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Beam Experiments with the MURA 50 Mev FFAG Accelerator*

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MASTER

I. INTRODUCTION

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In 1957 MURA began construction of a radial sector 50 Mev FFAG electron accelerator.¹ Its configuration was chosen in such a way that it could be operated as a two-way accelerator or by change of excitation a one-way accelerator of higher energy. The principal purposes for the construction of this accelerator were the investigation of many particle effects in accelerators, an experimental verification of radio-frequency accelerator methods, and a test of the two-way accelerator. Earlier results from MURA accelerators published in these Proceedings in 1959 and 1961,^{2,3,4} dealt with the verification of the theories of high efficiency beam stacking, multi-turn injection and operation of a two-way accelerator. Since this time, attention has been focused on a variety of accelerator problems which bear on the feasibility of the construction of high intensity, high energy proton accelerators and storage rings. These range from studies of single particle resonant effects to coherent beam phenomena.

II. BETATRON OSCILLATION FREQUENCY MEASUREMENTS

The radial betatron oscillation frequency, ν_r , and the vertical betatron oscillation frequency, ν_z , have been measured for small

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amplitude oscillations. The measurements were made at many radii in the accelerator for which computer calculated values of the tune have been obtained.

The computed tune values were obtained for small amplitude oscillations using measured magnetic field data and the Parmesh computer program. The results of this computer investigation of the tunes were reported by Parzen at the 1961 Brookhaven International Conference on High Energy Accelerators.⁵

The experimental tune data was obtained by the method of resonant driving of betatron oscillations with a localized radio-frequency voltage.⁶

In order to permit comparison of the measured tunes with the calculated tunes, the beam was limited in intensity and in radial and vertical amplitudes. The radial amplitudes were kept small by injecting at the minimum energy that the accelerator would accept. Vertical amplitudes were kept small by a vertical aperture limiting probe.

In order to obtain precision measurements of the frequencies that cause resonant growth of the betatron oscillation amplitudes, the knockout rf voltage is pulsed on during the time the beam is coasting at a constant revolution frequency.

For measurements at small radii the injection time is adjusted so that just enough energy is received from the injection betatron pulse to bring the beam out to the radius where the tune is to be measured. The beam circulates there until the analyzing betatron pulse gives it enough additional energy to bring it out onto a target.

At larger radii, the betatron pulse does not provide enough energy to bring the beam out to the desired radius, so rf acceleration is used to provide the necessary additional energy. Again the analyzing betatron pulse is used to bring the beam out to the target.

The knockout rf is pulsed on, in either case, shortly before the analyzing betatron comes on. At this time the particles are not under the influence of any accelerating voltage and have a constant revolution frequency. Resonant driving of the betatron oscillations is detected by observing the appearance of scintillation pulses during the time the knockout rf is pulsed on. These scintillation pulses are produced when the amplitudes of some of the beam particles grow large enough for the particles to strike a target located very near the circulating beam. When vertical tunes are measured, the target is located above or below the beam. When radial tunes are measured, the target is at a slightly larger radius than the circulating beam. The analyzing betatron pulse brings the remainder of the beam out to a target to provide an indication of the amount of beam being accelerated.

The betatron oscillation frequencies are obtained from the relation:⁶

$$\nu = \left| \frac{f_{KO}}{f_0} \pm N \right|. \quad (2.1)$$

Where ν is the betatron oscillation frequency in question, f_{KO} is the knockout rf frequency, f_0 is the particle revolution frequency and N is an integer.

Precision determinations of f_0 and f_{KO} along with a knowledge of ν to within ± 0.5 so that N can be determined permit the measurement of ν with an accuracy of $\pm 0.1\%$ or better.

Both the measured and the calculated values of ν_r versus radius are shown in Fig. 1. The agreement between the experimental data and the calculated frequencies is exceptionally good except in the large radius region. The discrepancy in this region is probably due to fast azimuthal variations of the magnetic field in the region of the nonscaling pole. Since there were only 10 mesh points per sector for the computer calculation, harmonics higher than the fourth sector harmonic could not be included. The fact that the tune variations can be so sudden is very surprising.

Measured and calculated values of ν_z versus radius are shown in Fig. 2. Again the agreement is generally good. The discrepancies at small radius are probably due to reluctance effects causing the positive to negative field ratio to be slightly different at the time the tunes were measured from what it was when the field data from which the tunes were calculated was taken. Again the coarseness of the computer mesh is probably responsible for the discrepancy above 190 cm. Figure 3 shows the experimental tune points plotted on the ν_r, ν_z plane. Each point represents the tune of the beam at one radius. A number of resonance lines are also shown on this plot. It can be seen that the tune crosses a number of nonlinear resonances during acceleration. Effects of these resonances are seen but no serious beam loss occurs.

In a scaling FFAG accelerator, such as the MURA 50 Mev electron accelerator, the nonlinear terms in the equation of motion of the particles produces a dependence of the betatron oscillation frequencies on the

amplitudes of the horizontal and vertical oscillations. The relations involved in the tune changes are complicated and evaluation of the tune changes is best done by digital computation. The development by Parzen of a computer technique to measure the nonlinear tune ν_r, ν_z to an accuracy of one part in 10^4 has permitted mapping on the ν_r, ν_z plane the dependence of ν_r, ν_z on the oscillation amplitudes A_r and A_z . This dependence can be presented in the form of a tune-space diagram. The method used to map out the tune space diagram has been to hold the vertical amplitude fixed and vary the radial amplitude in discrete steps, computing with the Parmesh program the radial and vertical betatron oscillation frequencies for each point. This results in a curve in ν_r, ν_z space for a given vertical amplitude. A number of such curves for different vertical amplitudes and the curves connecting points of equal radial amplitude then maps out a region in tune space in which the tune of particles, within the range amplitudes considered, will exist.

The tune-space diagram for the 50 Mev accelerator is shown in Fig. 4. The tunes for this diagram were calculated using the measured magnetic fields of the accelerator. A number of resonance lines are also drawn on this diagram.

The combination of the tune change due to amplitude changes and the tune change as a function of radius may make a resonance serious or negligible depending on the way the tune changes combine. If the tune changes are in opposite directions, the beam tends to remain on the resonance longer and the amplitude growth is larger. If the tune changes are

in the same direction, the beam moves through the resonance more rapidly and there is less effect on the amplitudes.

An experiment was performed on the 50 Mev accelerator to try to test the computer results on the tune change with amplitude. Since it was not possible, at the time this experiment was done, to obtain a beam having a small spread of vertical amplitudes at large vertical amplitude, we could only obtain good data for the case where the vertical amplitudes were small.

The experiment was conducted at a particle energy of 100 kev. By use of the pulsed bump injection system, particles could be injected on the 100 kev equilibrium orbit or with radial amplitudes up to 1.7 cm about the 100 kev equilibrium orbit. This is an amplitude of about 0.015 in units of the radius. Proper adjustment of the timing of the injector pulse and the bump pulse made possible the injection of a small spread of radial amplitudes about a desired amplitude. The vertical amplitudes were limited to small values between 0.0017 and 0.003 (in units of the radius) by a vertical aperture limiting probe. Tunes were measured by the rf knockout method to about 0.03% in both ν_r and ν_z . This precision of tune measurement is necessary since the maximum tune change over the range of amplitudes available is only about 1%.

