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SUMMARY

During the past year Geoscience investigated high-temperature liquid-metal heat and momentum transfer processes for the Atomic Energy Commission. The program was composed of analytical and experimental liquid-metal boiling and condensing studies with emphasis on high gravity or helical flow in the high vapor quality and superheated regions. From the research results, design criteria which are applicable to the SNAP Project are being developed so that nuclear reactor and heat exchanger performance can be predicted.

The research results for the past year (Phase III) are outlined below:

1. A boiling heat transfer analysis has been made for alternate slugs of liquid and vapor that are flowing through a boiler tube. Both viscous and turbulent flow phenomena were considered. The solutions are applicable to linear, forced flow boiling.
2. A two-phase flow frictional pressure drop analysis has been made for alternate slugs of liquid and vapor that flow through a boiler tube; local and overall pressure drops have been expressed in dimensionless form. The results are applicable to linear, forced flow boiling.
3. An analysis has been made of helical, forced flow pressure drop for single phase flow in tubes. The results can be used to predict frictional pressure drop in the superheated region of a boiler tube under helical flow conditions.
4. An equation has been derived for convective heat transfer in helical, single phase flow; the analysis accounts for improved heat transfer as a result of both forced and free convection phenomena. The results can be used to predict heat transfer in the superheated region of a boiler tube under helical flow conditions.
5. The stationary droplet vaporization model has been extended to the forced flow case. The resulting equations can be used to estimate boiling heat transfer in the initial region of a boiler tube with helical flow where droplets or rivulets exist at the wall.

6. Experimental forced flow boiling mercury heat transfer and pressure drop measurements were made under linear flow conditions. Several different boiling test section geometries and heating methods were used. The results were in general agreement with each other as well as analytical predictions.
7. Experimental forced flow boiling mercury heat transfer and pressure drop measurements have been made under helical flow conditions. Several different boiling test section geometries and heating methods were used. The pitch to diameter ratio was about 4. The asymptotic bounds of the experimental heat transfer data were in good agreement with predictions.
8. Forced flow boiling mercury heat transfer and pressure drop measurements in linear and helical flow have been completed. Significant improvements in heat transfer and exit vapor quality were demonstrated with helical flow boiling. Helical flow pressure drop measurements were observed to fall only slightly higher than corresponding values in linear flow, thus, a net increase in boiler performance will result with a helical flow mercury boiler.
9. Preliminary forced flow potassium boiling heat transfer measurements were obtained.
10. Droplet vaporization lifetimes were measured and compiled for mercury, Freon, and benzene. In the film boiling region, the experimental lifetimes were in general agreement with predicted values from the previously developed theory. Preliminary vaporization lifetime measurements were also obtained for potassium droplets.
11. Liquid-metal condensation analyses were made for idealized forced flow condensers. The results were used in the experimental potassium boiling and condensing system.
12. Forced flow Freon boiling measurements have been made; six different fluid flow regimes were identified and quantitative heat transfer conductances were obtained as a function of vapor quality and flow rate. The results can be used to interpret more completely some of the complex phenomena in forced flow boiling.

13. Comparisons have been made of some of the experimental forced flow boiling liquid-metal heat transfer data found in the literature; similarities and differences were pointed out.
14. The thermodynamic properties of potassium vapor in the superheated region were extended analytically to higher temperatures.
15. A kinetics study was made of the effect of a temperature perturbation at the coolant channel wall on the reactor criticality; the phase lag and amplitude reduction in the fuel region were considered.

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I. INTRODUCTION

Forced flow heat and momentum transfer processes in tubes under boiling conditions are more complex than single phase systems. There are numerous ways in which the two phases of a forced flow boiling system can distribute themselves. It is for this reason that there are more heat and momentum transfer regimes than encountered in single phase flow.

In order to be able to establish quantitative and general heat transfer and fluid flow design correlations which can be used to predict the thermodynamic behavior of compact high-temperature Rankine Cycle space power systems, a fundamental investigation consisting of both analytical and experimental methods of study is desirable. The analytical work helps to identify the important flow regimes and corresponding transport processes. This procedure is also necessary to design more meaningful boiling experiments. The experimental studies should yield information that is sufficiently general to cover reasonable ranges of the controlling variables or functions. To this end, both analytical and experimental studies have been carried out at Geoscience.

The main emphasis of the liquid metal research program has been that of phase separation by high acceleration or helical flow. It is being demonstrated analytically and experimentally that such a flow pattern can yield optimum heat and momentum transfer characteristics in boiling systems.

Presented in this document are Geoscience's liquid metal research results in the areas of 1) forced flow liquid metal boiling, 2) droplet vaporization, 3) forced flow liquid metal condensation, 4) forced flow Freon boiling, 5) interpretation of liquid metal data in the literature, 6) thermodynamic properties of alkali metal vapors, and 7) heat transfer influences on reactor kinetics. Some of this information has previously been outlined in quarterly reports. (1,2,3)

*The authors of this report are: H.F. Poppendiek, N.D. Greene, C.M. Sabin, F.R. MacDonald, L.V. Feigenbutz, R.K. Livett, P.E. Schwartz, A.S. Thompson, D.J. Connelly, and W.A. Morton; the total technical effort for the year was about five-man years.

II. FORCED-FLOW LIQUID METAL BOILING HEAT AND MOMENTUM TRANSFER ANALYSES

A number of forced-flow liquid metal boiling heat and momentum transfer studies have been conducted. Linear and helical flow boiling systems are analyzed. The results of experimental boiling mercury heat transfer data for different test section geometries and methods of heating are given for linear and helical flow. Some boiling potassium heat transfer results for linear flow are also presented. The predicted and experimental heat and momentum transfer results are compared and interpreted.

A. Analyses

1. Two-Phase Linear Flow Model (Slug Flow)

Some of the two-phase flow models described in previous project reports have been extended and used to predict heat transfer and pressure drop performance for mercury forced-flow boiling. Consider the slug flow model which is useful to define linear flow behavior. It is postulated that alternate slugs of liquid and vapor (with no liquid in the volume or at the wall) flow through the boiler tube. It is presumed that heat transfer from the boiler tube to the liquid and vapor slugs is defined in terms of the wall-saturation temperature difference and liquid and vapor convective conductances. It is postulated that the mixed mean or bulk temperature of a vapor slug is equal to the saturation temperature. The mean heat transfer conductance is then defined as,

$$h(x) = h_{\ell}(x) y_{\ell}(x) + h_v(x) y_v(x) \quad \text{II-(1)}$$

where h_{ℓ} and h_v are the liquid and vapor slug conductances, respectively, y_{ℓ} and y_v are the liquid and vapor volume fractions respectively, and x is the vapor quality. The relation between vapor quality and the volume fraction for this flow type can be shown to be

$$x = \frac{1}{1 + \frac{w_{\ell}}{w_v} \frac{y_{\ell}}{y_v}} = \frac{1}{1 + \frac{w_{\ell}}{w_v} \left(\frac{y_{\ell}}{1 - y_{\ell}} \right)} \quad \text{II-(2)}$$

where w_{ℓ} and w_v are the liquid and vapor weight densities, respectively. A valid simplification in Equation II-(1) (for practical, forced-flow boiling saturated liquid metals in a

tube) is that the liquid conductance is given by the classical viscous flow equation for uniform wall heat addition,

$$h_{\ell} = \frac{48}{11} \frac{k_{\ell}}{D} \quad \text{II-(3)}$$

where k_{ℓ} is the thermal conductivity of the liquid metal and D is the tube diameter. The convective conductance for a vapor slug is

$$h_v = \frac{48}{11} \frac{k_v}{D} \quad (\text{for viscous flow}) \quad \text{II-(4)}$$

and

$$\begin{aligned} h_v &= h_v(x=1) \left(\frac{W_v(x)}{W_v(x=1)} \right)^{0.8} \\ &= h_v(x=1) (x)^{0.8} \quad (\text{for turbulent flow}) \end{aligned} \quad \text{II-(5)}$$

where

$$h_v(x=1) = 0.023 \frac{k_v}{D} \text{Re}_v^{0.8}(x=1) \text{Pr}_v^{0.4} \quad (\text{turbulent flow}) \quad \text{II-(6)}$$

The quantity W_v is the vapor flow rate, k_v is the vapor thermal conductivity, Re_v is the vapor Reynolds modulus and Pr_v is the vapor Prandtl modulus. The substitution of Equations II-(2) to II-(6) into Equation II-(1) yields,

$$\frac{h(x)}{h_v(x=1)} = H = \frac{190}{(k_v/k_{\ell}) \text{Re}_v^{0.8}(x=1) \text{Pr}_v^{0.4}} \left(\frac{1-x}{1 + \left(\frac{W_{\ell}}{W_v} - 1 \right) x} \right) + x^{0.8} \left(1 - \frac{1-x}{1 + \left(\frac{W_{\ell}}{W_v} - 1 \right) x} \right) \quad \text{II-(7)}$$

for turbulent gas flow

and

$$H = \left(\frac{k_{\ell}}{k_v} - 1 \right) \left(\frac{1-x}{1 + \left(\frac{W_{\ell}}{W_v} - 1 \right) x} \right) + 1 \quad \text{II-(8)}$$

for viscous gas flow

Equations II-(7) and II-(8) have been evaluated and are shown in Figure II-1 for specified properties that are representative of mercury.

Figure II-1. Dimensionless boiling heat transfer conductances versus vapor quality for laminar and turbulent slug flow models.

The slug flow model has also been used to derive a frictional pressure drop relationship for linear flow by adding pressure drops in the liquid and vapor slugs. The continuity and single-phase gas flow pressure drop equations were combined with the two-phase frictional pressure drop expression yielding,

$$\frac{\Delta p_{\text{TPF}}}{\Delta p_{v_o}} = \frac{1 + \frac{\xi_\ell}{\xi_v} \frac{w_\ell}{w_v} \frac{y_\ell}{y_v}}{\frac{\xi_{v_o}}{\xi_v} y_v} \quad \text{II-(9)}$$

where Δp_{TPF} , frictional two-phase pressure drop

Δp_{v_o} , frictional pressure drop with vapor flowing alone

ξ_ℓ , ξ_v , liquid and vapor friction factors, respectively

ξ_{v_o} , friction factor for vapor flowing alone.

Equation II-(9) can be simplified by the approximation that the three friction factors given above are all equal and constant. This postulate would hold most nearly for turbulent flow. The resulting equation is

$$\frac{\Delta p_{\text{TPF}}}{\Delta p_{v_o}} = \mathbb{P}(x) = \frac{1 + \frac{w_\ell}{w_v} \left(\frac{y_\ell(x)}{1 - y_\ell(x)} \right)}{1 - y_\ell(x)} = \frac{1 + \left(\frac{w_\ell}{w_v} - 1 \right) x}{\frac{w_\ell}{w_v} x^2} \quad \text{II-(10)}$$

where the relation between y_ℓ and the quality x is given by Equation II-(2). For the case where physical fluid properties are invariant and the vapor quality varies linearly with axial distance (uniform wall heat addition), the total frictional pressure drop function along a boiler tube is obtained by the integration,

$$\frac{\Delta p_{\text{TPF}}}{\Delta p_{v_o \text{ exit}}} = \frac{\int_0^{L_o} \frac{\partial p_{\text{TPF}}}{\partial L} dL}{\Delta p_{v_o \text{ exit}}} = \frac{\int_0^{L_o} \frac{\frac{\partial p_{\text{TPF}}}{\partial L}}{\frac{\partial p_{v_o}}{\partial L}} \frac{\partial p_{v_o}}{\partial L} dL}{\Delta p_{v_o \text{ exit}}}$$

(continued on next page)

$$\begin{aligned}
 &= \int_0^1 \frac{\frac{\partial p_v}{\Delta p_{v_o \text{ exit}}} \frac{\partial L}{L_o} d\left(\frac{x}{x_{\text{exit}}}\right)}{\frac{\partial p_v}{\Delta p_{v_o \text{ exit}}} \frac{\partial L}{L_o}} = \frac{1}{x_{\text{exit}}^3} \int_0^{x_{\text{exit}}} P(x) x^2 dx \\
 &= \frac{w_v}{w_\ell} \left[\frac{1}{x_{\text{exit}}^2} + \frac{\left(\frac{w_\ell}{w_v} - 1\right)}{2x_{\text{exit}}} \right]
 \end{aligned}
 \tag{II-11}$$

where, $\Delta p_{v_o \text{ exit}}$, frictional pressure drop through tube on basis of exit vapor flow alone

L , axial tube position

L_o , tube length

x_{exit} , vapor quality at tube exit

The dimensionless pressure drop relation given by Equation II-(11) is shown in Figure II-2.

2. Single-Phase Helical Flow Model

A number of fluid dynamics analyses applicable to helical or rotational flow can be found in the literature. An interesting study has been reported by Smithberg and Landis⁽⁴⁾ for single-phase flow in a tube with a twisted divider. An approximate pressure drop expression is derived which accounts for axial and tangential flow and vortex mixing. The predicted results are in good agreement with several sets of experimental data. A corresponding heat transfer analysis was also made; however, the effect of free convection was not included.

The 100% vapor quality region of a boiler tube can be defined in terms of single-phase gas flow. A simplified expression for the adiabatic friction factor in the single-phase turbulent flow regime can be derived by considering only gross flow features. A large fraction of a fluid flowing in a tube exists in an annular region near the wall. If the fluid flows helically through the tube, the effective flow length has been increased by the ratio of the helical path at the tube wall per unit of axial distance, $(1 + (\pi D/p)^2)^{1/2}$ where p is the helix pitch. Similarly, the resultant fluid velocity in the annular region near

Figure II-2. Dimensionless frictional boiler tube pressure drop versus quality for the slug flow model.

the wall has increased approximately by the same ratio. Upon substituting these quantities into the pressure drop equation, a group of terms results which is defined as the friction factor for helical flow; this function is based on the flow length, equivalent diameter, and mean fluid velocity for an axial flow system and is given by the relation,

$$\frac{\zeta_h}{\zeta_\ell} = \left[1 + \left(\frac{\pi}{p/D} \right)^2 \right]^{1.5} \frac{D_\ell}{D_h} \quad \text{II-(12)}$$

where, ζ_h, ζ_ℓ , Weisbach friction factors for helical and linear flow systems, respectively,

D_h, D_ℓ , equivalent diameter for helical and linear flow systems, respectively.

Experimental adiabatic helical to linear flow friction factor ratios as a function of pitch to diameter ratio were found to agree with predicted values, using Equation II-(12) (Reference (5)).

An expression for the single-phase turbulent Nusselt modulus for helical flow (excluding free convection) can be derived on the same bases given for the friction factor relation, Equation II-(12). The ratio of helical to linear flow Nusselt moduli for ordinary liquids and gases is

$$\frac{Nu_{\text{forced}}}{Nu_\ell} = \left[1 + \left(\frac{\pi}{p/D} \right)^2 \right]^{0.4} \left(\frac{D_h}{D_\ell} \right)^{0.8} \quad \text{II-(13)}$$

where, Nu_{forced} , forced convection Nusselt modulus for helical flow system
 Nu_ℓ , forced convection Nusselt modulus for linear flow system.

Free convection heat transfer expressions for idealized natural convection cells in a horizontal fluid layer under unstable temperature gradient conditions have been derived. Typical power law Grashof-Prandtl modulus equations give results that are similar to the experimental expressions for turbulent free convection from horizontal cylinders. The relative importance of free convection to forced convection in turbulent helical flow with wall heat addition can be estimated by the expression,

$$\frac{Nu_{\text{free}}}{Nu_\ell} = 7.1 \frac{(\beta \Delta t)^{1/3}}{Re_\ell^{0.13} Pr^{0.07}} \left(\frac{\pi}{p/D} \right)^{2/3} \left(\frac{D_t}{D_\ell} \right)^{2/3} \quad \text{II-(14)}$$

where, Nu_{free} , free convection Nusselt modulus for helical flow system
 β , volumetric expansion coefficient of the coolant
 Δt , wall-fluid temperature difference
 Re_ℓ , Reynolds modulus for linear flow system
 Pr , Prandtl modulus
 D_t , equivalent diameter for tangential flow in a tube

Equation II-(14) is based on a one-third power Grashof-Prandtl relation and is valid only for gases and ordinary liquids. The acceleration field in the Grashof modulus is given by the ratio of the square of the tangential fluid velocity in the boundary layer to the tube radius. Also, the ratio of the tangential to linear velocity has been expressed in terms of the pitch to diameter ratio. Typical values for Nu_{free}/Nu_ℓ as a function of $\beta \Delta t$, p/D , Re_ℓ , and Pr are given in Table 1. The comparison made in Equation II-(14) implies that free convection can be superposed in a forced convection system. It is clear that the complicated simultaneous processes are only approximately additive. The sum of Equation II-(13) and II-(14), Nu_h/Nu_ℓ , defines the total increase in heat transfer for a helical flow pattern; experimental values of Nu_h/Nu_ℓ as a function of pitch to diameter ratio were found to be in reasonable agreement with predicted values⁽⁵⁾ based on wall heat transfer representative of the experimental measurements.

3. Droplet or Rivulet Flow Model

Heat transfer from relatively large liquid droplets contiguous to a heat transfer surface as well as heat transfer to fog flow have been discussed in a previous report⁽⁶⁾; droplet and fog flow models were developed and used to interpret some of the bounds on boiling heat transfer. The droplet vaporization model previously described has been used to examine helical forced flow boiling in the low vapor quality region in the hope of identifying some of the parameters that control boiling in this region.

The heat transfer from a boiler tube wall to vaporizing liquid droplets contiguous to the wall is given by the relation*

$$q = k_v \frac{(t_w - t_s)}{\delta} A_d \quad \text{II-(15)}$$

*It is postulated that all the liquid phase is in the form of droplets at the tube wall and that liquid and vapor velocities are equal.

