DEEP INELASTIC ELECTRON-PROTON AND
ELECTRON-NEUTRON SCATTERING

Avadis Hacinliyan

The Enrico Fermi Institute
and The Department of Physics
The University of Chicago, Chicago, Illinois 60637

SEPTEMBER, 1971

Contract No. AT (11-1)-264

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

MASTER

DEEP INELASTIC ELECTRON-PROTON AND
ELECTRON-NEUTRON SCATTERING ⁺₀

Avadis Hacinliyan

September, 1971

The Enrico Fermi Institute
and the Department of Physics
The University of Chicago, Chicago, Illinois 60637

ABSTRACT

Formulae for the structure functions W_1 and W_2 are obtained using the statistical approach to the Veneziano model together with Pomeron dominance. A form for W_2 is obtained that scales and agrees well with the data. The question of the scaling of W_1 is discussed in relation to the available data. Phase space corrections and generalized Glauber corrections are then discussed in connection with e-D scattering. These corrections are found to be much smaller than the experimentally observed difference between the deuteron cross section and twice the free proton cross section. Hence, contributions from both the Pomeron and lower trajectories are used to generalize the model. Satisfactory agreement is obtained.

⁺Supported in part by the U. S. Atomic Energy Commission.

⁰Submitted to the Department of Physics, The University of Chicago, in partial fulfillment of the requirements for the Ph.D. degree.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

I. INTRODUCTION

Inelastic electron-proton and electron-neutron scattering are extremely interesting because they enable us to use the electromagnetic interactions in order to probe the internal structure of strongly interacting particles, in particular protons and neutrons. In elastic scattering, the virtual photon interacts with the target nucleon in a coherent way and the form factors that we obtain are roughly related to the average shape of the nucleon. On the other hand, in the inelastic case, the virtual photon interacts in an incoherent fashion with the target nucleon, and the form factor that we obtain reflects the instantaneous composition of the target. Thus the inelastic experiments are a useful tool in understanding the structure of the hadrons. Furthermore, differences found between the e-p and e-n scattering can be extremely illuminating both in understanding the difference between the proton and the neutron and as a means to learn something about the deuteron as a bound state of the proton and neutron.

Perhaps the most important property of the data is that it exhibits scaling. Although we will go into the mathematical details later, the fact that photons exist in two helicity states means that deep inelastic scattering can be described by two Lorentz invariant structure functions, which, in turn, will depend on two independent Lorentz invariant variables. For instance, these variables can be chosen to be the four momentum squared of the virtual photon and the total center of mass energy squared in the final state. If one makes the assumption that there are no fundamental dimensions in the theory, then the structure functions

can only depend on the dimensionless ratio of the two independent variables. This phenomenon of scaling was first observed by Bjorken.¹

It is immediately obvious from unitarity that the deep inelastic scattering cross section can be related to the imaginary part of the forward Compton scattering amplitude for virtual photons. Thus, the structure functions may be replaced by the cross sections for transverse and longitudinal virtual photons. Experimentally, the ratio of the longitudinal to the transverse photon cross section turns out to be small. This implies that if one of the structure functions scales, the other must also scale within more or less reasonable limits. It also means that models that predict a high or unbounded value of this ratio, such as the Vector Meson Dominance Model², should be treated with caution.

In marked contrast to the elastic case, the data exhibit scaling and fail to show any significant decrease for intermediate values of the scaling variable (which is proportional to the center of mass energy). Therefore new theories must be formulated. It is immediately obvious that successful theories must, in general restrict the number of channels that will contribute to the process, since the threshold behaviour of a channel does not a priori scale. In other words, we must somehow prevent new channels from opening in an arbitrary fashion as the center of mass energy increases. For example, we can take a few restricted classes of channels as the principal contributors to the process, or open the new channels in a controlled fashion to ensure scaling.

One of the models of the former class is the parton model³. In this theory, the proton is assumed to consist of nearly free pointlike constituents with small transverse momentum. The total cross section is the incoherent sum of individual parton cross sections.⁴ This model does predict the scaling of the structure functions. Furthermore, the smallness of the ratio of longitudinal to transverse cross sections imply that most of the

partons should have half integer spin. A very tempting association here is to assume that the partons are the quarks.

One of the immediate consequences of this is the requirement that the neutron cross section be $2/3$ of the proton cross section. This does not seem to agree with experiment. The values of the ratio of the deuterium cross section to the hydrogen cross section tend to increase with the center of mass energy and asymptotically approach 2 instead of $5/3$.⁵ This casts some doubt on the simple quark-parton model and gives support to a Regge type Model.⁶ In a Regge model, we assume that there are contributions from a higher isoscalar (Pomeranchuk) trajectory and a lower isovector ($\rho - A_2$) trajectory to the structure functions. The difference between the neutron and proton structure functions is attributed to the lower trajectory. It is possible to modify the parton-quark model to increase compatibility with the data by introducing additional $q\bar{q}$ pairs. However the increase of the D/H ratios with energy can be much more naturally explained with a Regge type model.

The main purpose of this paper is to apply the Dual Resonance Model⁷ to deep inelastic processes. It is well known that⁸ the dual amplitude can be factorized by introducing an infinite set of harmonic oscillators. Since the deep inelastic scattering process probes the internal structure of hadrons, this model should be relevant here. The only complication is how to introduce electromagnetic interactions into the dual scheme. It is possible to write a Lagrangian for the ensemble of harmonic oscillators and introduce the electromagnetic interactions through the minimal coupling. Although at first sight it seems possible to write a gauge invariant interaction, there are several points that should be treated with caution.⁹ It is not even clear if the electromagnetic interactions themselves are dual. Several attempts

have been made to introduce electromagnetism into the dual scheme. A recent attempt by Landshoff and Polkinghorne¹⁰ has the desired poles, factorization and gives a scaling form factor resembling the shape of experimental data. Our point of view will be slightly different. We will break the strict duality both by using the high energy statistical approximation to the Veneziano Model¹¹ and by smearing out the interaction vertex from the well known singular form that gives strict duality.

According to the Freund-Harari conjecture¹² the Pomeranchuk Trajectory is excluded from the Born approximation to the dual resonance model. Therefore the success of dual models would mean that the main contribution to the structure functions comes from resonances (which are dual to the lower trajectories). A similar conclusion seems to be reached by Bloom and Gilman¹³ as well. On the other hand, the shape of the difference between the proton and neutron cross sections indicates that we need at least two trajectories with differing intercepts; the trajectory with the larger intercept being an isoscalar and the next trajectory being an isovector. The Pomeron and $\rho - A_2$ trajectories are the most logical choices. Therefore, our theory will be more satisfactory if we can include the Pomeron. This is done by taking the Pomeron as a statistical ensemble of resonances with an effective trajectory with arbitrary slope. Since the statistical approximation is asymptotic, the trajectory need not consist of poles; we can not distinguish them from cuts anyway. Therefore this approximation is not incompatible with the usual identification of the Pomeron as being dual to the background. In this way, we will seek to introduce both the Pomeranchuk and the lower trajectories into the theory.

It is also tempting to see if one can predict the exact form of the structure functions from such a model, and in particular, whether or not scaling follows from this.

Therefore, by taking the harmonic oscillator formalism as the physical basis of the dual resonance model and by using the statistical approximation to obtain and generalize the asymptotic properties of the model, we will be able to retain the Pomeranchuk trajectory in the dual scheme. We will get a Gaussian form factor which is typical of harmonic oscillators. This form factor will exhibit scaling and will fit data for W_2 successfully. Unfortunately, our results for the other structure function W_1 will be inconclusive; in particular, whether it scales will be questionable.

In order to apply our model to deep inelastic electron-neutron data, we must investigate the corrections that are necessary since the scattering is not off free neutrons but off a bound state. Hence, a second purpose of this paper will be to investigate the corrections due to the structure of the deuteron. We will study in some detail the phase space restrictions that exist, the corrections in the matrix element due to the fact that the target particle is no longer at rest, and the corrections stemming from shielding of one of the particles by the other (generalized Glauber corrections¹⁴). It will be found that these corrections alone are not enough to explain D/H experimental data. Therefore, we will apply our model to this situation by generalizing it through the addition of both the Pomeranchuk trajectory and the lower trajectories. We will thus be able to isolate the relative contributions from each of these trajectories.

II. PRELIMINARIES

This section will be devoted to exposition of preliminary material and to establishing the notation to be used throughout. First, we will set up the kinematics and structure functions for deep inelastic scattering. Then, we will briefly review the harmonic oscillator formalism and statistical approximation to the dual resonance model.

At the simplest level, the process of inelastic electron-proton scattering is treated in the lowest order as a one photon exchange process as shown in Figure 1.⁴ The notation that we shall describe here will be used throughout this paper: An electron of four momentum $k=(E_0, \vec{k})$ is scattered off a hadron to a final four momentum $k'=(E', \vec{k}')$. The four momentum of the virtual photon is denoted by $q=(q^0, \vec{q})$. The final hadronic state is identified by its energy squared $s=(P+q)^2 = M_f^2$ where M_f denotes the effective mass. All the non Lorentz covariants refer to the laboratory frame unless stated otherwise. In discussing scattering off deuterium, we will also need a notation for the momentum of the target hadron since it will no longer be at rest in the lab frame. For this purpose, we will occasionally use $P=(E_p, \vec{P})$. Frequently, this will be simply $(E_p=M, \vec{P}=0)$.

