SYMMETRIES AND NON-SYMMETRIES OF THE RELATIVISTIC QUARK MODEL*

J. L. Rosner

School of Physics and Astronomy
University of Minnesota
Minneapolis, Minnesota 55455

May 1, 1972

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Abstract

The dynamical successes of the relativistic quark model are separated from those due to symmetry arguments. Particular attention is paid to the example of the $q\bar{q}$; $L = 1$ meson decays. A relativistically invariant coplanar form of $U(3) \otimes U(3)$ is found to explain all predictions of the relativistic quark model for decays within a given multiplet except (a) the absence of spin-orbit effects and (b) relations between different partial waves, e.g., in $B \rightarrow \omega\pi$. The coplanar symmetry is expected to be as good as the quark model classification scheme itself. It preserves most of the better predictions of $SU(6)_W$, while avoiding some that are questionable. Some of the "good" predictions preserved are $\Gamma(A_2 \rightarrow \rho\pi) / \Gamma(A_2 \rightarrow K\bar{K}) = 6$, $(d/f)_{g(5/2^-) \rightarrow g(1/2^+)g(0^-)} = -1/3$ and $(d/f)_{g(5/2^+) \rightarrow g(1/2^+)g(0^-)} = 2/3$. Questionable predictions of $SU(6)_W$ avoided by the coplanar symmetry include the selection rules $A_1 \rightarrow f(\lambda = 0)\pi$, $B \rightarrow \omega(\lambda = 1)\pi$. A model of hadron decays which assumes them to occur via production of a 3P_0 $q\bar{q}$ pair is consistent with (but more specific than) the coplanar symmetry.

* Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-1764.

Alfred P. Sloan Foundation Research Fellow, 1971-73.

MASTER

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

I. INTRODUCTION

The quark model has had considerable success in classifying mesons and baryons, and also describes certain features of their interactions with one another fairly well. Many of the predictions of the quark model can be stated more abstractly in terms of various forms of $SU(6)$.¹

More recently it has been found that certain quark models which violate $SU(6)_W$ describe hadronic interactions quite a bit more accurately.²⁻⁴ The relativistic quark model,^{3,4} for example, has specific $SU(6)_W$ - violating interactions which correspond to quark recoil during meson emission. Another picture views hadron decays as proceeding via the creation of a 3P_0 $q\bar{q}$ pair.^{2,5-7} Matrix elements in this picture, which we shall call the " 3P_0 picture," bear striking similarity to those in the relativistic quark model.

One is thus led to ask whether there is a symmetry intermediate between $SU(6)_W$ and $SU(3)$ which is characteristic of the relativistic quark model and the 3P_0 picture. Such a symmetry in fact exists. It is a coplanar form of $U(3) \otimes U(3)$,⁸ originally proposed as a possible description of any process involving two directions. Its specific applicability to hadron decays has been suggested previously.⁹

The model presented in Ref. 2 is sufficiently general that it serves primarily as a parametrization of this symmetry. Our purpose here is mainly an expository one. We wish to present the rules for calculating directly using this symmetry (without the reference to additional parameters of Ref. 2) and to show its relation to other schemes. This will be done with particular reference to decays of the $q\bar{q}$; $L = 1$ mesons. We find, in order of increasing predictive power, the sets of predictions given in Eq. (1):

MASTER

$$\begin{aligned} \{ SU(3) \} &\subset \{ \text{coplanar } U(3) \otimes U(3) \} \subset \{ {}^3P_0 \text{ picture}^{2,5-7} \} \subset \\ &\subset \left\{ \begin{array}{l} \text{relativistic quark} \\ \text{model}^3 \end{array} \quad \text{or} \quad SU(6)_W \right\} \quad (1) \end{aligned}$$

(As mentioned, the last two are inconsistent with one another.) We shall give the physical assumptions that allow one to move to successively more predictive schemes in Eq. (1).

The symmetry $SU(6)_W$ is generated by those generators of $[U(6) \otimes U(6)]_\beta^{1,8,10}$ which commute with the generator α_z of Lorentz boosts in a given direction.¹¹ The coplanar $U(3) \otimes U(3)$ symmetry is generated by the subset which commute with Lorentz boosts in a plane. The motivation for applying such a symmetry to hadron decays is discussed in sec. II. Section III deals with the specific example of the decays of the lowest 2^+ , 1^+ , and 0^+ mesons, classified as $q\bar{q}$; $L = 1$ states. The predictions of the coplanar symmetry are compared there to those of other approaches. A brief discussion of more general predictions and of the physical differences between the various schemes is presented in section IV. Appendix A gives rules for classifying states under coplanar $U(3) \otimes U(3)$, while Appendix B deals with transformations between spin quantization axes. Appendix C shows how angular momentum conservation constrains various amplitudes.

II. $SU(6)_W$ VERSUS COPLANAR SYMMETRY

Consider, to begin, quarks and antiquarks at rest. The projection operator for a quark is $(1 + \beta)/2$; that for an antiquark is $(1 - \beta)/2$. These projections times the spin operators $(1, \vec{\sigma})$ times the nine λ_i matrices¹² of $U(3)$ then generate an algebra of $[U(6) \otimes U(6)]_\beta$. Particles may be classified according to this symmetry by building them up from quarks q and antiquarks $\bar{q}$:

$$\begin{aligned} q &\in (\underline{6}, \underline{1}) \\ \bar{q} &\in (\underline{1}, \underline{\bar{6}}) \end{aligned} \quad (2)$$

so that (for example) π and $\rho \in (\underline{6}, \underline{\bar{6}})$; N and $\Delta \in (\underline{56}, \underline{1})$; $\bar{N}$ and $\bar{\Delta} \in (\underline{1}, \underline{\bar{56}})$; etc. One can imagine the quarks to have some relative orbital angular momentum L , or introduce an abstract quantity with properties of L .¹⁰ The resulting scheme, $[U(6) \otimes U(6)]_\beta \otimes O(3)$, then describes the low-lying hadrons fairly well.¹³

Note at this point that a meson need not be purely $q\bar{q}$; L nor a baryon qqq ; L for the classification just mentioned to work. The same results will hold if the wavefunctions have admixtures of $SU(3)$ singlet $q\bar{q}$ pairs, for example a 3S_1 pair replacing a unit of L . Experiments on deep inelastic scattering of electrons indicate that the quark structure (if any) of protons and neutrons is more complicated¹⁴ than the naive picture.¹³ Hence we must be wary of any attempt to ascribe the success of $[U(6) \otimes U(6)]_\beta \otimes O(3)$ to any too-realistic quark model. We proceed further bearing such caution in mind.

The symmetry $[U(6) \otimes U(6)]_\beta \otimes O(3)$ must be violated if hadrons are to decay, as one can easily see by taking the example of $\Delta \rightarrow N\pi$. The initial state belongs to $(\underline{56}, \underline{1})$. The final state would belong to $(\underline{56} \otimes \underline{6}, \underline{\bar{6}})$ if all

the quarks were at rest. This is not the case, of course, since the N and π must be in a relative P-wave. Fortunately, a moving quark is not a member of a pure representation of $[U(6) \otimes U(6)]_B$. Of the generators of this "rest symmetry," only a subset commute with the "boost" operator $\exp\{\alpha_z \chi/2\}$ (where $\cosh \chi = [1 - v^2/c^2]^{-1/2}$). This latter set forms the algebra of $SU(6)_W$.¹⁵

$$SU(6)_W : \left\{ \begin{array}{l} \lambda_i \\ \lambda_i \sigma_z \\ \lambda_i \beta \sigma_x \\ \lambda_i \beta \sigma_y \end{array} \right\} \text{ commute with } \alpha_z. \quad (3)$$

A quark moving along the z -axis is then to be classified according to the 6-dimensional representation of $SU(6)_W$. Its "W-spin" (generated by $\sigma_z, \beta \sigma_x, \beta \sigma_y$) is the same as its quark spin. An antiquark moving along the z axis belongs to $\bar{6}$, and its x, y components of W-spin are opposite to those of its quark spin. Because of Eq. (3), these classifications are invariant under change of frame along the z axis.

What has happened to the moving quark under the "boost"? The quark number is the eigenvalue of $(1 + \beta)/2$, and the antiquark number is the eigenvalue of $(1 - \beta)/2$. Then, for a quark at rest, "boosted" by $e^{\alpha_z \chi/2}$,

$$\begin{aligned} N_q &= q_{\text{rest}}^+ e^{-\alpha_z \chi/2} \frac{1+\beta}{2} e^{\alpha_z \chi/2} q_{\text{rest}} \\ &= \frac{1}{2} \text{Tr} \left\{ \frac{1+\beta}{2} e^{-\alpha_z \chi/2} \frac{1+\beta}{2} e^{\alpha_z \chi/2} \right\} \\ &= \cosh^2 \frac{\chi}{2}, \end{aligned} \quad (4)$$

while
$$\bar{N}_q = \sinh^2 \frac{\chi}{2} \quad (5)$$

The quark has thus acquired a "cloud" of quark-antiquark pairs (in an SU(3) singlet state) as we move past it. Hence the decay of a hadron into two moving ones is under no obligation to conserve the number of quarks plus antiquarks.

Under SU(6)_W, since W = S for quarks, the N and Δ belong to a $\underline{56}$, the negative-parity baryons below ~ 1800 MeV belong to a $\underline{70}$, and so on. For mesons, the fact that $W_{x,y} = -S_{x,y}$ for antiquarks makes the classification less straightforward. The ρ and π have quark spin 1 and 0, respectively. However, the helicity-zero ρ has W = 0 while the π has W = 1. This phenomenon is known as W-S flip.^{15,16} The longitudinal ρ , with W = 0, can decay without trouble to two pions, each with W = 1. Similarly the Δ (W = 3/2) can decay to a π (W = 1) and a nucleon (W = 1/2). If quark spin conservation had been demanded instead, neither of these two decays could occur.

Let us now take L of the initial hadron to be not necessarily zero, while the final hadrons will still be taken to have L = 0. This situation describes most of the well-studied decays; the exceptions are few enough that a systematic treatment of them would be premature at present. SU(6)_W can still be applied, if one assumption is made. Since the z component of quark spin, S_z, is equal to W_z (which is conserved), and since J_z = S_z + L_z is conserved, L_z must also be conserved. Since the final state has L_z = 0, so must the initial state. The resulting symmetry¹⁷ has sometimes been called SU(6)_W $\otimes$ O(2)_{L_z},¹⁸ since it entails L_z conservation as well as W-spin conservation. These are not independent, of course.

The symmetry SU(6)_W $\otimes$ O(2)_{L_z} and models equivalent to it have been

applied to the decays of mesons,¹⁹ to the decays of baryons,²⁰ and to photo-production of baryon resonances.²¹ Some of the results indeed agree with experiment fairly well. Typical predictions of this scheme which work qualitatively include the following:

(1) Predictions for decays of $L = 0$ hadrons.

For $L = 0$, the question of what to do with L_z never arises. One then obtains relations such as²²

$$\Gamma(\Delta \rightarrow N\pi) = \frac{12}{25} \frac{g_{N\bar{N}\pi}^2}{4\pi} \frac{p^3}{m_\Delta^2} \quad (6)$$

Symmetry breaking corrections due to masses²² have been neglected here. Eq.

(6) then predicts a partial width of about 60 MeV, low by a factor of 2.²³

We will take this as an index of the quality of the "good" predictions of $SU(6)_W$. The "bad" ones turn out to be considerably worse.^{2,3}

A prediction analogous to Eq. (6) for mesons, which makes use of vector dominance, is based on an $SU(6)_W$ relation between $g_{\rho\pi\pi}$ and $g_{\omega\rho\pi}$.²⁴ It reads:

$$\frac{\tilde{\Gamma}(\omega \rightarrow 8\pi^0)}{\tilde{\Gamma}(\rho \rightarrow \pi\pi)} = \frac{2e^2}{g_{\rho\pi\pi}^2} \simeq 8 \times 10^{-3} \quad (7)$$

whereas the present experimental ratio is about²³ (1 MeV)/(150 MeV)

7×10^{-3} . Here $\tilde{\Gamma}$ denotes the partial width corrected for phase space and centrifugal carrier factors.