Figure 5 presents a comparison of the computer data for a vertical amplitude of 0.0027 in units of the radius and experimental data for vertical amplitudes of 0.0017 and 0.003. No significant difference is seen in the experimental data for the two amplitudes. Instead of being plotted in ν_r, ν_z space, the change in ν_r and the change in ν_z are plotted separately versus

the radial amplitude in the center of a positive magnet. The radial frequency increases but the vertical frequency decreases for this case where the vertical amplitudes are small. The computer curve and the experimental points agree quite well for the vertical tune change, but the agreement is not so good for the radial tune change. The radial frequencies are harder to measure than the vertical frequencies and although the percentage accuracies are about the same, the absolute accuracy for the radial measurements is worse than for the vertical measurements since the radial tune is about 4.41 while the vertical tune is about 2.75.

III. COUPLING RESONANCE STUDIES

Studies have been carried out in the 50 Mev accelerator to determine the effect of nonlinear coupling resonances on the amplitude of betatron oscillations. Two resonances were studied, the $2\nu_z + \nu_r = 10$ sum resonance and the $2\nu_z - \nu_r = 1$ difference resonance. To study the effect of these resonances on the amplitudes, the beam was accelerated and its vertical size measured as a function of the radius. Since single particle effects were being investigated, a low intensity beam was used. In order to maximize the effects of the resonances, a beam having only small vertical amplitudes and a small spread of large radial amplitudes was used.

When the magnetic guide field is set to its standard operating point, the beam twice crosses the $2\nu_z + \nu_r = 10$ sum resonance quite near injection. A beam was injected with 0.12 inch maximum vertical amplitude ($A_z/R = 0.0025$) and with 0.7 inch radial amplitude ($A_r/R = 0.015$). The spread in radial amplitudes was 0.14 inch ($A_r/R = 0.0029$). Figure 6 is a plot of the maximum vertical amplitude measured throughout the acceleration

cycle. The two abrupt increases in $A_{z \max}$ at 51.2 inches and 52.3 inches are due to the $2\nu_z + \nu_r = 10$ sum resonance. Only a small fraction of the particles grow to the maximum amplitude of 0.37 inch at the resonance. The range of vertical amplitudes after passing through the resonance is from 0.03 inch to 0.37 inch. Adiabatic damping of the betatron oscillation amplitudes is also evident in regions where the beam is free from the effect of resonances. No measurements were taken of the radial amplitudes near the resonance.

The study of the $2\nu_z - \nu_r = 1$ difference resonance proceeded in a similar manner. The operating point of the machine at injection was changed so the beam crossed this resonance near injection, at 50.4 inches and at 52.3 inches. A beam was injected with a maximum vertical amplitude of 0.17 inch ($A_z/R = 0.0035$) and with radial amplitudes of 0.8 inch ($A_r/R = 0.017$). The spread in radial amplitudes was 0.08 inch ($A_r/R = 0.0017$). The measured maximum vertical amplitudes in the region of interest are shown in Fig. 7. Vertical growth occurs each time the $2\nu_z - \nu_r = 1$ resonance is crossed. Measurements were also made of the radial amplitudes in this region. When the beam crosses the difference resonance, it is split into two components. One component has essentially the same vertical and radial amplitudes it had before it entered the resonance. The other component has increased vertical amplitudes and decreased radial amplitudes. This segment exhibits the expected coupling of energy from the radial into the vertical mode.

Since these studies indicated quite large growths at the sum resonance, the vertical amplitudes were measured using a "normal" beam. A high intensity beam, having a full range of vertical amplitudes ($A_{y \max} \approx 0.7$ inch) and a full range of radial amplitudes ($A_r \approx 0.8$ inch) was injected and accelerated. In this study interest was centered on what happened to the beam as a whole. Consequently, $A_{y \max}$ refers to the maximum amplitude of 90% of the beam. The growth of a minor portion of the beam is disregarded. The resulting data is plotted in Fig. 8. Note that the beam as a whole shows no pronounced growth at any stage. The line drawn through the measured points is at a slope of - 4.61, the adiabatic damping coefficient for the machine.

IV. BEAM LIFETIME STUDIES

In this accelerator one can expect the lifetime of a coasting beam to be limited by:

- a. Proximity of the operating point to resonances.
- b. Gas scattering.
- c. Radiation anti-damping of the radial betatron oscillations.

These effects are always present to some extent. In order to get an understanding of the importance of these factors on the lifetime, the experiments were performed in a manner that minimized effects of collective interactions and effects of the two accelerating systems on the lifetime.

For each measurement only a single pulse of electrons was injected; furthermore, this pulse was collimated so that its intensity was reduced by a factor of 10^{-2} to 10^{-3} (resulting in a beam cross section of approximately 1 mm^2 at the stacking energies). This beam was accelerated in the normal

way, except that at the end of the acceleration cycle, the rf accelerating voltage was kept on, and its frequency clamped so as to maintain a constant beam energy during the life of the beam. The frequency deviation during the holding time was less than 7 kc out of 25 Mc, corresponding to a radial motion of $1/2$ mm. The displacement acceleration system was not used. The beam was detected by the signal it induced in the electrostatic induction probe. The signal was amplified by wide band amplifiers, rectified, and displayed on an oscilloscope. In Fig. 9, the rectified signal is displayed vertically as a function of the time horizontally. The time scale is 1 second per major division. We see that the signal increases during the early life of the beam. This is presumed to be due to the energy damping resulting from synchrotron radiation. Because this damping occurs while the beam is inside an rf bucket and over a time long compared to a phase-oscillation period, it is accompanied by increased azimuthal bunching to which the sensitivity of the induction probe is higher. There is also a rather long time during which no beam is lost, and a comparable period of time over which beam is lost.

To obtain the dependence of the lifetime on aperture, the available radial and vertical apertures were varied by the means of probes. When this was done, a shortening of lifetime resulted, as indicated in Fig. 10. We have chosen the time for the beam to fall to half intensity as a representative parameter of each curve. We call this time the halftime.

The halftime was measured as a function of energy and the results are shown in Fig. 11. The halftime increases as a function of energy as

expected from gas scattering. At about 40 Mev there are two sharp dips in the halftime. These are due to several high order coupling resonances present in the machine, but which ordinarily cause no beam loss. The energy of the first dip (39 Mev) is where three resonances occur:

$\gamma_r + 2\gamma_z = 10$, $3\gamma_r + \gamma_z = 16$, and $2\gamma_r - \gamma_z = 6$. The other dip is where $2\gamma_z - \gamma_r = 1$. Above 45 Mev we leave the region of good magnetic field and the stability limits decrease. Also we expect anti-damping of the radial oscillations to become important.

In Fig. 12 the halftime is plotted horizontally, as a function of the vertical aperture. The expected curvature of the graphs is observed. Further, we note the saturation of the halftime at rather modest aperture. This is due to the presence of the clearing electrodes in the vacuum tank. We notice the difference between the curves at 24.5 Mev and 27.2 Mev, when we cross into the region where there are clearing electrodes. This vertical limitation presents a severe limitation on radial measurements, except for very small radial apertures. In Fig. 13 the halftime is plotted as a function of radial aperture. We see that the halftime saturates at very small amplitudes. This is due to the vertical limitation.

Each of the six energies shown in Figs. 12 and 13 was carefully chosen to avoid the halftime dips (see Fig. 3) caused by stacking on resonances. At these energies, one would then expect only gas scattering and radiation damping (of the vertical oscillations) to determine the halftime vs. aperture curves, with gas scattering predominating at sufficiently small amplitudes and/or energies.

Under these conditions the halftime would depend on the square of the available aperture. Then the slope of a plot of halftime vs. square of the aperture would give us directly the growth rate due to scattering.

