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Physics

PHOTOELASTIC ANALYSIS OF STRESS WAVES
RESULTING FROM EXPLODING WIRES (U)

W. H. Thompson

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UNION CARBIDE CORPORATION
NUCLEAR DIVISION
OAK RIDGE Y-12 PLANT

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With the US Atomic Energy CommissionPHOTOELASTIC ANALYSIS OF STRESS WAVES RESULTING
FROM EXPLODING WIRES

W. H. Thompson

This report is based on a study by the author as partial
fulfillment of requirements for the degree of Master of
Science in Physics from The University of Tennessee.

Oak Ridge, Tennessee

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ABSTRACT

This thesis involves the development of a photoelastic system capable of visualizing dynamic stress waves traveling through transparent plastics. Large amplitude stresses were created in plastics with relatively high elastic moduli by the force imparted by an exploding wire. A 42,000-joule energy-storage capacitor bank provided the energy necessary to explode the wire.

The type of wave traveling through the plastic plates was found to be analogous to Lamb wave propagation in plates. A piezoelectric transducer was used along with the photoelastic pictures to verify this analogy.

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CHAPTER I

INTRODUCTION

Photoelasticity has been applied to innumerable static stress problems since its discovery in England by Sir David Brewster well over a century ago, when stressed glass was found to exhibit colored patterns under polarized light. With the development of plastics, particularly those with low stress-optic constants and elastic moduli, the technique has grown immensely and has been used to study dynamic problems. However, little work has been done to study stresses and stress-wave propagation in plastics with large stress-optic constants and elastic moduli. These plastics are more closely related to actual fabrication materials, such as steel and aluminum, than the lower moduli plastics, therefore a study of their behavior under stress would be more useful.

A means of applying short-wavelength stresses of sufficient magnitude to produce interference fringes in these plastics is provided by an exploding wire. Energy stored in a capacitor bank can be discharged into a small wire causing it to explode in an extremely short time with sufficient force to fracture many materials in contact with the wire. The wave length of this pulse is short enough for the pulse to be followed through a plastic plate with dimensions greater than about two inches.

This thesis reports the development of a photoelastic system capable of visualizing the stress wave propagation in plastics, and the subsequent discovery

that waves analogous to Lamb waves were excited in plates of different thicknesses. A plastic with a large stress-optic constant and one with a medium constant were employed in the experiments. The velocities of the waves were computed for the two materials.

The individual stress fringes were indistinguishable because of the short wavelength of the pulse; therefore, no absolute measurements of stress could be made. However, calibration curves of stress-optic constants were made for the two materials used, and these place a lower limit on the stresses. A piezoelectric ultrasonic transducer was used to verify the type of wave propagation, and its calibration helped to ascertain peak stresses.

CHAPTER II

THEORY

Photoelasticity

The basis of photoelasticity is the temporary birefringence displayed by certain materials when stressed. When a beam of plane polarized light passes through a transparent solid which is under stress, the electric-field vector of the light beam splits into two beams polarized at right angles to each other and parallel to the directions of the principal stresses, as shown in Figure 1. It was established by Brewster that the index of refraction is different for the two directions; hence, one beam will take longer to traverse the medium than the other. The relative retardation between the two beams produces interference fringes in the medium which are proportional to the difference of the principal stresses. The two-dimensional stress-optic law (Frocht, 1949) has been shown to be:

$$n = \frac{t(p - q)}{2C}, \quad (1)$$

where:

n = the fringe order,

t = the thickness of the medium (in.),

p = the algebraically largest principal stress (p.s.i.),

q = the algebraically smallest principal stress (p.s.i.), and

C = the stress-optic constant $\left(\frac{\text{p.s.i.-in.}}{\text{fringe}} \right)$.

FIGURE 1

ILLUSTRATION OF THE BIREFRINGENCE PHENOMENON

Holding the stresses constant and doubling the thickness, therefore, doubles the number of fringes in the material. The stress-optic constant is a function of the material and its thickness. Materials with a small C will display more fringes per unit stress than those with a large C and equal thickness.

A calibration of the material to obtain C will allow measurements of principal stress differences by merely counting the fringes. The direction of the stresses may be determined by rotating the direction of the light polarization. When the polarization is along a principal stress direction, there will be no splitting of the E vectors and, consequently, no relative retardation of the light beams.

If the light beam consists of several wavelengths, the interference fringes will consist of color cycles corresponding to the wavelengths of the light source. A fringe order is thus obtained by counting the number of fringes of a given wavelength.

To avoid the disappearance of fringes when the polarization happens to be along a principal stress direction, circularly polarized light may be used rather than plane polarized light. This type of light propagation is usually accomplished with quarter-wave plates. These plates are anisotropic in two orthogonal directions. If plane polarized light enters the plate with the E vector at a 45-degree angle to these directions, the light will split into equal components along them. The velocity of the components is different, and they will emerge out of phase. The quarter-wave plate is of the proper thickness to produce a phase difference of

a quarter of a wavelength. Therefore, a rotating E vector of constant amplitude is produced, which is by definition, circularly polarized light. Since the thickness of the plate depends on the wavelength of the incident light, it is clear that only monochromatic light can be circularly polarized in this way.

