AUG 28 1964

LAAC-6515

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82576

TABLES FOR THE DETERMINATION OF THE SURFACE TENSIONS
OF LIQUID METALS BY THE PENDANT DROP METHOD*

by

Richard M. Bidwell
J. L. Duran, Jr.
Grant L. Hubbard**

University of California
Los Alamos Scientific Laboratory
Los Alamos, New Mexico

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ABSTRACT

Extended tables are presented for the reduction of measurements on drops, for the determination of surface tension by the pendant drop method. As they cover the range $0.20 \leq S \leq 1.10$, these new tables can be used for liquid metals as well as more conventional liquids, and probably cover all cases of practical interest. The significances of the various parameters used in the pendant drop method are clarified.

TABLES FOR THE DETERMINATION OF THE SURFACE TENSIONS OF LIQUID METALS BY THE PENDANT DROP METHOD

The pendant drop method seems one of the most promising for the measurement of interfacial tensions involving liquid metals. It has, in fact, been called the only method practicable for determining the surface tensions of the refractory metals.¹ Liquid metals are frequently easily oxidizable, and the new and renewable surface of a pendant drop is less likely to be affected by contamination than those of other techniques, particularly the commonly used sessile drop.

Data obtained on pendant drops are most conveniently reduced by the tables of Fordham.² On attempting to interpret measurements on liquid metals, however, it was found that their characteristically high surface tensions led to parameters outside the range of values tabulated. The tables were therefore recalculated and extended, using the DST code on an IBM 7090/7094 digital computer.

DERIVATION OF PARAMETERS

As was originally pointed out by Laplace,³ the surface configuration of a hanging or sessile drop of homogeneous liquid can be described exactly in mathematical terms. Tables relating the parameters involved in the shape of a drop were published by Bashforth and Adams⁴ and Andreas, Hauser and Tucker.⁵ The tables of the former investigators require a considerable amount of calculation for application to measurements on a given drop. The latter group, although they recognized that the fundamental equations relating curvatures to physical properties could be integrated numerically, based their tables on empirical observations on liquid of known surface tension. Fordham² actually carried out the integration and published tables to five significant figures for the reduction of measurements on hanging drops. These tables are simple to use and are adequate for calculating surface tensions of ordinary liquids, but are not generally applicable to liquid metals.

It was shown that the most easily measured significant dimensions of a pendant drop are its maximum diameter d_e and the diameter d_s at the height d_e above the lowest point of the drop (Fig. 1). The ratio $S = d_s/d_e$ is a function of the physical properties of the liquid, which can be used to calculate the surface tension. The derivation of the expressions used in the calculations will follow generally that of Fordham, with minor modifications in nomenclature.

The relationships between the dimensions of a drop can be handled most conveniently if expressed in dimensionless form, taking as a unit of length b , the radius of curvature at the origin, or lowest point of the drop. Considering a vertical plane through the center of the drop, the equations for the line of intersection of the plane with the drop then appear as:

$$\frac{b}{R} = \frac{d\varphi}{d(s/b)} = 2 + \beta(z/b) - \frac{\sin\varphi}{x/b} \quad (1)$$

$$\frac{d(x/b)}{d(s/b)} = \cos\varphi \quad (2)$$

$$\frac{d(z/b)}{d(s/b)} = \sin\varphi \quad (3)$$

where R is the radius of curvature at the point (x, z) , $\tan\varphi$ is the slope at point (x, z) , s is the arc distance from the origin at the bottom of the drop to point (x, z) , and

$$\beta = -g\rho b^2/\gamma,$$

where g is the acceleration of gravity, ρ is the difference in density across the surface and γ is the surface tension. The parameter β describes the shape of the drop (Fig. 2), all drops having the same value of β being geometrically similar. Liquid metals, having high values of surface tension, give drops more nearly spherical, corresponding to low values of β .

Equations 1 to 3 were solved simultaneously by the computer, using Gill's variation of the Runge-Kutta method.⁶ As in Fordham's work, x/b and z/b were calculated for assumed values of β and s/b , integrating along the profile of

Fig. 1. Dimensions Used in Determining Surface Tension by the Pendant Drop Method.

Fig. 2. Contours of Pendant Drops Described as a Function of β .

the drop from the origin. The maximum x_e/b were selected from the tabulated values, and the corresponding x_s/b obtained at the ordinates where $z = 2x_e$. Since $(x_s/b)/(x_e/b) = d_s/d_e$, S is thus determined.

The parameter $1/H$, a unique function of S , is defined by the equation:

$$1/H = -1/\beta(d_e/b)^2,$$

whence:

$$\gamma = g\rho d_e^2/H,$$

from which the surface tension is obtained directly.

