EXPERIMENTALLY DETERMINED SHAPE FACTORS FOR DEEP PART-THROUGH CRACKS IN A THICK-WALLED PRESSURE VESSEL*

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ABSTRACT

Fracture mechanics shape factors for two series of part-through cracks in thick-walled epoxy pressure vessels were determined experimentally and compared with theoretical estimates.

MASTER

INTRODUCTION

Approximately twenty, thick-walled, epoxy cylinders without nozzles were cast as control specimens for an investigation of nozzle corner cracks.¹ At the conclusion of this investigation some of these specimens were used to measure the shape factors for deep, part-through cracks. The shape factor, C , is implicitly defined by the equation,

$$K_I = C \sigma_n \sqrt{\pi a}, \quad (1)$$

where K_I is the stress intensity, σ_n is the nominal stress and a is the crack depth. For the thick-walled vessels σ_n is taken to be the average stress across the section.

There were two reasons for this study. First, if the experimentally determined values agree with theoretical predictions, then the confidence in the applicability of the experimental procedures to regions where current theory is inadequate is greatly increased. Second, experimental data on the influence of a stress-free back surface near the tip of a crack is actually quite scarce.

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PROCEDURE

Pressure vessels were fabricated from the cylindrical bodies by gluing on hemispherical end caps. (These caps were cast from the same epoxy as the cylinders.) The important dimensions of the vessel were as follows:

Inside radius: 2.69 in.

Outside radius: 3.23 in.

Wall thickness: 0.59 in.

Length cylindrical section: 9 in.

Following gluing the vessels were stress relieved, and then a slot was machined in the outside wall with a tiny cutting wheel. Two sizes of slots were used. For vessels destined for deep cracks the slots were approximately 1.1 in. long and 0.2 in. deep. For vessels destined for shallower cracks the slots were approximately 0.7 in. long and 0.1 in. deep.

After the slots had been completed the vessels were pressurized approximately three times per minute. It was found that because of self-heating, higher cycling rates would result in very low crack growth rates. Extreme care was required to select the correct cyclic pressure. If too high, the vessels would burst prematurely; if too low, no crack would grow.

Another difficulty related to the self-heating was "start-up" brittleness. When cyclic pressure was applied to the vessels after a shut down of more than an hour, overnight for example, the pressure needed to cause failure might be considerably less than what the vessel would tolerate after, say, 30 min of operation. Hence the vessels were cycled at reduced pressure whenever the equipment was started up after an extended shut down. This problem was eliminated when round-the-clock operation was adopted.

Because of the transparency of the epoxy the approximate size and shape of the growing fatigue cracks could be estimated. When the cracks had finally reached a suitable size the vessel was allowed to stand untouched for many hours, usually overnight. Note that this interval put the vessel in a state of start-up brittleness. The final step was a burst test in the start-up condition with pressure measured to the nearest psi.

Following the burst test the remains were cut up into small beams approximately one-half in. square and 6 in. long. These beams were then slotted and a fatigue crack was grown from the slot. The same cyclic rate of loading, 3 cpm, was used as with the vessels. The start-up problem for the beams was never completely solved. The best solution was, as implied above, to keep the machine going continuously. The beams required much closer attention than the vessels because a crack would often grow from an unacceptably small size through to failure in about 20 min. Once a crack had reached the desired depth, about 40 per cent of beam depth, the specimen was removed and like a pressure vessel was allowed to remain undisturbed for many hours. Finally, the little beams were loaded in four-point bending to destruction. Given the load at failure, the crack depth, and the geometry of the beam, the fracture toughness, K_{Ic} , was calculated using the standard procedure.²

Returning to the pressure vessels, it was now possible to solve Eq. (1) for C, the shape factor. The nominal stress term was found from Pr/T where P is pressure, r is the inside radius and T is wall thickness. The crack depth, a, was measured with a steel rule or a comparator to one hundredth of an inch. Since $K_I = K_{Ic}$ at failure, the value of K_{Ic} determined in the beam tests was substituted into Eq. (1) for K_I .

RESULTS

The raw data for the vessel burst tests are presented in Table 1. The actual shape of the cracks can be seen in Figs. 1 and 2. The specimens shown in these pictures were cut from the remains of each vessel after the burst test. The starter notches are on the outside surfaces and can be identified because they are darker than the actual fatigue cracks. The distance shown as surface length in Table 1 is the actual length of the starter notch along the outside surface of the vessel. This distance was used later (in the interpretation) as the major axis of a semiellipse. Note that for the shallower notches the fatigue crack does not grow out of the complete length of the starter notch. In both figures a small, white, paper wedge was placed on each specimen to show the exact point at which failure occurred.

The experimental data presented in Table 1 were used with the procedure described in the previous section to calculate a shape factor for each crack. The results are shown in the column marked, "Experiment" in Table 2.

Paris and Sih have suggested that the stress intensity for deep cracks can be determined from³

$$K_I = [1 + 0.12 (1 - \frac{a}{b})] \frac{\sigma_n \sqrt{\pi a}}{\Phi_0} \left(\frac{2T}{\pi a} \tan \frac{\pi a}{2T} \right), \quad (2)$$

where Φ_0 is the elliptic integral:

$$\Phi_0 = \int_0^{\pi/2} [1 - (\frac{b^2 - a^2}{b^2}) \sin^2 \theta]^{1/2} d\theta. \quad (3)$$

By grouping the appropriate terms it can be seen that the shape factor as previously defined is

$$C = [1 + 0.12 (1 - \frac{a}{b})] \left(\frac{2T}{\pi a} \tan \frac{\pi a}{2T} \right) / \Phi_0. \quad (4)$$

The column entitled "C Paris-Sih Equation" in Table 2 is simply a tabulation of the values obtained by substituting the appropriate values into Eq. (4).