Table 1. TYPICAL VALUES FOR Nu_{free}/Nu_{ℓ} AS A FUNCTION OF $\beta\Delta t$, Re_{ℓ} , p/D , AND Pr

Nu ratio	Re_{ℓ}	$\beta\Delta t$	Pr	p/D	Nu ratio	Re_{ℓ}	$\beta\Delta t$	Pr	p/D	Nu ratio	Re_{ℓ}	$\beta\Delta t$	Pr	p/D
1.694	10,000	.05	1	1	1.513	10,000	.05	5	1	1.442	10,000	.05	10	1
1.374	50,000	↓	↓	↓	1.227	50,000	↓	↓	↓	1.169	50,000	↓	↓	↓
1.256	100,000	↓	↓	↓	1.122	100,000	↓	↓	↓	1.069	100,000	↓	↓	↓
1.019	500,000	↓	↓	↓	.9101	500,000	↓	↓	↓	.8670	500,000	↓	↓	↓
2.134	10,000	.1	↓	↓	1.907	10,000	.1	↓	↓	1.816	10,000	.1	↓	↓
1.731	50,000	↓	↓	↓	1.547	50,000	↓	↓	↓	1.473	50,000	↓	↓	↓
1.582	100,000	↓	↓	↓	1.413	100,000	↓	↓	↓	1.346	100,000	↓	↓	↓
1.283	500,000	↓	↓	↓	1.147	500,000	↓	↓	↓	1.092	500,000	↓	↓	↓
3.649	10,000	.5	↓	↓	3.260	10,000	.5	↓	↓	3.106	10,000	.5	↓	↓
2.960	50,000	↓	↓	↓	2.645	50,000	↓	↓	↓	2.520	50,000	↓	↓	↓
2.705	100,000	↓	↓	↓	2.417	100,000	↓	↓	↓	2.302	100,000	↓	↓	↓
2.195	500,000	↓	↓	↓	1.961	500,000	↓	↓	↓	1.868	500,000	↓	↓	↓
4.598	10,000	1.0	↓	↓	4.108	10,000	1.0	↓	↓	3.913	10,000	1.0	↓	↓
3.730	50,000	↓	↓	↓	3.333	50,000	↓	↓	↓	3.173	50,000	↓	↓	↓
3.408	100,000	↓	↓	↓	3.045	100,000	↓	↓	↓	2.901	100,000	↓	↓	↓
2.765	500,000	↓	↓	↓	2.470	500,000	↓	↓	↓	2.352	500,000	↓	↓	↓

Table 1. (continued)

Nu ratio	Re_ℓ	$\beta\Delta t$	Pr	p/D	Nu ratio	Re_ℓ	$\beta\Delta t$	Pr	p/D	Nu ratio	Re_ℓ	$\beta\Delta t$	Pr	p/D
.8143	10,000	.05	1	3	.7275	10,000	.05	5	3	.6931	10,000	.05	10	3
.6606	50,000	↓	↓	↓	.5902	50,000	↓	↓	↓	.5622	50,000	↓	↓	↓
.6036	100,000	↓	↓	↓	.5393	100,000	↓	↓	↓	.5138	100,000	↓	↓	↓
.4897	500,000	↓	↓	↓	.4375	500,000	↓	↓	↓	.4168	500,000	↓	↓	↓
1.026	10,000	.1	↓	↓	.6522	10,000	.1	↓	↓	.8732	10,000	.1	↓	↓
.8323	50,000	↓	↓	↓	.5291	50,000	↓	↓	↓	.7084	50,000	↓	↓	↓
.7605	100,000	↓	↓	↓	.4835	100,000	↓	↓	↓	.6473	100,000	↓	↓	↓
.6170	500,000	↓	↓	↓	.3922	500,000	↓	↓	↓	.5251	500,000	↓	↓	↓
1.754	10,000	.5	↓	↓	1.567	10,000	.5	↓	↓	1.493	10,000	.5	↓	↓
1.423	50,000	↓	↓	↓	1.271	50,000	↓	↓	↓	1.211	50,000	↓	↓	↓
1.301	100,000	↓	↓	↓	1.162	100,000	↓	↓	↓	1.107	100,000	↓	↓	↓
1.055	500,000	↓	↓	↓	.9426	500,000	↓	↓	↓	.8979	500,000	↓	↓	↓
2.210	10,000	1.0	↓	↓	1.975	10,000	1.0	↓	↓	1.838	10,000	1.0	↓	↓
1.793	50,000	↓	↓	↓	1.602	50,000	↓	↓	↓	1.526	50,000	↓	↓	↓
1.639	100,000	↓	↓	↓	1.464	100,000	↓	↓	↓	1.395	100,000	↓	↓	↓
1.329	500,000	↓	↓	↓	1.188	500,000	↓	↓	↓	1.131	500,000	↓	↓	↓

Table 1. (continued)

Nu ratio	Re _ℓ	βΔt	Pr	p/D	Nu ratio	Re _ℓ	βΔt	Pr	p/D	Nu ratio	Re _ℓ	βΔt	Pr	p/D
.5793	10,000	.05	1	5	.5176	10,000	.05	5	5	.4930	10,000	.05	10	5
.4699	50,000	↓	↓	↓	.4198	50,000	↓	↓	↓	.4000	50,000	↓	↓	↓
.4294	100,000	↓	↓	↓	.3838	100,000	↓	↓	↓	.3655	100,000	↓	↓	↓
.3484	500,000	↓	↓	↓	.3113	500,000	↓	↓	↓	.2965	500,000	↓	↓	↓
.7298	10,000	.1	↓	↓	.6521	10,000	.1	↓	↓	.6212	10,000	.1	↓	↓
.5920	50,000	↓	↓	↓	.5290	50,000	↓	↓	↓	.5039	50,000	↓	↓	↓
.5410	100,000	↓	↓	↓	.4834	100,000	↓	↓	↓	.4605	100,000	↓	↓	↓
.4389	500,000	↓	↓	↓	.3921	500,000	↓	↓	↓	.3736	500,000	↓	↓	↓
1.248	10,000	.5	↓	↓	1.115	10,000	.5	↓	↓	1.062	10,000	.5	↓	↓
1.012	50,000	↓	↓	↓	.9045	50,000	↓	↓	↓	.8617	50,000	↓	↓	↓
.9252	100,000	↓	↓	↓	.8266	100,000	↓	↓	↓	.7874	100,000	↓	↓	↓
.7505	500,000	↓	↓	↓	.6705	500,000	↓	↓	↓	.6358	500,000	↓	↓	↓
1.572	10,000	1.0	↓	↓	1.405	10,000	1.0	↓	↓	1.338	10,000	1.0	↓	↓
1.276	50,000	↓	↓	↓	1.140	50,000	↓	↓	↓	1.086	50,000	↓	↓	↓
1.166	100,000	↓	↓	↓	1.041	100,000	↓	↓	↓	.9922	100,000	↓	↓	↓
.9456	500,000	↓	↓	↓	.8448	500,000	↓	↓	↓	.8048	500,000	↓	↓	↓

Table 1. (continued)

Nu ratio	Re_ℓ	$\beta \Delta t$	Pr	p/D	Nu ratio	Re_ℓ	$\beta \Delta t$	Pr	p/D	Nu ratio	Re_ℓ	$\beta \Delta t$	Pr	p/D
.4629	10,000	.05	1	7	.4136	10,000	.05	5	7	.3940	10,000	.05	10	7
.3764	50,000	↓	↓	↓	.3355	50,000	↓	↓	↓	.3196	50,000	↓	↓	↓
.3432	100,000	↓	↓	↓	.3066	100,000	↓	↓	↓	.2921	100,000	↓	↓	↓
.2784	500,000	↓	↓	↓	.2487	500,000	↓	↓	↓	.2369	500,000	↓	↓	↓
.5832	10,000	.1	↓	↓	.5210	10,000	.1	↓	↓	.4960	10,000	.1	↓	↓
.4731	50,000	↓	↓	↓	.4227	50,000	↓	↓	↓	.4027	50,000	↓	↓	↓
.4323	100,000	↓	↓	↓	.3863	100,000	↓	↓	↓	.3680	100,000	↓	↓	↓
.3507	500,000	↓	↓	↓	.3133	500,000	↓	↓	↓	.2985	500,000	↓	↓	↓
.9972	10,000	.5	↓	↓	.8910	10,000	.5	↓	↓	.8488	10,000	.5	↓	↓
.8090	50,000	↓	↓	↓	.7228	50,000	↓	↓	↓	.6885	50,000	↓	↓	↓
.7393	100,000	↓	↓	↓	.6605	100,000	↓	↓	↓	.6292	100,000	↓	↓	↓
.5997	500,000	↓	↓	↓	.5358	500,000	↓	↓	↓	.5104	500,000	↓	↓	↓
1.256	10,000	1.0	↓	↓	1.123	10,000	1.0	↓	↓	1.069	10,000	1.0	↓	↓
1.019	50,000	↓	↓	↓	.9106	50,000	↓	↓	↓	.8675	50,000	↓	↓	↓
.9314	100,000	↓	↓	↓	.8322	100,000	↓	↓	↓	.7928	100,000	↓	↓	↓
.7556	500,000	↓	↓	↓	.6751	500,000	↓	↓	↓	.6431	500,000	↓	↓	↓

Table 1. (continued)

Nu ratio	Re _ℓ	βΔt	Pr	p/D	Nu ratio	Re _ℓ	βΔt	Pr	p/D	Nu ratio	Re _ℓ	βΔt	Pr	p/D
.3649	10,000	.05	1	10	.3260	10,000	.05	5	10	.3106	10,000	.05	10	10
.2960	50,000	↓	↓	↓	.2645	50,000	↓	↓	↓	.2520	50,000	↓	↓	↓
.2705	100,000	↓	↓	↓	.2417	100,000	↓	↓	↓	.2303	100,000	↓	↓	↓
.2195	500,000	↓	↓	↓	.1961	500,000	↓	↓	↓	.1868	500,000	↓	↓	↓
.4598	10,000	.1	↓	↓	.4108	10,000	.1	↓	↓	.3913	10,000	.1	↓	↓
.3730	50,000	↓	↓	↓	.3332	50,000	↓	↓	↓	.3174	50,000	↓	↓	↓
.3408	100,000	↓	↓	↓	.3045	100,000	↓	↓	↓	.2901	100,000	↓	↓	↓
.2765	500,000	↓	↓	↓	.2470	500,000	↓	↓	↓	.2300	500,000	↓	↓	↓
.7862	10,000	.5	↓	↓	.7024	10,000	.5	↓	↓	.6692	10,000	.5	↓	↓
.6378	50,000	↓	↓	↓	.5698	50,000	↓	↓	↓	.5428	50,000	↓	↓	↓
.5825	100,000	↓	↓	↓	.5207	100,000	↓	↓	↓	.4961	100,000	↓	↓	↓
.4728	500,000	↓	↓	↓	.4224	500,000	↓	↓	↓	.4024	500,000	↓	↓	↓
.9906	10,000	1.0	↓	↓	.8850	10,000	1.0	↓	↓	.8431	10,000	1.0	↓	↓
.8035	50,000	↓	↓	↓	.7179	50,000	↓	↓	↓	.6839	50,000	↓	↓	↓
.7343	100,000	↓	↓	↓	.6561	100,000	↓	↓	↓	.6246	100,000	↓	↓	↓
.5957	500,000	↓	↓	↓	.5322	500,000	↓	↓	↓	.5070	500,000	↓	↓	↓

where, q , heat transfer rate

k_v , thermal conductivity of the vapor

$\bar{\delta}$, mean thickness of the vapor film under the droplet

t_w , wall temperature

t_s , saturation temperature

A_d , heat transfer area of the droplets

The heat transfer rate can also be defined in terms of a conductance by the common expression

$$q = hA_B(t_w - t_s) \quad \text{II-(16)}$$

where, h , heat transfer conductance

A_B , heat transfer area of the boiler tube

From Equations II-(15) and II-(16), the Nusselt modulus expression can be obtained,

$$\frac{hD}{k_v} = Nu = \frac{A_d D}{A_B \bar{\delta}} \quad \text{II-(17)}$$

where D is the diameter of the tube. The ratio A_d/A_B is defined by the relation,

$$\frac{A_d}{A_B} = \frac{n \pi r_d^2}{\pi D \cdot D} \quad \text{II-(18)}$$

where r_d , is the radius of the disk shaped droplets

n , is the number of droplets contiguous to the tube wall per diameter length of tube.

From continuity,

$$n \pi r_d^2 2r_d = \bar{A}_\ell D, \quad \text{II-(19)}$$

where the disk shaped droplets are $2r_d$ units thick and $\bar{A}_\ell$ is the mean liquid cross sectional area and $\bar{A}_\ell D$ the liquid volume per diameter length of tube. The liquid fraction y_ℓ can be expressed as

$$y_\ell = \frac{\bar{A}_\ell}{\frac{\pi D^2}{4}} \quad \text{II-(20)}$$

From Equations II-(19) and II-(20) it can be shown that

$$\frac{r_d}{D} = \sqrt[3]{\frac{y_\ell}{8n}} \quad \text{II-(21)}$$

From the previous droplet vaporization analysis, it was found that

$$\frac{\bar{\delta}}{D} = \left(\frac{\zeta}{80}\right)^{1/5} \left(\frac{r_d}{D}\right) \left(\frac{k_v \Delta t}{\sqrt{2} \left(\frac{r_d}{D}\right)^{1.5} L \rho_v \rho_\ell D^{1.5} \left(\frac{U_{tv}^2}{D/2}\right)^{1.5}} \right)^{2/5} \quad \text{II-(22)}$$

where, ζ , Weisbach friction factor

Δt , wall-saturation temperature difference

L , latent heat of vaporization

ρ_v, ρ_ℓ , vapor and liquid mass densities, respectively

U_{tv} , tangential component of the vapor velocity

Upon substituting Equations II-(18), II-(19), and II-(20) into Equation II-(17) and using a ζ of 0.1 there results,

$$Nu = \frac{6.6 n \left(\frac{1}{8n} y_\ell\right)^{8/15}}{B^{2/5}} \quad \text{II-(23)}$$

where

$$B = \frac{k_v \Delta t}{L \rho_v^{1/2} \rho_\ell^{1/2} U_{tv}^3} \quad \text{II-(24)}$$

The tangential vapor velocity can be expressed as follows:

$$\frac{U_{tv}}{\bar{U}_v} = \frac{\pi D}{p} \quad \text{II-(25)}$$

$$\bar{U}_v = x \bar{U}_{v(x=1)} = x \frac{W_t}{A_c w_v} \quad \text{II-(26)}$$

The combination of Equations II-(25) and II-(26) yields

$$U_{tv} = \frac{\pi D}{p} \frac{x W_t}{A_c w_v} \quad \text{II-(27)}$$

where p , is the helix pitch

$\bar{U}_v$, is the mean axial vapor velocity

W_t , is the total fluid flow rate (vapor and liquid)

A_c , flow cross sectional area of the boiler tube

The relationship for the liquid volume fraction in terms of the vapor quality can be obtained from Equation II-(2); the function is

$$y_\ell = \frac{1 - x}{1 + \left(\frac{w_\ell}{w_v} - 1 \right) x} \quad \text{II-(28)}$$

Thus, the resulting Nusselt modulus relation, Equation II-(23) is a function of n , x , w_ℓ , w_v , k_v , Δt , L , p/D , A_c , and W_t . A typical graph of Nu vs. x and B is shown in Figure II-3.

B. Mercury Experiments (Resistance Heating)

The low pressure Haynes 25 alloy system was used to obtain local boiling heat transfer conductances and over-all pressure drops. Linear and helical experiments were conducted. The maximum vapor quality was limited by the maximum allowable wall temperature for the Haynes alloy. A maximum wall temperature of 2100°F was chosen as a safe maximum for short term experiments.

Figure II-4 indicates a schematic diagram of the system components. Mercury was circulated by gravity or a small pump through an instrumented test section and a water cooled condenser to a Lucite reservoir. For gravity flow operation, the pump was used intermittently to transport the mercury from the lower to the upper reservoir. The boiler section (Figure II-5) consisted of a Haynes 25 alloy welded and drawn tube, 111 inches long, 0.25 inch OD with a 0.03 inch wall.

The system heating was accomplished by passing a current directly through the boiler tube. The present power supply is capable of supplying 1000 amps at 36 volts.

The boiler section was instrumented with 22 spot-welded chromel-alumel thermocouples spaced at equal increments along the tube. These temperatures were recorded on a 24 point Leeds and Northrup Speedomax potentiometer. The boiler inlet and outlet pressures were indicated on Bourdon gages and the pressure taps are connected to a standard Ashcraft gage protector. In the latter device, a rubber diaphragm separated the glycerin filled Bourdon gage from the mercury. This arrangement reduced possible

Figure II-3. Nu versus B and x

Figure II-4. Low pressure Haynes 25 alloy system.

Figure II-5. Haynes 25 alloy boiler.

errors in pressure measurements because of variable liquid mercury levels in the gages. The pressure gages and lines were mounted in such a manner that during the thermal expansion of the boiler tube gage positions relative to the boiler do not change.

The test section was insulated by means of a 14 inch diameter duct filled with a combination of Cerafelt and Refrasil insulation. Tests were performed to establish the heat loss of the boiler insulation as a function of the boiler tube temperature. Figure II-6 indicates the results of these tests. These data indicate that the heat loss through the insulation is 10% of the total heat input to the boiler section.

The previous mercury experiments (as well as the Freon experiments described in a subsequent section) indicated that in forced flow boiling, considerable superheat exists in the mercury vapor near the boiler wall. An estimate of this superheat temperature is obtained from a wall thermocouple located directly after the exit of the boiler section. Also, a mixing chamber was provided for accurate determination of the heat into the fluid.

The mercury flow rate was obtained by measuring the liquid level (or volume) change in the Lucite reservoir over a measured time interval.

The completed system is shown in Figure II-7.

Eleven complete linear flow boiling mercury heat transfer experiments were performed with the Haynes 25 alloy system;⁽²⁾ an additional number of duplicate or partially complete experiments were also conducted. Local conductance and total pressure drop measurements were made. Each experiment yielded approximately twenty local or point heat transfer conductances. In addition to the linear flow experiments, ten helical flow boiling mercury experiments were also conducted using a twisted divider having a pitch to diameter ratio of 4.23; additional duplicate or partially complete experiments were also obtained. Again, corresponding conductance and pressure drop measurements were made. Heat balances for the various experiments were found to be about 94% accurate. To date the following range of parameters have been investigated:

Linear Flow: Quality - 0 to 43%

Flow rate - 28 to 234 lb/hr

Heat flux - 4000 to 7300 Btu/hr ft²

Saturation pressure - 21 to 33 psia

Figure II-6. Heat loss calibration for the Haynes 25 alloy boiler.

Figure II-7. Photograph of the Haynes 25 alloy system.

Helical Flow: Quality - 0 to superheat

Flow rate - 39 to 220 lb/hr

Heat flux - 7000 to 36,000 Btu/hr ft²

Saturation pressure - 26 to 79 psia

The heat transfer results for these experiments are presented in Figures II-8, II-9, and II-10 where the boiling conductances are shown as functions of vapor quality and mercury flow rate. Note that the helical flow conductances are superior to those for linear flow over the entire quality range. In general, the helical flow data lie about two orders of magnitude above the linear flow values in the low quality region for the conditions shown. Both linear and helical flow experiments suggest that vapor films existed at the Haynes 25 alloy wall.*

Linear and helical flow frictional pressure drop data for the Haynes 25 alloy boiler tube are shown in Table 2 as functions of total flow rate, exit vapor quality, pressure at the point of boiling initiation, tube length in boiling state, total boiling pressure drop and frictional boiling pressure drop. In general, the helical flow frictional pressure drops fall above the linear flow values but not by as large a factor as the corresponding conductance values.

