3/19

Hr. 24

LA-5171-MS

INFORMAL REPORT

The n - d Breakup Reaction and the n - n Scattering Length

los alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87544

MASTER

UNITED STATES
ATOMIC ENERGY COMMISSION
CONTRACT W-7405-ENG. 36

In the interest of prompt distribution, this LAMS report was not edited by the Technical Information staff.

Printed in the United States of America. Available from
National Technical Information Service
U. S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151
Price: Printed Copy $3.00; Microfiche $0.95

LA-5171-MS
Informal Report
UC-34
ISSUED: February 1973

los alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87544

The n - d Breakup Reaction and the n - n Scattering Length

by

B. F. Gibson
G. J. Stephenson, Jr.

THE n - d BREAKUP REACTION AND THE n - n SCATTERING LENGTH

by

B. F. Gibson and G. J. Stephenson, Jr.

ABSTRACT

A simple approximation to the Faddeev theory of the n - d breakup reaction is reviewed in an attempt to emphasize the difficulty of obtaining a precision measurement of the n - n scattering length using this reaction mechanism.

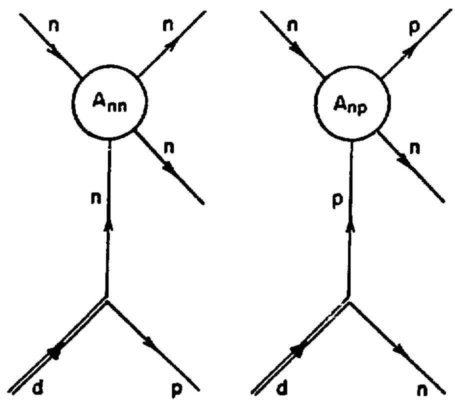
It has long been known that the nucleon-nucleon force is not charge independent; i.e., the neutron-proton singlet force is not the same as either the neutron-neutron force or the Coulomb corrected proton-proton force. Henley and Keliher[1] have recently proposed a form of charge symmetry breaking potential that would imply a difference between the n-n and Coulomb corrected p - p scattering lengths of approximately 0.8 fm. At present, however, the experimental measurement of the n - n scattering length is not precise enough to enable one to make any definite statement about charge symmetry breaking in the nucleon-nucleon force (see Appendix). Therefore, a precise determination of a_{nn} is of fundamental interest to physics.

Many attempts have been make to exploit the n - d breakup reaction

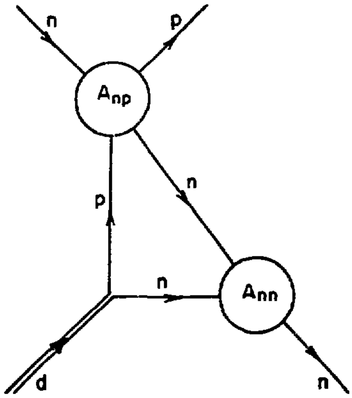
$$n + d \rightarrow n + n + p$$

to determine a_{nn}. At the recent Few Particle Conference held at UCLA[2] it was clear that sufficient accuracy in this measurement has not been attained, since the reported uncertainties in the neutron-neutron scattering lengths indicated that the results were accurate to only about ± 2 fm. Proposals submitted to LAMPF indicate that further attempts to measure a_{nn} by means of the n - d breakup reaction will be made. Therefore, we wish to review here a theoretical picture due to the several people noted below and summarized by Aitchison[3] that provides a good fit to and a simple understanding of the high energy peak in the proton spectrum from this reaction. It reproduces the three-body Faddeev result for low energy, displays explicitly the dependence of the spectrum on a_{nn}, and explains why the impulse approximation gives a much better fit to the data than does the Watson-Migdal approximation.

