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DAMPING OF DISLOCATION MOTION
 BY CONDUCTION ELECTRONS IN METALS*

K. M. Jassby and M. Shull

W. M. Keck Laboratories
 California Institute of Technology
 Pasadena, California 91109

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ABSTRACT

The rate of energy absorption from a moving dislocation by conduction electrons is recalculated using the semiclassical Boltzman transport equation and the model of a free electron gas. Temperature-independent values of the electronic damping coefficients B_{edge} and B_{screw} for edge and screw dislocations are found where B_{edge} and B_{screw} are similar in magnitude. The results are compared to previous work by Kravchenko and Brailsford.

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1. INTRODUCTION

In metals at very low temperatures, thermal vibrations of the crystal lattice are weakly excited and contribute little to the scattering of conduction electrons. In addition, they are ineffective in inhibiting the motion of dislocations or the propagation of ultrasonic waves. Hence, in this temperature region, any residual damping of acoustic disturbances such as moving dislocations or ultrasonic waves would indicate a dissipative mechanism caused by interaction with the conduction electrons. This fact has been well documented experimentally, as for example in superconductors by the difference in attenuation of ultrasonic waves between the normal and superconducting states.^{1,2}

The attenuation of both longitudinal and transverse phonons by a free electron gas was first considered theoretically by Pippard.³ The rate of energy dissipation by a longitudinal phonon in a free electron gas has been calculated using perturbation theory (where the mean free path of the electron gas, ℓ , is long with respect to the phonon wavelength, $1/q$) and by a semiclassical treatment employing the Boltzmann transport equation, while the latter method has also been used to determine the interaction between a transverse phonon and a free electron gas (see for example, Kittel⁴). It was shown that in the limit $q\ell \ll 1$, damping of phonons is due to the viscosity of the electron gas and is directly proportional to the electrical conductivity of the metal. In the limit $q\ell \gg 1$, the damping was found to be independent of the electronic relaxation time and hence invariant with temperature.

The transport method gives us a conceptual picture of the energy absorption process resulting from the conduction electron-phonon interaction:

An acoustic phonon travelling through a crystal lattice imparts a local, time-dependent displacement and velocity to the ions in the lattice. If we consider the conduction electrons to be a free electron gas, then the probability of finding an electron with coordinates $(\underline{r}, \underline{k})$, where $\underline{r}$ refers to ionic position and $\underline{k}$ is the electronic wavevector, is given by the Fermi-Dirac distribution function in the local coordinate system moving with the local lattice. This equilibrium distribution function can be disturbed in two ways: (i) by a perturbation in the local lattice velocity, which changes the relative velocity between electron and ion, and (ii) by a local compression or expansion of the lattice which locally increases or decreases the fermi energy.

To restore equilibrium after a small change in ionic coordinates has taken place, the electrons redistribute themselves in $(\underline{r}, \underline{k})$ space according to the new equilibrium distribution. The net current flow and power dissipation can be calculated by combining the Boltzmann transport equation for the distribution function with the Maxwell equations. It has been shown⁴ that in the linear approximation, when collision drag is neglected, the power dissipation for both longitudinal and transverse phonons is proportional to the second power of the amplitude of the local ionic velocity.

The displacement and velocity fields of a moving dislocation can be expanded in terms of a Fourier series, and thus the dislocation may be considered as a travelling wave packet of phonons. Dislocation damping is then due to the power dissipation from the sum of the individual phonons.

Mason^{5, 6} proposed a model for energy dissipation from a moving dislocation based on the viscosity of a free electron gas. However, as was pointed out by later investigators,^{7, 8} the requirement for viscous damping

$q\ell \ll 1$, is grossly violated for the typical Fourier component of the dislocation displacement field, especially at low temperatures. Kravchenko⁷ employed the Boltzmann transport equation to calculate the electronic damping coefficients, B_{edge} and B_{screw} , for edge and screw dislocations. He found temperature independent damping coefficients with B_{edge} somewhat less than B_{screw} . Holstein⁸ also determined a temperature-independent value for B_{edge} , using perturbation theory. This latter work was supported by the calculation of Brailsford⁹ using the Boltzmann equation analysis of Ref. 4. However, Brailsford found B_{screw} to be negligible with respect to B_{edge} , in contradiction to Kravchenko's results. An additional treatment of the problem by Huffmann and Louat using the Boltzmann equation¹⁰ indicated that B_{edge} was proportional to electrical conductivity, and B_{screw} negligibly small. Brailsford⁹ showed that these results came from an incomplete expression for the electric field in the Maxwell equations.

