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THE SIMULTANEOUS MEASUREMENT OF SURFACE
TEMPERATURE AND SURFACE EMISSIVITY

F. C. Wessling, Jr.

Final Report ME-56(72)SA-51-1984-5-217

October 1972

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THE SIMULTANEOUS MEASUREMENT OF SURFACE
TEMPERATURE AND SURFACE EMISSIVITY

ABSTRACT

A method is described and demonstrated for simultaneously determining the emissivity and the surface temperature of an opaque, thermally radiating body. The method utilizes a spherical mirror to focus emitted radiation upon its point of origin; this radiation is chopped to distinguish it from the radiation directly emitted by the body and to minimize the effect of noise. The ratio of the chopped reflected signal to the directly emitted signal yields the reflectivity, and hence, emissivity. Data are presented for the emissivity of graphite in agreement with published values.

TABLE OF CONTENTS

	PAGE
TITLE PAGE AND ABSTRACT	i
ACKNOWLEDGMENT	iii
SUMMARY	iv
LIST OF FIGURES	v
INTRODUCTION	1
I. RADIATION FROM SURFACES WITH COMBINED DIFFUSE AND SPECULAR REFLECTION COMPONENTS	3
II. RADIATIVE AND CONVECTIVE HEAT TRANSFER FROM AN INTERNALLY HEATED SLAB, CYLINDER OR SPHERE	8
III. SIGNAL ATTENUATION BY INTERVENING SURFACES	14
IV. HIGH TEMPERATURE RADIATION SOURCE	23
V. CALIBRATION OF THE DETECTION SYSTEM	26
VI. EXPERIMENTAL TESTING OF SIMULTANEOUS TEM- PERATURE AND EMISSIVITY MEASUREMENT	33
REFERENCES	38

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SUMMARY

A method for the simultaneous measurement of emissivity and surface temperature of opaque radiating solid bodies with both specular and diffuse components of reflection is described analytically and demonstrated experimentally. Basic to the method is the assumption that the emissivity is the complement of the reflectivity.

Radiation emitted over a given wavelength range from a specific area on the body is reflected from a spherical mirror and focused onto the area through an optical chopper. The chopped, reflected radiation incident upon a sensor focused at the same area on the surface is determined using a lock-in amplifier. This is compared to the signal measured when the chopper is closed. The reflectivity and hence the emissivity is thus determined. Planck's radiation law and the emissivity yield the surface temperature. The emissivity of graphite at a wavelength of 0.9μ is determined to be 0.90 ± 0.1 over the temperature range 1900-2400K in agreement with published values. For the experiments a radiating resistance heated graphite slab was designed and constructed utilizing an analysis of an internally heated slab cooled by radiative and convective heat loss.

LIST OF FIGURES

Figure		Page
1.	Solution of $T_s^4 + CT_s = F$	39
2.	Dimensionless Energy Transmitted Through a Dielectric	40
3.	Experimental Setup for Signal Attenuation	41
4.	Sensor Calibration for Attenuation Tests	42
5.	Experimental Transmissivity Through Pyrex Glass	43
6.	Heater Assembly	44
7.	Heater Support Leg	45
8.	Heater Support Back	46
9.	Heater Support Blocks	47
10.	Heater Support Straps	48
11.	Photograph of the Graphite Rod Resistance Heater	49
12.	Chopper Wheel Geometry	50
13.	Sensor Intensity Ratio	51
14.	PAR Output as a Function of Frequency Trim	52
15.	PAR Calibration	53
16.	Sensor Recalibration	54
17.	Experimental Setup for Measuring the Emissivity and the Surface Temperature of Zirconia	55
18.	Reference Signal from PAR Zirconia Tests	56
19.	Noise from Photodiode Zirconia Tests	56
20.	Output from Photodiode Zirconia Tests	56
21.	Emissivity of Graphite	57

INTRODUCTION

Optical pyrometry is generally employed for the measurement of surface temperature in the range 1000-3000°C. Inherent in the use of pyrometry as it is normally conducted is the assumption that the emissivity of the material is known as a function of temperature and wavelength. This assumption is often not valid, and can lead to significant errors in temperature.

The purpose of this two-year study was the development of a method for the simultaneous determination of emissivity and temperature. The results of the first year, reported in Reference [1], indicated that the goal of simultaneous measurement of emissivity and temperature was feasible. The present report presents further evidence for the practicability of the method, and extends the mathematical analysis to more complex geometries, surface emissions and reflection characteristics.

In Section I the analysis is extended to include surfaces with both diffuse and specular components of reflection. In Section II the steady-state temperature distribution is solved for a solid (slab, cylinder, or sphere) heated electrically and cooled by radiative and convective heat transfer. Section III describes the attenuation of radiation due to the presence of an optical window at an arbitrary angle to the direction of observation; this section should have application in the determination of surface temperature on samples heated in a plasmajet, for example. Section IV is a description of a high-temperature

radiation source, designed using the results of Section II, developed for the experiments discussed in Section VI. Section V contains the calibration procedures employed in those experiments.