We now define the two independent Lorentz-invariant variables at the γ -hadron vertex

$$q^2 = q^{\mu 2} - q^0^2 \quad (II.1)$$

$$v = P \cdot q / M = q^0 \quad (\text{in the lab frame}), \quad (II.2)$$

Except in the definition of q^2 , where because of conventional usage we have introduced an extra minus sign, scalar products of four vectors are defined as $A \cdot B = A_0 B_0 - \vec{A} \cdot \vec{B}$. The dimensionless scaling variable can now be introduced as

$$\omega = 2M\nu/q^2 \quad (II.3)$$

The following kinematical relations are useful

$$s = M^2 + 2M\nu - q^2 \quad (II.4a)$$

$$\text{or } s = M^2 + q^2(\omega - 1) \quad (II.4b)$$

If we introduce the scattering angle in the laboratory frame θ , we can write

$$q^2 = 4E_0 E' \sin^2 \theta / 2 \quad (II.5a)$$

$$\nu = q^0 = E_0 - E' \quad (II.5b)$$

We now proceed to calculate the graph in Fig 1, obtaining the results

$$M_{fi} = e[\bar{u}_e \gamma_\mu u_e] \langle P | J_\mu^h | n \rangle \quad (II.6a)$$

$$d\sigma = e^2 M / (4\pi E_0 E_{p \text{ rel}} q^4) m_{\mu\nu} W^{\mu\nu} E' dE' d\Omega \quad (II.6b)$$

where

$$\begin{aligned} m_{\mu\nu} &= \text{Tr} [(K+m_e) \gamma_\mu (K'+m_e) \gamma_\nu] \\ &= 2(k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k \cdot k') \end{aligned} \quad (II.7a)$$

The electron mass has been ignored in writing the last line. Also,

$$W_{\mu\nu} = \int d^3P' M / (2\pi M_f) \frac{1}{f} \delta^4(P+q-P') \langle P | J_\mu^h | P' \rangle \langle P' | J_\nu^h | P \rangle \quad (II.7b)$$

J_μ^h denotes the hadronic current. It is possible to write (II.7b) in the following equivalent forms

$$W_{\mu\nu} = \int \frac{d^4x}{4\pi} e^{iq \cdot x} \langle P | [J_\mu^h(x), J_\nu^h(0)] | P \rangle \quad (II.7c)$$

$$\propto \sum_{n, E_n = s} \langle P | J_\mu^h(-q) | n \rangle \langle n | J_\nu^h(q) | P \rangle \quad (II.7d)$$

Because of gauge invariance and Lorentz invariance, it is possible to write $W_{\mu\nu}$ in terms of only two structure functions W_1 and W_2 . These in turn can only depend on the two available Lorentz invariant variables ν and q^2 . We introduce them as follows:

$$W_{\mu\nu} = -[g_{\mu\nu} - q_\mu q_\nu / q^2] W_1(\nu, q^2) + (P_\mu - P \cdot q q_\mu / q^2) (P_\nu - P \cdot q q_\nu / q^2) W_2(\nu, q^2) / M^2 \quad (II.8)$$

The substitution of (II.8) and (II.7) in (II.6) gives the following formula for the cross section¹⁵.

$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha^2}{4E_0^2 \sin^4 \frac{\theta}{2}} \left[W_L(\nu, q^2) \cos^2 \frac{\theta}{2} + 2W_1(\nu, q^2) \sin^2 \frac{\theta}{2} \right]. \quad (\text{II.9})$$

There is a frequently used alternate notation. Since the deep inelastic cross section is related to the imaginary part of the forward Compton scattering amplitude for virtual photons, we can use the transverse and longitudinal virtual photon cross sections instead of the structure functions introduced above.¹⁶ It is easy to obtain these from Equation II.8. The relations between the two notations are:

$$W_1(\nu, q^2) = \frac{\nu - q^2/2M}{4\pi^2\alpha} \sigma_T(\nu, q^2) \quad (\text{II.10a})$$

$$W_2(\nu, q^2) = \frac{\nu - q^2/2M}{4\pi^2\alpha} \frac{q^2}{q^2 + \nu^2} (\sigma_T(\nu, q^2) + \sigma_L(\nu, q^2)) \quad (\text{II.10b})$$

Dividing these two relations by each other, setting $R = \sigma_L/\sigma_T$,

$$W_1/W_2 = (1 + \nu^2/q^2)/(1 + R). \quad (\text{II.10c})$$

Obviously $R \geq 0$, and we get a set of inequalities. They could have been also obtained through an application of the Schwartz Inequality to the quadratic form $\epsilon_\mu \epsilon_\nu W^{\mu\nu}$. These inequalities read:

$$W_2(1 + \nu^2/q^2) \geq W_1 \geq 0. \quad (\text{II.11})$$

At this point, we present a very brief discussion of the important properties of deep-inelastic data. Measurements of the inelastic electron-proton scattering cross sections at various angles permit a separation of the two structure functions in a limited kinematic region. νW_2 typically scales. As a function of ω , it increases steeply from zero at the threshold ($\omega = 1$) and later flattens to an approximate value of 0.3. There are indications that νW_2 may begin to decrease at large $\omega \geq 10$. The value of R seems to be small, 0.20 ± 0.10 . Because of the $\tan^2 \theta/2$ factor in Eq.(II.9) and the fact that the measurements have been done only up to $\theta = 34$ degrees, W_1 is more difficult to determine. Therefore it is more difficult to ascertain if it scales. Finally the ratios of cross sections for scattering off deuterium over that for scattering off hydrogen tend to increase with ω and eventually flatten to approximately 2.0. (This ratio is referred to as the D/H ratio.)

We now proceed to a discussion of the dual resonance model and the statistical approximation to it. For a simple case, the amplitude proposed by Veneziano is

$$A(s, t) = \int_0^1 x^{-\alpha_s-1} (1-x)^{-\alpha_t-1} dx. \quad (\text{II.12})$$

In order to factorize this amplitude, we rewrite the factor

$$\begin{aligned} (1-x)^{-\alpha_t-1} &= \exp [(2P \cdot P' + C) \ln(1/(1-x))] \\ &= \exp [(2P \cdot P' + C) \sum_n x^n/n] \end{aligned} \quad (\text{II.13})$$

We want to identify the coefficient of x^n with a form $\sum_p \langle 0 | \Gamma(p) | x \rangle \langle x | \Gamma(p') | 0 \rangle$. To this end, let us introduce oscillator-like

creation and annihilation operators, $a_\alpha^{+(n)}$, $\alpha=(1,2,3,4,5)$. The fifth dimension is needed if $C \neq 0$. For simplicity, we will ignore this dimension at present. The commutation relations are taken as

$$[a_\alpha^{(n)}, a_\beta^{+(m)}] = \delta_{nm} g_{\alpha\beta} \quad (II.14)$$

$$[a_\alpha^{(n)}, a_\beta^{(m)}] = [a_\alpha^{+(n)}, a_\beta^{+(m)}] = 0$$

Then we write the vertex as

$$\langle 0 | T(p) | n \rangle = \langle 0 | \exp i \sum_n \sqrt{\frac{2}{n}} (a_{n\mu}^+ - a_{n\mu}) \cdot p^\mu | n \rangle \quad (II.15)$$

This means that we can factorize the amplitude as

$$A(s,t) = \langle 0 | T(p) \Pi(\alpha_s) T(p') | 0 \rangle, \quad (II.16)$$

where

$$\Pi(\alpha_s) = (H - \alpha_s - i\epsilon)^{-1} = \int_0^1 x^{H - \alpha_s - i\epsilon - 1} dx \quad (II.17)$$

with

$$H = \sum_n n a_\alpha^{+(n)} a_\alpha^{(n)} \quad (II.18)$$

and $T(p)$ is given by II.15.

The appearance of these harmonic oscillators for the internal degrees of freedom of this problem is quite natural since the bound state picture of quarks in a harmonic oscillator potential would naturally give rise to families of linear trajectories. To make the analogy more explicit,

we introduce a Boson field $x_\mu(\xi, n)$ with the free Lagrangian

$$L = (1/4\pi) (\partial x_\mu / \partial \xi \partial x^\mu / \partial \xi - \partial x_\mu / \partial \eta \partial x^\mu / \partial \eta) \quad (II.19)$$

We have the equation of motion

$$\partial^2 x_\mu / \partial \xi^2 = \partial^2 x_\mu / \partial \eta^2 \quad (II.20)$$

Imposing periodic boundary conditions on x_μ in the form $x_\mu(\xi+2\pi, n) = x_\mu(\xi, n)$, we expand a solution of II.20 into plane waves

$$x_\mu(\xi, n) = \sum_{n=1}^{\infty} (2n)^{-1/2} (a_n^{(n)} \exp in\xi + a_n^{+(n)} \exp -in\xi), \quad (II.21)$$

where we have assumed that the center of mass motion is separated. The a 's and a^* 's are the annihilation and creation operators defined above. It is possible to obtain the Hamiltonian from II.19 and II.21; we obtain, naturally, II.18. The interaction vertex is just

$$: \exp ik_\mu x^\mu(0) : = : \exp ik_\mu \int x^\mu(\xi) \delta(\xi) d\xi : \quad (II.22)$$

This is the translation operator. The vertex function is dumping momentum k_μ at that vertex. This singular nature of the interaction causes the strict duality.

As a result of this factorization, the density of states for the hadrons grows exponentially with the energy. Since this growth is so fast, it is questionable if we should continue to distinguish between the individual particles at high energy. Rather, a statistical treatment seems to be more appropriate and we will describe the products of hadronic reactions by a

statistical ensemble average (e.g. temperature) instead of specifying the individual quantum numbers.

Our starting point is Eq.II.12. The factorization of this amplitude is shown in Equations II.15 thru 17. We observe that the absorptive part of Eq.II.17 is a delta function. This is reminiscent of the microcanonical ensemble. We pass to the canonical ensemble by the replacement

$$\Pi(\alpha_s) = (H - \alpha_s - i\epsilon)^{-1} \rightarrow C \exp[-\beta(H - \alpha_s)] \quad (II.23)$$

where β is related to the "temperature" that will be specified below. C is a constant factor which is not important for our purpose. If we insert Eq.II.23 into II.16 we obtain as $\beta \rightarrow 0$,

$$A(s,t) = C \exp(\beta \alpha_s) \beta^{-\alpha} t \quad (II.24)$$

To ensure Regge behavior we must have $\beta \alpha_s \rightarrow 0(1)$. Another way to obtain this would be to set $x = \exp(-\beta)$ in II.12 and perform a steepest descent estimate of the asymptotic behavior of the result as $\alpha_s \rightarrow -\infty$.

This constitutes the essence of the statistical approach to the Veneziano Model. As a further generalization, we may introduce a density matrix $\rho \exp(-\beta H)$ in order to perform sums over the intermediate states where the energy is specified. This is no more than another example of the transition from the microcanonical to the canonical ensemble.