$1/H$ was calculated for each assumed β and resulting x_e/b . The final value for $1/H$ in terms of S were calculated by Aitkin's iterative table interpolation method⁷ and are presented in Table I.

The initial calculations of x/b and z/b were carried out to eight figures, rather than the six reported by Fordham, and the final values of $1/H$ were tabulated to five decimal places (Table I), as in the previous work. The range covered was $0.20 \leq S \leq 1.10$.

RESULTS

At most points complete agreement to five decimal places was found with Fordham's tables. Where they existed, differences amounted usually to only one or two units in the fifth place. Somewhat larger deviations existed at the ends of Fordham's tables, which were admitted to be somewhat less accurate than the other parts. From a comparison of the methods used, it was concluded that the present values were in essential agreement with Fordham, being probably somewhat more accurate.

The range computed is believed to cover all practical cases to be encountered in the measurement of the surface tensions of liquid metals as well as ordinary liquids. Although the tables could in principle be calculated to any desired degree of precision, available methods of measuring drops do not seem to justify more than the five decimal places tabulated.

Table I

PARAMETERS FOR CALCULATING SURFACE TENSION
(1/H as a Function of S at Intervals of 0.001)

S	0	1	2	3	4	5	6	7	8	9
0.20	19.35325	19.15633	18.96089	18.76692	18.57443	18.38343	18.19390	18.00584	17.81927	17.63418
0.21	17.45056	17.26842	17.08776	16.90858	16.73087	16.55465	16.37990	16.20663	16.03484	15.86452
0.22	15.69569	15.52833	15.36245	15.19805	15.03513	14.87369	14.71372	14.55524	14.39823	14.24270
0.23	14.08864	13.93607	13.78497	13.63535	13.48721	13.34055	13.19537	13.05167	12.90944	12.76869
0.24	12.62942	12.49163	12.35531	12.22048	12.08712	11.95524	11.82484	11.69592	11.56848	11.44251
0.25	11.31802	11.19501	11.07348	10.95343	10.83485	10.71776	10.60214	10.48800	10.37534	10.26415
0.26	10.15445	10.04622	9.93947	9.83420	9.73041	9.62809	9.52726	9.42790	9.33002	9.23362
0.27	9.13870	9.04525	8.95328	8.86280	8.77379	8.68625	8.60020	8.51562	8.43253	8.35091
0.28	8.27077	8.19211	8.11492	8.03921	7.96499	7.89224	7.82028	7.74918	7.67920	7.61023
0.29	7.77429	7.70640	7.63914	7.57252	7.50655	7.44121	7.37651	7.31245	7.24903	7.18625
0.30	7.12411	7.06261	7.00175	6.94153	6.88195	6.82301	6.76471	6.70704	6.65002	6.59364
0.31	6.53790	6.48279	6.42833	6.37450	6.32132	6.26878	6.21687	6.16560	6.11498	6.06499
0.32	6.01565	5.96694	5.91887	5.87144	5.82466	5.77851	5.73300	5.68813	5.64390	5.60027
0.33	5.60579	5.56287	5.52033	5.47815	5.43635	5.39491	5.35385	5.31316	5.27284	5.23289
0.34	5.19331	5.15410	5.11526	5.07679	5.03869	5.00096	4.96361	4.92662	4.89000	4.85376
0.35	4.81788	4.78238	4.74725	4.71248	4.67809	4.64407	4.61042	4.57714	4.54423	4.51169
0.36	4.47952	4.44772	4.41629	4.38523	4.37119	4.34076	4.31058	4.28065	4.25096	4.22152
0.37	4.19232	4.16337	4.13466	4.10620	4.07798	4.05001	4.02229	3.99481	3.96758	3.94059
0.38	3.91385	3.88735	3.86110	3.83509	3.80933	3.78382	3.75855	3.73353	3.70875	3.68422
0.39	3.65993	3.63589	3.61210	3.58855	3.56524	3.54941	3.52652	3.50381	3.48127	3.45891
0.40	3.43672	3.41471	3.39288	3.37123	3.34975	3.32844	3.30732	3.28636	3.26559	3.24499
0.41	3.22457	3.20432	3.18425	3.16436	3.14464	3.12510	3.10573	3.08654	3.06753	3.04870
0.42	3.03003	3.01155	2.99691	2.97875	2.96073	2.94284	2.92508	2.90745	2.88996	2.87261
0.43	2.85538	2.83829	2.82133	2.80451	2.78782	2.77126	2.75484	2.73855	2.72239	2.70636
0.44	2.69047	2.67471	2.65909	2.64360	2.62824	2.61301	2.59792	2.58292	2.56801	2.55319
0.45	2.54081	2.52629	2.51187	2.49755	2.48334	2.46923	2.45523	2.44133	2.42754	2.41385
0.46	2.40026	2.38678	2.37341	2.36014	2.34697	2.33391	2.32096	2.30811	2.29536	2.28272
0.47	2.27018	2.25899	2.24664	2.23437	2.22218	2.21008	2.19806	2.18613	2.17428	2.16252
0.48	2.15084	2.13924	2.12773	2.11631	2.10497	2.09371	2.08254	2.07145	2.06045	2.04953
0.49	2.03870	2.02795	2.01809	2.00750	1.99698	1.98653	1.97614	1.96583	1.95559	1.94542
0.50	1.93532	1.92529	1.91532	1.90543	1.89561	1.88586	1.87617	1.86656	1.85702	1.84755

