A STUDY OF THE LIFE-I CODE

by

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NOMENCLATURE

A_n	Avogadro's number
C_p	Specific heat of coolant
d	Grain diameter of fuel
D	Fuel density in percent
D_e	Equivalent diameter (assumed to be four times coolant flow area associated with one element divided by wetted perimeter of element)
D_o	Thermal-diffusion coefficient
e_{cs}	Swelling strain of cladding
e_s	Total swelling strain
E	Young's modulus
E_f	Young's modulus of fuel
F	Axial sliding force between fuel and cladding
F_i	Radial distribution factor for heat-generation rate
F	Frictional force between fuel and cladding
g	Force of gravity
G	Shear modulus
G_c	Shear modulus of cladding
G_f	Shear modulus of fuel
$\dot{G}_p$	Total fission-gas production of fuel element
G_r	Fractional gas-release rate
G_T	Fission-gas release
G_P	Rate of fission-gas production
h_{cp}	Heat-transfer coefficient between coolant and capsule
h_g	Heat-transfer coefficient of gap between fuel and cladding
j^{Pu}	Flux of plutonium with respect to center-of-mass reference velocity
k_c	Thermal conductivity of cladding
k_{cp}	Thermal conductivity of capsule
k_f	Thermal conductivity of fuel
k_{Na}	Thermal conductivity of coolant
K	Boltzmann's constant
L_c	Cladding length
L_f	Fuel length
L_{pl}	Plenum length

NOMENCLATURE (contd)

n_a	Atoms of fission gas per fission
n_f	Number of fissions per kW-hr/l2
P	Porosity of fuel
P_e	Peclet number
P_{pl}	Plenum pressure
POW	Power generated
q	Linear power in fuel element
q_{avg}	Average linear power for element
Q	Activation energy
r_{ci}	Radius of inner surface of cladding
r_{co}	Radius of outer surface of cladding
r_{cpi}	Radius of inner surface of capsule
r_{cpo}	Radius of outer surface of capsule
r_{fo}	Radius of outer surface of fuel
R	Universal gas constant
Δt	Time interval
T_{av}	Volume average temperature of gas
T_{ci}	Temperature of inner surface of cladding
T_{co}	Temperature of outer surface of cladding
T_{cpi}	Temperature of inner surface of capsule
T_{cpo}	Temperature of outer surface of capsule
T_{in}	Inlet coolant temperature
T_{Na}	Coolant temperature
T_o	Room temperature
T_{out}	Outlet coolant temperature
T_{pl}	Plenum temperature
$T_{void(j)}$	Temperature of central void in axial section j
$\bar{T}$	Average temperature
u	Radial displacement of a cylinder
v	Migration velocity of fuel
V_f	Fuel volume
V_p	Total volume of gas in fuel element
V_{pl}	Plenum volume
$V_{void(j)}$	Volume of central void in axial section j

NOMENCLATURE (contd)

$\left(\frac{\Delta V}{V}\right)_{i_{\text{CRIT}}}$	Critical fractional volume increase due to fission-gas swelling in <u>i</u> th zone
$\left(\frac{\Delta V}{V}\right)_{i_{\text{gas}}}$	Fractional volume increase due to fission-gas swelling in <u>i</u> th zone
$\dot{w}$	Mass flow rate of coolant
W^{Pu}	Wt % of plutonium in fuel
W^{U}	Wt % of uranium in fuel
Z_6	Generation rate of Frenkel pair
α^*	Thermal diffusion factor
α_c	Thermal expansion coefficient of cladding
α_T	Thermal expansion
ϵ_p	Equivalent creep strain
$\epsilon_{r,\theta,z}^p$	Components of creep strain
$\epsilon_{r,\theta,z}^s$	Components of swelling strain
$\epsilon_{r,\theta,z}^T$	Components of total strain
μ	Coefficient of friction between fuel and cladding
ν	Poisson's ratio
ν_c	Poisson's ratio of cladding
ν_f	Poisson's ratio of fuel
ν_o	Poisson's ratio at room temperature
ρ_f	Density of fuel
ρ_o	Theoretical density of fuel
σ_e	Equivalent stress
$\sigma_{r,\theta,z}$	Components of stress
ϕ	Neutron flux
ϕt	Fast fluence

Subscripts 1, 2, 3 denote the columnar, equiaxed, and undisturbed regions of the fuel, and subscript suffixes a and b denote inside and outside radii of the designated region. A subscript 4, with suffix a or b, is sometimes used to denote the cladding.