Lamb Waves

In 1916, Horace Lamb published a paper in which he mathematically proved that, under certain conditions, an infinite number of vibrational waves could travel in an infinite plate. Little work had been done to experimentally generate Lamb waves in elastic media until the last decade, when investigations (Fitch, 1962 and Worlton, 1957) of the use of Lamb waves in nondestructive testing were begun.

Lamb waves (Grisby and Tajchman, 1960) are waves which can exist in an isotropic-lossless elastic plate of finite thickness but of infinite length and width which is bounded on both of its faces by vacuum, and which are independent of a direction which is parallel to the free surfaces and normal to the propagation direction. The classical method of obtaining the equation relating phase velocity to wave length is by using Hooke's law, Newton's second law of motion, and the boundary conditions that the tensional stress normal to the free surfaces and the shear stress tangential to the free surfaces must vanish.

Consider a differential element of volume, as shown in Figure 2. The stress on the surface of this element, in general, has components both normal to

FIGURE 2

STRESS COMPONENTS ACTING ON AN INFINITESIMAL
RECTANGULAR PARALLELEPIPED

the plane and tangential to it. The normal or tensional stresses are denoted by σ_{xx} , σ_{yy} , and σ_{zz} ; the tangential or shearing stresses, by σ_{xy} , σ_{yx} , σ_{xz} , σ_{zx} , σ_{yz} , and σ_{zy} . For equilibrium:

$$\sigma_{xy} = \sigma_{yx},$$

$$\sigma_{xz} = \sigma_{zx},$$

$$\sigma_{yz} = \sigma_{zy}.$$

Therefore, only six independent components of stress remain.

The displacement of any point in the element may be resolved into components u , v , and w , parallel to the x , y , and z axes, respectively. If a point, P , is displaced from coordinates (x, y, z) , its new position is $(x + u, y + v, z + w)$. To define the strain, consider a point close to P whose coordinates without strain are $(x + dx, y + dy, z + dz)$. Let the displacement under strain have components $(u + du, v + dv, w + dw)$. Then if dx , dy , and dz are small:

$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy + \frac{\partial u}{\partial z} dz, \quad (2)$$

$$dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy + \frac{\partial v}{\partial z} dz,$$

$$dw = \frac{\partial w}{\partial x} dx + \frac{\partial w}{\partial y} dy + \frac{\partial w}{\partial z} dz.$$

The three components of tensional and compressional strain (Kolsky, 1963) are:

$$\epsilon_{xx} = \frac{\partial u}{\partial x}, \quad \epsilon_{yy} = \frac{\partial v}{\partial y}, \quad \epsilon_{zz} = \frac{\partial w}{\partial z}. \quad (3)$$

The three components of shearing strain are:

$$\epsilon_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}, \epsilon_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \epsilon_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}. \quad (4)$$

In addition to the six components of strain, there are three components of rigid-body rotation given by:

$$\bar{W}_x = 1/2 \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right), \bar{W}_y = 1/2 \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right), \bar{W}_z = 1/2 \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right). \quad (5)$$

When these last three components are zero, the deformation is irrotational and is described as pure strain.

It is found experimentally that for most solids the strain is proportional to the stress below the elastic limit for the solid. Stated mathematically, Hooke's law is:

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \epsilon_{yz} \\ \epsilon_{zx} \\ \epsilon_{xy} \end{pmatrix}, \quad (6)$$

where the coefficients are the elastic constants of the material. In an isotropic solid, the values of the coefficients must be independent of the set of rectangular axes chosen. If this condition is applied to the previously listed equations, only two independent constants remain (denoted by λ and μ) and are known as Lamé's constants. Hooke's law then becomes:

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{pmatrix} = \begin{pmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \epsilon_{yz} \\ \epsilon_{zx} \\ \epsilon_{xy} \end{pmatrix} \quad (7)$$

To obtain the equations of motion, the variation in stress across a small element is considered, as shown in Figure 3. Let F_x be the resultant force, and a_x , the acceleration along the x axis. Then:

$$\begin{aligned}
 F_x = & \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} dx \right) dydz - \sigma_{xx} dydz \\
 & + \left(\sigma_{xy} + \frac{\partial \sigma_{xy}}{\partial y} dy \right) dx dz - \sigma_{xy} dx dz \\
 & + \left(\sigma_{xz} + \frac{\partial \sigma_{xz}}{\partial z} dz \right) dx dy - \sigma_{xz} dx dy,
 \end{aligned}$$

or:

$$F_x = \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} \right) dx dy dz. \quad (8)$$

Neglecting body forces such as gravity, Newton's second law of motion states:

$$\begin{aligned}
 F_x &= m a_x \\
 &= (\rho dx dy dz) \frac{\partial^2 u}{\partial t^2} = \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} \right) dx dy dz. \quad (9)
 \end{aligned}$$

