + (\underline{I} - \underline{D}) \cdot \underline{\hat{M}}_S. \quad (6)$$

With reference to the coordinate system of Fig. 1, we then have

$$\begin{aligned} \mathcal{B}_1 &= H \cos \theta + (1-D) \cos (\phi-\theta) \\ \mathcal{B}_2 &= H \sin \theta - (1/2)(1+D) \sin (\phi-\theta) \\ \mathcal{B}_3 &= 0. \end{aligned} \quad (7)$$

The absolute magnitude of $\underline{\mathcal{B}}$ and the angle χ between $\underline{\mathcal{B}}$ and the polar axis are

$$\begin{aligned} \mathcal{B} &= [\mathcal{B}_1^2 + \mathcal{B}_2^2]^{1/2}, \\ \chi &= \arctan \left(\frac{\mathcal{B}_2}{\mathcal{B}_1} \right). \end{aligned} \quad (8)$$

These equations for $\mathcal{B}$ and χ can be solved numerically by first computing ϕ from the Stoner-Wohlfarth Eq. (5).

The approximate formulae, which give the magnitude and the direction of the induction explicitly in terms of H , D , and θ , are

$$\mathcal{B}' = 1 - D + \mathcal{H} \frac{(\mathcal{H} + X)}{(\mathcal{H} + Y)}$$

and

(9)

$$\chi' = \frac{\theta Z \mathcal{H}}{(1 + Z \mathcal{H})}$$

where

$$X = (1/2)(1 - 3D)(3 \cos^2 \theta - 1)$$

$$Y = (1 - 3D) \cos^2 \theta$$

$$Z = \frac{4\theta}{(1 - 3D) \sin 2\theta}$$

(Gold, 1964); these formulae were constructed in such a way as to give the correct results for $\theta = 0, \pi/2$ irrespective of the values of D and $\mathcal{H}$, as well as for a sphere ($D = 1/3$).

COMPARISON OF THE HEURISTIC FORMULAE WITH THE EXACT RESULTS

The maximum differences between the approximate results (9) and the exact results (8) are found to occur for the two limiting cases of an infinite cylinder ($D = 0$) and a flat disc ($D = 1$). Figure 2 shows the fractional error $(\mathcal{B}' - \mathcal{B})/\mathcal{B}$ as a function of θ for $D = 0$ and $D = 1$, while Fig. 3 shows the corresponding angular error $\chi' - \chi$. By and large the results calculated explicitly from (9) are seen to be in remarkably close agreement with the exact results when $\mathcal{H}$ is greater than about 2 or 3, with the agreement improving as the value of $\mathcal{H}$ is increased.

Fig. 2. Fractional error $(\mathcal{B}'-\mathcal{B})/\mathcal{B}$ (or $B'-B/B$) as a function of θ for $D=0$ and $D=1$. The numbers refer to values of the reduced applied field H .

Fig. 3. Angular error $\chi' - \chi$ as a function of θ for $D=0$ and $D=1$. The numbers again refer to values of μ .

In recent de Haas-van Alphen studies with iron whiskers (Panousis, 1967), the applied field H_0 was of order 100 kG. while for iron $4\pi M_S$ is 21.8 kG, so that we have $H \geq 5$. The specimens had been electropolished into long ellipsoids of revolution with axial ratios of 30-50, and from the standard tables of demagnetizing factors (cf. Brown, 1962), the corresponding variation of D is 0.003-0.001. For such low values of D and $H \geq 5$, Figs. 2 and 3 show that θ' will never differ from θ by more than 0.1% while χ' will never deviate from χ by more than 0.2°.

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