I. RADIATION FROM SURFACES WITH COMBINED DIFFUSE
AND SPECULAR REFLECTION COMPONENTS

NOMENCLATURE

English Letters

r_c	radius of curvature of the mirror, cm
r_L	radius of the lens, cm
r_m	radius of the mirror, cm
r_s	radius of the sensor, cm
$d^3Q'_{\lambda, s+r}$	power received by the sensor with the mirror uncovered, watts
$d^3Q'_{\lambda, s}$	power received by the sensor with the mirror covered, watts

Greek Letters

β	polar angle, radians
β_r	polar angle of reflectance, radians
β_s	polar angle of the sensor viewed from the surface, radians
ϵ'_{λ}	angular emissivity at wavelength λ
θ	azimuthal angle of incidence, radians
θ_r	azimuthal angle of reflectance, radians
λ	wavelength, microns
μ	cosine of the polar angle of incidence
μ'	cosine of the polar angle of reflectance
ρ_D	diffuse reflectance of the hot surface
ρ_m	diffuse reflectance of the mirror
$\rho'_{\lambda d}$	hemispherical angular reflectance at wavelength λ
ρ''_{λ}	biangular reflectance at wavelength λ

$\rho_{\lambda d}''$

diffuse biangular reflectance at wavelength λ

ρ_s

specular reflectance

ω_m

solid angle intercepted by the mirror, steradians

ω_s

solid angle intercepted by the sensor, steradians

In Reference [1] was presented a technique for the simultaneous determination of surface emissivity and surface temperature of an opaque body. As previously discussed, the method was restricted to purely diffuse or purely specular reflectors. In this section the analysis is extended to consider a surface possessing both a diffuse and a specular component of reflection.

For materials which are neither purely specular nor purely diffuse, R. A. Seban in a discussion of a paper by Sparrow, Eckert and Jonsson [2] suggested that real surface reflectance might be approximated by assuming the reflection to be composed of a specular component and a diffuse component. Love [3] showed that in this case:

$$\epsilon_{\lambda}'(\mu, \theta) = 1 - \rho_s(\mu, \theta) - \frac{1}{\pi} \int_0^{2\pi} \int_0^1 \rho_D(\mu', \theta_r) \mu' d\mu' d\theta_r \quad (\text{I-1})$$

If the specular and diffuse components of reflection are invariant with direction, then

$$\epsilon_{\lambda}' = 1 - \rho_s - \rho_D.$$

The ratio of the power incident on the sensor with the mirror uncovered to the power incident with the mirror covered is given in Reference [1] as:

$$\frac{d^3 Q_{\lambda, s+r}'}{d^3 Q_{\lambda, s}'} = 1 + \int_{\omega_m} \rho_m \cos(\beta) \rho_{\lambda}''(\lambda, \beta_r, \theta_r, \beta, \theta) \frac{\epsilon_{\lambda}'(\lambda, \beta, \theta)}{\epsilon_{\lambda}'(\lambda, \beta_r, \theta_r)} d\omega \quad (\text{I-2})$$

Using the method suggested by Seban yields

$$\rho_{\lambda}''(\lambda, \beta_r, \theta_r, \beta, \theta) = \rho_{\lambda d}'' + \rho_s(\lambda, \beta, \theta) \quad (\text{I-3})$$

Again considering

$$\frac{\varepsilon_{\lambda}'(\lambda, \beta, \theta)}{\varepsilon_{\lambda}'(\lambda, \beta_r, \theta_r)} = 1 \quad (\text{I-4})$$

Thus one finds the power ratio to be given by Equation (I-5).

$$\begin{aligned} \frac{d^3 Q_{\lambda, s+r}'}{d^3 Q_{\lambda, s}'} &= 1 + \int_{\omega_m} \rho_m \cos(\beta) \rho_{\lambda d}''(\lambda, \beta_r, \theta_r, \beta, \theta) d\omega \\ &\quad + \int_{\omega_s} \rho_m \cos(\beta) \rho_s(\beta, \theta) d\omega \end{aligned} \quad (\text{I-5})$$

Assuming $\rho_m, \rho_{\lambda d}, \rho_s, \cos(\beta_s) = \text{constant}$ and integrating this equation yielded

$$\frac{d^3 Q_{\lambda, s+r}'}{d^3 Q_{\lambda, s}'} = 1 + \rho_m \cos(\beta_m) \rho_{\lambda d}' \left(\frac{r_m}{r_c} \right)^2 + \rho_m \cos(\beta_s) \frac{\rho_s}{4} \left(\frac{r_L}{r_s} \right)^2$$

Unless the sensor lies inside the specular angle, the term containing ρ_s does not appear. Hence, the sensor is placed outside the reflected solid angle of the mirror to determine $\rho_{\lambda d}'$. Then, ρ_s is obtained by setting the sensor at the specular angle and again measuring the power ratio.

Thus the reflectance for any type surface can be measured.