Figure 14 shows the halftime plotted against the square of the vertical aperture for each energy. Up to rather large apertures the points do lie on straight lines.

The table below gives the slopes of these lines and also the expected growth rates due to multiple scattering at a pressure of 3.5×10^{-7} Torr. The ratios of these are also given. They are far from unity. The measured pressure was 2.5×10^{-7} Torr, but the constituents of the gas, when weighted according to Z^2 , yielded an equivalent nitrogen pressure of 3.5×10^{-7} Torr. If we assume that the equivalent pressure is in error by a factor of 3, then the measured and expected slopes would be in agreement for the first four energies but not for the last two. Furthermore, the gas pressure and composition were carefully measured, giving an estimated error of 30% rather than 300%.

TABLE I

Energy (Mev)	Growth Rate (in^2/sec)		Ratio Theory/Exp't.
	Experiment	Theory	
19.0	0.062	0.171	2.8
24.5	.035	.105	3.0
27.2	.030	.0873	2.9
34.6	.016	.0552	3.5
39.8	.011	.0457	4.3
46.2	.0063	.0376	6.0

Another discrepancy between theory and experiment is the apparent absence of radiation damping at the larger amplitudes. A calculation of the combined effects of multiple scattering and radiation damping showed that the damping should have been observable even at 19 Mev, and at the highest energy, 46.2 Mev, should have limited y^2 in Fig. 14 to 0.003 in². A more accurate calculation is being made.

To check that the proper damping rates were used in this calculation, the rate of synchrotron radiation, dE/dt , was measured at five energies between 28 and 40 Mev. The damping rate is equal to $\frac{1}{2E} (dE/dt)$. In order to measure dE/dt , a beam was maintained at its stacking energy with the displacement rf system, the ordinary accelerating system being turned off after the beam was stacked. dE/dt was then obtained from the measured parameters of the displacement bucket. The theoretical and experimental values agreed to within the estimated error of $\pm 15\%$.

V. MULTITURN INJECTION

Electrons are captured at injection by a variety of mechanisms. Low intensity beams are accelerated away from the thin (0.010 inch) inflector septum by betatron or rf voltage with efficiencies up to several per cent. In addition, multiturn capture without aid of acceleration is accomplished with efficiencies up to 100% by use of other methods.

The beam emittance from the injector is a small fraction of the accelerator admittance. The radial emittance, a_x , is about 1% of the radial admittance, A_x , as shown in the phase plot for radial motion in the phase plot for radial motion in Fig. 15. Likewise for the vertical

motion, a_y is a similar fraction of A_y . The maximum number of turns which will fit into the accelerator admittance is $N = \frac{A_x A_y}{a_x a_y}$, although, in practice, the number of turns may be limited to a much smaller value by the nature of the injection mechanism. In the MURA accelerator, injection occurs on the median plane with an adjustable radial betatron oscillation amplitude so that effort is directed toward filling only the radial phase space.

The multiturn injection method which has been investigated the most is application of a time-varying guide field bump at the injection radius to gradually shift the equilibrium orbit away from the inflector septum during the interval of injection. The radial attractive field of a single bump electrode perturbs the equilibrium particle orbits as shown in Fig. 16. Injection is accomplished by turning off the bump adiabatically. Injected particles continue to oscillate with approximately constant amplitudes about the perturbed equilibrium orbit and thus move away from the inflector with the orbit.

In the absence of orbit movement, an electron returns to strike the septum in some n turns depending on its value of radial tune ν_x and its initial position and angle (x, x'). There is thus some number of turns during which the orbit must move a distance equal to the spread, ΔA_x , in oscillation amplitudes of the beam plus the septum thickness in order to effect capture of the total beam. The injected beam is matched to the accelerator to give approximately the minimum value of ΔA_x , which is a function of beam emittance.

In measurements to determine efficiency of injection, the bump electrode voltage was adjustable in amplitude from zero to 6 kv with a fall time adjustable in steps from 0.05 to 9μ sec. Assurance that space charge effects were negligible was obtained by setting the beam intensity from the electron gun well below the threshold for capture due to space charge damping of betatron oscillation amplitudes. Current injected during the bump fall time was captured and 100μ sec later accelerated out in radius by a betatron core to a current collecting probe. This current probe was calibrated against an inductive pickup probe giving a measure of the accelerated charge. Injected charge was determined by intercepting the first turn current with a Faraday cup. Efficiency was taken as the ratio of accelerated to injected charge.

A sample of some of the data is given in Fig. 17 where capture efficiency is plotted as a function of orbit displacement per turn for an injection energy of 95 kev. Also shown is the corresponding number of turns for which beam was captured. Beyond about 15 turns the efficiency drops off so that although more turns contribute, the total accelerated charge remains approximately constant. The general shape of the curve is characteristic of several runs taken at various energies and for different electron guns. The maximum efficiency fluctuated above and below 100% reflecting uncertainties in the measurement. Reduction of the beam size from the inflector by collimation resulted in high efficiency of capture up to 40 turns.

An example of the large range of betatron oscillation amplitude produced by the programmed field bump is shown by the oscilloscope traces

of Fig. 18. The observation is of accelerated current striking a target probe for injection before the betatron core is turned on. For one trace the field bump is off and there is just sufficient beam intensity to effect capture of a small amount of beam by space charge forces alone. For the other trace, the field bump is on with the same injected beam intensity. One notes the increased charge captured by use of the bump and smaller amplitudes filled in as seen from delay in arrival time at the target by much of the beam.

An indication of high capture efficiency from use of the field bump is indicated in Fig. 19. Here is shown the differentiated signal from an electrostatic pickup electrode as a 30 nsec beam pulse passes it on successive turns. Beam was injected for a time less than the 50 nsec revolution period by pulsing the inflector during the fall time of the bump. The beam normally hits the inflector after approximately 5 turns when the bump field is not used. The gradual decrease in signal with successive turns in this picture is principally due to space charge forces in the moderately intense beam pulse which increase the amplitude and energy spread of the particles and thereby the spread in their revolution frequencies.

A second method of multiturn beam capture which was demonstrated is rf "knock-in" whereby a radial rf electric field is applied at some point in the injection orbit. Betatron oscillations of some of the electrons have appropriate phases with respect to the applied rf field to have their amplitudes damped so that the electrons miss the inflector. Electrons at other phases have their amplitudes increased and become lost on the inflector.

Fewer than half of the particles can be captured in this manner. On the other hand, certain good features of the method are worth noting. Contributions to the motion due to nonlinearities of the perturbing field are averaged out. Also, the perturbing field does not cause a reduction of the stability limit due to nonlinearities of the guide field, in contrast with the programmed bump method.

In a test of the method, beam intensity was set low to eliminate space charge effects. Some current was captured over the complete rf frequency range available from 7 to 28 Mc/sec with the most knock-in occurring for frequencies near 8 and 12 Mc/sec. The particle revolution frequency was 20 Mc/sec. The betatron oscillation amplitude distribution was modified by the filling in of smaller amplitudes as in the case of the programmed bump injection.

A third method of multiturn injection which always plays a role for the higher intensity beams is capture through space charge effects. Various theories give oscillation damping for some electrons at the expense of oscillation growth for other electrons, or other electron loss. Interaction can occur between electrons in successive turns and within the same turn of beam.

