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TESTS FOR A WEAK NEUTRAL CURRENT IN

$l^{\pm} + N \rightarrow l^{\pm} + \text{anything}$ AT HIGH ENERGY*

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ABSTRACT

Tests for a weak neutral current in high energy inclusive $\ell^\pm - N$ scattering are discussed, where ℓ is a charged lepton and N a nucleon. If the polarization of the incoming lepton or the target nucleon can be varied, or the outgoing lepton's polarization observed, an unambiguous measurable consequence of such a current is parity violation. Some general expressions for parity violating correlations are derived, and their magnitude in particular models is calculated.

Further consequences of such a current are (i) a difference between $\ell^- - N$ and $\ell^+ - N$ cross sections and (ii) a deviation of the $\ell^- - N$ cross section from the form implied by one photon exchange only. Tests for these effects are similarly examined, but an accurate knowledge of higher order electromagnetic contributions to them is necessary before such tests can be implemented.

I. INTRODUCTION

Despite a variety of stimuli for conjecturing their existence, neither leptonic nor hadronic weakly coupled neutral currents have ever been detected¹. For semileptonic processes in particular, the miniscule experimental upper limit² for the decay $K_L^0 \rightarrow \mu^+ \mu^-$ effectively rules out any strangeness - nonconserving neutral current with appreciable coupling to leptons. The upper limit is far less stringent for $\Delta S=0$ semileptonic processes involving neutrinos³. For similar processes involving charged leptons, the upper limits are least severe because allowed electromagnetic effects are usually large enough to mask any sign of weak neutral currents.

In high energy large momentum transfer lepton-nucleon scattering however, weak cross sections can become comparable to electromagnetic ones because of the rapidly decreasing photon propagator. The advent of highly energetic polarized charged lepton beams at N.A.L. (and the possibility of even higher energy at P.E.P. or ISABELLE), together with the interest in deep inelastic $l^\pm - N$ scattering, might make feasible the search for weak neutral currents in such a process.

This paper contains a study of tests for a weak neutral current in the inclusive process

$$l^\pm + N \rightarrow l^\pm + \text{anything} \quad (1.1)$$

at high energy, as well as estimates of the size of possible effects. Since nothing really trustworthy is known about the form or strength of these unobserved currents, I have chosen a simple phenomenological interaction of variable strength (described below) in order to plausibly estimate the size of typical effects.

In what follows, the electromagnetic interaction Lagrangian is given by⁴

$$\mathcal{L}_{e.m.} = -e j_\mu^\gamma A^\mu - e_p j_\mu^x A^\mu \quad (1.2)$$

where e (e_p) is the electron's (proton's) charge, and A^μ the electromagnetic field.

J_μ^δ is the conserved hermitian hadronic electromagnetic current in units of e_p , and

$$j_\mu^\delta = \sum_{\ell=e,\mu} \bar{\Psi}_\ell \gamma_\mu \Psi_\ell \quad (1.3)$$

is the leptonic electromagnetic current. Time reversal invariance (and of course parity conservation) is assumed for the electromagnetic interaction.

For the semileptonic weak interaction, an effective neutral current - current interaction Lagrangian

$$\mathcal{L}_W = \frac{G}{\sqrt{2}} j_\mu^W J_\mu^W \quad (1.4)$$

is used. G is the analogue of the Fermi constant in β - decay, but of unknown strength. Motivated by the charged weak currents, the leptonic weak neutral current is chosen to be

$$j_\mu^W = \sum_{\ell=e,\mu} \bar{\Psi}_\ell \gamma_\mu (1 - \gamma_5) \Psi_\ell \equiv V_\mu - A_\mu \quad (1.5)$$

The hadronic weak neutral current J_μ^W is hermitian, but is otherwise presently left unspecified. $\mathcal{L}_W$ is assumed to be time reversal invariant. The effects of a weak interaction mediated by an intermediate vector boson will be considered when numerical estimates are made.

The presence of a weak neutral current produces characteristic effects which would be absent or suppressed if the interaction were purely electromagnetic. The most unequivocal of these is parity violation. Unless one of the initial particles in (1.1) is polarized, or the polarization of the final lepton is measured, a parity violating correlation cannot be observed. The main focus of this paper is on tests for, and estimates of the parity violating correlations in the cross section when either (i) the initial lepton's polarization is varied, (ii) the target nucleon's polarization is varied, or (iii) the final lepton's polarization is observed.

Tests for effects which, unlike parity violation, are absent only in the one photon

exchange electromagnetic contribution are more ambiguous. The one photon exchange contribution to the rate for (1.1) is independent of lepton charge, and also implies a characteristic energy dependence of the cross section. A weak neutral current destroys these properties, but so does the presence of two photon exchange effects. Tests for J_μ^W based on searching for such effects are also discussed below. Their inherent disadvantage (in comparison with the parity test above) is that without some accurate prior knowledge of the magnitude of two photon exchange effects, their observation is not an unequivocal indicator of such currents.

The paper is arranged as follows. Section II contains the cross section for (1.1) where the incoming lepton is polarized, in the presence of J_μ^W , in the zero lepton mass approximation. It is expressed in terms of nucleon structure functions describing the nucleon - current interaction. Their scaling properties and positivity constraints are considered in Appendix A. The cross section with all lepton mass - dependent terms, and an estimate of their magnitude is in Appendix B.

In section III tests for parity violation using a variable incoming lepton polarization are considered. A plausible form for J_μ^W is chosen, and parity violating effects are calculated in the generalized quark-parton model of Llewellyn - Smith⁵, and in the more specific model of Kuti and Weisskopf⁶. The dependence of such effects on energy, coupling strength and the mass of the hypothetical intermediate vector boson is displayed in a series of graphs. Finally, the case where instead the final lepton's polarization is measured as a test of parity violation is simply related to the effects already considered. Section IV then contains a similar treatment of tests for J_μ^W which utilize either the lepton charge dependence of reaction (1.1), or the deviation of the cross section from the energy dependence of the one photon exchange form. Section V contains the kinematics and cross section for (1.1) in the presence of J_μ^W when the target is polarized. In VI estimates of the parity violation when the target spin is varied are calculated in the Kuti - Weisskopf model. Tables of structure functions in the various models are given in Appendix C.

II. THE CROSS SECTION FOR $l^\pm + N \rightarrow l^\pm + \text{anything}$ WITH A POLARIZED LEPTON BEAM.

The cross section for inclusive polarized lepton - nucleon scattering (1.1) in the presence of both electromagnetic and weak neutral currents is presented below. Only the momentum of the final lepton is observed. The one photon exchange approximation is used for the electromagnetic contribution, and all the results are given in the limit that the lepton has zero mass. (See Appendix B for the cross section with all mass-dependent terms, and estimates of their size.)

Figure 1 displays the lowest order Feynman diagrams for inelastic $l^\pm - N$ scattering with the interactions (1.2) and (1.4). Throughout this paper the following notation is used:

k^μ (k'^μ) = four-momentum of incoming (outgoing) lepton;

P^μ = four-momentum of target nucleon;

m = lepton mass; M = nucleon mass;

$q = k - k'$, the four-momentum transfer;

$Q^2 = -q^2$; $\nu = q \cdot P / M$;

E = energy of the incoming lepton in the frame where the nucleon is at rest (usually the laboratory frame);

α = fine structure constant;

$\beta = \frac{GM^2}{4\pi\sqrt{2}\alpha}$ measures the ratio of weak to electromagnetic coupling;

s^μ = covariant average polarization vector of incoming lepton beam, $s \cdot k = 0$;

λ = average helicity of incoming lepton beam, $|\lambda| \leq 1$;

G_f = Fermi constant in β -decay $= 1.02 \times 10^{-5} M^{-2}$.

Apart from the lepton polarization, the independent variables are E , Q^2 and ν , with $Q^2 \leq 2M\nu \leq 2ME$. We shall always be concerned with the Bjorken limiting region⁷ $Q^2 \gg M^2$.

A. $\ell^- + N \rightarrow \ell^- + \text{anything}$

The cross section for $\ell^- + N \rightarrow \ell^- + \text{anything}$, for a lepton beam of polarization s^μ can be calculated from Figure 1. It can be written

$$d\sigma^- = d\sigma_g^- + d\sigma_Y^- + d\sigma_R^- \quad (2.1)$$

where $d\sigma_g^-$ is the electromagnetic contribution from Figure 1a, $d\sigma_Y^-$ the weak contribution from Figure 1b, and $d\sigma_R^-$ the interference contribution. There are discussed below separately. At high energy, where the lepton mass can be ignored, the spin-dependent part of the cross section in fact depends only upon the helicity λ of the lepton beam - i.e. transversely polarized leptons scatter as if unpolarized.

Electromagnetic contribution $d\sigma_g^-$

Subject to the above approximations, from Figure 1a in the laboratory frame

$$d\sigma_g^- = \frac{\pi e^4}{E q^4} \left[2m^2 j_{\mu\nu}^\gamma W^{\mu\nu}(q, P) \right] \frac{d^3 k'}{(2\pi)^3 k_0} \quad (2.2)$$

where the leptonic contribution is contained in the tensor

$$j_{\mu\nu}^\gamma = \frac{1}{2m^2} \{ k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k \cdot k' - i\lambda \epsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta \} \quad (2.3)$$

and the nucleon's interaction with the virtual photon is described by the tensor

$$W_{\mu\nu}(q, P) = \frac{4\pi^2 P_0}{M} \int d^4 x e^{iq \cdot x} \langle P_m | J_\mu^\gamma(x) J_\nu^\gamma(0) | P_m \rangle. \quad (2.4)$$

The omission of a spin index for the nucleon implies an average over nucleon spin.

$W_{\mu\nu}(q, P)$ depends only upon q^μ and P^μ and is examined in Appendix A.

From (A8),

$$W_{\mu\nu}(q, P) = -W_1 \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + \frac{W_2}{M^2} \left(P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \left(P_\nu - \frac{P \cdot q}{q^2} q_\nu \right) \quad (2.5)$$

where W_1 and W_2 , real scalar functions of q^2 and $q \cdot P$ of dimension $(\text{mass})^{-1}$, are the well known structure functions of deep inelastic $e^- - N$ scattering. In Appendix A the constraints⁸

on W_1 that follow from the semi-positive definite nature of $W_{\mu\nu}(q, P)$, and their behaviour in the Bjorken limit⁷ assuming scale invariance are discussed. W_1 and W_2 have been measured in the SLAC - M.I.T. experiments⁹ on $e^- + N \rightarrow e^- + \text{anything}$; νW_2 and MW_1 are found to scale⁷, and to be consistent with the behaviour $\nu W_2(x) = 2Mx W_1(x)$.

From (2.2), (2.3) and (2.5) one finds

$$\frac{d\sigma_g^-}{dQ^2 d\nu} = \frac{\pi\alpha^2}{E^2 Q^4} [2W_1 Q^2 + W_2 \{4E(E-\nu) - Q^2\}] \quad (2.6)$$

Note that this cross section is independent of lepton spin.

