MEASUREMENT OF THE $K_L^0 \rightarrow \pi e \nu$ ABSOLUTE DECAY RATE

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MEASUREMENT OF THE $K_L^0 \rightarrow \pi \nu$ ABSOLUTE DECAY RATE

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ABSTRACT

An analysis of 620 K_L^0 decays via the mode $K_L^0 \rightarrow \pi \nu$ yields a rate equal to $(7.81 \pm 0.56) \times 10^6 \text{ sec}^{-1}$. The data were collected from a low-energy K_L^0 experiment performed in the LRL 25-inch hydrogen bubble chamber. The K_L^0 flux was deduced from events of the reaction $K_L^0 p \rightarrow \Lambda \pi^+$ which was observed in the same experiment, using the cross section of the process $K^- p \rightarrow \Lambda \pi^0$ measured in a companion experiment, and assuming charge independence.

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I. INTRODUCTION

The theory of weak interaction originated in 1932-33 with the neutrino hypothesis of Pauli and the first successful theory of nuclear beta decay (Fermi). Rapid advances followed the discovery of parity-nonconservation in 1957. Lee and Yang formulated the two-component neutrino theory and this was further elaborated by the V-A theory of Feynmann and Gell-Mann, Marshak and Sudarshan in the same year. According to the V-A theory, weak interactions are described by the interaction of two currents. The Hamiltonian, $\mathcal{H}$, has the form:

$$\mathcal{H} = \frac{G}{\sqrt{2}} J_{\lambda}^{\ell} J_{\lambda}^{h+}.$$

Here J_{λ} is a current of a leptonic part J_{λ}^{ℓ} and a hadronic part J_{λ}^h , which operate on the weakly interacting and strongly interacting particles respectively, and is responsible for causing the transition between initial and final states.

$$J_{\lambda} = J_{\lambda}^h + J_{\lambda}^{\ell}.$$

The structure of the leptonic current is well defined by the V-A theory, while the behavior of J_{λ}^h must be understood by the study of certain selection rules of J_{λ}^h . The elementary particles are defined by a set of quantum numbers. Among these are the strangeness S , isotopic spin I and its third-component I_3 . These quantum numbers are conserved by strong interaction between particles, but not necessarily by weak interaction. In the strangeness conserving ($\Delta S = 0$) weak interactions, such as the decay $\pi \rightarrow \mu\nu$, only the $I = 1, I_3 = 1$ part of J_{λ}^h can contribute, and one can advance the hypothesis that this $\Delta S = 0$ part of J_{λ}^h always behaves as a particle with quantum numbers $I = 1, I_3 = 1$. The assumption is usually referred to as the $\Delta I = 1$ selection rule for $\Delta S = 0$ transitions. In the strangeness-changing ($\Delta S = 1$) decays, such as $K \rightarrow \mu\nu$ and $\Lambda \rightarrow p e \nu$, only the $I = 1/2$ piece of J_{λ}^h can contribute, and the hypothesis that the

$\Delta S = 1$ part of J^h always behaves in this way is called the $\Delta I = 1/2$ rule for $\Delta S = 1$ semi-leptonic decay. These two rules are important in supporting the hypothesis that the hadronic currents participating in weak interactions are members of an SU_3 octet in the more modern theory of hadron classification.

The leptonic $\Delta I = 1/2$ rule is challenged by the $K^+ \rightarrow \pi^0 e^+ \nu$ and $K_L^0 \rightarrow \pi e \nu$ decays which contain both $I = 1/2$ and $I = 3/2$ parts of the hadronic current. If the leptonic $\Delta I = 1/2$ rule holds, it is shown in Sec. VI that the decay of $K_L^0 \rightarrow \pi e \nu$ should be twice that of $K^+ \rightarrow \pi^0 e^+ \nu$. This experiment measures the decay rate of $K_L^0 \rightarrow \pi e \nu$ to test the $\Delta I = 1/2$ rule, in addition to the interest in intrinsic properties of a fundamental particle.

However, the long lifetime and neutral character of K_L^0 together made the measurement of its decay rates quite difficult, and the direct experimental results concerning the absolute decay rate, $\Gamma_{2\beta}$, of the three-body electronic decay of K_L^0 mesons has been measured by only one previous experiment.¹ In that experiment the K^0 particles were produced by charge-exchange of 800 MeV/c K^+ mesons using a separated beam at the CERN Synchrotron and the electronic decays were recognized by direct identification of the electron "spiralizing" in the Ecole Polytechnique heavy liquid bubble chamber. The analysis made use of a Monte-Carlo method, and their absolute decay rate $\Gamma_{2\beta}$ was reported to be $(7.52 \pm 0.5) \times 10^6 \text{ sec}^{-1}$, assuming $\Delta S = \Delta Q$ rule and CP conservation.

In the present experiment the above decay rate was measured by a different method. The bubble chamber was directly exposed to a low energy K_L^0 beam, thus avoiding K_S^0 background. The momentum spectrum of the K_L^0 particles extends from about 100-500 MeV/c (see Fig. 11). The secondaries of the decays were sufficiently slow that they usually could be identified by the bubble density measurements, and the absolute K_L^0 flux was obtained by the simultaneous observation of the strong interactions of K_L^0 in hydrogen. Measurement of the distribution of the beam momenta for the process $K_L^0 p \rightarrow \Lambda \pi^+$ will provide the K_L^0 pathlength distribution if the cross section for this process is known. No published data exist at present for this cross section at the laboratory momenta ranging from 100-500 MeV/c. In order to determine

this cross section, another experiment was done, using the same bubble chamber exposed to a K^- beam of low momenta, obtaining the reaction $K^-p \rightarrow \Lambda\pi^0$ in the momentum range of about 180-300 MeV/c. Outside this momentum region the cross section for this reaction has already been determined by other K^- experiments.²⁻⁴ The $\Lambda\pi^+$ cross section can be obtained indirectly from the K^- experiments by comparing the corresponding K_L^0p and K^-p reactions and using the principle of charge independence. Sections II and III will describe the K^- experiment. Sections IV and V will describe the K_L^0 experiment and the comparison of the partial decay rates of K_L^0 and K^+ mesons.

II. K^- EXPERIMENT

A. Beam and Chamber Exposure

The low energy K^- beam⁵ set up by J. Murray at the Bevatron in 1965 was very different in concept in that it used a septum separator to separate the wanted K^- and unwanted particles at essentially the momentum (≈ 400 MeV/c) desired for studying $\Lambda(1540)$ resonance by formation rather than at a much higher momentum (typically 800 MeV/c). and then reducing the beam momentum to this value by placing an absorber (degrader) in the beam. The great advantage of the method was that the momentum was determined to high precision ($\pm 1\%$) which is not possible with the use of the absorber.

This experiment used a relatively thin beryllium absorber to reduce the beam momentum at the entrance of the 25-inch LRL bubble chamber to (a) 310 ± 8 MeV/c for the higher momentum run and (b) 251 ± 17 MeV/c for the lower momentum run. In the higher energy run the K^- ^{momentum} decreases by about 60 MeV/c in traversing the chamber, due solely to energy loss. In low energy run many K^- mesons come to rest near the end of the chamber. These two runs together cover the range of momenta of about 0-300 MeV where more than half of our K_L^0 data lie. There is very little difficulty in distinguishing the heavily ionizing K^- mesons

from the minimum ionizing background of π^- and μ^- particles.

During a period of a month or so in 1967, 123 rolls of film were taken, amounting to nearly 200,000 pictures. These were divided into K^- energy intervals:

Rolls 800-875A	280 MeV/c	K^- momentum
Rolls 875A-923	245 MeV/c	K^- momentum .

B. Fiducial Volume

The coordinate system is defined in Fig. 1. An event is defined to be inside the fiducial volume for the following conditions:

- (1) The coordinates (X,Y,Z) of the primary vertex must satisfy

$$\sqrt{X^2 + Y^2} < 22 \text{ cm}$$

$$6.53 \text{ cm} < Z < 12.37 \text{ cm} .$$

- (2) The secondary decay vertex, if any, must satisfy

$$\sqrt{X^2 + Y^2} < 25 \text{ cm}$$

$$2 \text{ cm} < Z < 25 \text{ cm} .$$

C. Scanning and Measurement

Only three-prong and 0-prong with V^0 topologies were recorded. For the low-momentum rolls only the events lying approximately in the upstream half of the chamber were recorded⁶ in an attempt to reduce the number of events to be measured resulting from stopping K^- particles called "at rest" events. No attempt was made to determine whether the event was inside the fiducial volume at the scanning stage. The reactions and the decays were classified into the following event types (ET):

ET 2 $K^- \rightarrow \bar{K}^0 n$

ET 3 $K^- \rightarrow \pi^- \pi^- \pi^+$

Identification of event types 1 and 2 was made by scanners based on visual check of ionization of the positive V^0 secondaries, and in the few indistinguishable cases the events were assigned to be a category called ET 4. The K^- decays with a π^0 giving a Dalitz-pair can resemble the tau decay mode and if the recognition of the Dalitz pair was uncertain, the decay was called ET 3 and recorded. Such events were (easily) separated by measurement and kinematic fitting. Measurement was performed on film-plane digitizers ("Frankenstein" measuring projectors). Detailed scanning and measuring instructions may be found in Trilling-Goldhaber Technical Notes.⁶

Geometrical reconstruction and kinematic analysis were done using the standard Berkeley SIOUX (combined TVGP⁷ and SQUAW⁸). The beam momentum and momentum resolution were determined by means of τ^- decays. The fitted momenta of the incident tracks, corrected for the energy loss due to the hydrogen transversed from the entrance to the decay points provided the beam momentum

distribution which provide its mean value and width. The mean beam momentum and beam momentum width are 300 ± 18 MeV for higher momentum rolls and 250 ± 17 MeV/c for lower momentum rolls. These values were then used as input to the fitting procedure through the technique known as "beam averaging": for each event the measured incident K^- momentum at the interacting point and the beam spectrum propagated through the absorber (and hydrogen) to the interacting point were averaged with weights inversely proportional to their respective errors. The "beam-averaged" momentum was then used for the measured momentum of the beam track in kinematic analysis.