(2) Ratios $\Gamma(2^+ \rightarrow 1^- 0^-)/\Gamma(2^+ \rightarrow 0^- 0^-)$ for mesons.

A typical prediction is

$$\frac{\Gamma(A_2 \rightarrow \rho\pi)}{\Gamma(A_2 \rightarrow K\bar{K})} \simeq \frac{\tilde{\Gamma}(A_2 \rightarrow \rho\pi)}{\tilde{\Gamma}(A_2 \rightarrow K\bar{K})} = 6, \quad (8a)$$

which agrees very well with experiment.^{23,25} Such a relation can be generalized to arbitrary J and reads

$$\frac{\tilde{\Gamma} [A_2^*(J) \rightarrow \rho\pi]}{\tilde{\Gamma} [A_2^*(J) \rightarrow \kappa\bar{\kappa}]} = 4 \frac{J+1}{J} \quad (8b)$$

(3) Baryon f/d ratios.

The $SU(6)_W$ predictions for baryons

$$(f/d)_{5/2^-} = -1/3 \quad (9)$$

$$(f/d)_{1/2^+} = (f/d)_{5/2^+} = 2/3 \quad (10)$$

seem to be fairly consistent with the data.²⁷⁻²⁹ An example of this is the extreme inelasticity of the $\Lambda(1830) 5/2^-$ resonance. The $N\bar{K}$ branching ratio $\chi_{N\bar{K}}$ of this resonance is not quoted consistently from experiment to experiment, but most analyses agree that $\chi_{N\bar{K}}$ is no more than $\sim 10\%$.

On the other $\sqrt{\chi_{N\bar{K}} \chi_{\Sigma\pi}}$ is significantly larger than this. For $f/d = -1/3$, the $\Lambda(1830)$ would decouple altogether from $N\bar{K}$.

(4) Various photoproduction selection rules.²¹

These include the fact that the transition $\gamma N \rightarrow \Delta$ seems to be M1, the failure of $N^+(5/2^-)$ to be photoproduced from protons, and the failure of the $N^0(5/2^+)$ to be photoproduced in the $\lambda = 3/2$ state from neutrons.³⁰

On the other hand, there are predictions of $SU(6)_W$ which are poorer than those just mentioned. As an example, we consider the decays of $\underline{35}$, $L = 1$ mesons into 1^-0^- and 0^-0^- . Since the $\underline{35}$, $L = 1$ mesons have $J^P = 2^+, 1^+$, and 0^+ , the allowed partial waves in the final state are $\mathcal{L} = 0(S)$ and $\mathcal{L} = 2(D)$. $SU(6)_W$ relates these two. This effect is most

strongly felt in the decays $1^+ \rightarrow 1^- 0^-$ where both partial waves can occur. $SU(6)_W$ predicts that they will interfere in such a way as to produce vector mesons of a given helicity. For example, $SU(6)_W$ predicts $A_1 \rightarrow \rho(\lambda=0)\pi$ and $B \rightarrow \omega(\lambda=1)\pi$. Experimentally these decays are very much present and the latter indeed appears to be dominant.³¹ In this case, the admixture of D wave seems to have the opposite sign, relative to the S wave, from the $SU(6)_W$ prediction.²

Similar shortcomings are apparent in baryon decays into $3/2^+ 0^-$. For example, $SU(6)_W$ predicts that the $5/2^+ F_{15} N(1690)$ resonance should decay into $\Delta\pi$ only in the $\lambda = 1/2$ state. (This follows trivially from S_z conservation). The suppression of $\lambda = 3/2$ decays requires the P wave and F wave amplitudes to interfere in a very definite way. No evidence for such interference is seen.³²

If one treats photoproduction using vector dominance, so that the photon is equivalent to a transversely polarized 3S_1 $q\bar{q}$ pair, S_z conservation means that upon absorbing a photon a quark must flip its spin. Put another way, the quark cannot be given any L_z by the photon. This is very far from true. Such a rule would forbid all $\lambda = 3/2$ excitations of $S_q = 1/2$ resonances, whereas these seem to dominate in many instances.³³

These problems with $SU(6)_W$ are not characteristic of various recent quark models.²⁻⁷ We shall show that such models possess a well-defined symmetry, weaker than $SU(6)_W \otimes O(2)_{L_z}$ but stronger than $SU(3)$. The symmetry may be motivated on rather general grounds independent of the models.

As the reader may have suspected, the weak assumption in applying $SU(6)_W$ to decays of L-excited hadrons is the restriction $L_z = 0$. Physically

this corresponds to saying the quarks in the initial hadron have no transverse momentum relative to the eventual decay axis. In our opinion, there is no a priori reason for this assumption. We shall now see what happens when it is relaxed.

Consider a hadron with $L \neq 0$. Its quarks with spin S are coupled with L to total angular momentum J :

$$|JJ_z\rangle = \sum_{L_z S_z} (SS_z LL_z | JJ_z) |LL_z\rangle |SS_z\rangle \quad (11)$$

In $SU(6)_W$, matrix elements from states with $L_z \neq 0$ are assumed to vanish, as mentioned, so that $S_z = W_z$ may be conserved. Even when the matrix elements for $L_z \neq 0$ do not vanish, we shall continue to apply $SU(6)_W$ to the case $L_z = 0$, projecting out the appropriate piece of the initial state wavefunction using Eq. (11).

The decay helicity amplitude for $A(L_z = 0, \lambda) \rightarrow B(\lambda')C$ may then be written

$$M[A(L_z=0, \lambda) \rightarrow B(\lambda')C] = \sum_i \left(\begin{matrix} A \\ \alpha, a \end{matrix} \middle| \begin{matrix} B & C \\ \beta, b & \gamma, c \end{matrix} \right)_i \times$$

$$\times \left(\begin{matrix} \alpha \\ A \end{matrix} \middle| \begin{matrix} B & C \\ \beta & \gamma \end{matrix} \right) \times [A(L_z=0, \lambda) \rightarrow B(\lambda')C] \quad (12)$$

where

$$X[A(L_z=0, \lambda) \rightarrow B(\lambda')C] = (W^B \lambda' W^C \lambda - \lambda' | W^A \lambda) \times$$

$$\times (S^A \lambda L 0 | J \lambda) a^{(0)} [A(L) \rightarrow B C] \quad (13)$$

The z axis is taken to be direction of B. λ and λ' are the respective projections of J^A and J^B along this axis. The first term on the right hand side of Eq. (12) is an $SU(6)_W$ Clebsch-Gordan coefficient, with the Clebsch-Gordan coefficients of $SU(3)$ and $SU(2)_W$ factored out.³⁴ A, B, and C denote $SU(6)_W$ multiplets and $\alpha, a; \beta, b; \text{ and } \delta, c$ label the dimensionality of $SU(3) \otimes SU(2)_W$ multiplets. The second term is an isoscalar factor³⁵ with A, B, and C labelling specific isomultiplets. The sum over i corresponds to d and f couplings when α, β, δ are all octets. In the case of mixed $SU(3)$ representations (such as decays involving ω, ϕ, f , and f') two or more expressions (12) must be summed with appropriate mixing coefficients. For decays into specific charge states M in Eq. (12) is to be multiplied by an appropriate isospin Clebsch-Gordan coefficient. In Eq. (13) the first factor expresses the coupling of W-spin, while the second is the term coming from Eq.(11) that projects out $L_z = 0$. The reduced matrix element $a_{[A(L) \rightarrow B C]}^{(\omega)}$ depends only on the initial and final multiplets. For example, the decays of the $2^+, 1^+$, and 0^+ mesons $[35, L = 1]$ into $1^- 0^-$ or $0^- 0^-$ pairs $[35, L = 0 \otimes 35, L = 0]$ are described by a single reduced matrix element of $SU(6)_W \times O(2)_{L_z}$.

As mentioned previously, decays from $L_z = 0$ substates into two $L = 0$ hadrons cannot conserve $SU(6)_W$. In certain circumstances they can, however, conserve a "coplanar" symmetry defined by a subset of the generators in Eq. (3). To see this, consider the combinations

$$|\vec{L} = \hat{n}_x\rangle \equiv [|L=1, L_z=-1\rangle - |L=1, L_z=1\rangle] / \sqrt{2}, \quad (14)$$

$$|\vec{L} = \hat{n}_y\rangle \equiv i [|L=1, L_z=-1\rangle + |L=1, L_z=1\rangle] / \sqrt{2}, \quad (15)$$

$$|\vec{L} = \hat{n}_z\rangle \equiv |L=1, L_z=0\rangle \quad (16)$$

Together these span the states of $L = 1$; they correspond to "linear polarization" states. Now take a hadron with $\vec{L} = \hat{n}_x$. The orbital part of the quark wavefunction is proportional to $Y_1^{-1}(\theta, \phi) - Y_1^1(\theta, \phi) \sim \sin\theta \cos\phi$, corresponding to quarks which tend to move in the x - z plane. When a hadron containing such quarks decays to two $L = 0$ hadrons, these quarks must end up travelling essentially along the z axis. The decay Hamiltonian must then give them a "kick" in the x - z plane. We seek a symmetry invariant under such a "kick", i.e., under boosts in the x - z plane, as compared with $SU(6)_W$ which is defined by invariance under boosts in z alone.

A subset of the generators (3) commutes with both α_z and α_x . It forms an algebra of $U(3) \otimes U(3)$ defined by the y -component of W -spin, W_y :

$$[U(3) \otimes U(3)]_{\beta\sigma_y} : \left\{ \begin{matrix} \lambda_i \\ \lambda_i \beta\sigma_y \end{matrix} \right\} \text{ commute with } \alpha_z, \alpha_x. \quad (17)$$

This is a coplanar $U(3) \otimes U(3)^{8,9}$ whose applicability to decays of L -excited hadrons has been noted previously.⁹ Our purpose is to carry out such applications, and to show that the relativistic quark model and 3P_0 picture possess this symmetry.

In Appendix A we shall classify hadrons according to multiplets of $[U(3) \otimes U(3)]_{\beta\sigma_y}$: for example,

$$\Delta(W_y = \frac{3}{2}) \in (10, 1). \quad (18)$$

The first and second numbers here denote the dimension of the representation dealing with W_y up and down, respectively. These classifications of course, are relevant also for $[U(3) \otimes U(3)]_{\beta\pi_x}$ when W is quantized along the x axis. Since

$$|L=1, L_z = \pm 1\rangle = [\mp |\vec{L} = \hat{n}_x\rangle - i |\vec{L} = \hat{n}_y\rangle] / \sqrt{2}, \quad (19)$$

one needs both of the above coplanar groups, but matrix elements associated with the two will turn out to be related by angular momentum conservation. One must transform states of definite S_z to those of definite S_y and S_x in order to apply the symmetry to physical processes. This involves matrices M :

$$|Sm\rangle_z = M_{mm'}^{(S; z_y)} |Sm'\rangle_y \quad (20a)$$

$$= M_{mm'}^{(S; z_x)} |Sm'\rangle_x \quad (20b)$$

which are derived in Appendix B. (In Eq. (20) the subscript denotes the quantization axis, while S denotes the total spin.) Appendix C deals with constraints due to angular momentum conservation.