FIGURE 3

STRESSES ACTING ON A SMALL RECTANGULAR
PARALLELEPIPED

Substituting Hooke's law for an isotropic medium gives:

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x} \left[(\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}) \lambda + 2\mu \epsilon_{xx} \right] + \frac{\partial}{\partial y} (\mu \epsilon_{xy}) + \frac{\partial}{\partial z} (\mu \epsilon_{xz}). \quad (10)$$

By definition (Equations 3 and 4) this becomes:

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + \mu) \frac{\partial (\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz})}{\partial x} + \mu \nabla^2 u, \quad (11)$$

where:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

Similarly, for the y and z axes:

$$\rho \frac{\partial^2 v}{\partial t^2} = (\lambda + \mu) \frac{\partial (\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz})}{\partial y} + \mu \nabla^2 v, \quad (12)$$

$$\rho \frac{\partial^2 w}{\partial t^2} = (\lambda + \mu) \frac{\partial (\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz})}{\partial z} + \mu \nabla^2 w. \quad (13)$$

These are the equations of motion of an isotropic elastic solid in which body forces are absent.

Lamb wave motion is independent of z; i.e., all cross sections of the plate parallel to the xy plane are identical. The plate is assumed to be symmetrical about the y axis. Consequently, it is required to satisfy only Equations 11 and 12. The solution for these equations (Grigsby and Tajchman, 1960) is:

$$u = \frac{\partial \phi}{\partial x} + \frac{\partial \Psi}{\partial y} \quad \text{and} \quad v = \frac{\partial \phi}{\partial y} - \frac{\partial \Psi}{\partial x}, \quad (14)$$

where ϕ and Ψ satisfy the equations:

$$\rho \frac{\partial^2 \phi}{\partial t^2} = (\lambda + 2\mu) \nabla^2 \phi \quad \text{and} \quad \rho \frac{\partial^2 \Psi}{\partial t^2} = \mu \nabla^2 \Psi. \quad (15)$$

Since $V_l = \sqrt{\frac{\lambda + 2\mu}{\rho}}$ = the longitudinal wave velocity, and $V_s = \sqrt{\frac{\mu}{\rho}}$ = the shear wave velocity, Equation 15 may be written:

$$\frac{\partial^2 \phi}{\partial t^2} = V_l^2 \nabla^2 \phi \quad \text{and} \quad \frac{\partial^2 \Psi}{\partial t^2} = V_s^2 \nabla^2 \Psi.$$

Assuming periodicity with respect to x and t :

$$\phi = \phi(y) e^{i(\omega t - kx)}, \quad \text{and} \quad (16)$$

$$\Psi = \Psi(y) e^{i(\omega t - kx)},$$

where:

ω = the angular frequency = $2\pi f$,

k = the wave number = $\frac{2\pi}{\lambda'}$, and

λ' = the wavelength.

Therefore:

$$\frac{\partial^2 \phi}{\partial t^2} = -\omega^2 \phi = V_l^2 \nabla^2 \phi,$$

$$\frac{\partial^2 \phi}{\partial x^2} = -k^2 \phi, \quad \text{and}$$

$$\frac{\partial^2 \phi}{\partial y^2} = \left(k^2 - \frac{\omega^2}{V_l^2} \right) \phi.$$

Let V be defined as the phase velocity $= f\lambda'$.

Then:

$$\frac{\partial^2 \phi}{\partial y^2} = \left(k \sqrt{1 - \frac{V^2}{V_l^2}} \right)^2 \phi = \gamma^2 \phi. \quad (17)$$

Similarly,

$$\frac{\partial^2 \Psi}{\partial y^2} = \left(k \sqrt{1 - \frac{V^2}{V_s^2}} \right)^2 \Psi = \beta^2 \Psi. \quad (18)$$

The solutions to these equations are:

$$\phi(y) = ae^{\gamma y} + be^{-\gamma y}, \text{ and}$$

$$\Psi(y) = ce^{\beta y} + de^{-\beta y},$$

so that:

$$\phi(x, y, t) = (A \sinh \gamma y + B \cosh \gamma y) e^{i(\omega t - kx)}, \text{ and}$$

$$\Psi(x, y, t) = (C \sinh \beta y + D \cosh \beta y) e^{i(\omega t - kx)}.$$

To solve for the constants, the boundary conditions that $\sigma_{yy} = \sigma_{xy} = 0$ at $y = \pm d/2$ are applied, where d is the plate thickness. σ_{yy} and σ_{xy} are given by

Hooke's law:

$$\sigma_{yy} = \lambda (\epsilon_{xx} + \epsilon_{yy}) + 2\mu \epsilon_{yy}, \text{ but} \quad (19)$$

$$\epsilon_{xx} + \epsilon_{yy} = \nabla^2 \phi = (\gamma^2 - k^2) \phi, \text{ and}$$

$$\epsilon_{yy} \equiv \frac{\partial v}{\partial y} = \frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 \Psi}{\partial x \partial y} = \gamma^2 \phi - \frac{\partial^2 \Psi}{\partial x \partial y}.$$

$$\sigma_{yy} = \left[\lambda (\gamma^2 - k^2) (A \sinh \gamma y + B \cosh \gamma y) + 2\mu \gamma^2 (A \sinh \gamma y + B \cosh \gamma y) + 2ik C \beta \mu \cosh \beta y + 2ik D \beta \mu \sinh \beta y \right] e^{i(\omega t - kx)}. \quad (19a)$$

