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Period Equilibrium Effects in the Start-up of
Nuclear Rocket Reactors

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Abstract

It is shown that the small-signal transfer function of a nuclear reactor whose power is increasing at constant period is different from that of a reactor at constant power level. A reactor power control system designed for adequate stability using a power equilibrium reactor transfer function may therefore be unstable if the reactor power is forced to increase on a sufficiently short period. For a nuclear rocket reactor, such an instability is most likely to occur during the very rapid reactor startup to full power. The results of this analysis are applied to the KIWI B1-A log-power reactor control system, and it is shown that the system originally designed using a power equilibrium reactor transfer function was unstable for periods shorter than about two seconds. A re-analysis of the system using a period equilibrium transfer function resulted in a control system stable for any period longer than about 0.5 sec.

List of Notations

N	number of neutrons in the reactor
δK	excess reactivity
ℓ^*	mean fast neutron lifetime
β_i	fraction of total delayed neutrons from i^{th} group
β	fraction of total neutrons which are delayed $\approx .0075$
λ_i	probability per unit time for delayed neutron emission from i^{th} group
C_i	number of delayed neutron emitters in the i^{th} group
γ	constant relating neutron population to reactor power level
γ_i	$\gamma C_i / P$
δK_0	average excess reactivity
k	variation in excess reactivity
P	reactor power
P_0	initial power level
p	variation in power level
s	LaPlace transform variable
G_p	power equilibrium reactor transfer function
τ	reactor period
G_τ	Period equilibrium transfer function
α_0	inverse of the average period

I. INTRODUCTION

Because of the need to use fuel stores efficiently, it is commonly recognized that rocket propulsion engines must have the capability of being programmed from stand-by to full power in rather short times. Nuclear reactor propulsion systems should, in principle, be no exception to this rule; yet the problem of an automatic rapid startup of such a reactor may present considerable difficulties. Assuming that the hydrogen propellant has a specific impulse of 800 sec, and is heated to 4000°R in the reactor-heat exchanger, a rough calculation⁽¹⁾ shows that the reactor power required to develop 100,000 lbs. of thrust is approximately 2000 megawatts. Since this type of reactor rarely contains any buildup of fission fragments, photo-neutrons are nearly absent, and the startup-to-full-power range may exceed twelve decades. At least the last five decades require propellant flow to avoid excessive reactor core temperatures, so that the rapid startup must certainly cover five decades of power, and in a time measured in seconds. Yet experience obtained from large power reactors indicates that startup times of many minutes or even hours are required.

It is imperative that the system used to control the reactor power remain stable during a rapid startup. The test facility at the Nuclear Rocket Development Station (NRDS) at Jackass Flats, Nevada, for the ROVER KIWI prototype reactors uses a feedback control system to force the reactor power to follow the rapidly changing demanded power level (startup program typically lasts tens of seconds and involves periods which at times are as short as several seconds); and it appears reasonable to assume that a flight vehicle will also use a feedback power control

system. It follows that the dynamic behavior of the reactor as an element in these control systems will play an important role in determining the system stability for fast startups.

One of the standard problems involved in the stability analysis of reactor control systems is to determine the reactor power level response to transient variations in excess reactivity. An early step in this procedure is to consider only small variations in excess reactivity; the problem is then equivalent to finding the small signal open-loop reactor transfer function. Harrer, Boyer, and Krucoff⁽²⁾ have shown that this can be done by making a linear perturbation expansion of the space-independent reactor kinetic equations for the equilibrium condition where the average reactor power remains constant and the average excess reactivity is zero. Yet, during startup, the excess reactivity is not zero, and the power level is changing quite rapidly. Hence the dynamic behavior of the reactor under, for example, a constant period mode, is more pertinent for consideration of startup stability problems. We shall show that the reactor transfer function for such an equilibrium mode is different from the power equilibrium transfer function of Harrer, Boyer, and Krucoff; and that a reactor control system whose stability margin appears adequate when deduced from a power equilibrium analysis, may in fact become unstable if allowed to remain on a sufficiently short period.