Injection without aid of the betatron cores or other special mechanisms results in capture of a large circulating beam current. At sufficiently high injected beam intensity, little more beam is captured with the aid of the programmed field bump or of rf knock-in. Some of the beam which is lost on the septum lives for many hundreds of turns before loss. When the

captured beam is betatron accelerated onto a target probe, there is a spread in time of arrival at the probe resulting from a spread in radial oscillation amplitudes and energy produced by space charge forces. The observed threshold for beam capture is less than one ma of beam from the electron gun. It has been shown that some beam is captured even when less than one turn of beam is injected for sufficiently high intensity beam (~ 50 ma) from the gun.

The most circulating beam current which has been captured and accelerated from the injection radius by any means is ~ 150 ma. Estimates for the space charge limit for 100 kev injection give roughly 300 ma.

VI. LONGITUDINAL BUNCHING

Azimuthal bunching of the captured beam sets in at injection. It is seen to set in for a beam intensity not greatly different from that producing capture. The bunching is detected by an electrostatic induction electrode and appears as an amplitude modulated pattern. This pattern varies from one injected beam pulse to another and its general character changes with intensity and other parameters. Frequencies consist of a strong fundamental equal to the particle revolution frequency plus higher harmonics. With betatron acceleration to higher energy, bunching continues and frequently grows larger. Figure 20 gives samples of the induction signal from the bunched beam below transition energy.

As the beam is accelerated across transition energy into the region where the negative mass instability exists, bunching usually sets in with renewed intensity. An example of the signal in the vicinity of transition is

shown in Fig. 21. The fast sweep samples were taken just above transition.

Measurements have shown approximately 5% bunching of the beam below transition at the injection radius with two to three times this amount above transition.

Further study of bunching in the coasting beam at injection was made by pulsing the inflector to inject beam pulses variable in length from six turns down to one-half turn. With injection of a moderately high intensity beam (~ 30 ma) from the gun for one-half turn, the initial beam lump persists for the order of 100 turns by which time the lump structure has gradually disappeared. A low intensity beam lump persists much longer, thus indicating the role of space charge forces in spreading the revolution frequencies in the high intensity beam. After the circulating beam loses its longitudinal density variation almost completely, there begins a slow build-up of longitudinal bunching which reaches a maximum intensity in approximately $50 \mu\text{sec}$ as shown in Fig. 22. In marked contrast to the initial lumped structure, this later bunching is not repetitive in detailed structure from pulse to pulse, as shown in Fig. 23 where several beam clumps are in evidence. As the beam is allowed to circulate at the injection radius for many milliseconds, the bunching instability is observed to disappear with a lifetime of a few milliseconds and never to return. There is observed to remain much more circulating current in this old unbunched beam than that which is required to produce the instability in a new beam of low intensity. The beam thus seems to acquire, given sufficient time, an energy and amplitude distribution which is conducive to stability.

When a pulsed beam of a few turns is injected, it is impossible to isolate bunching due to inherent instability of the beam from the initial bunching (now smaller than for the half-turn beam) due to the injection process. The bunching is now nearly repetitive in structure from pulse to pulse for a much longer time than it is for the case of half-turn injection. Bunching is shown in Figure 24 starting with injection time for a six-turn beam, where relative motion of beam lumps cause modulation of the signal.

At energies below transition the longitudinal bunching has been qualitatively attributed to a two-stream or multistream instability in the revolution frequency distribution, since conditions appear to favor this possibility.

VII. RADIO-FREQUENCY EXPERIMENTS

The great versatility of the acceleration system of the 50 Mev accelerator with its separate betatron, rf acceleration, and rf displacement modes permits a great variety of experiments concerning the acceleration and stacking processes to be performed. Three of these experiments will be described. They concern programed acceleration over transition in an FFAQ accelerator, energy spread measurements on the accelerated beam, and beam stacking procedures.

The experiment on programed acceleration through the transition energy (which occurs at 1.13 Mev in this accelerator) used betatron acceleration to a few hundred kev to reduce the frequency range covered by the rf system. The beam was picked up by the accelerating rf at the betatron drop-off energy and accelerated to the transition energy. At this point an

increase of frequency of short duration produced the necessary phase shift of the rf with respect to the beam. The beam is then accelerated on to higher energy above transition.

The results of the experiment are illustrated in Fig. 25. Figure 25(b) is the frequency program for the rf system. It gives an increasing frequency up to transition, a sudden frequency jump of short duration at transition, and a slow decrease in frequency to accelerate the beam above transition. The adjustment of the rapid frequency jump is quite critical to obtain optimum acceleration through transition.

Figure 25(c) shows the amplitude program for the accelerating rf. The dip in the amplitude is to tailor the rf bucket to the beam as it crosses transition. In practice, it appears that the amount of beam carried across transition is rather insensitive to the lowering or raising of the dip.

Figure 25(d) is the signal from the electrostatic induction electrode. The beam moves into the electrode just below transition. At transition, there is a dip in the induction signal where some debunching of the beam occurs. On the high energy side of transition the beam is accelerated out until it strikes the rear of the induction probe causing the scintillation detector signal of Fig. 25 (a). The efficiency of acceleration through transition is estimated to be about 100%. The ripples in the induction probe signal above transition are assumed to be due to collective effects in the beam.

The measurements of the energy spread in the beam were made using two different techniques, one for the energy range where betatron acceleration is used and another for the energy range where radio-frequency acceleration is used.

In the first method the beam is accelerated to the desired energy with the betatron cores and left to circulate at that energy. The radio-frequency system was then turned on at very low voltage so as to produce an accelerating bucket of area small compared to that of the beam. In this way a small sample of the beam is captured and accelerated to a current probe. By varying the turn-on frequency, a density vs. frequency plot was obtained. This procedure was used for beams of two different intensities, one near the maximum current that could be obtained from the injector, and the other one-tenth that intensity. The frequency spread data was converted to the synchrotron phase space "energy" variable W through the relation

$$\Delta W = \frac{\Delta E}{f} = \frac{E df}{K f^2} \quad (7.1)$$

where

$$K = \frac{E}{f} \frac{df}{dE} \quad (7.2)$$

The results of these measurements are shown in Figs. 26, 27, and 28 for three different beam energies, 0.500 Mev, 0.880 Mev, and 1.5 Mev. The interval labeled "Max ΔW_1 " above the curves in each figure indicates the maximum energy spread for the injected beam for each case.

For the measurements at 0.500 Mev in Fig. 26, there are two interesting observations. First, for the smaller beam current the spread in W is just about that for the injected beam ΔW_1 . Second, for the larger current not only does the beam spread out in W but also moves to a lower value of W_0 . This result is in agreement with measurements of the betatron oscillation amplitudes and the relation between the betatron oscillation amplitudes and frequency:

$$df_{\beta} \approx \frac{A_r^2 \nu_r^2}{4} + \frac{A_z^2 \nu_z^2}{4} \quad (7.3)$$

These measurements indicate that larger injection currents generally result in large betatron amplitudes. These larger amplitudes mean a lower revolution frequency and therefore a lower rate of energy gain from the betatron core. Figure 27 shows the results of the same measurement repeated at 0.880 Mev. Again we see the same shift in W_0 , but a considerably smaller ratio of spreads in W . The conclusion we draw here is that the actual energy spread in both beams resulting from differential betatron acceleration due to the spread in amplitudes has become dominant.