After measuring and fitting, the events had to pass the fiducial criteria, and were either accepted, rejected, or remeasured. For an event to be accepted, it was required that: (a) it passed the fiducial volume criteria, and (b) all the tracks were well measured, as indicated by the spread of measured points from fitting curve. Using the chisquare value resulting from kinematic fit, a cut-off was arbitrarily made at the 1% confidence level. Events which were badly measured or which yielded no acceptable fits for ^{the} first good measurement were remeasured. Events which were well measured the second time but still yielded no acceptable fits were rejected as "no-fit" events. The number of accepted events and no-fit events are tallied in Table I. The other type of rejects are:

Non-beam track reject--the measured beam track momentum is required to be within 3 standard deviations from the mean value as input in the fitting program. Tracks off momentum will be rejected.

Immeasurable--beam tracks overlapped, chamber distortion, or film damage. The two types of rejects amount to 4% of the data, and no correction was made to these rejects as they are distributed approximately the same for the 3-prong and 0-prong with V^0 topologies.

Table I. Accepted and rejected events.

K^- process	High momentum roll		Low momentum roll	
	No. of fits	No. of no fits	No. of fits	No. of no fits
Neutral hyperon production	5448	126 (2.32%)	4384	39 (0.89%)
tau-minus decay	4750	144 (3.04%)	995	25 (2.6%)

Table II. Scanning efficiency for single scan.

K^- process	High momentum roll	Low momentum roll
Neutral hyperon production	95%	87.4%
Tau decay	97.2%	91.4%

The scanning efficiencies were determined by a rescan of 20% of the data. The rescan events were subject to the same acceptance criteria. Table II gives the scanning efficiencies.

III. ANALYSIS OF ^{THE} K^- EXPERIMENT

The reactions studied in the K^- experiment are:

- (1) $K^- p \rightarrow \Lambda p^0$
- (2) $K^- p \rightarrow \Sigma^0 \pi^0$
 $\quad \quad \quad \downarrow$
 $\quad \quad \quad \gamma \Lambda$
- (3) $K^- p \rightarrow \bar{K}^0 n$
- (4) $K^- \rightarrow \pi^- \pi^- \pi^+$ (tau decay)

A. K^- Pathlength and Tau Decay

The incident flux of kaons was computed from the number of observed τ^- decays, reaction (4) and the known rate for τ^- decay. As mentioned earlier, the K^- decays with a π^0 giving a Dalitz pair can resemble tau decay, and these decay modes are:

	Fraction
(5a) $K^- \rightarrow \pi^- \pi^0$	(20.93%)
(5b) $\rightarrow e^- \pi^0 \nu$	(3.18%)
(5c) $\rightarrow \mu^- \pi^0 \nu$	(4.85%)
(5d) $\rightarrow \pi^- \pi^0 \pi^0$	(1.7 %)

$\pi^0 \rightarrow \gamma e^- e^+$

If the recognition of the Dalitz pair events was uncertain, these decays were also recorded as ET 3. Only reaction (4) and reaction (5a) can be fitted kinematically, and the result of fitting events belonging to ET 3 showed that both reactions were unambiguously fitted, and only 1/2% of these events satisfied the hypothesis of reaction (5a). This indicates ^{that} the number of the Dalitz pair events not recognized by ^{the} scanners is small and negligible. Assuming that the scanners cannot recognize the Dalitz pair events resulting from reactions (5) with approximately the same inefficiency, and taking into account

their branching ratio, one estimates that less than 1/4% of the events belonging to ET 3 resulted from reactions (5b,c,d). These reactions account for part of the "no fit" events belonging to ET 3, as mentioned in the last section.

The momentum distribution of tau events is shown in Fig. 2, and L the pathlength at each momentum interval was calculated by the formula

$$L = \frac{Nt}{BR} r\beta c$$

where N is the number of taus at the momentum P_K ,

βc = velocity of K^- particles,

$$r = 1/\sqrt{1 - \beta^2},$$

$t = 1.235 \times 10^{-8}$ sec is K^- lifetime,

$BR = 0.0557$ is the tau branching ratio, and

$M_K = 493.82$ is the K^- mass.

The distribution of the pathlengths is shown in Fig. 3.

B. Neutral Hyperon Production

The charge-exchange reaction (3) and the neutral hyperon reactions (4) and (2) were completely separable after visual check of the bubble density and processing through kinematic fitting. The biases affecting the scanning and measuring of such events were similar and will be discussed jointly. The losses and corresponding corrections occurred in the following situations:

(1) Length and escape correction--A decays either outside the fiducial region or so close to the interacting point that the event appears as if it were a two-prong event. A procedure of correction was implemented as follows. Each accepted event was required to have a minimum trajectory for the A particle ($L_m = 2$ mm). The acceptance probability P_a was calculated as

$$P_a = e^{-L_m/L} - e^{-L_p/L}$$

where L_p is the potential length; i.e., the maximum possible trajectories of

the neutral particle before leaving the fiducial volume; and L is the mean decay length. The reciprocal of the factor P_a was used as a weighing factor for each event.

(2) Short proton loss--the positive track is too short to be detected. This occurs, when the proton from the Λ particle is emitted backwards in the rest frame with a velocity not far from that of the Λ particle. This loss depends directly on the angle θ of the positive decay tracks in the rest frame of the parent particle with respect to its line of flight and on the momentum distribution of the neutral particle. A slight loss is present near $\cos \theta = -1$ and for small values of the momentum. This was corrected by rejecting events with the proton momenta less than 100 MeV/c to insure the corresponding track length greater than 3 mm. This introduced a weight which was calculated from the above isotropic distribution in the angle θ .

C. Separation of Λ and Σ^0 Events

The dominant reactions producing neutral hyperons for this low-energy K^- beam are reactions (1) and (2) (as defined above). Multipion production with Λ particle is negligible because of the small cross section at this low energy. Only reaction (1) can be fitted kinematically. Reaction (2) is kinematically ambiguous with it. However, for both reactions the missing mass (MM) corresponding to all neutral particles accompanying the Λ particle was computed by the formula

$$(MM)^2 = (E_{cm} - E_{\Lambda})^2 - P_{\Lambda}^2$$

where E_{cm} is the total c.m. energy, and E_{Λ} and P_{Λ} are the c.m. energy and momentum of the Λ particle. The experimental distribution of the above quantity are shown in Fig. 4a,b,c for the three K^- momentum intervals: (a) 180-220 MeV/c, (b) 220-260 MeV/c and (c) 260-300 MeV/c. The finite experimental resolution broadens the π^0 line contributed by reaction (1), and this π^0

distribution is overlapped by the rectangular spectrum of $(\pi^0\gamma)$ from reaction (2), whose boundaries vary with incident K^- momentum (see Appendix A). The events which fitted the hypothesis of reaction (1) are shown shaded in the figures, and the MM of these events falls below 220 MeV.