It was shown by Popova[4] that, for low values of the relative neutron-neutron momentum, the matrix element describing the n - d breakup reaction is given to a reasonable approximation by the sum of the three diagrams:



(1) (2)



(3)

If we define $\vec{k}_0$ to be the momentum of the incident neutron, $\vec{k}$ to be the momentum of the outgoing proton, $\vec{p}$ to be the neutron-neutron relative momentum, $\vec{q} = (\vec{k}_0 - \vec{k})/2$, and $\alpha^2 = mB$ where B is the deuteron binding energy, then the amplitude represented by the above diagrams can be written as

$$A \sim A_{nn}(p)\,\frac{2}{\alpha^2 + k^2} + [A^s_{np}(\tfrac{1}{2}k) + A^t_{np}(\tfrac{1}{2}k)]/2 \times \left(\frac{1}{\alpha^2 + (\vec{q} - \vec{p})^2} + \frac{1}{\alpha^2 + (\vec{q} + \vec{p})^2}\right) + A_{nn}(p)\,\frac{1}{2iq}\,[A^s_{np}(\tfrac{1}{2}k) + A^t_{np}(\tfrac{1}{2}k)] \times \ln\left(\frac{i\alpha + p - q}{i\alpha + p + q}\right) \quad , \qquad (1)$$

as was shown by Voitovetskii et al[5] and by Popova and Muskalu.[6] Here A_{nn}, A^s_{np}, and A^t_{np} represent the nucleon-nucleon amplitudes, which for low energy can be expressed as

$$A_{nn}(p) = \left[-\frac{1}{a_{nn}} + \frac{1}{2}\,r^2_{nn}p^2 - ip\right]^{-1} \quad , \qquad (2)$$

etc. The use of Eq. 2 is not a good approximation for A^s_{np} and A^t_{np} at some of the higher energies considered below and will affect the absolute cross section; however, the relative dependence on a_{nn} will not be altered. Since we are interested in very small p, the relative n - p momentum in the n - p amplitudes appearing in diagram 3 does not vary much from its on-shell value. Consequently, the amplitudes were removed from the loop integral and evaluated at $k/2$.

Diagram 1 corresponds to the Watson-Migdal approximation. Since the factor $(\alpha^2 + p^2)^{-1}$ varies slowly for small p, the peak arising from the W - M contribution to the breakup amplitude comes entirely from the variation due to $A_{nn}(p)$. It is well known that this produces a peak in the cross section; however, the peak produced is not nearly sharp enough to account for the data. The second and third diagrams correspond to the impulse approximation with final state interactions. As we shall see below, it is these diagrams that dominate the cross section and produce the narrow peak.

For simplicity, let us limit our consideration to forward scattering of the proton. Then for $q \approx 0$, one can expand the functions in Eq. 1 to obtain

$$A \sim A_{nn}(p)\,\frac{2}{\alpha^2 + k^2} + [A^s_{np}(\tfrac{1}{2}k) + A^t_{np}(\tfrac{1}{2}k)](\alpha^2 + p^2)^{-1} + i\,A_{nn}(p)[A^s_{np}(\tfrac{1}{2}k) + A^t_{np}(\tfrac{1}{2}k)](p + i\alpha)^{-1} \qquad (3)$$

where $p^2 = k_0^2/4 - \alpha^2 - 3k^2/4 + k_0k/2$. It is the last term in Eq. 3 containing the factor $(p + i\alpha)^{-1}$ that dominates the amplitude in the region of the peak cross section. This term is approximately an order of magnitude larger than the W - M term. The additional p dependence produces a much more rapid variation in the amplitude than $A_{nn}(p)$ alone can generate. To demonstrate the excellent approximation to the Faddeev result, we reproduce part of a figure due to Aitchison.[3] (See Fig. 1). It is clear that the simple approximations outlined above reproduce the full theory quite well, and that the peak is much sharper than the Watson-Migdal approximation. It is also clear that, because the Watson-Migdal curve has been multiplied by approximately 50 to place it on the graph, the n - d breakup amplitude is determined primarily by the impluse approximation contribution.