III. DECAYS OF $q\bar{q}$, $L = 1$ MESONS

To begin with, we present the matrix elements for decays of the 2^+ , 1^+ , and 0^+ mesons (A) into 1^-0^- and 0^-0^- (B,C) pairs based on $SU(6)_W$ invariance. These matrix elements are taken to describe the decays of the $L = 1$ mesons from the $L_z = 0$ state.

Most of the transitions of interest describe the coupling $\underline{35} \rightarrow \underline{35} \otimes \underline{35}$. In order that the constraints of charge conjugation invariance be obeyed, this coupling must be symmetric. This will ensure³⁴ that the $SU(3)$ couplings of octets be pure F when the product of charge-parities of A, B, and C is -, and pure D when it is +.

Since particle C is always taken to be a member of the pseudoscalar octet, it will always belong to the $\underline{35}$ of $SU(6)_W$. On the other hand, both A and B can involve $SU(6)_W$ singlet states, e.g. in $f \rightarrow \pi\pi$ and $B \rightarrow \omega (= 0)\pi$. Hence, one would expect - a priori - three independent amplitudes, corresponding to $\underline{35} \rightarrow \underline{35} \otimes \underline{35}$, $\underline{1} \rightarrow \underline{35} \otimes \underline{35}$, and $\underline{35} \rightarrow \underline{1} \otimes \underline{35}$. The second and third are related to the first if we make the quark model assumptions $f \rightarrow \pi\pi$ and $B \rightarrow \phi\pi$, rules which are certainly well in agreement with present data. As our main concern in what follows is the spin structure of decay amplitudes, we shall concentrate on a decay which is pure $\underline{8} \rightarrow \underline{8} \otimes \underline{8}$ in $SU(3)$. The most convenient such decay is

$$"K" \rightarrow "K" \pi, \quad (21)$$

where the particle symbols in quotes denote the appropriate octet member. One can then obtain any matrix element involving pure octets in terms of those in Eq. (21). Some of the more useful are summarized in Table Ia.

With the additional assumptions $f' \rightarrow \pi\pi$, $B \rightarrow \phi\pi$ one can obtain as well decay amplitudes involving octet-singlet mixtures, some of which are compared to those of Eq. (21) in Table Ib.

The helicity amplitudes for $"K" \rightarrow "K"\pi$ are now taken from Eqs. (12) and (13), and summarized in the $a^{(0)}$ column of Table II. The W-spin assignments used are based on using $W^- = W_q^- + W_{\bar{q}}^- = S_q^- - S_{\bar{q}}^-$ to construct multiplets, so that

$$\begin{aligned} |11\rangle_S &= |11\rangle_W \\ |10\rangle_S &= -|00\rangle_W \quad \text{and} \quad |00\rangle_S = -|10\rangle_W \\ |1-1\rangle_S &= |1-1\rangle_W \end{aligned} \quad (22)$$

The partial widths are related to the values of m_λ by

$$\Gamma(A \rightarrow BC) = \frac{1}{2J_A + 1} \frac{p}{M_A^2} \sum_\lambda |m_\lambda|^2, \quad (23)$$

where p is the magnitude of the final CM 3-momentum.

At this point we remind the reader that the only experimental reason for doubting the predictions of $SU(6)_W$ comes from helicity distributions in $1^{++} \rightarrow 1^- 0^-$ decays (notably $B \rightarrow \omega\pi^2$). These data would not justify in themselves an attempt to "repair" $SU(6)_W$ at their present level of accuracy. Reliable studies of the contamination due to nonresonant effects in $\pi^\pm p \rightarrow \pi^\pm \omega p$ would be very helpful in clearing up this question.

Our prime interest here is a theoretical one: we have no compelling reason to restrict L_z to be zero. Hence, we should be surprised if all the

predictions of $SU(6)_W$ held true. As we shall see, some of them are preserved even when $L_z \neq 0$ decays are admitted.

The initial ($L = 1$) state of definite J, J_z may be written as

$$|JJ_z\rangle = \sum_{L_z} (SS_z | L_z | JJ_z) |SS_z\rangle |L_z\rangle \quad (24)$$

or, using Eqs. (14)-(16), as

$$\begin{aligned} |JJ_z\rangle = & |\vec{L} = \hat{n}_x\rangle \left\{ (S J_z + 1 \ 1 - 1 | JJ_z) |S J_z + 1\rangle + \right. \\ & \left. - (S J_z - 1 \ 1 - 1 | JJ_z) |S J_z - 1\rangle \right\} / \sqrt{2} + \\ & + |\vec{L} = \hat{n}_y\rangle \left\{ (S J_z + 1 \ 1 - 1 | JJ_z) |S J_z + 1\rangle + \right. \\ & \left. + (S J_z - 1 \ 1 - 1 | JJ_z) |S J_z - 1\rangle \right\} / i\sqrt{2} + \\ & + |\vec{L} = \hat{n}_z\rangle (S J_z \ 1 \ 0 | JJ_z) |S J_z\rangle. \end{aligned} \quad (25)$$

We have already calculated the decay from the $|\vec{L} = \hat{n}_z\rangle$ ($L_z = 0$) substate. Now it remains to treat the decays from $|\vec{L} = \hat{n}_{x,y}\rangle$ using $[U(3) \otimes U(3)]_{B\sigma_{y,x}}$, respectively, adding the resulting matrix elements in such a way as to ensure conservation of total angular momentum. Rather than presenting all results at every intermediate stage, we prefer to limit the discussion to some examples of how constraints on various

amplitudes arise. This discussion is presented in Appendix C. The net result is that there are two independent amplitudes describing $L_z \neq 0$ decays into $K^*\pi$ and $K\pi$, which we shall call $a^{(3)}$ and $a^{(8)}$, for reasons stated in Appendix C. Table II gives the amplitudes for " K " $\rightarrow$ " K " π in terms of these, as well as the $SU(6)_W$ -invariant amplitude $a^{(0)}$. The reader is reminded that other physical amplitudes are to be constructed using Table I with isospin Clebsch-Gordan coefficients for specific charge states.

As Table II gives seven amplitudes in terms of three parameters, it involves four relations among these amplitudes. Three of these allow direct comparison of partial widths. They are, for " K " $\rightarrow$ " K " π

$$m_1(2^{++} \rightarrow 1^-0^-) = (\sqrt{3}/2) m_0(2^{++} \rightarrow 0^-0^-) \quad (26a)$$

$$m_0(1^{++} \rightarrow 1^-0^-) = \sqrt{2} m_1(1^{++} \rightarrow 1^-0^-) \quad (26b)$$

and

$$\begin{aligned} \frac{1}{2} |m_0(2^{++} \rightarrow 0^-0^-)|^2 + 2 |m_1(1^{++} \rightarrow 1^-0^-)|^2 &= \\ &= |m_0(0^{++} \rightarrow 0^-0^-)|^2 + |m_0(1^{++} \rightarrow 1^-0^-)|^2 \end{aligned} \quad (26c)$$

A fourth relation can be tested via a triangle inequality: it can be written, for example,

$$m_1(2^{++} \rightarrow 1^-0^-) + m_1(1^{++} \rightarrow 1^-0^-) = \sqrt{2} m_0(1^{++} \rightarrow 1^-0^-). \quad (26d)$$

Let us discuss these relations briefly.

Relation (26) is one of the better $SU(6)_W$ predictions that we see continues to hold when coplanar symmetry is involved. It relates all the pure D-wave decays to one another, and has been shown to agree with experiment when a D-wave centrifugal barrier is used.²⁵

Relation (26b) involves $SU(6)_W$ -violating decays. It predicts, for example,

$$\frac{\tilde{\Gamma} [A_1 \rightarrow \rho(\lambda=0) \pi]}{\sum_{\lambda=\pm 1} \left\{ \tilde{\Gamma} [B \rightarrow \omega(\lambda) \pi] \right\}} = 2 \quad (27)$$

when one takes account of Table II. The quantities $\tilde{\Gamma}$ are partial widths uncorrected for any symmetry-breaking due to masses, and without any phase space factors. As the amplitudes $\mathcal{M}_0(1^{++} \rightarrow 1^{-}0^{-})$ and $\mathcal{M}_1(1^{+-} \rightarrow 1^{-}0^{-})$ contain both S and D wave contributions, an unambiguous prescription for dealing with centrifugal barrier effects is hard to find. Similar objections apply to the direct comparison of Eqs. (26c) and (26d) with experiment. On the other hand, there is a parametrization of the symmetry presented here for which comparison with experiment becomes notably easier. Let us decompose the final $1^{-}0^{-}$ states in 1^{+} decays into partial wave amplitudes $a^{(\ell)}$: $\mathcal{M}_\lambda = \sum_{\ell} N_\ell (\ell 0 | \lambda | 1 \lambda) a^{(\ell)}$, where N_ℓ are normalization factors chosen at convenience. Specifically,

$$\begin{aligned} \mathcal{M}_1(1^{++} \rightarrow 1^{-}0^{-}) &= S_A + D_A/2 \\ \mathcal{M}_0(1^{++} \rightarrow 1^{-}0^{-}) &= S_A - D_A \end{aligned} \quad (28)$$

$$\begin{aligned} \mathcal{M}_1(1^{+-} \rightarrow 1^{-}0^{-}) &= (S_B - D_B)/\sqrt{2} \\ \mathcal{M}_0(1^{+-} \rightarrow 1^{-}0^{-}) &= (S_B + 2D_B)/\sqrt{2} \end{aligned} \quad (29)$$

(These represent S wave and D wave contributions.) Eqs. (28) and (29) are completely general. We have chosen the normalizations in such a way that in the $SU(6)_W$ limit, $S_A = D_A = S_B = D_B$.

Now, Eq. (26b) implies that

$$S_A - D_A = S_B - D_B \quad (30)$$

Eq. (26d) implies that

$$\mathcal{M}_1(2^{++} \rightarrow 1^{-}0^{-}) = S_B + 2D_B - (S_A + D_A/2) \quad (31)$$

Together Eqs. (30) and (31) imply that

$$\mathcal{M}_1(2^{++} \rightarrow 1^{-}0^{-}) = 3(D_B - D_A/2), \quad (32)$$

whereby this amplitude is expressed (as it should be) in terms of purely D wave amplitudes. Similarly one can write an expression for $\mathcal{M}_0(2^{++} \rightarrow 0^{-}0^{-})$ expressed in terms of D waves:

$$\mathcal{M}_0(2^{++} \rightarrow 0^{-}0^{-}) = 2\sqrt{3} (D_B - D_A/2) \quad (33)$$

and an expression for $M_0(0^{++} \rightarrow 0^{-}0^{-})$ expressed in terms of S waves:

$$M_0(0^{++} \rightarrow 0^{-}0^{-}) = \sqrt{3/2} (S_B - 2S_A) . \quad (34)$$

Eqs. (28)-(34) are equivalent to Table II. One may then describe the symmetry breaking in a phenomenological way by setting

$$\begin{aligned} S_A &= a_A + b_A (p/p_0)^2 \\ S_B &= a_B + b_B (p/p_0)^2 \\ D_A &= d_A (p/p_0)^2 \\ D_B &= d_B (p/p_0)^2 , \end{aligned} \quad (35)$$

where p_0 is a suitably chosen scale factor which we find convenient to take equal to 0.5 GeV. Eq. (30) then implies

$$a_A = a_B = a \quad (36)$$

and

$$b_A - b_B = d_A - d_B \quad (37)$$

so that four free parameters are involved. In $SU(6)_W$ $a_A = a_B = 0$ and $b_A = b_B = d_A = d_B$.

Present data are inadequate to fix the parameters of Eq. (35) satisfactorily. A particular case seems to fit what data are available adequately. This is the case $b_A = b_B = 0$, for which the quality of the resulting fit has already been discussed in Ref. 2.

It is worth noting the types of improvement in data that would enable one to check the full range of possibilities suggested by Eqs. (35)-(37).