At 1.5 Mev the situation is complicated by the fact that the beam has crossed transition energy at 1.1 Mev and has been subject to the negative mass instability. As shown in Fig. 28, the smaller beam has about the same average spread in W as at 880 kev, but has grown a "tail" in the low energy direction. The larger beam has grown appreciably in W and also has a "tail." Since it is known that the beam crosses a number of coupling resonances at about 900 kev and that the effects of these resonances are strongly dependent on the betatron amplitudes, we conclude that these resonances are primarily responsible for the difference in spreads in W . In fact, the same measurement repeated at 900 kev shows the large beam to look essentially the same as at 1.5 Mev, but without the tail. The tails then are probably the result of the negative mass instability.

In the range of energies where the beam receives acceleration from the radio-frequency system, another method of energy spread measurement was used. To make this measurement at a given energy, the current probe

is pulled out to a radius corresponding to an energy about 1.5 Mev larger. At the time the beam reaches the desired energy, the accelerating bucket area is reduced by lowering the rf accelerating voltage until a small fraction of the beam is "squeezed" out. This is indicated by an output from the current probe at the next betatron pulse. From a measurement of the voltage and of the frequency, the accelerating bucket area may be computed. Figure 29 shows the results of these measurements made on the higher intensity beam. It may be seen here that the total area occupied by the beam is somewhat larger than before capture. This is probably due to nonadiabatic capture. However, it remains roughly constant throughout the rest of the rf acceleration phase. The dip between 20 and 30 Mev which in principle cannot occur is, unfortunately, reproducible. We are at a loss to explain this. We feel however that it is not real, but rather reflects an error in the calculations resulting from an unexpected variation in df/dE with radius. This could come about through second-order variations in orbit scalloping.

We feel that the methods described give a reasonably accurate and consistent measure of the phase area, and therefore the energy spread, of the beam in the MURA 50 Mev electron accelerator and that this energy spread is due mostly to the interaction of synchrotron and betatron motion of the particles during the first 2 Mev of acceleration.

In the experiment on beam stacking procedures, particles were injected into the machine with an energy of 92 kv. They were then brought to an energy of 1.5 Mev by means of a betatron core. At this energy, and

after the betatron core flux had decreased to zero, the particles were picked up and accelerated by the main radio-frequency accelerating system to the stacking energy which was 30 Mev. As this process was repeated, a circulating beam was built up. Phase displacement was used to restore energy lost through synchrotron radiation by the circulating beam. It is interesting to note that this energy restoration process also provided a convenient means for observing the growth of the stacked beam and for measuring the circulating current and energy spread of the stacked beam. In Fig. 30, we see traces corresponding to the amplitude program of the main accelerating system, trace (d), and the beam induced signal on an electrostatic pickup, trace (b). We will first consider the amplitude program. After the accelerating potential reaches its maximum value (in this case approximately 400 volts), it remains constant during the major part of the acceleration cycle. However, as the accelerated beam approaches the stacking energy interval, the accelerating potential is reduced adiabatically to a value of 175 volts. By adiabatically, we simply mean slowly in comparison to the angular frequency of synchrotron oscillation. As the accelerating potential is reduced, the rate of frequency modulation is also reduced so as to keep the sine of the stable phase angle, $\sin \phi_s$, essentially constant. By performing these operations carefully, it is possible to tailor the "bucket" so as to just fit the accelerated beam as it passes through the stacked beam. If this is done, then in theory there will result a stacked beam of the highest possible density; provided that the density of particles is constant in the accelerating bucket. The stacked beam will move down by just the energy width of the accelerated beam increment resulting in an

an energy continuum of particles. The stack would then continue to grow downwards in energy each time particles are added to it until either the range of energies over which the displacement system can restore synchrotron radiation losses is exceeded or the particles are lost to the stack through the growth of betatron oscillation amplitudes. These, of course, grow with time due to synchrotron radiation so that the particles at the low energy end of the stack have the largest amplitude.

Now turning to the electrostatic pickup signal shown in Fig. 30(b), we see the signal from the accelerated beam as it passes through the region of the pickup electrode and is intercepted by a target probe. (There is some betatron voltage pickup on this signal which helps to correlate the signals with the betatron timing.) We can also see that the reduction of accelerating potential referred to previously has actually "squeezed" some particles out of the bucket. Since the cavity voltage at which this occurs is quite well defined, we feel that the density of particles in the bucket is fairly constant. The apparent slow decay of the electrostatic pickup signal is actually the result of insufficient low frequency response in an amplifier which amplifies the rectified signal from the probe.

The remaining two traces in Fig. 30 are the frequency program, trace (a), and the amplitude program of the displacement system, trace (c). They are shown here only to make clear the sequence of events shown in Fig. 31. As indicated by the frequency program signal, the displacement system turns on at a lower frequency corresponding to an energy higher than the stacking energy and goes to a higher frequency corresponding to a lower

energy than the stacking energy. The displacement system turns off somewhat before the accelerating bucket reaches the lower energy bound of the stacking interval. Now if any particles were circulating in the machine at an energy within the stacking interval as defined by the displacement system frequency program as the displacement bucket passed through, these particles would "anti-bunch" and we would expect to see some signal on the electrostatic pickup.

Figure 32(a) illustrates the importance of reducing the accelerating voltage so the bucket just contains the particles while in the region of the stack. This is a multitrace photograph of the build-up of the stack in a case where the bucket is larger than necessary to enclose the accelerated particles. Each trace is the signal produced on the electrostatic probe as the displacement system sweeps through the stack. The first group of particles stacked is indicated by the smallest signal and as successive groups of particles are added to the stack the signal grows until equilibrium is established. The dips seen in the traces distinguish separate groups of particles added to the stack and indicate that the density of the stack has been reduced by not making the bucket small enough to just "fit" the accelerated beam. Comparison of the equilibrium stack with the stack shown in Fig. 31(b) shows that the density of the low energy part of the stack (farthest to the right) has been much reduced by poor drop-off procedure.

Figure 32(b) illustrates the decay of the stack after the injector is turned off and no more particles are added.

VIII. CLEARING ELECTRODE EXPERIMENTS

Estimates of the neutralized space charge limit in the MURA electron accelerator indicated that the limit might be reached at about 5 to 10 amperes with a 5 Mev energy spread. Since approximately 5 amperes of stacked beam had been obtained, it was important to find out if the stacked beam current could be increased by clearing the neutralizing ions from the neighborhood of the stacked beam.

Since the existing vacuum tank had no provision for clearing electrodes and construction of a new vacuum tank containing them was a costly and time consuming task, studies were made of the possibility of installing a simple electrode structure in the existing tank. The only convenient way to insert the electrodes into the tank was through four ports which were 4.5 inches wide. Therefore, a simple electrode 4 inches wide was considered which could be supported on insulators by the bottom of the vacuum tank and which would extend 2π in azimuth except for gaps for a few probes and the accelerating gaps. This electrode would be centered on the 35 Mev orbit and would extend from 26 Mev to 46 Mev with the edges scalloped to conform to the equilibrium orbit scalloping. Such an electrode could be made in sections and assembled inside the vacuum tank and then positioned in the required location.

The fields necessary to clear the beam were calculated for two cases. The first case was for a stacked current density of 2 amp/Mev (4 amp/cm of radius) and the second more optimistic case of 5 amp/Mev (10 amp/cm of radius). With a beam having 3 mm of vertical amplitude, these give current

densities of 6.5 amp/cm^2 and 16.5 amp/cm^2 respectively. The field strength necessary to clear the beam in the first case is 750 v/cm and for the second case 1900 v/cm . With a 3 cm spacing between the electrode and the top of the vacuum tank, 6 kv were required on the electrode to produce 2 kv/cm .