III. CALCULATION OF THE ASYMPTOTIC FORMULA FOR THE STRUCTURE FUNCTIONS AND COMPARISON WITH SLAC-MIT PROTON DATA

A. CALCULATION AND DISCUSSION

In order to obtain the asymptotic formula for the structure functions, all that we need to do is to take Eq. II.7d and perform the indicated sum over the intermediate states by introducing a density matrix.¹⁷ The sum has precisely the form of a sum over a microcanonical ensemble and we can replace it with a sum over a canonical distribution. There is a small ambiguity here. In introducing the density matrix, $\rho \exp(-\beta H)$, we have chosen $\beta(s-M^2) \rightarrow 0(1)$. This means that energy in excess of the nucleon mass has been chosen as the conjugate variable to the statistical temperature, while we could have chosen s as well. Asymptotically the two are indistinguishable, while in general, this later choice would correspond to the replacement $\omega \rightarrow \omega' = \omega + M^2/q^2$; the alternate scaling variable suggested by Gilman.¹³ We will consider this point when the comparison with experimental data is done, although there is no a priori justification for either choice.

One crucial obstacle that remains is the choice of the current operator $J_\mu^h(q)$. Without specifying this operator, we could not go any further. The most logical candidate would be the well known translation operator $\exp i q \cdot x(0)$ ¹⁸ if angular momentum is neglected.¹⁹ However, we will smear out the interaction by assuming that it takes place at a location

$$\bar{x}_\mu = \int_0^{2\pi} x(\xi) f(\xi) d\xi \quad (III.1a)$$

$$= x_{\mu 0} + 2 \sum_{n=1}^{\infty} x_{\mu n} f_n \quad (III.1b)$$

Therefore, the vertex function now assumes the form

$$\exp(iq \cdot \bar{x}) = \exp i q_{\mu} (x_{\mu 0} + 2 \sum_{\mu n} x_{\mu n} f_n) \quad (\text{III.2a})$$

$$= \exp(i q_{\mu} x_{\mu}(0)) \exp(i q_{\mu} \delta x_{\mu}) \quad (\text{III.2b})$$

$$\text{where } x_{\mu}(0) = x_{\mu 0} + \sum_{\mu n} x_{\mu n} \quad (\text{III.2c})$$

In order to ensure scaling, we have to resort to a somewhat arbitrary step at this point. This consists of removing a divergent renormalization factor from the unsmeared vertex of the form:

$$\begin{aligned} \langle \exp i q \cdot x(0) \rangle_0 &= \exp(2 \sum_n q^2 (f_n^2 - 1)/n) \\ &= \exp(2 q^2 \sum_n \Delta f_n^2) = \exp c q^2 \quad (\text{III.3}) \end{aligned}$$

We now assume that the current operator is such that it will produce a Regge factor $s^{\alpha(0)}$ or $(s-M^2)^{\alpha(0)}$ multiplied by the form factor obtained by substituting III.2b into II.7d. We get

$$\sum_n \langle P | j_{\mu}(q) | n \rangle \langle n | j_{\nu}(q) | P \rangle \alpha \beta^{-\alpha(0)} \exp(-2 q^2 \sum_{n=1}^{\infty} \frac{\Delta f_n^2}{n} (1 - e^{-n\beta})). \quad (\text{III.5})$$

Finally, we assume that $\sum_n \Delta f_n^2/n$ and $\sum_n \Delta f_n^2$ both converge absolutely. If we now expand the exponential $\exp(-n\beta)$, set $\lambda = 2 \sum_n \Delta f_n^2$, and compare leading coefficients of III.5 and II.8, we get²⁰,

$$W_1 = c_1 (s-M^2)^{\alpha(0)} \exp(-\lambda q^2/(s-M^2)) \quad (\text{III.6a})$$

$$W_2 = c_2 (s-M^2)^{\alpha(0)-2} \exp(-\lambda q^2/(s-M^2)) \quad (\text{III.6b})$$

If we had not called for an extra Regge type factor in the current operator, we would have implicitly chosen $\alpha(0) = 1$. This intercept

is the proper one for the Pomèranchuk Trajectory. (Because of kinematical reasons, W_2 behaves like $(s-M^2)^{\alpha(0)-2}$.) However, the next order term in the asymptotic expansion for W_2 would have gone like $(s-M^2)^{-2}$, while the next available lower trajectory has an intercept of $1/2$, not zero. To expedite the introduction of contributions from lower trajectories, we have instituted the extra Regge type factor. This factor may be generated, for instance, by introducing a fifth dimension. To get the leading behaviour of W_2 (assuming that it corresponds to the Pomeron), we do not need this extra assumption.

Before we compare this formula with experimental data and also discuss the possibility of contributions from lower trajectories, we will make a few important comments. We may immediately write $\sqrt{W_2}$ in the alternate form

$$\sqrt{W_2} = c_2/2M \left(\omega/(\omega-1) \right) \exp(-\lambda/(\omega-1)) \quad (\text{III.7})$$

This shows that $\sqrt{W_2}$ scales. However, as it stands, $2MW_1$ does not scale. The reason for this is the fact that what we have calculated here is the Regge limit and not the scaling limit. In simpler terms, we have calculated the limiting value for $s \rightarrow \infty$, $v \rightarrow \infty$ at fixed q^2 . But the scaling limit is defined as $s \rightarrow \infty$, $v \rightarrow \infty$, and $q^2 \rightarrow \infty$, with $\omega = 2Mv/q^2$ kept fixed. Unless we know in advance that the result scales (or does not scale), we can not necessarily draw a conclusion concerning the scaling limit from the Regge limit. For example, we may put

$$2MW_1 = c_1'' (\omega-1) \exp(-\lambda/(\omega-1)) \quad (\text{III.8})$$

and at fixed q^2 , this is completely consistent with the Regge limit. We have introduced a $1/q^2$ factor into c_1 . Because of the gauge conservation constraint reflected by Eq. II.8, W_1 is multiplied by a tensor of order $1/q^2$ as $q^2 \rightarrow 0$,

while W_2 is multiplied by a tensor of order $1/q^4$. Therefore, as $q^2 \rightarrow 0$, W_2/W_1 must be $O(q^2)$. Division of W_1 by q^2 is one possible way of achieving this, although, admittedly it is not the most logical way to satisfy gauge invariance. It must also be observed that we can not incorporate the gauge invariance condition during the process of taking an asymptotic limit. On general grounds, it can be shown that a dual, gauge invariant theory for electromagnetic currents can be constructed.⁹ The experimental basis -if any- for the scaling of W_1 will be discussed separately.

Another trouble with this model is the apparent inconsistency between the inequality II.11 and the asymptotic formulae III.6. This problem may be a unitarity trouble and this is characteristic of dual resonance models (or models derived therefrom). Alternatively, it may stem from the simple asymptotic approximation that we have been using. The Gaussian form factor is also totally unacceptable near the threshold and particularly, in the region of timelike q^2 . By sheer accident, the asymptotic formula cuts off at the threshold ($\omega \rightarrow 1$), where the statistical approximation is expected to break down. Of course, the exponential cutoff is too harsh. These points are mentioned here as limitations of our theory. If we just accept Eq. III.6 as asymptotic formulas (which they are) and then extend them in the intermediate region²¹, we will obtain useful agreement with experiment. Nearly all of presently available data is in such a region that the abovementioned limitations are only of academic interest.

The structure functions III.6 are probably more general than their derivation may seem to indicate, the Pomeranchuk theorem and the scaling hypothesis are the essential assumptions. Coupling these to other reasonable assumptions also yields Eqs. III.6²².

B. COMPARISON WITH SLAC-MIT PROTON DATA

In this subsection, we do not consider the contributions from lower trajectories. The simplest comparison that we make is to take the structure functions III.6a, III.6b and fit to data, obtaining the quantities c and λ from the fit. As we can see in Fig. 2a, except near threshold, the fit for W_2 is very good, as expected. The values for c_2 and λ are 0.74 (GeV) and 1.54 respectively. If we replace ω by ω' (corresponding to the choice $\beta s \rightarrow 1$ instead of $\beta(s-M^2) \rightarrow 1$), the agreement close to the threshold improves, but the overall chi-squared deteriorates. All this agrees with the findings of Bloom and Gilman (Ref 13). The values of c_2 and λ are 0.91 (GeV) and 2.30 when ω' is used. This fit is displayed in Fig. 2b.

Although the situation with respect to W_2 is simple, the situation with respect to W_1 is much more complicated. First of all Eq. III.6a does not seem to agree with data. Instead, a more satisfactory form seems to be

$$W_1 = C (s-M^2) \exp(-\lambda q^2) \quad (III.9)$$

First of all, this form does not show scaling. Secondly, it has the required Regge behavior, the additional q^2 dependent factor is reminiscent of a factor that we dropped in the previous subsection - albeit with the wrong sign!-. If a factor of the form $\exp -\lambda/(\omega-1)$ is added to III.9, the value of λ obtained from the fit is compatible with zero. The results of the fit to W_1 data separated both by the SLAC-MIT group and independently by the author are shown in Figs. 3a and 3b. In connection with the aforementioned possibility of nonscaling of W_1 , the following statements can be made.

The literature concerning the scaling of W_1 is contradictory and confusing. On one hand, the SLAC-MIT group claims that W_1 scales.²³ On the

other hand, Chen²⁴ has fitted SLAC-MIT W_1 data with a nonscaling form²⁵. If we look at Eq.II.10c, we see that scaling of νW_2 will imply scaling of $2M W_1$ either if $R=q^2/\nu^2$ or if R is a constant, provided that $2M/\nu$ is much less than one, or at least, not greater than the errors in $2M W_1$. In the presently measured region, $2M/\nu$ ranges between .06 and .30 while the errors range between 0.05 and 0.15. Since we do not know W_1 to sufficient accuracy because of the $\tan^2 \theta/2$ factor multiplying it, it therefore becomes possible that νW_2 scales while $2M W_1$ does not scale. Of course, if W_1 is determined to greater accuracy, we may be able to say something more definite. Provisionally, we find it more likely that W_1 does not scale.²⁶

Our conclusion from this analysis would be that νW_2 definitely scales and fits very well with a single trajectory contribution in our form, while the scaling of W_1 is questionable. We must agree that there are indications which imply that W_1 may not scale, or even it scales, it begins to do so at a higher value of the scaling variable (or more correctly, center of mass energy). If this latter alternative is true, it will then be very difficult to fit W_1 with a scaling form derived from the present theory, since the scaling limit and Regge limit will be even further apart.