Weak contribution $d\sigma_Y^-$

From Figure 1b one finds

$$d\sigma_Y^- = \frac{\pi G^2}{E} \left[m^2 j_{\mu\nu}^Y Y^{\mu\nu}(q, P) \right] \frac{d^3 k'}{(2\pi)^3 k'_0} \quad (2.7)$$

where the lepton contribution is

$$j_{\mu\nu}^Y = \frac{(1-\lambda)}{m^2} \{ k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k \cdot k' + i\epsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta \} \quad (2.8)$$

The nucleon's interaction with the weak leptonic current is described by

$$Y_{\mu\nu}(q, P) = \frac{4\pi^2 P_0}{M} \int d^4x e^{iq \cdot x} \langle P | J_\mu^W(x) J_\nu^W(0) | P \rangle \quad (2.9)$$

From (A13), the assumed properties of J_μ^W lead to the most general form

$$\begin{aligned} Y_{\mu\nu}(q, P) = & -(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) Y_1 + (P_\mu - \frac{P \cdot q}{q^2} q_\mu)(P_\nu - \frac{P \cdot q}{q^2} q_\nu) \frac{Y_2}{M^2} - i\epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta \frac{Y_3}{2M^2} \\ & + q^\mu q^\nu \frac{Y_4}{M^2} + (P_\mu q_\nu + P_\nu q_\mu) \frac{Y_5}{M^2} \end{aligned} \quad (2.10)$$

where the Y_i are real scalar structure functions depending on q^2 and $q \cdot P$. The positivity constraints on Y_i , and their behaviour in the Bjorken limit assuming scale invariance are discussed in Appendix A. MY_1 , νY_2 and νY_3 are found to scale, with

$$\nu Y_2 = 2Mx Y_1 \quad \text{in the spin - } \frac{1}{2} \text{ parton models of section III.}$$

(2.7), (2.8) and (2.10) lead to

$$\frac{d^2\delta_Y^-}{dQ^2 dy} = \frac{(1-\lambda)G^2}{16\pi E^2} \left[2Y_1 Q^2 + Y_2 \{4E(E-y) - Q^2\} - Y_3 \frac{Q^2(2E-y)}{M} \right] \quad (2.11)$$

The vanishing of $d\delta_Y^-$ for a fully polarized right handed lepton beam ($\lambda = 1$) is a simple consequence of the assumed V - A form of the leptonic weak current. Y_4 and Y_5 do not enter the cross section because in the limit of zero lepton mass which we consider, the lepton current is conserved.

Interference contribution $d\delta_R^-$

From the interference between the diagrams of Figures 1a and 1b,

$$d\delta_R^- = -\frac{G\pi E^2}{\sqrt{2}Q^2 E} \left[2M^2 R^{\mu\nu}(q,P) j_{\mu\nu}^R \right] \frac{d^3k'}{(2\pi)^3 k_0} \quad (2.12)$$

The lepton contribution is given by $j_{\mu\nu}^R = \frac{1}{2} j_{\mu\nu}^Y$. The nucleon's interaction with interfering weak and electromagnetic currents is specified by

$$R_{\mu\nu}(q,P) = \frac{4\pi^2 P_0}{M} \int d^4x e^{iq \cdot x} \langle P_{in} | J_\mu^\nu(x) J_\nu^\mu(0) + J_\mu^W(x) J_\nu^W(0) | P_{in} \rangle \quad (2.13)$$

The properties of the currents lead to the general form (A16):

$$R_{\mu\nu}(q,P) = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) R_1 + \left(P_\mu - \frac{P \cdot q}{q^2} q_\mu\right) \left(P_\nu - \frac{P \cdot q}{q^2} q_\nu\right) \frac{R_2}{M^2} - i\epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta \frac{R_3}{2M^2} + \frac{R_5}{M^2} \left(P_\mu q_\nu + P_\nu q_\mu - \frac{2P \cdot q}{q^2} q_\mu q_\nu\right) \quad (2.14)$$

where the $R_i(q^2, q, P)$ are interference structure functions. Positivity inequalities relating R_i to W_i and Y_i , and scaling behaviour in the Bjorken limit are deduced in Appendix A. MR_1 , $\sqrt{s}R_2$ and $\sqrt{s}R_3$ are found to scale, with $\sqrt{s}R_2 = 2MxR_1$ in the spin - $\frac{1}{2}$ parton model of section III.

From (2.12) and (2.13),

$$\frac{d\delta_R^-}{dQ^2 dy} = \frac{(1-\lambda)\alpha G}{4\sqrt{2}Q^2 E^2} \left[2R_1 Q^2 + R_2 \{4E(E-y) - Q^2\} - R_3 \frac{Q^2(2E-y)}{M} \right] \quad (2.15)$$

The cross section $d\sigma^- = d\sigma_{\gamma}^- + d\sigma_R^- + d\sigma_Y^-$.

The dimensionless parameter $\beta = \frac{GM^2}{4\pi\sqrt{2}\alpha}$ is a convenient measure of the ratio of weak to electromagnetic coupling. $\beta Q^2/M^2$ signifies the relative strength of the weak and electromagnetic coupling at momentum transfer Q^2 . For $G = G_f$,

$\beta = 7.9 \times 10^{-5}$. From (2.1), (2.6), (2.11) and (2.15) the cross section for inelastic inclusive $\ell^- N$ scattering for leptons of average helicity λ , in the limit $m = 0$ is

$$\frac{d^2\sigma^-(\lambda)}{dQ^2 d\nu} = \frac{\pi\alpha^2}{E^2 Q^4} \left\{ \begin{aligned} &2W_1 Q^2 + W_2 \{4E(E-\nu) - Q^2\} \\ &+ \left(\frac{\beta Q^2}{M^2}\right)(1-\lambda) \left[2R_1 Q^2 + R_2 \{4E(E-\nu) - Q^2\} - R_3 Q^2 \frac{(2E-\nu)}{M} \right] \\ &+ 2\left(\frac{\beta Q^2}{M^2}\right)^2(1-\lambda) \left[2Y_1 Q^2 + Y_2 \{4E(E-\nu) - Q^2\} - Y_3 Q^2 \frac{(2E-\nu)}{M} \right] \end{aligned} \right\} \quad (2.16)$$

For fixed Q^2 , ν and λ , $\left[\frac{E^2 Q^4}{\pi\alpha^2} \right] \frac{d^2\sigma^-(\lambda)}{dQ^2 d\nu}$ is a second order polynomial in E , whose coefficients are dependent on the dynamical structure functions.

Only three independent functions of Q^2 and ν can thus be measured.

Varying λ allows the extraction of two more.

B. $\ell^+ + N \rightarrow \ell^+ + \text{anything}$.

The cross section for $\ell^+ + N \rightarrow \ell^+ + \text{anything}$ under the same assumptions can be found from (2.16) as follows. Consider the matrix elements of the weak leptonic

current (1.5) between $|l^- \rangle$ states, $\langle l^- | V_\mu - a_\mu | l^- \rangle$. Under charge conjugation for the leptons, one finds $\langle l^- | V_\mu - a_\mu | l^- \rangle = -\langle l^+ | V_\mu + a_\mu | l^+ \rangle$. Since the leptonic tensors $j_{\mu\nu}^S$, $j_{\mu\nu}^V$ and $j_{\mu\nu}^R$ are bilinear in the lepton currents, the effect is simply to change the sign of all axial terms in these tensors. By inspection, this is equivalent to the transformation $\lambda \rightarrow -\lambda$ and $i \rightarrow -i$ in the leptonic tensors. Since the cross sections is real, imaginary leptonic terms always multiply only imaginary hadronic terms, and thus one can instead transform $i \rightarrow -i$ in the structure function representations for the hadronic tensors $W_{\mu\nu}$, $R_{\mu\nu}$ and $Y_{\mu\nu}$. This effectively changes the signs of R_3 and Y_3 .

So the cross section for $l^+ - N$ scattering is obtained by transforming $\lambda \rightarrow -\lambda$ and $R_3, Y_3 \rightarrow -R_3, -Y_3$ respectively in (2.16):

$$\frac{d^2\sigma^+(\lambda)}{dQ^2 d\nu} = \frac{\pi\alpha^2}{E^2 Q^4} \left[\begin{aligned} & 2W_1 Q^2 + W_2 \{4E(E-\nu) - Q^2\} \\ & + \frac{\beta Q^2}{M^2} (1+\lambda) \left[2R_1 Q^2 + R_2 \{4E(E-\nu) - Q^2\} + R_3 Q^2 \frac{(2E-\nu)}{M} \right] \\ & + 2 \left(\frac{\beta Q^2}{M^2} \right)^2 (1+\lambda) \left[2Y_1 Q^2 + Y_2 \{4E(E-\nu) - Q^2\} + Y_3 Q^2 \frac{(2E-\nu)}{M} \right] \end{aligned} \right] \quad (2.17)$$

The vanishing of the weak contributions to the cross section for left handed antileptons ($\lambda = -1$) is of course a consequence of the V - A nature of the leptonic weak current.

III. TEST FOR PARITY VIOLATION USING THE INCOMING LEPTON SPIN

As discussed in section I, a consequence of a weak neutral current would be the measurement of a non - zero value for some parity violating correlation function.

If one considers only longitudinally polarized leptons in the inclusive process of section II, all spin dependent effects are inherently parity violating¹⁰. A suitable test for a weak neutral current is thus to examine the change in $d^2\sigma/dQ^2 dy$ of (2.16) or (2.17) when the lepton helicity is reversed.

For convenience I always deal with a fully polarized ($|\lambda| = 1$) negatively charged lepton beam, and define the suitable "lepton spin asymmetry parameter" for the nucleon

$$a(Q^2, \nu) = \frac{d\sigma^-(+1) - d\sigma^-(-1)}{d\sigma^-(+1) + d\sigma^-(-1)} \quad (3.1)$$

where $d\sigma^-(\lambda)$ is given by (2.16). A test for parity violation and a weak neutral current is to measure non - zero $a(Q^2, \nu)$. From (2.16) one can write

$$a(Q^2, \nu) = -U/D \quad (3.2)$$

where

$$U = \frac{\beta Q^2}{M^2} \left[2R_1 Q^2 + R_2 \{4E(E-\nu) - Q^2\} - R_3 \frac{Q^2}{M} (2E-\nu) \right] + 2 \left(\frac{\beta Q^2}{M^2} \right)^2 \left[2\gamma_1 Q^2 + \gamma_2 \{4E(E-\nu) - Q^2\} - \gamma_3 \frac{Q^2}{M} (2E-\nu) \right] \quad (3.3)$$

and

$$D = 2W_1 Q^2 + W_2 \{4E(E-\nu) - Q^2\} + U$$

For values of Q^2 accessible at N.A.L. energies, say $Q^2 \lesssim 400 \text{ GeV}^2/c^2$, and for $G \lesssim G_f$, we find $\beta Q^2/M^2 \lesssim 0.03$; thus sufficient accuracy is obtained if terms of order β^2 are dropped. In this approximation

$$a(Q^2, \nu) \approx \frac{-\frac{\beta Q^2}{M^2} [2R_1 Q^2 + R_2 \{4E(E-\nu) - Q^2\} - R_3 \frac{Q^2}{M} (2E-\nu)]}{2W_1 Q^2 + W_2 \{4E(E-\nu) - Q^2\}} \quad (3.5)$$

In order to proceed further in estimating $a(Q^2, \nu)$, a reasonable form for J_μ^w must be used. $\Delta S \neq 0$ hadronic neutral currents which couple to leptons are experimentally excluded^{1,2}. A pure isovector weak neutral current

$$J_\mu^w = V_\mu^3 - A_\mu^3 \quad (3.6)$$

is chosen, where V_μ^3 and A_μ^3 are the 3- components of the vector and axial current octets of chiral $SU_3 \times SU_3$.

A model for the nucleon's structure functions is also needed. The Kuti - Weisskopf quark - parton model⁶ provides a very convenient parametrization of much of the available data for deep inelastic weak and electromagnetic processes. Because of the underlying physical picture, it furnishes a plausible way of trying to estimate similar but as yet unobserved effects. It will be used below in conjunction with (3.6) to obtain numerical estimates for $a(Q^2, \nu)$ in the Bjorken limit. The generalized quark - parton model of Llewellyn - Smith⁵ is also used below to predict upper bounds for $a(Q^2, \nu)$, and to predict it directly in terms of the ratio of ν - N and unpolarized ℓ^- - N scattering.

A crucial assumption in all the work below is that the effective weak Lagrangian is still valid in the kinematic region where the hadronic structure functions are assumed to scale.

