MASTER

AN EXPERIMENTAL HYDROSTATIC JOURNAL BEARING

by

A. E. Rains

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ABSTRACT

This report describes the design, fabrication, and testing of a typical hydrostatic spindle bearing. An extension of classic design to account for shaft angularity is presented.

INTRODUCTION

Because of the ever-increasing stringency of accuracy requirements, designers have been forced to take a long, hard look at old as well as new methods of supporting moving members. One outgrowth of this "accuracy pressure" has been serious development work with hydrostatic bearings which seemingly offer excellent possibilities (See References 1 and 2 for complete discussions of advantages and limitations of hydrostatic bearing application). While not a new concept, hydrostatic bearings have only recently been employed for precision support since they are relatively complex in nature. They are not yet available as "off-the-shelf" components because each bearing must be individually engineered to suit a given design situation. To our knowledge, only a few companies in the United States offer to design and fabricate a hydrostatic spindle to order and only a few more offer equipment incorporating any sort of hydrostatic bearing as a standard component. Their use is on the up-swing, however, and it is clear that future machines will make more and more use of hydrostatic bearing technology.

Thus it becomes necessary for potential users of hydrostatic bearings to be able to specify criteria for design purposes, and, likewise, to be able to evaluate the resulting design. This "feel" for the working of a hydrostatic bearing was felt to be developed best by actually designing and testing a typical bearing.

Because of special interest in low speed, intermittent rotational motion, it was decided to construct the hydrostatic spindle described

in the Apparatus section. Since the spindle was intended only for testing theory, such refinements as end-thrust bearings and shaft-journal oil seals were not included in the design.

THEORY

DESIGN THEORY

Most of the following concepts are contained in the comprehensive design manual published by the Cast Bronze Institute⁽¹⁾. The nomenclature used is essentially the same as the manual's and is presented in full as Appendix II.

A simple hydrostatic bearing is composed of three elements: the bearing pad - containing a recess area surrounded by a land or sill area; the runner or slide - the moving element; the fluid source - usually a pump and reservoir (see Figure 1). Fluid is pumped under pressure into the recess area and flows to atmospheric pressure between the runner and sill (provided the pressure is large enough to "lift" the runner and its associated weight). The load carrying capacity of the bearing is proportional to the total pad area (modified by the load coefficient a_f) which accounts for pressure drop across the sills) and the recess pressure, or

$$F = a_f A_p P_s.$$

The flow demanded by the bearing to support the load at a given clearance h is

$$Q = q_f \frac{F}{A_p} \frac{h^3}{\mu}$$

where q_f , the flow coefficient, is the proportionality constant.

These principles can be extended to apply to a four pad hydrostatic journal which supports a round shaft (see Figure 2). The pads are assumed to be identical and equally spaced at 90° . To insure the shaft lifting from all four pads, a compensating element or flow restrictor is included in the supply line to each recess. Because of these restrictions, the recess pressures are somewhat less than the supply pressure. For an individual pad, the recess pressure is expressed as

$$P_R = \beta P_S$$

where the pressure ratio β is determined by, among other things, the type of compensating element used. Commonly used are flow control valves, capillary tubes, and orifices. We will limit discussion to orifice compensation as was used on the experimental bearing.

In operation, each of the four pads produce on the shaft a force directed toward the center of the journal. These forces, plus the shaft's weight and any external load, determine the equilibrium position of the shaft within the journal. With no external load and a "weightless" shaft, the shaft will be concentric to the journal and the forces will be of a magnitude

$$F_B = F_T = F_L = F_R = q_f A_p P_S \beta.$$

The initial or no-load pressure ratio will be

$$\beta' = \frac{1}{\beta_S C^6} \left(\frac{K_o \mu}{a_f q_f} \right)^2 \left[\sqrt{1 + \frac{2}{\frac{1}{\beta_S C^6} \left(\frac{K_o \mu}{a_f q_f} \right)^2}} - 1 \right],$$

where

$$K_0 = \frac{\pi}{4} \frac{C_0 d_0^2}{\rho^{1/2}}$$

is the orifice coefficient and C is the radial clearance between shaft and concentric journal.