The electrodes were constructed of titanium and were run with negative polarity. It was hoped that sputtering of the titanium when ions were driven into the plates would produce some pumping. Actually no pumping action has been observed. A plan view of the electrodes is shown in Fig. 33.

Operation of the electrodes at 6 kv produces a 2.5 mm vertical displacement of the beam and some reduction in the vertical aperture at the stacking radius.

The most serious effect of the unsymmetrical nature of the electrodes is the field gradient at the edge of the electrodes which has both radial and vertical components. These gradients produce severe changes in both the radial and vertical betatron oscillation frequencies. Figures 34 and 35 show plots of computed and measured tune changes as functions of the radius. The computation was done using data obtained from a water tank study of the clearing electrode fields and the measured fields of the accelerator. The computed and measured tune changes have the same form but the measured changes are about a factor of 4 larger than the computed ones. The cause of this discrepancy has not yet been determined. The magnitudes of the tune changes are linear with the clearing electrode voltage.

These tune changes were large enough to cause the accelerated beam to cross several nonlinear resonances and to approach the $\nu_z = 3$ integral resonance while crossing the edge of the electrodes. As a result, most of the accelerated beam was lost at the edge of the electrodes at voltages above about 1.5 kv on the electrodes. A scheme of pulsing off the clearing electrode voltage for about 500 μ sec during the time the accelerating beam was crossing the edge of the electrodes permitted getting the major part of the beam across into the more uniform field region at the center of the electrodes. The clearing electrodes had the expected effect on the tune of the stacked beam. When the beam was stacked without clearing, both the vertical and radial betatron oscillation frequencies increased due to the attractive potential of the neutralizing ions. A large tune spread existed in the beam and the radial tune was hard to measure but the vertical tune of the low energy part of the beam increased to 2.86 from 2.82 for a low intensity beam, approaching the $\nu_z = 3$ resonance. Turning on the clearing electrodes to a voltage of 1000 v reduced the tune back to $\nu_z = 2.83$, close to the tune of a low intensity beam at 35 Mev. The tune spread measured in the stacked beam with clearing seems to be quite small. This factor contributes to the current limitation caused by the vertical instability.

The maximum current stacked using the clearing electrodes was 8 amperes over an energy spread of 7.3 Mev. At this point the beam became unstable due to a coherent vertical oscillation. However, this oscillation does not seem to be due to space charge tuning of the beam onto a

resonance but rather to interaction of the cleared beam and the walls of the vacuum chamber.

Under the same conditions that the 8 ampere beam was obtained, the maximum beam with the clearing electrode voltage turned off was 3 amperes.

IX. EXPERIMENTS ON THE STACKED BEAM INSTABILITY

During the experiments with the clearing electrodes, the intensity of the stacked beam was found to be limited by an instability which caused loss of a large fraction of the beam. The characteristic behavior of the stacked beam current is shown in Fig. 36. As indicated by a stacked beam current monitor which samples the current every $1/60$ of a second, the stack slowly fills (for ~ 1 sec) until a current is reached where the beam goes unstable and most of the stack disappears in less than $1/60$ of a second.

For a given set of machine parameters, the build-up and loss of the beam is periodic with the loss occurring each time the stack reaches a certain current. Changing the amount of accelerated current changes the repetition rate at which the loss occurs but does not change the maximum stacked current attained. The stacked current at which the instability occurs can be changed by changing the parameters of the displacement rf system. Changing these parameters results in changes of the energy of the stacked beam and its current density distribution. The relationship between the rf parameters and the current "threshold" at which the instability sets in has not been thoroughly studied.

Investigation of the rf signals produced by the instability provides the best means of determining the nature of the instability. When the beam goes unstable, rf signals resulting from the instability are easily detected on most probes located near the beam. However, when the induction probe is positioned to enclose the entire radial extent of the stacked beam and is sensitive only to longitudinal bunching, the signal picked up on it is negligible compared to the signals picked up on probes sensitive to transverse oscillations. Figure 37 shows the signals picked up on a probe sensitive to both transverse and longitudinal oscillations and the signal picked up by the induction probe which is sensitive only to longitudinal oscillations. The other probe picks up a clean sinusoidal signal while the induction probe detects very little signal at all, although its sensitivity for longitudinal signals is the greater of the two. From these observations it may be concluded that the instability is not of a longitudinal nature. This conclusion was verified by observing the signals induced on plates situated above and below the beam at the same azimuth in the accelerator. These signals proved to be 180° out of phase during the instability. Therefore, the instability involved a coherent vertical oscillation.

Measurements of the frequencies present in the signals from the instability have shown either a frequency of about 4.5 Mc or a frequency of about 28.7 Mc to be present. First measurements of the vertical frequencies showed only the 4.5 Mc to be present. After a short shutdown of the machine, only the 28.7 Mc frequency was seen.

Model of the Instability

The behavior of the beam during the instability is consistent with a simple physical picture of the process, based on the single particle dynamics in the accelerator guide field. In this picture, the beam appears at any instant of time as a wavy thread of electrons. For the purpose of describing the vertical position of the thread-like beam, we make use of a coordinate system which rotates with the beam at the revolution frequency f_0 . Each electron in the beam appears to remain fixed in azimuthal position $\bar{\theta}$ measured with respect of this rotating coordinate system.

The vertical position of the beam at any instant of time is periodic in $\bar{\theta}$ with a period 2π , and can be expressed with complete generality by the sum

$$z(\bar{\theta}, t) = \sum_{n=-\infty}^{+\infty} A_n \cos(n\bar{\theta} + 2\pi \nu_z f_0 t + \delta_n). \quad (9.1)$$

One sees from this relation that for $t = t_0$, the vertical position is a periodic function of $\bar{\theta}$. It is also evident that electrons at any particular azimuth in the rotating system (i.e., $\bar{\theta} = \bar{\theta}_0$) undergo vertical oscillations of frequency $\nu_z f_0$ (as required by single particle dynamics).

If we ask now for the vertical position of the beam at the azimuth of a particular probe (at the laboratory angle θ), we must convert back to the laboratory coordinate system. $\bar{\theta}$ is related to the laboratory angle θ by the relation $\bar{\theta} = \theta - 2\pi f_0 t$. Substituting this into the relation for $z(\bar{\theta}, t)$, we get

$$z(\theta, t) = \sum_n A_n \cos \left[n\theta - 2\pi(n - \nu_z)f_0 t + \phi_n \right]. \quad (9.2)$$

If we select a certain harmonic n , and a certain laboratory angle θ , we see that the z position of the beam appears to be oscillating at a frequency of

$$f_{TH} = f_0 |(\nu_z - n)|. \quad (9.3)$$

The majority of the observations have been made at a stacked beam energy of about 35 Mev, a revolution frequency $f_0 \approx 24.2$ Mc, and a vertical betatron oscillation frequency $\nu_z \approx 2.81$.

The frequencies below 100 Mc which result for particular harmonic number n are given in Table II.

n	$f_{TH}(\text{Mc})$	$f_{Exp}(\text{Mc})$	TABLE II Method of Inducing Instability			Scintillator Detects KO
			Spontaneous	Pulsed RF	CW RF	
-1	92.2	91.99	no	no	yes	yes
0	68.0	67.60	no	no	yes	yes
1	43.8	43.62	no	no	yes	yes
2	19.6		no	no	no	yes
3	4.6	4.51	yes	yes	yes	yes
4	28.8	28.6	yes	yes	yes	yes
5	53.0	52.74	no	yes	yes	yes
6	77.2	76.80	no	no	yes	yes

n is the harmonic number

f_{TH} is the frequency predicted by the model. It is equal to $f_0 (\nu_z - n)$.

f_{Exp} is the measured frequency.