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ABSTRACT

LIFE-I is a FORTRAN-IV computer code that is capable of predicting behavior of fuel elements in a fast reactor. The logic, the numerical techniques, and the methods of solution employed by the code are presented, and the functions of the MAIN program and each of the subroutines are discussed. The computer program examined is based on the version existing in December 1970.

I. INTRODUCTION

The LIFE-I code was developed by V. Z. Jankus and R. W. Weeks¹ to predict the inpile performance of fast-reactor fuel elements. It has been widely distributed. The purpose of this report is to present a detailed description of the code, including the numerical techniques used in it, to enable the users to have a better understanding of the code.

II. DESCRIPTION OF THE CODE

A. MAIN Program

A simplified flow chart for the LIFE-I code is given in Ref. 1.

The MAIN program reads the input by READ statements and by calling subroutine READ1. The variables read by the MAIN program are described in Ref. 1.

The initial parameters required by the code are defined both in the MAIN program and by calling subroutine BEGIN, and the initial values of the input are printed by calling subroutine RITE2.

The temperature distribution in each axial section of the fuel element then is obtained by calling subroutine GOLDN, and the average temperature and thermal expansion in the cladding are determined by calling subroutine CLTMP. When there is a gap between the fuel and the cladding, the effective heat conductivity of the gap (XKO) can be expressed as

$$XKO = A_3 + A_4 T_{ci} \quad [00017800],^* \quad (A-1)$$

where $A_3 = 0.08483 \text{ Btu-hr}^{-1}\text{-}^\circ\text{F}^{-1}\text{-ft}$, $A_4 = 9.111 \times 10^{-5} \text{ Btu-hr}^{-1}\text{-}^\circ\text{F}^{-2}\text{-ft}$, and T_{ci} is the temperature of the inside surface of the cladding. The heat-transfer coefficient of the gap (h_g) is found as

$$h_g = XKO + \left[XKO^2 + \frac{A_4 q(n)}{\pi} \ln(r_{ci}/r_{fo}) \right]^{1/2} \left[\ln(r_{ci}/r_{fo}) \right]^{-1} \quad (A-2)$$

[00018300],

where $q(n)$ is the linear power in axial section n , and r_{ci} and r_{fo} are the radii of the inside of the cladding and the outside of the fuel, respectively. The temperature of the outside of the fuel is found by using Eq. (8) of Ref. 1 and is given in [00018700] of the computer code. The MAIN program then calls subroutine FLTMP, which calculates the temperature, thermal expansion, and restructuring rate of the fuel. The mass transfers among the three regions (columnar, equiaxed, and undisturbed) of the fuel are calculated both in the MAIN program and by calling FLTMP.

The total gas release of the fuel then is calculated by subroutine GASOUT. From the release, the pressure in the plenum is found by calling

* Italicized numbers in brackets refer to the sequence numbers in the LIFE-I code.

subroutine PLENP. The material properties of the fuel and cladding are determined by calling subroutine XLAME. The results of all these calculations are used to predict the mechanical behavior of the fuel element.

The stress and strain in the fuel element are determined by calling subroutines SWELL, HKNS, CREEF, CREEC, VZJRW, STRESS, and STRAIN. The estimated and the calculated results of stress and strain are compared in subroutine COMPO. If the convergence criterion required by the code is satisfied, the results are printed out by calling subroutine RITE2, and the calculation proceeds to the next time step. If the convergence criterion is not satisfied, a smaller time interval is used, and the stress and strain of the fuel element are reevaluated.

B. Subroutines READ1, RITE1, RITE2, and RITE3

Subroutine READ1 reads the initial values of the following parameters: the total strains (EPR, EPC, EPZ); the creep-strain increments (DPR, DPC, DPZ); the total creep strains (TPR, TPC, TPZ); the swelling-strain increments (DSR, DSC, DSZ); the total swelling strain (TSR, TSC, TSZ); the stress components (SIGR, SIGC, SIGZ); and the displacements (WDISP, UDISP). It also reads the radius of the fuel void (RVB), the radii of the columnar (RIB), equiaxed (REB), and undisturbed (RUB) regions of the fuel, the inside and outside radii of the cladding (RCA, R4B) and the capsule (RCPI, RCPO), and the sodium flow area associated with the fuel element (AFNA).

Subroutines RITE1, RITE2, and RITE3 write the initial input values and the fuel-element conditions at a particular time. Detailed functions of these subroutines are given in Ref. 1.

C. Subroutine BEGIN

1. Purpose

This subroutine defines some of the initial values of the parameters required by the MAIN program and also calculates the mass of the fuel and the generation rate of the Frenkel pair.

