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TYPE EQUILIBRIA AS A FUNCTION
OF CROSS SECTIONAL SHAPE

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Vertical Stability of Tokamak Type Equilibria
as a Function of Cross Sectional Shape

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The fact that tokamak type devices with elongated cross section appear to have improved plasma parameters leads to the question of the stability of these configurations against vertical translation. This study investigates the vertical stability of both elliptical and rectangular cross section equilibria as a function of elongation. The stability analysis was done numerically through the use of a computer code.

The model used for this study is shown in schematic in Fig. 1. The solution has two parts, the details of which are given in the Appendix. First, an arbitrary current is distributed over the plasma region and the external shell currents are computed which make the edge of the plasma region the $\psi = 0$ magnetic surface.

The second part of the solution involves translating the plasma region while holding the external shell currents constant. A new plasma current distribution is computed which maintains all the flux values on the edge and inside the plasma region constant. This new plasma current distribution interacts with the magnetic field of the external current shell to give a net force on the plasma related to the displacement. This force is

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either in the same or opposing direction as the translation indicating instability or stability, respectively.

Solutions were carried out for both elliptical and rectangular cross section plasma regions with both uniform and parabolic current distributions. The results are shown in Fig. 2. The restoring force per unit displacement is plotted vs the semi-axis ratio. The major radius of these toroidal equilibria is 1.275 meters and the area of the plasma cross section is 0.98 square meters. The graph shows that for a uniform plasma current the ellipse becomes unstable at a semi-axis ratio of 1.5 while the rectangle becomes unstable at 4.2. The graphs for a parabolic plasma current distribution show that the ellipse goes unstable at 1.6 and the rectangle at 3.2.

The results of this study indicate that if tokamak type devices are to be modified to have elongated cross sections the elongation should be rectangular not elliptical in nature.

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APPENDIX

ψ_{seij} Flux at plasma edge point i due to a unit current at external shell point j.

ψ_{peij} Flux at plasma edge point i due to a unit current at plasma region point j.

ψ_{spij} Flux at plasma region point i due to a unit current at external shell point j before translation.

ψ'_{spij} Flux at plasma region point i due to a unit current at external shell point j after translation.

ψ_{ppij} Flux at plasma region point i due to a unit current at plasma region point j.

I_{pi} Current at plasma region point i before translation.

I'_{pi} Current at plasma region point i after translation.

I_{si} Current at external shell point i.

m Number of points on external shell.

n Number of points in plasma region.

k Number of points on plasma edge.

I. Simultaneous Linear Equations to Solve for External Shell Currents

$$\psi_{sell} I_{s1} + \psi_{sel2} I_{s2} + \dots + \psi_{selm} I_{sm} = - \sum_{j=1}^n \psi_{pelj} I_{pj}$$

$$\psi_{se21} I_{s1} + \psi_{se22} I_{s2} + \dots + \psi_{se2m} I_{sm} = - \sum_{j=1}^n \psi_{pe2j} I_{pj}$$

•
•
•

$$\psi_{sekl} I_{s1} + \psi_{sek2} I_{s2} + \dots + \psi_{sekm} I_{sm} = - \sum_{j=1}^n \psi_{penj} I_{pj}$$

II. Simultaneous Linear Equations to Solve for Plasma Region Currents After Translation

$$\begin{aligned} \psi_{pp11} I'_{p1} + \psi_{pp12} I'_{p2} + \dots + \psi_{pp1n} I'_{pn} \\ = \sum_{j=1}^n \psi_{pp1j} I_{pj} + \sum_{j=1}^n \psi_{sp1j} I_{sj} - \sum_{j=1}^n \psi'_{sp1j} I_{sj} \end{aligned}$$

$$\begin{aligned} \psi_{pp21} I'_{p1} + \psi_{pp22} I'_{p2} + \dots + \psi_{pp2n} I'_{pn} \\ = \sum_{j=1}^n \psi_{pp2j} I_{pj} + \sum_{j=1}^n \psi_{sp2j} I_{sj} - \sum_{j=1}^n \psi'_{sp2j} I_{sj} \end{aligned}$$

⋮

$$\begin{aligned} \psi_{ppn1} I'_{p1} + \psi_{ppn2} I'_{p2} + \dots + \psi_{ppnn} I'_{pn} \\ = \sum_{j=1}^n \psi_{ppnj} I_{pj} + \sum_{j=1}^n \psi_{spnj} I_{sj} - \sum_{j=1}^n \psi'_{spnj} I_{sj} \end{aligned}$$

a. EQUILIBRIUM

b. DISPLACEMENT WITH FLUXES HELD CONSTANT

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Fig. 1. Schematic of Solution Model.

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Fig. 2. Restoring Force per unit Displacement vs Semi-axis Ratio.

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