The column under "Scintillator detects KO" indicates if some beam was knocked out by the rf and detected by a scintillator at the f_{Exp} frequency.

The frequencies corresponding to $n = 3$ and $n = 4$ have been observed to occur naturally in the unstable beam. The energy spread of the beam contributes an angular velocity spread of the particles in the beam which should remove the coherence of the oscillations. Also the spread of the betatron oscillation frequencies due to the spread in amplitudes or to tune changes as a function of the radius should affect the coherence. In certain cases, however, these effects can cancel the oscillations of particles of different energies can be coherent.

Neglecting tune spreads due to amplitude spreads, the value of $\frac{d\gamma_z}{dE}$ which leads to coherence has been calculated for different values of n . For $n = 3$, $\frac{d\gamma_z}{dE} = -0.00053$. For $n = 4$, $\frac{d\gamma_z}{dE} = -0.00335$. For $n = 5$, $\frac{d\gamma_z}{dE} = -0.00617$. In the accelerator, near 35 Mev, $\frac{d\gamma_z}{dE}$ is about -0.003 . This could explain why the $n = 4$ mode is most often seen.

Other frequencies have been observed by initiating an unstable oscillation of the beam through the application of rf to the knockout probe. When the applied rf has the frequencies specified by Eq. (9.3), the betatron oscillations of the particles are driven in resonance.⁶ This results in a coherent oscillation of the beam of the form described by the model. After the oscillation has been driven over a threshold by the rf, it becomes unstable and continues to grow in amplitude even if the rf is turned off.

To study the modes of the instability that could be driven in this way, the stacked beam intensity was kept just below the level at which the instability occurred spontaneously. Then rf voltages having mainly a vertical field component were applied to the rf KO plates as either pulses

of about 500μ sec duration or as a CW signal. The peak-to-peak rf voltage applied to the probe was about 100 v at frequencies below 60 Mc but was down an order of magnitude by 80 Mc due to cutoff of the driver amplifier.

At the frequencies given in Table II, the beam is driven into an unstable oscillation having the same frequency as the applied rf. In the case of the pulsed rf, the main part of the vertical growth due to the instability occurs after the applied rf is turned off. The behavior of the instability after the rf is turned off is discussed in a later section of the paper.

Since the accelerating rf frequency passes through 28.7 Mc and the accelerating gap is slightly misaligned so that there is a vertical component to the accelerating field, the beam could be driven across the threshold by this means. This might also explain why the 28.7 Mc frequency is most often seen in the instability.

The model which describes the type of oscillation the beam undergoes makes no attempt at explaining the phenomenon responsible for driving this coherent oscillation.

Only one frequency is observed experimentally at any given time. In this case n , the harmonic number, specifies a closed wave with n sinusoidal oscillations per revolution. At a given instant in time, a shift of θ degrees in machine azimuth should mean a shift of $\alpha = n \theta$ in the phase of the beam oscillation.

In order to test the validity of the model, the phase shift between the signals from two probes $22-1/2^\circ$ apart in machine azimuth was measured. With the instability signal having a frequency of 28.7 Mc, a phase shift of approximately 90° was observed. This is in agreement with the phase shift expected for an n value of 4.

Growth Rate of the Instability

To study the growth rate of the instability, the stacked beam current was again set just below the threshold where the instability would go spontaneously. The instability was then initiated by pulsing an rf voltage across the rf knockout probe. Figure 38(a) shows the 28.7 Mc voltage which initiates the instability. It is pulsed on for 60μ sec. Figure 38(b) shows typical rf signals from the instability. Note that, though the instability is initiated by the rf voltage, the growth is quite small during the time the driving voltage is on. Most of the growth takes place hundreds of microseconds after the driving voltage has been turned off. Apparently the pulsed rf moves the beam over a threshold, after which other forces are strong enough to sustain and drive the oscillation. Figure 38(b) shows a smooth growth, except during its early stages. Figure 39(b), taken at an earlier date, shows more extreme variation in the growth. Here the coherent oscillation markedly decreases at times, and then grows again. For the more smooth growths, the average time for the amplitude to double is about 200μ sec. During the decay of the signal, the average time for the amplitude to halve is about 100μ sec. The average time from the turn-on of the driving rf until the oscillation reaches its peak amplitude is about 1.3 msec.

To correlate growth of the coherent beam oscillation with actual loss of beam, the signal from a scintillation detector positioned to detect lost beam was displayed simultaneously with the coherent beam signal. Figure 40(a) shows the growth and decay of the coherent oscillation initiated by an rf knockout pulse. Trace (b) shows the beam loss as detected by the scintillator. Little beam is lost during the first 600 μ sec of the oscillation. Almost all is lost in the final 600 μ sec, when the coherent oscillation is strongest. The three spikes on trace (b) do not give an accurate representation of beam lost at that time. They are the result of overdriving the scintillation detector.

The instability was initially observed with 0.8 kv to 1.5 kv on the clearing electrodes. Later it was obtainable with from 0.4 kv to 4.6 kv. It has never been obtained, driven or undriven, with the clearing electrode voltage off. Attempts to drive the instability with an rf pulse while the clearing electrode voltage is off have given rise to detectable coherent oscillations, but these die out quickly as the driving rf voltage is turned off. They never grow and develop into an instability.

An experiment was performed to study the effect of various clearing electrode voltages on the growth rate of the instability. The instability was again triggered by an rf voltage on the rf knockout probe. A series of pictures of the growth rate were then taken with 1 kv, 2 kv, 3 kv and 4 kv on the clearing electrodes. The pulse-to-pulse variation in growth rates at a given voltage setting proved to be larger than any discernible

trend due to changing the voltage. The pulse-to-pulse total growth time varied by as much as 600μ sec at 1 kv on the clearing electrodes.

An investigation was carried out to determine the effect of pulsing off the clearing electrode voltage at various times during the instability. In this experiment the stacked beam current was again set just below the threshold of spontaneous instability and the instability initiated with an rf pulse. The clearing electrode voltage was pulsed off for a few milliseconds at different times with respect to the turn-on of the rf driving pulse. If the clearing electrode voltage is turned off before the rf pulse comes on to initiate the instability, the instability will not develop. Once the instability has begun, turning off the clearing electrode voltage will not stop the instability. This effect may be related to the time necessary for the beam to neutralize after the clearing electrode voltage is turned off.

Two possible ways of raising the current threshold of the instability have been considered. One is to detect the beginning of the growth of the instability and to apply a feedback-out-of-phase rf signal to damp the oscillations. The other method is to change $\frac{dV_z}{dE}$ in the stacking region to wash out the coherence of the instability oscillation for particles of different energies and thus raise the threshold.

APPENDIX

Beam Parameters for Instability Studies

The values listed in Table III cover the parameters of the stacked beam for three cases of interest in relation to the coherent transverse instability.

One set of parameters is for a wide beam (~ 7.8 Mev) having a total current of about 8 amperes. This case was measured just before the instability was first observed.

The second set of parameters is for a narrower beam (~ 5.4 Mev) with a total current of about 6 amperes. The current of this beam is just below the amount necessary to produce the instability under the conditions given.

The third set of parameters is for the minimum beam at which the instability has been seen.