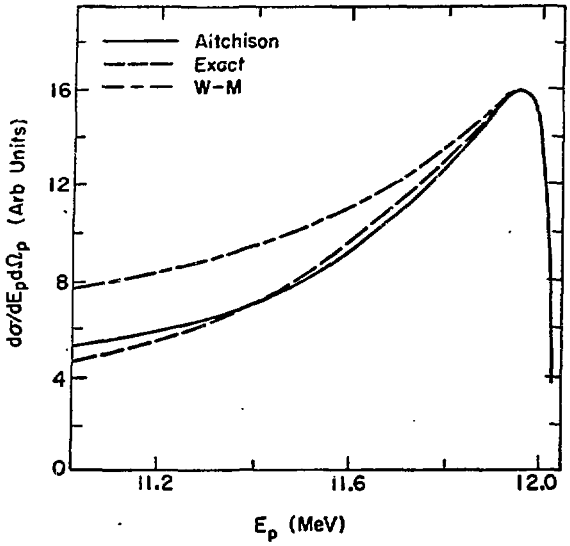


Fig. 1. The n-d spectra from Ref. 3; $E_n^0 = 14.4$ MeV and $\theta_p = 0^o$.

In Fig. 2-4 we have plotted the cross section

$$\frac{d\sigma}{dE_p d\Omega_p} (\theta = 0^0) \sim |A|^2 kp$$

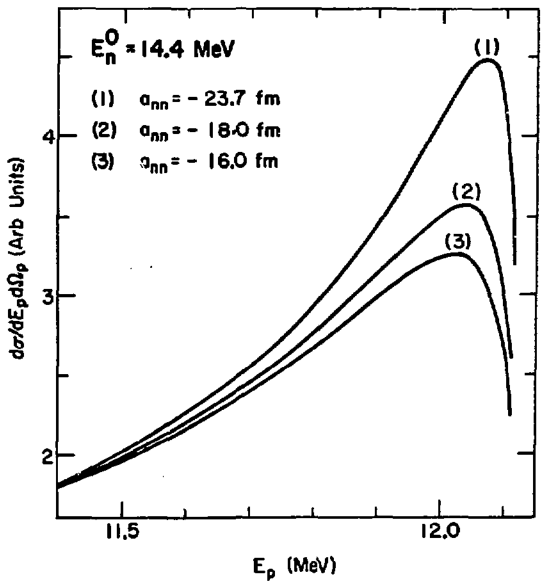


Fig. 2. n-d breakup cross section; for all curves $a_{np}^s = -23.7$ fm , $r_{np}^s = 2.8$ fm , $a_{np}^t = 5.4$ fm , $r_{np}^t = 1.7$ fm , $r_{nn} = 2.8$ fm , and $\theta_p = 0^o$.

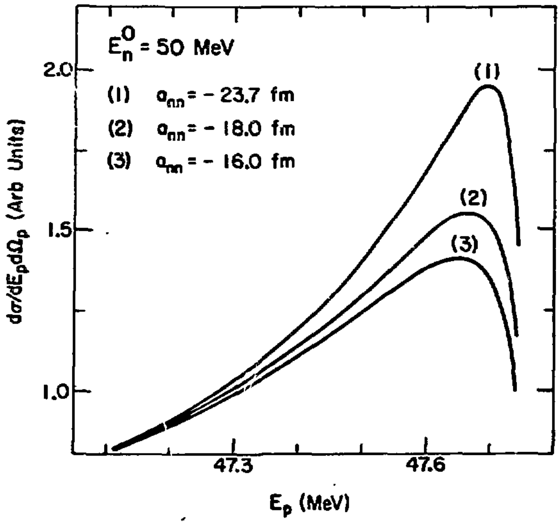


Fig. 3. n-d breakup cross section; parameters identical to those of Fig. 2.