II. THE PERIOD EQUILIBRIUM TRANSFER FUNCTION

The dynamics of a nuclear reactor at low power (where reactivity contributions due to core temperature changes are negligible) is often

described by the "point" reactor kinetic equations:⁽³⁾

$$\frac{dN}{dt} = \frac{\delta K - \beta}{\ell^*} N + \sum_{i=1}^6 \lambda_i C_i \quad (1a)$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\ell^*} N - \lambda_i C_i, \quad i \dots 6 \quad (1b)$$

By making the transformations

$$\begin{aligned} P &= \gamma N \\ Y_i &= \gamma \frac{C_i}{P}, \end{aligned} \quad (2)$$

equations (1) and (2) can be written in the form

$$\frac{d}{dt} (\log P) = \frac{\delta K - \beta}{\ell^*} - \sum_{i=1}^6 \lambda_i Y_i \quad (3a)$$

$$\frac{dY_i}{dt} = \frac{\beta_i}{\ell^*} - \lambda_i Y_i - Y_i \frac{d}{dt} (\log P) \quad (3b)$$

The logarithmic forms of these equations are used here because the KIWI-BLA reactor control system (to be described below) controlled the logarithm of the power level.

In the standard small-signal analysis, the equilibrium mode is taken as one where the average power P_0 is constant and the average excess reactivity K_0 is zero. The perturbation equations are then

$$P = P_0 + p \quad (4a)$$

$$\delta K = \delta K_0 + k, \quad \delta K_0 = 0$$

$$Y_i = Y_i^0 + y_i, \quad Y_i^0 = \frac{\beta_i}{\ell^* \lambda_i} \quad (4b)$$

Substituting equation 4 into equation 3 and taking the Laplace transform, one obtains the standard constant-power reactor transfer function:

$$G_p(s) = \frac{\delta[\log P(s)]}{k(s)} = \frac{1}{\ell^*} \cdot \frac{1}{s \left[1 + \sum_{i=1}^6 \frac{\beta_i}{\ell^*(s + \lambda_i)} \right]} \quad (5)$$

Now, the instantaneous reactor period τ is defined by

$$\frac{1}{\tau} = \frac{1}{P} \frac{dP}{dt} = \frac{d}{dt} (\log P) \quad (6)$$

Then equation 3 may be written as

$$\frac{1}{\tau} = \frac{\delta K - \beta}{\ell^*} - \sum_{i=1}^6 \lambda_i Y_i \quad (7a)$$

$$\frac{dY_i}{dt} = \frac{\beta_i}{\ell^*} - \lambda_i Y_i - Y_i \cdot \left(\frac{1}{\tau} \right) \quad (7b)$$

If one were to make a perturbation expansion of equation 7, equation 5 will result if the average value of $1/\tau$ is zero, since this is equivalent to power equilibrium. Since the dynamics of the reactor is strongly influenced by that of the delayed neutron emitters, it is tempting to extend the concept of equilibrium to one where the existence of an equilibrium mode depends on whether the Y_i can achieve a steady state constant value. From equation 7(b), a constant power mode is an equilibrium mode because $(1/\tau)_{ave.} = 0$ and $(Y_i)_{ave.} = \text{const} = \frac{\beta_i}{\lambda_i \ell^*}$ is a solution of the equation. However, the same situation exists provided $(1/\tau)_{ave.}$ is a constant (which need not be zero). This is the period equilibrium mode; the reactor period is on the average a constant τ , the power level on the average changes exponentially.

$$P = P_0 e^{t/\tau},$$

$$\text{and} \quad (Y_i)_{ave} = \frac{\beta_i}{\ell^*} \frac{1}{\lambda_i + 1/\tau}.$$

Upon carrying through the usual perturbation approximations, one obtains for the period equilibrium transfer function

$$G_T(s) = \frac{1}{\ell^*} \frac{1}{s \left[1 + \sum_{i=1}^3 \frac{\beta_i}{\ell^*} \frac{\lambda_i}{\lambda_i + \alpha_0} \frac{1}{\lambda_i + \alpha_0 + s} \right]} \quad (8)$$

It is evident that $G_T \rightarrow G_p$ as the average inverse period $\alpha_0 \rightarrow 0$.