To estimate the actual number of $\Sigma^0\pi^0$ and $\Lambda\pi^0$ events, the real events with MM above 220 MeV are assumed to be $\Sigma^0\pi^0$ events. The angular distributions of the Λ particles in the K^-p c.m. system for the $\Sigma^0\pi^0$ events are observed to be isotropic at the three K^- momentum intervals. The Σ^0 has a typical momentum of 210 MeV/c in the K^-p center of mass, and the Λ resulting from the decay is expected to deviate from the Σ^0 direction by at most 20 degrees. Since neither the Σ^0 nor the π^0 particles are seen directly or indirectly for this channel, the corresponding angular distribution of the real Σ^0 particles will be sufficiently well approximated by an isotropic distribution for the generation of simulated $\Sigma^0\pi^0$ events. The $\Sigma^0\pi^0$ events were simulated by a Monte Carlo program, PHONY,⁹ which generated the events according to an isotropic c.m. production angle. These "fake" events were analyzed just as were the real events by using the program SIOUX, and the previously described criteria. Resolutions were checked for satisfactory agreement between real and fake events. At each interval of incident momentum the fraction of $\Sigma^0\pi^0$ fake events with MM above 220 MeV was obtained, and using this fraction the number of real $\Sigma^0\pi^0$ events was determined from using real events MM greater than the above value. The chi-squares resulting from fitting the "fake" $\Sigma^0\pi^0$ as $\Lambda\pi^0$ are large, and in order to reduce the percentage of contamination of $\Lambda\pi^0$ by $\Sigma^0\pi^0$, the chi-square cutoff was made at the 5% confidence level. With this chi-square cutoff 12% of the fake $\Sigma^0\pi^0$ events fitted the hypothesis of $\Lambda\pi^0$. Furthermore no dependence on K^- momentum in our range of momenta was observed. The information found from the fake $\Sigma^0\pi^0$ events allows one to

estimate the background contributed by reaction (2) and thus determine the true number of $\Lambda\pi^0$ events. The number of $\Lambda\pi^0$ and $\Sigma^0\pi^0$ events thus determined is shown in Table III along with the number of τ^- events for the corresponding intervals. The events obtained were divided into 40 MeV/c momentum intervals for the K^- and K_L^0 exposures. These were chosen to make best use of the finite number of K_L^0 events available.

Table III. Summary of events as a function of K^- momentum.

K^- laboratory momentum interval (MeV/c)	Number of events (weighted for corrections mentioned above)		
	$K^-p \rightarrow \Lambda\pi^0$	$K^-p \rightarrow \Sigma^0\pi^0$	$K^- \rightarrow \pi^-\pi^-\pi^+$
180-220	200.6	146.6	249
220-260	1048.2	910.3	1607
260-300	1533.9	1455.6	2852

D. Cross Sections

The pathlength L was obtained earlier from the τ^- events. The cross section σ is given by

$$\sigma = \frac{\text{number of events}}{nL}$$

where $n = 3.705 \times 10^{22}$ hydrogen atoms/cm³ is obtained from the density of hydrogen in the chamber. After making corrections for scanning efficiency and the no-fit events, the cross sections for hyperon production were computed. In these calculations the branching ratio of the Λ particle decaying into the $p\pi^-$ final state was taken to be 0.653. In Table IV the cross section for the $\Sigma^0\pi^0$ reaction is compared with that given by Kim².

Table IV. $K^-p \rightarrow \Sigma^0 \pi^0$ cross section.

K ⁻ laboratory momentum interval (MeV/c)	Cross section	
	This exp.	Kim ²
180-220	9.30±1.05	10.2±3.6
220-260	7.47±0.34	13.00±4.9
260-300	5.77±0.24	---

In Table V is shown a compilation of the cross section for the $\Lambda \pi^0$ final state as determined from this experiment as well as from other experiments. The data in the momentum range of 120-300 MeV/c from this experiment and Kim's were fitted by the cross-section formula

$$\sigma = \frac{2\pi}{k} \left[\frac{\epsilon_A^2 b^2}{(1 + kb)^2 + (ka)^2} \right],$$

where k is the c.m. energy of K^-p system, ϵ_A is the ratio of $\Lambda \pi^0$ production to the total hyperon production in the $I = 1$ state, and $A = a + ib$ is the complex scattering length. The formula is derived using a constant S-wave scattering length parametrization.¹⁰ The values of the parameters obtained were used to extrapolate the cross section to 80 MeV/c.

IV. K_L^0 EXPERIMENT

A. Beam

The K_L^0 particles were generated by K^+ charge exchange on a solid platinum target located 22 inches from the center of the chamber. Figure 5 shows schematically the experimental layout at the Bevatron. The K^+ beam was produced at 0° from a platinum target placed in the Bevatron external beam, and was focused to a small spot on the charge-exchange target. The charge-exchange target was of rectangular shape, 5/16-inch

Table V. $K^-p \rightarrow \Lambda\pi^0$ cross sections for the determination of K_L^0 flux.

K^- momentum (MeV/c)	Cross section (mb)	Reference
60-100	40.8 ± 15	Extrapolated
100-140	23.0 ± 7	Kim ²
140-180	14.2 ± 5	
180-220	12.0 ± 1.17	
220-260	8.6 ± 0.34	This experiment
260-300	6.08 ± 0.19	
300-310	5.76 ± 0.6	
310-320	5.45 ± 0.47	Tripp et al. ³
320-330	5.87 ± 0.45	
330-340	5.78 ± 0.40	
340-350	4.70 ± 0.40	
350-360	4.37 ± 0.31	
360-370	4.21 ± 0.18	
370-380	4.12 ± 0.16	
380-390	4.41 ± 0.16	
390-400	4.03 ± 0.16	
400-410	4.47 ± 0.18	
410-420	3.67 ± 0.21	
420-430	3.73 ± 0.31	
430-440	3.68 ± 0.35	
440-450	3.81 ± 0.38	
455	4.03 ± 0.45	Armenteros et al. ⁴
475	2.97 ± 0.30	
495	3.26 ± 0.38	

high by 1-inch wide (horizontally), by 1-1/2-inch thick (beam direction). The K^+ beam was operated at an intensity of about 1000 K^+ per pulse at a momentum of about 830 MeV/c. There was a single stage of separation using a ten-foot separator operating at a field of approximately 100 kV/cm. Although there was only a single vertical focus, i.e., at the charge-exchange target, there were two horizontal foci--the intermediate horizontal focus being at a uranium collimator for the purpose of momentum definition. In addition, most of the protons in the beam were separated at the collimator, in spite of the fact that no vertical focus existed here. Following the collimator were two additional magnets which acted to sweep out beam particles that were scattered in the first part of the beam, or in the collimator. This was felt to be important in eliminating potential background. In addition, these two magnets served to remove the dispersion introduced by the first magnet so as to obtain a momentum focus at the final target. A sextupole component was shimmed into the intermediate bending magnet in order to correct for the chromatic aberration in the entire beam system.

Most of the area between the chamber and the external beam was filled with concrete shielding. This was because the chamber was located only about 25 feet from the backstop for the external beam where nearly 10^{11} protons/pulse were stopped. Due to the importance of protecting the chambers from sources of background caused by the external beam, nearly every non-essential space was filled with loaded concrete, steel, lead and uranium.

The K_L^0 particles which entered the chamber were produced at a $90^\circ \pm 20^\circ$ to the K^+ direction to avoid most of the neutron induced background, since neutrons dislodged by K^+ interaction were expected to be produced mainly in the forward direction. Moreover there is evidence that there is little sacrifice in K_L^0 flux in going from a forward production angle to one at 90° ,

when the production is by a relatively low momentum K^+ particle on a heavy nucleus. The large angle also allowed one to obtain a lower-momentum K_L^0 beam. Producing K 's by means of medium-energy K^+ beam, rather than by some other incident particle, such as π^- , has the advantage of a relatively higher K^0 production cross section, as well as fewer background reactions that accompany the K^0 production process.

At the charge-exchange target, the π^+ , K^+ , and protons are spatially separated, the protons passing above the target by about 1-1/2 inches, the π^+ passing below about 1/2 inch, and the K^+ being directed onto the target. About one-half million π^+ and protons passed by the charge-exchange target each pulse, orbited through the bubble chamber fringe field, and were stopped far enough away so as not to cause appreciable background. The target was only twelve inches from the internal window of the chamber, and subtended $\pm 2^\circ$ horizontally and $\pm 0.4^\circ$ vertically at the center of the chamber. In this experiment about one K_L^0 entered the chamber for each pulse, and a K_L^0 decay or interaction occurred on the average every 17 pictures. The principal background consisted mainly of some recoil protons resulting from neutrons entering the chamber, π^+ , protons and K^+ particles scattered from the target, and π^- from K_S^0 decays near the target. In the front part of the chamber there were quite a number of very low-energy electrons that were Compton recoils from soft γ -rays, which probably resulted from capture of neutrons ejected from the target by the beam. These had the appearance of many random bubbles resulting from the short Compton tracks, usually only one bubble in length.

B. Fiducial Volume

The coordinate system is defined precisely as in the K^- experiment. An event is defined to be inside the fiducial volume, if

(1) The coordinates of the primary vertex satisfy $\sqrt{x^2 + y^2} < 20$ cm.