(a) $0^{++} \rightarrow 0^-0^-$ decays [Eq. (34)].

One must ask whether the 0^+ mesons form a true nonet. This requires, in particular, determination of the $\pi\pi/K\bar{K}$ coupling ratios of the $I = Y = 0$ members. Otherwise, one can deal only with the π -like member (" δ "?) and the K-like member (seen at various masses in different $K\pi$ phase shift analyses). Then, assuming SU(3) to hold in relating these two, one could solve for both the constant and the momentum-dependent term given sufficiently accurate partial widths. This is not yet possible.

(b) $1^{+-} \rightarrow 1^-0^-$ decays [Eq. (29)].

At present, the decay $B \rightarrow \omega\pi$ is the only candidate. The mass and width of the B vary somewhat from experiment to experiment.²³ The other nonet members are either missing ($I = Y = 0$ ones) or mix with the 1^{++} nonet ($|S| \neq 0$ ones). Agreement on the exact values of $|m_\lambda(B \rightarrow \omega\pi)|$ (and their relative phase, if possible) then becomes of prime importance.

(c) $1^{++} \rightarrow 1^-0^-$ decays [Eq. (28)].

Here the situation is similar to that just mentioned, with the decay $A_1 \rightarrow \rho\pi$ the only candidate. The added problems of a large nonresonant Deck effect in $\pi p \rightarrow A_1 p$ and a possibly substantial $\sigma\pi$ mode make the A_1 parameters difficult to determine. It is hoped that experiments

on backward production of A_1 will clarify the situation. A Dalitz plot analysis to determine m_1/m_0 is of particular importance in such cases.

(d) 2^{++} decays [Eqs. (32), (33)].

Improvements possible here include such partial widths as

$$K^{*+} \rightarrow \rho K, \quad f' \rightarrow \bar{K}^* K + K^* \bar{K}, \quad \text{and} \quad f' \rightarrow K \bar{K}.$$

By and large, however, properties of these mesons are much better known than those of the 1^+ and 0^+ mesons at present.

(e) Interference between 2^+ and 0^+ amplitudes.

It has been suggested³⁶ that the phase of any S-wave $\pi\pi$ resonance under the f_0 would provide a test of the $\Delta L_z = 0$ assumption. Let us assume, with Ref. 36, that there existed an S wave $\pi\pi$ resonance exactly under the f_0 and with identical nonet properties. Then, in the narrow-width approximation, the expected $\pi\pi$ angular distribution would take the form

$$W(\cos \theta) \sim \left[\cos^2 \theta - \frac{4\sqrt{2}}{3} \frac{a^{(8)}}{a^{(0)}} \sin^2 \theta \right]^2 \quad (38)$$

For $a^{(8)}/a^{(0)} = 0$, this would give an angular distribution vanishing at $\theta = 90^\circ$, in accord with observation.³⁷ To determine if the nonet structure of the S wave resonance claimed in Ref. 37 is really the same as that of the f_0 , one should perform similar analyses on $K^+ K^-$ and $K_1^- K_1$ final states, with truly adequate statistics.

It is interesting to see how Table II relates to other approaches intermediate between $SU(6)_W$ and $SU(3)$. To do this, we may consider a relation derived in Ref. 2, which is also true in the relativistic quark model:³

$$2(m_1/m_0)_{A_1 \rightarrow \rho\pi} = (m_0/m_1)_{B \rightarrow \omega\pi} + 1 \quad (39a)$$

This expression relates the helicity distribution in $A_1 \rightarrow \rho\pi$ to that in $B \rightarrow \omega\pi$. Instead of (38), Table II implies

$$2(m_1/m_0)_{A_1 \rightarrow \rho\pi} = (m_0/m_1)_{B \rightarrow \omega\pi} + a^{(8)}/a^{(3)} \quad (39b)$$

The ratio $a^{(8)}/a^{(3)}$ is thus an extra degree of freedom as compared with the model giving Eq. (39a)² (the " 3P_0 model"^{2,5-7}). In the 3P_0 model there is only one amplitude describing decays from the $L_z \neq 0$ substate. It corresponds to the additional 3P_0 $q\bar{q}$ pair being produced with $L_z \neq 0$.

To interpret the extra degree of freedom of the coplanar symmetry we return to the graphical language of Ref. 2. There, the $L = 1$ mesons were described by 4×4 Dirac matrices with a Lorentz index μ for orbital angular momentum. This Lorentz index could be saturated in three distinct ways to form a Lorentz-invariant coupling as described in Fig. 1:

- (a) Contraction with a final momentum: coupling c_0 .
- (b) Contraction with γ^μ on the final $q\bar{q}$ pair: coupling c_1 .
- (c) Contraction with γ^μ one of the initial quark lines (spin-orbit effect): coupling c_2 .

Table III contains helicity amplitudes calculated in terms of c_0 , c_1 , and c_2 . By comparing it with Table II, one finds the two descriptions are equivalent to one another with

$$\begin{bmatrix} a^{(3)} \\ a^{(8)} \\ a^{(0)} \end{bmatrix} = \begin{bmatrix} A_1^{(3)} & 0 & 0 \\ A_1^{(3)} & A_2^{(8)} & 0 \\ A_1^{(0)} & A_2^{(0)} & A_3^{(0)} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_0 \end{bmatrix}.$$

(40)

(The exact values of the $A_j^{(i)}$ do not matter here). Thus, $a^{(i)}$ and c_j are related so that when $c_2 = 0$, $a^{(3)} = a^{(8)}$, and conversely. The results of the 3P_0 model then follow from coplanar symmetry when spin-orbit effects are neglected. When both c_2 and c_1 vanish, $a^{(3)}$ and $a^{(8)}$ vanish, leaving the $SU(6)_W$ conserving term $a^{(0)}$.

IV. DISCUSSION

Our experience with the mesons points to certain $SU(6)_W$ predictions that will continue to hold in the 3P_0 model and under (even less restrictive) coplanar symmetry. Consider hadrons with a certain L, S such that $J = L + S$, and let them decay to states for which only one partial wave is allowed. Examples are:

$$\underline{8}(2^+) \rightarrow 1^- 0^-, 0^- 0^- \quad (L=S=1; \text{ D wave }) \quad (41)$$

$$\underline{8}(5/2^-) \rightarrow \frac{1}{2}^+ 0^- \quad (L=1, S=\frac{3}{2}; \text{ D wave }) \quad (42)$$

$$\underline{8}(5/2^+) \rightarrow \frac{1}{2}^+ 0^- \quad (L=2, S=\frac{1}{2}; \text{ F wave }) \quad (43)$$

and the Regge recurrences of these. $SU(6)_W$ predicts Eqs. (8)-(10) and relations (9), (10) for Regge recurrences as well. First, we relax $SU(6)_W$ to the 3P_0 model, allowing for $L_{\pm} \neq 0$ decays. Since the final state contains only one ℓ ($\ell = L + 1$ here), it turns out that this relaxation does not affect anything. This is because it has been shown⁷ that the 3P_0 model preserves $SU(6)_W$ relations among processes with the same final ℓ .

Next, we relax the 3P_0 model to allow for spin-orbit effects. A unit of L gets "absorbed" by a quark as in Fig. 1(c). We can see that this is impossible for the sequences $A_2(J=2), A_2^*(J=4), \dots$, and $\underline{8}(5/2^-, 9/2^-, \dots)$, since all the quark spins and units of L are coupled up to the maximum possible J . The absorption of a unit of L by a quark is impossible since each unit of L and each quark form a state of $J = 3/2$.

Hence, we expect the predictions (8), (9), as well as

$$(f/d)_{9/2^-, L=3} = (f/d)_{13/2^-, L=5} = \dots = -1/3 \quad (44)$$

to remain valid when spin-orbit effects of the type shown in Fig. 1(c) are admitted. This has already been shown for Eq. (8a). Furthermore, explicit calculation using results of the Appendices allows one to conclude that coplanar symmetry implies Eq. (9) as well. This happens for a rather pretty reason. By virtue of angular momentum Clebsch-Gordan coefficients, one finds that an octet member with $L = 1$, $J^P = 5/2^-$, helicity $\pm 1/2$ belongs only to $(\underline{8}, \underline{1})$ or $(\underline{1}, \underline{8})$. The decay of such a particle to a $1/2^+$, $L = 0$ baryon and a 0^- , $L = 0$ meson, belonging respectively to $(\underline{6}, \underline{3})$ or $(\underline{3}, \underline{6})$ and $(\underline{3}, \underline{\bar{3}})$ or $(\underline{\bar{3}}, \underline{3})$, is thus characterized by a single independent amplitude and hence by a unique f/d . This, of course, must be the f/d of $SU(6)_W$, and we have verified that such is indeed the case.

On the other hand, for $L \geq 3$, we cannot justify the use of coplanar symmetry at present, and hence, have no way of proving Eqs. (8b) or (44) except by direct reference to the absence of graphs such as Fig. 1(c), as mentioned above. When $L \leq 2$, a fortunate circumstance allows one to prove Eq. (10). Since the decay of a quark-spin $1/2$ resonance with $L_z = 2$ to $1/2^+ 0^-$ is forbidden (remember the z -axis is the decay axis, so the final J_z must be $\pm 1/2$), decays such as $N(5/2^+; 1688) \rightarrow N\pi$ involve initial $L_z \leq 1$. As usual, one can treat the $L_z = 0$ substate via $SU(6)_W$. The states with $L_z = \pm 1$ can be written

$$\begin{aligned} |L=2, L_z = \pm 1\rangle &= \frac{1}{\sqrt{2}} \left[|L=1, L_z = \pm 1\rangle \otimes |L=1, L_z = 0\rangle + \right. \\ &\quad \left. + |L=1, L_z = 0\rangle \otimes |L=1, L_z = \pm 1\rangle \right], \end{aligned} \quad (45)$$

or, symbolically, recalling Eqs. (14)-(16), as

$$|L=2, L_z=\pm 1\rangle = \frac{1}{2} \left[(\mp \hat{n}_x - i \hat{n}_y) \hat{n}_z + \hat{n}_z (\mp \hat{n}_x - i \hat{n}_y) \right]. \quad (46)$$

Hence these states are linear superpositions of ones involving only one direction besides the z direction. We then expect coplanar symmetry to be applicable. Since only the transition $[(6,3) \text{ or } (3,6)] \rightarrow [(6,3) \text{ or } (3,6)] \otimes [(3,3) \text{ or } (3,3)]$ is involved, a unique f/d (that of Eq. (10)) results.

There are predictions of f/d ratios made by $SU(6)_W$ and the 3P_0 model but not by coplanar symmetry: notably, that all $70, L=1 \rightarrow 56 \otimes 35$ decays should have

$$f/d = 5/3 \quad \text{for} \quad \underline{8} (S_q = 1/2) \rightarrow \underline{8} \otimes \underline{8} \quad (47)$$

$$f/d = -1/3 \quad \text{for} \quad \underline{8} (S_q = 3/2) \rightarrow \underline{8} \otimes \underline{8} \quad (48)$$

The reason unique f/d values do not result in coplanar symmetry is that-- except for the states with $J^P = 5/2^-$, mentioned above -- all the

$J_z = \pm 1/2$ states are mixtures of $(8,1), (6,3), (\bar{3},3) + (W_y \rightarrow -W_y)$.

There are thus, in principle, three independent $|L_z| = 1$ amplitudes

describing decays of such resonances: $M_{1,33}^{8,63}$, $M_{3,63}^{6,33}$,

and $M_{3,63}^{\bar{3},33}$. (See Appendix C for notation). There is another

$|L_z| = 1$ amplitude describing decays to $10 \otimes 8$, namely $M_{3,13}^{6,10\bar{3}}$.