IV. CALCULATION OF CORRECTIONS DUE TO THE STRUCTURE OF THE DEUTERON

Before we embark on a detailed investigation of the deep inelastic electron-deuterium scattering data from the SLAC-MIT group, we must decide if the characteristic shape of the Deuterium cross sections divided by the Hydrogen cross sections (D/H ratios) can be attributed solely to the corrections which arise because the neutron is not free. In other words, we wish to determine the extent to which the simple statement,

$$\sigma_D = \sigma_P + \sigma_N \quad (IV.1)$$

should be amended. This statement has been very frequently assumed in the literature. Let us first list the possible candidates that should be considered in this regard:

- a- Phase space restrictions due to the fact that the neutron is not free, but is part of a bound state.
- b- The matrix element in Eqs. II.6 contains $\vec{p}$ dependent terms both explicitly through the tensor $W_{\mu\nu}$, and implicitly through the phase space factor.
- c- The "Generalized Glauber Corrections". The Glauber Corrections arise (Ref.14) because one of the nucleons casts a shadow over the other. This means that Equation IV.1 implies double-counting. Although at first sight, the double-counting would be proportional to the product of the two cross sections, and hence of order α^4 , we will generalize the model in a manner to be described below, so that the corrections we propose will be of order α^2 .

In the first subsection, we will consider items a and b above, deferring consideration of c to the next subsection.

A. PHASE SPACE CORRECTIONS AND CORRECTIONS DUE TO THE MOTION OF THE TARGET

The phase space corrections were first calculated by West²⁷. His apparent conclusion is that the well known shape of the SLAC-MIT D/H data could be explained as a phase space correction, if a suitable deuteron wave function were chosen. Upon recalculation of this correction for the SLAC-MIT data points, we have found the result to be quite small, if the Hamada-Johnston²⁸ deuteron wave function is used. An analytic form for this wave function given by I. J. McGee²⁹ was found to be extremely convenient for our calculations. The form of this function in momentum space,

$$\psi(\vec{p}) = \sum_{n=1}^{10} c_n (\vec{p}^2 + \alpha_n^2)^{-1}, \quad (IV.2)$$

is very simple. Occasionally, we will refer to the quantities α_n and c_n as "poles" and "residues" in a loose sense.

In spite of the smallness of the corrections with this wave function, we should remark that by making some changes in the parameters of the wave function, the corrections can be made to increase. For example, instead of the two single poles at approximately $\alpha_n = 0.90$ and 0.97 , if we introduce a double pole at the nucleon mass, the corrections substantially increase. Still, it is quite difficult to explain the entire shape of the curve in this way. The crucial dependence of these corrections on the deuteron wave function (See West, Ref.27) opens the way to speculation, as long as the relativistic deuteron problem remains unsolved. Furthermore, our knowledge of the deuteron wave function is derived from low energy data, while the dependence is on high momentum components in the wave function. Presumably we can have a sufficient amount of such components to give sufficient corrections for D/H data, without affecting the low energy

behavior of the wave function.

If we accept the non-relativistic wave function as an approximation, there is a conceptual difficulty as to what this means. Fortunately, regardless of this difficulty, the calculated corrections in the region of SLAC-MIT D/H data are small. However, as we shall see, the results of the three alternative interpretations are widely different close to the threshold ($\omega = 1$). If more data become available at ω close to 1, we could be able to better decide among the alternatives.

We now present the details of this calculation. The phase space corrections result from the fact that when the virtual photon hits the bound nucleon, the available energy must be at least equal to -or greater than- the lowest threshold energy, namely M . Admittedly, there is an ambiguity here; since this process is inelastic, and the inelastic threshold starts at $(M+m_\pi)$, this value could have been used. For simplicity, we will adopt the former choice. The situation is shown in Fig.4. The threshold condition reads,

$$(\vec{p} + \vec{q})^2 \geq M^2, \quad (IV.3a)$$

or upon simplification

$$2 \vec{p} \cdot \vec{q} - q^2 \geq 0. \quad (IV.3b)$$

Clearly, this relation restricts the region over which the phase space integral must be performed, since we get

$$2 E_p v - 2 |\vec{p}| |\vec{q}| \cos \theta_{pq} - q^2 \geq 0; \quad (IV.3c)$$

or

$$\cos \theta_{pq} \leq [2 E_p v - q^2] / [2 |\vec{p}| |\vec{q}|] \quad (IV.3d)$$

We first notice that if we write (IV.3c) as

$$E_p/M - |\vec{p}| \cos \theta_{pq} [2M/(v^2+q^2)/q^2]/M \geq 1 \quad (\text{IV.4a})$$

and take the scaling limit, we get the inequality,

$$(\omega/M)(E_p - |\vec{p}| \cos \theta_{pq}) \geq 1 \quad (\text{IV.4b})$$

This shows that the phase space corrections scale. Essentially, this is all we need to set up and calculate the relevant phase space integral. Before proceeding, we should discuss the ambiguity in the interpretation of the non-relativistic deuteron wave function. Presumably, we will have to perform integrals of the form,

$$\int_R M(\vec{p}, E_p, v, q^2 \dots) |\psi(\vec{p})|^2 d^3\vec{p}, \quad (\text{IV.5})$$

where $M(\vec{p}, E_p, v, q^2 \dots)$ is the matrix element for the process (essentially Eq. II.6b evaluated in a general frame and not in the rest frame of the proton). R denotes the restricted phase space region and $|\psi(\vec{p})|^2$ is a momentum probability distribution. We will take the non-relativistic wave function as $\psi(\vec{p})$.

The question now arises, how do we interpret the nonrelativistic wave function $|\psi(\vec{p})|^2$. The approach suggested by West is to assume energy conservation at the n-p-d vertex. (For convenience, we will label this as case C.) This means that

$$M_D = P^0 + \sqrt{(\vec{p}^2 + M^2)} \quad (\text{IV.6})$$

where P^0 is the energy of the participating nucleon, $\vec{p}$ is the common momentum of the two nucleons. This puts the spectator on mass shell and the participant

off mass shell. (We are neglecting the deuteron binding energy in the kinematic relations.) That approach would have been strictly true if the wave function were relativistically correct. But, treating the problem unsymmetrically in this fashion is contrary to the spirit of the nonrelativistic calculation that gave us the wave function. Therefore, until we solve the relativistic problem, there are two other alternatives. We can assume that both of the particles share the energy at the n-p-d vertex equally. This puts both of the particles off mass shell, (labelling this as case B) i.e.

$$P_n^0 = P_p^0 = M_D/2 \quad (\text{IV.7})$$

Alternatively, we can put both particles on mass shell and interpret the wave function as a statistical momentum distribution, effectively containing the relevant energy denominator. (Note that we are relaxing energy conservation at the n-p-d vertex. We will label this as case A.) Accordingly, we set

$$P_n^0 = P_p^0 = \sqrt{(\vec{p}^2 + M^2)} \quad (\text{IV.8})$$

We therefore have to compute the region for the phase space integral in each case. We will first consider Case B which is the simplest. Obviously, if the right hand side of Eq. IV.3d is larger than one, there will be no restriction, while if it is less than one, $\cos \theta$ must go from -1 to $(M_D v - q^2)/(2|\vec{p}||\vec{q}|) \geq 0$.

Therefore, the integral will appear as

$$\int_R = \int_{\vec{p}=0}^{\vec{p}=(M_D v - q^2)/2|\vec{q}|} \frac{2\pi|\vec{p}|^2 d|\vec{p}|}{4\pi|\vec{p}|^2 d|\vec{p}|} \left(1 + \frac{M_D v - q^2}{2|\vec{p}||\vec{q}|} \right) \quad (\text{IV.9})$$

We deal with Case A next. The restriction is derived in the same manner as before. If we set $\cos \theta_{pq} \leq 1$ in Eq. IV.3d and use Eq. IV.8

the following quadratic is obtained,

$$4 q^2 E_p^2 + 4 E_p q_0 q^2 - q^4 + 4 M^2 q_0^2 \leq 0 \quad (IV.10)$$

The quadratic form has zeros at

$$E_p^{r\pm} = -q_0/2 \pm \sqrt{[1/q^2(1+4M^2/q^2)]/2} \quad (IV.11)$$

Notice that both roots are real, one being always negative and the other being always positive and $\geq M^{30}$. The quadratic form is negative between $E_p = M$ ($\vec{p}=0$) and $E_p = E_p^{r+}$ ($\vec{p}=\vec{p}^r = \sqrt{(E_p^{r+2} - M^2)}$). It is positive for $E_p > E_p^{r+}$.

The integration region is therefore given by,

$$\begin{aligned} \int_R = \int_{E_p=M}^{E_p=E_p^{r+}} 4\pi |\vec{p}|^2 d|\vec{p}| \\ + \int_{E_p=E_p^{r+}}^{\infty} 2\pi |\vec{p}|^2 d|\vec{p}| \left(1 + \frac{2E_p q_0 - q^2}{2|\vec{p}||\vec{q}|}\right) \end{aligned} \quad (IV.12)$$

Finally, we consider case C. Substitution of IV.6 into Eq. IV.3c

gives,

$$\cos \theta_{pq} \leq (2M_D v - 2v \sqrt{(\vec{p}^2 + M^2)} - q^2) / (2|\vec{p}||\vec{q}|) \quad (IV.13a)$$

We define $\omega = 2M_D v / q^2$, $\omega_d = 2M_D v / q^2 - 1$ and obtain, as before

$$|\vec{p}|^2 - |\vec{p}_d|^2 + q^2(\omega_d^2 - \omega^2)/4 \leq 0 \quad (IV.13b)$$

This quadratic has zeros at

$$2|\vec{p}|^{\pm} = \omega_d |\vec{q}| \pm \sqrt{(\omega_d^2 v^2 + \omega^2 q^2)} \quad (IV.13c)$$

It can be easily verified that for $\omega \geq 1$, the two roots are positive and there is a restriction on the angle integral between these limits.