Collecting terms and noting that:

$$-\lambda k^2 + \lambda \gamma^2 + 2\mu \gamma^2 \equiv \mu (k^2 + \beta^2),$$

$$\sigma_{yy} = \left[A \mu (k^2 + \beta^2) \sinh \gamma y + B \mu (k^2 + \beta^2) \cosh \gamma y + C (2ik \beta \mu) \cosh \beta y + D (2ik \beta \mu) \sinh \beta y \right] e^{i(\omega t - kx)} \text{ is obtained.} \quad (19b)$$

Also:

$$\begin{aligned} \sigma_{xy} &= \mu \epsilon_{xy}, \\ &= \mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) = \mu \left(\frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \Psi}{\partial y^2} \right), \text{ or} \\ &= \left[A (-2ik \gamma \mu) \cosh \gamma y + B (-2ik \gamma \mu) \sinh \gamma y + C \mu (k^2 + \beta^2) \sinh \beta y + D \mu (k^2 + \beta^2) \cosh \beta y \right] e^{i(\omega t - kx)}. \end{aligned} \quad (20)$$

Applying the conditions that $\sigma_{yy} = \sigma_{xy} = 0$ at $y = \pm d/2$, four simultaneous equations are obtained in the unknown constants A, B, C, and D. In order to obtain a nontrivial solution, the determinant of the coefficients must vanish:

$$\begin{vmatrix} qs_1 & qc_1 & nc_2 & ns_2 \\ pc_1 & ps_1 & qs_2 & qc_2 \\ -qs_1 & qc_1 & nc_2 & -ns_2 \\ pc_1 & -ps_1 & -qs_2 & qc_2 \end{vmatrix} = 0, \quad (21)$$

where:

$$n = 2ik\beta\mu,$$

$$p = -2ik\gamma\mu,$$

$$q = \mu(k^2 + \beta^2),$$

$$s_1 = \sinh \frac{\gamma d}{2},$$

$$s_2 = \sinh \frac{\beta d}{2},$$

$$c_1 = \cosh \frac{\gamma d}{2},$$

$$c_2 = \cosh \frac{\beta d}{2}.$$

Manipulation of the determinant yields:

$$\begin{vmatrix} qs_1 & ns_2 & 0 & 0 \\ pc_1 & qc_2 & 0 & 0 \\ 0 & 0 & nc_2 & qc_1 \\ 0 & 0 & qs_2 & ps_1 \end{vmatrix} = 0, \text{ or}$$

$$\begin{vmatrix} qs_1 & ns_2 \\ pc_1 & qc_2 \end{vmatrix} \cdot \begin{vmatrix} nc_2 & qc_1 \\ qs_2 & ps_1 \end{vmatrix} = 0. \quad (22)$$

The system determinant will therefore be zero if either of these second-order determinants is zero. It is then possible to separate the original system into two independent second-order systems.

Assuming $A = D = 0$, the period equations yield:

$$\begin{aligned}\phi &= B \cosh \gamma y e^{i(\omega t - kx)}, \text{ and} \\ \Psi &= C \sinh \beta y e^{i(\omega t - kx)}.\end{aligned}\tag{23}$$

These vibrations are referred to as the symmetrical modes since the motion is symmetrical about the y axis.

Assuming $B = C = 0$, the functions are:

$$\begin{aligned}\phi &= A \sinh \gamma y e^{i(\omega t - kx)}, \text{ and} \\ \Psi &= D \cosh \beta y e^{i(\omega t - kx)}.\end{aligned}\tag{24}$$

Since these functions define a motion which is asymmetrical about the y axis, the solutions are referred to as asymmetrical modes. Usually the two modes have different velocities for a given frequency so that they are physically distinct.

The period equation (Grigsby and Tajchman, 1960) for the symmetrical mode is:

$$n p c_2 s_1 = q^2 c_1 s_2, \text{ or}$$

$$\frac{\tanh \gamma d/2}{\tanh \beta d/2} = \frac{(k^2 + \beta^2)^2}{4k^2 \gamma \beta}.$$

Substituting for γ and β gives:

$$\frac{\tanh \pi f d \sqrt{\frac{V_l^2 - V^2}{V^2 V_l^2}}}{\tanh \pi f d \sqrt{\frac{V_s^2 - V^2}{V^2 V_s^2}}} = \frac{\left(2 - \frac{V^2}{V_s^2}\right)^2}{4 \sqrt{1 - \frac{V^2}{V_l^2}} \sqrt{1 - \frac{V^2}{V_s^2}}} . \quad (25)$$

Similarly for the asymmetrical mode gives:

$$\frac{\tanh \pi f d \sqrt{\frac{V_l^2 - V^2}{V^2 V_l^2}}}{\tanh \pi f d \sqrt{\frac{V_s^2 - V^2}{V^2 V_s^2}}} = \frac{4 \sqrt{1 - \frac{V^2}{V_l^2}} \sqrt{1 - \frac{V^2}{V_s^2}}}{\left(2 - \frac{V^2}{V_s^2}\right)^2} . \quad (26)$$

Note that for certain values of phase, longitudinal, and shear velocity, the period equations contain circular tangents. This relationship leads to an infinite number of modes for both the symmetric and asymmetric vibrations.