A. Generalized quark - parton model⁵ predictions for $a(Q^2, \nu)$.

In the particular infinite momentum frame where the parton model is valid, the nucleon consists of arbitrary numbers of quarks and gluons. Let $G_i^N(x)$ be the probability distribution for finding a quark of type "i" in the nucleon N, with the quark's longitudinal momentum a fraction x of the nucleon's momentum. $i = 1, 2, 3$ denote the usual p , n and λ quarks of charge e_i and 3 - component of isotopic spin t_i , and $\bar{i}$ the corresponding antiquarks. The G_i^N are constrained to give the correct nucleon quantum numbers, and to respect SU_2 invariance.

In terms of the quark fields ψ_i ,

$$J_\mu^\sigma = \sum_{i=1}^3 e_i \bar{\psi}_i \gamma_\mu \psi_i \quad (3.8a)$$

$$J_\mu^W = \sum_{i=1}^3 t_i \bar{\psi}_i \gamma_\mu (1 - \gamma_5) \psi_i \quad (3.8b)$$

Within the framework of the parton model with these currents, $W_{\mu\nu}(q, P)$, $R_{\mu\nu}(q, P)$ and $Y_{\mu\nu}(q, P)$ of section II, and their corresponding structure functions, may now be found in terms of the G_i^N . Table 2 in Appendix C contains all the scaling structure functions expressed in the parton model with spin $-\frac{1}{2}$ quarks.

Because of SU_2 invariance, the distribution functions for proton and neutron are related by an isotopic rotation - e.g. $G_1^p = G_2^n$, etc. Thus only the six proton distribution functions G_i^p , $G_{\bar{i}}^p$ are in fact independent.

An upper bound for $a(Q^2, \nu)$.

To order $\beta Q^2/M^2$, $a(Q^2, \nu)$ is given by (3.5). Use of the kinematical inequalities $\nu \leq E$, $Q^2 = 2M\nu x \leq 2M\nu$, and the spin $-\frac{1}{2}$ parton model predictions that $\nu R_2 = 2MxR_1$, $\nu W_2 = 2MxW_1$ (Table 2, Appendix C), leads to the inequality

$$|a(Q^2, \nu)| \leq \left| \frac{\beta Q^2}{M^2} \right| \frac{|R_2| + x|R_3|}{W_2} \quad (3.9)$$

in the Bjorken limit. From Table 2, Appendix C, in terms of the G_i^P defined above,

$$\left. \begin{aligned} \nu W_2^P &= x \left[\frac{4}{9}(G_1^P + G_1^P) + \frac{1}{9}(G_2^P + G_2^P) + \frac{1}{9}(G_3^P + G_3^P) \right] \\ \nu R_2^P &= \frac{x}{3} \left[2(G_1^P + G_1^P) + (G_2^P + G_2^P) \right] \\ \nu R_3^P &= \frac{-1}{3} \left[2(G_1^P - G_1^P) + (G_2^P - G_2^P) \right] \end{aligned} \right\} \quad (3.10)$$

with similar relations for the neutron. Substituting (3.10) into (3.9) leads to the upper bound for the proton

$$|a(Q^2, \nu)| \leq \left| \frac{6\beta Q^2}{M^2} \right| \quad (3.11)$$

An identical upper bound is found for the neutron. For $G = G_f$, the upper bound is $\sim 5\%$ at $Q^2 = 100 \text{ GeV}^2/c^2$.

The validity of this bound to order $\beta Q^2/M^2$ depends upon: (i) the assumed forms of $\mathcal{L}_W$, j_μ^W and J_μ^W , (ii) being in a region where both $\mathcal{L}_W$ and scaling are valid, and (iii) the use of the generalized quark - parton model.

The determination of $a(Q^2, \nu)$ from neutrino scattering.

The cross section for deep inelastic $\nu - N$ scattering is

$$\frac{d^2\sigma^{\nu N}}{dQ^2 d\nu} = \frac{G_f^2}{8\pi E^2} \left[2W_1^{\nu N} Q^2 + W_2^{\nu N} \{4E(E-\nu) - Q^2\} - (2E-\nu) \frac{Q^2}{M} W_3^{\nu N} \right] \quad (3.12)$$

where $W_i^{\nu N}$ are the nucleon's structure functions discussed in reference 7, and depend upon the charged weak hadronic current $J_\mu^\dagger$. In the quark model, with the approximation of zero Cabibbo angle θ_c , $J_\mu^\dagger = \bar{p} \gamma_\mu (1 - \gamma_5) n$, and the $W_i^{\nu N}$ in the generalized quark - parton model can be found in terms of G_1^P , G_1^N , G_2^P and G_2^N alone, since only p and n quarks are involved. From (3.10), the νR_i^N are also functions of these G_i^P alone. These relations can be inverted to yield

$$\begin{aligned}
R_2^P &= \frac{1}{12} [3W_2^{pp} + 3W_2^{pn} + xW_3^{pp} - xW_3^{pn}] \\
R_3^P &= \frac{-1}{12x} [W_2^{pn} - W_2^{pp} - 3xW_3^{pn} - 3xW_3^{pp}] \\
R_2^N &= \frac{1}{12} [3W_2^{pn} + 3W_2^{pp} + xW_3^{pn} - xW_3^{pp}] \\
R_3^N &= \frac{-1}{12x} [W_2^{pp} - W_2^{pn} - 3xW_3^{pn} - 3xW_3^{pp}]
\end{aligned} \tag{3.13}$$

with $\nu R_2^N = 2MxR_1^N$ as usual. The future measurement of all the structure functions of $\nu - N$ scattering thus predicts the functions R_i^N via (3.13). Since the electromagnetic structure functions W_i^N are already known from reference 9, it will be possible to predict $a(Q^2, \nu)$ to order $\beta Q^2/M^2$ in (3.5) as a function of only the unknown coupling strength G .

A more directly applicable formula for $a(Q^2, \nu)$ follows. Consider inelastic lepton scattering on an unpolarized target consisting of equal numbers of protons and neutrons (isotopic spin zero), and use $\langle \rangle$ to denote the average over p and n for this target. (3.10) and (3.13) then imply^{11,12}

$$\langle W_i^{\nu} \rangle = 2 \langle R_i \rangle \tag{3.14}$$

Substitution of (3.14), (3.12) and (2.16) into (3.5) gives the simple relation

$$\langle a(Q^2, \nu) \rangle = \frac{-p^2 \left\langle \frac{d^2 \sigma^{\nu}}{dQ^2 d\nu} \right\rangle}{8 \left(\frac{\beta Q^2}{M^2} \right) \left\langle \frac{d^2 \sigma^L}{dQ^2 d\nu} \right\rangle} \tag{3.15}$$

where $\rho = G/G_f$, $\langle d\sigma^\nu \rangle$ is the neutrino cross section and $\langle d\sigma^l \rangle$ the unpolarized charged lepton cross section for scattering on the above target. $a(Q^2, \nu)$ can thus be predicted as a function of G from these directly measurable quantities. (3.15) is valid to order $(\beta Q^2/M^2)$, and depends upon the assumption listed beneath (3.11), as well as the approximation $\theta_c = 0$.

B. Numerical estimates of $a(Q^2, \nu)$ in the Kuti - Weisskopf parton model ⁶.

The Kuti - Weisskopf model (KWM) is a more specific version of the generalized model of section IIIA above; the distribution functions $G_i^N(x)$ are determined by some heuristic arguments, and the remaining adjustable constants fixed by comparison with experiment. Table 3 in Appendix C lists the independent $G_i^N(x)$ in the KWM; Table 4 lists the structure functions explicitly.

The parity violating asymmetry parameter $a(Q^2, \nu)$ defined in (3.1 - 4) can now be computed in the KWM. Figures 2a - d display the contour lines of constant percentage asymmetry $[a(Q^2, \nu) \times 100]$ for proton targets, for values of $G = \pm G_f$. Incoming lepton energies of 100, 200 and 4260 GeV are used, the latter corresponding to the C.M. energy of a 200 GeV proton - 10 GeV electron colliding beam machine. At the two lower energies, the G^2 terms may be neglected, and $a(Q^2, \nu)$ is linear in G ; only graphs for $G = +G_f$ are therefore displayed. At 4260 GeV, the G^2 terms are important at large Q^2 , and the effect depends appreciably on the sign of G - compare Figures 2c and 2d. The solid lines (denotes $M_z = \infty$) in the figures are those for the current - current Lagrangian of (1.4), used until now.

As long as the lepton mass is negligible, the effect of a neutral weak interaction mediated by an intermediate vector boson of mass M_z is simply to replace β in all the above formulae by $\beta / (1 + Q^2/M_z^2)$. The limit $M_z \rightarrow \infty$ then

reduces to the Lagrangian (1.4). The effect of such a boson is always to reduce the asymmetry from its $M_Z = \infty$ value, although for $Q^2 \ll M_Z^2$, the difference is small. Figures 2a-c include the effects for bosons of mass 37.3 and 74.6 GeV.

$a(Q^2, \nu)$ can also be calculated in the KWM for Weinberg's theory¹ of weak and electromagnetic interactions, when the existence of strange particles is totally ignored. The weak neutral currents and their couplings to the intermediate vector boson in his theory depend upon a parameter Θ_W . In the limit $\Theta_W \rightarrow 0$, Weinberg's Lagrangian becomes identical to the effective Lagrangian considered above for $M_Z = \infty$ with $G = +G_f$. The graphs displayed for $G = +G_f$ and $M_Z = \infty$ thus also represent this particular limit of Weinberg's theory.

For a neutron target the graphs of $a(Q^2, \nu)$ differ in no significant way from those for the proton, and have not been displayed.

C. Measurement of the polarization of the outgoing lepton as a test for parity violation.

An alternative indicator of a weak neutral current is the measurement of a non-zero longitudinal polarization for the outgoing lepton in unpolarized $\ell^\pm - N$ scattering.

In this case, define the longitudinal polarization of the final lepton in (1.1) to be

$$a'(Q^2, \nu) = \frac{d\sigma(+1) - d\sigma(-1)}{d\sigma(+1) + d\sigma(-1)} \quad (3.16)$$

where $d\sigma(\lambda)/dQ^2 d\nu$ is the cross section for producing a lepton of helicity λ in the scattering. In the limit of zero lepton mass, in the Born approximation,

$$a'(Q^2, \nu) \equiv a(Q^2, \nu) \quad (3.17)$$

where $a(Q^2, \nu)$ is the lepton spin asymmetry parameter of (3.1). Thus all estimates and bounds derived above for a in various models hold equally for a' . (3.17) is easily deduced from the fact that lepton helicity is conserved in the limit of zero lepton mass.

IV. TESTS FOR A WEAK NEUTRAL CURRENT BASED ON DEVIATIONS FROM THE ONE PHOTON EXCHANGE APPROXIMATION.

A. Charge dependence of the cross section

The one photon exchange electromagnetic contribution to $\ell^\pm - N$ scattering is independent of lepton charge. Both the two photon - one photon interference effects and the presence of a weak neutral current destroy this symmetry. A naive order - of - magnitude estimate, for $G = G_f$, suggests that such two - photon effects are probably ten times smaller than those of the weak neutral current at $Q^2 = 200 \text{ GeV}^2/c^2$, and decreasing as Q^{-2} . If this were true, the detection of larger lepton charge dependent effects would indicate some anomaly, perhaps a weak neutral current. Without accurate prior knowledge of two photon - one photon interference effects however, such a test is inconclusive, especially at low energy. Nevertheless, the charge dependence of unpolarized $\ell^\pm - N$ scattering due to the presence of a weak neutral current alone is estimated below, using the methods of section III.