Table I (Continued)

PARAMETERS FOR CALCULATING SURFACE TENSION
($1/H$ as a Function of S at Intervals of 0.001)

S	0	1	2	3	4	5	6	7	8	9
0.51	1.83814	1.82881	1.81955	1.81089	1.80175	1.79267	1.78365	1.77468	1.76578	1.75693
0.52	1.74815	1.73942	1.73075	1.72214	1.71358	1.70509	1.69666	1.68828	1.67996	1.67170
0.53	1.66351	1.65536	1.64766	1.63963	1.63164	1.62371	1.61583	1.60799	1.60021	1.59248
0.54	1.58479	1.57716	1.56958	1.56204	1.55456	1.54713	1.53975	1.53241	1.52513	1.51790
0.55	1.51099	1.50386	1.49676	1.48971	1.48270	1.47574	1.46882	1.46194	1.45510	1.44831
0.56	1.44156	1.43486	1.42820	1.42158	1.41500	1.40847	1.40198	1.39554	1.38934	1.38297
0.57	1.37665	1.37036	1.36410	1.35789	1.35171	1.34557	1.33947	1.33341	1.32738	1.32139
0.58	1.31544	1.30953	1.30365	1.29781	1.29201	1.28641	1.28067	1.27498	1.26931	1.26368
0.59	1.25808	1.25252	1.24699	1.24149	1.23602	1.23059	1.22519	1.21983	1.21449	1.20920
0.60	1.20393	1.19882	1.19361	1.18844	1.18329	1.17818	1.17309	1.16803	1.16301	1.15801
0.61	1.15304	1.14810	1.14319	1.13831	1.13346	1.12864	1.12385	1.11918	1.11444	1.10973
0.62	1.10505	1.10039	1.09576	1.09115	1.08657	1.08202	1.07749	1.07299	1.06851	1.06407
0.63	1.05964	1.05525	1.05095	1.04661	1.04228	1.03798	1.03371	1.02946	1.02523	1.02102
0.64	1.01684	1.01268	1.00855	1.00444	1.00035	0.99629	0.99225	0.98829	0.98430	0.98032
0.65	0.97637	0.97244	0.96853	0.96464	0.96077	0.95692	0.95310	0.94929	0.94551	0.94175
0.66	0.93801	0.93434	0.93064	0.92696	0.92330	0.91965	0.91603	0.91243	0.90884	0.90528
0.67	0.90173	0.89821	0.89470	0.89121	0.88774	0.88434	0.88091	0.87749	0.87410	0.87072
0.68	0.86736	0.86401	0.86069	0.85738	0.85409	0.85081	0.84756	0.84432	0.84110	0.83793
0.69	0.83474	0.83157	0.82842	0.82528	0.82216	0.81905	0.81596	0.81288	0.80982	0.80678
0.70	0.80375	0.80074	0.79775	0.79480	0.79183	0.78888	0.78595	0.78303	0.78012	0.77723
0.71	0.77435	0.77149	0.76864	0.76581	0.76299	0.76019	0.75743	0.75465	0.75189	0.74914
0.72	0.74641	0.74369	0.74098	0.73828	0.73560	0.73293	0.73028	0.72764	0.72501	0.72242
0.73	0.71982	0.71723	0.71466	0.71209	0.70954	0.70700	0.70447	0.70196	0.69946	0.69697
0.74	0.69449	0.69203	0.68959	0.68715	0.68472	0.68231	0.67990	0.67750	0.67512	0.67275
0.75	0.67039	0.66804	0.66570	0.66338	0.66108	0.65877	0.65648	0.65420	0.65193	0.64966
0.76	0.64741	0.64517	0.64294	0.64073	0.63852	0.63632	0.63413	0.63197	0.62980	0.62765
0.77	0.62550	0.62336	0.62124	0.61912	0.61701	0.61491	0.61283	0.61075	0.60868	0.60663
0.78	0.60459	0.60255	0.60051	0.59849	0.59648	0.59448	0.59248	0.59050	0.58852	0.58656
0.79	0.58460	0.58266	0.58072	0.57879	0.57687	0.57495	0.57305	0.57115	0.56926	0.56738
0.80	0.56551	0.56365	0.56179	0.55995	0.55812	0.55629	0.55446	0.55265	0.55084	0.54904
0.81	0.54725	0.54547	0.54370	0.54193	0.54017	0.53843	0.53669	0.53495	0.53322	0.53150