Figure 4 is a physical picture (Bradfield, 1964) of some of these modes.

(a) First Symmetrical Mode

(c) First Asymmetrical Mode

(b) Second Symmetrical Mode

(d) Second Asymmetrical Mode

T - Tension
C - Compression

FIGURE 4
LAMB WAVE MODES

CHAPTER III

EXPERIMENTAL ARRANGEMENT

In order to photograph high-speed stress waves, an extremely fast camera or a light source with a short-duration pulse is required. Since the velocity of sound waves in Plexiglas is about 100,000 in./sec., a camera with a framing rate capable of following the waves would require a very intense light source. Less difficulty is encountered if a pulsed light source is used. However, only a single picture of an event can be photographed. To photograph an event completely, pictures must be made at different times in carefully repeated events. The light source selected for the experiment was the "Microflash" by Edgerton, Germeshausen, and Grier, Inc. This light emits a fifty-million-beam-candlepower flash of light with a duration of one-half microsecond. The flash is started by the breakdown of an air gap with a high-voltage pulse. The unit has multiple triggering methods and permits delays from five microseconds to one millisecond. The light was triggered by the magnetic field from the exploding wire.

To enlarge the beam of light so as to cover most of the ten-inch-square plastic plates and to collimate the beam, a Fresnel lens was used. Thin sheets of twelve-inch-square Polaroid plastic were used for the polarizer and analyzer. Quarter-wave plates were employed unsuccessfully since the light source was not monochromatic. The camera was a Polaroid Model 110A which takes $3\frac{1}{4}$ by $4\frac{1}{4}$ -inch pictures. A diagram of the experimental arrangement appears in Figure 5.

FIGURE 5

DIAGRAM OF PHOTOELASTIC EQUIPMENT

The capacitor unit consisted of twenty-eight paralleled capacitors rated at twenty kv. and seven and one-half microfarads each. The power supply is capable of charging the bank from zero to twenty kv. continuously. Therefore, the maximum energy available is 42,000 joules as computed from the relation $E = 1/2 CV^2$. The bank is discharged by actuating a solenoid which pulls a copper disc into a circular air gap in the copper bus of the bank. This action completes the circuit and the capacitors discharge through the load. The load for the experiments was a one-inch length of twenty-gauge copper wire.

To shield the photoelastic equipment from the light emitted by the exploding wire, a light-tight box was placed around the wire. The plastic plates were placed against the wire with about three inches of the plate inside the box. The wire was positioned on the centerline of one edge of the plates so that the wire explosion would produce a radial wave in the plates.

A photocell monitored the arcing across the solenoid-activated disc and triggered an oscilloscope, while another photocell monitored the firing of the light source. An oscilloscope trace of these monitors was used to verify the delay time for the photoelastic picture. A typical oscilloscope trace appears in Figure 6.

FIGURE 6

OSCILLOSCOPE TRACE OF EXPLODING
WIRE AND LIGHT SOURCE

CHAPTER IV

EXPERIMENTAL RESULTS

So that the stress waves in all directions could be visualized, quarter-wave plates were employed in the photoelastic device to obtain circularly polarized light. However, since the light source was not monochromatic and insufficient light was available to permit filtering, the stress fringes were not sharply defined, and the quarter-wave plates were omitted. Photoelastic pictures were taken with the polarizer and analyzer crossed (dark background) and parallel (white background). The clearest fringes were obtained with the dark background. Therefore, this technique was used with the polarizing axis at a 45-degree angle to the symmetry axis of the plate so that the stress along this line could be studied. A comparison between a dark-background picture and one with quarter-wave plates is shown in Figure 7. These pictures were taken of the stress in one-eighth-inch polystyrene resulting from a 6.7-kilojoule discharge.