Figure II-11 compares some experimental helical flow heat transfer data for a pitch to diameter ratio of 4.23 with corresponding experimental linear flow measurements, as well as with predicted functions. Note that the experimental and predicted data for linear flow are in general agreement. Comparisons for helical flow have only been made at 0 and 100% vapor quality. Some of the interesting conductance variations in the low and high vapor quality region for helical flow can be interpreted with the aid of liquid annulus and fog flow models. Figure II-12 presents a comparison of experimental linear and helical flow mercury boiling frictional pressure drops as a function of vapor quality; predicted values for linear flow based on the slug flow model are also shown. Note that

*It is recognized that when no vapor films exist at the boiler tube wall (wetted walls with superheated liquid films), the boiling heat transfer conductances in the low vapor quality regions would be higher and more nearly equal in both linear and helical flow. In the more important high vapor quality regions, however, film boiling phenomena are always expected to be present. The effects of surface phenomena on the conductance variations in the low vapor quality region are under study.

TABLE 2. LINEAR AND HELICAL FLOW FRICTIONAL PRESSURE DROP
FOR MERCURY BOILING IN THE HAYNES SYSTEM

Run No.	Flow Type	Total Flow Rate #/hr	Exit Vapor Quality %	Pressure at Boiling Initiation psia	Boiling Length inches	ΔP_{total} psi	$\Delta P_{\text{frictional}}^*$ psi
9	linear ↓ helical (P/D=4.23) ↓	110	6.1	25.6	31	2.9	1.98
10		106	7.1	29.1	38.5	8.4	7.54
12		107	6.1	34.1	34.6	3.5	2.92
13		107.5	9.7	37.3	58.5	5.6	4.77
14		130	7.4	38	57.5	7.3	6.25
15		126	8.7	no pressure data			
16		123	9.8	35.7	62.5	4.0	2.68
17		234	7.0	24.4	91	5.6	5.167
18		60.5	27.6	22.1	103.5	3.0	1.72
19		53.3	23.7	24.2	88.5	0.90	.092
20		28.7	43.51	28.31	108	4.11	1.83
22		80	21.3	31.7	81	11.5	10.22
23		62	55	49.2	88.5	24.2	22.83
24		220	18.6	72	94.5	32.3	27.40
25		114	40.8	55.6	78.5	24.3	20.60
26		90.5	79.1	78.3	95	39.1	35.63
27		93	100	100	95.5	50.8	47.30
28		115.2	81.1	105.3	86.5	50.6	46.04
29		38.9	super heat	57.6	100	26.4	--
30		41	100	56.6	100	25.4	13.9
31		49.3	97.7	59.5	95.5	27.6	26.24

* Acceleration pressure drops were calculated presuming that liquid and vapor velocities were equal (see Reference 2). These pressure drops were then subtracted from the total measured values to obtain the frictional pressure drops.

Figure II-8. Linear flow boiling mercury heat transfer conductances versus vapor quality (pipe diameter = 0.190" ID).

Figure II-9. Helical flow boiling mercury heat transfer conductances versus vapor quality (pipe diameter = 0.190" ID, divider with $p/D = 4.23$).

Figure II-10. Helical flow boiling mercury heat transfer conductances versus vapor quality (pipe diameter = 0.190" ID, divider with $p/D = 4.23$).

Figure II-11. Comparison of experimental and predicted boiling mercury heat transfer conductances for linear and helical flow.

Figure II-12. Comparison of experimental linear and helical flow boiler tube frictional pressure drops for mercury with some predictions.

the helical flow pressure drop ratio at 100% vapor quality is somewhat higher than the adiabatic prediction (deduced from Equation II-(12)) as is expected.

C. Mercury Experiments (Radiant Heating)

The radiantly heated boiler section was installed in the tantalum, high-pressure boiling facility. This system was an open circuit, i.e., the mercury was not recirculated continuously by means of a pump, but was forced from one reservoir through the test section, vapor separator and cooling sections to another set of reservoirs by means of argon pressure. The reservoirs were of sufficient size to provide a running time in excess of two hours. The reservoirs were mounted on load cells which indicate the weight in terms of a recorded electrical signal. The mass flow rate was obtained by measuring the slope from the weight versus time recording. The heated section was attached to the system by means of a tantalum flange, fitted with stainless steel gasket. This design allowed changing the helical inserts, boiler geometry or heating method. The pressurization and flow measuring systems, flanges and containment vessel have been previously described⁽⁶⁾. The system schematic is shown in Figure II-13. The condenser circuit valve was inoperable in these experiments because of a fracture of the tungsten tip of the valve. This line was sealed and the system operated without the separator; thus the vapor and liquid both passed through the subcooler circuit.

The radiantly heated boiler test section was a 30.5 inch long, 0.50 ID, 0.06 inch wall tantalum tube. A tantalum twisted divider with a 4.5 pitch to diameter ratio was inserted in the total length of the boiler. Small cavities (hohlraums) were provided at five stations along the boiler in order that the temperature measurements obtained from the optical pyrometer would not be subject to corrections due to non-black body conditions.

The heater consisted of a 1.75 inch ID, 3.00 inch OD cylindrical Agot grade graphite tube divided axially to allow installation around the tantalum boiler tube without removal of the boiler tube from the system. The initial radiation shield consisted of three concentric refractory metal and stainless cylinders spaced around the outside of the graphite heater by means of alumina spacer blocks. The cylinders were axially cut into two sections. Small holes were drilled through the shields and the graphite heater, so that the temperature of the boiler tube wall could be measured with an optical pyrometer.

Figure II-13. Schematic diagram of the radiantly heated boiling system.

During the initial tests, thermally induced differential expansion caused the warping of several shields. Large convection currents were then produced creating excessive transfer of heat from the radiant heater to the containment vessel. In order to alleviate this condition, two refractory metal shields were eliminated and a water cooled copper jacket provided. Several layers of stainless shim stock were positioned between the copper jacket and the inner tantalum shield. The use of the water cooled jacket successfully removed the heat from the containment vessel allowing for a longer operation time, as well as providing a means for establishing the heat loss by the measurement of the water flow rate and inlet and outlet temperatures.

The electrical power system consisted of a large saturable reactor capable of supplying current in excess of the 1000 amps for the low resistance graphite heater. Eight 200-amp Conax connectors were used to duct current through the containment vessel. A large copper ring was silver soldered to the inner leads and the final connection made to the graphite heater by sixteen braided copper connectors. Figure II-14 is a photograph of the radiantly heated boiler assembly and containment vessel.

The graphite and tantalum reactions with air were minimized by evacuating the containment vessel and then flushing with Argon. An 80-cfm Stokes Model pump was used for this purpose. A tantalum-heated tungsten getter was also provided to help reduce the oxygen content within the containment vessel.

The heater current was measured by a current transformer and the test section voltage by a vacuum tube voltmeter. The mercury inlet and outlet temperatures were measured by means of chromel-alumel thermocouples and the pressures by means of Bourdon type gages. A Micro Optical Pyrometer manufactured by the Pyrometer Instrument Co. was used to measure the boiler wall temperatures.

The range of parameters studied are given below

Heater current: 300 - 980 amp

Heater power: 3 - 10 kw

Mercury flow rate: 10-32 lb/hr

(continued)

Figure II-14. Photograph of the radiantly heated boiler assembly.

Outlet vapor quality > 50%

Outlet vapor superheat < 150°F

Maximum boiler wall temperature < 2930°F

Saturation pressure: 20-30 psig

Helix Pitch to Diameter: 4.5

Figure II-15 indicates the temperature profile for the radiantly heater and the boiler sections for several high temperature runs. The corresponding heat flux curves are given in Figure II-16. The heat flux was calculated from the equation

$$q/A_B = F \sigma \left[T_H^4 - T_B^4 \right] \quad \text{II-(29)}$$

$$\text{where } F = \frac{1}{\frac{1}{\epsilon_B} + \frac{A_B}{A_H} \left(\frac{1}{\epsilon_H} - 1 \right)}$$

ϵ , emissivity

A_B , heat transfer area of the boiler tube

A_H , heat transfer area of the heater

σ , Stefan-Boltzman constant

The experimental value of 0.52 was determined for the function F from several nonboiling runs by relating the heat obtained from Equation II-(29) to that of the sensible heat of the liquid.

The heat loss for the tubing between boiler outlet and the condenser inlet was also obtained from the nonboiling experiments. This heat loss plus the condenser load agreed with the heat input for the superheat experiments within ten per cent; where the heat input was calculated both by Equation II-(29) and by the latent and sensible heat gains.

It was necessary to correct the pyrometer readings for attenuation and scattering affects of the three glass surfaces between the pyrometer and the boiler. This was accomplished by varying the temperature of a small filament over the range of interest.

Figure II-15. Temperature profiles for the heater and test section for the radiantly heated boiler.

Figure II-16. Heat flux profiles in the radiantly heated boiler.

Readings were taken with and without the glass at three axial stations. Figure II-17 indicates the resulting correction curve.

The shapes of the local boiling conductance curves for the present and previous (tube resistance heating) experiments were similar although the boiler geometries, materials, outlet Reynolds numbers and heat fluxes differed considerably between experiments. However, the absolute heat transfer conductances for the present experiments were lower than for the previous experiments. The ratios of the local experimental heat transfer conductance to the linear flow vapor conductance at 100 per cent quality are shown in Figure II-18. The measurements for both systems lie above $H = 1.5$ and have the same features with the exception of a few points for the present experiments in the 50 per cent vapor quality region; these specific data correspond to low vapor velocity conditions where not all the advantages of helical flow are in operation. Predicted values of the function H at 100 per cent vapor quality (Equations II-(13) and II-(14)) are also presented in Figure II-18.

Some of the physical parameters for the two boiling systems were quite different. Table 3 defines these differences.

Table 3

<u>Parameter</u>	<u>Radiantly Heated Boiler</u>	<u>Resistance Heated Boiler</u>
Boiler length	30.5 "	111 "
Boiler ID	0.50 "	0.190 "
Boiler L/D	61	584
Mean vapor Reynolds number at exit	5000	20,000
Wall-Saturation temperature difference range	1500 - 2300° F	200-1200
Flow rate	17 to 32 lb/hr	38 to 115

Figure II-17. Pyrometer corrections.

Figure II-18. Comparison of two sets of boiling mercury heat transfer measurements. (H versus x).

D. Potassium Experiments

The direct resistance heated potassium system has been completed. Heat transfer and fluid flow data for linear and helical boiling will be obtained within the following estimated parameter ranges:

Heat fluxes:	5000 Btu/hr ft ² - 50,000 Btu/hr ft ²
Boiler pressure:	20 psia - 150 psia
Vapor outlet qualities:	0 - 100%
Boiler wall temperatures:	to 2100° F
Potassium flow rate:	to 40 lb/hr
Boiler power (maximum):	10 kw

The boiler section consists of ten-foot length of 1/4 inch drawn Haynes 25 alloy tubing with a wall thickness of 0.030 inches. The subcooled potassium enters the boiler at about 1000° F and is preheated and boiled in one continuous length of tubing. From the boiler outlet, potassium vapor enters a condenser in which heat is transferred through a high thermal resistance wall to cooling water. The various system components are depicted schematically in Figure II-19. Subcooling of the condensed potassium is also accomplished in the condenser by means of its extended heat transfer surface. A stainless steel, remote head, oil actuated, positive displacement pump withdraws liquid potassium from the condenser through a surge tank and forces the fluid into the boiler through a second surge tank. A low voltage ac power supply (~ 24 volts 400 amp) generates heat within the boiler wall by resistance heating. The boiler section is center tapped and each end is at ground potential.

Boiler wall temperatures are sensed by twenty-two chromel alumel thermocouples which are spot-welded to the exterior surface. The boiler tube is insulated uniformly with eight inches of Refrasil insulation through which the heat losses have been accurately determined over the temperature range of operation. Heat balances are ascertained by the same procedure used in the mercury boiling system. Potassium flow rates are measured to within $\pm 1\%$ because of the accurately known frequency and displacements inherent to the positive displacement pump. A further verification of the flow rates are made by means of a heat balance method. The total quantity of potassium in the system is approximately one pound.

Figure II-19. Potassium boiling system (resistance heating).

Figure II-20. Mobile liquid metal facility.

Figure II-21. Liquid metal pump calibration system.

Because of the dangerous nature of alkali liquid metal experiments, it was decided to perform the work at a remote site. Accordingly, a test facility has been made available by the U.S. Air Force at the Sycamore Canyon testing site in San Diego. A war surplus truck, obtained through the auspices of the AEC, has been modified to contain and transport the liquid potassium heat transfer system. A photograph of the completed instrument cab and associated apparatus can be seen in Figure II-20.

1. Filling Procedure and Pump Calibration

A working model of the flow system to be used for the potassium forced-convection boiling experiments was built, using water as the working fluid. The flow system contained the positive displacement Lapp Pulsafeeder metering pump. Because the pump is a reciprocating unit which operates at about sixty strokes per minute, surge tanks are required at the inlet and outlet to maintain steady state flow in the circuit. The working model (see Figure II-21) was constructed to test the design of the surge tanks, as well as to test the transient response of the flow rate to sudden changes in pressure drop. This model was also used to establish the charging, draining and cleaning methods that were utilized in the actual potassium system.

After complete assembly of the potassium system, a second check of the filling operation was made, substituting reagent grade acetone for the potassium. A glass float type flowmeter was inserted in the system, thereby allowing a further calibration of the Lapp pump flow-rate characteristics. The flow rate versus metering position curve was linear and within 8% of the curve supplied by the manufacturer. This value was within the accuracy of the flow-meter used.

The acetone was removed from the system by drawing a vacuum and heating the system. A liquid nitrogen cold trap was used between the system and the vacuum pump to remove the acetone. The final operation before filling consisted of bake-out of the system for several days at a vacuum of less than 10 microns and at temperatures from 300 to 1500° F.

The potassium was introduced into the preheated, evacuated system by pressurizing the potassium container and subsequently opening an intermediate valve. After filling the system, the boiler insulation heat losses were obtained by varying the electrical power (with zero flow rate) and noting the corresponding axial boiler temperature profile. An equivalent

insulation conductance was obtained which was within 10% of the value previously obtained for the mercury system insulation losses. The same type of insulation was used in both systems.

2. Initial Experiments

The initial nine linear flow boiling experiments for the potassium system were carried out over the flow rate range of 5.0 to 10 lb/hr and an outlet vapor quality range of 2 to 40%. A typical set of experimental axial wall temperature profiles can be seen in Figure II-22; the several sets of data were obtained for consecutive time periods noted on the figure. These results were obtained for a coolant flow rate of 5.25 lbs/hr and an exit vapor quality of 35%. Note that with the exception of a narrow region near the center of the boiler where boiling began, the low wall temperatures suggest that a nucleate type of boiling existed. A graph of the time-temperature variation of the highest temperature station shown in Figure II-22 can be noted in Figure II-23.

During two of the forced flow boiling potassium experiments, the high wall temperature region or peak (as noted in Figure II-23) in the central region of the boiler became excessively hot indicating that probably intermittent film boiling occurred. This behavior is currently under study. It is believed that this result may reflect the low boiling conductances which can occur in linear flow if vapor exists at the boiler tube wall; somewhat similar features were noted in the previous linear flow mercury experiments.

Figure II-22. Boiler wall temperature profile.

Figure II-23. Transient temperature profile of the highest temperature station in Figure II-22.

III. DROPLET VAPORIZATION

Analytical and experimental studies were continued on the vaporization of droplets suspended above a hot heat transfer surface by issuing vapor films.

A. Mercury

The previously described apparatus (Reference 6), which was used to measure vaporization lifetimes of water droplets, was modified to permit the measurement of the lifetimes of mercury droplets. The modification was necessary to accomplish the safe removal of generated mercury vapor, the production of mercury droplets of a uniform size and the confinement of droplets on the plate. To accomplish removal of mercury vapor, the plate on which the drops were heated was enclosed in a metal box, the inside of which was maintained at a negative gage pressure by a small, air-driven jet pump. Maintaining the box at a negative pressure insured the influx of air rather than the egress of mercury vapor.

To produce drops of a uniform size, a heated stainless steel tube (OD = 0.125 inches) with one end sealed was cooled with the open end submerged in mercury. As the tube cooled, the air within the tube contracted causing the mercury to be drawn into the tube. When a drop was desired, the tube was reheated until a drop was produced. The mercury-drop producing tube was water cooled at the ejection end to insure a constant size drop (i.e., constant surface tension and density).

The droplets were confined in an indentation in the plate. The indentation was produced by a 1 9/16 inch diameter steel ball.

The mercury droplet lifetime data are plotted on Figure III-1. All data having the same symbol were taken sequentially. The theoretical lifetimes for droplets of the size used in the experiments are also indicated.

At temperatures below 950°F, the data from the two runs are vastly different. These data were taken on subsequent days and the difference can only be explained by a change in the interaction between the mercury and the heated surface (wetting and non-wetting). Unfortunately, the data represented by the square symbols were not extended to higher temperatures, so that a transition, if any, from wetting to non-

Figure III-1. Experimental and predicted mercury droplet vaporization lifetimes.

wetting could have been observed. Presumably, if such a transition had occurred, the data at temperatures above the transition would have fallen in proximity to the other data.

At plate temperatures above 950°F, the data are fairly reproducible and in agreement with the analytical model. The good agreement is considered to be somewhat fortuitous, because the theoretical model is based on film boiling, whereas, in fact, either nucleate or unstable film boiling occurred for all drops for a substantial portion of their respective lifetimes. Under these boiling conditions, relatively small mercury droplets were emitted from the upper unstable surface of the large drop probably because of internal vertical mercury vapor flow.

The behavior of the boiling mercury droplets was substantially different from that of the water droplets. This difference in behavior is attributable to the difference between the surface tension and density of the two fluids. The water droplets were far more physically stable than the mercury droplets. The water droplet boiling mechanism was stable film boiling and the droplets had the appearance of shiny glass spheroids (see illustration IV of Figure III-2).