(2) The lines connecting the center of the target to the primary vertex have to all pass through the inner window of the chamber, or equivalently the azimuthal (ϕ) and the dip (λ) of the above lines are required to be within the following minimum and maximum calculated quantities. Let

$D(22.86 \text{ cm})$ = the distance from the target to the inner window;

$DXW(21.59 \text{ cm})$ = the width of the beam window;

$DZW(6.35 \text{ cm})$ = the height of the beam window;

$DXT(3.81 \text{ cm})$ = the length of the target;

$DZT(0.794)$ = the thickness of the target;

$ZT(0.75 \text{ cm})$ = the Z coordinate of the target relatively to the center of the beam window; and

$XT(-0.64 \text{ cm})$ = the X coordinate of the target, then

$$\text{Dipmin} = -A \tan\left[\sin(\phi) \cdot \left(\frac{DZW + DZT}{2} + ZT\right)\right]$$

$$\text{Dipmax} = -\text{Dipmin}$$

and

$$\phi_{\min} = A \tan\left[\frac{D}{\frac{DXW + DXT}{2} - XT}\right]$$

$$\phi_{\max} = A \tan\left[\frac{D}{\frac{DXW + DXT}{2} + XT}\right]$$

(3) In addition, the secondary decay vertex, if any, must satisfy

$$\sqrt{x^2 + y^2} < 25 \text{ cm}$$

$$2 \text{ cm} < Z < 23 \text{ cm} .$$

C. Scanning and Measurement

The main decay modes and interactions of K_L^0 were classified into the following event types:

(a) ET 17

This represents the topology of K_L^0 interactions producing an associated one prong with V^0 .

(b) ET 07

This represents the topology of K_L^0 decays:

This category will also include ET 17 events with unseen V^0 origins, giving $K_L^0 \rightarrow \pi^+ \pi^-$ and $\Lambda \rightarrow p \pi^-$, and also more subtle reactions in which Track 3 is not a negative track going out from vertex, but rather a positive track coming in and decaying, see ET 12.

(c) ET 12

This represents the following reactions:

Topologically, type 07 and type 12 look the same; they can only be distinguished by kinematic fitting, or by determining which way the ambiguous track is moving. When there was doubt, the event was called an ET 07.

In this experiment 246 rolls of film or about 400,000 pictures were scanned for the above event types. The events were recorded on IBM cards with sketches. All interaction events and about one-fourth of the decay

events were measured for two chosen views on measuring projectors ("Frankenstein"), and kinematically fitted by PACKAGE,¹¹ a program for event reconstruction and kinematic analysis. An event was decided by this program if it occurred outside the fiducial volume. From the results of the PACKAGE program and the appearance of the event on the scan table (bubble density, possible stopping or scattering, etc.) a decision was made as to which process the event belonged. The chi-square cutoff was made at 1% confidence level. A no-fit event would not fit the hypothesis of the above process even if it had two good measurements. About 6% of the film was rescanned, and the efficiencies were 92.6%, 91% and 89% for K_L^0 decay modes, $\Lambda\pi^+$ or $\Sigma^0\pi^+$ final states and K_{Sp}^0 reactions respectively.

V. ANALYSIS OF THE K_L^0 EXPERIMENT

A. K_L^0 Interaction

The interactions involved in the Determination of K_L^0 flux mentioned earlier were

$$K_L^0 p \rightarrow K_{Sp}^0 \quad (6)$$

$$\rightarrow \Lambda\pi^+ \quad (7)$$

$$\rightarrow \Sigma^0\pi^+ \\ \quad \quad \quad | \rightarrow \Lambda\gamma \quad (8)$$

Because of the low energies involved, the K_S^0 and Λ^0 particles could be readily recognized and distinguished at the scan table in nearly every instance, using the bubble densities of the secondary tracks. This fact, in combination with kinematic fitting, yielded essentially a unique identification of the decay particles. However events with $\Lambda^0\pi^+$ and $\Sigma^0\pi^+$ final states had identical appearances at the scan table, and separation depended exclusively on kinematic fitting.

The direction of K_L^0 line of flight from the center of the target to

the interaction position was computed, and this information was used in the fitting procedure. However, K_L^0 particles left the target in all directions. A fraction of K_L^0 particles were scattered by the steel walls of the chamber or the pole faces of the bubble chamber magnet into the fiducial volume. Information on such processes was provided by reactions (6) and (7) which could be fitted without assuming the K_L^0 origin at the target. The K_L^0 direction thus derived from the fits of these events were analyzed and used as input for generating Monte Carlo events discussed later. For convenience, those events which fitted the interaction hypotheses using K_L^0 direction from the target are referred to as "unscattered events" while those events which would not fit a K_L^0 direction from the target, but fitted some other direction were called "scattered events." Those which occurred inside the fiducial volume and fitted unambiguously for a certain hypothesis are tallied in Table VI. In Table VI there were no scattered $\Sigma^0 \pi^+$, because this process was kinematically underconstrained in the fitting procedure unless the computed K_L^0 direction was used. Length and escape corrections were made for the events with K_S^0 and Λ and the minimum V^0 pathlength was chosen as 3 mm. Correction was also made for the loss of events resulting from the short protons (the positive secondaries of the Λ decays). The procedure of the corrections have been discussed thoroughly in the section about the K^- experiment. The correction for short proton loss in reaction (6) was unnecessary since this reaction was used to find the ratio of fluxes in different portions of the data (discussed later in the following section) and any such correction would simply cancel out in this ratio.

Kinematic separation of reactions (7) and (8) was incomplete; there were a total of 100 ambiguities, i.e., about 5% of these reactions. These 100 events were divided into two categories:

Table VI. Unambiguous K_L^0 interactions.

Final state	Non-scattered events	Scattered events	Total
K_{SP}^0	620	164	784
$\Lambda\pi^+$	696	309	1005
$\Sigma^0\pi^+$	694	-	694

(1) 57 events which fitted both as unscattered $\Lambda\pi^+$ and unscattered $\Sigma\pi^+$ events; and

(2) 43 events which fitted both as scattered $\Lambda\pi^+$ and unscattered $\Sigma^0\pi^+$ events.

To aid the assignment of ambiguities, events were simulated by the PHONY program for these interactions. In order to simulate the scattered events, the K_L^0 directions were distributed just as were the K_L^0 directions found from fitting scattered $\Lambda\pi^+$ and K_{SP}^0 events. Table VII summarizes the Monte Carlo results, which, together with the unambiguous events (see Table VI) allowed one to study the sources of ambiguities. Contributions from the first category ($\Lambda\pi^+/\Sigma^0\pi^+$ ambiguities) were estimated to be distributed as follows:

52 $\Sigma^0\pi^+$ fits to real $\Lambda\pi^+$ events, based upon the 696 real unscattered $\Lambda\pi^+$ events.

2 $\Lambda\pi^+$ fits to real $\Sigma^0\pi^+$ events, based upon the 694 real unscattered $\Sigma^0\pi^+$ events.

Since essentially all such ambiguities arise from events that really are $\Lambda\pi^+$, all 57 real ambiguities were assumed to be real unscattered $\Lambda\pi^+$ events for simplicity, as this procedure results in negligible error. Contribution of the second category of ambiguities were estimated to be distributed as follows:

Table VII. Results of Monte Carlo events.

Final state generated	% fitting the hypothesis		
	Unscattered $\Lambda\pi^+$	Scattered $\Lambda\pi^+$	Unscattered $\Sigma^0\pi^+$
Unscattered $\Lambda\pi^+$	100%		7.5%
Scattered $\Lambda\pi^+$	--	100%	1.5%
Unscattered $\Sigma^0\pi^+$	0.3%	5%	100%

35 scattered $\Lambda\pi^+$ fits to real $\Sigma^0\pi^+$ events, based upon the 694 real unscattered $\Sigma^0\pi^+$ events.

5 $\Sigma^0\pi^+$ fits to real scattered $\Lambda\pi^+$ events, based upon the 309 real scattered $\Lambda\pi^+$ events.

Since essentially such ambiguities arise from events that really are $\Sigma^0\pi^+$, all 43 real ambiguities were assumed for convenience to be all real unscattered $\Sigma^0\pi^+$ events. The laboratory momenta of K_L^0 particles in the reaction (6), (7) and (8) were plotted in Figs. 6, 7, 8 respectively. The portion of scattered events were shaded in the figures.

The number of events that were well measured but failed to fit any hypothesis were 27 K_{sp}^0 and 178 $\Lambda\pi^+/\Sigma^0\pi^+$ events, amounting to 3.4% and 9.6% respectively. The high percentage of no-fit events in hyperon production results mostly from the failure of scattered $\Sigma^0\pi^+$ events to be fitted to the target (i.e., as unscattered $\Sigma^0\pi^+$ events). The Monte Carlo program predicts that a fraction of these scattered $\Sigma^0\pi^+$ events would fit the hypothesis of unscattered $\Sigma^0\pi^+$ events. The number of scattered $\Sigma^0\pi^+$ was estimated by assuming the fraction of scattered events to be the same for both hyperon reactions, and this gave the result that 128 $\Sigma^0\pi^+$ events would

not fit. The remaining 50 events out of $178 \Lambda\pi^+/\Sigma\pi^+$ events are not accounted for by the scattered $\Sigma\pi^+$ events. This amounts to 2.7% of all accepted $\Lambda\pi^+/\Sigma\pi^+$ events. Due to finite measurement errors, 3.5% of fake events generated by PHONY program for these interactions failed to be fitted. This agrees well within statistical uncertainty with 2.7% of real no-fit events, and thus these 50 events were apportioned according to the ratio of the numbers of accepted real events for these two reactions.