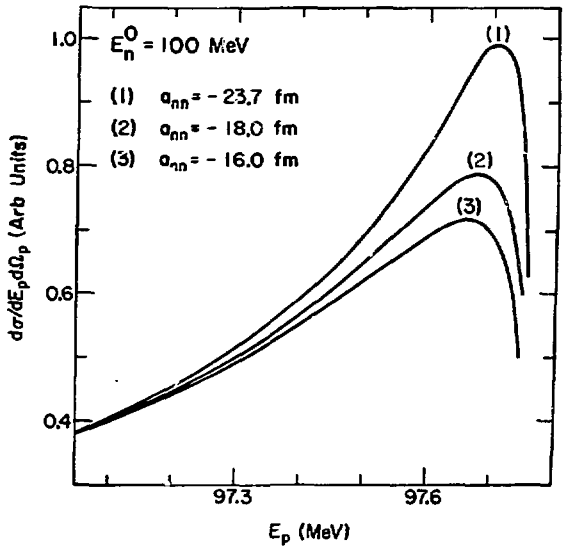


Fig. 4. n-d breakup cross section; parameter identical to those of Fig. 2.

for incident neutron energies of 14.4, 50, and 100 MeV , where we have used Eq. 1 to determine A. (Note the suppressed zeros.) Each set of curves is calculated for neutron-neutron scattering lengths of -23.7, -18.0, and -16.0 fm. We point out again that a much more careful treatment of A^s_{np} and A^t_{np} is required to obtain accurate absolute cross sections; in fact the model needs to be compared with the Faddeev calculation for the higher incident energies. To eliminate problems of absolute cross sections, we have plotted corresponding proton spectra normalized at the peak cross section in Fig. 5-7.

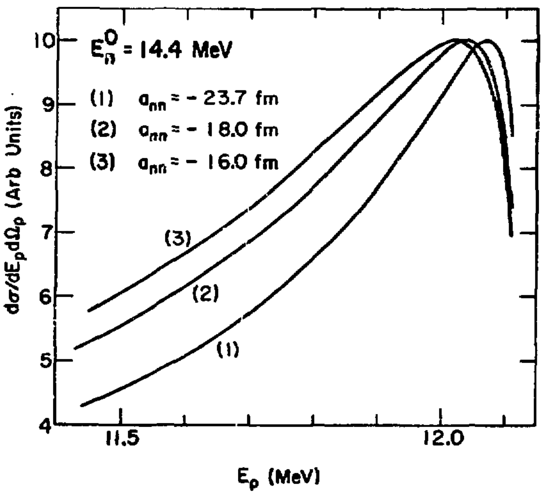


Fig. 5. Normalized proton spectra corresponding to Fig. 2.

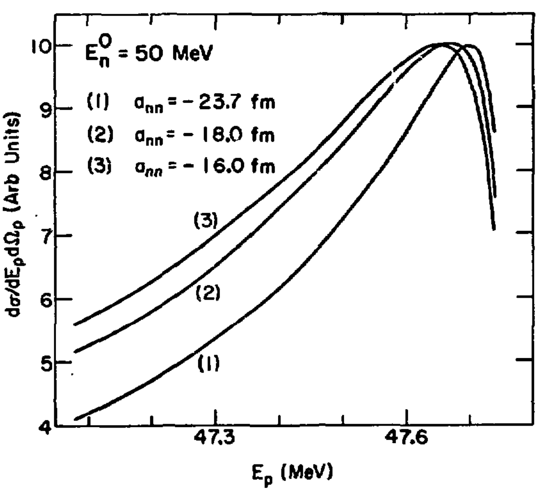


Fig. 6. Normalized proton spectra corresponding to Fig. 3.