A set of thirty pictures was made of the stress in a 10 by 10 by 1/8-inch sheet of polystyrene to study the dispersion of the stress wave. The energy stored in the capacitor bank for each wire explosion was 6,700 joules. This low energy was selected for two reasons: (1) at higher energies, the explosion was so forceful that light was emitted from the shielding box; (2) it was felt that at lower energies the stress fringes could be more easily distinguished since the stress wave would have a longer width at these stress levels. The pictures follow the wave from the

Dark Background

118916(U)

Quarter-Wave Plates

118917(U)

FIGURE 7

COMPARISON BETWEEN TWO PHOTOELASTIC TECHNIQUES

time of arrival of the first white fringe at the edge of the light-tight box until this fringe has reflected from the opposite edge of the plate and traveled half the distance back to the exploded edge—a period of about 130 microseconds. The reflections from the edges blur the original fringe at this point, and it is impossible to follow the separate fringes. At least twenty-two fringes can be noted in these photographs. Some of the pictures from this sequence are shown in Figure 8. A plot was made of the first, second, fifth, and shear fringes as a function of the distance from the explosion to determine the dispersion of the wave. This curve is shown in Figure 9, and it indicates that the dispersion is negligible over the length of the plate. The curves do not go through the origin because all distances were measured to the center of the fringes. From the slopes of the curves, the velocities of these fringes were computed as follows:

- 1st fringe - 72,400 in./sec.,
- 2nd fringe - 71,600 in./sec.,
- 5th fringe - 70,900 in./sec., and
- Shear fringe - 44,700 in./sec.

The velocity of the shear fringe is extremely close to the published value of 44,100 in./sec. The velocities of the other fringes, however, are only about 80 per cent of the longitudinal velocity of sound in polystyrene—88,200 in./sec. The decrease in velocity with increasing fringe order indicates a slight dispersion of the wave.

FIGURE 8

A SEQUENCE OF PHOTOELASTIC PICTURES DEPICTING
STRESS FRINGES IN POLYSTYRENE

FIGURE 9
FRINGE DISPLACEMENT IN POLYSTYRENE

The regular spacing of the fringes and their low velocity were the first indications that the stress wave was not a cylindrical longitudinal wave. In addition, the fringes were not colored when photographed on color film. The only color was seen in the shear fringe which contained several colored fringes. This situation indicated that the other white fringes were not regular stress fringes; i.e., they could not just be counted to determine the stress magnitude. Therefore, the white fringes were thought to contain the regular fringes which would be colored, but the Fresnel lens did not collimate the light sufficiently to reveal these fringes.

To determine that the photoelastic system was capable of detecting colored fringes, a static load was applied to a plastic tensile specimen. This photograph, shown in Figure 10, indicates that the system can detect colored fringes.

Photographs were taken of the stress wave in a 10 by 10 by 1/4-inch Plexiglas plate. The number of fringes decreased as expected due to the difference in the stress-optic constants between Plexiglas and polystyrene. However, the phase velocity, though sufficient sequences for a very accurate measurement were not made, was found to be about 15 per cent lower than longitudinal velocity. Again, the fringes did not disperse appreciably over the length of the plate.

Since it had been determined that the stress could not be evaluated by counting fringes, another method for visualizing the shape and magnitude of the stress wave was sought. A semiconductor strain gage was mounted on one of the one-eighth-inch-thick polystyrene plates. It was hoped that the strain gage would

118953(U)

FIGURE 10

STATIC STRESS FRINGES IN A TENSILE SPECIMEN
(The Fringes Are Shown in Their True Colors
in the Original Photograph)

allow for the calculation of the peak stresses in the plate, thus furnishing a limit to the number of regular, colored stress fringes which were blurred together in the white fringes of the photoelastic pictures. However, the induced current from the field associated with the wire explosions (6.7 kilojoules) was superimposed on the strain gage signal, making it impossible to distinguish the two signals. Figure 11 shows the oscilloscope trace of the strain gage signal and the photoelastic picture indicating stress was in the plate. The picture was taken just as the shear fringe was leaving the gage. Because it is practically impossible to shield a strain gage completely from magnetic fields, particularly those of the magnitudes encountered in this experiment, another means of evaluating the stress wave was desired.

A piezoelectric ultrasonic transducer was attached to the edge of a one-quarter-inch-thick Plexiglas plate opposite the wire explosion and along the center-line of the plate. This transducer was a commercially manufactured, 2.25 megacycle, barium titanate crystal, 0.375 inch in diameter. The output of this transducer was connected to the oscilloscope. The energy stored in the capacitor bank was 2,600 joules. This lower value was used to protect the transducer. The oscilloscope trace indicated a high-frequency vibration in the plate which coincided with the fringes in the photoelastic experiment.

The transducer gave definite proof, then, that the fringes were not regular photoelastic fringes but, rather, they contained the photoelastic fringes. Experiments were conducted with the transducer to determine the type of wave motion.

118950(U)

50 μ sec./cm.