It is convenient to define the "lepton charge asymmetry parameter"

$$C(Q^2, \nu) = \frac{d\sigma^- - d\sigma^+}{d\sigma^- + d\sigma^+} \quad (4.1)$$

$d\sigma^\pm$ are the unpolarized cross section for $\ell^\pm - N$ scattering given by (2.16) and (2.17) with $\lambda = 0$. In terms of the hadronic structure functions

$$c(Q^2, \nu) = \frac{-N}{D + N} \quad (4.2)$$

where

$$N = \frac{\beta Q^2}{M^2} \left[\frac{(2E - \nu) Q^2}{M} \right] \left[R_3 + 2 \frac{\beta Q^2}{M^2} Y_3 \right] \quad (4.3)$$

D is defined in (3.4), and lepton mass ignored. Neglecting two photon effects, a consequence of a weak neutral current is a non zero value for c .

Generalized quark - parton model predictions for $c(Q^2, \nu)$

Techniques analogous to those of section IIIA are employed here; the same notation and form (3.6) for J_μ^W is assumed. To lowest order in $\beta Q^2/M^2$,

$$c(Q^2, \nu) = \frac{-\frac{\beta Q^2}{M^2} \left[\frac{(2E-\nu)Q^2}{M} \right] R_3}{2W_1 Q^2 + W_2 \{4E(E-\nu) - Q^2\}} \quad (4.4)$$

From the kinematical relations $Q^2 \leq 2M\nu \leq 2ME$, in the Bjorken limit we find

$$(2E - \nu) Q^2 \leq 2MxE^2 \cdot \min\{1, 2\nu/E\} \quad . \quad \text{Together with } \nu W_2 = 2MxW_1 \text{ in (4.4), this implies}$$

$$|c(Q^2, \nu)| \leq \left| \frac{\beta Q^2}{M^2} \cdot \frac{xR_3}{W_2} \right| \cdot \min\left\{1, \frac{2\nu}{E}\right\}$$

(3.10) leads to the upper bound, to order $\beta Q^2/M^2$ in the Bjorken limit,

$$|c(Q^2, \nu)| \leq \left| \frac{6\beta Q^2}{M^2} \right| \cdot \min\left\{\frac{1}{2}, \frac{\nu}{E}\right\} \quad (4.5)$$

The validity of the bound depends on the conditions mentioned below (3.11).

Comparing (4.5) with (3.11), it is clear that for $\frac{\nu}{E} \ll 1$, $c(Q^2, \nu)$ has a much smaller upper bound than $a(Q^2, \nu)$. This is due to the fact that a in (3.5) has both its numerator and denominator containing E^2 terms, whereas c in (4.4) has the same denominator but only a linear dependence on E in the numerator. This difference leads one generally to expect larger values for the parity violating asymmetry than the lepton charge asymmetry for $\nu/E \ll 1$, independent of the detailed form of J_μ^W .

$c(Q^2, \nu)$ can also be related to the cross sections for $\nu - N$ and $\bar{\nu} - N$ inelastic scattering on isotopic spin zero targets. The relation $2\langle R_3 \rangle = \langle W_3^\nu \rangle$ that follows from isotopic spin considerations^{11,12} in the approximation $\theta_c = 0$ leads to

$$\langle c(Q^2, \nu) \rangle = \frac{-\rho^2 \left[\left\langle \frac{d^2 \sigma^{\bar{\nu}}}{dQ^2 d\nu} \right\rangle - \left\langle \frac{d^2 \sigma^\nu}{dQ^2 d\nu} \right\rangle \right]}{16 \left(\frac{\beta Q^4}{M^2} \right) \left\langle \frac{d^2 \sigma^L}{dQ^2 d\nu} \right\rangle} \quad (4.6)$$

accurate to order $\beta Q^2/M^2$, where the notation is the same as that given below (3.15). Recall again the total neglect of two photon contributions.

Numerical estimates of $c(Q^2, \nu)$ in the KWM.

With the structure functions given in Table 4 of Appendix C, $c(Q^2, \nu)$ can be calculated. Figures 3a-d display the contour lines of constant percentage asymmetry $[c(Q^2, \nu) \times 100]$ for proton targets, with the same values of G and E as were used for computing $a(Q^2, \nu)$. The G^2 term again becomes important only at high E and high Q^2 , as witnessed by the difference between Figures 3c and 3d. The effect of an intermediate vector boson of mass M_Z is shown. The $M_Z = \infty$ solid lines, for $G = +G_f$, are also the predictions of Weinberg's theory in the limit mentioned in section IIIB.

The values of $c(Q^2, \nu)$ at a given energy are generally smaller than those of $a(Q^2, \nu)$. The most striking difference between Figures 2a-d and Figures 3a-d lies in the slope of the contour lines. $c(Q^2, \nu)$ decreases sharply for small ν , whereas $a(Q^2, \nu)$ is relatively independent of ν . Such behaviour is explained by the remarks following (4.5).

B. Energy dependence of the cross section

From (2.16), the unpolarized ($\lambda = 0$) cross section for $\ell^- - N$ inelastic scattering in the Born approximation can be written in the form

$$\left(\frac{E^2 Q^4}{\pi \kappa^2} \right) \frac{d^2 \sigma^-}{dQ^2 d\nu} = AE^2 + BE + C \quad (4.7)$$

where A , B and C are function of Q^2 and ν depending on the dynamical structure function W_i , R_i and Y_i . In the absence of a weak neutral current, $R_i = Y_i = 0$, and since there are only two independent W_i , the extant one photon exchange electromagnetic contribution leads to the ratio $B/A = -\nu$.

In the presence of a weak neutral current, one finds from (2.16) that to lowest order in $\beta Q^2/M^2$,

$$\frac{B}{A} = -\nu [1 + \Delta] \quad (4.8)$$

where Δ is the "deviation" from the one photon exchange value, here given by

$$\Delta(Q^2, \nu) = \frac{\beta Q^2}{M^2} \cdot \frac{\chi R_3}{W_2} \quad (4.9)$$

Δ also receives contributions from higher order electromagnetic effects, so non-zero Δ is not an unequivocal proof of the existence of J_μ^w . Since such effects were estimated to be small (about 10% of J_μ^w effects) at

$Q^2 \approx 200 \text{ GeV}^2/c^2$, large values of Δ would indicate some anomaly, perhaps a weak neutral current. The deviation Δ has therefore been calculated for the proton in the KWM, with $G = G_f$ and J_μ^w given by (3.6). The result is shown in Figure 4 for $2M\nu \leq 375.3 \text{ GeV}^2/c^2$, the maximum value attainable for $E = 200 \text{ GeV}$. In this range G^2 terms are unimportant.

V. THE CROSS SECTION FOR $\ell^- - N \rightarrow \ell^- + \text{anything}$ WITH A POLARIZED TARGET.

The cross section for inclusive $\ell^- - N$ scattering with a polarized target is presented below. Figure 1 again shows the lowest order Feynman diagrams, in the presence of interactions (1.2) and (1.4). The notation is identical to that of section II except that S^μ denotes the nucleon's covariant polarization vector, $S \cdot P = 0$. All results are given in the limit $m = 0$, and in the laboratory frame.

Completely analogous to (2.1), the cross section may be written

$$d\sigma^- = d\sigma_{\gamma}^- + d\sigma_{\gamma}^- + d\sigma_{\pi}^- \quad (5.1)$$

These are discussed individually below.

Electromagnetic contribution $d\sigma_{\gamma}^-$

The contribution corresponding to Figure 1a is

$$d\sigma_{\gamma}^- = \frac{\pi e^4}{E q^4} \left[2m^2 j_{\mu\nu}^{\gamma} W^{\mu\nu}(q, P, S) \right] \frac{d^3 k'}{(2\pi)^3 k'_0} \quad (5.2)$$

where the leptonic contribution $j_{\mu\nu}^{\gamma}$ is given by (2.3) with $\lambda = 0$, and the virtual photon's interaction with a nucleon of spin S is described by the tensor

$$W_{\mu\nu}(q, P, S) = \frac{4\pi^2 P_0}{M} \int d^4 x e^{iq \cdot x} \langle PS | j_{\mu}^{\gamma}(x) j_{\nu}^{\gamma}(0) | PS \rangle \quad (5.3)$$

$W_{\mu\nu}(q, P, S)$ depends only upon q^μ , P^μ and S^μ and is discussed in Appendix A. From (A7 - 10)

$$\begin{aligned} W_{\mu\nu}(q, P, S) = & -W_1 \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + \frac{W_2}{M^2} \left(P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \left(P_\nu - \frac{P \cdot q}{q^2} q_\nu \right) \\ & + \frac{iW_3}{2M^3} (q \cdot S) \epsilon_{\mu\nu\alpha\beta} q^\alpha P^\beta + \frac{iW_4}{M} \epsilon_{\mu\nu\alpha\beta} q^\alpha S^\beta \end{aligned} \quad (5.4)$$

where W_3' and W_4' , the spin dependent structure functions of pure electromagnetic

ℓ^- - N scattering ¹³, are real scalar functions of q^2 and $q \cdot P$ analogous to the usual W_i introduced in (2.5). The consequences of positivity and scaling for W_i' are given in Appendix A. Here it is sufficient to note that in the spin - $\frac{1}{2}$ parton model of section VI, $\nu^2 W_3'/M$ and $\nu W_4'$ scale in the Bjorken limit.

From (5.2) and (5.4),

$$\frac{d\sigma_8^-}{dQ^2 d\nu d\phi} = \frac{\alpha^2}{2E^2 Q^4} \left[2W_1 Q^2 + W_2 \{4E(E-\nu) - Q^2\} \right] \quad (5.5)$$

where ϕ is the azimuthal angle of the outgoing lepton's momentum relative to the spin of the target. Note that this contribution to the cross section is independent of S^μ , and hence of ϕ .

Weak contribution $d\sigma_7^-$

From Figure 1b one finds

$$d\sigma_7^- = \frac{\pi G^2}{E} \left[m^2 j_{\mu\nu}^Y Y^{\mu\nu}(q, P, S) \right] \frac{d^3 K'}{(2\pi)^3 K_0'} \quad (5.6)$$

where the leptonic contribution $j_{\mu\nu}^Y$ is given by (2.8) with $\lambda = 0$, and the hadronic contribution by

$$Y_{\mu\nu}(q, P, S) = \frac{4\pi^2 P_0}{M} \int d^4 x e^{iq \cdot x} \langle PS_m | J_\mu^W(x) J_\nu^W(0) | PS_m \rangle \quad (5.7)$$

From (A12-14) one finds the most general form

$$\begin{aligned} Y_{\mu\nu}(q, P, S) = & - \left[Y_1 + Y_1' \frac{q \cdot S}{M} \right] (g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + \frac{1}{M^2} \left[Y_2 + Y_2' \frac{q \cdot S}{M} \right] (P_\mu - \frac{P \cdot q}{q^2} q_\mu) (P_\nu - \frac{P \cdot q}{q^2} q_\nu) \\ & - \frac{1}{2M^2} \left[Y_3 + Y_3' \frac{q \cdot S}{M} \right] \epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta + \frac{Y_4}{M^2} q_\mu q_\nu + \frac{Y_5}{M^2} (P_\mu q_\nu + P_\nu q_\mu) \\ & + \frac{i Y_6}{M} \epsilon_{\mu\nu\alpha\beta} q^\alpha S^\beta + \frac{Y_5'}{M} \left[(P_\mu - \frac{P \cdot q}{q^2} q_\mu) (S_\nu - \frac{S \cdot q}{q^2} q_\nu) + (P_\nu - \frac{P \cdot q}{q^2} q_\nu) (S_\mu - \frac{S \cdot q}{q^2} q_\mu) \right] \\ & + \frac{i Y_6'}{M^3} (q_\mu \epsilon_{\nu\alpha\beta\gamma} - q_\nu \epsilon_{\mu\alpha\beta\gamma}) q^\alpha P^\beta S^\gamma + \frac{Y_7'}{M^3} (q \cdot S) q_\mu q_\nu \\ & + \frac{Y_8'}{M^3} (q \cdot S) (P_\mu q_\nu + P_\nu q_\mu) + \frac{Y_9'}{M} (q_\mu S_\nu + q_\nu S_\mu) \end{aligned} \quad (5.8)$$

where the Y_i are the same structure functions as those of section II, and the $Y_i'(q^2, q, P)$ are similar real structure functions describing the spin-dependent part of the scattering.