2. Equations

The masses per unit length of the columnar, equiaxed, and undisturbed regions of the fuel at axial section n are

$$XM6(n,j) = \pi \rho_f(j) [r_o^2(j) - r_i^2(j)] \quad [00061700 \text{ to } 00061900], \quad (C-1)$$

where ρ_f , r_o , and r_i are the density, inside radius, and outside radius, respectively, of each region of the fuel, and $j = 1, 2, 3$ denotes the columnar, equiaxed, and undisturbed regions of the fuel, respectively.

The generation rate of the Frenkel pair in section n is

$$\left. \begin{aligned} Z_6(n) &= Z6FLX(N) \cdot \phi, \\ Z6FLX(N) &= (1.053 \times 10^{-22}) [1 - 0.018z(n)] \quad [00062400], \end{aligned} \right\} \quad (C-2)$$

where

ϕ is the neutron-flux input from the MAIN program, and z is the distance from the fuel midplane (in cm).

D. Subroutine GOLDN

1. Purpose

For encapsulated fuel elements, this subroutine determines the temperatures of the inner and outer capsule surfaces (T_{cpi} and T_{cpo}) and the temperature of the outer cladding surface (T_{co}) at each axial section of the fuel element.

For unencapsulated fuel elements, this subroutine determines the outside cladding temperature at each axial section.

2. Equations and Solutions

The sodium-coolant temperature T_{Na} at the middle of axial section n is determined from Eq. (D-1) (whose terms are defined on p. 11 of Ref. 1):

$$\int_{T_{in}}^{T_{Na}(n)} C_p dT = \left[\frac{\sum_{i=1}^n QSF(i) - QSF(n)/2}{\sum_{i=1}^{NF} QSF(i)} \right] \int_{T_{in}}^{T_{out}} C_p dT, \quad (D-1)$$

$$\text{with} \quad C_p = C_0 + C_1 T_{Na}(n) + C_2 T_{Na}^2(n), \quad (D-2)$$

where C_0 , C_1 , and C_2 are constants defined in this subroutine. T_{out} , T_{in} , and $QSF(i)$ are input from the MAIN program.

Both sides of Eq. (D-1) are a function of $T_{Na}(n)$; an iteration technique is used to solve for $T_{Na}(n)$. Let us define:

$$\left. \begin{aligned} \Delta T_1(n) &= T_{Na}^{(1)}(n) - T_{in}; \Delta T_2(n) = T_{Na}^{(2)}(n) - T_{in} - - - - \\ \Delta T_i(n) &= T_{Na}^{(i)}(n) - T_{in}; \Delta T_{i+1}(n) = T_{Na}^{(i+1)}(n) - T_{in}, \end{aligned} \right\} \quad (D-3)$$

where i is the number of the iteration. By substituting Eq. (D-3) into Eq. (D-2), we find that

$$C_p = B_0 + B_1 \Delta T_i + B_2 \Delta T_i^2 \quad [00228500], \quad (D-4)$$

where

$$\left. \begin{aligned} B_0 &= C_0 + C_1 T_{in} + C_2 T_{in}^2 \quad [00116000], \\ B_1 &= C_1 + 2C_2 T_{in} \quad [00116100], \\ B_2 &= C_2. \end{aligned} \right\} \quad (D-5)$$

and

By substituting Eqs. (D-3) and (D-4) into Eq. (D-1) and iterating, we obtain the following for the temperature difference after the (i+1)th iteration:

$$\Delta T_{i+1} = \left[\frac{\sum_{i=1}^n QSF(i) - QSF(n)/2}{\sum_{i=1}^n QSF(i)} \int_{T_{in}}^{T_{out}} C_p dT + \frac{1}{2} B_1 \Delta T_i^2(n) + \frac{2}{3} B_2 \Delta T_i^3(n) \right] / [B_0 + B_1 \Delta T_i(n) + B_2 \Delta T_i^2(n)] \quad [00118600], \quad (D-6)$$

where

$$\int_{T_{in}}^{T_{out}} C_p dT = CPDT = B_0 (T_{out} - T_{in}) + \frac{B_1}{2} (T_{out} - T_{in})^2 + \frac{B_2}{3} (T_{out} - T_{in})^3 \quad [00116400]. \quad (D-7)$$