Two distinct life histories were observed for the mercury droplets depending upon the plate temperature, T_w . At plate temperatures below approximately 1000°F, a droplet had the appearance sequence I, II, III, illustrated in Figure III-2, whereas, at plate temperatures above 1000°F, the I, II, IV sequence was observed.

Appearance I is characterized by 1) a high frequency, low displacement, vertical oscillation, 2) nucleate or unstable film boiling, 3) a relatively stable convex lower drop surface and 4) ejection of relatively small liquid droplets from the upper surface probably by vapor bubbles as they pass through the upper surface of the droplet. Nucleate boiling may be occurring when the droplet contacts the wall during the oscillation.

Appearance II is characterized by a high frequency vertical oscillation with a displacement of approximately five diameters. The frequency of oscillation was so fast that it was impossible to observe other characteristics.

Small droplets (less than approximately one sixteenth of an inch) oscillated with

Figure III-2. Sketches of various mercury droplet states on the surface of a hot plate during the vaporization process.

an amplitude of at least ten diameters, the oscillation having a sinusoidal appearance, see III of Figure III-2. The sinusoidal appearance is attributable to the dish shaped indentation in which the drops were evaporated.

Appearance IV is characterized by a shiny spheroid floating on a layer of mercury vapor.

It is thought that Appearance I may be somewhat representative of the first stages of forced flow mercury boiling in a boiler tube. Appearance IV is envisioned to describe vaporization of small mercury droplets contiguous to a boiler tube wall in the exit or fog flow region.

Further studies of these mechanisms are being made; vaporization lifetime measurements of potassium droplets are also being initiated.

The droplet vaporization mechanism becomes important in forced-flow boiling of a saturated liquid with net generation of vapor as the volume fraction of liquid becomes small. Without some consideration, it is often not appreciated that the volume fraction of liquid decreases very rapidly as the quality of the mixture increases. The volume fraction of liquid for slug flow or fog flow where local liquid and vapor velocities are equal is related to vapor quality by Equation II-(2). For mercury at a vapor quality of 0.1, the liquid volume fraction is about 2.5×10^{-3} . The results for water are similar to those for mercury.

These considerations suggest that heat transfer by the vaporization of liquid slugs or drops may become important at a fairly low vapor quality.

The tendency of the liquid in two-phase flow to form droplets is probably related to the liquid volume fraction and the surface tension of the liquid, as well as other variables. On the basis of surface tension alone, one would expect that liquid mercury layers at the boiler tube wall would more readily form droplets than would water layers. This reasoning seems to be supported by experimental evidence.

The initially measured vaporization lifetimes of mercury⁽¹⁾ were determined for one drop size (radius ~ 0.054 "). More measurements were subsequently made for a smaller size (radius ~ 0.048 "). In Figure III-3 the predicted lifetimes, based on the model described previously, are plotted versus the measured values for all mercury data taken

Figure III-3. A comparison of predicted and measured vaporization lifetimes for mercury droplets in the film boiling regime.

to date in the film boiling regime. The vaporization lifetimes for the smaller droplets were less accurately determined since the lifetimes were shorter and the measurement of droplet size more difficult. It is noted from Figure III-3 that 87% of the data fall within 20% of the predicted values.

Figure III-4 shows a photograph of a mercury droplet boiling on a nickel-plated copper plate having a temperature of approximately 1100°F. The droplet appearance is characterized by 1) a high frequency, low displacement, vertical oscillation, 2) nucleate or unstable film boiling, 3) a relatively stable convex lower drop surface, and ejection of relatively small liquid droplets from the upper surface, probably by vapor bubbles as they pass through the upper surface of the droplet. Nucleate boiling occurs when the droplet contacts the wall during the oscillation. When the droplet reached a smaller size it became a stable, shiny spheroid floating on a layer of mercury vapor.

To demonstrate the effect of wetting, a droplet was placed on a mercury wetted, indented copper plate heated to 1100°F. The lifetime of this droplet was not measurable since nucleate boiling almost instantaneously vaporized the droplet.

B. Potassium

A number of potassium droplet vaporization experiments have been performed to determine the range of surface temperatures through which transition from nucleate to stable film boiling occurs. In these experiments boiling took place on a stainless steel cylinder with a one inch radius indentation in the upper surface. This heated cylinder is enclosed in a windowed container so that an argon atmosphere may be maintained during the experiments. This container is shown in Figure III-5. The surface is heated by either a platinum resistance element or by induction.

The transition from nucleate boiling to stable film boiling for one-eighth inch diameter droplets at one atmosphere pressure occurs over the range of approximately 1750°F to approximately 2000°F. Below the temperature of 1750°F, all droplets observed immediately flashed into vapor, with no observable lifetime. Of the approximately eight droplets vaporized in the 1750°F to 1850°F range, one was in stable film boiling while the remainder flashed into vapor with no observable lifetime.

At 2000°F surface temperature, droplets of one-eighth inch diameter appear

Figure III-4. Vaporization of a mercury droplet.

Figure III-5. Windowed capsule.

to be in stable film boiling, while one-sixteenth inch droplets oscillate vertically with an amplitude of some 4 to 8 diameters.

Several observations have been made of potassium droplets vaporizing on a surface at 2200°F. These droplets were subcooled below the boiling point (at about 1200°F) and when dropped onto the hot surface vaporized almost instantaneously. No film boiling was observed with these subcooled droplets. Apparently the potassium vaporized almost simultaneously throughout its volume so that no insulating vapor film was ever formed on the lower surface. The sudden release of such a large amount of vapor caused a marked pressure rise in the container in spite of the venting system.

Some difficulty has been experienced with observation of the droplets in this series of experiments. The argon atmosphere in which the experiments are being performed is at a temperature considerably lower than the equilibrium vapor temperature of potassium at one atmosphere pressure, so that the potassium vapor rapidly condenses into a dense fog which obscures the droplet on the heated surface. Difficulty has also been experienced with the formation of potassium oxide in the dropper assembly during its transfer from the filling container to the vaporization experimental apparatus. Modifications are being made to the apparatus to eliminate both of these difficulties and to facilitate performance of the experiments.

In order to eliminate the obscuring of the droplet by the fog and to decrease the magnitude of the pressure rise in the event of the release of large amounts of vapor, a larger container is being constructed which will have the argon injected in such a way as to clear the condensate without disturbing the droplet on the surface. This new container will be constructed of stainless steel to facilitate cleaning. A new dropper filling apparatus is also being constructed which will be integral with the vaporization vessel, so that the transfer of the filled dropper from one container to another will no longer be necessary.

C. Benzene

Reference (7) reports the vaporization lifetimes of benzene droplets. A mean curve, which closely represents some of these data, is plotted in Figure III-6. The analytical predictions for the droplet lifetimes are shown. Reasonable agreement between predictions and experiment is observed in the temperature range from 200 to 350°C where film boiling probably exists.

D. Freon 11

A series of experiments was performed to determine the lifetime of Freon 11 droplets vaporizing on a hot nickel surface with a 3/4 inch radius indentation on the upper surface. The results of these experiments, which were performed with droplets of 9.5×10^{-2} inch diameter, are compared to the analytical prediction in Figure III-7. The experimental points lie somewhat above the predicted lifetime curve, but parallel to it in the established film boiling region. In one set of experiments, in which the nickel surface was highly polished, transition from nucleate to stable film boiling took place at higher temperature and more uniformly than in the other set, in which a thin layer of smooth oxide was present on the surface. In the presence of the oxide, a sudden jump in the lifetime curve is observed at a temperature difference of about 110°F. Transition on the clean surface does not appear to exhibit this sudden jump. The temperature at which these experiments could be performed was limited by decomposition of the Freon 11 vapor to below 700°F.

The atmospheric pressure boiling process of Freon 11 below 140°F is relatively quiet, and no sputtering or violent agitation is observed. At temperatures up to 140°F, corresponding to surface to saturation temperature differences up to 65°F, the droplets spread out into a thin film, completely wetting the surface. This film then evaporates rapidly from the free surface. Since very few gas bubbles come through the liquid, it is concluded that vaporization must occur almost entirely by conduction of heat across the Freon film. This is consistent with observations made in the forced-convection flow visualization experiment (see section V).

At surface temperatures between 140°F and 220°F, the droplets act either like those at lower temperature, or remain in film boiling with a spherical form for a short

Figure III-6. Predicted and measured vaporization lifetimes of benzene.

Figure III-7. Experimental and predicted Freon 11 droplet vaporization lifetimes.

period of time, and then spread out into an irregular mass which nucleate boils with considerably more agitation and bubble evolution than at lower temperatures.

Above 220°F the droplets are in stable film boiling with either a spherical or oblate spheroidal form. These droplets vaporize quietly until the diameter decreases to the order of half the original size, when the droplet begins to oscillate vertically with increasing amplitude as the diameter decreases. This oscillation reaches a maximum amplitude of about one-fourth of an inch, when the droplet is of vanishing size.

E. Influence of Radiation on Vaporization

In the previously developed model (Reference 6) of the vaporization of a liquid drop on a horizontal hot surface, it was postulated that conduction was the sole mode of heat transfer from the hot surface to the drop through the film formed by the vapor issuing from the bottom of the drop. To find the influence of radiant heat transfer, Equation (15) of Reference 6 was modified to include the radiation term. The resulting equation is as follows:

$$\frac{L G}{\pi r_d^2} = \frac{k_v (T_w - T_s)}{\bar{\delta}} + \frac{\sigma (T_w^4 - T_s^4)}{\frac{1}{\epsilon_w} + \frac{1}{\epsilon_d} - 1} \quad \text{III-(1)}$$

where,

- L, latent heat of vaporization
- G, total weight flow rate of the vapor beneath the drop
- r_d , drop radius
- k_v , vapor thermal conductivity
- T_w , absolute temperature of the heat transfer surface
- T_s , absolute saturation temperature
- $\bar{\delta}$, average thickness of vapor film beneath the drop
- σ , Stephan - Boltzmann constant
- ϵ_w , emissivity of heat transfer surface
- ϵ_d , emissivity of the drop

Similarly, a more general equation for vaporization lifetime results, namely,

$$\Theta_{\ell} = 6 \gamma_{\ell} L \int_0^{r_d} \frac{dr_d}{\frac{k_v (T_w - T_s)}{\bar{\delta}} + \frac{\sigma (T_w^4 - T_s^4)}{\frac{1}{\epsilon_w} + \frac{1}{\epsilon_d} - 1}} \quad \text{III-(2)}$$

where γ_{ℓ} is the weight density of the liquid. By combining Equation (18) in Reference 6 with Equation III-(1), an expression for the unknown average thickness of the vapor film, $\bar{\delta}$, results; it has the following form:

$$A + B \bar{\delta} + C \bar{\delta}^2 - D \bar{\delta}^5 = 0 \quad \text{III-(3)}$$

Because a numerical solution of Equations III-(2) and III-(3) is required, the lifetime for only one set of conditions was determined. The following parameters were used for a water droplet: $r_{d_o} = 0.10$ inches, $k_v = 1.4171 \times 10^{-2}$ Btu/hr/ft $^{\circ}$ R, $\epsilon_w = \epsilon_d = 1.0$, $L = 970$ Btu/lb, $T_s = 672^{\circ}$ R and $T_w = 1472^{\circ}$ R. Water was used because its high emissivity was thought to represent a more extreme test of the importance of the radiation term than would mercury.

The computed drop lifetime for the above conditions considering conduction only is 52.7 seconds, whereas the computed lifetime considering radiation and conduction is 51.3 seconds. It is concluded that this 2.5% difference demonstrates that the radiation component of the total heat transferred from a hot plate to a drop is generally not important for most practical conditions. Since mercury has a low thermal emissivity, it may be further concluded that the radiation component for a boiling mercury drop would be even less important.

IV. LIQUID METAL CONDENSATION

A. Literature Review and Analysis

The classical Nusselt condensation theory has been experimentally verified for many non-liquid metal working fluids. Equation IV-(1) is the conventional Nusselt equation for a vertical plate derived on the basis that the main thermal resistance is in a laminar condensate layer contiguous to the condenser wall,

$$\bar{h} = 0.943 \sqrt[4]{\frac{g_c \gamma^2 k^3 \lambda}{\mu L \Delta T_f}} \quad \text{IV-(1)}$$

where $\bar{h}$, mean axial heat transfer conductance

g_c , gravitational constant

γ , condensate density

k , condensate thermal conductivity

λ , latent heat of vaporization

μ , condensate viscosity

L , axial length

ΔT_f , wall to vapor temperature difference

Equation IV-(1) is derived presuming negligible vapor velocity. For a forced flow condenser with a relatively large vapor velocity, however, the liquid layer is affected by the liquid-vapor shear force. For a vertical, downflow condenser, the laminar liquid layer thickness is reduced by the shear force, resulting in an increased conductance over that calculated from Equation IV-(1). For a constant vapor velocity, the mean heat transfer conductance for a tube can be calculated from equation (2) (Reference 8),

$$\bar{h} = 1.04 \sqrt[3]{\frac{\gamma \lambda k^2}{\mu L \Delta T_f} \frac{4f \rho_v V_v^2}{2}} \quad \text{IV-(2)}$$

where f , Fanning friction factor

ρ_v , vapor density

V_v , vapor velocity

In a condenser, the vapor velocity changes throughout the condensing length because of the decrease in vapor quality. For a constant heat flux condenser, the variable vapor velocity can be accounted for by relating the vapor quality to the condenser length. Equation IV-(3) for such a tube system is derived in Appendix A,

$$\bar{h}_{0 \text{ to } L} = k \left(\frac{\lambda \gamma f}{4\mu \rho_v \ell_o} \right)^{1/2} \frac{1}{(q/A)^{1/2}} \frac{m_{t,i} x_i \left[2 - \frac{2}{3} \left(\frac{L}{\ell_o} \right) \right]}{A_c \left(\frac{L}{\ell_o} \right)^{1/2}} \quad \text{IV-(3)}$$

Equation IV-(3) may be compared to Equation IV-(2) on the basis of equal heat flux or equal mean temperature difference. For the latter case the heat transfer coefficient Equation IV-(3) is approximately 60% of that of Equation IV-(2).

Equation IV-(3) was used to predict the maximum values of the wall temperatures occurring in the condenser discussed in the next section.

Condensation experiments performed with liquid metals yield lower coefficients (by an order of magnitude or more) than those calculated from the Nusselt Analysis. Work done by Seban, Reference (9), shows that for low Prandtl number fluids the condensing conductances may be decreased if the condensate layer is turbulent rather than laminar as postulated in the Nusselt Analysis. Thicker turbulent condensate layers have poorer heat conduction. For the higher Prandtl number fluids, the increased eddy conductivity term yields increased heat transfer with the onset of turbulence. The Seban analysis neglects the effect of the vapor velocity on the condensate layer thickness.

Modification of Nusselt's theory to account for momentum forces, subcooling of the condensate, variable film temperature drop and turbulent condensate flow are summarized in Reference (10); it is indicated that these factors cannot account for the low experimental values for the liquid metal condensing conductances. It is postulated in Reference (10) that the major thermal resistance in liquid metal condensing is at the condensate-vapor interface. This resistance may be caused by the presence of non-condensable gases or by non-equilibrium thermodynamic conditions at the gas-liquid interface. Work performed in Reference (10) with mercury at low pressure indicates good agreement with theory for a

condensing accommodation coefficient in the order of 0.45 .

Equation (8) of Reference (10) can be written in terms of the heat transfer conductance as follows:

$$h = \left\{ \frac{\sigma}{2 - \sigma} \right\} \left\{ \frac{M}{R} \right\}^{1/2} \frac{P_v}{T_v} \lambda^2 [g_c]^{1/2} \quad \text{IV-(4)}$$

where M , molecular weight
 R , universal gas constant
 T_v , saturation temperature

P_v , saturation pressure
 σ , condensation accommodation coefficient

In order to obtain condensation conductances that are of the same magnitude as those that have been reported for experimental potassium systems⁽¹¹⁾, accommodation coefficients of the order of 0.1 are required. Such values are significantly lower than the value of 0.45 suggested in Reference (10).

B. Experimental

The experimental condenser assembly drawing for the Geoscience forced flow potassium boiling system is shown in Figure IV-1. This unit is presently installed in the potassium boiling system in the vertical position. The condenser assembly consists of two sections which are clamped to the 0.25 "OD Haynes tubing. The clamp-on design allows the condenser-boiler assembly to consist of one continuous piece of tubing thus eliminating need for a high temperature joint between the boiler and condenser. Each condenser subassembly consists of a T-shaped section. When attached to the tube, the heat flows radially to the water cooled tubes. A two dimensional analog analysis was performed to insure that the heat is divided equally to all four radial sections.

The heat flux is measured at different axial stations by means of radially spaced thermocouples. The temperature difference between these couples combined with the known thermal resistance will be used to calculate the heat flux. Thermocouples are spot-welded to the wall and lead out between the two sections through ceramic insulators.

Figure IV-1. Potassium condensate assembly.

Knowing the heat fluxes, wall temperatures and saturation temperatures, the condensation conductances can be calculated. It is expected that experimental information from this system would give reasonable results in the region of thick condensate layers (or lower conductance regions) and not in the entrance region where the conductances are extremely high. Two dimensional heat flow analyses of possible thermocouple errors in the measurement of condenser wall temperatures are being initiated.

V. FORCED FLOW BOILING FREON HEAT TRANSFER

A series of simple experiments in boiling heat transfer has been performed using Freon 11 as the working fluid. The purpose of these experiments was to allow visualization and photography of the boiling fluid at the same time that quantitative measurements were being made. This procedure made it possible to relate the fluid mechanics of the flow to the variation in heat transfer conductance. It is expected that this relationship will be of considerable aid in interpreting the data from systems in which it is impossible to observe the boiling fluid.

The working fluid, Freon 11, was chosen because of its convenient boiling temperature, about 75°F, and relatively small latent heat of vaporization. The room temperature boiling point minimizes insulation requirements, and the small latent heat minimizes power requirements. Freon 11 wets the glass boiler tube wall so one supposes that its behavior in regimes where the wall is normally wet is similar to that of water and potassium, rather than mercury.

A photograph of the Freon boiling system is shown in Figure V-1. The boiler consists of a 3/8 inch OD, 1/4 inch ID Pyrex tube with three Alchrome wire heaters wound along it. The maximum heated length is 33 inches. This boiler tube is insulated by several aluminum foil radiation shields and a two-inch OD Pyrex outer tube. It has been found that the heat losses from this boiler are negligible compared to the heat input during boiling and no correction is made for them. At the outlet of this boiler is a specially designed phase separator, which removes the liquid from the gas in a length of about three inches. The purpose of this unit is to allow the measurement of the gas temperature independently from that of the liquid. The remainder of the flow system consists of a pressurized supply tank, an inlet subcooler, a tapered tube flow meter, and a condenser.