This means four amplitudes describe $|L_z| = 1$ decays and one (the $SU(6)_W$

invariant one) describes $L_z = 0$ decays of the $70, L=1$ into $56 \otimes 35$. This

total of five amplitudes makes for trouble in comparison with experiment! ³⁸

Experimentally there does seem to be some evidence for the existence of a $J^P = 3/2^-$ baryon octet, containing the $N(1520)$, with f/d close to the value of $5/3$ that would be predicted by $SU(6)_W$, the 3P_0 model, or the relativistic quark model for a pure $S_q = 1/2$ state.²⁷⁻²⁹ Speaking optimistically, this may mean one or more of these more specialized schemes is correct. Speaking pessimistically, one is disturbed by the lack of clear separation of the $\Sigma(1660)$ states, two of which seem to be $3/2^-$, the arbitrariness of mixing of the Λ state with the unitary singlet, and the uncertain status of the $\Xi(1820)$ as a $3/2^-$ resonance.

It is amusing that the prediction $f/d = -1/3$ for the $5/2^-$ octet, which is stronger than the other $SU(6)_W$ predictions of f/d ratios as just mentioned, is also the most reliable such prediction in various studies based on duality.³⁹

At present, coplanar symmetry does not seem to say very much regarding photoproduction. It forbids

$$\gamma p \rightarrow N^{*+} [1670, 5/2^-; \lambda = \pm 1/2] \quad (49)$$

but does not seem to say anything regarding excitation of the $\lambda = \pm 3/2$ state. Other selection rules of the relativistic quark model³ do not seem to follow readily. This suggests that-especially for photoproduction--the relativistic quark model may embody some very desirable dynamical features which our weak symmetry arguments are unable to provide.

We have steered clear of $SU(6)_W$ ⁴⁰ and coplanar $U(3) \otimes U(3)$ ⁴¹ for $2 \rightarrow 2$ reactions, in view of their more questionable applicability in such cases.⁴² Nonetheless, if one applied coplanar symmetry only to $K^\pm N$ and πN reactions,

one might expect relations of roughly the same quality as the Johnson-Treiman relations. We should note, moreover, that the apparent violation of $SU(6)_W$ in comparing (6) with experiment is probably as bad as that of the Johnson-Treiman relations. Thus, without a more sophisticated understanding of "kinematic" symmetry breaking effects, $SU(6)_W$ for $L = 0$ decays and consequently coplanar symmetry for $L = 1$ decays may be no more accurate than $\pm 50\%$, and their violation may indeed spring from the same unknown mechanism as the violation of the Johnson-Treiman relations. The violation of (6) enhances the contribution of the Δ pole over the N pole in $\pi N \rightarrow \pi N$. This turns out, by duality, to push the D/F ratio of the crossed channel nonflip coupling of the tensor and vector trajectories to octet baryons from 0 to a negative value.⁴³ This is indeed the direction required by experimental data.^{5,43,44} It is notable that some saturation schemes for superconvergence relations⁴⁵ indeed violate $SU(6)_W$ in places where ΔL_z must be zero, e.g. in relations such as (7).

The physical circumstances under which the various symmetries presented here are likely to be valid are summarized in Figure 2. We believe the results of coplanar symmetry to be as valid as the particle classification itself. The most likely source of deviation from such relations as Eq. (6) and Eqs. (28)-(34) would thus be mixing of multiplets: e.g., $\underline{56}$, $L = 0$ mixing with $\underline{56}$, $L = 2$.⁴⁶ At present no evidence has appeared for spin-orbit effects,^{2,3,7} so that even the 3P_0 model may be a good approximation to decays. In decays of 2^+ , 1^+ , and 0^+ mesons to $1^- 0^-$ or $0^- 0^-$, and in baryonic decays to $\frac{1}{2}^+ 0^-$ and $\frac{3}{2}^+ 0^-$, the 3P_0 model involves two independent partial wave amplitudes.^{2,7} These in turn are related to one another in more restrictive symmetries: apparently incorrectly in $SU(6)_W$

and correctly in the relativistic quark model. This model thus seems to contain a couple of non-trivial dynamical statements. We have shown that some but by no means all of its relations follow from symmetry arguments alone.

ACKNOWLEDGMENTS

I wish to thank Peter Freund and Jeffrey Mandula for encouragement in relating the 3P_0 model to a relativistic symmetry. Professor George Zweig extended a much appreciated invitation to visit Caltech, where this work was begun. I am grateful to my colleagues there and at Minnesota for many helpful discussions.

APPENDIX A. CLASSIFICATION OF HADRONS IN MULTIPLTS OF $[U(3) \otimes U(3)]_{\beta \sigma_y}$

1. Quarks

The projection operators

$$\Sigma_y^{\pm} \equiv \frac{1 \pm \beta \sigma_y}{2} \quad (A.1)$$

have the following effect on quarks and antiquarks:

$$\Sigma_y^{\pm} \left\{ \begin{matrix} q \\ \bar{q} \end{matrix} \right\} (S_y = \pm \frac{1}{2}) = + \left\{ \begin{matrix} q \\ \bar{q} \end{matrix} \right\} (S_y = \pm \frac{1}{2}), \quad (A.2)$$

$$\Sigma_y^{\mp} \left\{ \begin{matrix} q \\ \bar{q} \end{matrix} \right\} (S_y = \pm \frac{1}{2}) = 0. \quad (A.3)$$

Then

$$\begin{aligned} q, S_y = +\frac{1}{2} & \in (\underline{3}, \underline{1})_y \\ \bar{q}, S_y = +\frac{1}{2} & \in (\bar{\underline{3}}, \underline{1})_y \\ q, S_y = -\frac{1}{2} & \in (\underline{1}, \underline{3})_y \\ \bar{q}, S_y = -\frac{1}{2} & \in (\underline{1}, \bar{\underline{3}})_y \end{aligned} \quad (A.4)$$

Specifically, we shall denote (say) a u quark⁴⁷ with $S_y = \frac{1}{2}$ by the symbol $(u, -)$, where the dash indicates a unitary singlet.

2. Mesons

The states of $q\bar{q}$ are 3S_1 ($S_y = 1, 0, -1$) $\equiv V$ and $^1S_0 \equiv P$. In terms of quark spin, one may write:

$$\begin{aligned}
 V(S_y = 1) &= q(S_y = 1/2) \bar{q}(S_y = 1/2) \\
 V(S_y = 0) &= [q(S_y = 1/2) \bar{q}(S_y = -1/2) + q(S_y = -1/2) \bar{q}(S_y = 1/2)]/\sqrt{2} \\
 V(S_y = -1) &= q(S_y = -1/2) \bar{q}(S_y = -1/2)
 \end{aligned} \tag{A.5}$$

and

$$P = [q(S_y = 1/2) \bar{q}(S_y = -1/2) - q(S_y = -1/2) \bar{q}(S_y = 1/2)]/\sqrt{2} \tag{A.6}$$

These then belong to the following multiplets of $[U(3) \otimes U(3)]_{\beta\sigma_y}$:

$$\begin{aligned}
 V(S_y = 1) &\in (\underline{8}, \underline{1})_y \oplus (\underline{1}, \underline{1})_y \\
 V(S_y = 0) &\in [(\underline{3}, \underline{\bar{3}})_y + (\underline{\bar{3}}, \underline{3})_y]/\sqrt{2} \\
 V(S_y = -1) &\in (\underline{1}, \underline{8})_y \oplus (\underline{1}, \underline{1})_y
 \end{aligned} \tag{A.7}$$

and

$$P \in [(\underline{3}, \underline{\bar{3}})_y - (\underline{\bar{3}}, \underline{3})_y]/\sqrt{2} \tag{A.8}$$

The total multiplicity of V and P is that of $(\underline{8}, \underline{1}) \oplus (\underline{1}, \underline{1}) \oplus (\underline{3}, \underline{\bar{3}}) \oplus (\underline{\bar{3}}, \underline{3}) + (\underline{1}, \underline{8}) + (\underline{1}, \underline{1})$, i.e. 36. The corresponding $q\bar{q}$ multiplets in $SU(6)_W$ are an $SU(6) \downarrow$, containing the $S_z = 0$ vector meson unitary singlet and a $\underline{35}$, containing all the remaining particles.

The specific members of the multiplets (A.7) and (A.8) are easily written down. We shall adopt notation whereby an octet member is labelled by the corresponding pseudoscalar meson: e.g.,

$$\rho^+(S_y=1) = (\pi^+, -)_y, \quad (\text{A.9})$$

$$\rho^+(S_y=0) = [(u, \bar{d})_y + (\bar{d}, u)_y] / \sqrt{2} \quad (\text{A.10})$$

$$\rho^+(S_y=-1) = (-, \pi^+)_y, \quad (\text{A.11})$$

while

$$\pi^+(S_y=0) = [(u, \bar{d})_y - (\bar{d}, u)_y] / \sqrt{2} \quad (\text{A.12})$$

The multiplet assignments for octet-singlet mixtures are straightforward except in the case of $S_y = \pm 1$, for which

$$\omega(S_y=1) = \sqrt{1/3} (\eta, -)_y + \sqrt{2/3} (-, -)_y, \quad (\text{A.13})$$

$$\phi(S_y=1) = \sqrt{2/3} (\eta, -)_y - \sqrt{1/3} (-, -)_y. \quad (\text{A.14})$$

3. qq states

In building the baryons it will be helpful to define members of the $\underline{21}$ and $\underline{15}$ of $SU(6)$, which are formed by respective symmetric and anti-symmetric combinations of $\underline{6} \otimes \underline{6}$. Under $SU(3) \otimes SU(2)$, these multiplets reduce as follows:

$$\underline{21} = (\underline{6}, \underline{3}) \oplus (\underline{\bar{3}}, \underline{1}) \quad (A.15)$$

$$\underline{15} = (\underline{6}, \underline{1}) \oplus (\underline{\bar{3}}, \underline{3}) \quad (A.16)$$

The $SU(3)$ representations of non-zero triality indicated on the right-hand side occur often enough in our calculations that we find it helpful to define them in Figure 3.

The reduction of the multiplets under $[U(3) \otimes U(3)]_{\beta\sigma_y}$ is as follows:

$$(qq)_{\underline{21}}, S_y = 1 \in (\underline{6}, \underline{1})_y \quad (A.17)$$

$$(qq)_{\underline{21}}, S_y = 0 \in (\underline{3}, \underline{3})_{y, \underline{21}} \quad (A.18)$$

$$(qq)_{\underline{21}}, S_y = -1 \in (\underline{1}, \underline{6})_y \quad (A.19)$$

while

$$(qq)_{\underline{15}}, S_y = 1 \in (\underline{\bar{3}}, \underline{1})_y \quad (A.20)$$

$$(qq)_{\underline{15}}, S_y = 0 \in (\underline{3}, \underline{3})_{y, \underline{15}} \quad (A.21)$$

$$(qq)_{\underline{15}}, S_y = -1 \quad \in \quad (\underline{1}, \underline{\bar{3}})_y \quad (\text{A.22})$$

Assignments are unambiguous except for the $S_y = 0$ states. Here one has the following examples (see Fig. 3 for the state labels):

$$A(S=1, S_y=0) (\epsilon \underline{21}) = (p, p)_{y, \underline{21}} \quad (\text{A.23})$$

$$A(S=0, S_y=0) (\epsilon \underline{15}) = (p, p)_{y, \underline{15}} \quad (\text{A.24})$$

$$\chi(S=1, S_y=0) (\epsilon \underline{15}) = [(p, n)_{y, \underline{15}} - (n, p)_{y, \underline{15}}] / \sqrt{2} \quad (\text{A.25})$$

$$\chi(S=0, S_y=0) (\epsilon \underline{21}) = [(p, n)_{y, \underline{21}} - (n, p)_{y, \underline{21}}] / \sqrt{2} \quad (\text{A.26})$$

to show the ambiguity that arises if the origin of the $(\underline{3}, \underline{3})$ representation is not labelled. Such a (purely formal) labelling allows one to keep track of the symmetry or antisymmetry of the $SU(6)$ part of the quark wavefunction. One could imagine this to be important in a "realistic" quark calculation for which such symmetry or antisymmetry would determine permutation properties of the spatial part of the wavefunction.