If now an external load R is applied to the shaft so that its line of action passes (say) through the center of two opposed pads, the shaft will move a distance e in the direction of the force until a new equilibrium position is reached (see Figure 3). At this position, the pad toward which the shaft has moved will have increased its reaction because of the reduced clearance; conversely, the opposite pad will have decreased its reaction because of the increased clearance. The difference of these two reactions is equal to the applied load, or

$$R = F_B - F_T.$$

For values of displacement small in comparison to the radial clearance, the reactive forces produced by the two "side" recesses will remain essentially unchanged from their initial value and add nothing to the load-carrying ability of the bearing.

Based on these concepts, a "load vs deflection" relationship can be predicted for the bearing. Stated dimensionlessly, the relationship is

$$\frac{R}{a_f A_p P_s} = \frac{\beta'}{2(1-\beta')} \left\{ \frac{\sqrt{(\beta')^2 + 4(1-\beta')(1-\epsilon)^6} - \beta'}{(1-\epsilon)^6} - \frac{\sqrt{(\beta')^2 + 4(1-\beta')(1+\epsilon)^6} - \beta'}{(1+\epsilon)^6} \right\}$$

where

$$\epsilon = \frac{e}{c},$$

the dimensionless deflection. A complete derivation of this equation is presented in Appendix III. As can be seen on plots of the equation for different initial pressure ratios (see Figure 4), maximum bearing stiffness is obtained when $\beta' = 0.5$ (this is not the case for other types of compensation) so it is this value that is generally aimed for in design.

EXTENSION OF THE DESIGN THEORY TO ACCOUNT FOR RUNNER ANGULARITY

One of the basic assumptions used to derive the load-deflection relationship above is that the flow coefficient q_f remains constant for all conditions of load. This is approximately true if the shaft's centerline remains parallel to the journal's, but shows serious error if any angularity is encountered. To explain the differences between theory and the experimental results noted later, the following analysis is presented.

Two basic modifications to the classic theory are needed: first, a load-deflection relationship which does not assume a constant q_f and, second, a functional relationship for q_f versus angularity. The modified load-deflection relationship as derived in Appendix III is

$$\frac{R}{a_f A_p B_s} = \frac{\beta'}{2(1-\beta')} \left\{ \left(\frac{q_f}{q_{fB}} \right)^2 \frac{\sqrt{(\beta')^2 + 4(1-\beta')(1-\epsilon)^6 (q_{fB}/q_f)^2 - \beta'}}{(1-\epsilon)^6} - \left(\frac{q_f}{q_{fT}} \right)^2 \frac{\sqrt{(\beta')^2 + 4(1-\beta')(1+\epsilon)^6 (q_{fT}/q_f)^2 - \beta'}}{(1+\epsilon)^6} \right\};$$

q_f is the "classic" flow coefficient, assumed the same for both the top and bottom pads and of constant value. The terms q_{fT} and q_{fB} are the variable flow coefficients for the top and bottom pads respectively. For the case

$$q_{fT} = q_{fB} = q_f ,$$

the load deflection relationship reverts to the form presented earlier.

The variation of q_{fB} and q_{fT} proposed for the experimental bearing is based on idealized flow through the trapezoidal spaces formed between the tilted shaft and its supporting pads. The resulting functions may or may not be applicable to another bearing configuration; however, it is felt that the method used in determining these functions is general. This method is outlined in Appendix IV.

Both flow factors are of the same form

$$q_{fN} = \frac{1}{6a_f} \left\{ \left[1 + 3\alpha^2 \left(\frac{H}{h_N} \right)^2 \right] \frac{a}{l} + \left[1 + \alpha^2 \left(\frac{H}{h_N} \right)^2 \right] \frac{b}{L} \right\}$$

where h_N is the centerpoint clearance between the shaft and pad N (see Figure 5) and H is the maximum amount h_N can be changed at the sill by tilting the shaft until it touches the pad. The factor α is the ratio of actual shaft tilt to this maximum possible tilt. For the condition $\alpha = 0$ (no angularity), the function reduces to

$$q_{fT} = q_{fB} = \frac{1}{6a_f} \left[\frac{a}{l} + \frac{b}{L} \right] = q_f ,$$

the classic flow coefficient.