The positivity constraints and scaling behaviour in the Bjorken limit are discussed in

Appendix A. In spin - $\frac{1}{2}$ parton models, MY_1 , νY_2 , νY_3 , $\nu Y_1'$, $\frac{\nu^2}{M} Y_2'$, $\frac{\nu^2}{M} Y_3'$, $\nu Y_4'$ and $\nu Y_5'$ scale, with $\nu Y_2 = 2MxY_1$ and $\frac{\nu^2 Y_2'}{2Mx} - \nu Y_1' + \frac{\nu Y_5'}{x} = 0$.

From (5.6) and (5.8),

$$\frac{d\sigma_W}{dQ^2 dy d\phi} = \frac{G^2}{32\pi^2 E^2} [F_Y] \quad (5.9)$$

where

$$[F_Y] = \left[\begin{aligned} & 2(Y_1 + Y_1' \frac{q \cdot s}{M}) Q^2 + (Y_2 + Y_2' \frac{q \cdot s}{M}) \{4E(E-\nu) - Q^2\} - (Y_3 + Y_3' \frac{q \cdot s}{M}) Q^2 \frac{(2E-\nu)}{M} \\ & - 2Y_4' \frac{Q^2}{M} [s \cdot (k+k')] + 4Y_5' [E s \cdot k' + (E-\nu) s \cdot k] \end{aligned} \right] \quad (5.10)$$

In the limit $m = 0$ used here, Y_4 , Y_5 , Y_6' , Y_7' , Y_8' and Y_9' do not enter the cross section; these are the structure functions which would be absent if the hadronic weak current were conserved, and thus there is no observable difference between a conserved and non-conserved hadronic current.

Interference contribution $d\sigma_R^-$

From the interference between the diagrams of Figures 1a and 1b,

$$d\sigma_R^- = -\frac{G\pi e^2}{\sqrt{2} q^2 E} \left[2m^2 j_{\mu\nu}^R R^{\mu\nu}(q, P, S) \right] \frac{d^3 k'}{(2\pi)^3 k_0} \quad (5.11)$$

where $j_{\mu\nu}^R$ is given by one half of (2.8) with $\lambda = 0$, and

$$R_{\mu\nu}(q, P, S) = \frac{4\pi^2 p_0}{M} \int d^4 x e^{iq \cdot x} \langle PS_{in} | J_\mu^V(x) J_\nu^W(0) + J_\mu^W(x) J_\nu^V(0) | PS_{in} \rangle \quad (5.12)$$

The properties of the currents lead to the general form described below (A16):

$$\begin{aligned}
R'_{\mu\nu}(q, p, s) = & -[R_1 + R'_1 \frac{q \cdot s}{M}](g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + \frac{1}{M^2} [R_2 + R'_2 \frac{q \cdot s}{M}](p_\mu - \frac{p \cdot q}{q^2} q_\mu)(p_\nu - \frac{p \cdot q}{q^2} q_\nu) \\
& - \frac{i}{2M^2} [R_3 + R'_3 \frac{q \cdot s}{M}] \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta + \frac{R_5}{M^2} (p_\mu q_\nu + p_\nu q_\mu - \frac{2p \cdot q}{q^2} q_\mu q_\nu) \\
& + \frac{iR'_4}{M} \epsilon_{\mu\nu\alpha\beta} q^\alpha s^\beta + \frac{R'_5}{M} [(p_\mu - \frac{p \cdot q}{q^2} q_\mu)(s_\nu - \frac{s \cdot q}{q^2} q_\nu) + (p_\nu - \frac{p \cdot q}{q^2} q_\nu)(s_\mu - \frac{s \cdot q}{q^2} q_\mu)] \\
& + \frac{iR'_6}{M} (q_\mu \epsilon_{\nu\alpha\beta\gamma} - q_\nu \epsilon_{\mu\alpha\beta\gamma}) q^\alpha p^\beta s^\gamma + \frac{R'_7}{M^3} [q \cdot s] q_\mu q_\nu - \frac{q^2}{2} (q_\mu s_\nu + q_\nu s_\mu) \\
& + \frac{R'_8}{M^3} [(q \cdot s)(p_\mu q_\nu + p_\nu q_\mu) - p \cdot q (q_\mu s_\nu + q_\nu s_\mu)] \quad (5.13)
\end{aligned}$$

where the R'_i are additional spin dependent interference structure functions. Positivity and scaling are discussed in Appendix A. In spin - $\frac{1}{2}$ parton models, $M R_1$, νR_2 , νR_3

$\nu R'_1$, $\frac{\nu^2}{M} R'_2$, $\frac{\nu^2}{M} R'_3$, $\nu R'_4$ and $\nu R'_5$ scale, with $\nu R_2 = 2Mx R_1$

and $\frac{\nu^2 R'_2}{2Mx} - \nu R'_1 + \frac{\nu R'_5}{x} = 0$.

The above are the only relevant functions

in the $m = 0$ limit for the cross section, which from (5.11 - 13) is given by

$$\frac{d\sigma_R^-}{dQ^2 d\nu d\phi} = \frac{G\alpha}{8\pi\sqrt{2}Q^2 E^2} [F_R] \quad (5.14)$$

where $[F_R]$ is given by (5.10) with R_i and R'_i replacing Y_i and Y'_i respectively.

The cross section $d\sigma^- = d\sigma_S^- + d\sigma_R^- + d\sigma_Y^-$.

Analogous to (2.16), the laboratory cross section for (1.1) with a nucleon of spin S^H is

$$\frac{d^2\sigma^-(s)}{dQ^2 d\nu d\phi} = \frac{\alpha^2}{2E^2 Q^4} \left[2W_1 Q^2 + W_2 \{4E(E-\nu) - Q^2\} + \frac{\beta Q^2}{M^2} [F_R] + 2\left(\frac{\beta Q^2}{M^2}\right)^2 [F_Y] \right] \quad (5.15)$$

Since all lepton currents are conserved when $m = 0$, only those structure functions corresponding to conserved hadron currents enter the cross section.

VI. TEST FOR PARITY VIOLATION USING THE NUCLEON SPIN

In the inclusive process considered in V, a weak neutral current manifests itself in some parity violating correlation. If it exists, this is best detected by using a nucleon polarized longitudinally (i.e. parallel or antiparallel to the incoming lepton's laboratory momentum) for two reasons. Firstly, as in section III, in this case all spin dependent effects are inherently parity violating¹⁰. Secondly, at high energy ($E \sim E' \gg M$) in the Bjorken limit, the spin dependent or parity violating terms like $S \cdot k'$ in the Born approximation to the cross section given by (5.15) are smaller for transversely polarized nucleons than for longitudinally polarized ones by a factor $\sim \sqrt{M/E'}$. This is because at high energy the laboratory scattering angle $\theta \leq \sqrt{2M/E'}$ (see Appendix B). A suitable test for a weak neutral current is therefore to examine the change in $\frac{d^2\sigma(s)}{dQ^2 d\nu d\phi}$ of (5.15) when the longitudinally polarized nucleon's spin is reversed.

The "nucleon spin asymmetry parameter" is therefore defined as

$$A(Q^2, \nu) = \frac{d\sigma^-(\uparrow) - d\sigma^-(\downarrow)}{d\sigma^-(\uparrow) + d\sigma^-(\downarrow)} \quad (6.1)$$

where $d\sigma^-/dQ^2 d\nu$ is given by (5.15), and $\uparrow (\downarrow)$ denotes the nucleon's spin parallel (antiparallel) to the incoming lepton momentum in the laboratory. A test for parity violation is to measure a non-zero $A(Q^2, \nu)$. From (5.15) one can write

$$A(Q^2, \nu) = \frac{\frac{BQ^2}{M^2} [H_R] + 2 \left(\frac{BQ^2}{M^2} \right)^2 [H_Y]}{D} \quad (6.2)$$

where

$$[H_R] = \frac{q^5}{M} \left[2R_1' Q^2 + R_2' \{ 4E(E-\nu) - Q^2 \} - R_3' Q^2 (2E-\nu) \right] - \frac{2R_4' Q^2}{M} S \cdot (k+k') + 4R_5' [E S \cdot k' + (E-\nu) S \cdot k] \quad (6.3)$$

$[H_Y]$ is given by (6.3) with all R_i' replace by Y_i' , and S^μ here is the rest-frame polarization vector for a nucleon with spin parallel to the lepton's laboratory momentum.

D is given by (3.4).

Numerical estimates of $A(Q^2, \nu)$ in the KWM⁶

In order to estimate $A(Q^2, \nu)$ numerically, the specific form (3.6) for J_μ^w is again chosen. In the quark parton model, the currents are given by (3.8a,b). Since the spin dependent structure functions are required, in the parton model it is necessary to guess at details of the parton angular momentum states. Even then, calculating a spin dependent nucleon structure functions as a sum of contributions from moving partons in various spin and relative orbital angular momentum states is a tortuous endeavour.

For the purposes of this section, it is simplest to eschew such complexity, and in the most naïve way consider the partons to have no transverse momentum at all, so that in the rest frame of the nucleon, all partons are at rest. A parton which carries a fraction x of the nucleon's momentum in the infinite momentum frame then actually has a rest mass which is a fraction x of the nucleon's mass. (This picture is in some respects similar to that of a liquid drop model for the nucleon.) Following the KWM, one also assumes that only the three "valence quarks" that carry the nucleon's quantum numbers carry a net spin, and that their wave function in the nucleon's ground state is dictated by $SU(6)$. This allows one to determine the probability for finding a parton i with spin parallel or antiparallel to the nucleon spin, which together with the KWM's momentum distribution functions for the valence quarks, is sufficient to calculate all spin dependent structure functions by summing over those of the partons. There are no orbital angular momenta.

By analogy with IIIA, let $G_{iV}^N(x)$ be the probability distribution for finding a valence quark of type i with longitudinal momentum a fraction x of that of the N - type nucleon. Let the $SU(6)$ value for the i - quark's average spin component parallel to the nucleon spin be denoted σ_i^N . These are listed in Table 5 of Appendix C. In the simple model described above the structure functions can now be calculated for one parton, and simply summed over spin and momentum distributions to yield the nucleon's

function. The resultant spin independent structure functions are the same as those of section II, and the new spin dependent functions are explicitly listed in Table 6, Appendix C.

$A(Q^2, \nu)$ in (6.2) can now be calculated using these structure functions.

Figures 5a-d show the contour lines of constant percentage asymmetry $[A(Q^2, \nu) \times 100]$ for proton targets, with the same values of G and E as were used in II. The G^2 term is again only important at both high E and high Q^2 (Figures 5c and 5d). $A(Q^2, \nu)$ is in general about a factor of two smaller than $a(Q^2, \nu)$, as is evident from comparing their contour plots, but since the spin dependent parton model used is extremely naive, this is not significant. $A(Q^2, \nu)$ for a neutron target is generally smaller than for a proton, and has not been displayed.