II. RADIATIVE AND CONVECTIVE HEAT TRANSFER FROM AN
INTERNALLY HEATED SLAB, CYLINDER, OR SPHERE

NOMENCLATURE

b	αT_o
B	$hL/k, hR/k$, Biot number
C	Table I and Equation (II-13)
E	$\epsilon \sigma L T_o^3/k, \epsilon \sigma R T_o^3/k$
F	Table I and Equation (II-14)
H	the internal energy generation per unit volume and time, watts/cm ³
h	surface convective heat transfer coefficient, watts/cm ² /K
k	thermal conductivity, watts/(cm K)
L	width of the slab, cm
N	1 for slab; 2 for cylinder; 3 for sphere
Q	$HL^2/kT_o, HR^2/kT_o$
r	radial coordinate, cm
R	radius of the cylinder or sphere, cm
T	absolute temperature, Kelvin (K)
x	spatial coordinate, cm
z	defined in Equation (II-9)
α	temperature coefficient of internal energy generation, K ⁻¹
ϵ	emissivity of the surface $\eta = 1$
η	x/L or r/R
T	T/T_o
σ	Stefan Boltzmann constant, 5.729×10^{-12} watts/(cm ² K ⁴)

Subscripts

s	at location of radiating or convecting surface $\eta = 1$
o	ambient condition

Transient solutions for slabs, cylinders and spheres with internal energy generation and convective boundary conditions are well known [4]. An approximate transient solution for the combined boundary condition of heat transfer by both convection and radiation into black isothermal surroundings has been discussed by Zyszkowski [5] using Biot's variational method. In this brief, the exact steady-state solution for the combined boundary condition is presented; the results are utilized in the development of the radiation source employed in this investigation. Internal energy generation is assumed to be a linear function of temperature. Uniform energy generation is included as a special case.

The convective heat-transfer coefficient "h" and the emissivity "ε" are uniform on the surface located at $\eta = 1$. The surface $\eta = 0$ has a zero temperature derivative. The non-dimensionalized steady-state equations for the problem with the radiation and convection boundary conditions and constant thermal conductivity are:

$$\eta \frac{d^2 T}{d\eta^2} + (N-1) \frac{dT}{d\eta} + \eta Q[1+bT] = 0 \quad 0 \leq \eta \leq 1 \quad (\text{II-1})$$

$$\frac{dT}{d\eta} = 0 \quad @ \quad \eta = 0 \quad (\text{II-2})$$

$$-\frac{dT}{d\eta} = E(T^4-1) + B(T-1) \quad @ \quad \eta = 1 \quad (\text{II-3})$$

Here "N" equals one for the slab, two for the infinitely long cylinder and three for the sphere. The solutions of these equations are

for the slab, $N = 1$:

$$T = \frac{(1+bT_s) \cos (\eta\sqrt{Qb})}{b \cos \sqrt{Qb}} - \frac{1}{b} \quad (\text{II-4})$$

for the infinitely long cylinder, $N = 2$:

$$T = \frac{(1+bT_s)}{b} \frac{J_0 (\eta\sqrt{Qb})}{J_0 (\sqrt{Qb})} - \frac{1}{b} \quad (\text{II-5})$$

for the sphere, $N = 3$:

$$T = \frac{(1+bT_s) \sin (\eta\sqrt{Qb})}{b\eta \sin \sqrt{Qb}} - \frac{1}{b} \quad (\text{II-6})$$

where in equations (II-4), (II-5) and (II-6) T_s is the solution of:

$$T_s^4 + CT_s = F \quad (\text{II-7})$$

The quantities "C" and "F" are given in Table I for each of the three geometries.

Table I
Expressions for C and F of Equation (II-7)

N	C	F
1	$\frac{B}{E} - \frac{\sqrt{Qb} \tan \sqrt{Qb}}{E}$	$1 + \frac{B}{E} + \frac{\sqrt{Qb}}{bE} \tan (\sqrt{Qb})$
2	$\frac{B}{E} - \frac{\sqrt{Qb}}{E} \frac{J_1 (\sqrt{Qb})}{J_0 (\sqrt{Qb})}$	$1 + \frac{B}{E} + \frac{\sqrt{Qb}}{bE} \frac{J_1 (\sqrt{Qb})}{J_0 (\sqrt{Qb})}$
3	$\frac{B}{E} + \frac{\sqrt{Qb} \cot (\sqrt{Qb}) - 1}{E}$	$1 + \frac{B}{E} - \frac{(\sqrt{Qb} \cot (\sqrt{Qb}) - 1)}{bE}$

The solutions for T_s as a function of C and F are given in Figure 1 and can be calculated from Cardano's formulae [6], which are as follows:

$$T_s = -(z/2)^{1/2} + \left[(F + z^2)^{1/2} - z/2 \right]^{1/2} \quad (\text{II-8})$$

where z is given by:

$$z = \left[\frac{C^2}{16} - \left[\left(\frac{C^2}{16} \right)^2 + \frac{F^3}{27} \right]^{1/2} \right]^{1/3} + \left[\frac{C^2}{16} + \left[\left(\frac{C^2}{16} \right)^2 + \frac{F^3}{27} \right]^{1/2} \right]^{1/3} \quad (\text{II-9})$$