To obtain some intuitive feeling for the effect of a non-zero α_0 on the reactor transfer function, it is illustrative to examine equation 8 for a one delayed-group model:

$$G_T(s) \Big|_{i=1} = \frac{1}{\ell^*} \frac{\alpha_0 + \lambda}{\alpha_0 + \lambda + \frac{\beta}{\ell^*} \cdot \frac{\lambda}{\alpha_0 + \lambda}} \frac{1 + \frac{s}{\alpha_0 + \lambda}}{s \left[1 + \frac{s}{\alpha_0 + \lambda + \frac{\beta}{\ell^*} \frac{\lambda}{\alpha_0 + \lambda}} \right]} \quad (9)$$

For comparison, when $\alpha_0 = 0$ (power equilibrium),

$$G_p(s) \Big|_{i=1} = \frac{1}{\ell^*} \frac{\lambda}{\lambda + \beta} \frac{1 + \frac{s}{\lambda}}{s \left[1 + \frac{s}{\lambda + \beta} \right]} \quad (10)$$

$G_p(s)$ can be characterized as follows: (a) in the low frequency region, $|G_p|$ falls at 20 db per decade and the phase of G_p lags by 90° . (b) At a frequency $f = \lambda/2\pi$, the lead term begins to be effective; $|G_p|$ levels off at a value approximately $1/\beta$, and the magnitude of the phase approaches zero. (c) At a frequency $f = \frac{1}{2\pi} \frac{\beta}{\ell^*}$, the lag term becomes effective, $|G_p|$ again begins to fall at 20 db/decade and the phase lag again approaches 90° . The effect of $\alpha_0 \neq 0$ is then (1) the lead term breaks in at the higher frequency $f' = (\lambda + \alpha_0)/2\pi$; (2) in the mid-frequency range, $|G_T|$ is larger than $|G_p|$ by the ratio

$$\frac{G_T}{G_p} \approx 1 + \left\{ \frac{\alpha_0 \ell^*}{\beta} - \frac{\alpha_0}{\lambda + \alpha_0} \right\}, \quad \left(\frac{\beta}{\ell^*} \gg \lambda \right) \quad (11)$$

and (3) the lag term becomes important at a lower frequency, at $f = \frac{1}{2\pi} (\alpha_0 + \lambda + \frac{\beta}{\ell^*} \frac{\lambda}{\alpha_0 + \lambda})$. In the limit as α_0 becomes large,

$$G_T(s) \rightarrow \frac{1}{\ell^* s}$$

Thus for intermediate values of α_0 , the magnitude of G_T and of its phase lag increases (a most unwelcome combination), and as α_0 becomes large, the delayed neutron emitters play an increasingly smaller role in the period equilibrium reactor dynamics.

The results of a more exact calculation⁽⁴⁾ are shown in Figures 1 and 2. An IBM 7090 digital computer was used to obtain the magnitude and phase shift of $G_T(s)$ as defined in equation 8 for values of ℓ^* from 10^{-3} to 10^{-6} sec for periods from 0.01 sec to infinity. Figures 1 and 2 are the results for $\ell^* = 5 \times 10^{-5}$ sec, and for periods of ∞ , 30, 15, 10, 5, 1, 0.5, 0.1, and 0.01 sec.

III. THE KIWI BLA POWER CONTROL SYSTEM

The result of using the incorrect equilibrium reactor transfer function can be appreciated by reviewing the experience with the KIWI BLA reactor control system.

In the initial analysis, the reactor was indeed represented by the power equilibrium transfer function, equation 5. During the installation and check-out of this system, the performance was evaluated using a three delayed group reactor log power simulator and a typical reactor control rod. The open and closed loop system transfer function (at constant power), and the step response agreed quite satisfactorily with the predicted values. Nevertheless, when placed on fast periods (~ 2 sec), the system became unstable after a few seconds, in that 4 cps sinusoidal oscillations with rapidly diverging amplitude appeared

on the log power signal. At first, it seemed unreasonable that a system with all of the stability margin that our measurements indicated could be unstable, and the effect was attributed to improper operation of the reactor kinetics simulator. Careful investigation ruled out this possibility, and the discovery that the power and period equilibrium transfer functions can differ appreciably followed shortly thereafter. When the results of the period equilibrium analysis were applied to the simulated power loop, the reactor control system was seen to be marginally stable in the region 4 - 6 cps. As a result, the system compensation (which included proportional plus integral terms, and lead compensation for the reactor control rods) was modified by reducing the proportional term by 6 db, and by altering the phase compensation to reduce the phase lag in the vicinity of several cycles per sec.