B. K_L^0 Flux

The spectrum of the beam momenta for the process $K_L^0 p \rightarrow \Lambda\pi^+$ was now available and if the cross section for this interaction is known, the distribution of the pathlength L can then be obtained by the formula

$$L = \frac{N}{\rho\sigma}$$

where N is the number of $\Lambda\pi^+$ events corrected for the unseen neutral decay mode of Λ using the branching ratio of Λ decaying into $p\pi^-$ final state to be 0.653. Here $\rho = 3.705 \times 10^{22}$ hydrogen atoms/cm³ is obtained from the density of hydrogen in the chamber and σ is the cross section of the process $K_L^0 p \rightarrow \Lambda\pi^+$. As there were no data for the cross section of $K_L^0 p \rightarrow \Lambda\pi^+$ at this low energy, the cross section was deduced from the reaction $K^- p \rightarrow \Lambda\pi^0$. Both reactions possess equal amplitudes expressed as

$$\langle K_L^0 p | T | \Lambda\pi^+ \rangle = \frac{1}{2} \langle K^- p | T | \Lambda\pi^+ \rangle = \frac{1}{2} A$$

$$\langle K^- p | T | \Lambda\pi^+ \rangle = \frac{1}{2} A$$

where A is the strangeness -1, isotopic spin-1 amplitude. Their cross sections are approximately equal, the small difference arising from the slightly unequal phase spaces caused by the mass differences. The K^- experiment just described provided the cross sections in the 180-300 MeV/c momentum range where most of the K_L^0 data lay while the cross sections at other momenta were taken from other experiments as shown earlier in Table V.

The K_L^0 decay modes were carefully identified for a portion of the data. The corresponding K_L^0 flux was determined from the interactions in all the available data and thus has a smaller statistical error. The sample of interactions occurring in the rolls where K_L^0 decay modes were carefully analyzed is referred to as sample A, while the remaining part of the interactions is referred to as sample B. The pathlength was determined directly by Λx^+ events from sample A, and also indirectly from Λx^+ events in sample B as well as all K_{SP}^0 events using the relation:

$$\frac{N_{KA}}{N_{KB}} = \frac{N_{AA}}{N_{AB}}$$

where N_K , N_A stand for the numbers of events at a certain momentum for the processes $K_L^0 p \rightarrow K_S^0 p$, Λx^+ respectively, and subscripts A or B denote sample A or B. The right or left-hand side of this equation is the ratio of fluxes at a certain momentum for A and B, and the relation is the statement that this ratio is the same for any process. The statistical average of the pathlengths obtained in these different ways was performed, and this pathlength was used later to calculate the rate of K_{e3}^0 decays. The results are shown in Table VIII and in Fig. 9. The K_L^0 pathlength was deduced from events of the reaction $K_L^0 p \rightarrow \Lambda x^+$, using the cross section obtained from the reaction $K^- p \rightarrow \Lambda x^0$, and the use of charge independence. Since, for the same c.m. momenta, the difference of the laboratory momenta for the above two reactions is less than 0.3%, cross sections were taken equal at the laboratory momenta instead of the c.m. momenta for practical convenience.

As was mentioned earlier that there are mass differences for $K_L^0 p \rightarrow \Lambda x^+$ and $K^- p \rightarrow \Lambda x^0$ interactions, and it is equally valid to determine the K_L^0 flux by assuming the cross sections to be equal at the same total c.m.

Table VIII. K_L^0 pathlengths (cross sections assumed equal at equal K_L^0 and K^- laboratory momenta).

K_L^0 laboratory momenta (MeV/c)	Pathlength (km)		
	Direct method	Indirect method	Weighted average
60-100	0.06 ± 0.03	--	0.06 ± 0.03
100-140	0.29 ± 0.10	--	0.29 ± 0.10
140-180	0.92 ± 0.35	--	0.92 ± 0.36
180-220	1.49 ± 0.27	1.80 ± 0.41	1.66 ± 0.23
220-260	2.40 ± 0.33	4.05 ± 0.76	3.15 ± 0.36
260-300	4.21 ± 0.55	3.92 ± 0.71	4.12 ± 0.45
300-340	1.86 ± 0.36	2.75 ± 0.67	2.22 ± 0.35
340-380	2.95 ± 0.51	1.26 ± 0.38	2.49 ± 0.40
380-420	0.71 ± 0.27	1.73 ± 0.64	1.23 ± 0.35
420-460	1.79 ± 0.46	1.03 ± 0.61	1.67 ± 0.40
460-500	1.06 ± 0.35	1.46 ± 0.58	1.24 ± 0.33

energies instead of the c.m. momenta. Table IX gives the results of this new procedure, and the pathlength distribution thus obtained is shown in Fig. 10. It should be noted that the results are presented in the K^- momentum intervals. The pathlength was obtained by using $K_L^0 p$ interactions in the same c.m. total energy intervals corresponding to the K^- laboratory momentum intervals where cross sections of K^- experiments are presented.

C. K_{e3}^0 Decays

Fifty-five rolls of film, amounting to 22% of the data were carefully analyzed to identify the common K_L^0 decay modes:

$$K_L^0 \rightarrow \pi^\pm e^\mp \nu \quad (9)$$

$$\rightarrow \pi^\pm \mu^\mp \nu \quad (10)$$

$$\rightarrow \pi^+ \pi^- \pi^0 \quad (11)$$

Table IX. K_L^0 pathlengths (cross sections assumed equal at equal K_L^0 and K^- c.m. energies).

K^- laboratory momenta (MeV/c)	Pathlengths (km)		
	Direct method	Indirect method	Weighted average
60-100	--	--	--
100-140	0.17±0.08	--	0.17±0.08
140-180	0.69±0.28	--	0.69±0.28
180-220	1.67±0.29	1.95±0.42	1.78±0.24
220-260	2.61±0.37	4.72±0.78	3.50±0.39
260-300	2.92±0.47	3.58±0.78	3.21±0.40
300-340	2.42±0.44	4.08±0.97	3.04±0.45
340-380	2.87±0.53	1.48±0.41	2.44±0.39
380-420	0.98±0.32	1.18±0.39	1.08±0.25
420-460	1.01±0.35	1.12±0.68	1.03±0.31
460-500	1.20±0.41	0.91±0.36	1.08±0.28
500-540	0.16±0.16	0.24±0.16	0.22±0.12

Although the K_L^0 momentum is not well known, the direction is well determined from the known position of the small target and the observed decay point except for a small fraction of events which are scattered K_L^0 particles. A kinematic solution is only possible when the K_L^0 direction is used, and the K_L^0 origin is always assumed to be at the target. Due to the zero-constraint nature of the solutions, there will, in general, be more than one decay mode which has a kinematic solution. If the identity of the decay secondaries was known, the decay might then be completely reconstructed except for the two-fold ambiguity, resulting from the zero-constraint solutions.

The correct decay mode could be identified by means of bubble density in a large fraction of the cases, since K_L^0 momentum, and therefore the momenta of the secondary particles, were quite low. The average electron or positron momentum from K_{e3}^0 decays was about 140 MeV/c (see Fig. 12) and the average measured track length was about 10 centimeters (Fig. 13).

In hydrogen the bubble density k is known to vary with the particle velocity β ¹² as

$$k = k_0 / \beta^2$$

where k_0 = the bubble density of a relativistic track. The PACKAGE program used this relation to compute the bubble density of each secondary track from the measured momentum and assumed identity as π , μ and e respectively. The computed bubble density was normalized to unity for relativistic track by putting $k_0 = 1$. The appropriate conical projection of the track in each view was used to calculate the bubble density of the track exactly as it would appear on the scanning table for each of the three views. This information was available at the time when the visual identification of the decay mode was made at the scanning table. The observed bubble density of one of the secondary tracks was compared with that of the other. The comparison was repeated using the predicted bubble densities for each possible K_L^0 decay mode and in each of the three views. Consistency with a specific decay mode was determined by requiring agreement between observed and predicted bubble densities within two standard deviations for each of the three views. In case one of the secondary tracks could not be used for bubble density information (e.g., when the track is too steep), the bubble density of the other secondary track was compared with that of a nearby minimum ionizing track closely. When a minimum ionizing track was not available in the same frame of film, it was looked for in a nearby frame. In the instance where the track is uniquely identified

as a μ or e , the decay mode is unambiguously identified. For many events, an unambiguous identification was made solely from a visual estimate of the bubble density. In more difficult cases, usually when the secondary momentum was high, a gap-counting technique was also used. In the diagram below the sequence of dots represents a track to which a ruler is applied.