The line $C = 0$ is the limiting value for pure radiation with no convection and is given by:

$$T_s = (F)^{1/4} \quad (\text{II-10})$$

The dashed extensions of the family of "C" curves are the solutions for pure convection with no radiation and are given by:

$$T_s = F/C \quad (\text{II-11})$$

The broken lines labeled 1% and 5% delimit the region below which the error in T_s is less than 1% and 5%, respectively, when T_s is calculated by equation (II-11) rather than equations (II-8) and (II-9).

For the case of $b = 0$, that is, for uniform internal heat generation, the solution simplifies for all three values of N to:

$$T = T_s + \frac{Q}{2N} [1 - \eta^2] \quad (\text{II-12})$$

$$C = \frac{B}{E} \quad (\text{II-13})$$

$$F = 1 + \frac{B}{E} + \frac{Q/N}{E} \quad (\text{II-14})$$

The solutions presented in equations (II-12), (II-13) and

(II-14) agree with the asymptotic solutions for time approaching infinity as given by Zyszkowski.

III. SIGNAL ATTENUATION BY INTERVENING SURFACES

NOMENCLATURE

A	the absorbed energy divided by the incident energy for "N" internal reflections of one polarization component
AI	the angle of incidence measured from the normal to the surface, radians
AR	the angle of reflectance measured from the normal to the surface, radians
AT	the absorbed energy divided by the incident energy for an infinity of internal reflections of one polarization component
C_2	14388 micron K
g	$\exp(-t)$
h	the thickness of the dielectric plate, cm
K	a dummy variable in a summation
N	the number of internal reflections from the second surface of the dielectric
R	the reflected energy divided by the incident energy for "N" internal reflections of one polarization component
RT	the reflected energy divided by the incident energy for an infinity of internal reflections of one polarization component
t	$h/\cos(AR)$, cm
T	the transmitted energy divided by the incident energy for "N" internal reflections of one polarization component

T_H	indicated temperature of the tungsten filament without the plate glass
T_G	indicated temperature of the tungsten filament with the plate glass
TF	the transmitted energy divided by the incident energy for "N" internal reflections of both polarization components
TFT	the transmitted energy divided by the incident energy for an infinity of internal reflections of both polarization components
TR	transmissivity
TT	the transmitted energy divided by the incident energy for an infinity of internal reflections of one polarization component

Greek letters

α	the absorption coefficient, cm^{-1}
λ	wavelength, microns
ρ	the reflection coefficient from one surface

Subscripts

i	internal
o	external
$\perp$	perpendicular component of polarization
$\parallel$	parallel component of polarization

The measurement of temperature by optical means often requires looking through a window in order to view with a sensing device the object under consideration. The window reflects, absorbs and transmits the radiation incident upon it. Thus the intensity of the beam of energy measured by the sensing device is decreased. The transmission of energy is often reported in the literature for a beam striking the window normal to the surface of the window. Often, however, the experimental arrangement requires the measured beam to be incident upon the window at some angle different from normal. The necessary equations were derived by Stokes [7] and by Fresnel [8] more than one hundred years ago.

The derivation originally given by Stokes was for the general case of M parallel flat plates stacked with air gaps between them. No emissions from the plates due to their temperatures were considered.

The results are presented here for one component of polarization in equations (III-1) through (III-7) and for initially unpolarized light in equations (III-8) through (III-12). Only one plate is considered. The fraction of energy T, transmitted through the plate due to direct transmission and to internal reflections for a total of N internal reflections from the second surface is given by equation (III-1),

$$T = (1-\rho_o)(1-\rho_i) e^{-\alpha t} \sum_{K=0}^N (\rho_i e^{-\alpha t})^{2K}. \quad (\text{III-1})$$

As N approaches infinity this becomes

$$TT = \frac{(1-\rho_o)(1-\rho_i) e^{-\alpha t}}{1-\rho_i^2 e^{-2\alpha t}} . \quad (\text{III-2})$$

The fraction of energy, R, reflected from the plate due to a total of N internal reflections from the second surface is given in equation (III-3),

$$R = \rho_o + \frac{(1-\rho_o)(1-\rho_i)}{\rho_i} \sum_{K=1}^N (\rho_i e^{-\alpha t})^{2K} . \quad (\text{III-3})$$

and again as N approaches infinity this becomes

$$RT = \rho_o + \frac{(1-\rho_o)(1-\rho_i) \rho_i e^{-2\alpha t}}{1-\rho_i^2 e^{-2\alpha t}} . \quad (\text{III-4})$$

The fraction of energy, A, absorbed by the plate under these same conditions is given by equation (III-5).