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APPENDIX A: STRUCTURE FUNCTIONS AND THEIR PROPERTIES

This Appendix contains an account of the relevant properties of the hadronic tensors

$$W_{\mu\nu}(q, P, S) = \frac{4\pi^2 P_0}{M} \int d^4x e^{iq \cdot x} \langle PSin | J_\mu^\gamma(x) J_\nu^\gamma(0) | PSin \rangle \quad (A1)$$

$$Y_{\mu\nu}(q, P, S) = \frac{4\pi^2 P_0}{M} \int d^4x e^{iq \cdot x} \langle PSin | J_\mu^W(x) J_\nu^W(0) | PSin \rangle \quad (A2)$$

$$R_{\mu\nu}(q, P, S) = \frac{4\pi^2 P_0}{M} \int d^4x e^{iq \cdot x} \langle PSin | J_\mu^\gamma(x) J_\nu^W(0) + J_\mu^W(x) J_\nu^\gamma(0) | PSin \rangle \quad (A3)$$

that describe the interaction of J_μ^γ and J_μ^W with a nucleon in deep inelastic $\ell^\pm - N$ scattering. $|PSin\rangle$ denotes an incoming nucleon state of momentum P^μ and covariant axial spin vector S^μ such that $S \cdot P = 0$.

(A1-3) are second rank Lorentz tensors depending only upon q^μ , P^μ and S^μ .

From the reduction formula, $W_{\mu\nu}(q, P, S) = \bar{u}(P, S) F_{\mu\nu} u(P, S)$ where $u(P, S)$ is a Dirac spinor⁴ and $F_{\mu\nu}$ a 4×4 matrix constructed from q^μ , P^μ and the Dirac matrices γ^μ . Use of the spin projection operator then shows that (A1) may be written

$$W_{\mu\nu}(q, P, S) = W_{\mu\nu}(q, P) + W'_{\mu\nu}(q, P, S) \quad (A4)$$

where

$$W_{\mu\nu}(q, P) = \frac{1}{2} [W_{\mu\nu}(q, P, S) + W_{\mu\nu}(q, P, -S)] \quad (A5)$$

is a second rank tensor depending only upon q and P , and

$$W'_{\mu\nu}(q, P, S) = \frac{1}{2} [W_{\mu\nu}(q, P, S) - W_{\mu\nu}(q, P, -S)] \quad (A6)$$

is a tensor depending upon q , P and S , and linear in S . $Y_{\mu\nu}(q, P, S)$ and

$R_{\mu\nu}(q, P, S)$ may be analogously decomposed into tensors independent of and linear in S .

There are six linearly independent tensors of second rank that can be constructed

from q^μ and P^μ , and fourteen from q^μ , P^μ and S^μ that are linear in S . The linearly independent bases used here are presented in Tables 1a and 1b below, classified according to their symmetry, whether they are conserved, and their behaviour under reflection.

	CONSERVED	NON - CONSERVED
POLAR SYMMETRIC	$g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} ; (P_\mu - \frac{P \cdot q}{q^2} q_\mu)(P_\nu - \frac{P \cdot q}{q^2} q_\nu)$	$q_\mu q_\nu ; P_\mu q_\nu + P_\nu q_\mu$
POLAR ANTI- SYMMETRIC	—	$P_\mu q_\nu - P_\nu q_\mu$
AXIAL SYMMETRIC	—	—
AXIAL ANTI- SYMMETRIC	$\epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta$	—

TABLE 1a: Properties of the second rank tensors constructed from P^μ and q^μ .

	CONSERVED	NON - CONSERVED
POLAR SYMMETRIC.	$(P_\mu - \frac{P \cdot q}{q^2} q_\mu) \epsilon_{\nu\alpha\beta\gamma} P^\alpha q^\beta S^\gamma + \{\mu \leftrightarrow \nu\}$	$q_\mu \epsilon_{\nu\alpha\beta\gamma} P^\alpha q^\beta S^\gamma + \{\mu \leftrightarrow \nu\}$
POLAR ANTI- SYMMETRIC	$(q \cdot S) \epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta$ $\epsilon_{\mu\nu\alpha\beta} q^\alpha S^\beta$	$q_\mu \epsilon_{\nu\alpha\beta\gamma} P^\alpha q^\beta S^\gamma - \{\mu \leftrightarrow \nu\}$
AXIAL SYMMETRIC	$(q \cdot S)(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})$ $(q \cdot S)(P_\mu - \frac{P \cdot q}{q^2} q_\mu)(P_\nu - \frac{P \cdot q}{q^2} q_\nu)$ $(P_\mu - \frac{P \cdot q}{q^2} q_\mu)(S_\nu - \frac{S \cdot q}{q^2} q_\nu) + \{\mu \leftrightarrow \nu\}$	$(q \cdot S) q_\mu q_\nu$ $(q \cdot S)(P_\mu q_\nu + P_\nu q_\mu)$ $q_\mu S_\nu + q_\nu S_\mu$
AXIAL ANTI- SYMMETRIC	$(P_\mu - \frac{P \cdot q}{q^2} q_\mu)(S_\nu - \frac{S \cdot q}{q^2} q_\nu) - \{\mu \leftrightarrow \nu\}$	$(q \cdot S)(P_\mu q_\nu - P_\nu q_\mu)$ $q_\mu S_\nu - q_\nu S_\mu$

TABLE 1b: Properties of the second rank, linear - in - S tensors constructed from P^μ , q^μ and S^μ .

The most general form of (A1-3) is a linear combination of the twenty tensors in Tables 1a and 1b, with coefficients which are scalar functions of q^2 and $q \cdot P$, subject to the constraints of current conservation, parity conservation, time reversal invariance and hermiticity, where they exist.

$$\underline{W_{\mu\nu}(q, P, S)}$$

From (A4-6),

$$W_{\mu\nu}(q, P, S) = W_{\mu\nu}(q, P) + W'_{\mu\nu}(q, P, S) \quad (A7)$$

$W_{\mu\nu}(q, P)$ alone is the spin-averaged hadronic tensor relevant to deep inelastic $\ell^\pm - N$ scattering on an unpolarized target. The electromagnetic interaction is parity conserving and time-reversal invariant. Also $\partial^\mu J_\mu^\delta(x) = 0$ and $J_\mu^\delta = J_\mu^{\delta\dagger}$.

These conditions imply that (i) $W_{\mu\nu}(q, P)$ is a real polar symmetric conserved tensor and (ii) $W'_{\mu\nu}(q, P, S)$ is an imaginary, polar, antisymmetric conserved tensor linear in S . The Tables above give the most general forms

$$W_{\mu\nu}(q, P) = -W_1(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + \frac{W_2}{M^2} (P_\mu - \frac{P \cdot q}{q^2} q_\mu)(P_\nu - \frac{P \cdot q}{q^2} q_\nu) \quad (A8)$$

$$W'_{\mu\nu}(q, P, S) = \frac{iW_3}{2M^3}(q \cdot S) \epsilon_{\mu\nu\alpha\beta} q^\alpha P^\beta + \frac{iW_4}{M} \epsilon_{\mu\nu\alpha\beta} q^\alpha S^\beta \quad (A9)$$

where $W_i = W_i(q^2, q \cdot P)$ and $W'_i = W'_i(q^2, q \cdot P)$ are real scalar functions of dimensions $(\text{mass})^{-1}$.

By inserting a complete set of states between the currents in (A1), one can establish the semi-positive definite nature of $W_{\mu\nu}(q, P, S)$: for any complex four vector n^μ ,

$$n^\mu W_{\mu\nu}(q, P, S) n^\nu \geq 0 \quad (A10)$$

Since $W_{\mu\nu}(q, P, S)$ is hermitian, the necessary and sufficient condition for this⁸ is

that all principal minors of $W_{\mu\nu}(q, p, s)$ be non-negative. It is most convenient to implement this by transforming $W_{\mu\nu}(q, p, s)$ to the helicity basis in the laboratory frame, where it is then reducible according to the conservation of spin components parallel to $\underline{q}$ ¹⁴.

The resultant positivity constraints are

$$\begin{aligned} (1 - \frac{\nu^2}{q^2}) W_2 &\geq W_1 \geq 0 \\ \left| \frac{\nu W_4'}{M} + \frac{(\nu^2 - q^2) W_3'}{2M^2} \right| &\leq W_1 \\ \left[(1 - \frac{\nu^2}{q^2}) W_2 - W_1 \right] \left[W_1 + \left\{ \frac{\nu W_4'}{M} + \frac{(\nu^2 - q^2) W_3'}{2M^2} \right\}^2 \right] &\geq \frac{2Q^2 W_4'^2}{M^2} \end{aligned} \quad (\text{A.11})$$

The consequences of scale invariance in the Bjorken limit⁷ for $W_{\mu\nu}(q, p, s)$ and all subsequent tensors will be determined by requiring that the total helicity cross sections for current-nucleon scattering decrease as Q^{-2} , as suggested by the dimension of cross section. This is what one means by point-like behaviour. For W_i and W_i' this leads to Bjorken-type scaling⁷ for νW_2 , $M W_1$ and the combination $(\nu W_4' + \nu^2 W_3'/2M)$. The positivity constraints of (A11) in the Bjorken limit then set rigorous bounds¹⁵ on the large ν -dependence (i.e. scaling behaviour) of W_4' and W_3' individually, namely $W_4' \lesssim O(\nu^{-1/2})$ and $W_3' \lesssim O(\nu^{-3/2})$.

Spin - $\frac{1}{2}$ parton models, which are always used for any specific predictions in this paper, lead to more stringent scaling behaviour than that required above. These models do predict scaling for νW_2 and $M W_1$ (see Appendix C) with the additional relation $\nu W_2 = 2Mx W_1$, consistent with the SLAC - M.I.T. experiments⁹. However in such models $\nu W_4'$ and $\frac{\nu^2 W_3'}{2M}$ scale individually - i.e. $W_4' \approx O(\nu^{-1})$ and $W_3' \approx O(\nu^{-2})$, a scaling behaviour more stringent than that required by positivity alone.

$$\underline{Y_{\mu\nu}(q, P, S)}$$

$Y_{\mu\nu}(q, P, S)$ is similar to $W_{\mu\nu}(q, P, S)$, and may be decomposed analogously to (A7),

$$Y_{\mu\nu}(q, P, S) = Y_{\mu\nu}(q, P) + Y'_{\mu\nu}(q, P, S) \quad (A12)$$

where $Y_{\mu\nu}(q, P)$ is the spin - averaged hadronic tensor of section II. Here however,

$\partial^\mu J_\mu^W \neq 0$ and parity is not conserved. The remaining conditions of time - reversal invariance and hermiticity constrain (i) $Y_{\mu\nu}(q, P)$ to be constructed from real, polar, symmetric or imaginary, axial, antisymmetric tensors, and (ii) $Y'_{\mu\nu}(q, P, S)$ to be constructed from imaginary, polar, antisymmetric or real, axial symmetric tensors linear in S . From Tables 1a and 1b therefore,

$$\begin{aligned} Y_{\mu\nu}(q, P) = & -Y_1(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + \frac{Y_2}{M^2}(P_\mu - \frac{P \cdot q}{q^2} q_\mu)(P_\nu - \frac{P \cdot q}{q^2} q_\nu) - \frac{iY_3}{2M^2} \epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta \\ & + \frac{Y_4}{M^2} q_\mu q_\nu + \frac{Y_5}{M^2} (P_\mu q_\nu + P_\nu q_\mu) \end{aligned} \quad (A13)$$

and

$$\begin{aligned} Y'_{\mu\nu}(q, P, S) = & -Y'_1 \frac{q \cdot S}{M} (g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + Y'_2 \frac{q \cdot S}{M^3} (P_\mu - \frac{P \cdot q}{q^2} q_\mu)(P_\nu - \frac{P \cdot q}{q^2} q_\nu) - \frac{Y'_3}{2M^3} q \cdot S \epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta \\ & + i \frac{Y'_4}{M} \epsilon_{\mu\nu\alpha\beta} q^\alpha S^\beta + \frac{Y'_5}{M} [(P_\mu - \frac{P \cdot q}{q^2} q_\mu)(S_\nu - \frac{S \cdot q}{q^2} q_\nu) + (P_\nu - \frac{P \cdot q}{q^2} q_\nu)(S_\mu - \frac{S \cdot q}{q^2} q_\mu)] \\ & + \frac{iY'_6}{M^3} [q_\mu \epsilon_{\nu\alpha\beta\gamma} - q_\nu \epsilon_{\mu\alpha\beta\gamma}] q^\alpha P^\beta S^\gamma + \frac{Y'_7}{M^3} (q \cdot S) q_\mu q_\nu \\ & + \frac{Y'_8}{M^3} (q \cdot S) (P_\mu q_\nu + P_\nu q_\mu) + \frac{Y'_9}{M} (q_\mu S_\nu + q_\nu S_\mu) \end{aligned} \quad (A14)$$

with Y_i and Y'_i analogous to W_i and W'_i .¹⁶

As in (A10), $\eta_\mu^* Y^{\mu\nu}(q, P, S) \eta_\nu \geq 0$; in the Bjorken limit in which we are interested the resultant positivity constraints take the form

$$\nu Y_2 \geq 2Mx Y_1 \geq x |\nu Y_3|$$

$$|\nu Y_1'| \leq M Y_1$$

$$\left| \frac{\nu^2 Y_3'}{2M} + \nu Y_4' \right| \leq M Y_1$$

$$\left| \frac{\nu^2 Y_2'}{2Mx} - \nu Y_1' + \frac{\nu Y_5'}{x} \right| \leq \frac{\nu Y_2}{2x} - M Y_1$$