The convergence criterion (ϵ_1) used in the problem is

$$\epsilon_1 = \frac{\Delta T_{i+1} - \Delta T_i}{\Delta T_{i+1}} \leq 10^{-5} \quad [00118700], \quad (D-8)$$

and the coolant temperature found at the nth section is

$$T_{Na}(n) = T_{in} + \Delta T(n) \quad [00119000]. \quad (D-9)$$

If the fuel element is encapsulated, the temperature of the outer capsule surface at section n can be found by

$$T_{cpo}(n) = T_{Na}(n) + \frac{q(n)}{2\pi r_{cpo} h_{cp}}, \quad (D-10)$$

where r_{cpo} is the outside radius of the capsule,

$$h_{cp} = \frac{k_{Na}}{D_e} [5 + 0.025 (P_e)^{0.8}] \quad [00120900], \quad (D-11)$$

$$k_{Na} = a_0 + a_1 T_{Na}(n) + a_2 T_{Na}^2(n) \quad [00120300], \quad (D-12)$$

$$P_e = \frac{4\dot{w}C_p}{2\pi r_{cpo} k_{Na}} \quad [00120500], \quad (D-13)$$

and

$$q(n) = q_{avg} \cdot NF \left[\frac{QSF(n)}{\sum_{i=1}^{NF} QSF(i)} \right] \quad [00119300]. \quad (D-14)$$

In Eqs. (D-10) through (D-14), h_{cp} is the heat-transfer coefficient between coolant and capsule, k_{Na} is the thermal conductivity of the coolant, D_e is the equivalent diameter (assumed to be four times the coolant flow area associated with one element divided by the wetted perimeter of the element), P_e is the Peclet number, $\dot{w}$ is the mass flow rate of the coolant, q_{avg} is the average linear power for the element (which is input from the MAIN program), and a_0 , a_1 , and a_2 are constants defined in this subroutine.

The inside capsule temperature (T_{cpi}) at an axial section n can be expressed as

$$T_{cpi}(n) = T_{cpo} + \frac{[q(n)/\pi] \ln(r_{cpo}/r_{cpi})}{k_{cp} + \left[k_{cp}^2 + \frac{2A_2}{\pi} \ln(r_{cpo}/r_{cpi}) \right]^{1/2}} \quad [00122500], \quad (D-15)$$

with

$$k_{cp} = A_1 + A_2 T \quad [00122400], \quad (D-16)$$

where r_{cpi} and r_{cpo} are the radii of the inner and outer capsule surfaces, k_{cp} is the thermal conductivity of the capsule, and the constants A_1 and A_2 are defined in this subroutine.

The temperature (T_{co}) of the outside surface of the cladding in axial section n is first determined by

$$\int_{T_{cpi}}^{T_{co}} k_{Na} dT = \frac{q(n)}{2\pi} \ln(r_{co}/r_{cpi}), \quad (D-17)$$

where r_{co} is the radius of the inside surface of the cladding.

By an iteration technique similar to that used in determining the coolant temperature, the cladding temperature after the (i+1)th iteration is calculated from

$$T_{co}(n) = T_{cpi}(n) + \left\{ [q(n)/2\pi] \ln(r_{cpi}/r_{co}) + \frac{B_1}{2} \Delta T_1^2(n) + \frac{2}{3} B_2 \Delta T_1^3(n) \right\} / [B_0 + B_1 \Delta T_1(n) + B_2 \Delta T_1^2(n)]$$

[00123500, 00124100, 00124600].

(D-18)

The convergence criterion in this iteration is

$$\epsilon_2 = \frac{T_{co}^{(i+1)}(n) - T_{co}^i(n)}{T_{co}^i(n)} \leq 10^{-4} \quad [00121500]. \quad (D-19)$$

If the fuel element is unencapsulated, the temperature of the outside surface of the cladding can be found by an equation similar to Eq. (D-10):

$$T_{co}(n) = T_{Na}(n) + \frac{q(n)}{2\pi r_{co} h_c} \quad (D-20)$$

3. Constants Defined by Subroutine

$$\begin{aligned} a_0 &= 4.5255 \text{ Btu-hr}^{-1}\text{-}^\circ\text{F}^{-1}\text{-in.}^{-1} \\ a_1 &= -0.001565 \text{ Btu-hr}^{-1}\text{-}^\circ\text{F}^{-2}\text{-in.}^{-1} \\ a_2 &= 1.7428 \times 10^{-7} \text{ Btu-hr}^{-1}\text{-}^\circ\text{F}^{-3}\text{-in.}^{-1} \\ A_1 &= 0.765 \text{ Btu-hr}^{-1}\text{-}^\circ\text{F}^{-1}\text{-in.}^{-1} \\ A_2 &= 3.508 \times 10^{-5} \text{ Btu-hr}^{-1}\text{-}^\circ\text{F}^{-2}\text{-in.}^{-1} \end{aligned}$$

$$\begin{aligned} C_0 &= 0.345696 \text{ Btu-lb}^{-1} \\ C_1 &= -0.79211 \times 10^{-4} \text{ Btu-lb}^{-1} \text{ } ^\circ\text{F}^{-1} \\ C_2 &= 3.41178 \times 10^{-8} \text{ Btu-lb}^{-1} \text{ } ^\circ\text{F}^{-2} \end{aligned}$$

E. Subroutine CLTMP

1. Purpose

This subroutine determines the average temperature and thermal expansion of the cladding in each axial section.