Although the theoretical calculations^{7, 8, 9} support a temperature-independent value for B_{edge} , the predictions for B_{screw} are in disagreement with one another. The present authors reexamined the problem employing the formalism of Ref. 4, with the aim of clarifying the inconsistency of the different calculations.

2. THEORETICAL CONSIDERATIONS

2.1 Formalism of the Electron-Phonon Interaction

Equations 1 to 8 serve as a brief review of the Boltzmann transport equation analysis of Kittel.⁴ Let us consider a travelling phonon with wave-vector $\underline{q}$ and velocity of propagation $\underline{v}$. Then the corresponding ionic velocity is given by $\dot{\underline{u}}_{\underline{q}} \exp -i\underline{q} \cdot (\underline{r} - \underline{v}t)$ where t is equal to time. Generally $\dot{\underline{u}}_{\underline{q}}$

contains both longitudinal and transverse components, $\dot{\underline{u}}_{\underline{q}\parallel}$ and $\dot{\underline{u}}_{\underline{q}\perp}$. The power dissipated by the interaction between the phonon and free electron gas is of the form

$$P_{\underline{q}} = A_{\parallel} \left| \dot{\underline{u}}_{\underline{q}\parallel} \right|^2 + A_{\perp} \left| \dot{\underline{u}}_{\underline{q}\perp} \right|^2 \quad (1)$$

The proportionality factors, $A_{\parallel}$ and $A_{\perp}$, are given by¹¹

$$A_{\parallel} = \frac{n_o m V}{2\tau} \operatorname{Re} \left[\frac{g(\sigma_o)}{|g(\sigma')|^2} \left(\frac{\sigma_o}{\sigma'} - 1 \right) \right] \quad (2)$$

and

$$A_{\perp} = \frac{n_o m V}{2\tau} \operatorname{Re} \left(\frac{\sigma_o}{\sigma_{xx}} - 1 \right) \quad (3)$$

where n_o and m are the free electron density and mass, τ is the electronic relaxation time, V is the volume of metal, and σ_o is the dc conductivity.

The expression for $g(x)$ is given by

$$g(x) = 1 + \frac{i\omega}{4\pi x} \quad (4)$$

where $\omega = \underline{q} \cdot \underline{v}$ is the frequency of the phonon. The relationship for σ' is

$$\sigma' = \frac{\sigma_{zz}}{\left(1 + \frac{iqv_F a \sigma_{zz}}{3\sigma_o \omega} \right)} \quad (5)$$

where v_F is the Fermi velocity and

$$a = \frac{q\ell}{1 - i\omega\tau} \quad (6)$$

Finally, the conductivity tensor components σ_{xx} and σ_{zz} are given by

$$\sigma_{xx} = \frac{\sigma_o}{1 - i\omega\tau} \cdot \frac{3}{2a^3} \left[(1 + a^2) \tan^{-1} a - a \right] \quad (7)$$

and

$$\sigma_{zz} = \frac{\sigma_o}{1 - i\omega\tau} \cdot \frac{3}{a^3} \left[(a - \tan^{-1} a) \right]. \quad (8)$$

Equations (2) and (3) have two asymptotic forms, when $q\ell \ll 1$ and $q\ell \gg 1$, for each of the two limiting cases $\omega\tau \ll 1$ and $\omega\tau \gg 1$. Also the relations $\frac{\sigma_o}{2} \gg q$ and $\sigma_o \gg \omega$ are generally valid for any q as large as the Debye radius, q_D .

In both limits $\omega\tau \ll 1$ and $\omega\tau \gg 1$ and with the additional condition $q\ell \gg 1$ ($\tan^{-1} a$ has only a small imaginary component in both cases), $A_{||}$ and $A_{\perp}$ reduce to

$$A_{||} \rightarrow \frac{\pi n_o m v_F V}{12} q \quad (9)$$

and

$$A_{\perp} \rightarrow \frac{2n_o m v_F V}{3\pi} q, \quad (10)$$

which are independent of temperature. (We shall show in Section 2.3 that the Fourier components of the moving dislocation that dominate in the energy absorption process both when $\omega\tau \ll 1$ and $\omega\tau \gg 1$ are those for which $q\ell \gg 1$.)