Temperatures are measured at six stations along the tube by copper-constantan thermocouples on the outside of the wall. A theoretical correction dependent upon the heat flux is used to determine the inner wall temperature. The exit gas temperature is measured by a thermometer in a well in the separator, and the power is controlled by

Figure V-1. Freon 11 boiling system.

a Variac auto transformer.

This heat transfer system is quite flexible. Any combination of the heaters may be used to vary the heated length, the flow rate is reproducible from run to run, and the entire boiler assembly may be turned horizontally to change the direction of the gravitational field with respect to the boiler.

The experimental program consisted of two parts. The first was a qualitative survey to observe the different regimes of flow in the tube under boiling conditions. The second part of the experimental program was a combined qualitative-quantitative one in which heat transfer conductances were measured and the type of flow at each thermocouple station was noted. Data were taken at two flow rates, 18.6 lb/hr and 51 lb/hr. The heat fluxes over which the data were taken varied from approximately 1000 to 35,000 Btu/hr ft².

A. Qualitative Results

In addition to the two end conditions, heating of the pure liquid or dry vapor, six fairly distinct regimes of fluid flow have been observed in the boiler for linear flow. These regimes do not necessarily all occur during any one run, although generally several are present along the length of the tube. These six regimes for linear flow are described briefly as follows:

1. Subcooled Liquid with Nucleate Boiling In this case, the mixed mean fluid temperature is below the boiling point, and the wall temperature is above it. Streams of small bubbles form at discrete points on the wall and rise through the fluid.
2. Homogeneous Mixture of Agitated Liquid and Vapor This regime is characterized by an apparently uniform mixture of liquid and vapor extending across the tube. The agitation is violent but of small amplitude. The wall is wet at all heat fluxes.
3. Mixture at Wall, Vapor Blowing up the Center with Violent, Large Oscillations The proportions of the vapor is higher than in (2), and an

almost continuous vapor channel has formed up the center. The violent agitation is caused by occasional slugs of mixture being driven up the vapor column. The frothy liquid vapor mixture next to the wall is washed violently back and forth. The wall is wet at all heat fluxes.

4. Vapor Interspersed with Droplets Flowing in the Center of the Tube, and a Violently Agitated Film of Liquid Adjacent to the Wall

In this regime, surface waves wash up and down the inner surface of the liquid on the wall. Few vapor bubbles are seen in the film.

5. Dry Wall with Droplets Entrained in the Fluid

The vapor velocity is high. Droplets are carried up the tube by the vapor, occasionally hitting the wall, wetting it locally and vaporizing. This fog flow was discussed previously in Reference 6.

6. Film Boiling In this case, the wall temperature is high enough to maintain a stable vapor film. This regime can occur anywhere along the length of the boiler. When it occurs near the entrance, where a large portion of the flow is liquid, large irregular streams and drops of liquid are entrained in the vapor and carried out of the heater tube unboiled. Heat transfer conductances are low. If film boiling occurs near the beginning of the boiler, the exit qualities are very low.

Several important observations can be made from the experiments. The first is that in flow regimes (2), (3), and (4) the wall is wet, although the heat transfer rates as high as $30,000 \text{ Btu/ft}^2 \text{ hr}$ and temperature differences as high as 150°F have been observed. Thus the liquid in contact with the wall must be considerably superheated, and vaporization must occur from the free surface of the liquid film.

The second observation is that when the wall is dry and droplets are entrained in the flow, (Regime (5)) the liquid vaporization cannot control the temperature of the vapor. The vapor temperature can be far above the mixed mean temperature corresponding to thermodynamic equilibrium of the components. If the mixture is allowed to come to equilibrium in the exit duct, a significant portion of the "boiling" actually

occurs outside of the heated tube. Gas superheat temperatures of 100°F above the Freon 11 boiling point have been observed with a significant liquid flow from the separator.

The third observation is that gravity has a large effect upon the boiling process. When the boiler is in the horizontal position, two obvious conditions can occur. In one, which occurs when the flow rate is small, the liquid simply flows down the bottom of the tube, the gas flows above it, and the phases are mostly separate. The vapor bubbles which form at the lower wall rise through the liquid and break through into the gas layer. The processes appear similar to those observed in pool boiling. The other condition occurs when the flow rate is large enough so that the liquid fills the tube. Then the first vapor formed simply ejects the liquid from the tube. One would expect that in a space power system in which the boiler operated in the absence of a gravitational field, a process somewhat similar to that just described would result, and the attainment of high vapor qualities would be quite difficult in linear flow.

B. Quantitative Results

Normalized local heat transfer conductances as a function of quality for Freon 11 are shown in Figures V-2 and V-3. The conductances are normalized on the computed heat transfer conductance computed for fully-established turbulent flow at $x = 1.0$. The variation in flow pattern with quality is shown parallel to the horizontal coordinate. Points to observe are: 1) the conductances are highest when a thin film of fluid is adjacent to the wall; 2) for the higher flow rate the homogenous mixture flow pattern ends at lower quality and the continuous thin film extends to a slightly higher quality; 3) in the normalized coordinates, the 51 lb/hr conductances are smaller than the 18.6 lb/hr conductances. The actual 51 lb/hr conductances are larger than the 18.6 lb/hr conductances. This decrease in H with increased flow rate is also observed with some other fluids (e.g. water) since the controlling mechanisms for heat transfer with wet walls are not necessarily related to the mechanisms controlling gaseous heat transfer, and are probably less sensitive to changes in flow rate. In the intermittently dry wall region, which lies in the approximate range $0.8 < x < 0.92$, the wall is wetted by the large slugs of fluid which are driven along the boiler intermittently in the mixed phase region. The beginning of this region is of course wet almost all of

Figure V-2. Freon 11 linear forced convection boiling, flowrate 18.6 lb/hr.

Figure V-3. Freon 11 linear forced convection boiling, flowrate 51.0 lb/hr.

the time, the end of the region is wet in the order of one-half to two-thirds of the time. The region where all trace of the film disappears is fairly well determined and is less than one half a tube diameter long. In this short distance the heat transfer conductance drops by a factor of almost 20, and since this is a constant wall heat flux experiment, the temperature difference increases by a factor of 20 in the same distance.

As pointed out before, the exit gas component can superheat considerably at qualities significantly less than 1.0. Observations during this series of experiments show that the gas component superheats even while the wall is still wet with fluid. A few representative points are shown in Figure V-4. This superheating of the gas in the presence of the liquid film is taken to be further evidence that the liquid is superheated significantly above the normal boiling point, and that large departures from thermodynamic equilibrium exist. In most of the runs in which $x = 1.0$ was attained at the exit, there was still a small flow of liquid from the separator, as would be expected with superheated gas at this quality.

Figure V-5 shows a comparison of the present data for Freon 11 with data for Freon 22 taken from Reference (12). Although the agreement between the sets of data is fair, the shapes of the two sets at high quality are different, and thus some comment on the method of data taking is necessary. The data of Reference (12) are "local averages". That is, they are average heat conductances taken over a small range in quality, in this case $\Delta x = 0.2$. Thus, the heat conductance is measured for the full length of a boiler tube with say, $x = 0.8$ at the inlet and $x = 1.0$ at the outlet. A sharp peak in the conductance and an order of magnitude decrease at $x \cong 0.9$, such as has been observed in the present experiments, would be observed as a rounded and much lower peak in the "local average" experiment. Thus, the variation in conductance actually existing in the data of Reference (12) is probably quite similar to that observed here.

Two other dissimilarities between the two experiments are in the wall boundary condition and the initial condition. The present experiments were performed with constant wall heat flux and with subcooled liquid entering the boiler. The Freon 22 data were obtained with constant wall temperature and with a mixture of liquid and vapor entering the test section.

Figure V-4. Freon 11 linear forced convection boiling, superheat of gas component at exit of boiler.

Figure V-5. Linear forced convection boiling data for Freon 11 and 22.

VI. SOME INTERPRETATIONS OF LIQUID METAL BOILING DATA REPORTED IN THE LITERATURE

The forced flow boiling heat transfer characteristics in the initial or low vapor quality region of a boiler tube can vary significantly depending on the presence of superheated layers or vapor films at the boiler tube wall. The boiling mercury heat transfer measurements obtained in the Haynes 25 alloy system described previously, as well as the mercury measurements obtained in the radiantly heated boiler discussed in an earlier section of this report, suggest that mercury vapor films existed contiguous to the boiler tube wall of these systems. Forced flow boiling sodium measurements for low vapor qualities reported in the literature have been analyzed and compared to the predictions.

The General Electric Co. has made forced flow boiling sodium measurements under helical flow conditions⁽¹³⁾. The boiler tube of the 100 kw system consisted of a 30-inch long tube with a 0.423 inch ID made of columbium and one per cent zirconium; heat was added to the boiler by radiation. Typical boiling heat transfer measurements for helical flow with a pitch to diameter ratio of about 9.5 obtained for this system are shown in Figure VI-1 in the zero to twenty per cent vapor quality region. In the very low vapor quality region, the Nusselt moduli are nearly uniform with vapor quality. As the quality increases, the Nusselt moduli increase rapidly by as much as an order of magnitude with only a few per cent increase in vapor quality. The greater the sodium flow rate, the lower the vapor quality at which this rapid heat transfer increase occurs. In general, the heat transfer again decreases as vapor quality is increased subsequent to the region of high heat transfer noted in Figure VI-1.

As discussed in previous quarterly reports, it is possible to define an upper bound for forced flow boiling heat transfer in helical flow in the low vapor quality region by presuming that superheated liquid layers exist at the boiler tube walls. It is postulated that no vapor films are present between the liquid and wall. The Nusselt moduli for such layers based on simple heat conduction have been calculated for the ratio of layer thickness to tube radius. This latter ratio was further expressed in terms of vapor quality using the two-phase-flow model described previously for the

Figure VI-1. Some forced-flow boiling sodium measurements found in the literature.
(Ref. 13)

case of separated viscous-viscous flow (liquid annulus at the wall with a vapor core). From these calculations, it was possible to deduce the Nusselt modulus as a function of vapor quality in the initial, low quality region of a boiler tube; the results for sodium are also shown in Figure VI-1. The asymptotic, established flow Nusselt moduli for viscous and turbulent flow convection at the tube entrance are also indicated.

The results shown in Figure VI-1 indicate that most of the experimental data fall below predicted values. Although no experimental data are reported below a vapor quality of three per cent, it appears that the experimental heat transfer measurements, which lie in the transition flow region, fall below the fully established Nusselt modulus limits at the boiler tube entrance by a factor of two or three. Although some of the experimental heat transfer peaks approach the predicted values for the superheated liquid annulus model, in general, the experimental data fall far below the predictions. This result suggests that vapor films or some other thermal resistance may have existed at the boiler tube wall. Because of the excellent heat transfer characteristics of liquid sodium, such interface resistances could play a dominant role.

In addition to the low vapor quality boiling heat transfer data for helical flow discussed in the previous paragraphs, several additional sets of boiling data over a wider vapor quality region are available (see Figure VI-2). Boiling potassium, mercury, and sodium are shown for helical and serpentine flow; the references and the specific flow rates, tube diameters and p/D ratios are also noted. With the exception of Reference 15, similar boiling conductance characteristics are noted in the intermediate vapor quality region (about $0.3 < x < 0.6$). Generalization of these data are in progress. Further, two additional sets of data recently made available will be included in a subsequent report (i.e. ORNL and Aerojet measurements).

Figure VI-2. Helical and serpentine flow liquid metal forced-flow boiling measurements.

VII. THERMODYNAMIC PROPERTIES OF POTASSIUM VAPOR IN THE SUPERHEATED REGION

A modest theoretical supporting study of the thermodynamic properties of potassium vapor in the superheated region was conducted. These properties are needed for this research program. Once the computational procedure had been devised and completed for potassium, it was simple and inexpensive to also evaluate the properties for lithium and sodium; these three vapors are of interest in space power systems.

The vapors are an ideal mixture of gaseous diatomic and monatomic molecules which are in chemical equilibrium. The ideal thermodynamic properties of the gaseous diatomic and monatomic molecules used were those given by W. H. Evans et al.⁽¹⁶⁾. In this study the properties of Li_2 , Na_2 , K_2 , Rb_2 , and Cs_2 were extended from 1500°K to 2500°K using the equations given in Mayer and Mayer⁽¹⁷⁾ for a rigid rotator - harmonic oscillator with corrections for rotational stretching, vibrational anharmonicity, and rotational - vibrational interaction. The fundamental spectroscopic constants used for Li_2 , Na_2 , K_2 , Rb_2 , and Cs_2 were obtained from Reference 16.

The pertinent chemical reaction energies used are presented in Table VII-1. The enthalpy base for the vapor is based upon solid lithium, sodium, and potassium having zero enthalpy at 0°K .

The thermodynamic properties were calculated using the thermodynamic formulas for an ideal gaseous mixture in chemical equilibrium. The mass fraction, α , of the monatomic molecule (the monatomic molecule will be represented by the symbol A and the diatomic molecule by A_2) was calculated using Equation VII-(1),

$$\alpha_A = \sqrt{\frac{K_p}{4P + K_p}} \quad \text{grams A / gram mixture} \quad \text{VII-(1)}$$

where K_p is the equilibrium constant, a function of temperature, for the reaction

and P is the pressure. The mass fraction of the diatomic molecule was obtained from Equation VII-(2),

$$\alpha_A + \alpha_{\text{A}_2} = 1 \quad \text{VII-(2)}$$

Table VII-1

Energies of Reaction at $T = 0^\circ\text{K}$

<u>Reaction</u>	<u>$\Delta H^\circ_{0^\circ\text{K}}$, cal/mole</u>
Li (solid) $\rightarrow$ Li (gas)	38,050
2Li (solid) $\rightarrow$ Li ₂ (gas)	50,340
Li ₂ (gas) $\rightarrow$ 2Li (gas)	25,760
Na (solid) $\rightarrow$ Na (gas)	25,920
2Na (solid) $\rightarrow$ Na ₂ (gas)	34,310
Na ₂ (gas) $\rightarrow$ 2Na (gas)	17,530
K (solid) $\rightarrow$ K (gas)	21,710
2K (solid) $\rightarrow$ K ₂ (gas)	31,570
K ₂ (gas) $\rightarrow$ 2K (gas)	11,850
Rb ₂ (gas) $\rightarrow$ 2Rb (gas)	11,300
Cs ₂ (gas) $\rightarrow$ 2Cs (gas)	10,380

Table VII-1 (continued)

Base Enthalpy at $T = 0^\circ\text{K}$

<u>Species</u>	<u>$\Delta H^\circ_{0^\circ\text{K}}$, cal/mole</u>
Li (solid)	0
Li (gas)	38,050
Li ₂ (gas)	50,340
Na (solid)	0
Na (gas)	25,920
Na ₂ (gas)	34,310
K (solid)	0
K (gas)	21,710
K ₂ (gas)	31,570

The corresponding mole fractions were calculated by the relationship given by Equations VII-(3) and VII-(4),

$$X_A = \frac{2\alpha_A}{\alpha_A + 1} \quad \frac{\text{moles A}}{\text{mole mixture}} \quad \text{VII-(3)}$$

$$X_A + X_{A_2} = 1 \quad \text{VII-(4)}$$

The average molecular weight, M , of the gaseous mixture was calculated using Equation VII-(5),

$$M = X_A M_A + X_{A_2} M_{A_2} \quad \frac{\text{grams mixture}}{\text{mole mixture}} \quad \text{VII-(5)}$$

where M_A and M_{A_2} are the molecular weights of the monatomic and diatomic molecules respectively.

The enthalpy, H°_T , of the gaseous mixture was calculated using Equation VII-(6),

$$H^\circ_T = \frac{1}{M} \left\{ X_A (H^\circ_T - H^\circ)_A + X_A (H^\circ)_A + X_{A_2} (H^\circ_T - H^\circ) + X_{A_2} (H^\circ)_{A_2} \right\} \frac{\text{cal}}{\text{gram}} \quad \text{VII-(6)}$$

where $(H^\circ_T - H^\circ)_A$ and $(H^\circ_T - H^\circ)_{A_2}$ are the sensible enthalpies, which are functions of temperature and are given in Reference 16. The enthalpies have been extended in temperature for this study. The absolute values are given in Table VII-1.

The entropy, S°_T , of the gaseous mixture was calculated using Equation VII-(7)

$$S^\circ_T = \frac{1}{M} \left\{ X_A (S^\circ_T)_A + X_{A_2} (S^\circ_T)_{A_2} - R(X_A \ln X_A + X_{A_2} \ln X_{A_2} + \ln P) \right\} \frac{\text{cal}}{\text{gram } ^\circ\text{K}}, \quad \text{VII-(7)}$$

where $(S^\circ_T)_A$ and $(S^\circ_T)_{A_2}$ are the entropies of molecules A and A_2 which are functions of temperature and are given in Reference 16. The entropies have been extended in temperature for this study. R is the universal gas constant.

The equilibrium specific heat, C°_p , of the gaseous mixture was calculated using Equation VII-(8),

$$C^\circ_p = \frac{1}{m} \left\{ 2\alpha_A (C^\circ_p)_A + (1 - \alpha_A)(C^\circ_p)_{A_2} + \frac{1}{2}(1 - \alpha_A^2)(\alpha_A)R \left(\frac{\Delta H^\circ_T}{RT} \right)^2 \right\} \frac{\text{cal}}{\text{gram } ^\circ\text{K}} \quad \text{VII-(8)}$$

where $(C_p^\circ)_A$ and $(C_p^\circ)_{A_2}$ are the specific heats of molecules A and A_2 , which are functions of temperature and are given in Reference 16. The specific heats have been extended to higher temperatures for this study. The total mass of the gaseous mixture is m and the heat of reaction ΔH_T° , is given by Equation VII-(9),

$$\Delta H_T^\circ = 2(H_T^\circ - H_0^\circ)_A - (H_T^\circ - H_0^\circ)_{A_2} + 2(H_0^\circ)_A - (H_0^\circ)_{A_2} \quad \text{VII-(9)}$$

Figure VII-1 gives the vapor pressure of potassium versus temperature. The data were obtained from Reference 18 and extrapolated from 1430°K to 1500°K. Figure VII-(2) presents the enthalpy of potassium vapor versus temperature with the pressure as a parameter. The saturated vapor line is included as a dashed line. Figure VII-3 gives the equilibrium specific heat of potassium vapor versus temperature with the pressure as a parameter. The large value of C_p° from 1000°K to 2000°K results from the additional energy absorbed in dissociating K_2 as the temperature is increased. The saturated vapor line is included as a dashed line. Figure VII-4 shows the equilibrium composition of potassium vapor versus temperature with the pressure as a parameter. The composition is given in terms of the mass fraction, α , of monatomic potassium, K, in grams of K per gram of gaseous mixture. The mass fraction of K_2 is $1 - \alpha$. The saturated vapor line is included as a dashed line. Figure VII-5 shows a mollier diagram of potassium vapor in the superheat region.