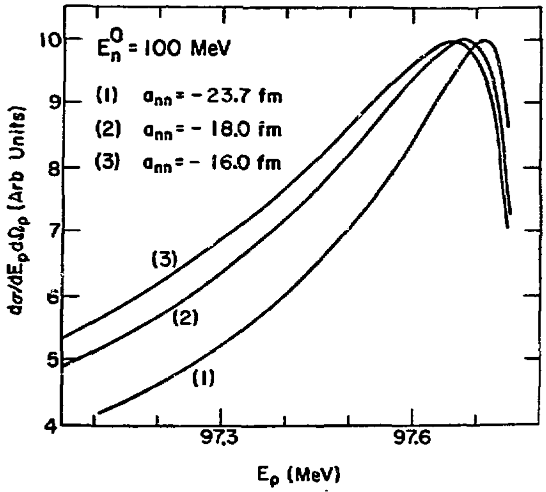


Fig. 7. Normalized proton spectra corresponding to Fig. 4.

From these figures, we wish to emphasize the difficulty of determining a_{nn} precisely. The difference between a_{nn} = -18 and -16 fm is only some 10% ---- for the cross sections in Fig. 2-4 this occurs only in an energy range of less than 0.2 MeV in the proton energy. Such a difference is measurable. However, we need to know a_{nn} to an uncertainty equal to that occurring for the Coulomb corrected a_{pp} , or some ±0.3 fm (see Appendix). Such precision would seem difficult to achieve at present: a spectrum measured to better than 0.2%.

Finally, we should point out that, in the region of the peak cross section, the relative momentum of the two neutrons is sufficiently low that small uncertainties in r_{nn} will not produce measureable effects. Differences in spectra for r_{nn} of 2.6 and 2.8 fm could not be distinguished in the figures above. Therefore, a reasonable assumption for r_{nn} in any experiment to examine charge symmetry breaking effects is that it is identical to the Coulomb corrected proton-proton effective range; i.e., one should assume charge symmetry in the effective range until charge asymmetry is established for the scattering lengths.

APPENDIX

The low energy singlet scattering parameters for the proton-proton and neutron-proton systems are now known to relatively good accuracy:[7]

$$a^C_{pp} = -7.82 + .01 \text{ fm}, \qquad r^C_{pp} = 2.80 \pm .02 \text{ fm};$$

$$a^s_{np} = -23.72 \pm .02 \text{ fm}, \qquad r^s_{np} = 2.73 \pm .03 \text{ fm};$$

where the superscript C indicates that these are the measured values including Coulomb effects. The uncertainty associated with the p - p scattering length is of the order of 0.01 fm. However, it is the "Coulomb corrected" value of the p - p scattering length that we need for comparison with the n - n scattering length. Removal of the Coulomb effects is a model dependent process.[8] Hence, the uncertainty quoted for a_{pp} is larger than the experimental uncertainty in a^C_{pp}. A generally accepted value for the Coulomb corrected p - p scattering length is[9]

$$a_{pp} = -17.1 \pm .3 \text{ fm} \quad .$$

The n - n scattering length is very uncertain. A value of $a_{nn} = -17 \pm 1$ fm is quoted in Ref. 7. However, results reported at the recent UCLA conference[2] indicated an uncertainty more of the order of 2 fm. It is clear that a precise measurement of a_{nn} is lacking.

REFERENCES

1. E. M. Henley and T. E. Keliher, Nucl. Phys. A189, (1972) 632.
2. International Conference on Few Particle Problems in the Nuclear Interaction, (1972), edited by I. Slaus, to be published by North Holland Publishing Co.
3. I. J. R. Aitchison, Nucl. Phys. A148, (1970) 457.
4. A. M. Popova, Nucl. Phys. 82, (1966) 209.
5. V. K. Voitovetskii, I. L. Korsunskii, and Y. F. Pazhin, JETP 20, (1965) 1084; Nucl. Phys. 69, (1965) 513.
6. A. M. Popova and S. Muskalu, SJNP 1, (1965) 555.
7. H. P. Noyes and H. M. Lipinski, Phys. Rev. C4, (1971) 995.
8. L. Heller, P. Signell, and N. R. Yoder, Phys. Rev. Letters 13, (1964) 577.
9. R. V. Reid, Ann. Phys. 50, (1968) 411.

HK:631(370)