Table I (Continued)

PARAMETERS FOR CALCULATING SURFACE TENSION
(1/H as a Function of S at Intervals of 0.001)

S	0	1	2	3	4	5	6	7	8	9
0.82	0.52979	0.52808	0.52638	0.52469	0.52300	0.52133	0.51966	0.51800	0.51635	0.51470
0.83	0.51306	0.51142	0.50979	0.50817	0.50656	0.50495	0.50335	0.50176	0.50017	0.49860
0.84	0.49703	0.49546	0.49390	0.49234	0.49080	0.48926	0.48772	0.48619	0.48467	0.48315
0.85	0.48165	0.48015	0.47865	0.47716	0.47568	0.47420	0.47272	0.47126	0.46980	0.46834
0.86	0.46689	0.46545	0.46402	0.46259	0.46116	0.45974	0.45833	0.45692	0.45551	0.45412
0.87	0.45272	0.45134	0.44995	0.44858	0.44722	0.44585	0.44449	0.44314	0.44179	0.44044
0.88	0.43910	0.43777	0.43644	0.43512	0.43380	0.43249	0.43118	0.42988	0.42858	0.42729
0.89	0.42600	0.42472	0.42344	0.42216	0.42089	0.41963	0.41837	0.41712	0.41587	0.41462
0.90	0.41338	0.41214	0.41091	0.40968	0.40846	0.40724	0.40602	0.40481	0.40361	0.40241
0.91	0.40121	0.40002	0.39883	0.39764	0.39646	0.39529	0.39411	0.39294	0.39178	0.39062
0.92	0.38947	0.38831	0.38716	0.38602	0.38488	0.38374	0.38261	0.38147	0.38035	0.37922
0.93	0.37811	0.37700	0.37588	0.37477	0.37367	0.37257	0.37147	0.37037	0.36928	0.36819
0.94	0.36711	0.36602	0.36495	0.36388	0.36280	0.36174	0.36067	0.35960	0.35854	0.35749
0.95	0.35643	0.35538	0.35433	0.35328	0.35225	0.35121	0.35017	0.34913	0.34810	0.34707
0.96	0.34604	0.34501	0.34399	0.34296	0.34195	0.34094	0.33992	0.33891	0.33790	0.33689
0.97	0.33588	0.33487	0.33387	0.33286	0.33186	0.33086	0.32988	0.32888	0.32788	0.32689
0.98	0.32587	0.32489	0.32391	0.32293	0.32194	0.32095	0.31996	0.31897	0.31797	0.31697
0.99	0.31597	0.31497	0.31397	0.31296	0.31195	0.31094	0.30992	0.30891	0.30854	0.30747
1.00	0.30636	0.30519	0.30398	0.30273	0.30142	0.30007	0.29867	0.29722	0.29573	0.29419
1.01	0.29260	0.29096	0.28581	0.28415	0.28254	0.28097	0.27945	0.27796	0.27652	0.27513
1.02	0.27377	0.27246	0.27119	0.26996	0.26878	0.26764	0.26654	0.26548	0.26447	0.26350
1.03	0.26403	0.26309	0.26216	0.26123	0.26031	0.25939	0.25848	0.25757	0.25667	0.25577
1.04	0.25487	0.25398	0.25309	0.25221	0.25133	0.25046	0.24959	0.24874	0.24788	0.24703
1.05	0.24617	0.24533	0.24448	0.24364	0.24281	0.24197	0.24115	0.24032	0.23950	0.23869
1.06	0.23788	0.23707	0.23626	0.23546	0.23467	0.23389	0.23311	0.23232	0.23154	0.23076
1.07	0.22999	0.22922	0.22846	0.22769	0.22694	0.22618	0.22543	0.22468	0.22394	0.22320
1.08	0.22246	0.22173	0.22101	0.22029	0.21957	0.21885	0.21813	0.21742	0.21671	0.21601
1.09	0.21530	0.21460	0.21391	0.21322	0.21253	0.21184	0.21116	0.21048	0.20981	0.20913
1.10	0.20846	0.20780	0.20714	0.20648	0.20582	0.20517	0.20452	0.20387	0.20323	0.20259

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