Figure 6 is a graphical representation of the modified load-deflection relationship including the above flow-factors as computed for the experimental bearing. As can be seen, angularity reduces the load carrying-capacity of the bearing a great deal, particularly when severe at low load levels. The practical aspect of these curves is realized when one considers that any hydrostatic bearing shaft subjected to a transverse load will bend. This is, in effect, a tilt which will modify the deflection of the shaft in the bearing. If the bending of a given shaft is predicted for a number of loads using beam deflection theory, the curves may be entered to predict the resulting deflection of the shaft at the pad. This is illustrated by Figure 7 which shows the predicted load-deflection curve for the experimental bearing. Briefly, procedure is as follows: (1) Assume a load. (2) Predict the shaft slope at the pad centerpoint due to this load and assume it constant over the pad length. (3) Calculate the factor α for a number of different ϵ 's using this slope; locate points corresponding to these values on a graph similar to Figure 6 and complete a curve through the points. (4) Determine the intersection of a horizontal line drawn from the assumed load and the curve drawn in (3) above; this point fixes the value of deflection for the load. (5) Repeat the above procedure for other loads to obtain a continuous load-deflection plot.

APPARATUS

A diagrammatic sketch of the experimental hydrostatic journal is shown in Figure 8 which is largely self-explanatory. Two identical bearings, one at each end of the journal, support the shaft. Oil is supplied to both the bearings and the loading arrangement at a

regulated pressure and feeds back to the reservoir via a gravity drain system. Separate pressure gages measure the individual pad pressures and a mercury-in-glass thermometer measures oil temperature. Coolant is circulated in the reservoir to maintain a stable operating temperature near ambient.

Equal loads are applied to shaft ends by the cylinder, cable, and pulley arrangement. Load magnitude is controlled by bleed-off through the bypass valve. Deflection of each end of the shaft under load is measured by Sheffield jets. The shaft's perimeter is marked with 30° increments near one end to provide reference for runout measurements. A magnetic base tooling ball is centered on this end and its eccentricity is measured by two Airborne Instrument Laboratories (AIL) "feather-touch" gages at right angles to each other and to the shaft's centerline.

PROCEDURE

After an initial warm up period, a number of runs were made in which the load was varied while maintaining a constant source pressure. Load cylinder pressure, deflection of each end of the shaft, and oil temperature were recorded as each increment of load was applied.

Actual load on each bearing was determined from a load cylinder pressure versus load calibration obtained beforehand. Readings of shaft deflection, etc., were made only with increasing loads since this corresponded to the way the load cylinder was calibrated. Dimensionless loads were calculated by dividing the actual load per bearing by the measured source pressure and theoretical effective pad area.

Shaft deflection at the pad's centerpoint was obtained from the Sheffield jet readings by correcting for bend in the shaft with beam deflection theory. Dimensionless deflections (or eccentricities) were then calculated by dividing the corrected readings by the radial clearance predetermined from shaft and journal diameter measurements. High and low readings on each of the AIL gages were noted as the shaft and tooling ball were rotated as a unit. The ball's location on the shaft end was then adjusted until these differences were minimized. The limiting condition, where no further improvement in centering of the ball could be obtained, measured the runout or centerline-wander of the shaft in the bearing.

RESULTS AND DISCUSSION

Typical results for the load-deflection portion of the experiment are shown graphically in Figure 8, which includes the theoretically predicted curve for comparison. Only one curve, that for $\beta' = 0.5$ is presented since the others obtained showed no significant differences. Note that shaft bending limited the range over which the bearing could be tested. In general, experimental deflection exceeded theoretical predictions over the entire range. At the maximum load point, experimental deflection was approximately 20 percent more than predicted by classic methods and 7 percent more than predicted by the modified theory presented here. These figures decrease for smaller loads, however, and in the mid-load range typical values are 13 percent and 4 percent, respectively. Maximum bearing stiffness was approximately 4×10^6 lbs/in. The large deviations from theory are due, no doubt, to basic theoretical assumptions becoming less and less applicable as shaft tilt increases. In particular, the uniform

pressure distribution and idealized flow assumptions are seriously in question when value of the "tilt ratio" α become large. Regardless, it is seen that the modified theory does represent actual conditions more closely than the classic theory. A conservative design approach is justifiable, however, and should include minimizing shaft-bend wherever possible. Measurements indicate that runout is of the same order of magnitude as the shaft roundness and increases slightly with increasing load. With no load, maximum runout was found to be 10 microinches and increased to 15 microinches with maximum load. The shaft was round within 50 microinches.