2.2 Fourier Decomposition of the Dislocation Velocity Field

We will consider a dislocation lying parallel to the z -axis in an (x, y, z) coordinate system and moving in the positive x -direction with uniform velocity v_D . The slip plane is defined by $y = 0$. For a screw dislocation,

we will denote the ionic displacement field by u_z , while for an edge dislocation, the displacements will be given by u_x and u_y . When the dislocation velocity is small with respect to the shear wave velocity, and for the screw dislocation in an isotropic linear elastic continuum,¹²

$$u_z = \frac{b}{2\pi} \tan^{-1} \left(\frac{y}{x'} \right), \quad (11)$$

where b is the dislocation Burgers vector and $x' = x - v_D t$ is fixed to the dislocation. Then the ionic velocity is

$$\dot{u}_z = \frac{du_z}{dt} = \frac{bv_D}{2\pi} \frac{y}{x'^2 + y^2}. \quad (12)$$

Similarly, for the edge dislocation

$$u_x = \frac{b}{2\pi} \left[\tan^{-1} \frac{y}{x'} + \frac{x'y}{2(1-\nu)(x'^2 + y^2)} \right], \quad (13)$$

$$u_y = -\frac{b}{8\pi(1-\nu)} \left[(1-2\nu) \ln(x'^2 + y^2) - \frac{2y^2}{x'^2 + y^2} + 1 \right], \quad (14)$$

and

$$\dot{u}_x = \frac{bv_D}{4\pi(1-\nu)} \left[(1-2\nu) \frac{y}{x'^2 + y^2} - \frac{2x'y}{(x'^2 + y^2)^2} \right], \quad (15)$$

$$\dot{u}_y = \frac{bv_D}{4\pi(1-\nu)} \left[(1-2\nu) \frac{x'}{x'^2 + y^2} + \frac{2x'y^2}{(x'^2 + y^2)^2} \right], \quad (16)$$

where ν is Poisson's ratio.

According to Eqs. 12, 15, and 16, the velocity components diverge at the center of the dislocation, $(x', y=0, 0)$. This is not the case in a crystal, where the high strain region at the center of a dislocation the material

behaves in a nonlinearly elastic manner and all dislocation properties remain finite. It is customary to consider a small cylindrical core region of radius $r_0 \sim b$ about the center of the dislocation, within which the material properties are nonlinear. Then the ionic velocity components can be expanded in terms of a set of phonon wavevectors $\underline{q}$, all with propagation velocity v_D , such that

$$\begin{aligned} \dot{u}_j(x', y) &= \sum_{\underline{q}} \dot{u}_{\underline{q}j} \exp - i \underline{q} \cdot (\underline{r} - \underline{v}_D t) \\ &= \sum_{\underline{q}} \dot{u}_{\underline{q}j} \exp - i (q_1 x' + q_2 y), \end{aligned} \quad (17)$$

where $j = x, y, \text{ or } z$. The components of the wavevector $\underline{q}$ in the x and y directions are q_1 , and q_2 respectively. The ionic velocity amplitude, $\dot{u}_{\underline{q}j}$, is given by

$$\dot{u}_{\underline{q}j} = \frac{1}{(2C)^2} \int_{y=-C}^C \int_{x'=-C}^C \dot{u}_j(x', y) \exp i (q_1 x' + q_2 y) dx' dy, \quad (18)$$

where $2C$ is the dimension of the material in both x' and y directions, and $q_1, q_2 = 0, \pm \frac{\pi}{C}, \pm \frac{2\pi}{C}, \dots \pm q_{\max}$.

The magnitude of $q_{\max}$ can be estimated in two limiting cases. In the first case, the elastic properties of the dislocation are assumed to be constant within the small core region which is assumed to support phonon motion. Then, in the Debye approximation, $q_{\max}$ is equal to the Debye radius q_D , which is inversely proportional to the interatomic distance. In the second case, electrons are assumed not to interact with the core directly and then $q_{\max}$ is inversely proportional to $2r_0$. We see that $q_{\max}$ is overestimated by the first condition and underestimated by the second.