It is interesting to observe that the customary techniques for observing the dynamic response of a control system, i.e., sinusoidal and step measurements, can be misleading unless the information is obtained with the system in the proper equilibrium mode. During the first start-up of the KIWI-BIA reactor at the NRDS, the performance of the log power control system was evaluated at constant power for sinusoidal and step function perturbations. Figure 3 is a reproduction of an on-line strip chart recording (taken during the step perturbation portion of the test) of the following variables: demanded log power increment, measured log power increment, log power error signal (demanded log power minus measured log power), measured linear power

increment, position increment command to all reactor rods, measured rod position increment (of one rod), and the total demanded log power. For the latter variable, full scale represents nearly five decades of linear power. The measured open loop transfer function for this system is shown in Figure 4. The gain and phase margin are 14 db and 75 degrees, respectively, and from Figure 3, the step response shows virtually no overshoot. The stability margin appears quite adequate; yet here, just as for the simulated system, the existence of a potential instability is masked because the data were taken in the power equilibrium mode. Despite the loop gain reduction described above, the BLA system was not unconditionally stable. Assuming that $t^* \approx 5 \times 10^{-5}$ sec, the data of Figure 1 and 2 are applicable, and the BLA open loop transfer functions are plotted in Figure 4 for periods of 10, 1, 0.5, and 0.1 sec. Instability is indicated for periods shorter than about 0.5 sec.

IV. CONCLUSION

In designing control systems for reactors intended to operate at fixed power levels, the question as to whether the system is stable for severe perturbations in power level is somewhat academic, since period and/or power level scrams would normally then be expected to bypass the control system and scram the reactor. For rocket propulsion reactors such as those used in the ROVER program, power level changes of many decades from startup levels to full operating levels are expected to occur in times measured in tens of seconds, and in a controlled and stable fashion. In the analysis of such systems, it is important to realize that another "equilibrium" mode, period equilibrium, exists for

reactors, and that the power equilibrium transfer function, when used, can lead to invalid conclusions concerning the stability of feedback reactor power control systems during rapid increases in power level.

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Figure 1

Title: Period equilibrium reactor transfer function amplitude vs.
frequency

Caption: The amplitude response of the period equilibrium reactor
transfer function of equation 8 is plotted for a variety
of periods. The curve numbers increase sequentially from
top to bottom.

Figure 2

Title: Period equilibrium reaction transfer function phase shift vs.
frequency

Caption: The phase shift of the period equilibrium reactor transfer
function of equation 8 is plotted for a variety of periods.
The curve numbers increase sequentially from top to bottom.

Figure 3

Title: Step response of the Improved KIWI BL-A log power control system

Caption: A real time strip-chart recording of the step response of the complete KIWI BL-A log power control system as measured during the first reactor start-up test at the NRDS in September 1961. The variables are, from top to bottom, the step demand in log power (on an expanded scale); the observed response in the logarithm of reactor power (also on an expanded scale); the control system error signal, i.e., the algebraic difference between log P demand and log P measured; the observed response in the power itself (on an expanded scale); the electrical rod position command sent to the reactor control rods; the observed response in reactor rod position (of one reactor rod); and finally the log power demand on a non-expanded scale, showing the magnitude of the square wave perturbation on a scale where a full deflection corresponds to the full range of the control system (~ 5 decades). In the top recording, deviation from the quiescent state is in the positive (upward) direction; in all others it is in the downward direction. Note the change in chart speed, resulting in a time-expanded graph of the system response to one complete square wave.

Figure 4

Title: Gain-phase shift plot for the KIWI Bl-A log power control system

Caption: The response of the log power control system is shown here as a plot of the system open loop gain vs the open loop phase shift, for several different equilibrium periods. The curve for $T = \infty$ is obtained from the (power equilibrium) transfer function measurements from the first KIWI Bl-A start-up of September 1961; the other curves are derived from the $T = \infty$ curve using figures 1 and 2. Instability is indicated for $T \lesssim 0.5$ sec.