$$A = (1-\rho_o)(1-e^{-\alpha t}) \sum_{K=0}^N (\rho_i e^{-\alpha t})^K \quad (\text{III-5})$$

as N approaches infinity:

$$AT = \frac{(1-\rho_o)(1-e^{-\alpha t})}{1-\rho_i e^{-\alpha t}} \quad (\text{III-6})$$

where

$$t = \frac{h}{\cos(AR)} . \quad (\text{III-7})$$

For a dielectric in air ρ_o and ρ_i are given by the Fresnel equations and are identical. Because there are two components to a light beam, these equations must be applied to each polarized component and the results added. Thus the ratio, TF, of the transmitted energy to the incident energy

becomes:

$$T_F = \frac{g}{2} \left\{ (1-\rho_{\perp})^2 \sum_{K=0}^N (\rho_{\perp} g)^{2K} + (1-\rho_{||})^2 \sum_{K=0}^N (\rho_{||} g)^{2K} \right\} \quad (\text{III-8})$$

for N approaching infinity this becomes

$$T_{FT} = \frac{g}{2} \left\{ \frac{(1-\rho_{\perp})^2}{1-\rho_{\perp}^2 g^2} + \frac{(1-\rho_{||})^2}{1-\rho_{||}^2 g^2} \right\} \quad (\text{III-9})$$

The reflection coefficients are given by the Fresnel equations (III-10) and (III-11):

$$\rho_{\perp} = \left(\frac{n_1 \cos(AI) - n_2 \cos(AR)}{n_1 \cos(AI) + n_2 \cos(AR)} \right)^2 \quad (\text{III-10})$$

$$\rho_{||} = \left(\frac{n_2 \cos(AI) - n_1 \cos(AR)}{n_2 \cos(AI) + n_1 \cos(AR)} \right)^2 \quad (\text{III-11})$$

$$g = \exp(-\alpha t) \quad (\text{III-12})$$

The resulting equations (III-9) through (III-12) are shown on Figure 2 for an index of refraction of 1.5, for various values of the product $\bar{h} = h\alpha$ and for angles of incidence from zero to ninety degrees. The results for other values of index of refraction are similar. The figure shows that the transmission function is strongly dependent upon the angle of incidence. The angular variation must be considered for values of AI greater than approximately twenty degrees. In an actual situation, the number of terms evaluated in equation (III-8) would be less than infinity. The number of terms depends upon the solid angle of view of the sensor and upon the

angle of incidence to the window. For the values of n and $\bar{n}$ considered, however, the results of equations (III-8) and (III-9) are identical to five significant figures if " N " is as small as five.

A Thermogauge Inc. miniature pyrometer was mounted on an optical bench. The lens furnished with the instrument was replaced with a type B, 40 mm diameter, 126 mm focal length coated objective. The pinhole of the sensor was located 85 cm from a calibrated tungsten ribbon filament lamp, at the image of the lamp. The experimental setup was as shown in Figure 3.

The sensor was calibrated using the lamp. The calibration data are shown in Figure 4.

For these experiments a 1/4 inch Pyrex glass plate was employed. A protractor was manufactured and attached to an optical mount, allowing the angle between the incident beam and the glass surface to be measured. The lamp intensity was measured without the glass plate. Then the glass plate was positioned to intercept the energy incident on the sensor from the lamp. The output of the sensor was recorded for various angles, θ_i , made by the incident beam and a normal to the glass plate surface. As the angle changed, the ribbon image moved from the pinhole. Thus the lamp was moved slightly until the maximum signal was obtained. The maximum signal was recorded in all cases.

The ratio of the power received with and without the glass plate was calculated. The sensor peak sensitivity was 0.9 microns with a bandwidth of 0.1 μ . The intensity ratio was

calculated from Planck's law. The temperatures used in Planck's law were the apparent temperatures of the filament as indicated by the sensor (the sensor is calibrated in terms of brightness temperature vs. voltage output). Thus the transmissivity was given by equation (III-13)

$$TR \approx \exp \left(- \frac{C_2}{\lambda} \frac{T_H - T_G}{T_H T_G} \right) \quad (III-13)$$

where

$C_2 = 14388$ micron K

λ = wavelength in microns

TR = transmissivity

T_H = indicated temperature of the tungsten filament
without the plate glass

T_G = indicated temperature of the tungsten filament
with the plate glass.

The transmissivity as a function of angle was plotted in Figure 5. The solid line was calculated from equations (III-9) through (III-12) for $n = 1.5$. The dashed curve was calculated similarly but for $n = 1.4$. The absorption coefficient was taken to be zero in both cases. The enclosed points were calculated from the experimental data using equation (III-13).

The theoretical calculations yielded excellent agreement with the experimental data for all angles less than eighty degrees. From this experiment it appeared as if the index of refraction of the Pyrex plate was closer to 1.4 than to

1.5. Thus equations (III-9) through (III-12) can be used to calculate attenuation of an optical signal by a Pyrex or other dielectric plate.