We can also see that $\cos \theta_{pq} \geq -1$. Hence, the phase space integral has the form

$$\int_0^{p^+} 4\pi |\vec{p}|^2 d|\vec{p}| + \int_{p^-}^{p^+} 2\pi |\vec{p}|^2 d|\vec{p}| \left[1 + \frac{2M_D v - q^2 - 2v \sqrt{\vec{p}^2 + M^2}}{2|\vec{p}||\vec{q}|}\right] \quad (IV.14)$$

Before we evaluate the integrals indicated, we must calculate the P dependence of the matrix element in Eq. II.6b. The essential task is to evaluate the expression $m W^{\mu\nu}$ directly from the definitions II.7a and II.7c. The result of this evaluation, (dropping some numerical constants) is³¹

$$\begin{aligned} \frac{d^2\sigma}{d\Omega dE'} \propto \frac{E'}{q^2} \left[-\frac{q^2 W_1}{ME} + \frac{W_2}{M^2} (2ME - 2Mv + M^2 q^2 / 2ME) \right] \\ + \frac{E'}{q^4} \left[\left(\frac{q^2 W_1}{ME} - \frac{q^2 W_2}{2ME} \right) \left(\frac{k \cdot P - ME}{k \cdot P} \right) + \frac{W_2}{M^2} (2k \cdot P - 2ME + 2Mv - 2q \cdot P) \right] \end{aligned} \quad (IV.15)$$

This expression was deliberately written as a sum of two terms. The first bracketed term is just the standard expression for the cross section in the laboratory frame when the target is at rest. The second term is the correction. For purposes of this section, let us assume that the free proton and neutron cross sections are equal. Hence, we have the result,

$$(d^2\sigma / d\Omega dE')_{APP} = (d^2\sigma / d\Omega dE')_{FREE} [f_R + f_R \epsilon(\vec{p}, v, q^2)] d^3\vec{p} \quad (IV.16)$$

The subscripts APP and FREE denote the apparent cross section for the bound particle and the free particle cross section respectively. Their ratio corresponds to the SLAC-MIT D/H -1 in the absence of other

effects. $\epsilon(\vec{p}, v, q^2)$ is simply the ratio of the second term to the first in Eq. IV.15 and $\int_R$ denotes the phase space integral discussed above. Although the first integral can be evaluated in closed form and the second integral can be numerically approximated, we can easily demonstrate that the second integral is small. (In all the integrals that we have written, a common factor of $|\psi(P)|^2$ is dropped for simplicity.) To do this, we expand $\epsilon(\vec{p}, v, q^2)$ in powers of $\vec{p}$ to $\vec{p}^2$ (non-relativistic approximation). This will definitely over-estimate the correction. The final result is

$$(d^2\sigma/d\Omega dE')_{\text{APP}} = (d^2\sigma/d\Omega dE')_{\text{FREE}} \left[\int_R 1 + \int_R (d^3\vec{p}/2M^2) \cdot (W_2/(2W_1 \sin^2 \theta/2 + W_2 \cos^2 \theta/2)) \right] \quad (\text{IV.17})$$

The bracketed quantity in the second term above is roughly equal to one since the $\sin^2 \theta/2$ term is small. Therefore, the corrections due to the explicit and implicit P dependence in the matrix element are of the order $\langle \vec{p}^2 \rangle / 2M^2$ where $\langle \vec{p}^2 \rangle$ is the mean Fermi momentum. Unless the wave function is dominated by unusually high momentum components, this quantity is expected to be small³². Once more, we see the crucial dependence of the correction on the exact wave function of the deuteron. To demonstrate this dependence further, we take the limits $\omega \rightarrow 1$ and $\omega \rightarrow \infty$ in the three expressions for the phase space integral. For convenience, let us define

$$R' = (d^2\sigma/d\Omega dE')_{\text{APP}} / (d^2\sigma/d\Omega dE')_{\text{FREE}} = D/H - 1. \quad (\text{IV.18})$$

We summarize our results in the table below:

TABLE 1 - VALUES OF R' UNDER VARIOUS HYPOTHESES

	$R' (\omega \rightarrow 1)$	$R' (\omega \rightarrow \infty)$
CASE A Both Part. on M. S.	$1/2 \leq R' < 1$ depending on q^2 indep. of W. Function.	$R' = 1$
CASE B Both Part. off M. S.	$R' = 1/2$	$R' = 1 - 0.5 \int_{2M}^{\infty} (1 - 2M/P) \psi(\vec{p})^2 d^3\vec{p}$
CASE C Spectator on M.S.	$-1 < R' \leq 0$ depending on q^2 indep. of W. Function. ³³	$R' = 1 - 0.5 \int_{3M}^{\infty} (1 - 3M/P) \psi(\vec{p})^2 d^3\vec{p}$

Assuming that we know the free neutron cross section, we see that the threshold behaviour of the D/H ratio (which is so far unknown) will determine which, if any, of these hypotheses is true. On the other hand, the behaviour at large ω depends on the high momentum components of the deuteron wave function (barring Case A). With presently available deuteron wave functions, the corrections shown are negligible.

If we assume that the free neutron and proton cross sections are equal, we can first determine which alternative is true from the threshold behaviour (or by exactly solving the relativistic deuteron problem!). Then, from the behaviour at large ω , we can presumably determine the high momentum components of the wave function. Although this may sound speculative, it is nevertheless a valid viewpoint. Essentially, it is the point of view taken

by West²⁷ (except that he assumes Case C).

Plots of the expected D/H ratios ($=R'+1$) on the basis of corrections outlined in this subsection versus ω are shown in Fig 5. Curves A, B and C are the results under the alternatives A, B and C. Curves D and E are obtained from a modified deuteron wave function (replacing the two single poles close to the nucleon mass with a double pole at the nucleon mass, with a residue determined by a chi-squared minimization) under the alternatives A and C. We see that the corrections fall short of explaining the shape of data, although they increase towards the threshold.

B. GENERALIZED GLAUBER CORRECTIONS IN e-D SCATTERING

As is well known, the Glauber corrections involve the case in which one of the participants casts a shadow on the other and blocks it. If we then simply add $\sigma_D = \sigma_N + \sigma_p$, we will be overcounting. This effect has been first evaluated by Glauber³⁴ using the eikonal approximation, which should be good for small momentum transfers.³⁵

Since the cross section correction involves double scattering, it would be tempting to say that the correction is of order α^4 and hence, we should neglect those corrections in a first order calculation. As mentioned above, we will take a different view of this correction by assuming vector meson dominance. The virtual photon incident on the deuteron will convert itself to a vector meson (this being the only electromagnetic process). The rescattering will be done by the reaction products of the vector meson nucleon scattering.³⁶ Therefore, this correction will be only first order in the electromagnetic interactions. This is just a special case of similar corrections that occur in γ -nuclide interactions.³⁷

We first cast the relevant formulae derived by Glauber and Franco in a form suitable to our calculation. Then we will estimate the vector meson dominance amplitude for rescattering.

The result obtained by Glauber and Franco is the following

$$F_{fi}(\vec{q}, \nu) = \langle f | F(\vec{q}, \vec{s}, \nu) | i \rangle \quad (\text{IV.19})$$

where F_{fi} is the transition amplitude between initial and final states i and f , $\vec{s}$ is the component of the separation $\vec{r}$ between the neutron and proton in the impact plane (perpendicular to $\vec{q}$) and $F(\vec{q}, \vec{s}, \nu)$ is given by

$$F(\vec{q}, \vec{s}, \nu) = e^{i \frac{1}{2} \vec{q} \cdot \vec{s}} f_n(\vec{q}, \nu) + e^{-i \frac{1}{2} \vec{q} \cdot \vec{s}} f_p(\vec{q}, \nu) \\ + \frac{i}{2\pi\nu} \int f_n(\vec{q}' + \frac{1}{2} \vec{q}, \nu) f_p(-\vec{q}' + \frac{1}{2} \vec{q}, \nu) e^{i \vec{q}' \cdot \vec{s}} d^3 \vec{q}' . \quad (IV.20)$$

Here f_n and f_p are the individual scattering amplitudes from the proton and the neutron. For this subsection only, we will choose the rather unorthodox normalization

$$\frac{d^2\sigma}{d\Omega dE'} = \sum_i \sum_f |F_{fi}(\vec{q}, \nu)|^2 , \quad (IV.21)$$

and similar relations for f_n and f_p .

In performing the sum over the intermediate states in this case, there is a small point which should be kept in mind. Since this process is inelastic, some of the terms in the $\sum_f |F_{fi}|^2$ can not contribute because of charge conservation.³⁸ Taking full account of this constraint, substituting Eq. IV.21 into Eq. IV.20 and performing the sum over the intermediate states by using the completeness of the final states, we obtain

$$\frac{d^2\sigma_d}{d\Omega dE'} = \frac{d^2\sigma_p}{d\Omega dE'} + \frac{d^2\sigma_n}{d\Omega dE'} \\ - \frac{i}{\pi\nu} \int |\Psi(r)|^2 e^{-i \frac{1}{2} \vec{q} \cdot \vec{s}} d^3 \vec{r} f_p(\vec{q}, \nu) \int e^{i \vec{q}' \cdot \vec{s}} f_n(\vec{q}' + \frac{1}{2} \vec{q}, \nu) f_p(-\vec{q}' + \frac{1}{2} \vec{q}, \nu) d^3 \vec{q}' . \quad (IV.22)$$

We introduce the deuteron form factor

$$S(\vec{q}) = \int |\Psi(\vec{r})|^2 e^{i \vec{q} \cdot \vec{r}} d^3 \vec{r} \quad (IV.23a)$$

$$= \int \Psi(\vec{p}) \cdot \Psi(\vec{p}-\vec{q}) d^3 \vec{p} \quad (IV.23b)$$

$$= N \sum_{m,n} c_m c_n \arctan |\vec{q}/(\alpha_m + \alpha_n)| \quad (IV.23c)$$

where the last line is obtained by use of the McGee wave function Eq. IV.2 and N is the normalization factor. Let us now view the correction term as though the vector meson first struck the proton and then the neutron. Thus f_n will be replaced by the amplitude for ρ meson nucleon scattering which we will define shortly. For the present we introduce the notation

$$R(\vec{q}') = f_p(\vec{q}')/f_p(\vec{q}) \quad (IV.24)$$

We let $K(\vec{q})$ be the vector meson nucleon scattering amplitude. Under the assumption that the amplitude is basically imaginary, $R(\vec{q})$ can be estimated by

$$R(\vec{q}') = f_p(\vec{q}')/f_p(\vec{q}) = \sqrt{W_2(\vec{q}')/W_2(\vec{q})} \quad (IV.25)$$

Then, we can write Eq. IV.22 as follows

$$d^2\sigma_d/d\Omega dE' = d^2\sigma_p/d\Omega dE' + d^2\sigma_n/d\Omega dE' - x d^2\sigma_p/d\Omega dE' \quad (IV.26)$$

with x , the correction factor being given by

$$x = \frac{1}{\pi\nu} \int d^3\vec{q}' S(1-\vec{q}' + \frac{\vec{q}}{2}) R(1\vec{q}' + \frac{\vec{q}}{2}) K(1-\vec{q}' + \frac{\vec{q}}{2}) \quad (IV.27)$$