118951(U)

FIGURE 11

STRESS FRINGES IN POLYSTYRENE WITH A CORRESPONDING
OSCILLOSCOPE TRACE OF THE STRAIN-GAGE SIGNAL

Polystyrene plates with thicknesses of $1/8$, $1/2$, $3/4$, and 1 inch; and Plexiglas plates with thicknesses of $1/8$, $1/4$, $3/8$, $1/2$, $3/4$, and 1 inch were employed. If the stress wave were only a reflected longitudinal wave, an increase in thickness would cause an apparent decrease in velocity due to the longer transit distance. In addition, the fringe spacing should increase with increasing thickness. Figures 12 and 13 show the photoelastic pictures and transducer signals resulting from wire explosions into the different thicknesses of polystyrene and Plexiglas, respectively. The transducer signals indicate that the wavelength of the waves in the plates varied directly with thickness up to three-quarter-inch-thick Plexiglas and one-half-inch-thick polystyrene. At these thicknesses, a higher-frequency component predominates. This component may be seen on the transducer signal from the thinner plates, and its dominance increases with increasing thickness. The photoelastic pictures also indicate this change in frequency. The change may occur in polystyrene below the one-half-inch thickness, but plates in this thickness range could not be obtained. The higher-frequency wave appears to be a different mode of propagation similar to the different modes of the Lamb waves.

Due to its low stress-optic constant, some of the thick polystyrene plates contained stress patterns resulting from their fabrication. Figure 14 shows this stress in a one-half-inch-thick plate and the same plate after it had been heated to 100°C for three days to remove the stresses.

Wires were exploded at three different energies against a one-quarter-inch-thick Plexiglas plate to ascertain variations in the wave pattern. The oscilloscope

1/8-Inch Polystyrene

118945(U)

118932(U)

Horizontal — 50 μ sec./cm.
Vertical — 0.2 v./cm.

118939(U)

3/4-Inch Polystyrene
Horizontal — 50 μ sec./cm.
Vertical — 1 v./cm.

1/2-Inch Polystyrene

118944(U)

118931(U)

Horizontal — 50 μ sec./cm.
Vertical — 0.5 v./cm.

118938(U)

1-Inch Polystyrene
Horizontal — 50 μ sec./cm.
Vertical — 1 v./cm.

FIGURE 12

STRESS FRINGES IN POLYSTYRENE WITH CORRESPONDING
OSCILLOSCOPE TRACES OF THE PIEZOELECTRIC
TRANSDUCER SIGNALS

1/8-Inch Plexiglas

118948(U)

118930(U)

Horizontal - 50 μ sec./cm.
Vertical - 0.5 v./cm.

3/8-Inch Plexiglas

118949(U)

118929(U)

Horizontal - 50 μ sec./cm.
Vertical - 0.5 v./cm.

118941(U)

1/2-Inch Plexiglas
Horizontal - 50 μ sec./cm.
Vertical - 0.5 v./cm.

1/4-Inch Plexiglas

118947(U)

118934(U)

Horizontal - 50 μ sec./cm.
Vertical - 0.5 v./cm.

3/4-Inch Plexiglas

118942(U)

118943(U)

Horizontal - 50 μ sec./cm.
Vertical - 1.0 v./cm.

118937(U)

1-Inch Plexiglas
Horizontal - 50 μ sec./cm.
Vertical - 2.0 v./cm.

FIGURE 13

STRESS FRINGES IN PLEXIGLAS WITH CORRESPONDING
OSCILLOSCOPE TRACES OF THE PIEZOELECTRIC
TRANSDUCER SIGNALS

Internal Stresses

118946(U)

Internal Stresses Removed and External Stress Applied

118944(U)

FIGURE 14

REMOVAL OF INTERNAL STRESSES IN
POLYSTYRENE BY HEATING

traces of the transducer signal appear in Figure 15. No noticeable variations were found, other than an increase in amplitude with increasing energy.

The transducer was mounted on the side of a Plexiglas plate to determine whether vibration normal to the plate thickness existed. The transducer output indicated that there were vibrations in this direction, but their amplitude was about 10 per cent of the amplitude parallel to the plate thickness. Part of this amplitude decrease could be attributed to the poorer coupling of the transducer to the plate.

The velocity, thickness, and frequency values for one-quarter-inch-thick Plexiglas and one-eighth-inch-thick polystyrene were substituted into the Lamb wave-period equation for symmetric vibrations (Equation 25, page 18). The reported values of longitudinal and shear velocity were used, while the phase velocity indicated by the photoelastic experiment was employed. The frequency necessary to solve the equation was 165,000 c.p.s. for one-quarter-inch Plexiglas and 240,000 c.p.s. for one-eighth-inch polystyrene. These values compare favorably with the values of 100,000 and 160,000 c.p.s., respectively, which were estimated from the oscilloscope trace of the transducer output, since these signals indicate that several frequencies were propagated.

It was deemed necessary to calibrate the piezoelectric transducer so that an idea of the peak stresses generated in the plates might be obtained. Attempts to calibrate the transducer statically failed because of the low capacitance of the transducer. This capacitance was calculated from the manufacturer's standard

2.6 Kilojoules
Horizontal - 50 μ sec./cm.
Vertical - 0.5 v./cm.
118934(U)

6.7 Kilojoules
Horizontal - 50 μ sec./cm.
Vertical - 1.0 v./cm.
118933(U)

10.5 Kilojoules
Horizontal - 50 μ sec./cm.
Vertical - 5.0 v./cm.
118935(U)

FIGURE 15
VARIATION OF PIEZOELECTRIC TRANSDUCER SIGNAL
WITH THE AMOUNT OF ENERGY
DEPOSITED IN PLEXIGLAS

data to be 620 picofarads. Even with high resistances in series with the output, no calibration was obtained statically. To calibrate the transducer dynamically, a cylindrical Plexiglas sample, 0.375 inch in diameter, was fabricated and attached to the transducer. Two semiconductor strain gages were attached to the Plexiglas sample to measure the stress in the sample. The force was supplied by accelerating a long bar with an air gun and causing this bar to strike the sample and transducer. The calibration derived from this experiment was considerably in error. The stresses obtained were low by a factor of about six, probably because of the better coupling which was obtained with the sample than that obtained with the plates used in the experiment. The error was known because the stress-optic constants give the minimum values of the stresses.