4. Baryons as qqq states

The qqq baryons encompass the $SU(6)$ representations $\underline{56} \oplus \underline{70} = \underline{21} \oplus \underline{6}$ and $\underline{70} \oplus \underline{20} = \underline{15} \oplus \underline{6}$. Since under $[U(3) \otimes U(3)]_{\beta\sigma_y}$ one has

$$\underline{21} = (\underline{6}, \underline{1})_y \oplus (\underline{3}, \underline{3})_{y, \underline{21}} \oplus (\underline{1}, \underline{6})_y \quad (\text{A.27})$$

$$\underline{15} = (\underline{\bar{3}}, \underline{1})_y \oplus (\underline{3}, \underline{3})_{y, \underline{15}} \oplus (\underline{1}, \underline{\bar{3}})_y \quad (\text{A.28})$$

$$\underline{6} = (\underline{3}, \underline{1})_y \oplus (\underline{1}, \underline{3})_y, \quad (\text{A.29})$$

the following decompositions can be extracted:

$$\underline{56} = (\underline{10}, \underline{1})_y \oplus (\underline{6}, \underline{3})_{y, \underline{56}} \oplus (\underline{3}, \underline{6})_{y, \underline{56}} \oplus (\underline{1}, \underline{10})_y \quad (\text{A.30})$$

$$\underline{70} = (\underline{8}, \underline{1})_y \oplus (\underline{6}, \underline{3})_{y, \underline{70}} \oplus (\underline{\bar{3}}, \underline{3})_{y, \underline{70}} \oplus (\underline{3}, \underline{\bar{3}})_{y, \underline{70}} \oplus (\underline{3}, \underline{6})_{y, \underline{70}} + (\underline{1}, \underline{8})_y \quad (\text{A.31})$$

$$\underline{20} = (\underline{1}, \underline{1})_y \oplus (\underline{\bar{3}}, \underline{3})_{y, \underline{20}} \oplus (\underline{3}, \underline{\bar{3}})_{y, \underline{20}} \oplus (\underline{1}, \underline{1})_y \quad (\text{A.32})$$

Some ambiguities arise in identifying multiplets of definite quark spin.

For this reason we shall be more explicit in our construction.

The $S_y = \pm 3/2$ states are clearly identified in each of Eqs. (A.30)-(A.32) since the $S=3/2$ members of the $\underline{56}$, $\underline{70}$, $\underline{20}$ are respectively $\underline{10}$, $\underline{8}$ and $\underline{1}$ of $SU(3)$. The $S_y = \pm 1/2$ states, however, pose more of a problem.

Let us take $\{\Delta^{++}, S_y = 1/2\} = (A, u)$. Then acting with I_- we have $\{\Delta^+, S_y = 1/2\} = \sqrt{2/3} (B, u) + \sqrt{1/3} (A, d)$. The orthogonal state is a proton: $\{p, S_y = 1/2\} = \sqrt{2/3} (A, d) - \sqrt{1/3} (B, u)$. Then acting with I_- we construct the neutron:

$$\{n, S_y = 1/2\} = \sqrt{1/3} (B, d) - \sqrt{2/3} (C, u).$$

Note, however, that the members of the $(\underline{6}, \underline{3})$ representation of $[U(3) \otimes U(3)]_{B\sigma_y}$ we have just constructed have nothing to label them as having come from the $\underline{56}$ of $SU(6)_W$. A similar $(\underline{6}, \underline{3})$ comes from the $\underline{70}$.

For this $(\underline{6}, \underline{3})$, however, the decimet has $S = 1/2$ and the octet is an (a priori) arbitrary mixture of $S = 1/2$ and $S = 3/2$.

Our practical problem is this: given an assignment of quark spin S to a member of the $\underline{70}$ (no firm evidence for the $\underline{20}$ exists), what does this say about the $[U(3) \otimes U(3)]_{B\sigma_y}$ assignment? For example, suppose we take the $D_{13} (1520) 3/2^- \pi N$ resonance to have $S_q = 1/2$. Does the $S_y = 1/2$ state of this particle then fit into $(\underline{6}, \underline{3})_{\underline{70}}$ into $(\bar{\underline{3}}, \underline{3})_{\underline{70}}$, or into some mixture?

At present we suspect the answer to this question is non-trivial and would involve assumptions which are more model-dependent than those we choose to make. For the time being, therefore, we can only write

$$\{\underline{70} : (\underline{8}, \underline{4}), S_y = 1/2\} \in \cos \theta (\underline{6}, \underline{3})_{y, \underline{70}} + \sin \theta (\bar{\underline{3}}, \underline{3})_{y, \underline{70}} \quad (A.33)$$

$$\{\underline{70} : (\underline{8}, \underline{2}), S_y = 1/2\} \in -\sin \theta (\underline{6}, \underline{3})_{y, \underline{70}} + \cos \theta (\bar{\underline{3}}, \underline{3})_{y, \underline{70}} \quad (A.34)$$

APPENDIX B. TRANSFORMATION BETWEEN STATES OF DEFINITE S_z AND THOSE OF DEFINITE S_y, S_x .

The symbols $|S m\rangle_{x,y,z}$ will denote spin eigenfunctions when x, y , and z are the respective quantization axes. For definiteness in phases we shall build spin eigenfunctions from those with $S = \frac{1}{2}$. Define

$$|\frac{1}{2} \frac{1}{2}\rangle_y \equiv R_x(-\frac{\pi}{2}) |\frac{1}{2} \frac{1}{2}\rangle_z \quad (\text{B.1})$$

$$|\frac{1}{2} \frac{1}{2}\rangle_x \equiv R_y(\frac{\pi}{2}) |\frac{1}{2} \frac{1}{2}\rangle_z \quad (\text{B.2})$$

where $R_i(\theta)$ is a rotation by θ about the i axis. In terms of Pauli spinors this means that with

$$|\frac{1}{2} \frac{1}{2}\rangle_z = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (\text{B.3})$$

one has

$$|\frac{1}{2} \frac{1}{2}\rangle_y = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix} = \frac{1}{\sqrt{2}} \left[|\frac{1}{2} \frac{1}{2}\rangle_z + i |\frac{1}{2} -\frac{1}{2}\rangle_z \right] \quad (\text{B.4})$$

$$|\frac{1}{2} \frac{1}{2}\rangle_x = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \left[|\frac{1}{2} \frac{1}{2}\rangle_z + |\frac{1}{2} -\frac{1}{2}\rangle_z \right] \quad (\text{B.5})$$

We shall then define

$$|\frac{1}{2} -\frac{1}{2}\rangle_{x,y} \equiv S_{x,y}^{-1} |\frac{1}{2} \frac{1}{2}\rangle_{x,y} \quad (\text{B.6})$$

where

$$\begin{aligned} S_y^- &= S_z - iS_x \\ &= S_z - \frac{i}{2}(S_z^+ + S_z^-) \end{aligned} \quad (\text{B.7})$$

and

$$\begin{aligned} S_x^- &= S_y - iS_z \\ &= \frac{i}{2}(S_z^- - S_z^+) - iS_z \end{aligned} \quad (\text{B.8})$$

Applying these to (B.4) and (B.5), one has

$$|\frac{1}{2} - \frac{1}{2}\rangle_y = \frac{1}{\sqrt{2}} \left[|\frac{1}{2} \frac{1}{2}\rangle_z - i |\frac{1}{2} - \frac{1}{2}\rangle_z \right] \quad (\text{B.9})$$

and

$$|\frac{1}{2} - \frac{1}{2}\rangle_x = \frac{1}{\sqrt{2}} \left[-|\frac{1}{2} \frac{1}{2}\rangle_z + |\frac{1}{2} - \frac{1}{2}\rangle_z \right] \quad (\text{B.10})$$

The states with spin 1 are defined by setting

$$|11\rangle_{x,y,z} \equiv |\frac{1}{2} \frac{1}{2}\rangle_{x,y,z} \otimes |\frac{1}{2} \frac{1}{2}\rangle_{x,y,z} \quad (\text{B.11})$$

and then applying $S_{x,y,z}^-$ successively. Similarly we define

$$|\frac{3}{2} \frac{3}{2}\rangle_{x,y,z} \equiv |11\rangle_{x,y,z} \otimes |\frac{1}{2} \frac{1}{2}\rangle_{x,y,z} \quad (\text{B.12})$$

In proceeding from (B.11) or (B.12) we assume all phases are positive:

$$S_{x,y,z}^- |S m\rangle_{x,y,z} = + \sqrt{(S+m)(S-m+1)} |S m-1\rangle_{x,y,z} \quad (B.13)$$

We shall also define the spin-0 states in terms of spin $\frac{1}{2}$:

$$|00\rangle_{x,y,z} = \frac{1}{\sqrt{2}} \left[|\frac{1}{2}\frac{1}{2}\rangle \otimes |\frac{1}{2}-\frac{1}{2}\rangle - |\frac{1}{2}-\frac{1}{2}\rangle \otimes |\frac{1}{2}\frac{1}{2}\rangle \right]_{x,y,z} \quad (B.14)$$

Expressions for $|S m\rangle_z$ in terms of $|S m'\rangle_{x,y}$ are given in Tables IV, V, for $S = 0, \frac{1}{2}, 1$, and $3/2$. These tables are in the form of matrices (essentially Wigner D-functions, which we have thus calculated for specific cases):

$$|S m\rangle_z = M_{mm'}^{(S;zy)} |S m'\rangle_y \quad (B.15)$$

$$= M_{mm'}^{(S;zx)} |S m'\rangle_x \quad (B.16)$$

These transformations are unitary, so that

$$|S m'\rangle_y = M_{mm'}^{*(S;zy)} |S m\rangle_z \quad (B.17)$$

$$|S m'\rangle_x = M_{mm'}^{*(S;zx)} |S m\rangle_z \quad (B.18)$$

APPENDIX C. CONSTRAINTS DUE TO ANGULAR MOMENTUM CONSERVATION.

Consider an amplitude for the decay $(\alpha_1, \alpha_2) \rightarrow (\beta_1, \beta_2) \otimes (\gamma_1, \gamma_2)$. We shall label such an amplitude by $M_{\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2}^{x \text{ or } y}$, depending on whether the $U(3) \otimes U(3)$ generated by λ_i and $\lambda_i \beta \sigma_x$ or $\lambda_i \beta \sigma_y$ is being considered.

We are treating the decays $"K" \rightarrow "K" \pi$. In order to do this most simply we shall consider a particular charge state, $"K^+ \rightarrow "K^0 \pi^+$, and then multiply by $-\sqrt{3/2}$, the inverse of the isospin Clebsch-Gordan coefficient, to obtain an isoscalar factor. The isoscalar factors for other decays may then be gleaned from Table II.

In the decay $"K^+ \rightarrow "K^0 \pi^+$ from $L_z \neq 0$ substates the following amplitudes contribute: $M_{1,3\bar{3}}^{8,3\bar{3}}(x,y)$, $M_{8,3\bar{3}}^{1,3\bar{3}}(x,y)$, $M_{3,1\bar{3}}^{\bar{3},8\bar{3}}(x,y)$, and $M_{\bar{3},8\bar{3}}^{3,1\bar{3}}(x,y)$. By considering enough examples, we shall show that in fact, only two of these eight are independent.