The results of the thermodynamic property calculation for Li and Na are presented in Appendix B ; the mass fractions for Cs and Rb vapor are also shown. A comparison of the mass fraction versus vapor pressure is shown in Figure VII-6.

Figure VII-1. Vapor pressure versus temperature for potassium.

Figure VII-2. Enthalpy of potassium vapor (vapor in chemical equilibrium between K_2 and K).

Figure VII-3. Specific heat of potassium vapor (vapor in chemical equilibrium between K_2 and K).

Figure VII-4. Composition of potassium vapor (vapor in chemical equilibrium).

Figure VII-5. Mollier diagram for potassium vapor in superheat region.

Figure VII-6. Composition of alkali vapors versus vapor pressure.

VIII. EFFECTS OF HEAT TRANSFER ON REACTOR KINETICS

In a previous report⁽¹⁹⁾ an analysis was presented of the effects of voids formed, because of boiling, in the liquid metal used for a reactor coolant, on the reactivity of a nuclear reactor. The analysis was carried out, in the interest of simplicity, for bare, homogeneous reactors. It was found that the class of compact reactors for space, cooled by boiling liquid metals, tends to have a positive void coefficient of reactivity. In other words, if a void in the coolant increases in size due, for instance, to an increase in reactor power, the result is an increase in the nuclear reactivity of the reactor, and hence a further increase in power. This effect is, of course, in a direction to cause instability of the operating power level of such a reactor, so that an increase in its power level results in further increases in reactor power, limited only by some change in reactor configuration. It appears necessary in such a case to find some other effect of an increase in the reactor power level which will provide a negative power coefficient capable of overriding the effect of this positive void coefficient.

A second previous report⁽⁶⁾ considered the possibility that boiling, and the concomitant formation of voids, in a single fuel coolant channel, representing a small fraction of the total volume of the reactor, might lead to a local instability, which would increase in degree. The possibility considered was that in such a single fuel channel, an incidental increase in its power generation might result in further unstable increases in the local rate of power generation, leading eventually perhaps to a melt down of that one fuel element. This study resulted in an indication that the stability of a single cooling channel could become an important consideration only if there existed an inherent tendency toward instability of the heat transfer process itself.

The current study presents the results of a preliminary analysis of the effects of heat transfer on reactor kinetics. With a change in the operating power level of a nuclear reactor during a rapid transient, the first effect generally is a change in the temperature of the fuel element. Because it takes a finite time for heat to be transferred by conduction through a material, the change in temperature at the fuel element is felt at more remote parts of the reactor only at a later time and at a reduced magnitude. Time lags due to either

mechanical inertia of the reactor parts or due to the heat transfer process may result in significant alterations in the transient response of reactor power to changes in its operating conditions. For instance, the time required for a change in the power level in the reactor to affect the size of voids in the reactor coolant comprises the time for heat to be transferred from the fuel element into the coolant, and the time for the coolant itself to move within the reactor to adapt to the change in void size. For the analysis of such a system to be feasible requires its pursuit on at least two different levels.

These levels are 1) an overall analysis to show the effects on system stability of individual coefficients of reactivity which act at different time rates, and 2) the evaluation of the size, sign (positive or negative) and effective rate of these individual coefficients.

The details of this study are shown in Appendix C. The report is the result of a preliminary investigation of the effects of the finite time required for the transfer of heat on the kinetics of nuclear reactors.*

When the power in a nuclear reactor changes from its steady state value, the most immediate change which follows is generally a change in the temperature of the fuel elements. Because of the time required for the conduction (and convection) of heat through the materials in the reactor, such a transient change in the temperature of the fuel element will be experienced later, and in a reduced amount, in parts of the reactor at a distance from the fuel

*A second study relating to the influence of coolant void perturbations on local neutron and heat flux variations was initiated; the scope is briefly outlined. In previous work⁽¹⁹⁾ an investigation was made of the effect of the formation of voids in the boiling, liquid-metal coolant of a uniform, bare nuclear reactor on the reactivity of the reactor. It was concluded that in such a reactor the void coefficient could generally be expected to be positive, resulting in a destabilizing effect on reactor power. As a continuation of this work an analysis is being made to establish practical procedures for the consideration of the formation of voids over arbitrarily chosen sections of non-uniform, spherically symmetrical reactors, and particularly to consider the effects of the formation of large-scale voids in a compact, reflected reactor cooled by boiling mercury. A procedure for handling the multi-group calculations required has been developed. This procedure is such that numerical work on a desk computer can be used for preliminary analyses of a survey nature. An analysis is in progress of a specific configuration of a reactor cooled by boiling liquid metal, to determine the effects of voids in the coolant.

element, for instance, in a separate moderator or coolant. In the case of a power oscillation, at a frequency, ω , the fuel elements will experience an oscillation in temperature which is generally in phase with the oscillation in transient stored energy. A moderator or coolant, at some distance from the fuel elements, will experience the same oscillation, but reduced in amplitude below that in the fuel element and displaced behind it by phase angle. The reduction in amplitude and the size of the phase angle depend both on the frequency, ω , and the distance of separation, x , between the oscillating source of energy and the point in the reactor under consideration. The equation for the temperature difference between the transient and the steady state, θ , is of the form

$$\theta = e^{-nx} \cos(\omega t - nx) \quad \text{VIII-(1)}$$

where

$$n = \sqrt{\frac{\omega}{2D}} \quad ,$$

D being the thermal diffusivity for the material through which heat is transferred. Hence the characteristics of the propagation of the temperature wave can be expressed entirely as a function of the frequency of oscillation, the distance of separation and the physical characteristics of the heat transfer material. If the distance is reduced to zero the temperature oscillation occurs with the fuel temperature amplitude and with no phase lag, as determined by the oscillation in stored energy.

Any reactor in which the level of operating power is statically stable, that is, a reactor which has a negative power coefficient of reactivity, is characterized in part by what may be called a nuclear frequency⁽²⁰⁾ whose value is related to the size of the negative coefficient of reactivity and to the power density in the reactor. Let this frequency be designated by the symbol, ω_{n2} . Because of the phase lag caused by the finite time required for transferring heat, between the energy source in the reactor and some part of the reactor whose temperature change is responsible for a negative coefficient of reactivity, an altered value of the nuclear frequency results. Let this altered frequency be designated by the symbol, ω . A power oscillation at the altered frequency, ω , can be further characterized by a damping factor, β , which is negative if the amplitude of the oscillation is decreasing, and positive if the amplitude is increasing. The

ratio

$$e^{2\pi\beta/\omega} - 1$$

gives a direct measure of the fractional increase (positive sign) or decrease in amplitude per cycle. Therefore, the stability of an oscillation depends on having a negative sign attached to β .

Conversely, any reactor in which the power level is statically unstable, that is, a reactor which has a positive power coefficient, is characterized by a power curve which changes with time according to an exponential relationship

$$e^{\alpha_{n2} t}$$

Again α_{n2} is related to the size of the positive coefficient of reactivity, and to the power density in the reactor. Because of the phase lag caused by the finite time required to transfer heat, a physical distance between the energy source in the reactor and that part of the reactor which is responsible for the existence of a positive coefficient, results in a modification which permits a frequency, ω , and an accompanying damping factor, β , which may be positive or negative.

Figure VIII-1 shows, for a wide range of values of the distance, ξ , where, for negative coefficients of reactivity,

$$\xi = \sqrt{\frac{\omega_{n2}}{2D}} \times \quad \text{VIII-(2)}$$

and for positive coefficients

$$\xi = \sqrt{\frac{\alpha_{n2}}{2D}} \times \quad \text{VIII-(3)}$$

the frequency ratios and damping ratios which would apply. It is apparent that any situation which has a negative damping coefficient is not of importance because the oscillation will die out unless some forcing function exists.

Figure VIII-2 shows, for the same range of ξ , the combinations of frequency and positive damping which remain, after cases involving negative damping are discarded.

The nuclear natural frequency, ω_{n2} , of a particular, stable nuclear reactor is proportional to the square root of its operating power level, P .

$$\omega_{n2} \propto \sqrt{P}$$

Figure VIII-1. Oscillation characteristics as a function of the heat transfer distance.

Figure VIII-2. Oscillation characteristics (for positive damping) as a function of the heat transfer distance.

Therefore, as the reactor is brought up to operating power, its nuclear natural frequency covers a range from zero to some maximum value. Is there, within this range of nuclear frequencies, a region which may be particularly susceptible to augmentation of power oscillation because of the heat transfer effect?

A nuclear reactor can be characterized also by other vibratory natural frequencies, such as those associated with mechanical or acoustic oscillations in the reactor structure. It can be shown that⁽²¹⁾ when the nuclear natural frequency, increasing as the power level of the reactor is raised, becomes of the same order as one of these mechanical frequencies, a coupling may occur in which unstable oscillations arise. Anything which increases the nuclear frequency of a particular reactor tends toward the possibility of such an unstable coupling, while anything which decreases the natural frequency would appear to be in the direction of stability. It can be seen in Figure VIII-2 (or Figure VIII-1) that, in all cases, the effect of heat transfer is to decrease the frequency (for all cases, $\omega/\omega_{n2} < 1$). The greater the value of the distance from the fuel element, ξ , the greater is the decrease in frequency. Therefore, it would be expected that any possible instability would appear at low, rather than at high values of ξ . It appears then that only the curves designated by the number, 1, in Figure VIII-2 need be considered, and these only for low values of ξ . These curves are shown repeated in Figure VIII-3, for values

$$\xi < 1.$$

Now, what interpretation is to be made of the analysis represented by Figure VIII-3? Suppose a reactor exists having a nuclear natural frequency, ω_{n2} , and a dimensionless heat transfer distance, $\xi = 0.2$. The value of ω/ω_{n2} then is 0.90, so that the reactor would tend to have an oscillation at a frequency of $0.90 \omega_{n2}$, particularly if this frequency were to coincide with one of the mechanical or acoustical modes of vibration of the reactor. The ratio of β/ω is + 0.10, so that the amplitude of vibration will be increased by about 90 percent per cycle. This situation would correspond to a runaway of reactor power, with the heat transfer process providing the mechanism for instability, in an oscillatory mode of increasing amplitude.

It appears that, in the design of reactors, particularly compact, high-power density reactors, one should consider the possibility of oscillatory power instabilities,

Figure VIII-3. First term, damping and frequency ratios versus ξ .

where the heat transfer process provides the mechanism for instability. Based on the analysis presented in this report it is concluded that the transient behavior of a nuclear reactor may be significantly affected by the finite time required for heat to be transferred within the reactor, from the regions at which it is generated in fuel elements to regions in which changes in reactivity are affected by temperature, such as the moderator or coolant. Specifically the finite time required for heat transfer may provide a mechanism by which power oscillations are made unstable.

It is recommended that analyses be made of existing and planned reactors, particularly where power oscillations are known to occur, to find whether the heat-transfer mechanism considered in this report may be operative. It is also recommended that a study be carried out to determine whether, in the design of a nuclear reactor, it may be possible to use the heat transfer process to assure that any oscillations which appear may be damped out.

IX. RESULTS AND CONCLUSIONS

Results and conclusions for the high-temperature liquid-metal research conducted in the past year (Phase III) are listed below:

1. A boiling heat transfer analysis has been made for alternate slugs of liquid and vapor that are flowing through a boiler tube. Both viscous and turbulent flow phenomena were considered. The solutions are applicable to linear, forced flow boiling.
2. A two-phase flow frictional pressure drop analysis has been made for alternate slugs of liquid and vapor that flow through a boiler tube; local and overall pressure drops have been expressed in dimensionless form. The results are applicable to linear, forced flow boiling.
3. An analysis has been made of helical, forced flow pressure drop for single phase flow in tubes. The results can be used to predict frictional pressure drop in the superheated region of a boiler tube under helical flow conditions.
4. An equation has been derived for convective heat transfer in helical, single phase flow; the analysis accounts for improved heat transfer as a result of both forced and free convection phenomena. The results can be used to predict heat transfer in the superheated region of a boiler tube under helical flow conditions.
5. The stationary droplet vaporization model has been extended to the forced flow case. The resulting equations can be used to estimate boiling heat transfer in the initial region of a boiler tube with helical flow where droplets or rivulets exist at the wall.
6. Experimental forced flow boiling mercury heat transfer and pressure drop measurements were made under linear flow conditions. Several different boiling test section geometries and heating methods were used. The results were in general agreement with each other as well as analytical predictions.

7. Experimental forced flow boiling mercury heat transfer and pressure drop measurements have been made under helical flow conditions. Several different boiling test section geometries and heating methods were used. The pitch to diameter ratio was about 4. The asymptotic bounds of the experimental heat transfer data were in good agreement with predictions.
8. Forced flow boiling mercury heat transfer and pressure drop measurements in linear and helical flow have been completed. Significant improvements in heat transfer and exit vapor quality were demonstrated with helical flow boiling. Helical flow pressure drop measurements were observed to fall only slightly higher than corresponding values in linear flow, thus, a net increase in boiler performance will result with a helical flow mercury boiler.
9. Preliminary forced flow potassium boiling heat transfer measurements were obtained.
10. Droplet vaporization lifetimes were measured and compiled for mercury, Freon, and benzene. In the film boiling region, the experimental lifetimes were in general agreement with predicted values from the previously developed theory. Preliminary vaporization lifetime measurements were also obtained for potassium droplets.
11. Liquid-metal condensation analyses were made for idealized forced flow condensers. The results were used in the experimental potassium boiling and condensing system.
12. Forced flow Freon boiling measurements have been made; six different fluid flow regimes were identified and quantitative heat transfer conductances were obtained as a function of vapor quality and flow rate. The results can be used to interpret more completely some of the complex phenomena in forced flow boiling.
13. Comparisons have been made of some of the experimental forced flow boiling liquid-metal heat transfer data found in the literature; similarities and differences were pointed out.

14. The thermodynamic properties of potassium vapor in the superheated region were extended analytically to higher temperature.
15. A kinetics study was made of the effect of a temperature perturbation at the coolant channel wall on the reactor criticality; the phase lag and amplitude reduction in the fuel region were considered.

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XI. RESEARCH AND DEVELOPMENT TO BE UNDERTAKEN NEXT YEAR

A. Analytical Studies

Further derivations to improve the fog flow model will be made to include a more accurate accounting of the axial variation in droplet distribution for linear, as well as helical flow (numerical solutions). Also, a more sophisticated heat sink term will be utilized. The superheated liquid annulus model for rotational flow will be extended with the hope of accounting for the critical film thickness at which droplets or rivulets are formed. Some liquid-metal condensation analyses will also be conducted.

B. Experimental Studies

The main forced-flow boiling experiments for potassium will be completed for the 0 to 100 per cent vapor quality and superheat region. The helical flow geometries will include single and double fluted helices. In addition to the boiling results, condensing measurements and experimental wall temperature fluctuation information in the mercury and potassium systems will be presented.

C. Report Covering Work to Date

A report covering the results of the research program for Phases I, II, III and IV will be prepared. It will also contain conclusions and recommendations on the application of high-acceleration or helical flow in mercury and potassium systems for Rankine cycle power plants.

XII. APPENDICES

APPENDIX A - DERIVATION OF CONDENSING COEFFICIENT EQUATION

In this part the condensing conductance is derived for a constant heat flux vertical condenser tube having a varying internal downward vapor velocity which results from the axial change in quality due to condensation. This derivation is based on the following postulates:

1. The major resistance to the radial flow of heat is offered by the condensate layer.
2. The condensate layer flow is laminar.
3. The radial heat flux in the tube is invariant with axial position.
4. There is no subcooling of the condensate.
5. The momentum forces are negligible.
6. The fluid properties are invariant.
7. The condensate layer thickness is small compared to the tube diameter.

Guided by the above postulates, one can express two equations for the differential radial condensate flow (Reference 1). One equation results from a balance of forces and is given by

$$d\omega' = \frac{\pi D \gamma^2}{\mu} \left(Y^2 + \frac{3bY}{4} \right) dY, \quad (1)$$

where,

$$b = \frac{4f}{3\gamma} \frac{\rho_v V_v^2}{2}.$$

The other equation results from a heat balance and is given by

$$d\omega' = \frac{k \pi D \Delta T_f}{\lambda Y} d\ell. \quad (2)$$

Equations (1) and (2) may be equated to yield the following equation:

$$dY \frac{\pi D \gamma^2}{\mu} \left(Y^2 + \frac{3bY}{4} \right) = \frac{k \pi D \Delta T_f}{\lambda Y} d\ell \quad (3)$$

Since this derivation postulates a constant heat flux, q/A , the following relation must hold:

$$\frac{k \Delta T_f}{Y} = \text{constant} = q/A \quad .$$

Substituting this later relation into Equation (3) yields

$$\frac{dY \pi D \gamma^2}{\mu} \left(Y^2 + \frac{3bY}{4} \right) = (q/A) \frac{\pi D d\ell}{\lambda} \quad (4)$$

Integrating Equation (4) yields

$$\frac{\pi D \gamma^2}{\mu} \left(\frac{Y^3}{3} + \frac{3bY^2}{8} \right) = \frac{\pi D (q/A)}{\lambda} \ell \quad (5)$$

Equation (5) may be expressed by

$$\frac{\ell}{a_1} = Y^3 + b_1 Y^2 \quad , \quad (6)$$

where,

$$a_1 = \frac{\lambda \gamma^2}{3\mu (q/A)} \quad , \quad (7)$$

$$b_1 = \frac{3}{4} \frac{f \rho_v V_v^2}{\gamma} \quad . \quad (8)$$

The factor b was evaluated for ranges of the values of its constituent parameters, the selected ranges being typical of liquid metal condensing systems. From this evaluation it was found that for liquid metals as well as some other liquids

$$\frac{Y^3}{3} \ll \frac{3bY^2}{8} \quad .$$

Therefore, Equation (6) may be approximated by

$$\frac{\ell}{a_1} \cong b_1 Y^2 \quad , \text{ or,} \quad (9)$$

$$Y = \sqrt{\frac{\ell}{a_1 b_1}} \quad . \quad (10)$$

The local heat transfer conductance, h , may now be expressed by

$$h = \frac{k}{Y} = k \left[\frac{\lambda \gamma f \rho_v V_v^2}{4 \ell \mu (q/A)} \right]^{1/2} \quad (11)$$

By assuming that the condensate layer thickness, Y , is very small in relation to the tube diameter, D , the following relation may be written for the vapor velocity, V_v :

$$V_v = \frac{m_t x}{\rho_v A_c}$$

This latter expression may then be substituted into Equation (11) to yield

$$h = \frac{k}{2} \left(\frac{\lambda \gamma f}{\mu \ell \rho_v} \right)^{1/2} \frac{1}{(q/A)^{1/2}} \frac{m_t x}{A_c} \quad (12)$$

For a constant heat flux condenser which requires length ℓ_o to condense all the vapor, the following expression may be written for the vapor quality, x :

$$x = x_i (1 - \ell/\ell_o)$$

Substituting this latter expression into Equation (12) yields

$$h = k \left(\frac{\lambda \gamma f}{4 \mu \rho_v \ell_o} \right)^{1/2} \frac{1}{(q/A)^{1/2}} \frac{m_t x_i (1 - \ell/\ell_o)}{A_c (\ell/\ell_o)^{1/2}} \quad (13)$$

To find the average heat transfer conductance, $\bar{h}$, over a length of the tube, L , the following integration was performed:

$$\bar{h} = \frac{1}{L} \int_0^L h d\ell$$

This integration yields

$$\bar{h}_{0 \text{ to } L} = k \left(\frac{\lambda \gamma f}{4 \mu \rho_v \ell_o} \right)^{1/2} \frac{1}{(q/A)^{1/2}} \frac{m_t x_i \left[2 - \frac{2}{3} \left(\frac{L}{\ell_o} \right) \right]}{A_c \left(\frac{L}{\ell_o} \right)^{1/2}} \quad (14)$$

APPENDIX B - THERMODYNAMIC VAPOR
PROPERTIES OF Li, Na, Rb, AND Cs

The vapor pressures, mass fractions, enthalpies, specific heats and Mollier diagrams are shown in Figure 1 to 10 for Li and Na. The mass fractions for Rb and Cs are shown in Figures 11 and 12 respectively.