The number of times when the bubbles along the track were intercepted by the mm divisions was counted and called H for "hits." The corresponding total number of ruler marks is called T and the number of "misses" is called M: $H + M = T$. In order to use this information, it was necessary to know the relationship between the predicted bubble density of the particles and the number of misses M for the track. This relation is derived in Appendix B and is

$$k = \frac{1}{a} \ln \frac{T}{M}$$

and the statistical error on k is given by

$$\delta k = \frac{1}{a} \sqrt{\frac{H}{MT}}$$

where k is the bubble density,

"a" is the size of the bubble image combined with the width of mm divisions on the ruler (see sketch)

$$a = r + W$$

where r = actual bubble

image radius

W = width of milli-

meter marking on

ruler

This quantitative method was used when visual estimation seemed uncertain, primarily for secondary particles of higher momenta, and since bubble densities were compared among two tracks, a determination of "a" was not needed, since it would divide out in the ratio of track densities. The bubble density measurement technique was found to agree well with the quantitative predictions for events where a unique visual identification could be made with certainty. Most of the more difficult judgments were made after independent measurements by different physicists whose conclusions nearly always were the same. Using these methods it appeared that reliable identification of electrons and positrons could be made on tracks having momenta up to about 200 MeV/c. Each event in the sample was resolved either uniquely, or to the minimum number of ambiguities consistent with the estimated uncertainty of judgment. Before any selection criteria were imposed, 842 events were reconstructed and consistent with K_{e3}^0 decay mode, 593 of which had identified electron or positron secondaries. In addition there were 134 events which had electron or positron secondaries identified, but which were not kinematically reconstructable.

To reduce the ambiguities for the K_{e3}^0 sample, the following criteria were used:

- (1) electron or positron momentum was required to be less than 200 MeV,
 - (2) the dip angles of all tracks were required to be within ± 80 degrees of the chamber midplane,
 - (3) the opening angle of the V^0 to be between 10 degrees and 154 degrees.
- The third criterion decreased the data by less than 10% but eliminated any possible bias arising from the similarity between a certain few K_{e3}^0 decays and electron pairs, or π - μ decays in-flight. Thus the K_{e3}^0 sample imposed by these criteria consisted of 476 identified reconstructable events, 108 identified but not reconstructable events, and 36 reconstructable

events which could not be identified. This sample of events were used for the determination of the form factor and the decay rate of K_{e3}^0 discussed later (Sec. VD)

D. Form Factor of K_{e3}^0

The K_{e3}^0 form factor $f_+(q^2)$ was parametrized in the conventional way, which assumes an energy dependence:

$$f_+(q^2) = f_+(0) (1 + \lambda q^2 / m_\pi^2) .$$

Here, q^2 is the square of four-momentum transfer between K and π mesons. The procedure of determining λ is recapitulated as its value has already been published.¹³ Several sets of K_{e3}^0 decays were simulated by using FAKE,¹⁴ a Monte Carlo program. Each set consisted of 4000 events, generated with a different value of λ and according to the hypothesis of pure vector interaction in the decays (see Appendix C). The K_L^0 momentum was generated in the FAKE program for a Gaussian distribution having a 220-MeV/c full width at half maximum and peaking at 280 MeV/c. For the purpose of this analysis the representation was found to be a good approximation to the actual spectrum.

As is well known, two valid solutions generally are obtained from solving the zero-constraint fit, which correspond to two distinct values of K_L^0 momentum. There are different ways to handle this problem, but as long as both real and simulated events are treated identically, no biases should result in the determination of λ , and only different degrees of sensitivity to λ will result. In practice this was found to be the case. The method chosen was to select the solution having the greater normalized weight factor, which was equal to the product of two factors:

(a) the relative likelihood for obtaining the K_L^0 momentum for the particular

solution as deduced from the known K_L^0 spectrum, and (b) the relative probability in the Dalitz plot. By subjecting the samples of real and simulated events to exactly the same criteria as described above, and by comparing the Dalitz distributions for the sample of real events with that of the Monte Carlo events, the best value of λ was obtained:

$$\lambda = + 0.01 \pm 0.015 .$$

The functional dependence on λ of the chi-squared probability of fitting is shown in Fig. 14. The world average of λ for K_{e3}^0 decays compiled by Gaillard and Chounet¹⁵ in 1970 was 0.017 ± 0.008 .

It should be noted that in this analysis, events occurring in all regions of the Dalitz plot were used in fitting for λ . No portion of the Dalitz plot was removed from the sample. The c.m. electron spectrum shown in Fig. 15 indicated no need for any additional correction for the variation of electron detection efficiency other than that provided by the selection criteria. Figure 16 gave the c.m. pion energy distribution for the real events compared with the Monte Carlo prediction for $\lambda = 0.01 \pm 0.015$. A search for other possible systematic effects has revealed none larger than about half the quoted statistical error.

E. The Absolute Decay Mode of K_L^0 Into $\pi^\pm e^\mp \nu$

For a particle of rest mass m , moving with velocity v (or having momentum p), $N(p)$ the number of decays observed in the time t is expressed as:

$$N(p) = \frac{\Gamma}{\gamma} t$$

where Γ is the absolute decay rate defined in the rest frame and $\gamma = 1/\sqrt{1 - (v/c)^2}$ is the relativistic kinematic factor. Since the time is measured as the pathlength $L(p)$ divided by the velocity and $\frac{1}{\gamma v} = \frac{m}{p}$,

the formula can be re-expressed as

$$N(p) = \frac{\Gamma}{\gamma} \frac{L(p)}{v}$$

$$= \Gamma \frac{m}{p} L(p) .$$

Since the decays were obtained under certain selection criteria, the righthand side of the equation was multiplied by $\text{Eff}(p)$, the detection efficiency of decays

$$N(p) = \Gamma \text{Eff}(p) \frac{m}{p} L(p) .$$

The detection efficiency was determined as a function of the beam momentum from the Monte Carlo events subject to the same criteria as real events, and is discussed in the next paragraph. A complication arose on the application of the equation due to the difficulty in assigning the number of decays in a momentum interval for two reasons: firstly, the quadratic ambiguities of the K_L^0 beam momenta for reconstructable events, and secondly, the beam momenta of events not reconstructable due to the scattered origins. As mentioned earlier, the pathlength distribution on the right-hand side of the above equation was obtained from $K_L^0 p \rightarrow \Lambda \pi^+$ interaction. One can sum the equation over all the momentum intervals and express the absolute decay rate as

$$\Gamma = \frac{N_{\text{tot}}}{\sum_i \text{Eff}(p_i) \cdot \frac{m}{p_i} \cdot L(p_i)}$$

where $N_{\text{tot}} = \sum_i N(p_i)$. In this expression, the decay rate was obtained without needing to assign the number of decays in a momentum interval.

The detection efficiency was obtained by a statistical method. The K_{e3}^0 events at each momentum interval were simulated according to the experimental condition by the FAKE program mentioned earlier. The events were simulated according to the pure vector interaction with the λ parameter

in the form factor equal to 0.01 as determined in this experiment. $\text{Eff}(p)$ was defined as the percentage of Monte Carlo events surviving the selection criteria and is given in Tables X and plotted in Figs. 17 and 18. Criteria A are the same as the selection criteria discussed above, in addition to the requirement that K_L^0 momentum be within 100-500 MeV/c. Criteria B are defined by changing two conditions of criteria A, that is, (1) the electron or positron momenta less than 180 MeV/c and (2) the dip angles of secondaries tracks within $\pm 70^\circ$.

Table X. Detection efficiency of K_{e3}^0 decay mode as a function of K_L^0 momenta.

K_L^0 momentum (MeV/c)	Detection efficiency	
	Criteria A	Criteria B
60-100	0.340	0.313
100-140	0.542	0.465
140-180	0.664	0.542
180-220	0.710	0.572
220-260	0.715	0.581
260-300	0.710	0.580
300-340	0.690	0.567
340-380	0.665	0.550
380-420	0.623	0.520
420-460	0.570	0.483
460-500	0.480	0.424
500-540	0.300	0.250

The number of events selected by Criteria A was 620 events, of which 476 events were identified and reconstructable, 108 were identified but not reconstructable, and 36 were reconstructable but not identified.

The K_L^0 pathlength has already been determined and discussed in Sec. VB. The pathlength distribution was obtained in two different ways: first by assuming cross sections for $K_L^0 p \rightarrow \Lambda \pi^+$ and $K^- p \rightarrow \Lambda \pi^0$ interaction equal at the same laboratory momenta, and secondly by assuming them equal at equal total c.m. energies. The first method gave the result:

$$\Gamma = (7.81 \pm 0.56) \times 10^6 \text{ sec}^{-1}.$$

The decay rate was recalculated by the second method giving

$$\Gamma = (7.82 \pm 0.56) \times 10^6 \text{ sec}^{-1}.$$

Since no significant change occurs in the decay rate as a result of these alternative methods, the $K_L^0 K^-$, $\pi^+ \pi^0$ mass splittings cause no difficulty in the determination of this value. Furthermore the change of the decay rate by subjecting the events to more stringent criteria was studied by using 521 events selected by criteria B, of which 404 were identified and reconstructed, 94 were identified but not reconstructed, and 23 not identified but reconstructed. The decay rate resulting from this analysis was:

$$\Gamma = (7.99 \pm 0.59) \times 10^6 \text{ sec}^{-1}.$$

This only shifted the decay rate by 2%, a small change relative to the statistical error. The contributions by individual terms in the denominator of the decay rate formula are shown in Table II.