TABLE III

Date Data was Taken	12-15-62	1-4-63	4-15-63
Pressure	7×10^{-7} mm Hg	5×10^{-7} mm Hg	3×10^{-7} mm Hg
Beam Current	8 amp	6.1 amp	1.7 amp
ΔE	7.31 Mev	5.4 Mev	3.6 Mev
ΔR	1.49 in.	1.17 in.	0.75 in.
f_1	24.551 Mc	24.450 Mc	24.302 Mc
f_2	24.068 Mc	24.078 Mc	24.053 Mc
Γ	0.329	0.272	0.169
a	0.0418 volt sec	0.0437 volt sec	0.0515 volt sec

ΔE is the energy spread of the stacked beam.

ΔR is the radial spread in beam equilibrium orbits corresponding to ΔE .

The average radius of the 35 Mev orbit is 75.85 in.

f_1 is the frequency of the low energy edge of the stacked beam.

f_2 is the frequency of the high energy edge of the stacked beam.

$\Gamma = \sin \phi$, ϕ is the stable phase angle.

a is the bucket area.

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FIGURE CAPTIONS

- Fig. 1. Calculated and measured radial betatron oscillation frequency versus radius.
- Fig. 2. Calculated and measured vertical betatron oscillation frequency versus radius.
- Fig. 3. Measured betatron oscillation frequencies plotted in the ν_r, ν_z plane. Radii of selected points are indicated.
- Fig. 4. Tune-space diagram indicating computed dependence of tune on amplitude. The vertical amplitude (A_y/R) for each curve is indicated at the point of zero radial amplitude. Other points on the curves indicate the tune change for increments of 0.004 in radial amplitude.
- Fig. 5. Comparison of computed and measured change of vertical and radial tune as a function of radial betatron oscillation amplitude.
- Fig. 6. Measured maximum vertical amplitude throughout the accelerating cycle showing vertical growth at the $2\nu_z + \nu_r = 10$ sum resonance. Lines are drawn at a slope of - 4.61, the coefficient for the adiabatic damping of the betatron oscillations.
- Fig. 7. Vertical growth at the $2\nu_z - \nu_r = 1$ nonlinear difference resonance. Measured maximum vertical amplitude is plotted versus radius and energy.
- Fig. 8. Measured maximum vertical amplitude at high intensity plotted on log-log scale to show correspondence with adiabatic damping coefficient. The slope of the solid line is determined by the calculated damping coefficient.

Fig. 9. Signal from beam monitor indicating beam lifetime at 40 Mev.

Sweep speed 1 second per major division.

Fig. 10. Beam lifetime for various vertical apertures to 40 Mev. Each

trace indicates surviving beam for a given available aperture.

From left to right, the traces are for vertical apertures of 0.066 in,

0.106 in, 0.149 in, 0.191 in, 0.276 in, and 0.436 in. Sweep speed

1 second per major division.

Fig. 11. Beam halftime versus energy.

Fig. 12. Beam halftime versus vertical aperture.

Fig. 13. Beam halftime versus radial aperture.

Fig. 14. Beam halftime versus the square of the vertical aperture.

Fig. 15. Radial accelerator phase space at injection showing injector emittance.

Fig. 16. Perturbed equilibrium orbit. A radial betatron oscillation about the

unperturbed orbit position diminishes in amplitude with time.

Fig. 17. Beam capture efficiency from use of a programed field bump.

Fig. 18. Accelerated beam current striking a target probe. The smaller

pulse is current captured through slight space charge effects. The

larger pulse shows increased intensity and a large range of radial

oscillation amplitudes produced by the field bump.

Fig. 19. Electrostatic induction probe signal for injection of a 30 nanosecond

beam pulse. From top to bottom, sweep speeds are 100, 200 and

500 nanosecond per major division.

Fig. 20. Single sweep traces of electrostatic induction probe signal below transition energy with multiturn injection. In (a) injection time is indicated by the arrow. The signal on the bottom trace shows the start of betatron acceleration which continues just past the center of the photograph beyond which the beam is decelerated back to the injection radius.

Fig. 21. Beam bunching in the vicinity of transition. In (a) the arrow denotes the time of transition with times above transition occurring to the right. The faster sweep traces of (b) are for a time interval shortly after transition.

Fig. 22. Induction signal of a coasting beam at the injection radius for injection of a 30 nsec beam pulse. Sweep speeds are 1, 5, and 10 μ sec per major division. Injection times for the last two traces are at the left-hand border of the photograph.

Fig. 23. Induction signal for 30 nsec beam pulse 120 μ sec after injection showing lumpy structure of the beam which varies from pulse to pulse of injected beam. Sweep speed is 100 nsec per major division. Particle revolution period is 50 nsec.

Fig. 24. Induction signal of coasting beam at injection for six turns of injected beam. Sweep speed is 10 μ sec per major division. Top, center and bottom traces are for high, intermediate and low intensity beams respectively.

Fig. 25. Radio-frequency acceleration over transition. (a) Signal from scintillation detector as beam strikes rear of induction probe. (b) Frequency program for accelerating rf. Spike in the program indicates point where phase is shifted at transition energy.

Fig. 25. (c) Amplitude program for accelerating rf. (d) Rectified signal from induction probe.

Fig. 26. Beam density versus W at 0.500 Mev.

Fig. 27. Beam density versus W at 0.800 Mev.

Fig. 28. Beam density versus W at 1.50 Mev.

Fig. 29. Total beam area as a function of energy.

Fig. 30. Program sequence for 50 Mev accelerator. (a) Frequency program for displacement rf system. (b) Signal from induction probe indicating accelerated beam. No stack present because beam is accelerated into target. (c) Amplitude program for displacement rf system. (d) Amplitude program for accelerating rf system.

Fig. 31. Program sequence for 50 Mev accelerator showing stacked beam. (a) Frequency program for displacement rf system. (b) Signal from the induction probe with stacked beam. The signal from the accelerated beam has almost disappeared because its density is the same as that in the stacked beam. (c) Amplitude program for displacement rf system. (d) Amplitude program for accelerating rf system.

Fig. 32. Multitrace photograph of the build-up and decay of the stacked beam. (a) Build-up of the stacked beam in the case where the accelerating bucket is larger than necessary to enclose the accelerated beam. (b) Decay of the stack after no more particles are added.

Fig. 33. Plan view of the clearing electrodes as installed in the vacuum tank.

Fig. 34. Change in radial tune at the edge of the clearing electrode showing computed and measured values.

Fig. 35. Change in vertical tune at the edge of the clearing electrodes showing computed and measured values.

Fig. 36. Signal monitoring the total current in the stacked beam once every $1/60$ sec. Shows the slow build-up and then sudden destruction of the beam due to the instability. Sweep speed 0.2 sec per major division.

Fig. 37. (a) Signal from a probe sensitive to longitudinal and transverse oscillations. (b) Negligible signal from the induction probe which is sensitive only to longitudinal oscillations. Sweeps triggered simultaneously. Sweep speed 100 nsec per major division.

Fig. 38. (a) Displays the rf voltage which initiates the instability. (b) Displays the signal from the instability initiated by the rf voltage. Both traces were triggered simultaneously. Sweep speed 0.2 msec per major division.

Fig. 39. Examples of non-uniform growth of coherent beam oscillations. Traces are of separate events occurring within an interval of 5 sec. Instability triggered by rf Sweep speed 0.2 msec per major division.

Fig. 40. (a) Growth of the coherent beam oscillation. (b) Beam loss monitored by a scintillation detector. Sweep speed 0.2 msec per major division.