Figure 1. Vapor pressure versus temperature for lithium vapor.

Figure 2. Composition of lithium vapor (vapor in chemical equilibrium).

Figure 3. Enthalpy of lithium vapor (vapor in chemical equilibrium between Li and Li₂).

Figure 4. Specific heat of lithium vapor (vapor in chemical equilibrium between Li_2 and Li).

Figure 5. Mollier diagram of superheated lithium vapor.

Figure 6. Vapor pressure versus temperature for sodium vapor.

Figure 7. Composition of sodium vapor (vapor in chemical equilibrium).

Figure 8. Enthalpy of sodium vapor (vapor in chemical equilibrium between Na_2 and Na).

Figure 9. Specific heat of sodium vapor, (vapor in chemical equilibrium between Na_2 and Na).

Figure 10. Mollier diagram of superheated sodium vapor.

Figure 11. Composition of rubidium vapor (vapor in chemical equilibrium).

Figure 12. Composition of cesium vapor (vapor in chemical equilibrium).

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APPENDIX C - EFFECTS OF HEAT TRANSFER ON REACTOR KINETICS

1. The Dynamic Equation for Reactor Kinetics

The equation which expresses the thermodynamic response of the average density of a reactor material to transient changes in reactor power can be written⁽¹⁾,

$$\frac{1}{\omega_m^2} \left(\frac{d^3 \gamma_1}{dt^3} + C \frac{d^2 \gamma_1}{dt^2} \right) + \frac{d\gamma_1}{dt} + \frac{\alpha}{C_p V} (P - P_0) = 0 \quad (1)$$

where ω_m is the mechanical (or acoustic) frequency of the reactor as a vibrating system, C is a mechanical damping factor, γ_1 is the transient variation in average density, P is the transient reactor power measured as a departure from the steady-state power P_0 , α is the volume coefficient of thermal expansion of the reactor material, and V represents the total volume of the reactor core.

The power, P , is related to the change in density, γ_1 , also because of the dependence of the reactivity in the reactor on the density of its materials. If the reactor power increases, this results in changes in the temperatures of the components of the reactor. The excess power which is generated in a fuel element results in an essentially immediate increase in the temperature of the fuel element at the point at which the energy is generated. By contrast, a point in the reactor coolant, more remote from the source at which the heat is generated, experiences a change in temperature only at some time after the excess heat was generated in the fuel element, because of the finite time required for the transfer of heat from one point to another by conduction or convection. Let us consider this effect.

A convenient form, for our present purpose, of the equation relating transient reactor power to changes in the reactivity is

$$\frac{1}{\lambda P} \frac{dP}{dt} = (\delta - \beta) + \beta \frac{P_0}{P} \quad (2)$$

where λ is the average cycle frequency for neutrons (from the birth of a neutron in fission until its absorption in creating a new fission), δ is the excess reactivity, and β is the fraction of fission neutrons which are delayed in formation for times after fission which are significant. The excess reactivity is related to changes in both the temperature and

the density of the reactor materials, and the temperature and density are of course interrelated by the thermodynamics of the reactor system. We can write for the dependence of reactivity on reactor density*

$$\delta = \delta_o + \alpha_\gamma \gamma_1 \quad (3)$$

where α_γ is the density coefficient of reactivity. Combining these equations gives for dependence of the transient power on density transients

$$\frac{1}{\lambda} \frac{dP}{dt} = (\delta_o + \alpha_\gamma \gamma_1 - \beta) P + \beta P_o \quad (4)$$

Assuming δ_o independent of time, and differentiating

$$\frac{1}{\lambda} \frac{d^2(\ln P)}{dt^2} = \alpha_\gamma \frac{d\gamma_1}{dt} - \beta \frac{P_o}{P^2} \frac{dP}{dt} \quad (5)$$

In linearized form

$$\frac{1}{\lambda} \frac{d^2 P}{dt^2} + \beta \frac{dP}{dt} = \alpha_\gamma P_o \frac{d\gamma_1}{dt} \quad (6)$$

Equation (1) and (6) are to be combined to eliminate the term γ_1 . This combination must be carried out recognizing that, because of the finite time required to transfer heat by conduction from the fuel element in which the heat is generated to the part of the reactor in which the temperature coefficient is effective, a phase lag and a reduction in temperature are involved. This can be accomplished by specifying a time lag, τ , and a reduction in amplitude by a factor, r , so that Equation (1) must be written

$$\frac{1}{\omega_m^2} \left(\frac{d^3 \gamma_1}{dt^3} + C \frac{d^2 \gamma_1}{dt^2} \right) + \frac{d\gamma_1}{dt} + \frac{\alpha r}{C_p V} \left\{ P(t - \tau) - P_o \right\} = 0 \quad (7)$$

Combining this with Equation (6) to eliminate γ_1 , gives

*A term involving a temperature coefficient of reactivity, not dependent on density changes (for instance, the Doppler coefficient) has been omitted, because its inclusion does not change significantly the results of this analysis. The Doppler coefficient, depending as it does on the temperature, and not the density, of the fuel element, is immediate in its effect, not being subject to mechanical inertia of lags due to heat transfer.

$$\frac{1}{\omega_m^2} \left\{ \frac{d^4 \rho}{dt^4} + (C + B) \frac{d^3 \rho}{dt^3} \right\} + \frac{d^2 \rho}{dt^2} + B \frac{d\rho}{dt} + r \frac{\alpha \alpha_\gamma \lambda P_0}{C_p V} \rho(t - \tau) = 0 \quad (8)$$

where

$$\rho = \frac{P}{P_0} - 1$$

and

$$B = \beta \lambda$$

The last term in this equation has the dimensions of a square of frequency, and depends on the nuclear characteristics of the reactor materials and the operating power level. Let

$$\omega_{n2}^2 = \frac{\alpha \alpha_\gamma \lambda P_0}{C_p V}$$

where the sign of α_γ becomes negative (corresponding to a positive power coefficient of reactivity) it is necessary, in ordinary to avoid imaginary numbers, to define

$$\alpha_{n2} = i\omega_{n2}$$

and to reverse the sign of the last term in the transient operation. Then

$$\frac{1}{\omega_m^2} \left\{ \frac{d^4 \rho}{dt^4} + (C + B) \frac{d^3 \rho}{dt^3} \right\} + \frac{d^2 \rho}{dt^2} + B \frac{d\rho}{dt} + r\omega_{n2}^2 \rho(t - \tau) = 0 \quad (9)$$

To investigate the stability of this equation we let

$$\rho = \rho_0 e^{St}$$

where, in general, the term S is a complex number. The criterion for stability of this equation is that the complex number, S , must not have a positive real part. Substituting this expression into Equation (9) gives for the stability equation

$$\frac{1}{\omega_m^2} \left\{ S^4 + (C + B) S^3 \right\} + S^2 + B S + r\omega_{n2}^2 e^{-S\tau} = 0 \quad (10)$$

In order to solve Equation (10) it is necessary to form an expression for the magnitude of the term

$$e^{-S\tau}$$

For this purpose it is necessary to evaluate the time lag caused by the process of transferring heat by conduction.

2. Time Lag Due to Heat Transfer

Heat transferred by conduction in a solid substance according to the equation*
(for linear heat transfer)

$$D \frac{\partial^2 \theta}{\partial x^2} = \frac{\partial \theta}{\partial t} \quad (11)$$

where D is the coefficient of thermal diffusivity for the material through which heat is being transferred, and θ is the difference between the actual temperature at a point and a reference equilibrium temperature. This equation can be solved by a separation of variables. Let

$$\theta = X(x) \cdot T(t)$$

Substituting this into Equation (11) and separating terms involving space from those involving time

$$\frac{D}{X} \frac{d^2 X}{dx^2} = \frac{1}{T} \frac{dT}{dt} = \beta \quad (12)$$

where β may be a positive or negative, real or imaginary number. Solving the second of these equations

$$T = e^{\beta t}$$

Solving the first equation

$$X = e^{\pm \sqrt{\frac{\beta}{D}} x} \quad (13)$$

The solutions to Equation (11) are then of the form

$$\theta = e^{\beta t} \left(A e^{\sqrt{\frac{\beta}{D}} x} + B e^{-\sqrt{\frac{\beta}{D}} x} \right) \quad (14)$$

This equation is subject to the spatial boundary condition that at large distance from the origin the value of θ must be finite, so that the positive real parts of the spatial exponents must disappear. Within the above limitation the solution, subject to appropriate boundary conditions at $t = 0$ and $x = 0$, can be expressed in terms like

$$\theta = \sum_{\ell=1}^{\infty} A_{\ell} e^{\left(\beta_{\ell} t - \sqrt{\frac{\beta_{\ell}}{D}} x \right)} + \sum_{m=1}^{\infty} A_m e^{-\beta_m t} \cos \sqrt{\frac{\beta_m}{D}} x + \sum_{n=1}^{\infty} A_n e^{-\sqrt{\frac{\beta_n}{2D}} x} \cos \left(\beta_n t - \sqrt{\frac{\beta_n}{2D}} x \right) \quad (15)$$

*A more detailed consideration is required to account for the time delay required to transfer heat by combined conduction and convection.

We can consider the above terms on the right hand side one by one. The first represents the exponential increase with time and the exponential decrease with distance from a fuel element of the reactor temperature in a runaway. The phase lag is

$$\lambda_l = \sqrt{\frac{1}{\beta_l D}} x \quad (16)$$

The second term represents the distribution in space and time around a fuel element during reactor shutdown. The third term represents a steady oscillation of the temperature at the boundary of a fuel element as it is propagated out into the reactor material. The phase angle due to heat transfer is

$$\lambda_n = \sqrt{\frac{1}{2\beta_n D}} x \quad (17)$$

Another solution, of direct interest here, is

$$\theta = Ae^{(\gamma t - \beta x)} \cos(\omega t - nx) \quad (18)$$

subject to the conditions

$$\gamma = D(\beta^2 - n^2) \quad (19)$$

and

$$\omega = 2D\beta n \quad (20)$$

This solution represents the temperature distribution around a fuel element whose surface temperature is being oscillated at a frequency ω , and with a magnitude which may be increasing (unstable power oscillation) or decreasing (decaying power oscillation). The phase angle for the exponential increase or decrease is

$$\lambda_1 = \frac{\beta x}{\gamma} \quad (21)$$

and for the oscillation is

$$\lambda_2 = \frac{nx}{\omega} \quad (22)$$

When $\gamma = 0$ (steady oscillation), this equation becomes identical to the third term of the previous solution, with

$$\beta = n = \sqrt{\frac{\omega}{2D}} \quad (22)$$

and

$$\lambda_1 = \infty \quad (23)$$

$$\lambda_2 = \sqrt{\frac{1}{2\omega D}} x \quad (24)$$

Apparently the problem of the time lag due to heat transfer can be handled by considering that any transient temperature change in a fuel element, x , at a time, τ , later, where

$$\tau = \sqrt{\frac{1}{2\omega D}} x \quad (25)$$

and with a reduced magnitude by a factor, r , due to the exponential decay with distance, where approximately

$$r = e^{-\sqrt{\frac{\omega}{2D}} x} \quad (26)$$

It is desired now to find the effects of this phase lag and reduced temperature effect. Combining the above equation to eliminate x

$$r = e^{-\omega\tau} \quad (27)$$

3. The Effect of Heat Transfer on Reactor Transients

The effects of heat transfer on reactor transients will be considered for three different situations, namely 1) a negative power coefficient, 2) a positive power coefficient, and 3) combined coefficients with different time lags.

Negative Power Coefficient

A solution to Equation (9), for the transient change in reactor power, as affected by the time required for heat transfer to take place, can be found rather easily for one special case in which damping effects, both mechanical and neutronic, and mechanical inertia in the reactor are neglected. For this case Equation (9) becomes

$$\frac{d^2 \rho}{dt^2} + \omega_n^2 \rho(t - \tau) = 0 \quad (28)$$

where $\omega_n^2 = r\omega^2$. The solution can then be expressed in the form

$$\rho = Ae^{\beta t} \cos \omega t + \frac{B}{\omega} e^{\beta t} \sin \omega t \quad (29)$$

where β and ω are taken to be real numbers and a positive value is always assigned to ω . The form of Equation (29) is such that in the limit, as $\omega \rightarrow 0$, the equation becomes

$$\rho = Ce^{\beta t} \quad (30)$$

Substituting the expression for ρ from Equation (29) into Equation (28) gives the following conditions on β and ω

$$\beta^2 - \omega^2 = -\omega_n^2 e^{-\beta \tau} \cos \omega \tau \quad (31)$$

and

$$2\beta\omega = \omega_n^2 e^{-\beta \tau} \sin \omega \tau \quad (32)$$

Taking the ratio of the terms in Equation (32) to those in Equation (31)

$$\frac{2\beta\omega}{\beta^2 - \omega^2} = -\tan \omega \tau \quad (33)$$

Solving for the ratio, β/ω

$$\frac{\beta}{\omega} = \frac{1 - \cos \omega \tau}{\sin \omega \tau} \quad (34)$$

Substituting this value into Equation (32) and solving for ω/ω_n

$$\frac{\omega}{\omega_n} = e^{-\frac{\beta \tau}{2}} \frac{|\sin \omega \tau|}{\sqrt{2(1 - \cos \omega \tau)}} \quad (35)$$

From Equation (27) and (28)

$$\frac{\omega_n}{\omega_{n2}} = e^{-\frac{\omega \tau}{2}} \quad (36)$$

The ratio, ω/ω_{n2} , is then given by

$$\frac{\omega}{\omega_{n2}} = \frac{\omega}{\omega_n} \frac{\omega_n}{\omega_{n2}} \quad (37)$$

Let us define a dimensionless distance from the point at which energy is generated, as

$$\xi = \sqrt{\frac{\omega}{n_2}} \frac{x}{2D} \quad (38)$$

From Equation (25), then

$$\xi = \frac{\omega \tau}{\sqrt{\frac{\omega}{n_2}}} \quad (39)$$

The quantities of most importance here are β/ω , ω/ω_{n2} , and ξ . The quantity β/ω gives a direct measure of the rate at which the amplitude of an oscillation in power is increasing or decreasing. Let us represent the transient power at time t by

$$\rho = \rho_0 e^{\beta t} \sin \omega t \quad (40)$$

At a time later by one cyclic period, namely

$$t + \frac{2\pi}{\omega} \quad (41)$$

the power is given by

$$\rho \left(t + \frac{2\pi}{\omega} \right) = \rho_0 e^{\beta t} e^{\frac{2\pi\beta}{\omega}} \sin \omega t \quad (42)$$

The ratio of amplitudes is then

$$\frac{\rho \left(t + \frac{2\pi}{\omega} \right)}{\rho} = e^{\frac{2\pi\beta}{\omega}} \quad (43)$$

The ratio, ω/ω_{n2} , is important because it relates the actual frequency of oscillation, as determined by heat transfer considerations to the nuclear natural frequency, as defined in Equation (9).

It is desired here to obtain sufficient analytical data to plot β/ω and ω/ω_{n2} as functions of ξ . The procedure used was to assign values to $\omega\tau$ in Equations (34), (35), (36), and (37) and to calculate the quantities desired at these values of $\omega\tau$. A partial tabulation of the results is shown in Table 1.

The results summarized in Table 1 are shown in Figures VIII-1, VIII-2, and VIII-3, where β/ω and ω/ω_{n2} are plotted as functions of ξ .

It is interesting to note the repetitive nature of some of the characteristics of β/ω and ω/ω_{n2} shown in Table 1, as particular values of $\omega\tau$ are increased by the addition of multiples of 2π .