APPENDIX I

REFERENCES

1. Rippel, H. C., Cast Bronze Hydrostatic Bearing Design Manual, Cleveland, Cast Bronze Institute, Inc., 1964.
2. Hahn, R. S., Some Advantages of Hydrostatics in Machine Tools, American Society of Lubrication Engineers, presented at the 1964 Annual Meeting, Chicago, Illinois, Paper No. 301.

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APPENDIX II

NOMENCLATURE

- F - Force exerted by a single pad on a runner or shaft - (lbs)
 R - External force supported by two or more pads of a journal bearing (lbs)
 p_s - Pump or source pressure (psi)
 p_R - Recess pressure of a compensated pad (psi)
 β - Ratio of recess to source pressure of a compensated pad (dimensionless)
 β' - Value of recess/source pressure ratio for all pads in journal bearing subjected to no external load (dimensionless)
 ρ - Density of hydraulic fluid - $\frac{\text{lb-sec}^2}{\text{in}^4}$
 μ - Absolute viscosity of hydraulic fluid - $\frac{\text{lb-sec}}{\text{in}^2}$
 Q - Fluid flow - (in^3/sec)
 g - Acceleration of gravity ($386 \text{ in}/\text{sec}^2$)
 a_f - Load coefficient of a pad (dimensionless)
 q_f - Flow coefficient of a pad; (dimensionless) with additional subscript, a variable quantity.
 A_P - Total pad area including recess (in^2)
 $\left. \begin{array}{l} L \\ l \\ a \\ b \end{array} \right\}$ - Characteristic pad dimensions (in)
 d_o - Orifice diameter (in)
 C_D - Discharge coefficient for orifice (dimensionless)
 k_o - Orifice coefficient ($\text{in}^4/\text{lb}^{1/2} \text{ sec}$)
 C - Radial clearance between journal-bearing pads and aligned, centered shaft (in)

- h - Clearance between journal bearing pad and eccentric shaft; variable clearance in flat bearing. Measured at pads centerpoint (in)
- e - Deflection of shaft in journal bearing; distance from shaft centerline to bearing centerline at pad's centerpoint (in)
- ϵ - Eccentricity; ratio of shaft deflection to original radial clearance, or e/c (dimensionless)
- Δh - Amount clearance is changed at center of sill due to tilt of shaft (in)
- H - Maximum possible value of h for a given shaft deflection (in)
- α - Ratio of actual to maximum values of Δh , or $\frac{\Delta h}{H}$ (dimensionless)

SUBSCRIPTS

- B - Refers to bottom pad of 4-pad journal bearing.
- T - Refers to top pad of 4-pad journal bearing.
- L - Refers to left pad of 4-pad journal bearing.
- R - Refers to right pad of 4-pad journal bearing.
- N - Refers to any pad of 4-pad journal bearing, but usually used in equations that can be applied to either the top or bottom pad.

APPENDIX III

DERIVATION OF THE GENERAL LOAD-DEFLECTION RELATIONSHIP

As stated in the text, the difference between the bottom and top pad reactions is equal to the external load, or

$$R = F_B - F_T$$

(see Figure 3). Each pad reaction is

$$F_N = a_{fN} A_{pN} P_{sN} / \beta_N$$

(where N can be either B or T) and, if equal and constant pad areas, load coefficients, and source pressures are assumed, the expression for load may be written

$$R = a_f A_p P_s (\beta_B - \beta_T)$$

or

$$\frac{R}{a_f A_p P_s} = \beta_B - \beta_T ,$$

the dimensionless form.