The last column in Table 2 is a tabulation of the values of shape factor as determined from the Irwin⁴ equation:

$$K_I = \frac{1.1 \sigma_n \sqrt{\pi a}}{\Phi_o} \quad (5)$$

Thus C, according to Eq. (5), is

$$C = 1.1/\Phi_o \quad (6)$$

(The value of C in Eq. (6) as in Eq. (4) is strictly speaking only applicable at the very deepest part of the crack.)

The predicted shape factors tabulated in Table 2 are shown plotted in Fig. 3. In each case every experimentally determined point is shown plotted as a function of the predicting theory. The circles are for the Paris-Sih theory and the squares are for the Irwin theory. Inspection of the figure suggests that for very deep cracks, such as those shown in Fig. 2, the Paris-Sih equation results in shape factors slightly too high and the Irwin equation in values a little too low. The Paris-Sih values, however, are somewhat closer to the experimentally determined values. Furthermore, the Paris-Sih values are generally within the 10 per cent limits suggested by these authors.

For smaller cracks, such as those shown in Fig. 1, both theories give an excellent prediction. This result was particularly gratifying in view of the difficulties involved in controlling scatter.

DISCUSSION

The reliability of the experimental method of determining shape factors described in this paper depends on controlling the scatter in the K_{Ic} determinations. To control scatter the specimens must be precisely machined and symmetrically loaded without friction. Furthermore the environment must be carefully controlled. In our first determinations of K_{Ic} for the nozzle investigations, 49 specimens were tested. The cracks were grown under mineral oil. The average K_{Ic} for these specimens was $922 \text{ psi}\sqrt{\text{in.}}$. For nine specimens which were cycled in air and then tested the average was $929 \text{ psi}\sqrt{\text{in.}}$. It should be added that the original intention was to test at least several dozen specimens with air-grown cracks. Unfortunately, nine specimens failed prematurely at one time in the fatigue machine during start up. A similar number had to be rejected because of eccentric crack growth. Hence the value of $929 \text{ psi}\sqrt{\text{in.}}$ as determined from the nine good specimens was used to calculate the experimental shape factors.

One other point about the interpretation of the results should be mentioned: The cracks are not true ellipses. Note especially the regions in Fig. 1 where the cracks have not grown out of the complete notch. It is, however, unlikely that the idealization of the cracks has a significant effect on the usefulness of the results.

The reader's attention should again be drawn to the white paper wedge in the photographs. It is noteworthy that no crack failed at the exact middle. This is particularly surprising in view of the fact that the cracks are projecting into a region of increasing stress. A detailed study of this observation was beyond the scope of the project as were several other interesting phenomena.

Every time the pressure cycling device was turned off after some hours of operation a distinct mark was left on the fatigue crack surface. These marks are not always visible with the naked eye, but are always found easily with an optical microscope under low magnification. With good lighting the lines can be photographed with a studio camera as in Fig. 4. Equally interesting is the fracture surface. In the 60 or so vessels tested to destruction the pattern was always the same.

In the region of initiation the surface is very shiny. Under the microscope tiny lines can be found pointing to the exact point of failure. As the distance from this point increases the lines become more and more coarse until they can be seen by the naked eye (see Fig. 4, particularly just to the right of the crack and also above the label). Just beyond the region of radiating lines the fracture surface becomes very coarse or knurled. This region is clearly seen on the far right of Figs. 1 and 4. It is usually only about 1/2 in. long. Beyond the knurled area the fracture surface becomes very smooth and shiny. This last region is not shown in any of the figures.

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FIGURE CAPTIONS

Fig. 1. First series of part-through cracks in a pressure vessel wall.

Fig. 2. Second series of part-through cracks in a pressure vessel wall.
Note that the cracks in this series are considerably deeper than those in the first series.

Fig. 3. Comparisons between experimentally determined shape factors and the predictions of two theories.

Fig. 4. Enlarged view of a fatigue crack and fracture surface. Note the beach markings on the fatigue crack. See text for discussion of fracture surface.

Table 1. Raw Data

Vessel Number	Failure Pressure (psi.)	Crack Depth* (in.)	Surface Length** (in.)
104	158	0.52	1.15
103	175	0.48	1.12
102	180	0.44	1.14
105	180	0.43	1.14
108	180	0.41	1.25
99	184	0.45	1.13
106	194	0.42	1.14
110	275	0.24	0.75
109	285	0.27	0.65
112	300	0.29	0.65
114	320	0.21	0.67

*"a" in Equation 1.

**"2b," major axis of ellipse.

Table 2. Comparison Between Experiment and Theory

Vessel Number	a/T	a/b	C Paris-Sih ^a Equation	C Experiment	C Irwin ^b Equation
104.	0.88	0.90	1.32	0.999	0.735
103	0.81	0.86	1.12	0.939	0.753
102	0.75	0.77	1.05	0.954	0.787
105	0.73	0.75	1.03	0.965	0.794
108	0.69	0.66	1.05	0.988	0.836
99	0.76	0.80	1.06	0.922	0.777
106	0.71	0.74	1.02	0.906	0.801
110	0.41	0.72	0.821	0.845	0.810
109	0.46	0.83	0.781	0.769	0.763
112	0.49	0.89	0.765	0.705	0.739
114	0.36	0.63	0.853	0.776	0.849

a. Reference 3.

b. Reference 4.

0

V-109
285 psi

V-110
275 psi

V-112
300 psi

V-114
320 psi

V-103
175 psi

V-104
158 psi

V-105
180 psi

V-106-1
194 psi

V-108
180 psi

V-99
184 psi

Fig 3

ORNL-DWG 71-14089

701-A
858
i sd

1-1
ch.
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