If we observe that the $\vec{q}'$ integration is to be performed in a plane

perpendicular to $\vec{q}$, we have $|\pm \vec{q}' + \vec{q}/2| = \sqrt{(\vec{q}'^2 + \vec{q}^2/4)}$

and $d^2\vec{q}' = 2\pi |\vec{q}'| d\vec{q}'$. If we let $\xi = \sqrt{(\vec{q}'^2 + \vec{q}^2/4)}$

the integral becomes,

$$x = 2\nu^{-1} \int_{|\vec{q}/2|}^{\infty} \xi d\xi S(\xi) R(\xi) K(\xi) \quad (IV.28)$$

Our task is now to derive the factor $K(\xi)$. To do this, we write the

diagram for the vector meson dominance contribution (Fig. 6) to deep

inelastic scattering. This graph has the contribution

$$m_{fi} = e[\bar{u}_e \gamma_\mu u_{e'}] \frac{1}{q^2} \frac{m_\rho^2}{f_\rho} \frac{1}{q^2 + m_\rho^2} \bar{u}_p \gamma_\mu u_n \quad (IV.29)$$

The total deep inelastic cross section can be written as

$$d\sigma = \frac{1}{v_{rel}} \frac{M}{EE_p} \frac{1}{q^4} \left(\frac{m_\rho^2}{q^2 + m_\rho^2} \right)^2 \frac{e^2}{f_\rho^2} m_{\mu\nu} A^{\mu\nu} E' dE' d\Omega \quad (IV.30)$$

where all the factors except f_ρ^2 and $A^{\mu\nu}$ are the same as those defined in Section I, f_ρ^2 is the ρ dominance constant ($f_\rho^2/4\pi = 2.5$) and $A^{\mu\nu}$ is

$$A^{\mu\nu} = \sum_n \int \frac{d^3P'}{E_{P'} M} \frac{1}{2\pi} \delta^4(P+q-P_n) |\bar{u}_p \gamma_\mu u_n| |\bar{u}_n \gamma_\nu u_p| \quad (IV.31)$$

The right hand side of Eq. IV.31 is closely related to the imaginary

part of the $\rho P \rightarrow \rho P$ scattering amplitude. ($\epsilon_{\mu\nu} A^{\mu\nu}$ is the imaginary part of this amplitude). Since P_μ is the only vector available

$$A^{\mu\nu} = P^\mu P^\nu K(\rho P \rightarrow \rho P) \quad (IV.32)$$

This gives the following formula for the deep inelastic scattering cross section

$$\frac{d\sigma}{d\Omega dE'} = \frac{e^2}{f_\rho^2} \frac{1}{4E_0^2 \sin^4 \frac{\theta}{2}} \left(\frac{m_\rho^2}{q^2 + m_\rho^2} \right)^2 \cos^2 \frac{\theta}{2} K(\rho P \rightarrow \rho P) \quad (IV.33)$$

Comparing Eq. IV.33 with Eq. II.9, we have an expression for $K(\rho P \rightarrow \rho P)$.

(We have assumed that $\sin^2 \theta/2 < 1$.)

$$K(\rho P \rightarrow \rho P) = f_\rho^2 W_2 [1 + q^2/m_\rho^2]^2 \quad (IV.34)$$

We are now prepared to evaluate Eq. IV.28. For W_2 , we always use Eq. III.6b

with the experimentally determined constants. The integral can be evaluated

numerically. We first make a few observations. First of all the result

does not scale. Secondly, it is possible to derive an upper bound to

the integral, exploiting the fact that $S(\xi)$ is a decreasing function.

Therefore, we can write

$$x \leq 2\nu^{-1} S(|\vec{q}|/2) \int_{|\vec{q}|/2}^{\infty} \xi d\xi R(\xi) K(\xi) \quad (IV.35)$$

This exhibits the crucial dependence of this correction on the deuteron form factor.

A plot of the Glauber corrections against ω is shown in Fig. 7. Since these corrections depend on the momentum transfer and do not scale, the corrections can have different values at a given value of ω depending on the momentum transfer. Therefore, two continuous curves appear in Fig. 7, one through the minimum corrections encountered among data points at that ω and the other through the maximum corrections. A plot of the corrected D/H values as a function of $|\vec{q}|^2$ is appended to the graph. In spite of the non-scaling behavior, the shape of the curve roughly resembles the shape of data. However, the magnitude of the correction is too small. Since the present result involves a number of assumptions, we may be tempted to think that the shape of D/H data could be explained by a model along these lines. However, both the nonscaling behavior of the result and the fact that we are performing a semiclassical calculation to which objections have been raised (Ref. 35), imply that these corrections would probably not explain the entire behavior of data.

Therefore, the proper conclusion to draw from the calculations in this section is that neither the phase space corrections nor the generalized Glauber corrections can exactly explain the behavior of D/H data. The crucial dependence on the deuteron wave function in both cases makes a completely clear-cut decision difficult. In light of these findings, we will attempt to explain the difference between proton and neutron data on the basis of contributions from lower trajectories.

V. ANALYSIS OF POSSIBLE LOWER TRAJECTORY CONTRIBUTIONS TO THE STRUCTURE FUNCTIONS

So far, we have only considered the corrections due to the structure of the deuteron, which implicitly meant that we assumed the structure functions of the free proton and neutron to be equal. This would be true if no isospin bearing trajectory contributed to the structure functions. However, there is no physical basis for this assumption. An analysis of this situation was first done by Moffat and Snell⁶ using simple Regge type power behavior. Use of our formula instead of the arbitrary threshold behavior that has been introduced by the above mentioned authors improves the fit appreciably (except near the threshold - of course). This recalculation will also enable us to say something more about W_{1p} .

Our model can be very easily visualised if we recall that our structure function looked like $(S - M^2)^{\alpha(0)} e^{-\lambda/(\omega-1)}$ and that the two dominant trajectories are the Pomeron with $\alpha_p(0) = 1$ and the isovector $\alpha_p(0) = 1/2$. Although there is the further possibility of a lower isoscalar trajectory, $\alpha_\omega(0) = 1/2$ it is quite difficult to isolate its contribution using only presently available data. Thus we set it aside. This means that our structure functions should behave as follows:

$$2M W_{1p} = e^{-\lambda/(\omega-1)} [c_1 \pm c_2(\omega-1)^{-1/2}] (\omega-1) \quad (V.1)$$

$$(s-M^2) W_{2p} = e^{-\lambda/(\omega-1)} [c_3 \pm c_4(\omega-1)^{-1/2}] \quad (V.2)$$

where the c's are to be determined from data.³⁹ In the proton case, since separated W_1 and W_2 data are available, the constants can be determined directly from experimental data with a nonlinear chi-squared

minimization. For the neutron data, it is impossible to separate W_1 and W_2 from presently available data, (except at 2 or 3 points with very low accuracy) therefore we simply use Eqs. V.1 and V.2 to reconstitute the SLAC-MIT D/H ratios and fit data in this manner. Of course, the fit is highly insensitive to the W_1 value.⁴⁰ The coefficients thus determined could be compared for consistency.

The graphs of the W_{1p} , W_{2p} and D/H ratios together with the fit obtained is demonstrated in Figs. 8, 9, 10 respectively. In the table below, we compare the values of the coefficients obtained from proton data and from D/H data.

TABLE 2 - TABLE OF FITTED COEFFICIENTS FOR EQ. V.1 and V.2.

Coefficient	from Proton data	from D/H data
λ	1.78 ± 0.08	1.67 ± 0.05
c_3	0.69 ± 0.03	0.70 ± 0.03
c_4	0.22 ± 0.10	0.22 ± 0.01
λ'	1.10 ± 0.40	1.36 ± 0.70
c_1	0.14 ± 0.04	0.75 ± 0.25
c_2	0.72 ± 0.12	0.38 ± 0.30

The agreement for W_2 is seen to be very impressive.

As far as the coefficients of W_1 are concerned, we have learned very little from the determination using the D/H ratios since that ratio is highly insensitive to the values of W_1 . The values of the coefficients from the W_{1p} data are surprising. We find that the secondary trajectory contribution is much larger than the Pomeron contribution to W_1 . This

is probably a manifestation of the nonscaling of W_1 . It was virtually impossible to fit W_{1p} with a single trajectory contribution and the two trajectory fit is probably accidental. Alternatively, we might take the fit seriously and claim that although W_2 is principally dominated by the Pomeron, W_1 is principally dominated by lower trajectories. In fact, there are two lower trajectories, the isovector ρ - A_2 and isoscalar P' - ω . For the proton W_1 contribution, it is impossible to distinguish between the two without clouding the picture further by cuts. This means that we run into no difficulty with the neutron data by adopting this alternate explanation. Another way to see this would be to recall that at low ω $2M W_{1p}$ is numerically much larger than νW_{2p} , meaning that the Pomeron contributions to the two functions are probably comparable. But W_1 also has substantially more non-diffractive contributions (and therefore is the more likely candidate for nonscaling behavior - if any). This conjecture is also compatible with Compton scattering data.⁴¹

A further comment here is that if $(\nu W_2)_p$ as given by Eq. V.2 is rewritten as a function of $X = 1/(\omega-1)$, we have

$$\nu W_{2p} = e^{-\lambda X} \sqrt{2M} [c_3 \pm \sqrt{X} c_4] (1+X) \quad (V.3)$$

In this form, we see that as X varies from 0 to ∞ (i.e. ω varies from 1 to ∞) this expression for νW_{2p} first increases and then begins to decrease, with the maximum occurring at a point close to $\omega = 10$. This apparently agrees with experiment. However, following the maximum, the curve approaches an asymptote as $\omega \rightarrow \infty$, rather than decreasing uniformly. It is unclear whether or not this agrees with experiment. Because of the sign difference in νW_{2n} , the turning point there does not occur at positive X . Therefore,

if the dip at large ω is caused by contributions from secondary trajectories, W_{2n} should not show such a dip.