The pressure ratio β_N for a given pad is a function of the clearance between the pad and shaft. The form of the function is determined by the type of compensation employed and may be found by equating flow through the clearance space to flow through the compensator, or

$$Q_{PADN} = Q_{COMPENSATORN}$$

Flow through the clearance can be expressed

$$Q_{PADN} = q_{fN} \frac{WN}{A_P} \frac{h_N^3}{\mu} = \frac{a_f P_s}{\mu} q_{fN} h_N^3 \beta_N$$

where q_{fN} is not assumed constant as is the usual case. Based on the familiar $\sqrt{2g\Delta\rho}$ law for orifices, the flow through an orifice compensator can be expressed

$$Q_{COMPENSATORN} = K_0 \sqrt{2P_s(1-\beta_N)}$$

where

$$K_0 = \frac{\pi C_d d_o^2}{4 \sqrt{e}},$$

the "orifice coefficient". (This expression includes the usual assumptions - constant density, relatively constant discharge coefficient, etc.) Thus, the flow equality can be written

$$\frac{a_f P_s}{\mu} q_{fN} h_N^3 \beta_N = K_0 \sqrt{2P_s(1-\beta_N)}$$

from which three important relations can be obtained;

(1) If the equation is solved for β_N , the result is

$$\beta_N = \frac{1}{P_s} \left(\frac{K_0 \mu}{a_f} \right)^2 \frac{1}{h_N^6 q_{fN}^2} \left[\sqrt{1 + \frac{2 h_N^6 q_{fN}^2}{\frac{1}{P_s} \left(\frac{K_0 \mu}{a_f} \right)^2}} - 1 \right];$$

this could be used in the dimensionless load equation as is, but a more convenient form can be obtained by further manipulation as shown below.

(2) If the initial, unloaded condition is assumed, the following statements hold:

$\beta_N = \beta'$; the initial pressure ratio.
 $h_N = C$; the radial clearance
 $q_{fN} = q_f$; the same valve for all pads

Thus, the initial pressure ratio is found to be

$$\beta' = \frac{1}{\beta_s} \left(\frac{K_o M}{a_f} \right)^2 \frac{1}{C^6 q_f^2} \left[\sqrt{1 + \frac{2C^6 q_f^2}{\frac{1}{\beta_s} \left(\frac{K_o M}{a_f} \right)^2}} - 1 \right]$$

by appropriate substitution into the expression for β_N .

(3) Finally, the initial pressure ratio expression can be solved for the term

$$\frac{1}{\beta_s} \left(\frac{K_o M}{a_f} \right)^2 = \frac{(\beta')^2}{2(1-\beta')} q_f^2 C^6 ;$$

this result can be substituted into the expression for β_N to get

$$\beta_N = \frac{\beta'}{2(1-\beta')} \left(\frac{q_f}{q_{fN}} \right)^2 \left[\frac{\sqrt{(\beta')^2 + 4(1-\beta')(1 \pm \epsilon)^6 (q_{fN}/q_f)^2} - \beta'}{(1 \pm \epsilon)^6} \right]$$

where the substitution

$$\frac{h}{C} = \frac{C \pm e}{C} = 1 \pm \frac{e}{C} = 1 \pm \epsilon$$

has been made. (The sign is + for the top pad, - for the bottom if the shaft deflects downward); it is this expression for β_N that is used to obtain the final form of the dimensionless load - deflection equation, which is then

$$\frac{R}{a_f A_P \beta_s} = \frac{\beta'}{2(1-\beta')} \left\{ \left(\frac{q_f}{q_{fB}} \right)^2 \frac{\sqrt{(\beta')^2 + 4(1-\beta')(1-\epsilon)^6 (q_{fB}/q_f)^2} - \beta'}{(1-\epsilon)^6} - \left(\frac{q_f}{q_{fT}} \right)^2 \frac{\sqrt{(\beta')^2 + 4(1-\beta')(1+\epsilon)^6 (q_{fT}/q_f)^2} - \beta'}{(1+\epsilon)^6} \right\}$$

Even though dimensionless, this equation is not truly universal for all bearings of the type considered here because of differences in the variations of flow coefficients. However, it is probably the

"least common denominator" between them all. If the flow coefficients are assumed constant regardless of deflection or shaft "tilt", the terms q_{fB} , q_{fT} , and q_f are equal and the equation reduces to

$$\frac{R}{Q_f A_P P_S} = \frac{\beta'}{2(1-\beta')} \left\{ \frac{\sqrt{(\beta')^2 + 4(1-\beta')(1-\epsilon)^6} - \beta'}{(1-\epsilon)^6} - \frac{\sqrt{(\beta')^2 + 4(1-\beta')(1+\epsilon)^6} - \beta'}{(1+\epsilon)^6} \right\}$$

which is the "classic" form as presented in Reference 1.