IV. A HIGH TEMPERATURE RADIATION SOURCE

In order to adequately test the theory of Section I, it was necessary to design and construct a radiation source. The desired characteristics of the source were that: (1) it have a maximum surface temperature of at least 2500°K; (2) that the reflectivity at 0.9 μ be sufficient to yield a measurable reflected, chopped signal; (3) that it be free of surface temperature fluctuations, particularly at frequencies near the chopper frequency; (4) that set points be repeatable; and (5) that it possess a reasonable lifetime. The source described below meets each of these requirements satisfactorily.

Graphite was chosen for the radiating element because of its high sublimation temperature ($\sim 4000^\circ\text{K}$), its reflection characteristics at temperatures in the range 1500-2500°K, and because it could be conveniently heated internally by electrical resistance heating. The element was designed, using the analysis of Section II, to provide the desired surface temperature range when powered by an existing Meaker 100KW, 80 volt DC power supply.

The ends of the graphite rod are held in water-cooled copper electrodes. A Pyrex tube channels inert gas around the element in order to prevent oxidation. A slit in the Pyrex tube serves to exhaust the inert gas, in addition to providing a viewing port. The graphite rod-electrode assembly is shown schematically in Figure 6.

Drawings of the remaining components of the source appear in Figures 7-10. Figure 11 is a photograph of the complete system in operation. The rod design can easily be

varied, if necessary, in order to obtain compatibility with a different power supply.

V. CALIBRATION OF THE DETECTION SYSTEM

NOMENCLATURE

English Letters

A	angle subtended by the segment of the circle, radians
AR	radius of the mirror, cm
n	dummy index
NA	normalized mirror area uncovered by the chopping wheel
P	peak to peak value of NA
R	distance from the centerline of the chopper to the centerline of the mirror, cm
X	defined on Figure 12, cm

Greek Letters

θ	angle of rotation of the mirror chopping wheel, radians
----------	---

In general, the magnitude of the chopped, reflected radiation was a few percent or less of the emitted radiation from the source. In many cases it was less than the noise present in the source emission. Hence, phase-sensitive detection was employed to extract the reflected signal from the background noise.

The unit employed was a Princeton Applied Research Model HR-8 Lock-In amplifier. The reference signal was chosen to be the chopper frequency, such that only radiative fluctuations at that frequency were amplified.

The PAR Lock-In amplifier yields a DC signal output that is proportional to the root mean square (RMS) value of the fundamental frequency of the Fourier expansion of the input signal. The input signal magnitude is a function of the position and geometry of the chopper opening in front of the mirror (see Figure 12). The resultant expression for the signal waveform is derived and verified experimentally with the experimental setup described in Reference [1]. In this setup, the sensor receives light from the tungsten lamp via direct reflection from the spherical mirror. A Fourier analysis is performed on the resultant waveform expression, permitting the RMS value of the fundamental frequency wave to be evaluated. Then by working backward from the measured RMS value, the peak-to-peak value of the original input signal is obtained. A description of the geometric and Fourier analyses follows.

The area of the segment of a circle is

$$\frac{1}{2} (AR)^2 (A - \sin(A)) \quad (V-1)$$

where A is the angle subtended by the segment of the circle, and AR is the mirror radius. In the limit $AR \rightarrow 0$, the input signal is a square wave. As AR increases in size, the rising portion of the square wave becomes more rounded. Equation (V-1), for the segment of the circle, is valid for less than one-half of the mirror being exposed. Hence this expression is modified to describe the entire covering or uncovering sequence. The final expression is divided by $\pi(AR)^2$ to yield an attenuation that varies from zero to one, and 0.5 is subtracted so that the Fourier expansion is a sine wave. Note that the peak-to-peak value is still 1.0. The final expression describing the wave is given below.

Angle value	Shifted, normalized attenuation (NA)
$0 \leq \theta \leq \arcsine (AR/R)$	$X = R \sin (\theta)$ $A = 2 \arccos (X/AR)$ $NA = 0.5(1. - (A-\sin(A))/\pi)$
$\arcsine (AR/R) \leq \theta \leq \pi/3 - \arcsine (AR/R)$	$NA = +0.5$
$\pi/3 - \arcsine (AR/R) \leq \theta \leq \pi/3 + \arcsine (AR/R)$	$X = R \sin (\pi/3 - \theta)$ $A = 2 \arccos (X/AR)$ $NA = 0.5 - 0.5 \sin (A-\sin(A))/\pi$
$\pi/3 + \arcsine (AR/R) \leq \theta \leq 2\pi/3 - \arcsine (AR/R)$	$NA = -0.5$
$2\pi/3 - \arcsine (AR/R) \leq \theta \leq 2\pi/3$	$X = R \sin (2\pi/3 - \theta)$ $A = 2 \arccos (X/AR)$ $NA = 0.5(A\sin(A))/\pi - 0.5$

The results of this expression are checked against the experimental waveform received from the sensor. Light from a tungsten filament lamp is reflected by the spherical mirror and focused onto the sensor. The chopping shutter, placed in its normal position, chops the light.