Positive Power Coefficient

With a positive sign attached to the power coefficient of reactivity Equation (9) becomes

$$\frac{d^2 \rho}{dt^2} - \alpha_n^2 \rho(t-\tau) = 0 \quad (44)$$

The solution can again be written in the form

$$\rho = Ae^{\beta t} \cos \omega t \quad (45)$$

which provides the condition equations

$$\beta^2 - \omega^2 = \alpha_n^2 e^{-\beta\tau} \cos \omega\tau$$

and

$$2\beta\omega = -\alpha_n^2 e^{-\beta\tau} \sin \omega\tau \quad (46)$$

From these equations, the damping ratio is

$$\frac{\beta}{\omega} = -\frac{\cos \omega\tau + 1}{\sin \omega\tau} \quad (47)$$

The frequency ratio is

$$\frac{\omega}{\alpha_{n2}} = e^{-\frac{\beta\tau}{2}} \frac{|\sin \omega\tau|}{\sqrt{2(\cos \omega\tau + 1)}} \quad (48)$$

with

$$\frac{\alpha_n}{\alpha_{n2}} = e^{-\frac{\omega\tau}{2}} \quad (49)$$

so that

$$\frac{\omega}{\alpha_{n2}} = \frac{\omega}{\alpha_n} \frac{\alpha_n}{\alpha_{n2}} \quad (50)$$

Table 1. CALCULATION FOR NEGATIVE COEFFICIENT

$\frac{\omega\tau}{2\pi}$	β/ω	ω/ω_{n2}	ξ
0	0	1.	0
.125	0.414	0.530	1.08
.250	1.00	0.147	4.10
.375	2.414	0.006,8	28.6
.500	∞	0	∞
.500	$-\infty$	∞	0
.625	-2.414	6.12	1.58
.75	-1.00	0.700	5.62
.875	-0.414	0.184	12.8
1.000	0	0.043	30.4
1.125	0.414	0.006,3	89.
1.250	1.00	0.000,27	467.
1.375	2.414	0.000,--	--
1.500	∞	0	∞
1.500	$-\infty$	∞	0
1.625	-2.414	528.	0.44
1.750	-1.00	0.700	13.1
1.875	-0.414	0.026,7	72.
2.000	0	0.001,9	290.
2.125	0.414	0.000,075	1,540.
2.250	1.00	0.000,--	19,700.
2.375	2.414	0.000,--	--
2.500	∞	0	∞
2.500	$-\infty$	∞	0
2.625	-2.414	139,000.	0.044
2.750	-1.000	0.700	20.7
2.875	-0.414	0.004,65	267.
3.000	0	0.000,082	2,100.

and

$$\xi = \frac{\omega\tau}{\sqrt{\frac{\omega}{\alpha_{n2}}}} \quad (51)$$

where ξ is defined as

$$\xi = \sqrt{\frac{\alpha_{n2}}{2D}} x \quad (52)$$

The results of the solution of these equations are shown in Figures VIII-1, VIII-2, and VIII-3, where β/ω and ω/α_{n2} are plotted as a function of ξ .

An abbreviated set of values, as functions of $\omega\tau$, are shown in Table 2, in which the repetitive nature of certain conditions is again apparent.

Combined Coefficients with Different Time Lags

Let us consider a reactor which is characterized by two negative power coefficients, one being prompt while the other is delayed by heat transfer considerations. Equation (28) can then be written

$$\frac{d^2\rho}{dt^2} + \omega_1^2 \rho(t) + \omega_2^2 \rho(t - \tau) = 0 \quad (53)$$

where ω_1 is the nuclear natural frequency which accompanies the prompt coefficient, while ω_2 is the nuclear frequency, modified by the appropriate heat transfer lag, which accompanies the delayed coefficient. The solution is again taken to be of the form

$$\rho = Ae^{\beta t} \cos \omega t \quad (54)$$

The condition on β and ω is now

$$\beta^2 - \omega^2 + \omega_1^2 + \omega_2^2 e^{-\beta\tau} \cos \omega\tau = 0 \quad (55)$$

and

$$-2\beta\omega + \omega_2^2 e^{-\beta\tau} \sin \omega\tau = 0 \quad (56)$$

From these equations the damping ratio can be written

$$\frac{\beta}{\omega} = \frac{a - \cos \omega\tau}{\sin \omega\tau} \quad (58)$$

where a is determined by the size of the prompt coefficient, ω_1 , and is given by

Table 2. CALCULATION FOR POSITIVE COEFFICIENT

$\frac{\omega\tau}{2\pi}$	β/ω	ω/α_{n2}	ξ
0	$-\infty$	0	0
.25	-1.0	0.707	1.87
.50	0	0.208	6.9
.75	1.0	0.006,5	58.3
1.00	∞	0	∞
1.00	$-\infty$	∞	0
1.25	-1.0	0.707	9.30
1.50	0	0.009	99.4
1.75	1.0	0.000,013	9,630.
2.00	∞	0	∞
2.00	$-\infty$	∞	0
2.25	-1.0	0.707	16.8
2.50	0	0.000,39	796.
2.75	1.0	0.000,000	110,000.
3.00	∞	0	∞

$$a = \sqrt{1 - \left(\frac{\omega_1}{\omega} \sin \omega \tau\right)^2} \quad (59)$$

To avoid imaginary numbers for frequencies, it is required that

$$1 > a > \cos \omega \tau \text{ as } 0 < \frac{\omega_1}{\omega_2} < \infty$$

The frequency ratios are now

$$\frac{\omega}{\omega_2} = e^{-\frac{\beta \tau}{2}} \frac{|\sin \omega \tau|}{\sqrt{2(a - \cos \omega \tau)}} \quad (60)$$

with

$$\frac{\omega_2}{\omega_{n2}} = e^{-\frac{\omega \tau}{2}} \quad (61)$$

and

$$\frac{\omega}{\omega_{n2}} = \frac{\omega_2}{\omega_{n2}} \frac{\omega}{\omega_2} \quad (62)$$

The value of ξ is

$$\xi = \frac{\omega \tau}{\sqrt{\frac{\omega}{\omega_{n2}}}} \quad (63)$$

where by definition

$$\xi = \sqrt{\frac{\omega_{n2}}{2D}} \times \quad (64)$$

The procedure for the solution of these equations is similar to that used for the case of the single negative coefficient, except that now it is necessary to assign values also to "a" as a parameter in addition to $\omega \tau$. A sample calculation, for

$$\omega \tau = (2n + \frac{7}{4})\pi, \quad n = 0, 1, 2 \dots$$

is shown in Table 3 in summary form.

The first column in Table 3, for $a = 0.707$, corresponds to the case where $\omega_2 = 0$, and hence $\xi = 0$. Because there is no heat transfer lag, this column describes a pure sinusoidal oscillation, with $\beta = 0$. The last column, for $a = 1.0$, corresponds to the case where $\omega_1 = 0$, and is therefore identical to the case previously solved where the

Table 3. CALCULATION FOR COMBINED COEFFICIENTS

$$\omega\tau = (2n + \frac{7}{4})\pi$$

0.707	0.8	0.9	1.0
	β/ω			
0	0	-0.130	-0.272	-0.414
1	0	-0.130	-0.272	-0.414
2	0	-0.130	-0.272	-0.414
	ω/ω_{n2}			
0	∞	0.150	0.155	0.184
1	∞	0.008,83	0.015,7	0.026,7
2	∞	0.000,64	0.001,59	0.004,65
	ξ			
0	0	14.2	14.0	12.8
1	0	126.	94.5	72.
2	0	720.	456.	267.

entire negative temperature coefficient was subject to a heat transfer lag.

Rather than present the details of the calculations for the case of the combined coefficients, a summary of the results is given here.

1. The calculated values of β/ω , ω/ω_{n2} , and ξ , at $\omega\tau = (2n)\pi$ and at $\omega\tau = (2n + 1)\pi$ are unaffected by the value of a .

2. For all other values of $\omega\tau$ the magnitudes calculated are affected by the value taken for a , but not the positive or negative signs attached.

3. Because of the above facts, the repetitive nature of the solution, as a function of $\omega\tau$ is preserved, and the general conclusions concerning the effect of heat transfer on stability are unchanged from those based on Figure VIII-3.

It is of interest to consider in more detail the case of the combined coefficients for small values of ξ , corresponding also to small values of the phase angle, $\omega\tau$.

A special treatment can be made of the equations for combined coefficients for small phase angles

$$\omega\tau = \delta \quad (65)$$

where δ is a small number, so that approximately

$$\sin\omega\tau = \delta \quad (66)$$

and

$$\cos\omega\tau = 1 - \frac{\delta^2}{2} \quad (67)$$

Now the approximate expression can be written, for the damping ratio

$$\frac{\beta}{\omega} = \frac{y}{\delta} \quad (68)$$

for the frequency ratios

$$\frac{\omega}{\omega_{n2}} = \frac{(1 - \frac{\delta}{2}) \delta}{\sqrt{2y}} \quad (69)$$

and

$$\frac{\omega_{n1}}{\omega_{n2}} = \frac{(1 - \frac{\delta}{2}) \sqrt{\frac{\delta}{2} - y}}{\sqrt{y}} \quad (70)$$

For the distance from the source of heat

$$\xi = \frac{\sqrt{\delta} \sqrt[4]{2y}}{1 - \frac{\delta}{4}} \quad (71)$$

where

$$y = a - 1 + \frac{\delta^2}{2} \quad (72)$$

To avoid imaginary numbers, y must be held between the limits

$$0 < y < \frac{\delta^2}{2} \quad (73)$$

as

$$\infty > \frac{\omega_{n1}}{\omega_{n2}} > 0 \quad (74)$$

It is desired to investigate the relationship for small angles, δ , of β/ω and ω/ω_{n2} to ξ for constant values of the parameter ω_{n1}/ω_{n2} . From Equation (70), for constant ω_{n1}/ω_{n2} ,

$$y = \frac{(1 - \delta) \delta^2}{2 \left\{ \left(\frac{\omega_{n1}}{\omega_{n2}} \right)^2 + (1 - \delta) \right\}} \quad (75)$$

Now Equation (68) can be written

$$\frac{\beta}{\omega} = \frac{(1 - \delta) \delta}{2 \left\{ \left(\frac{\omega_{n1}}{\omega_{n2}} \right)^2 + (1 - \delta) \right\}} \quad (76)$$

Equation (69) becomes

$$\frac{\omega}{\omega_{n2}} = \sqrt{\left(\frac{\omega_{n1}}{\omega_{n2}} \right)^2 + (1 - \delta)} \quad (77)$$

Equation (71) becomes

$$\xi = \frac{\delta}{4 \sqrt{\left(\frac{\omega_{n1}}{\omega_{n2}} \right)^2 + (1 - \delta)}} \quad (78)$$

Approximately

$$\frac{\beta}{\omega} = \frac{\delta}{2\Omega^2} \left\{ 1 - \delta \left(1 - \frac{1}{\Omega^2} \right) \right\} \quad (79)$$

$$\frac{\omega}{\omega_{n2}} = \Omega \left(1 - \frac{\delta}{2\Omega^2} \right) \quad (80)$$

$$\xi = \frac{\delta}{\sqrt{\Omega}} \left(1 + \frac{\delta}{4\Omega^2} \right) \quad (81)$$

where

$$\Omega^2 = \left(\frac{\omega_{n1}}{\omega_{n2}} \right)^2 + 1 \quad (82)$$

Solving the last of these equations for δ in terms of ξ , gives

$$\delta = \frac{\sqrt{\Omega} \xi}{1 + \frac{\sqrt{\Omega}}{4\Omega^2} \xi} \quad (83)$$

or, for small values of δ

$$\delta = \sqrt{\Omega} \xi \quad (84)$$

Combining Equations (79) and (84) to eliminate δ , approximately

$$\frac{\beta}{\omega} = \frac{\sqrt{\Omega}}{2\Omega^2} \xi \quad (85)$$

Combining Equations (80) and (84)

$$\frac{\omega}{\omega_{n2}} = \Omega \left(1 - \frac{\sqrt{\Omega}}{2\Omega^2} \xi \right) \quad (86)$$

It is necessary to establish limits on the validity of Equations (85) and (86). In the approximations made, a value of $\delta = 0.5$ results in about a four percent error in the evaluation of $(\sin \omega \tau)$, about the same in the evaluation of $\left(e^{\frac{\omega \tau}{2}} \right)$, and less than two percent in $(1 - \cos \omega \tau)$. From Equation (84) holding δ within these limits requires that ξ be held to the limits

$$\xi < \frac{0.5}{\sqrt{\Omega}} \quad (87)$$

In Figure 1 the values of β/ω and ω/ω_{n2} are shown as functions of ξ according to Equations (85) and (86) with the limits according to Equation (87) indicated. As is

Figure 1. Functions β/ω and ω/ω_{n2} vs. ξ [Equations (85) and (86)].

indicated, in the limit, when $\Omega = \infty$, the only values possible are

$$\xi = 0$$

$$\frac{\beta}{\omega} = 0$$

$$\frac{\omega}{\omega_{n2}} = \Omega$$

because this limit corresponds to the assumption that the entire negative coefficient of reactivity is prompt (no heat transfer effect). Inspection of Figure 1 shows that its results are consistent with those of Figure VIII-3 for the special case in which $\Omega = 1$, corresponding to

$$\frac{\omega_{n1}}{\omega_{n2}} = 0$$

SYMBOLS

a	Equation (59)
A	constant
A_ℓ, A_m, A_n	series coefficients
B	$\beta \lambda$, a constant
C	mechanical damping factor, a constant in Equation (30)
C_p	specific heat of the reactor material
D	coefficient of thermal diffusivity
i	$\sqrt{-1}$
P	transient reactor power
P_o	steady state reactor power
r	temperature amplitude reduction factor
S	complex number
t	time
m, n	integers
V	total volume of the reactor cove

x	distance
α	volume coefficient of thermal expansion of the reactor material
α_γ	density coefficient of reactivity
α_{n2}	$i\omega_{n2}$
β	damping factor, fraction of fission neutrons which are delayed in formation for time after fission which are significant, constant in Equations (12), (29), and (45)
$\beta_\ell, \beta_m, \beta_n$	series coefficients
γ_1	transient variation in average density
δ	excess reactivity, small number in Equation (65)
δ_0	steady state excess reactivity
θ	difference between the actual temperature at a point and a reference equilibrium temperature
λ	average cycle frequency for neutrons (from the birth of a neutron in fission until its absorption in creating a new fission)
λ_ℓ	phase lag due to heat transfer Equation (16)
λ_n	phase angle due to heat transfer Equation (17)
λ_1	phase angle Equation (21)
λ_2	phase angle Equation (22)
ρ	$(P/P_0) - 1$
τ	time lag
Ω	Equation (82)
ω	frequency
ω_m	mechanical or acoustic frequency of the reactor as a vibrating system
ω_1	nuclear natural frequency which accompanies the prompt negative power coefficient
ω_2	nuclear natural frequency which accompanies the delayed negative power coefficient
ω_{n2}	natural nuclear frequency

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Thompson, A.S., "Study of Reactor Kinetics, " ASME, Paper 62-WA-218, November 1962.

XIII. NOMENCLATURE

Letters

A	area and monatomic molecule
A_2	diatomic molecule
A_c	flow cross sectional area of boiler tube
A_d	heat transfer area of the droplets
a	acceleration of gravity
A_B	heat transfer area of the boiler tube
A_H	heat transfer area of the heater
$\bar{A}_l$	mean liquid cross-sectional area
B	see Equation II-(24)
C_p°	equilibrium specific heat
D	tube diameter and thermal diffusivity
D_h	equivalent diameter for helical flow system
D_l	equivalent diameter for linear flow system
D_t	equivalent diameter for tangential flow in tubes
e	base of natural logarithms
G	total weight flow rate of the vapor beneath the drop
g_c	gravitational constant
h	heat transfer conductance
$\bar{h}$	average heat transfer conductance
H	ratio of heat transfer conductances, see Equation II-(7)
h_l	liquid slug conductance
H_o°	absolute enthalpy at $T = 0^\circ$ Kelvin
H_T°	absolute enthalpy at temperature T

h_v	vapor slug conductance
K	constant
k	thermal conductivity of condensate
k_ℓ	thermal conductivity of liquid metal
K_p	equilibrium constant
k_v	thermal conductivity of vapor
L	latent heat of vaporization, axial tube position or total length
L_o	tube length
M	molecular weight
m	mass flow rate and mass of gaseous mixture
n	number of droplets contiguous to tube wall per diameter length of tube
P	pressure
p	helix pitch
P_v	saturation pressure
q	heat transfer rate
R	universal gas constant
r_d	radius of disk shaped droplets
r_{do}	initial radius of disk shaped droplets
S°_T	entropy at temperature T
T	absolute temperature
T_B	temperature of boiler tube
T_H	temperature of heater
T_s	absolute saturation temperature
t	time
t_s	saturation temperature
T_v	saturation temperature

T_w	absolute temperature of the heat transfer surface
t_w	wall temperature
U_{tv}	tangential component of the vapor velocity
$\bar{U}_v$	mean axial vapor velocity
V_v	vapor velocity
V_{vi}	initial vapor velocity
w_ℓ	liquid weight density
W_t	total fluid flow rate (vapor and liquid)
w_v	vapor weight density
W_v	vapor flow rate
x	distance of separation
α_{n2}	a constant related to the magnitude of the positive coefficient of reactivity
β	volumetric expansion coefficient of the coolant and damping factor
γ	condensate density
γ_ℓ	weight density of liquid
γ_v	weight density of vapor
ΔH°_o	energy of reaction at $T = {}^\circ K$
ΔH°_T	energy of reaction at temperature T
Δp_{TPF}	frictional two-phase pressure drop
Δp_{v_o}	frictional pressure drop with vapor flowing alone
$\Delta p_{v_o \text{ exit}}$	frictional pressure drop on basis of exit vapor flow alone
Δt	wall fluid temperature difference
ΔT_f	wall to vapor temperature difference

$\bar{\delta}$	mean thickness of the vapor film under the droplet
θ	difference between the transient and steady state temperatures
θ_e	droplet vaporization lifetime
λ	latent heat of vaporization
μ	condensate viscosity
ρ_l	liquid mass density
ρ_v	vapor mass density
σ	Stefan-Boltzman constant or condensation accommodation coefficient
ω	frequency
ω_{n2}	nuclear natural frequency

Moduli

F	dimensionless factor to allow for radiation interchange between gray surfaces
f	Fanning friction factor
Nu_{forced}	forced convection Nusselt modulus for helical flow system
Nu_{free}	free convection Nusselt modulus for helical flow system
Nu_h	forced convection Nusselt modulus for helical flow system
Nu_l	forced convection Nusselt modulus for linear flow system
Pr	Prandtl modulus
Pr_v	Prandtl modulus for vapor
Re_l	Reynolds modulus for linear flow system
Re_v	Reynolds modulus for vapor
X	mole fraction
x_{exit}	vapor quality at tube exit
y_l	liquid volume fraction
y_v	vapor volume fraction

α	mass fraction
ϵ	emissivity
ϵ_d	emissivity of the drop
ϵ_w	emissivity of heat transfer surface
ζ	Weisbach friction factor
ζ_h	Weisbach friction factor for helical flow
ζ_l	Weisbach friction factor for liquid
ζ_v	Weisbach friction factor for vapor
ζ_{v_o}	Weisbach friction factor for vapor flowing alone
ξ	the dimensionless distance between the energy source in the reactor and that part of the reactor which is responsible for the existence of a positive coefficient of reactivity

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