APPENDIX IV

DERIVATION OF THE FLOW-COEFFICIENT RELATIONSHIP FOR THE EXPERIMENTAL BEARING

The flow coefficient q_f relates flow through the bearing to the load and operating clearance. For an individual pad N, it can be calculated from the equation

$$q_{fN} = \frac{Q}{q_f P_R \frac{hN^3}{\mu}}$$

provided the flow Q can be estimated or experimentally measured. The most common way of estimating the flow, and the method employed here, is to consider the flow to be one-dimensional flow through a slot. This type of flow is characterized by the equation

$$Q = \frac{\Delta p h^3 M}{12 \mu N}$$

where Δp is the assumed linear pressure drop along the flow length N, M and h are the slot width and height respectively, and μ is the fluid's absolute viscosity.

For the experimental bearing, flat projections of the curved portions of the pad are used for estimating the flow (see Figure 5). The slot height varies along the length of the pad (parallel to the shaft's centerline) if the shaft is tilted in the bearing, but is assumed constant across the pad. Thus there are four flow areas formed: two identical, trapezoidal areas along the pad sides; and two rectangular areas of equal widths but different heights at the pad ends. Pressure is assumed uniform over the entire recess area. Flow for each area is

calculated from the "slot" equation and the results summed to give the total flow through the bearing. Accordingly, the flow through the smaller end-clearance is

$$Q_1 = \frac{PRa}{12\mu L} (h - \Delta h)^3 .$$

The flow through the larger end-clearance is

$$Q_2 = \frac{PRa}{12\mu L} (h + \Delta h)^3 .$$

If these terms are expanded, added together, and simplified, the result is

$$Q_{ENDS} = Q_1 + Q_2 = \frac{PRa}{12\mu L} [2h^3 + 6h(\Delta h)^2]$$

which can be rearranged to

$$Q_{ENDS} = \frac{PRah^3}{6\mu L} \left[1 + 3\left(\frac{\Delta h}{h}\right)^2 \right] .$$

Flow through the trapezoidal areas can be estimated by dividing the area into elemental slots of infinitesimal width and varying height, calculating the flow through each, and summing the result (see sketch below):

Thus, for both sides,

$$Q_{SIDES} = 2 \frac{PR}{12\mu L} \int_0^b [h(x)]^3 dx = \frac{PR}{6\mu L} \int_0^b \left[2 \frac{\Delta h}{b} x + (h - \Delta h) \right]^3 dx$$

which, when integrated, simplified, and rearranged, gives

$$Q_{SIDES} = \frac{P_R b h^3}{6 \mu L} \left[1 + \left(\frac{\Delta h}{h} \right)^2 \right]$$

Total flow from the pad is, then,

$$Q = Q_{ENDS} + Q_{SIDES} = \frac{P_R h^3}{\mu} \cdot \frac{a}{6l} \left[1 + 3 \left(\frac{\Delta h}{h} \right)^2 \right] + \frac{P_R h^3}{\mu} \cdot \frac{b}{6L} \left[1 + \left(\frac{\Delta h}{h} \right)^2 \right]$$

and the factor q_{fN} is

$$q_{fN} = \frac{Q}{a_f \frac{P_R h_N^3}{\mu}} = \frac{\frac{P_R h_N^3}{\mu} \left\{ \frac{a}{6l} \left[1 + 3 \left(\frac{\Delta h}{h} \right)^2 \right] + \frac{b}{6L} \left[1 + \left(\frac{\Delta h}{h} \right)^2 \right] \right\}}{a_f \frac{P_R h_N^3}{\mu}} \\ = \frac{1}{6a_f} \left\{ \left[1 + 3 \left(\frac{\Delta h}{h} \right)^2 \right] \frac{a}{l} + \left[1 + \left(\frac{\Delta h}{h} \right)^2 \right] \frac{b}{L} \right\}$$

The flow coefficient is thus seen to be of the same form for both the top and bottom pads, the only difference being $h_N = h_T$ for the top pad and $h_N = h_B$ for the bottom pad. If the effects of tilting are neglected the equation reduces to

$$q_{fT} = q_{fB} = \frac{1}{6a_f} \left[\frac{a}{l} + \frac{b}{L} \right] = q_f ,$$

equal and constant for both pads regardless of shaft deflection. This is the form usually assumed in classic theory.