In the following we assume $qC \gg 1$ which is the case for crystals of macroscopic dimensions. Substituting $\dot{u}_x$, $\dot{u}_y$, and $\dot{u}_z$ from Eqs. 12, 15, and 16 separately into Eq. 18, we find for a screw dislocation

$$\dot{u}_{qz} = \frac{ibv_D}{(2C)^2} \frac{q_2}{q_1^2 + q_2^2}, \quad (19)$$

and for an edge dislocation,

$$\dot{u}_{qx} = \frac{ibv_D}{2(2C)^2(1-\nu)} \frac{q_2}{q_1^2 + q_2^2} \left(1 - 2\nu + \frac{q_2^2 - q_1^2}{q_1^2 + q_2^2} \right) \quad (20)$$

and

$$\dot{u}_{qy} = \frac{ibv_D}{2(2C)^2(1-\nu)} \frac{q_1}{q_1^2 + q_2^2} \left(1 - 2\nu + \frac{q_1^2 - q_2^2}{q_1^2 + q_2^2} \right). \quad (21)$$

For the edge dislocation the longitudinal and transverse amplitude components of ionic velocity are given in cylindrical coordinates by

$$\begin{aligned} \dot{u}_{q||} &= \dot{u}_{qx} \cos \theta + \dot{u}_{qy} \sin \theta \\ &= \frac{ibv_D}{(2C)^2} \left(\frac{1-2\nu}{1-\nu} \right) \frac{\sin \theta \cos \theta}{q}, \end{aligned} \quad (22)$$

and

$$\begin{aligned} \dot{u}_{q\perp} &= \dot{u}_{qx} \sin \theta - \dot{u}_{qy} \cos \theta \\ &= -\frac{ibv_D}{(2C)^2} \frac{\cos 2\theta}{q}, \end{aligned} \quad (23)$$

where θ is the angle between $\underline{q}$ and the x' axis.

2.3 Calculation of the Dislocation Damping Coefficients.

The total power dissipation for a screw dislocation is given by

$$P_{\text{screw}} = \int_{\underline{q}} A_{\perp} \left| \dot{u}_{qz} \right|^2 D(\underline{q}) d^2 \underline{q}, \quad (24)$$

and for an edge dislocation

$$P_{\text{edge}} = \int_q [A_{\parallel} |\dot{u}_{\underline{q}\parallel}|^2 + A_{\perp} |\dot{u}_{\underline{q}\perp}|^2] D(\underline{q}) d^2 \underline{q}, \quad (25)$$

where $D(\underline{q})$ is the density of phonon states in two-dimensional $\underline{q}$ -space, equal to $(2C/2\pi)^2$, and q assumes values from 0 to q_{max} . The dislocation damping coefficients are defined by

$$B_{\text{screw}} = \frac{P_{\text{screw}}}{Lv_D^2} \quad (26)$$

and

$$B_{\text{edge}} = \frac{P_{\text{edge}}}{Lv_D^2}, \quad (27)$$

where L is the length of the dislocation and is equal to $V/(2C)^2$.

In the following, we shall determine the magnitude of the dislocation damping coefficients in the limiting cases $\omega\tau \ll 1$ and $\omega\tau \gg 1$.

We use the asymptotic forms for $A_{\parallel}$ and $A_{\perp}$ for $q\ell \gg 1$ (Eqs. 9 and 10) in the formulas for power dissipation (Eqs. 24 and 25). This can be justified as follows: Kittel⁴ has shown that if $q\ell \ll 1$, then

$$A_{\parallel}' \propto n_0 m v_F^2 \tau V q^2. \quad (28)$$

(The prime denotes the case $q\ell \ll 1$) The power dissipated by longitudinal phonons with wave vectors whose magnitudes lie between q and $q+dq$ is given by

$$P_{\parallel}' \propto n_0 m v_F^2 \tau V q dq. \quad (29)$$

This contrasts with

$$P_{\parallel} \propto n_0 m v_F V dq \quad (30)$$

in the case $q\ell \gg 1$ for the same change in the magnitude of the wave-vector dq . Taking the ratio,

$$\frac{P_{\parallel}}{P_{\perp}} \propto q\ell \ll 1, \quad (31)$$

which shows that the power dissipated by short wavevector longitudinal phonons may be neglected. A similar result holds for transverse phonons.