F. Comparison of Γ_{03} for K_L^0 and K^+ Mesons

To describe the semi-leptonic decays of neutral and charged mesons, three amplitudes $A(\frac{1}{2}, \frac{1}{2})$, $A(\frac{3}{2}, \frac{1}{2})$, $A(\frac{3}{2}, \frac{3}{2})$ are needed in principle. The numbers in the parenthesis denote the isotopic spin and its z-component

Table XI. The decomposition of the denominator in the decay rate formula.

K_L^0 momenta (MeV/c)	$\text{Eff}(p) \cdot \frac{m}{p} \cdot L(p) \cdot 10^{-6} \text{ sec}$	
	Criteria A ($N_{\text{tot}} = 626$)	Criteria B ($N_{\text{tot}} = 521$)
60-100	0.43 ± 0.24	0.39 ± 0.22
100-140	2.18 ± 0.78	1.87 ± 0.67
140-180	6.34 ± 2.50	5.17 ± 2.04
180-220	9.79 ± 1.34	7.89 ± 1.08
220-260	15.57 ± 1.81	12.60 ± 1.47
260-300	17.35 ± 1.88	14.17 ± 1.54
300-340	7.97 ± 1.26	6.55 ± 1.03
340-380	7.65 ± 1.24	6.33 ± 1.02
380-420	3.17 ± 0.91	2.65 ± 0.76
420-460	3.60 ± 0.85	3.05 ± 0.72
460-500	2.06 ± 0.55	1.82 ± 0.49

respectively. The three-body electronic decay rates for K_1^0 , K_2^0 , and K^+ mesons are expressed in terms of the above amplitudes as

$$\begin{aligned} \Gamma_{2\beta} &= \left| \sqrt{\frac{2}{3}} A(\frac{1}{2}, \frac{1}{2}) + \sqrt{\frac{1}{3}} A(\frac{3}{2}, \frac{1}{2}) - A(\frac{3}{2}, \frac{3}{2}) \right|^2 \\ \Gamma_{1\beta} &= \left| \sqrt{\frac{5}{3}} A(\frac{1}{2}, \frac{1}{2}) + \sqrt{\frac{1}{3}} A(\frac{3}{2}, \frac{1}{2}) + A(\frac{1}{2}, \frac{1}{2}) \right|^2 \\ \Gamma_{\beta}^+ &= \left| \sqrt{\frac{1}{3}} A(\frac{1}{2}, \frac{1}{2}) - \sqrt{\frac{2}{3}} A(\frac{3}{2}, \frac{1}{2}) \right|^2. \end{aligned}$$

The $\Delta S = \Delta Q$ rule implies $A(\frac{3}{2}, \frac{3}{2}) = 0$ and thus $\Gamma_{2\beta} = \Gamma_{1\beta}$ while $\Delta I = 1/2$ rule implies $A(\frac{3}{2}, \frac{1}{2}) = A(\frac{3}{2}, \frac{3}{2}) = 0$ and consequently $\Gamma_{1\beta} = \Gamma_{2\beta} = 2\Gamma_{\beta}^+$. Trilling¹⁶ has suggested the use of $\Gamma_{K_{e3}}$ rates to test the leptonic $\Delta I = 1/2$ rule:

$$\Gamma_{K_{e3}^0} / 2\Gamma_{K_{e3}^+} = 1 \quad |\Delta I| = 1/2 \text{ rule} .$$

Due to mass differences in (K^0 , K^+) and (π^- , π^0 , π^+) multiplets, the ratio should deviate from unity even if the rule is valid. The small deviation is evaluated in Appendix C where the effect of the masses on the decay rates are calculated. One finds after the correction:

$$\Gamma_{K_{e3}^0} / 2\Gamma_{K_{e3}^+} = 1.012 .$$

The experimental situation of the leptonic $\Delta I = 1/2$ rule is summarized in Table XII. In the XIVth International Conference on High-Energy Physics at Vienna, J. W. Cronin presented his analysis of the current data and remarked the result he obtained supported this rule, while the data from the Review of Particle Properties (1968) show a 3σ deviation from obeying the rule. The results from the latter issues of the Particle Properties show a disagreement of less than 2σ with the prediction.

Table XIII. Recent experimental determinations of $\Gamma_{K_{e3}^0}$ and test of the leptonic $\Delta I = 1/2$ rule for K mesons²³

Reference	$\Gamma_{K_{e3}^0} (10^6 \text{ sec}^{-1})$	$\Gamma_{K_{e3}^0} / 2\Gamma_{K_{e3}^+}$
Vienna Conference ¹⁷ (1968)	7.65 ± 0.30	0.99 ± 0.04
Review of Particle Properties ¹⁸ (1968)	6.62 ± 0.29	0.85 ± 0.05
Review of Particle Properties ¹⁹ (1970)	7.22 ± 0.29	0.94 ± 0.04
Review of Particle Properties ²⁰ (1971)	7.52 ± 0.14	0.976 ± 0.022

The decay rate of K_{e3}^0 obtained from the sample selection with criteria A is discussed in the previous section and is

$$\Gamma_{2\pi} = (7.81 \pm 0.56) \times 10^6 \text{ sec}^{-1} \text{ (this experiment)}$$

and is compatible with another direct measurement of this rate

$$\Gamma_{2\beta} = (7.52 \pm 0.85) \times 10^6 \text{ sec}^{-1} \text{ (Aubert et al.)}$$

and is less than one standard deviation from the rate obtained by the overall fit from Review of Particle Properties (see Table XII) and, when compared with the rate of K_{e3}^+ $(3.92 \pm 0.061) \times 10^6 \text{ sec}^{-1}$, yield

$$\Gamma_{K_{e3}^0} / 2\Gamma_{K_{e3}^+} = 0.994 \pm 0.073 .$$

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APPENDIX A

The MM^2 Distribution of $\Sigma^0 \pi^0$ Channel

The spectrum of the missing-mass square contributed by the process

is investigated in this Appendix. The missing mass (MM) in this reaction is computed as follows:

$$\begin{aligned} (MM)^2 &= (E_{cm} - E_{\Lambda})^2 - P_{\Lambda}^2 \\ &= E_{cm}^2 + m_{\Lambda}^2 - 2E_{cm} E_{\Lambda} \end{aligned} \quad (A.1)$$

where E_{cm} is the total c.m. energy, and E_{Λ} and P_{Λ} are the c.m. energy and momentum of Λ , which is the only secondary particle that can be measured, using its charged decay secondary track. The E_{Λ} are related to E' and P' , the constant energy and momentum of Λ in the Σ^0 rest frame, as

$$E_{\Lambda} = \gamma(E' + \beta P'_{\parallel}) \quad (A.2)$$

where $\parallel$ means parallel to $\vec{\beta}$ or $\vec{P}_{\Sigma}$ of the Σ^0 particle, $\gamma = E_{\Sigma}/m_{\Sigma}$ and $\gamma\beta = P_{\Sigma}/m_{\Sigma}$ are the relativistic factors defined in terms of the c.m. energy and momentum of Σ^0 particle. Rewriting (A.2)

$$E_{\Lambda} = \left[\frac{E_{\Sigma}}{m_{\Sigma}} E' + \frac{\vec{P}_{\Sigma} \cdot \vec{P}'_{\Lambda}}{m_{\Sigma}} \right]$$

and substituting into (A.1)

$$MM^2 = E_{cm}^2 + m_{\Lambda}^2 - 2E_{cm} \left[\frac{E_{\Sigma}}{m_{\Sigma}} E' + \frac{\vec{P}_{\Sigma} \cdot \vec{P}'_{\Lambda}}{m_{\Sigma}} \right] \quad (A.3)$$

The Σ^0 can be produced polarized in the direction of the normal to the production plane. Unless one correlates the angle variables in each decay with the normal to the production plane, all Σ^0 's are "effectively" unpolarized. Consequently, the angle θ between $\vec{P}'_{\Lambda}$ and $\vec{P}_{\Sigma}$ is distributed isotropically, and the spectrum of MM^2 is uniform between kinematic boundaries, which expand as P_{Σ} increases with higher E_{cm} . The shape of the spectrum is independent of the production angle.