We have a final remark concerning the value of R for proton and neutron data. If we accept the results of the fit and say that W_{1p} has a greater contribution from lower trajectories than W_{1n} , (assuming that both the isoscalar and isovector trajectories contribute) it can be easily seen that, at least for low ω , (W_{1p}/W_{2p}) must be larger than (W_{1n}/W_{2n}) . This implies that $R_{\text{neutron}} > R_{\text{proton}}$ which seems to reflect the behavior of data since (see footnote 39) R_{neutron} appears to be 0.6 ± 0.4 while $R_{\text{proton}} = 0.18 \pm 0.05$. Because of the high uncertainty in the former value, the result is definitely inconclusive. If this were true, it may be explained in terms of the generalized parton model.⁴² Exact helicity conservation requires that R be proportional to the number of partons with integer spin, and it is entirely plausible to have more integer spin partons in the neutron than in the proton.

VI. CONCLUSIONS

In this section, we will summarize our essential findings in this paper. It is unfortunate that the lack of sufficient experimental data has forced many of our conclusions not to be as clear cut as they might have been. We can, however, reach a number of conclusions with presently available data. When more data become available, a few of our conclusions could be reinforced (particularly those pertaining to W_{1p} , R_{neutron} and D/H ratio close to the threshold).

First of all, we have applied the statistical approach to the Veneziano model to describe this electromagnetic process successfully. We have obtained a formula for νW_2 , which, in spite of limitations, agrees nicely with experimental data, and scales. We have incorporated the Pomeron trajectory in an approximate way into the dual resonance model. The Pomeron contribution alone seems to fit νW_2 very well.

Secondly, the scaling of the proton W_1 structure function is definitely uncertain. Presently available data indicates that either W_1 does not scale, or more probably, scaling of W_1 sets in at a higher value of s than it does for W_2 . One way this could happen, is if W_1 had bigger contributions from lower trajectories than W_2 . This is entirely possible and is also compatible with D/H data.

Thirdly, because of our lack of knowledge in high energy components of the deuteron wave function, as well as because of the fact that we are not close enough to the threshold value, the arbitrary assumption $\sigma_D = \sigma_p + \sigma_n$ seems to be good to within a few percent. It should be understood that the corrections would probably be small anyway at reasonably moderate and large $\omega \geq 2$, as long as any reasonable assumption is made for the deuteron wave function. The crucial dependence of both the phase

space and the Glauber corrections on the exact form of the deuteron wave function is a regrettable setback, and any corrections calculated from a given deuteron wave function are as good as the function used. It is important to note that, barring the unlikely appearance of high momentum components in the deuteron wave function, the formula $\sigma_D = \sigma_N + \sigma_P$ is probably as good as anything else in the presently measured region. Close to the threshold the corrections become important regardless of what we use for the deuteron wave function. There the abovementioned simple assumption must be treated with suspicion. As a matter of fact, provided that we know the free proton and neutron cross sections, ambiguities concerning the proper interpretation of the deuteron wave function may be clarified. In short, in the presently measured region, contributions from secondary trajectories are definitely necessary to explain the behavior of the data. As we move closer to the threshold, Glauber and phase space corrections will eventually become important and even dominate. The model dependent feature is where this transition will occur.

Fourthly, contributions from a lower isospin bearing trajectory are sufficient to explain the behavior of SLAC-MIT D/H data. Use of the statistical approach to the Veneziano Model has improved the fits substantially without requiring arbitrary threshold factors and pole terms that have been used in conjunction with Regge type fits. Admittedly, the Gaussian form factor is in theoretical trouble - particularly in the timelike region - but for the region data is presently available in, the formula is very successful. Recalling that we derived the formula asymptotically, its success in the intermediate and low ω region is impressive. As far as the relative contribution of the Pomeron and

lower trajectories are concerned, νW_2 is definitely dominated by the Pomeron contribution with roughly 30% secondary trajectory contributions. There are some indications, however, that W_1 may have greater contributions from lower trajectories than νW_2 . This may be a reason why it could begin to scale at a higher value of the energy.

A slightly more speculative conclusion along these lines is that R for the neutron will probably be higher than R for the proton. This, if verified, can help us clear the air regarding whether the Pomeron or lower trajectories dominate the process.

Finally, use of the alternate scaling variable ω' suggested by Gilman (Ref. 13) improves the agreement with experimental data close to the threshold, but deteriorates the overall fits.

VII. ACKNOWLEDGEMENTS

It is a pleasure to express my thanks and deep gratitude to my advisor Professor Y. Nambu both for his guidance and help during my graduate studies in general, and specifically for suggesting to me this problem, exposing me to most of the material in Section III of this paper, encouraging me during the progress of the calculations, and reading the manuscript critically at several stages. Thanks are also due to Professor L. Mo for making available 18°, 26° and 34° deep inelastic scattering data prior to their publication. I have also benefited from discussions with Dr. K. Fujikawa and from a comment of Dr. E. Schonberg.

Finally, members of the University of Chicago Computation Center and the Bubble Chamber group have been of great help during the progress of the numerical calculations.

REFERENCES AND FOOTNOTES

- 1) J.D. Bjorken, Phys. Rev. 179, 1547 (1969).
- 2) J.J. Sakurai, Phys. Rev. Lett. 22, 981 (1969).
- 3) R.P. Feynman, Phys. Rev. Lett. 23, 1415 (1969).
- 4) For a detailed discussion of this calculation as well as for a thorough review of all the theories that have been mentioned, see;
  - a. C.H. Llewellyn-Smith, "Introduction to Highly Inelastic Lepton Scattering and Related Processes", CERN Pub. TH 1188.
  - b. Lowell S. Brown, "Causality in Electroproduction at High Energy", Lectures given at the Summer Institute for Theoretical Physics, is a slightly older reference.
- 5) The following references would constitute a reasonably complete listing for experimental data mentioned in this paper.
  - a. L.W. Mo, "Inelastic Electron Proton Scattering", lecture delivered at the Stony Brook Conference on High Energy Collisions, C.N. Yang, Edit. Gordon and Breach (1969).
  - b. R.E. Taylor and F.J. Gilman, Proceedings, 4th International Symposium on Electron and Photon Interactions at High Energy, Liverpool (1969).
  - c. E.D. Bloom, D.H. Coward, H. DeStaeblcr, J. Drees, G. Miller, L.W. Mo, R.E. Taylor, M. Breidenbach, J.I. Friedman, G.C. Hartmann and H.W. Kendall, Phys. Rev. Lett. 23, 930 (1968).
  - d. G. Miller, E.D. Bloom, G. Buschhorn, D.H. Coward, H. DeStaeblcr, J. Drees, C.L. Jordan, L.W. Mo, R.E. Taylor, J.I. Friedman, G.C. Hartmann, H.W. Kendall and R. Verdier, "Inelastic Electron-Proton Scattering at Large Momentum Transfers", SLAC preprint.
  - e. E.D. Bloom, G. Buschhorn, R.L. Cottrell, D.H. Coward, H. DeStaeblcr, J. Drees, C.L. Jordan, G. Miller, L. Mo, H. Piel, R.E. Taylor, M. Breidenbach, W.R. Ditzler, J.I. Friedman, G.C. Hartmann, H.W. Kendall and J.S. Poucher, Contributed paper to the 15th International Conference

References and Footnotes (continued)

- e. on High Energy Physics, Kiev (1970) SLAC Pub. 796.
- 6) J. W. Moffat and V. G. Snell, Phys. Rev. D3, 2848 (1971) as well as various unpublished Univ. of Toronto preprints. Reference 4 also contains information concerning this model.
- 7) G. Veneziano, Nuovo Cimento 57A, 190 (1968).
- 8) Y. Nambu, Proceedings of International Conference on Symmetries and Quark Models, Wayne University (1969).
- 9) Y. Nambu, Electromagnetic Currents in Dual Hadrodynamics, EFI Preprint 71-15, to be published in the Physical Review.
- 10) P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. B19, 432 (1970) See also, Y. Nambu, Ref. 9 above.
- 11) L. N. Chang, P. G. O. Freund and Y. Nambu, Phys. Rev. Letters. 24 628 (1970).
- 12) H. Harari, Phys. Rev. Letters, 22, 1078 (1969); P.G. O. Freund, Phys. Rev. Letters 20, 235 (1968).
- 13) D. Bloom and F. J. Gilman, Phys. Rev. Lett. 25, 1140 (1970).
- 14) R. J. Glauber, in Lectures in Theoretical Physics, Vol. 1, p. 315, Interscience Publishers (1959); R. J. Glauber, in High Energy Physics and Nuclear Structure; p. 311, G. Alexander Ed., North Holland Publishing Co. (1967); R. J. Glauber, High Energy Physics and Nuclear Structure, p. 207, S. Devons Edit. Plenum Press (1970).
- 15) S. Drell and J. D. Walecka, Annals of Physics 28, 18 (1964). Most standard QED textbooks also discuss this calculation, at least for the elastic case.

Refs. and Footnotes (continued)

- 16) L. N. Hand, Phys. Rev. 129, 1834 (1963).
- 17) Calculation of the sum in Eq. II.7d as the absorptive part of the forward Compton Scattering Amplitude $\langle P | j_\mu^h(-q) | \pi \rangle$
 $\langle n | j_\nu^h(q) | P \rangle = \text{Im} \langle P | j_\mu^h(-q) \Pi(s) j_\nu^h(q) | P \rangle$
 and subsequent use of the statistical approximation would produce the identical result.
- 18) See Equation II. 15 where the Harmonic Oscillator formulation of the dual resonance model is discussed.
- 19) The vector character of the interaction could be obtained through the minimal coupling, $e^{iq \cdot \psi(0)} | P \rangle = e^{iq \cdot \psi(0)} | P_\mu | 0 \rangle$
 $\rightarrow e^{iq \cdot \psi(0)} (P_\mu - \sum_n \frac{f_n}{\sqrt{n}} (a_{n\mu}^\dagger - a_{n\mu})) | 0 \rangle$
 The f_n 's are produced by smearing out the interaction in a manner to be described.
- 20) For details of this calculation, see Y. Nambu and A. Hacinliyan, "Veneziano Model and Inelastic e-p scattering" EFI Preprint 70-67, unpublished, or S. Matsuda and J. Manassah, Princeton Advanced Study Preprints.
- 21) This extension will implicitly assume scaling in the sense that, just as for W_1 , we could divide through by q^2 without violating the Regge limit while inducing scaling, similarly we could introduce extra q^2 dependences into W_2 before extrapolating back from the Regge limit. That we choose not to do so is an indication that we implicitly assume scaling of νW_2 .