The sensor output is recorded on an oscilloscope. The resulting experimental data and the theoretical prediction are shown in Figure 13.

Fourier Analysis of the Signal Wave Form

The odd function waveform of Figure 13 is expanded in a Fourier sine series [9]. Thus

$$NA(\theta) = \sum_{n=1}^{\infty} A_n \sin(3n\theta) \quad (V-2)$$

$$A_n = \frac{3P}{\pi} \int_0^{\pi/3} NA(\theta) \sin(3n\theta) d\theta \quad (V-3)$$

P is the peak-to-peak value of $NA(\theta)$. Only the value of A_1 is required since A_1 is the amplitude of the fundamental frequency. The expression for A_1 is integrated numerically. The numerical integration yields

$$\begin{aligned} A_1 &= \frac{3P}{\pi} \int_0^{\pi/3} NA(\theta) \sin(3\theta) d\theta \\ &= .238108 \quad \text{for } n = 1. \text{ and } P = 1. \end{aligned} \quad (V-4)$$

The RMS value of a sine wave is $\sqrt{2}/2 A_1$; then the peak-to-peak-value of the original input signal wave is calculated as follows:

$$A_1 = \frac{2}{\sqrt{2}} \text{ RMS} \quad (V-5)$$

Thus

$$P = \frac{\frac{2}{\sqrt{2}} \text{ RMS}}{\frac{3}{\pi} \int_0^{\pi/3} \text{NA}(\theta) \sin(3\theta) d\theta} \quad (\text{V-6})$$

$$= (5.95) \text{ RMS} \quad (\text{V-7})$$

Thus by measuring the RMS of the fundamental frequency of the sine wave Fourier expansion the peak-to-peak value is obtained.

The theoretical prediction of the RMS value of the Fourier fundamental frequency is checked experimentally. The peak-to-peak value of the wave from the sensor is observed on an oscilloscope. The indicated RMS value of the wave, as indicated by the PAR, is obtained from the DC level of the PAR output. The frequency trim of the PAR is adjusted to obtain the maximum value of the DC output, as the operating instructions suggest. The maximum value occurred for the frequency trim at approximately 4.5. The output with the frequency trim at 5 differed less than 1% from the maximum value.

A plot of output versus frequency trim is shown in Figure 14. This plot verifies the maximum value occurring at a frequency trim setting of approximately 4.5. Because the set point of five was easily repeatable, the PAR is calibrated for the set point of five.

The peak-to-peak value of the input signal from the sensor is shown as a function of the wave RMS value as indicated by the PAR in Figure 15. The equations which fitted the data are:

$$P = (5.95) (\text{RMS}) ; \text{ theoretical} \quad (\text{V-8})$$

$P = (4.75) (\text{RMS})$; for frequency trim equal to zero
(V-9)

$P = 3.36) (\text{RMS})$; for frequency trim equal to five
(V-10)

The theoretical line is plotted for reference. These results demonstrate that the gain of the PAR is greater than the manufacturer stated. The author assumes that these expressions are valid also in the microvolt range. No calibration equipment was available in the microvolt range.

The Thermogauge sensor with the UNM lens was recalibrated using the same procedure described earlier. The Thermogauge pinhole was 16.7 cm from the centerline of the lens. The thick side of the lens was toward the sensor. The results are shown in Figure 16.

VI. EXPERIMENTAL TESTING OF SIMULTANEOUS TEMPERATURE
AND EMISSIVITY MEASUREMENT

The theory of measuring the temperature and the emissivity of a hot surface simultaneously [1] was tested experimentally utilizing the apparatus shown in Figure 17. A one inch-square piece of zirconia 1/16" thick, (A), was heated by the flame of the oxyacetylene torch, (B). The zirconia was placed at the center of curvature of a spherical mirror (G). Part of the energy emitted by the zirconia into the hemisphere in front of the zirconia was reflected back to the zirconia by the spherical mirror. The energy traversing the path between the mirror and the zirconia was chopped by a two-blade rotating disc; a fraction of that energy was subsequently reflected by the zirconia into the hemisphere in front of the zirconia. The lens (D) collected part of the reflected and directly emitted radiation and focused it onto a photodiode (E). The photodiode converted the light energy into an electrical potential. The reflected chopped portion of the light caused an alternating electrical potential, and the emitted steady portion caused a steady potential. A Princeton Applied Research (PAR) lock-in amplifier model #HR-8 and a PAR type A preamplifier were utilized to separate the alternating from the steady signal. The two signals were then connected to individual amplifiers of an oscilloscope. A reference light beam and photocell (C) were utilized to obtain the frequency of the chopping disc (120 Hz). This reference signal served as the synchronization signal for the PAR amplifier.

Results

The synchronization signal, or reference signal for the PAR amplifier, produced an output reference signal from the PAR amplifier as shown in Figure 18.