(A.15)

$$\left\{ \left(Y_1 \pm \frac{\nu Y_3}{2M} \right) \pm \frac{\nu}{M} \left(Y_1' \pm \frac{\nu Y_3'}{2M} \right) + \frac{\nu Y_4'}{M} \right\} \left\{ \left(\frac{\nu Y_2}{2Mx} - Y_1 \right) \mp \frac{\nu}{M} \left(\frac{\nu Y_2'}{2Mx} - Y_1' \right) \mp \frac{\nu Y_5'}{Mx} \right\} \geq 2 \left[\sqrt{\frac{2x\nu}{M}} Y_4' \pm \sqrt{\frac{\nu}{2Mx}} Y_5' \right]^2$$

In the limit of zero lepton mass usually considered in this paper, $Y_4, Y_5, Y_6', Y_7', Y_8'$

and Y_9' do not enter the cross section, and positivity constraints on them are not presented.

In the Bjorken limit, in spin - $\frac{1}{2}$ parton models, $M Y_1, \nu Y_2, \nu Y_3, \nu Y_1', \frac{\nu^2 Y_3'}{M}, \frac{\nu^2 Y_2'}{M}, \nu Y_4'$ and $\nu Y_5'$ scale. Such dependence on ν is easily consistent with the inequalities (A15). Simply requiring that the helicity cross sections decrease as Q^{-2} in the Bjorken limit, together with (A15), in fact leads to scaling requirements which are less stringent for the Y_i' than those just listed, as was the case for W_i' . Additionally, in spin - $\frac{1}{2}$ models, $\nu Y_2 = 2Mx Y_1$, and therefore from (A15),

$$\frac{\nu^2 Y_2'}{2Mx} - \nu Y_1' + \frac{\nu Y_5'}{x} = 0$$

$$\underline{R_{\mu\nu}(q, P, S)}$$

As for $Y_{\mu\nu}(q, P, S)$ in (A12), one may write

$$R_{\mu\nu}(q, P, S) = R_{\mu\nu}(q, P) + R_{\mu\nu}'(q, P, S) \quad (A16)$$

where $R_{\mu\nu}(q, P)$ is the spin averaged hadronic tensor relevant to section II. $R_{\mu\nu}(q, P)$ has exactly the same form as $Y_{\mu\nu}(q, P)$ in (A13) with R_i replacing Y_i as the tensor coefficients, except for one additional restriction. Since $R_{\mu\nu}(q, P, S)$ depends linearly on the conserved J_μ^α , $q^\mu q^\nu R_{\mu\nu}(q, P, S) = 0$, and thus $R_5 = \frac{-q^2}{2P \cdot q} R_4$.

There are only four truly independent spin - averaged structure functions. Similarly

$R'_{\mu\nu}(q, P, S)$ has the same form as $Y'_{\mu\nu}(q, P, S)$ in (A14), with the constraint

$$R'_9 = -\frac{q^2}{2M^2} R'_7 - \frac{P \cdot q}{M^2} R'_8.$$

Because it involves the interference of J_μ^S and J_μ^W in (A3),

$R_{\mu\nu}(q, P, S)$ is not semi - positive definite. But $L_{\mu\nu} = W_{\mu\nu} + \lambda R_{\mu\nu} + \lambda^2 Y_{\mu\nu}$

is semi-positive definite for all real λ . This leads to a complicated set of Schwartz - type inequalities. For the spin averaged structure functions that still contribute to the cross section in the limit $m = 0$, they are

$$\begin{aligned} \left[R_1 \pm \frac{R_3}{2M} \sqrt{\nu^2 - q^2} \right]^2 &\leq 4W_1 \left[Y_1 \pm \frac{Y_3}{2M} \sqrt{\nu^2 - q^2} \right] \\ \left[R_2 \left(1 - \frac{\nu^2}{q^2} \right) - R_1 \right]^2 &\leq 4 \left[W_2 \left(1 - \frac{\nu^2}{q^2} \right) - W_1 \right] \left[Y_2 \left(1 - \frac{\nu^2}{q^2} \right) - Y_1 \right] \end{aligned} \quad (A17)$$

Constraints on the spin dependent R'_i are too awkward and numerous to list conveniently.

In spin - $\frac{1}{2}$ parton models, MR_1 , νR_2 , νR_3 , $\nu R'_1$, $\frac{\nu^2 R'_2}{M}$, $\frac{\nu^2 R'_3}{M}$, $\nu R'_4$ and $\nu R'_5$ scale, analogous to Y_i and Y'_i . Of course the dependence of R_i and R'_i on ν can never violate the behaviour allowed by the positivity inequalities; note that the behaviour assumed for all structure functions here is compatible with (A17).

Furthermore, positivity inequalities like (A17) show that spin - $\frac{1}{2}$ behaviour for the W_i and Y_i leads to similar behaviour for the R_i and R'_i : $\nu R_2 = 2Mx R_1$

and $\frac{\nu^2 R'_2}{2Mx} - \nu R'_1 + \frac{\nu R'_5}{x} = 0$.

APPENDIX B

In section II, the kinematics of $\ell^- + N \rightarrow \ell^- + \text{anything}$ was considered in the approximation $m = 0$. Here the cross section is presented keeping all terms dependent on the lepton mass m . An estimate of the magnitude of these additional terms is given.

Figure 1 describes the process. Let $\underline{k}, E$ ($\underline{k}', E'$) denote the laboratory momentum and energy of the incoming (outgoing) lepton. Let $\underline{\Lambda}$ be the rest frame average polarization vector of the incoming lepton ($|\underline{\Lambda}| \leq 1$) with component $\underline{\Lambda}_\perp$ orthogonal to $\underline{k}$, and component λ parallel to $\underline{k}$. (λ is the average helicity defined in section II.) Define ϕ to be the azimuthal angle for $\underline{k}'$, where $\underline{k}$ defines the z -axis and $\underline{\Lambda}_\perp$ the x -axis in the laboratory. Let $v = (E^2 - m^2)^{1/2}/E$, the incoming lepton velocity.

The exact cross section of II with all lepton mass - dependent terms, for ℓ^- , is

$$\frac{d\sigma^-}{dQ^2 d\nu d\phi} = \frac{\alpha^2}{2(E^2 - m^2)Q^4} \left[W + \frac{\beta Q^2 R}{M^2} + 2 \left(\frac{\beta Q^2}{M^2} \right)^2 Y \right] \quad (B1)$$

where

$$W = 2W_1(Q^2 - 2m^2) + W_2 \{ 4E(E-\nu) - Q^2 \}, \quad (B2)$$

$$\begin{aligned} R = & 2R_1 \left\{ Q^2 \left(1 - \frac{\lambda}{v} \right) - 2m^2 \left(1 + \frac{\lambda v}{vE} \right) + 2m \underline{\Lambda}_\perp \cdot \underline{k}' \right\} \\ & + R_2 \left\{ [4E(E-\nu) - Q^2] (1 - \lambda v) + \frac{\lambda m^2 (Q^2 + 2E\nu)^2}{v Q^2 E^2} - 2m \underline{\Lambda}_\perp \cdot \underline{k}' \left(1 + \frac{2E\nu}{Q^2} \right) \right\} \\ & - \frac{R_3}{M} \left\{ (2E-\nu) Q^2 (1 - \lambda v) + \frac{\lambda m^2 \nu (Q^2 + 2E\nu)}{v E^2} - 2m \nu \underline{\Lambda}_\perp \cdot \underline{k}' \right\} \\ & + \frac{2(R_5 + R_6)}{M} \left\{ \frac{\lambda m^2 (Q^2 + 2E\nu)}{v E} - 2m E \underline{\Lambda}_\perp \cdot \underline{k}' \right\} \end{aligned} \quad (B3)$$

and

$$\begin{aligned}
 Y = & 2Y_1 \left\{ Q^2 \left(1 - \frac{\lambda}{V}\right) + m^2 - \frac{2m^2 \nu \lambda}{EV} + 2m \lambda_{\perp} \cdot k' \right\} \\
 & + Y_2 \left\{ [4E(E-\nu) - Q^2] (1-\lambda V) + \frac{\lambda m^2 (Q^2 + 2E\nu)^2}{V Q^2 E^2} - 2m^2 \left(1 + \frac{\nu^2}{Q^2}\right) - \left(1 + \frac{2E\nu}{Q^2}\right) 2m \lambda_{\perp} \cdot k' \right\} \\
 & - \frac{Y_3}{M} \left\{ (2E-\nu) Q^2 (1-\lambda V) - \frac{\lambda m^2 (E-\nu)(Q^2 + 2E\nu)}{VE^2} + 2m \lambda_{\perp} \cdot k' (E-\nu) \right\} \\
 & + 2Y_4 \frac{m^2 Q^2}{M^2} - \frac{4Y_5}{M} \left\{ m^2 \nu \left(1 - \frac{\lambda}{V}\right) - \frac{\lambda m^2 Q^2}{2VE} + m E \lambda_{\perp} \cdot k' \right\}
 \end{aligned} \tag{B4}$$

$R_6(q^2, q, P)$ in (B3) is a new real structure function, only appearing in $d\sigma^-$ when $m \neq 0$, and defined by

$$\begin{aligned}
 \tilde{R}_{\mu\nu}(q, P) & \equiv \frac{4\pi^2 P_0}{M} \int d^4x e^{iq \cdot x} \langle P_{in} | J_{\mu}^{\delta}(x) J_{\nu}^W(0) - J_{\mu}^W(x) J_{\nu}^{\delta}(0) | P_{in} \rangle \\
 & = \frac{R_6}{M^2} (P_{\mu} q_{\nu} - P_{\nu} q_{\mu})
 \end{aligned}$$

The last equality follows from time reversal invariance and the hermiticity of the currents.

In the Bjorken limit, ν scales. As $m \rightarrow 0$, $\nu \rightarrow 1$, and (B1) $\rightarrow$ (2.16).

Error Estimates

The error in setting $m = 0$ in (B1) is considered in the Bjorken limit at high energy. "High energy" in this context means $E' \sim E \gg M$, so that in examining orders of magnitude, E' can be replaced by E . Similarly we keep x finite, so that Q^2 and $2M\nu$ are the same order of magnitude. In this region, $Q^2 = 4EE' \sin^2(\frac{1}{2}\theta)$ and

$\nu = E - E'$, where θ is the laboratory scattering angle for the lepton. Since $Q^2 \leq 2M\nu$, $\theta \lesssim \sqrt{\frac{2M}{E'}}$, and scattering is very forward-peaked.