Consider a 2^+ meson with $J_z = \pm 1$ (recall we are taking the z axis to be the direction of the first final particle, in this case the $"K^0"$). Then it is in the state

$$\begin{aligned} |J=2, J_z=\pm 1\rangle &= \frac{1}{\sqrt{2}} |S=1, S_z=0\rangle |L=1, L_z=\pm 1\rangle + \\ &+ \frac{1}{\sqrt{2}} |S=1, S_z=\pm 1\rangle |L=1, L_z=0\rangle \\ &= \pm \frac{i}{2\sqrt{2}} [|11\rangle_y - |1-1\rangle_y] |L=\hat{n}_x\rangle + \\ &- \frac{i}{2\sqrt{2}} [|11\rangle_x + |1-1\rangle_x] |L=\hat{n}_y\rangle + \\ &+ \frac{1}{\sqrt{2}} |1\pm 1\rangle_z |L=\hat{n}_z\rangle \end{aligned} \tag{C.1}$$

where the $\langle S_m \rangle_{x,y,z}$ on the right hand side refer to quark spin.

We have used Tables IV, V to quantize the quark spin along the appropriate axes for each $\vec{L}$ substate. Similarly a 1^- meson with $J_z = 1$ can be represented as

$$\begin{aligned} |J=1, J_z = \pm 1\rangle &= \frac{1}{2} |11\rangle_y + \frac{1}{\sqrt{2}} |10\rangle_y + \frac{1}{2} |1-1\rangle_y \\ &= \frac{1}{2} |11\rangle_x + \frac{i}{\sqrt{2}} |10\rangle_x - \frac{1}{2} |1-1\rangle_x \end{aligned} \quad (C.2)$$

and a 0^- meson as

$$|00\rangle_z = i |00\rangle_y = -i |00\rangle_x, \quad (C.3)$$

again using Tables IV, V.

Eqs. (C.1) - (C.3) may be translated into $[U(3) \otimes U(3)]_{B\sigma_{x,y}}$ representations as follows, using Appendix A:

$$\begin{aligned} | "K^+", J=2, J_z = \pm 1, \vec{L} = \hat{n}_x \rangle &= \\ &= \pm \frac{i}{2\sqrt{2}} [(K^+, -)_y - (-, K^+)_y] \end{aligned} \quad (C.4)$$

$$\begin{aligned} | "K^+", J=2, J_z = \pm 1, \vec{L} = \hat{n}_y \rangle &= \\ &= -\frac{i}{2\sqrt{2}} [(K^+, -)_x + (-, K^+)_x] \end{aligned} \quad (C.5)$$

$$| "K^0", J=1, J_z=1 \rangle =$$

$$= \frac{1}{2} (K^0, -)_y + \frac{1}{2} (d, \bar{s})_y + \frac{1}{2} (\bar{s}, d)_y + \frac{1}{2} (-, K^0)_y \quad (C.6)$$

$$= \frac{1}{2} (K^0, -)_x + \frac{i}{2} (d, \bar{s})_x + \frac{i}{2} (\bar{s}, d)_x - \frac{1}{2} (-, K^0)_x \quad (C.7)$$

$$|\pi^+\rangle = \frac{i}{\sqrt{2}} (u, \bar{d})_y - \frac{i}{\sqrt{2}} (\bar{d}, u)_y \quad (C.8)$$

$$= -\frac{i}{\sqrt{2}} (u, \bar{d})_x + \frac{i}{\sqrt{2}} (\bar{d}, u)_x \quad (C.9)$$

$$|K^0\rangle = \frac{i}{\sqrt{2}} (d, \bar{s})_y - \frac{i}{\sqrt{2}} (\bar{s}, d)_y \quad (C.10)$$

$$= -\frac{i}{\sqrt{2}} (d, \bar{s})_x + \frac{i}{\sqrt{2}} (\bar{s}, d)_x \quad (C.11)$$

In calculating the overlap of such states we need the following Clebsch-Gordan coefficients of SU(3):

$$\langle d\bar{d} | - \rangle = 1/\sqrt{3} \quad (C.12)$$

$$\langle \bar{s}u | K^+ \rangle = 1 \quad (C.13)$$

and elsewhere we will also need

$$\langle K^0 \bar{d} | \bar{s} \rangle = 1/\sqrt{3} \quad (C.14)$$

To obtain constraints we simply calculate some decays forbidden by J^P conservation and demand that they be forbidden. For example,

$$\begin{aligned} \mathcal{M} ["K^+" (J=2, J_z = \pm 1) \rightarrow K^0 \pi^+] &= 0 \\ &= \frac{i}{4\sqrt{6}} \left\{ \pm [\mathcal{M}_{1, \bar{3}\bar{3}}^{8, \bar{3}\bar{3}}(y) - \mathcal{M}_{8, \bar{3}\bar{3}}^{1, \bar{3}\bar{3}}(y)] + \right. \\ &\quad \left. - [\mathcal{M}_{1, \bar{3}\bar{3}}^{8, \bar{3}\bar{3}}(x) + \mathcal{M}_{8, \bar{3}\bar{3}}^{1, \bar{3}\bar{3}}(x)] \right\} \end{aligned} \quad (C.15)$$

which requires

$$\begin{aligned} \mathcal{M}_{1, \bar{3}\bar{3}}^{8, \bar{3}\bar{3}}(y) &= \mathcal{M}_{8, \bar{3}\bar{3}}^{1, \bar{3}\bar{3}}(y) \\ \text{and} \quad \mathcal{M}_{1, \bar{3}\bar{3}}^{8, \bar{3}\bar{3}}(x) &= -\mathcal{M}_{8, \bar{3}\bar{3}}^{1, \bar{3}\bar{3}}(x) \end{aligned} \quad (C.16)$$

Moreover with

$$\begin{aligned} \mathcal{M} ["K^+" (J=2, J_z = -1) \rightarrow K^{*0} (J_z = 1) \pi^+] &= 0 \\ &= \frac{1}{8\sqrt{3}} \left\{ - [\mathcal{M}_{1, \bar{3}\bar{3}}^{8, \bar{3}\bar{3}}(y) + \mathcal{M}_{8, \bar{3}\bar{3}}^{1, \bar{3}\bar{3}}(y)] \right. \\ &\quad \left. - i [\mathcal{M}_{1, \bar{3}\bar{3}}^{8, \bar{3}\bar{3}}(x) - \mathcal{M}_{8, \bar{3}\bar{3}}^{1, \bar{3}\bar{3}}(x)] \right\} \end{aligned} \quad (C.17)$$

we now have [using (C.16)]

$$\mathcal{M}_{1, \bar{3}\bar{3}}^{8, \bar{3}\bar{3}}(x) = i \mathcal{M}_{1, \bar{3}\bar{3}}^{8, \bar{3}\bar{3}}(y) \quad (C.18)$$

Hence, with (C.16), this relates four amplitudes to one another. Similar relations may be shown to hold among four other amplitudes one of which is $m_{3,13}^{\bar{3},8\bar{3}}(y)$. We shall define

$$a^{(8)} \equiv m_{1,3\bar{3}}^{8,\bar{3}\bar{3}}(y)$$

$$a^{(3)} \equiv m_{3,13}^{\bar{3},8\bar{3}}(y) .$$

(C.19)

All the $L_z \neq 0$ decays of "K" into "K" π are then described in terms of these two amplitudes, as noted in Table II.

REFERENCES

1. For a good review of the assumptions leading up to $SU(6)_W$, as well as an extensive bibliography on early work in the subject, see the article by B.W. Lee, in "Particle Symmetries," 1965 Brandeis University Institute in Theoretical Physics, edited by M. Chré́tien and S. Deser, New York, Gordon and Breach, 1966, p. 325.
2. E. W. Colglazier and J. L. Rosner, Nucl. Phys. B27, 349 (1971);
E. W. Colglazier, Thesis, California Institute of Technology, 1971 (unpublished).
3. R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D3, 2706 (1971).
4. D. L. Katyal and A. N. Mitra, Phys. Rev. D1, 338 (1970);
D. K. Choudhury and A. N. Mitra, Phys. Rev. D1, 351 (1970);
Rashmi Mehrotra and A. N. Mitra, Phys. Rev. D4, 1409 (1971).
5. J. L. Rosner, Phys. Rev. Letters 22, 689.
6. L. Micu, Nucl. Phys. B10, 521 (1969).
7. W. P. Petersen and J. L. Rosner, Phys. Rev., to be published.
8. R. F. Dashen and M. Gell-Mann, Phys. Letters 17, 142, 146 (1966).
9. Peter G. O. Freund, Nuovo Cimento 5A, 9 (1971).
Related problems have been considered by D. V. Volkov, Ukr. Fiz. Zh. 11, 1296 (1966), a work which came to our attention after the present paper was written.
10. M. Gell-Mann, Phys. Rev. Lett. 14, 77 (1965).
11. For a decay $A \rightarrow B + C$, the z axis is defined as the direction of particle B in the rest frame of A.
12. M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

13. Many good reviews exist. Two are: H. Harari, in Proceedings of the XIV International Conference on High Energy Physics, Vienna, 1968, edited by J. Prentki and J. Steinberger, CERN, Geneva, 1968, p. 195, and R. H. Dalitz, in "Symmetries and Quark Models," edited by Ramesh Chand, New York, Gordon and Breach, 1970, p. 355.
14. J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969); J. Kuti and V. Weisskopf, Phys. Rev. D4, 3418 (1971).
15. R. Dashen and M. Gell-Mann, Ref. 8; H. J. Lipkin and S. Meshkov, Phys. Rev. Letters 14, 670 (1965), and Phys. Rev. 143, 1269 (1966); K. J. Barnes, P. Carruthers, and F. von Hippel, Phys. Rev. Letters 14, 82 (1965); K. J. Barnes, Ibid., 798; R. Delbourgo, M. A. Rashid, Abdus Salam, and J. Strathdee, in "High Energy Physics and Elementary Particles", IAEA, Vienna, 1965, p. 455; B. Sakita and K. C. Wali, Phys. Rev. 139, B1355 (1965); P. G. O. Freund, R. Oehme, and P. Rotelli, Nuovo Cimento 51A, 217 (1967). For further references, see the bibliography in Ref. 1.
16. H. Harari, D. Horn, M. Kugler, H. J. Lipkin, and S. Meshkov, Phys. Rev. 146, 1052 (1966).
17. Delbourgo, et al., Ref. 15; G. Costa, M. Tonin, and G. Sartori, Nuovo Cimento 39, 352 (1965); P. G. O. Freund, A. N. Maheshwari, and E. Schonberg, Phys. Rev. 159, 1232 (1967); H. J. Lipkin, Phys. Rev. 159, 1303 (1967); H. J. Lipkin, Phys. Rev. 176, 1709 (1968); Q. Shafi, Nuovo Cimento 62A, 290 (1969).
18. Freund, et al., Ref. 17.
19. Costa, et al., Ref. 17; Shafi, Ref. 17.
20. A. N. Mitra and M. Ross, Phys. Rev. 158, 1630 (1967); D. Faiman and A. W. Hendry, Phys. Rev. 173, 1720 (1968); D. Faiman, Nucl. Phys. B32, 573 (1971).