By the substitution

$$\Delta h = \alpha h$$

where H is the maximum value Δh may assume for any given deflection

(see Figure 5), the flow coefficient equation becomes

$$q_{HN} = \frac{1}{6a_f} \left\{ \left[1 + 3\alpha^2 \left(\frac{H}{h_N} \right)^2 \right] \frac{a}{l} + \left[1 + \alpha^2 \left(\frac{H}{h_N} \right)^2 \right] \frac{b}{l} \right\}.$$

The factor H may be derived by simple geometric considerations (not presented here) and is

$$H = \frac{b}{2} \left\{ \sqrt{\frac{(b+l)^2(2C+D_s-2e)^2}{[D_s^2-(b+l)^2]^2}} - \frac{D_s^2-(2C+D_s-2e)^2}{D_s^2-(b+l)^2} \right. \\ \left. - \frac{(b+l)(2C+t-2e)}{t^2-(b+l)^2} \right\}.$$

APPENDIX V
FIGURES 1-8

Figure 1. SIMPLE FLAT-PAD HYDROSTATIC BEARING

Figure 2. FOUR-PAD HYDROSTATIC JOURNAL BEARING WITH NO APPLIED LOAD

Figure 3. FOUR-PAD HYDROSTATIC JOURNAL BEARING WITH APPLIED LOAD

$$\frac{R}{a_f A_p P_s} = \frac{\beta'}{2(1-\beta')} \left\{ \frac{-\sqrt{(\beta')^2 + 4(1-\beta')(1-\epsilon)^6} - \beta'}{(1-\epsilon)^6} - \frac{-\sqrt{(\beta')^2 + 4(1-\beta')(1+\epsilon)^6} - \beta'}{(1+\epsilon)^6} \right\}$$

Figure 4. LOAD VERSUS DEFLECTION OF A FOUR-PAD HYDROSTATIC JOURNAL BEARING FOR VARIOUS INITIAL PRESSURE RATIOS. LOAD IS APPLIED IN-LINE WITH TWO OPPOSING PADS AND SHAFT CENTERLINE REMAINS PARALLEL TO JOURNAL CENTERLINE FOR ALL LOADS

Figure 5. FOUR-PAD HYDROSTATIC JOURNAL BEARING WITH ECCENTRIC APPLIED LOAD. SHAFT'S CENTERLINE DOES NOT REMAIN PARALLEL TO JOURNAL CENTERLINE

$$\frac{R}{q_f A_p P_s} = \frac{\beta'}{2(1-\beta')} \left\{ \left(\frac{q_f}{q_{fb}} \right)^2 \frac{\sqrt{(\beta')^2 + 4(1-\beta')(1-\epsilon)^6 (q_{fb}/q_f)^2} - \beta'}{(1-\epsilon)^6} - \left(\frac{q_f}{q_{fr}} \right)^2 \frac{\sqrt{(\beta')^2 + 4(1-\beta')(1+\epsilon)^6 (q_{fr}/q_f)^2} - \beta'}{(1+\epsilon)^6} \right\}$$

Figure 6. LOAD VERSUS DEFLECTION OF A FOUR-PAD HYDROSTATIC JOURNAL BEARING LOADED IN-LINE WITH TWO OPPOSING PADS SHOWING EFFECTS OF SHAFT TILT. FOR INITIAL PRESSURE RATIO EQUAL 0.5 ONLY

Figure 7. COMPARISON OF PREDICTED AND EXPERIMENTAL LOAD VERSUS DEFLECTION CURVES FOR INITIAL PRESSURE RATIO OF 0.5

Figure 8. EXPERIMENTAL APPARATUS