Refs. and Footnotes (continued)

- 22) For example, we can write $W_1 = s^{\alpha_1(s, q^2)}$. The Pomerenchuk theorem says that $\alpha_1(s, t) \rightarrow 1$ for W_1 and -1 for W_2 as $s \rightarrow \infty$. Now if we hold s large and fixed and write a dispersion relation for α_1 in q^2 , it is obvious that the relation should be subtracted at least once. It is assumed that this subtraction is done at the vector meson mass (or any fixed point $\ll s$). If one now passes to the scaling limit, assuming that the results scale, form factors of the form (III.6) are obtained. The author would like to thank Dr. K. Fujikawa for pointing this out. As a general reference of the phase representation, see, for instance, Y. Nambu and M. Sugawara, Phys. Rev. 132, 2724 (1963).
- 23) J.I. Friedman, H.W. Kendall, E.D. Bloom, D.H. Clward, H.DeStaebler, J.Drees, C.L. Jordan, G. Miller and R.E. Taylor, SLAC preprint "Behavior of the Electromagnetic Inelastic Structure Functions of the Proton".
- 24) Fong Ching Chen, Phys. Lett. 34B, 625 (1971).
- 25) Although Chen does not indicate the theory behind his nonscaling form, one method of obtaining it consists of starting from the phase representation as in footnote 22, but then taking the imaginary part of $\alpha_1(s, q^2)$ as a constant in the scaling limit, instead of assuming overall scaling.
- 26) Another fact that supports the possible existence of nonscaling contributions to W_1 becomes apparent if the Cottingham formula (Ann. Phys. (N.Y.) 25, 424 (1963)) is used to calculate the neutron-proton mass difference. See, for example, J.W. Moffat, V.G. Snell and A.C.D. Wright, "Proton neutron mass difference and scale invariance", University of Toronto preprint.

Refs. and Footnotes (cont.)

- 27) G. B. West, Stanford Preprint "Total Neutron Cross Sections may not be what they seem to be".
- 28) T. Hamada and I. D. Johnston, Nucl. Phys. 34, 382 (1962).
- 29) I. J. McGee, Phys. Rev. 151, 772 (1966).
- 30) To demonstrate this we just set up the inequality $E_p^{r(+)} \geq M$ which we will just label as E_p^r for convenience. This expands into $\omega \geq 1$.
- 31) The existence of terms depending on θ_{pq} in (IV.15) below while the phase space integrals were written for matrix elements without such dependence could be confusing. We wish to indicate that the terms with angular dependence were numerically integrated (with required precision $\leq 0.05\%$) over the regions indicated. We will actually see that the contribution from P-dependent terms in the matrix element are small.
- 32) Direct calculation with the Hamada-Johnston wave function limits this to 3% at most.
- 33) Although at first sight this may sound unphysical, this simply means that the total deuteron cross section happens to be even less than the free proton cross section. As such it is totally plausible. Very obviously, $\omega = 1/2$ is the threshold for the whole deuteron and at that value the total deuteron cross section is zero. If we unphysically insist that $\sigma_D = \sigma_N + \sigma_p$, so close to the threshold, we will obviously reach the conclusion that $\sigma_N < 0$! What is unphysical is $\sigma_p + \sigma_N = 0$, not the fact that $\sigma_D < \sigma_p$ or σ_N . The latter is perfectly possible.

Refs. and Footnotes (cont.)

- 34) R.J. Glauber, Phys. Rev. 100, 242 (1955); R.J. Glauber and V. Franco, Phys. Rev. 142, 1195 (1966).
- 35) Blackenbecker and Gunion claim that the standard Glauber treatment fails here since the momentum transfers are large. However, we can easily see that for large ω (where the c.m. energy is $\gg$ momentum transfer) this approach should be reliable and in this region the corrections are very small. At lower ω , where the momentum transfer becomes appreciable, the calculated cross section corrections also become higher. Therefore, this limitation does not affect our main conclusion that these corrections are quite small. At any rate, as far as orders of magnitude are concerned, Gunion and Blackenbecker's result is not greatly different from the result obtained by straightforward Glauber theory. The only significant difference is a phase factor. Thus our calculation clearly overestimates these corrections, except very close to the threshold. For further details, J.F. Gunion and R. Blackenbecker, Phys. Rev. D3, 2125 (1971) could be consulted.
- 36) Although the vector meson dominance also fails for high momentum transfers, this is not really crucial insofar as we can presumably use any strong amplitude to get an order of magnitude for these corrections. The undesirable nature of VMD will become apparent in Eq. IV.34 since it violates the unitarity bound. However, the integral will converge since both the deuteron form factor and the Gaussian formula for W_2 will cut off rapidly. Clearly, in this way, we will over-estimate the corrections. Therefore, what we calculate must be regarded as an upper bound to the corrections, since both the use of classical Glauber theory and VMD will overestimate the corrections.

Refs. and Footnotes (cont.)

- 37) K. Gottfried, in Proceedings of the Stony Brook Conference. (Ref. 5a).
- 38) For instance, terms like $\sum_f \langle i | f p | f \rangle \langle f | f n | i \rangle$ or $\sum_f \langle i | f n | f \rangle \langle f | f p | n | i \rangle$ can not contribute because one of the factors requires $\langle i |$ and $|f\rangle$ to have the same charge while the other factor requires $\langle i |$ and $|f\rangle$ to differ in charge by one unit. Actually, this is merely of academic interest since if one blindly forms the incoherent sum as $|\langle i | F | i \rangle|^2 - |\langle i | F | i \rangle|^2$ where F is given by (5.2), the "forbidden" terms will appear with an additional coefficient $S^2 |\vec{q}|$ where $S |\vec{q}|$ is the deuteron form factor. This suppresses them heavily.
- 39) The Regge behavior has been written in terms of the scaling combination $\omega - 1 = (S - M^2)/q^2$ instead of $(S - M^2)$ directly. Use of the latter variable would have required an extra scaling coefficient (since $\omega - 1$ is dimensionless) for dimensional considerations. Furthermore, as far as data fitting is concerned, there is very little to choose between the two.
- 40) Paranthetically, we can remark that initially an attempt was made to separate W_{1n} and W_{2n} from the data by fitting SLAC-MIT D/H data with large order polynomials assumed for W_{1n} and W_{2n} . Separation could be done at a few points only. A Monte Carlo calculation yielded the value of 0.6 ± 0.4 for the R value of the neutron. (The assumption that R was constant was used in the calculation). Both the restricted region where the data overlaps and the reasonably large error bars on the experimental data prevented a more definite calculation. It is hoped that the advent of larger angle deuterium data will enable a straightforward separation of W_1 and W_2 for the neutron as well.

Refs. and Footnotes (cont.)

- 41) The fact that a rather large contribution from the lower trajectories is required for W_{1p} is compatible with Compton scattering data (reported in Phys. Rev. Lett. 25, 1218 (1970), and Phys. Rev. Lett. 26, 1600 (1971)). It is well known that the deep inelastic scattering cross section is related to the imaginary part of the forward Compton scattering cross section. If we simply assume that the latter is purely imaginary near the forward direction, we can generalize our structure function away from $t = 0$. A typical contribution will be

$$\begin{aligned} & \langle P | j_\mu^h(q) \pi(s) j_\nu^h(q') | P \rangle \\ & \sim e g g' \sum_n \frac{|\Delta f_n|^2}{n} e^{\beta g g' \sum_n |\Delta f_n|^2} \\ & \sim e^{At} e^{-\lambda t / 4M^2} \end{aligned}$$

The second term is in the same form as what we had in the case of deep inelastic scattering. Because of the inequality of the two momenta, we now have an additional zero order term e^{At} . Unfortunately, the kinematics makes it very difficult to verify contributions from the second factor. Combining the first factor above with the Regge factor $v^{\alpha(t)}$, we essentially have a form $c(v/v_0)^{\alpha(t)}$, a typical Regge fit. Such fits have previously appeared in the literature (e.g. Moffat and Snell, Ref. 6). We have attempted to fit both the neutron and proton data. In either case, the slope of the Pomeron comes out as $0.54 / \text{GeV}^2$, the slope of the lower trajectory around $1/\text{GeV}^2$ with 20% uncertainty. The combined neutron and proton data requires only a 30% contribution from lower trajectories. For the proton data only, an alternate fit requires a contribution from lower trajectories which is 3

Refs. and Footnotes (cont.)

- times that from the Pomeron. The chi-squared values are roughly 1 per d.f. in the first case and 0.53 per d.f. in the second fit. This may enhance our finding that there may be substantial contributions to the proton W_1 form factor from lower trajectories. A crucial assumption here is the fact that near-forward Compton scattering is assumed to be dominated by its imaginary part which is related to W_1 . This assumption has been used in the literature.
- 42) See p. 10 of Ref. 4a for example

FIGURE CAPTIONS

Fig. 1. The basic diagram for inelastic e-p scattering. The notation is that used in the text.

Fig. 2 - a) Plot of W_{2p} against ω showing the agreement of our vertex function with the data.

b) Same plot with ω' used instead of ω .

Fig. 3 - a) Plot of SLAC-MIT separated W_{1p} against $S-M^2$.

b) Plot of independently separated W_{1p} against $S-M^2$. The auxiliary lines drawn are at constant ω , they should be horizontal if scaling is perfect.

Fig. 4 - Deep inelastic scattering of electrons from deuterium.

Figure Captions (continued)

Fig. 5 - Calculation of the expected D/H ratios on the basis of phase space and internal motion corrections. Lines A, B and C are the predictions from hypothesis A, B and C discussed in the paper. Curves D and E are calculated from a modified wave function with hypothesis A and C respectively.

Fig. 6 - Vector meson dominance graph for deep inelastic scattering.

Fig. 7 - The Glauber corrections plotted over the data. Since the corrections do not scale, two curves, one showing the maximum, the other showing the minimum correction are plotted. The Glauber corrections plotted against $|\vec{q}|^2$ are also shown in the graph.

Fig. 8 - The SLAC-MIT W_{1p} data fitted with a two trajectory form

Fig. 9 - " " " W_{2p} " " " " " " "

Fig. 10- " " " D/H " " " " " " "

Fig.1

Fig.2(a)

Fig.2(b)

Fig.3(a)

Fig.3(b)

Fig.4

Fig.5

Fig.6

Fig.7

Fig.8

Fig. 9

Fig.10