The output from the photodiode when no light was incident upon it is reproduced in Figure 19. The amplitude of this noise was approximately four percent of the amplitude of the alternating signal produced by the photodiode when the light was incident on it from the zirconia.

The photodiode signal is reproduced in Figure 20. The DC level of the signal was approximately 15 millivolts. As can be seen in Figure 20, the output from the photodiode varied greatly in time. This was caused by fluctuation in the surface temperature of the zirconia. When one viewed the zirconia through a smoked glass, the fluctuations in brightness could readily be observed indicating that the surface temperature of the zirconia was not constant. Hence, a new high temperature source was designed and built, as described in Section IV.

Currents of approximately 500 amps and voltage of approximately 6 volts were required. This power was furnished by a Meaker 100KW DC power supply. The zirconia experiments were performed at 120 Hz. However the Meaker power supply was a full wave three-phase rectifier. This caused an oscillation of 360 Hz in the temperature of the graphite. Thus a new chopper blade was built and a gear box assembled to operate the sensor at 18.5 Hz. The ripple of the output from the

Meaker was large enough to cause the output from the sensor to vary by several hundred microvolts. The oscillating portion of the sensor output was in the tens of microvolts. Consequently, part of the oscillation of the surface temperature came through the PAR. This caused a 10% oscillation in the signal from the PAR. Efforts to smooth out the ripple in the power supply were fruitless.

The experiments performed on the zirconia were repeated with the graphite rod in its place. Hickok DC voltmeters indicated the output from the PAR and the sensor DC level. The emissivity was calculated from the following expression:

$$\epsilon_{\lambda d} = 1 - \rho_{\lambda d} = 1 - 0.925 \frac{\Delta y}{y} \left(\frac{r_c}{r_m} \right)^2 \frac{1}{\rho_m \cos \beta_o} \quad (\text{VI-1})$$

The results for emissivity versus true temperature are presented in Figure 21. The true temperature was calculated using the measured brightness temperature and the experimentally determined emissivity. Two different samples were machined from ATJ graphite. The results for the emissivity compared well with other published data for graphite [10]. The reference gives the total emissivity of ATJ as 0.86 in the temperature range of interest. Other data in the same reference indicate that the spectral emissivity value at 0.9μ is approximately 0.9 for graphite. This demonstrated the experimental feasibility of the method. The 5% scatter and total estimated uncertainty of $\pm 10\%$ was probably due to variation in the surface temperature and to mechanical vibra-

tions. Less variation in the emissivity should be obtainable with a lower ripple power supply since it may be shown that a $\pm 20\text{K}$ ripple in surface temperature yields a noise level equal to the observed reflected signals.

Conclusions

1. The concept of measuring surface temperature and surface emissivity simultaneously was proven experimentally.
2. Care must be taken to minimize vibration of the test setup.
3. The time response of the system should be either much faster or much slower than the other changes in the system.
4. A 20K variation in the surface temperature caused a signal change of the same order of magnitude as the desired signal.
5. Replacing the mirror by some independently powered optical source (e.g. a laser) should raise the level of the return signal, and thus improve the signal-to-noise ratio.

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Figure 1. Solution of $T_s^4 + CT_s = F$. C and F are defined in Table I.

Figure 2. Dimensionless Energy Transmitted Through a Dielectric

Figure 3. Experimental Setup for Signal Attenuation

Figure 4. Sensor Calibration for Attenuation Tests

Figure 5. Experimental Transmissivity
Through Pyrex Glass

Figure 6. Heater Assembly

SUPPORT LEG - ALUMINUM, 3 EACH

Figure 7. Heater Support Leg

46

SUPPORT BLOCK - MICARTA, 2 EACH

Figure 9. Heater Support Blocks

SUPPORT STRAP – MICARTA, 2 EACH

Figure 10. Heater Support Straps

Figure 11. The Graphite Rod Resistance Heater. The Hot Center Section of the Rod is Seen as a Bright Spot.

Figure 12. Chopper Wheel Geometry

SENSOR INTENSITY RATIO
 DATA SHIFTED FOR FOURIER ANALYSIS
 ORDINATE - INTENSITY RATIO
 ABSCISSA - CHOPPER ANGLE
 ○ EXPERIMENTAL
 — THEORETICAL

Figure 13. Sensor Intensity Ratio

Figure 14. PAR Output as a Function of Frequency Trim

Figure 15. PAR Calibration

Figure 16. Sensor Recalibration

Figure 17. Experimental Setup for Measuring the Emissivity and the Surface Temperature of Zirconia

Figure 18. Reference Signal. Zirconia Tests. Vertical gain was one volt per division. Sweep rate was 10 milliseconds per division.

Figure 19. Noise from Photodiode. Zirconia Tests. Vertical gain was 5 microvolts per division. Sweep rate was 10 milliseconds per division.

Figure 20. Output from Photodiode. Zirconia Tests. Vertical gain was 5 microvolts per division. Sweep rate was 10 milliseconds per division.

Figure 21. Emissivity of Graphite