The m -dependent terms occurring in the coefficients of W_i , R_i and Y_i in (B2) can be examined. For a given W_i , the "relative error" ΔW_i is defined to be the ratio of the largest m -dependent term to the m -independent term. For Y_4 (or R_5) whose

coefficient consists only of m - dependent terms, the relative error is defined as the ratio of these terms to the Y_1 (or R_1) term, assuming scaling to compared the magnitudes of the different structure functions.

Define the C.M. energy squared of the $\ell^- - N$ system to be $t \sim 2ME$.

The orders of magnitude of the relative errors for the structure functions are found to be

$$\Delta W_1 \sim \frac{m^2}{Q^2}$$

$$\Delta W_2 = 0$$

$$\Delta V_1 \sim \Delta R_1 \sim \Delta V_3 \sim \Delta R_3 \sim \frac{m}{\sqrt{Q^2}} \sqrt{\frac{t}{Q^2}}$$

$$\Delta V_2 \sim \Delta R_2 \sim \frac{m}{\sqrt{t}}$$

$$\Delta V_4 \sim \frac{m^2}{Q^2}$$

$$\Delta V_5 \sim \Delta R_5 \sim \Delta R_6 \sim \frac{m}{\sqrt{Q^2}} \left(\frac{t}{Q^2} \right)^{3/2}$$

The dominant error in the high energy region comes from ignoring V_5 , R_5 and R_6 in the cross section, assuming scaling, and is usually small.

APPENDIX C

STRUCTURE FUNCTION	PARTON MODEL FORMULA
$2MxW_1^N = \nu W_2^N$	$x \sum_{i=1}^3 e_i^2 (G_i^N + G_{\bar{i}}^N)$
$2MxR_1^N = \nu R_2^N$	$2x \sum_{i=1}^3 e_i t_i (G_i^N + G_{\bar{i}}^N)$
νR_3^N	$-2 \sum_{i=1}^3 e_i t_i (G_i^N - G_{\bar{i}}^N)$
$2MxY_1^N = \nu Y_2^N$	$2x \sum_{i=1}^3 t_i^2 (G_i^N + G_{\bar{i}}^N)$
νY_3^N	$-2 \sum_{i=1}^3 t_i^2 (G_i^N - G_{\bar{i}}^N)$

TABLE 2. The scaling structure functions of section II in the generalized quark - parton model of section IIIA. G_i^N are the quark distribution functions for a nucleon of type N, $N = (p, n)$. (1.2), (1.4) and (3.6) specify the assumed interactions.

G_1^P	$\frac{35}{16}c + \frac{1}{6}v$
$G_{\bar{1}}^P$	$\frac{1}{6}v$
G_2^P	$\frac{35}{32}c + \frac{1}{6}v$
$G_{\bar{2}}^P$	$\frac{1}{6}v$
G_3^P	$\frac{1}{6}v$
$G_{\bar{3}}^P$	$\frac{1}{6}v$

TABLE 3. $G_i^P(x)$ in the Kuti - Weisskopf parton model⁶. $c \equiv (1-x)^3/x^{1/2}$; $v \equiv (1-x)^{3/2}/x$. The adjustable parameters of Ref. 6 have the values $g = 1$, $g' = 2$, $\alpha = \frac{1}{2}$. G_i^n can be found by an isotopic rotation.

PROTON	NEUTRON
$\nu W_2^p = \frac{35}{32} x^{1/2} (1-x)^3 + \frac{2}{9} (1-x)^{7/2}$	$\nu W_2^n = \frac{35}{48} x^{1/2} (1-x)^3 + \frac{2}{9} (1-x)^{7/2}$
$2MW_1^p = \frac{35}{32} \frac{(1-x)^3}{x^{1/2}} + \frac{2}{9} \frac{(1-x)^{7/2}}{x}$	$2MW_1^n = \frac{35}{48} \frac{(1-x)^3}{x^{1/2}} + \frac{2}{9} \frac{(1-x)^{7/2}}{x}$
$\nu Y_2^p = \frac{105}{64} x^{1/2} (1-x)^3 + \frac{1}{3} (1-x)^{7/2}$	$\nu Y_2^n = \frac{105}{64} x^{1/2} (1-x)^3 + \frac{1}{3} (1-x)^{7/2}$
$2MY_1^p = \frac{105}{64} \frac{(1-x)^3}{x^{1/2}} + \frac{1}{3} \frac{(1-x)^{7/2}}{x}$	$2MY_1^n = \frac{105}{64} \frac{(1-x)^3}{x^{1/2}} + \frac{1}{3} \frac{(1-x)^{7/2}}{x}$
$\nu Y_3^p = \frac{-105}{64} \frac{(1-x)^3}{x^{1/2}}$	$\nu Y_3^n = \frac{-105}{64} \frac{(1-x)^3}{x^{1/2}}$
$\nu R_2^p = \frac{175}{96} x^{1/2} (1-x)^3 + \frac{1}{3} (1-x)^{7/2}$	$\nu R_2^n = \frac{35}{24} x^{1/2} (1-x)^3 + \frac{1}{3} (1-x)^{7/2}$
$2MR_1^p = \frac{175}{96} \frac{(1-x)^3}{x^{1/2}} + \frac{1}{3} \frac{(1-x)^{7/2}}{x}$	$2MR_1^n = \frac{35}{24} \frac{(1-x)^3}{x^{1/2}} + \frac{1}{3} \frac{(1-x)^{7/2}}{x}$
$\nu R_3^p = \frac{-175}{96} \frac{(1-x)^3}{x^{1/2}}$	$\nu R_3^n = \frac{-35}{24} \frac{(1-x)^3}{x^{1/2}}$

TABLE 4 . The scaling structure functions of section II in the Kuti - Weisskopf parton model⁶. (1.2) , (1.4) and (3.6) specify the assumed interactions.

G_{iv}^p	σ_i^p	G_{2v}^p	σ_2^p
$\frac{35}{16} \frac{(1-x)^3}{x^{1/2}}$	$2/3$	$\frac{35}{32} \frac{(1-x)^3}{x^{1/2}}$	$-1/3$

TABLE 5: Non - zero G_{iv}^p and σ_i^p of section VI in the KWM⁶.

The corresponding neutron values can be found by an isotopic rotation.

STRUCTURE FUNCTION	PARTON MODEL FORMULA	PROTON	NEUTRON
$\nu W_4'$	$\frac{1}{2} \sum_{i=1}^2 e_i^2 \sigma_i G_{iv}$	$\frac{175}{576} c$	0
$\nu Y_1'$	$-\sum_{i=1}^2 \sigma_i t_i^2 G_{iv}$	$-\frac{35}{128} c$	$-\frac{35}{128} c$
$\frac{\nu^2 Y_3'}{M}$	$\sum_i \sigma_i t_i^2 G_{iv}$	$\frac{35}{128} c$	$\frac{35}{128} c$
$\nu Y_4'$	$\frac{1}{2} \sum_i \sigma_i t_i^2 G_{iv}$	$\frac{35}{256} c$	$\frac{35}{256} c$
$\nu Y_5'$	$-x \sum_i \sigma_i t_i^2 G_{iv}$	$-\frac{35}{128} x c$	$-\frac{35}{128} x c$
$\nu R_1'$	$-\sum_i \sigma_i e_i t_i G_{iv}$	$-\frac{245}{576} c$	$-\frac{35}{288} c$
$\nu R_4'$	$\sum_i \sigma_i e_i t_i G_{iv}$	$\frac{245}{576} c$	$\frac{35}{288} c$
$\nu R_5'$	$-x \sum_i \sigma_i e_i t_i G_{iv}$	$-\frac{245}{576} x c$	$-\frac{35}{288} x c$

TABLE 6 : The spin - dependent scaling structure functions in the version of the KWM used in section VI. $c = (1-x)^3/x^{1/2}$. Only those non-zero functions which contribute in the limit $m=0$ are shown. G_{iv} and σ_i are listed in Table 5. (1.2), (1.4) and (3.6) specify the assumed interactions.

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10. The only possible s - dependent term in the cross section that is not parity violating is $\epsilon_{\mu\nu\alpha\beta} p^\mu s^\nu k^\alpha k'^\beta$. For a longitudinally polarized particle, this is identically zero. For arbitrary polarizations, from time reversal invariance, this term is still absent in the Born approximation used in this paper.

11. For $\theta_c = 0$, simple isotopic spin considerations¹² alone lead to the weaker version of (3.14), $\langle R_1 \rangle \leq \langle W_1^y \rangle$, $\langle R_2 \rangle \leq \langle W_2^y \rangle$, and $2\langle R_3 \rangle = \langle W_3^y \rangle$. The stronger assumption of asymptotic chiral symmetry in the Bjorken limit, with $\theta_c = 0$, is enough to ensure (3.14) itself. The quark parton model, with free currents and negligible masses in this limit, satisfies exact chiral symmetry.
12. The method is analogous to that of A. Pais and S.B. Treiman, Rockefeller Report No. COO - 3505 - 25.
13. All spin - dependent structure functions are denoted by a prime. In terms of the d and g functions of Kuti and Weisskopf⁶, $d = 4\pi W_4'$, $g = \frac{2\pi W_3'}{M^2}$.
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FIGURES

Figure 1 . Feynman diagrams for inelastic $\ell^+ - N$ scattering in the presence of a weak neutral current interacting as in (1.4). (a) electromagnetic contribution; (b) weak contribution.

Figures 2a-d. The lepton spin asymmetry. Contour lines of constant percentage asymmetry $[a(Q^2, \nu) \times 100]$ for various energies E and coupling constants G , for a proton target, calculated in the Kuti - Weisskopf parton model, are shown. The effect of an intermediate vector boson of mass M_Z is indicated.

- (a) $E = 100 \text{ GeV}$, $G = G_f$. (b) $E = 200 \text{ GeV}$, $G = G_f$
 (c) $E = 4260 \text{ GeV}$, $G = G_f$ (d) $E = 4260 \text{ GeV}$, $G = -G_f$.

Figures 3a-d. The lepton charge asymmetry. Contour lines of constant percentage asymmetry $[c(Q^2, \nu) \times 100]$ for various energies E and coupling constants G , for a proton target, calculated in the Kuti - Weisskopf parton model, are shown. The effect of an intermediate vector boson of mass M_Z is indicated.

- (a) $E = 100 \text{ GeV}$, $G = G_f$ (b) $E = 200 \text{ GeV}$, $G = G_f$
 (c) $E = 4260 \text{ GeV}$, $G = G_f$ (d) $E = 4260 \text{ GeV}$, $G = -G_f$.

Figure 4. Percentage deviation Δ (equation 4.9) attainable for $E = 200 \text{ GeV}$, $G = G_f$. Contour lines of constant percentage asymmetry $[\Delta(Q^2, \nu) \times 100]$ for a proton target, calculated in the Kuti - Weisskopf model, are shown.

Figures 5a-d. The nucleon spin asymmetry. Contour lines of constant percentage asymmetry $[A(Q^2, \nu) \times 100]$ for various energies E and coupling constant G , for a proton target, calculated in the model of section VI, are shown. The effect of an intermediate vector boson of mass M_Z is indicated.

- (a) $E = 100 \text{ GeV}$, $G = G_f$. (b) $E = 200 \text{ GeV}$, $G = G_f$
 (c) $E = 4260 \text{ GeV}$, $G = G_f$ (d) $E = 4260 \text{ GeV}$, $G = -G_f$.

$$q \cdot P = M\nu; \quad q^2 = -Q^2$$

(a)

+

(b)

FIGURE 1

FIGURE 2a

FIGURE 2b.

FIGURE 2c

FIGURE 2d

FIGURE 3a

FIGURE 3b

FIGURE 3c

FIGURE 3d

FIGURE 4

FIGURE 5a

FIGURE 5b

FIGURE 5d.