Substituting the appropriate ionic velocity amplitudes from Eqs. 19, 22, and 23 into Eqs. 24 and 25, and using Eqs. 9 and 10 for $A_{\parallel}$ and $A_{\perp}$, we obtain

$$B_{\text{screw}} = \frac{n_o m v_F b^2 q_{\text{max}}}{6\pi^2} \quad (32)$$

and

$$B_{\text{edge}} = \left[\frac{1}{6\pi^2} + \frac{1}{96} \left(\frac{1-2\nu}{1-\nu} \right)^2 \right] n_o m v_F b^2 q_{\text{max}} \quad (33)$$

DISCUSSION

Our calculations indicate that B_{screw} and B_{edge} are of the same order of magnitude. The damping coefficient is temperature-independent in both cases. The results are in agreement with the original work of Pippard,¹³ who found that the attenuation coefficients for longitudinal and shear waves in the short wavelength limit were temperature-independent and of similar magnitude.

We note that the contribution of longitudinal phonons to the damping coefficient of an edge dislocation (second term of Eq. 29 with q_{max} equal to q_D) agrees exactly with Brailsford's previous result.⁸ The screw

dislocation damping coefficient (Eq. 28 with $q_{\max}$ inversely proportional to the width of the dislocation core) is in accord with Kravchenko's result.⁶ Kravchenko's expressions for the Fourier components of the velocity field of the edge dislocation were not complete.

Since $\omega = \underline{q} \cdot \underline{v}_D$, we can write

$$\omega \tau \leq q \ell \left(\frac{v_D}{v_F} \right). \quad (34)$$

In the low velocity region, $v_D \leq 10$ cm/sec, we can expect to find that $\omega \tau < 1$ for all q even in pure metals at temperatures close to absolute zero. However, in the high velocity region ($v_D \sim 10^4$ cm/sec) at low temperatures, $\omega \tau > 1$ for the important wavevectors of large magnitude. Our calculations indicate that the magnitude of the damping mechanism is the same in both limits $\omega \tau \ll 1$ and $\omega \tau \gg 1$. However, we see from Eqs. 32 and 33 that the damping coefficients are sensitive to the choice of $q_{\max}$.

If we postulate that the dislocation core supports phonon motion, then $q_{\max}$ is equal to q_D for both edge and screw dislocations, where in the Debye approximation, $q_D = (6\pi^2)^{1/3}/g$, where g^3 is the atomic volume. On the other hand, if we stipulate that conduction electrons do not interact directly with the dislocation core then $q_{\max} \sim \frac{1}{2r_0}$, where $2r_0$ is the core diameter. In terms of the Peierls model of a dislocation¹², the dislocation 'width' gives a rough measure of the core diameter and is larger for an edge than for a screw dislocation. The values are $2r_0 \sim d/(1-\nu)$ and $2r_0 \sim d$ respectively, where d is the spacing between slip planes. Numerical calculations of B_{edge} and B_{screw} for aluminum, copper, and lead are presented in Table 1 for the two limiting cases of $q_{\max}$. We note from

Table I that in the limit $q_{\max} = q_D, B_{\text{edge}} > B_{\text{screw}}$, while in the case $q_{\max} \sim \frac{1}{2r_0}, B_{\text{screw}} > B_{\text{edge}}$. We would expect the correct magnitude of the damping coefficients to lie between the two limiting cases.

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TABLE I

Values of B_{edge} and B_{screw} for aluminum, copper and lead calculated from Eqs. 32 and 33. The material parameters are from Ref. 13.

MATERIAL	$B_{\text{screw}} (10^{-5} \text{ dyne sec/cm}^2)$		$B_{\text{edge}} (10^{-5} \text{ dyne sec/cm}^2)$	
	$q_{\text{max}} = q_D$	$q_{\text{max}} = 1/d$	$q_{\text{max}} = q_D$	$q_{\text{max}} = \frac{1-\nu}{d}$
Aluminum	7.2	1.4	8.2	1.0
Copper	2.3	0.4	2.6	0.3
Lead	5.6	1.1	6.1	0.7

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11. In Ref. 3, Kittel (P. 331, Eq. 36) finds $A_{||} = n_0 m V \text{Re} \left(\sigma_o / \sigma' - 1 \right) / 2\pi$.
However, this is actually the value for $A_{||}$ in the extreme case $q \ell \gg 1$.
Brailsford (Ref. 5, Eq. 6) gives the correct expression for $A_{||}$
(except for a typographical error of a factor of 1/2) in the general
case, $0 < q \ell < \infty$, which is given above in Eq. 2. Brailsford indicates
that $A_{\perp}$ was negligible, although this was not proved in his paper.
An independent calculation by the present authors gave the same result
as determined by Kittel (Eq. 3 above).
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