A more accurate calibration was obtained by mounting a semiconductor strain gage on a one-eighth-inch-thick polystyrene plate, attaching the transducer to the end of the plate, and striking the opposite edge of the plate with a hammer. The stress level was increased to the point where the plate broke. The calibration did not achieve the desired stress levels, but the curve could be extrapolated as a straight line on log-log graph paper. The calibration curve and the extrapolated stress values for the experiment appear in Figure 16 and Table 1, respectively. The number of fringes blurred together in the photoelastic pictures is obtained from Equation 1, page 3.

The stress was obtained from the strain-gage signal by the following expressions:

FIGURE 16

PIEZOELECTRIC TRANSDUCER CALIBRATION ON POLYSTYRENE

TABLE I
PLATE STRESSES OBTAINED USING TRANSDUCER
CALIBRATION CURVE

Energy (kj.)	Material	Thickness (in.)	Peak Stress (p.s.i.)
2.6	Plexiglas	1/8	6,000
2.6	Plexiglas	1/4	6,000
2.6	Plexiglas	3/8	6,000
2.6	Plexiglas	1/2	6,000
2.6	Plexiglas	3/4	10,000
2.6	Plexiglas	1	20,000
2.6	Polystyrene	1/8	2,500
2.6	Polystyrene	1/2	2,500
6.7	Plexiglas	1/4	15,000
10.5	Plexiglas	1/4	35,000

$$\epsilon = \frac{\Delta V}{I R (G. F.)} , \text{ and} \quad (27)$$

$$\sigma = Y\epsilon , \quad (28)$$

where:

ϵ = the strain,

ΔV = the voltage change on the gage,

I = the current in the gage,

R = the resistance of the gage,

$G. F.$ = the gage factor,

σ = the stress, and

Y = Young's Modulus of Elasticity.

Equation 27 will hold when a constant current supply is used with the strain gage.

The stress-optic constants for Plexiglas and polystyrene were obtained by statically loading tensile specimens of the two materials and viewing the stress in the specimens with polarized light as the load was increased. The load required to change from one white fringe to the next divided by the area of the specimen is the stress-optic value for that thickness of that material. The two calibration curves are shown in Figure 17.

FIGURE 17

CALIBRATION OF STRESS-OPTIC CONSTANTS

CHAPTER V

SUMMARY AND CONCLUSIONS

It has been demonstrated that stresses may be visualized photoelastically in a material with a high stress-optic constant such as Plexiglas. However, at the high stress rates obtained with the exploding wire, a better lens system than a Fresnel lens is required to reveal the individual fringes and, hence, determine the stress in the material. With the aid of a piezoelectric transducer it has been shown that the waves generated in the plastic plates and visualized photoelastically are analogous to Lamb waves. Further proof of this analogy might be obtained by using plates of different thickness than those employed here to determine the cutoff frequencies for the Lamb wave propagation, since these waves are quite analogous to electromagnetic waves traveling through a wave guide. Work with two transducers mounted symmetrically on opposite sides of a plate might be done to determine what modes, symmetric and/or asymmetric, can be propagated.

The reflection, rather than transmission, photoelastic technique may be applied to opaque substances such as metals, if they are coated with a thin photoelastic layer of plastic. With an improved lens system, this method could be applied to actual structures to measure the stress level as a function of time.

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SYMBOLS AND NOMENCLATURE

Symbol

a	acceleration
C	stress-optic constant $\left(\frac{\text{p.s.i.} \cdot \text{in.}}{\text{fringe}} \right)$
d	plate thickness (in.)
ϵ	strain (in./in.)
F	force
f	frequency (cycles/sec.)
G. F.	strain gage factor
I	strain gage current (amps.)
k	wave number
λ	Lamé elastic constant
λ'	wavelength
μ	Lamé elastic constant
n	fringe order
p	algebraically largest principal stress (p.s.i.)
q	algebraically smallest principal stress (p.s.i.)
ρ	density
R	strain gage resistance (ohms)
σ	stress (p.s.i.)
t	plate thickness (in.)

V	phase velocity (in./sec.)
V_l	longitudinal wave velocity (in./sec.)
V_s	shear wave velocity (in./sec.)
ΔV	strain gage voltage change (volts)
ω	angular frequency (sec. ⁻¹)
Y	Young's Modulus of Elasticity (p.s.i.)