APPENDIX B

Gap Counting Technique

The technique described here is a quantitative method to estimate the bubble density. It is employed when visual inspection at the scan table is unable to resolve the identity of the tracks. In the diagram below the sequence of dots represents a track to which a ruler is applied.

where r = actual bubble image radius

w = width of millimeter marking on ruler

Each mm division on the ruler is the scan line. The size of the bubble image combined with the width of the mm division yields the effective scan width (see sketch above). "Hit" or "miss" is determined whether the bubble is intercepted by the mm division. The statistical significant number of bubbles found in the interval of length x along the track is given by kx where k is the mean bubble density along the track. The statistical fluctuation in the number of bubbles found in the interval x is given by P_m , the Poisson distribution

$$P_m = (kx)^m e^{-kx}/m!$$

which is the probability that m bubbles will be found in the interval x where kx bubbles are found on the average. When the length of the track L is sufficiently long with respect to $1/k$, the bubble formation length,

P_m is normalized to one

$$\sum_1^{kL} P_m \approx 1$$

The probability that no bubble will be found in x is given by

$$P_0 = e^{-kx}$$

and the probability that some bubbles will be found is given by

$$1 - P_0 = 1 - e^{-kx}$$

For an effective scan width " a ", the probability of a hit on the mm division is

$$1 - e^{-ka}$$

and the probability of a miss is

$$e^{-ka}$$

For a track with T scans which include M misses and H hits, the likelihood function is

$$L(ka) = (e^{-ka})^M (1 - e^{-ka})^H$$

The maximum likelihood solution $(ka)^*$ is obtained from the relation

$$\frac{\partial L(ka)}{\partial (ka)} = 0$$

and the solution is

$$(ka)^* = \ln \frac{T}{M}$$

and its variance is defined as

$$\overline{\delta(ka)} = - \left(\frac{\partial^2 L(ka)}{\partial (ka)^2} \right)^{-1} = \sqrt{\frac{H}{MT}}$$

A relativistic track with a length of 10 cm in this chamber has about 60 misses out of 100 scans, and this track will yield

$$ka = 0.51$$

$$\overline{\delta(ka)} = 0.82$$

$$\frac{\delta(ka)}{ka} \approx 16\%$$

The percentage of the statistical error will be less than 16% if the track length is greater than 10 cm. The bubble density is known to vary with the particle velocity in the hydrogen as

$$k = k_0/\beta^2$$

where k_0 = the bubble density of a relativistic track. At a momentum of 200 MeV/c, the bubble density of electron and muon are k_0 and $1.275 k_0$ respectively--a deviation of about 28% for muon to be different from electron. In the case just mentioned it is about two standard effect of the measured bubble density.

APPENDIX C

K_{e3} Decay Probability

In the V-A theory of weak interaction, the matrix for the leptonic K_{e3} decays is generally written²¹

$$\sim (f_+(q^2)(P_K + P_\pi)_\mu + f_-(q^2)(P_K - P_\pi)_\mu) \bar{u}_\ell \gamma_\mu (1 + \gamma_5) u_\nu$$

where P_K and P_π are the four momenta of K and π, and f₊(q²) and f₋(q²) are called the form factors for the decay process. The matrix element yields the Dalitz plot density of the decays in terms of pion and electron energy in the kaon's rest frame.

$$\frac{d^2\Gamma}{dE_\pi dE_e} \sim m_K |f_+(q^2)|^2 \{2E_e(m_K - E_\pi - E_e) - m_K(W_0 - E_\pi)\} \quad (C.1)$$

where m_K, m_π and m_e are the rest masses of K, π, e respectively, q² = (P_K - P_π)² = m_K² + m_π² - 2m_KE_π is the square of the four momenta transfer, and W₀ = (m_K² + m_π² - m_e²)/2m_K is the maximum pion energy. Terms involving f₋ have been neglected since they are multiplied by (m_e²/m_K²). The relative population in the Dalitz plot can be used to determine the form factor. Monte Carlo events were generated according to this distribution for K_{e3} decays.

Integrating this Dalitz plot distribution with the maximum and minimum electron energies as

$$\begin{aligned} E_e \text{ max} \\ E_e \text{ min} \end{aligned} = \frac{(m_K - E_\pi \pm P_\pi)^2 + m_e^2}{(m_K - E_\pi \pm P_\pi)}$$

the pion spectrum is

$$\frac{d\Gamma}{dE_\pi} \sim m_K |f_+(q^2)|^2 \frac{P_\pi^3}{3} \quad (C.2)$$

Terms involving powers of (m_e/m_π)² are neglected in Eq. (C.2). The distri-

bution may also be used to determine the form factor, which assumes a small energy dependence:

$$f_+(q^2) = f_+(0)(1 + \lambda q^2/m_\pi^2) \quad (C.3)$$

The partial decay rate is obtained by integrating (C.2) over the pion energy spectrum

$$\Gamma = |f_+(0)|^2 \left\{ \frac{m_K^5}{192} F\left(\frac{m_\pi}{m_K}\right) + \lambda \left[\frac{m_K^2 + m_\pi^2}{m_\pi^2} \frac{m_K^5}{192} F(m_\pi/m_K) - \frac{1}{240} \left(\frac{m_K}{m_\pi}\right)^2 \left(\frac{m_K^2 - m_\pi^2}{2m_K}\right) m_K^5 \right] \right\} \quad (C.4)$$

where $F(X = \frac{m_\pi}{m_K}) = 1 - 8X^2 + 8X^6 - X^8 - 24X^4 \ln(X)$ is a function of the ratio of pion to kaon masses, resulting from the integration. From the formula (C.4) one calculates the ratio of $\Gamma_{K_{e3}^0}/2\Gamma_{K_{e3}^+}$ due to the mass differences:

λ	$\Gamma_{K_{e3}^0}/2\Gamma_{K_{e3}^+}$
0.00	1.012
0.01	1.008
0.02	1.004
0.03	1.0001
0.04	0.9963

The values of masses were taken from the Review of Particle Properties²⁰ (1971).

FOOTNOTES AND REFERENCES

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FIGURE CAPTIONS

Fig. 1. Definition of the 25-inch bubble chamber coordinate system.

Fig. 2. Momentum distribution of $K^- \rightarrow \pi^- \pi^- \pi^+$ decays.

Fig. 3. Pathlength distribution for K^- beam corresponding to Fig. 2.

Fig. 4. (a,b,c) Distributions of the missing mass squared for K^- neutral hyperon production for three intervals of K^- momenta: 180-220 MeV/c, 220-260 MeV/c, and 260-300 MeV/c respectively. The shaded areas correspond to the events which fit the hypothesis of the interaction $K^- p \rightarrow \Lambda \pi^0$.

Fig. 5. K_L^0 beam layout.

Figs. 6, 7, and 8. Momentum distributions for $K_L^0 p \rightarrow K_S^0 p$, $\Lambda \pi^+$, and $\Sigma^0 \pi^+$ interactions respectively from all data. The shaded events correspond to those which failed to be fitted with the K_L^0 direction assumed to be from the target.

Fig. 9. Pathlength distribution for K_L^0 beam for the portion of films where the K_{e3}^0 sample was obtained.

Fig. 10. The above distribution (Fig. 9) obtained in a different way by assuming cross sections for the interactions $K_L^0 p \rightarrow \Lambda \pi^+$ and $K^- p \rightarrow \Lambda \pi^0$ equal at their total c.m. energies.

Fig. 11. K_L^0 momentum distribution obtained from the pathlength distribution (Fig. 9). The portion of events accepted by selection Criteria A is shaded in the figure.

Fig. 12. Distribution of the laboratory momenta for electrons or positrons. Events which are not identified by bubble density are shaded and events with momentum greater than 200 MeV are included.

Fig. 13. The measured length distribution for electrons or positrons of the K_{e3}^0 sample selected by Criteria A.

Fig. 14. χ^2 vs λ , from fitting the Dalitz plot distribution. There are ten degrees of freedom. λ is determined to be 0.01 ± 0.015 .

Figs. 15 and 16. Distributions in the c.m. system of the electron and pion energy spectra respectively, using the selection Criteria A. The dashed curves are the Monte Carlo predictions for $\lambda = 0.01$, using the same criteria, and normalized to the same number of events.

Figs. 17 and 18. Detection efficiencies of K_{e3}^0 decays estimated by the FAKE program using selection Criteria A and B respectively.

Definition of the 25" Bubble Chamber Coordinate System

XBL 749-1424

Fig. 1

XBL 719-1425

Fig. 2

K⁻ PATHLENGTH DISTRIBUTION

XBL 749-1426

Fig. 3

$K^-p \rightarrow \Lambda$ (neutrals)

XBL 749-1427

Fig. 4(a)

$K^-p \rightarrow \Lambda$ (neutrals)

XBL 719-1428

Fig. 4(b)

XBL 719-1429

Fig. 4(c)

Fig. 5

$K_{2p}^0 \rightarrow K_{1p}^0$

XBI 719-1430

Fig. 6

XBL 719-1431

Fig. 7

XBL 749-1432

Fig. 8

K_2^0 PATHLENGTH DISTRIBUTION

XBL 719-1433

Fig. 9

K_2^0 PATHLENGTH DISTRIBUTION

XBL 719-1434

Fig. 10

K_2^0 MOMENTUM DISTRIBUTION

XBL 719-2435

Fig. 11

K_{e3}^0 Decays

XBL 749-1436

Fig. 12

K_{e3}^0 Decays

XBL 719-1437

Fig. 13

XBL 677-3536 A

Fig. 14

XBL677-3537

Fig. 15

Fig. 16

XBL 877-3538

CRITERIA A

XBL 719-1438

Fig. 17

CRITERIA B

XBL 719-1439

Fig. 18