21. See R. L. Walker, in Proceedings of the 4th International Symposium on Electron and Photon Interactions at High Energies, Liverpool, September 14-20, 1969, edited by D. W. Braben and R. E. Rand, Daresbury Nuclear Physics Laboratory, 1969, p. 23, for a review and further references.
22. F. Gursey, A. Pais, and L. Radicati, Phys. Rev. Lett. 13, 299 (1964).
23. Particle Data Group, Rev. Mod. Phys. 44, (1972).
24. Sakita and Wali, Ref. 15.
25. J. Rosner, in "Phenomenology in Particle Physics-1971", edited by C. B. Chiu, G. C. Fox, and A. J. G. Hey, Pasadena, California Institute of Technology, 1971, p. 387.
26. This relation was obtained by D. V. Volkov, E. A. Kuraev, and V. I. Tkach, Yad. Fiz. 4, 601 (1966) [Sov. J. Nucl. Phys. 4, 426 (1967)] in a slightly different but equivalent formalism.
27. R. Tripp, in 1968 Vienna Conf. op. cit., p. 173.
28. N. P. Samios, Rapporteur's talk, XV International Conference on High Energy Physics, Kiev, 1970, Moscow, Atomizdat, 1971.
29. D. E. Plane, et al., Nucl. Phys. B22, 93 (1970).
30. The photon is treated as transforming like the appropriate linear combination of transversely polarized vector mesons.
31. G. Ascoli, H. B. Crawley, D. W. Mortara, and A. Shapiro, Phys. Rev. Letters 21, 1411 (1968); G. Ascoli, et al., presented to the 1970 Kiev Conf., op.cit.
32. A. D. Brody, et al., Phys. Lett. 34B, 665 (1971); Phys. Rev. D4, 2693 (1971).
33. R. L. Walker, Ref. 21.

34. C. L. Cook and G. Murtaza, *Nuovo Cimento* 39, 531 (1965).
35. J. J. de Swart, *Rev. Mod. Phys.* 35, 916 (1963). We use the tables of Ref. 23.
36. H. J. Lipkin, in *Proceedings, of the Irvine Conference on Particle Physics*, December 1971, to be published. I thank Professor Lipkin for a discussion of this result.
37. J. T. Carroll, *et al.*, *Phys. Rev. Letters* 28, 318 (1972).
38. A similar total is obtained by E. W. Colglazier, Ref. 2 and private communication, by counting the graphs of Fig. 1(d)-(f). For $\underline{70}$, $L = 1$ decays there is one graph of the type in Fig. 1(d), one of the type 1(e), and three of the type 1(f). The assumption that spin-orbit effects are negligible is thus quite strong for $\underline{70}$, $L = 1$ decays, but seems all the same to be reasonably consistent with experiment (See Refs. 3,7). Five amplitudes are also obtained by Volkov. (See Ref. 9).
39. J. Rosner, *Phys. Rev.* D1, 2701 (1970); J. Mandula, J. Weyers, and G. Zweig, *Ann. Rev. Nucl. Sci.* 20, 289 (1970).
40. K. Johnson and S. B. Treiman, *Phys. Rev. Letters* 14, 189 (1965).
41. H. Ruegg and D. V. Volkov, unpublished (P. Freund, private communication).
42. J. D. Jackson, *Phys. Rev. Lett.* 15, 990 (1965).
43. J. Rosner, *Phys. Rev. Lett.* 24, 173 (1970).
44. J. Rosner, *Phys. Rev. Lett.* 21, 950, 1422 (E) (1968).
45. F. Gilman and H. Harari, *Phys. Rev.* 165, 1803 (1968).
46. H. J. Lipkin, *Phys. Letters* 35B, 534 (1971).
47. M. Gell-Mann, *Phys. Lett.* 8, 214 (1964).
48. G. Zweig, CERN reports TH-401 and TH-412, 1964 (unpublished).

TABLE I(a) Ratios of isoscalar factors for some meson decays

Decay ^a	F or D	Isoscalar factor ($\begin{smallmatrix} 8 & 8 & 8 \\ A & B & C \end{smallmatrix}$)	Ratio to ($\begin{smallmatrix} 8 & 8 & 8 \\ K & K & \pi \end{smallmatrix}$) [F: $\frac{1}{2}$; D: $3/2\sqrt{5}$]
" π " $\rightarrow$ " π " " π " ($A_{1,2} \rightarrow \rho\pi$)	F	$\sqrt{2/3}$	$2\sqrt{2/3}$
" π " $\rightarrow$ $K\bar{K}$ (e.g. $A_2 \rightarrow K\bar{K}$)	D	$-\sqrt{3/10}$	$-\sqrt{2/3}$
" π " $\rightarrow$ $\eta\pi$ (e.g. $A_2 \rightarrow \eta\pi$)	D	$1/\sqrt{5}$	$2/3$

Table I(b) Ratios of amplitudes to those for " K " $\rightarrow$ " K " π for some decays involving octet-singlet mixtures.^b

Decay ^a	Ratio to $M(K \rightarrow K\pi)$
$A \rightarrow BC$	
$f \rightarrow \pi\pi$ ^b	$-\sqrt{2}$
$f \rightarrow K\bar{K}$ ^b	$-\sqrt{2/3}$
$f' \rightarrow K\bar{K}$ ^b	$2/\sqrt{3}$
$B \rightarrow \omega\pi$	$2/\sqrt{3}$

a) For specific charge states, multiply by the appropriate isospin Clebsch-Gordan coefficient.

b) We take $f = (1/3)^{1/2} |1\rangle + (2/3)^{1/2} |8\rangle$; $f' = (2/3)^{1/2} |1\rangle - (1/3)^{1/2} |8\rangle$, etc., and have assumed $f' \rightarrow \pi\pi$, $B \rightarrow \phi\pi$. Octet couplings in this sub-table are all D-type.

TABLE II. Amplitudes for $K \rightarrow K \pi$ decays. Coefficients of $a^{(0)}$, $a^{(3)}$, $a^{(8)}$ shown.

$J^{PC}_{(A)}$	$J^{PC}_{(B,C)}$	λ	$a^{(0)}$	$a^{(3)}$	$a^{(8)}$
2^{++}	1^-0^-	1	$-3/16$	0	$-1/2\sqrt{2}$
	0^-0^-	0	$-\sqrt{3}/8$	0	$-1/\sqrt{6}$
1^{++}	1^-0^-	1	$-3/16$	0	$1/2\sqrt{2}$
		0	0	$1/\sqrt{2}$	0
1^{+-}	1^-0^-	1	0	$1/2$	0
		0	$-3/8\sqrt{2}$	0	0
0^{++}	0^-0^-	0	$(1/8)\sqrt{3/2}$	0	$-1/\sqrt{3}$

Other amplitudes may be obtained using Table I .

For specific charge states, multiply by the appropriate Clebsch-Gordan coefficient.

TABLE III. Parametrization of $q\bar{q}$; $L = 1$ decays in terms of the quark graphs of Figure 1.^{2,39} This formalism is equivalent to coplanar symmetry. Here m_1, μ are the mean masses of the $q\bar{q}$; $L = 1$ and 0 multiplets, respectively and $p^2 = m_1^2/4 - \mu^2$.

Decay	Coefficient of		
	c_0 (Fig. 1(a))	c_1 (Fig. 1(b))	c_2 (Fig. 1(c))
$2^{++} \rightarrow 1^-0^-(\lambda=1)$	$2\sqrt{2}(p/\mu)^2(1+2\mu/m_1)$	$-4\sqrt{2}(p/\mu)^2$	0
$0^-0^-(\lambda=0)$	$4\sqrt{2/3}$ " "	$-8\sqrt{2/3}$ "	0
$1^{++} \rightarrow 1^-0^-(\lambda=1)$	$2\sqrt{2}$ " "	$4\sqrt{2}(1+m_1/2\mu)^2$	$16\sqrt{2}(p/\mu)^2$
$(\lambda=0)$	0	$2\sqrt{2}m_1(m_1+2\mu)/\mu^2$	0
$1^{+-} \rightarrow 1^-0^-(\lambda=1)$	0	$2m_1(m_1+2\mu)/\mu^2$	0
$(\lambda=0)$	4 " "	$4(m_1+2\mu)/\mu$	$16(p/\mu)^2$
$0^{++} \rightarrow 0^-0^-$	$-4/\sqrt{3}$ " "	$-(4/\sqrt{3})(2+\frac{m_1}{\mu})(1+\frac{m_1}{\mu})$	$-16\sqrt{3}(p/\mu)^2$

These are amplitudes for " K " $\rightarrow$ " K " π decays.

Other amplitudes may be obtained using Table I. For specific charge states, multiply by the appropriate isospin Clebsch-Gordan coefficient.

TABLE IV. Expressions for $|Sm_z\rangle$ in terms of $|Sm_y\rangle$ for $S = 0, \frac{1}{2}, 1$, and $\frac{3}{2}$. Matrices $M_{mm'}^{(S;zy)}$.

$S = 0$

$|00\rangle_z = i |00\rangle_y$

$S = \frac{1}{2}$

$m \backslash m'$	1/2	-1/2
1/2	$\sqrt{1/2}$	$\sqrt{1/2}$
-1/2	$-i\sqrt{1/2}$	$i\sqrt{1/2}$

$S = 1$

$m \backslash m'$	1	0	-1
1	$1/2$	$\sqrt{1/2}$	$1/2$
0	$-i\sqrt{1/2}$	0	$i\sqrt{1/2}$
-1	$-1/2$	$\sqrt{1/2}$	$-1/2$

$S = \frac{3}{2}$

$m \backslash m'$	3/2	1/2	-1/2	-3/2
3/2	$\sqrt{1/8}$	$\sqrt{3/8}$	$\sqrt{3/8}$	$\sqrt{1/8}$
1/2	$-i\sqrt{3/8}$	$-i\sqrt{1/8}$	$i\sqrt{1/8}$	$i\sqrt{3/8}$
-1/2	$-\sqrt{3/8}$	$\sqrt{1/8}$	$\sqrt{1/8}$	$-\sqrt{3/8}$
-3/2	$i\sqrt{1/8}$	$-i\sqrt{3/8}$	$i\sqrt{3/8}$	$-i\sqrt{1/8}$

TABLE V. Expressions for $|S m\rangle_z$ in terms of $|S m'\rangle_x$ for $S = 0, \frac{1}{2}, 1$, and $\frac{3}{2}$. Matrices $M_{mm'}^{(S; z x)}$.

$S = 0$

$$|00\rangle_z = -i |00\rangle_x$$

$S = \frac{1}{2}$

$m \backslash m'$	1/2	-1/2
1/2	$\sqrt{1/2}$	$i\sqrt{1/2}$
-1/2	$\sqrt{1/2}$	$-i\sqrt{1/2}$

$S = 1$

$m \backslash m'$	1	0	-1
1	$1/2$	$i\sqrt{1/2}$	$-1/2$
0	$\sqrt{1/2}$	0	$\sqrt{1/2}$
-1	$1/2$	$-i\sqrt{1/2}$	$-1/2$

$S = \frac{3}{2}$

$m \backslash m'$	3/2	1/2	-1/2	-3/2
3/2	$\sqrt{1/8}$	$i\sqrt{3/8}$	$-\sqrt{3/8}$	$-i\sqrt{1/8}$
1/2	$\sqrt{3/8}$	$i\sqrt{1/8}$	$\sqrt{1/8}$	$i\sqrt{3/8}$
-1/2	$\sqrt{3/8}$	$-i\sqrt{1/8}$	$\sqrt{1/8}$	$-i\sqrt{3/8}$
-3/2	$\sqrt{1/8}$	$-i\sqrt{3/8}$	$-\sqrt{3/8}$	$i\sqrt{1/8}$

FIGURE CAPTIONS

Figure 1: Graphs describing independent decay amplitudes in the model of Ref. 2. (a)-(c): $L = 1$ mesons; (d)-(f): $L = 1$ baryons. (a) coupling constant c_0 ; (b) coupling constant c_1 ; (c) coupling constant c_2 .

Figure 2: Hierarchy of symmetries expressed by Eq. (1). The labels on the arrows denote sets of assumptions that lead to successively more restrictive symmetries.

Figure 3: (a) Quark states; (b) qq states belonging to $\underline{6}$ of $SU(3)$; (c) qq states belonging to $\overline{3}$ of $SU(3)$.

+ 2 OTHERS

Figure 1

Figure 2

Figure 3