2. Equations and Solutions

The cladding temperature at point i in axial section n can be written as

$$T_c(n,i) = T_{co}(n) + \frac{[q(n)/\pi] \ln[r_{co}/r(i)]}{k_c + \left\{ k_c^2 + \frac{2A_2}{\pi} \ln[r_{co}/r(i)] \right\}^{1/2}} \quad [0023700], \quad (E-1)$$

where $r(i)$ is the radius of the cladding at point i , $q(n)$ and k_c (the thermal conductivity of the cladding) are calculated in subroutine GOLDN, and A_2 is defined in subroutine GOLDN.

The average cladding temperature (TBC) and thermal expansion (ATBC) in section n are defined by

$$TBC(n) = \frac{\int_{r_{ci}}^{r_{co}} T_c(n,i) r dr}{\int_{r_{ci}}^{r_{co}} r dr} \quad [00237500] \quad (E-2)$$

and

$$ATBC(n) = \frac{\int_{r_{ci}}^{r_{co}} \alpha_c T_c(n,i) r dr}{\int_{r_{ci}}^{r_{co}} r dr} \quad [00237600], \quad (E-3)$$

where the thermal expansion coefficient, α_c , of the cladding is

$$\alpha_c = C_{A1} + C_{A2} \Delta T. \quad (E-4)$$

The integrals appearing in Eqs. (E-2) and (E-3) were numerically obtained using Weddle's rule. The constants C_{A1} and C_{A2} are input from the MAIN program; they are $8.778 \times 10^{-6} \text{ } ^\circ\text{F}^{-1}$ and $1.466 \times 10^{-10} \text{ } ^\circ\text{F}^{-2}$, respectively.

F. Subroutine FLTMP

1. Purpose

This subroutine determines the temperature and the restructuring rate of the fuel.

2. Equations and Solutions

In each axial section, the fuel is divided into N_1 radial elements, and the mass increment of each is defined as

$$dm = \frac{\sum_{i=1}^3 m(i)}{2 N_1} \quad [00097300], \quad (F-1)$$

where $m(1)$, $m(2)$, and $m(3)$ are the masses in the columnar, equiaxed, and undisturbed regions, respectively. The mass increment can be expressed in terms of radius increment as

$$dm_i = \rho_i \pi [(r_i + dr_i)^2 - r_i^2], \quad (F-2)$$

where ρ_i is the fuel density in the i th region. If dr_i^2 is neglected, the radius increment is

$$dr_o = \frac{dm_i}{2\pi\rho_i r_i} \quad [00099000], \quad (F-3)$$

where dr_o denotes the approximate quantity of dr .

Solving of Eq. (F-2) gives

$$dr = \frac{dm}{\pi \rho_1 \left[r + \sqrt{r^2 + \frac{dm}{\rho_1}} \right]} \quad (F-4)$$

By substituting Eq. (F-3) into Eq. (F-4), we find that

$$dr_1 = \frac{dr_o}{0.5 + \sqrt{0.25 + \frac{dr_o}{2r_1}}} \quad [00100200]. \quad (F-5)$$

When $\left(0.25 + \frac{dr_o}{2r_1}\right)$ is less than zero, the central hole in the columnar region is assumed to be closed [00099200, 00099600, 00099700].

The heat-generation rate within element dm_1 is

$$dq_1 = q_1 \frac{F_1 dm_1}{\sum_{i=1}^{2N_1} F_i dm_i} \quad [00100700], \quad (F-6)$$

where F_1 , the radial distribution factor for the heat-generation rate, is calculated in the MAIN program.

The temperature distribution of the fuel can be found from

$$dT = \frac{-1}{k_f} \left(J dr + \frac{1}{2} \frac{dJ}{dr} dr^2 + \frac{1}{2} \frac{dk_f}{dT} \frac{J^2}{k_f} dr^2 \right)^* \quad [00102000], \quad (F-7)$$

where k_f is thermal conductivity of the fuel,

$$\left. \begin{aligned} J &= \frac{W}{2\pi r} \quad [00100800], \\ \frac{dJ}{dr} &= \frac{1}{2\pi r^2} \left(\frac{dW}{dr} r - W \right) \quad [00101000], \\ \frac{dW}{dr} &= W_1 \frac{dm}{dr} \quad [00100900], \end{aligned} \right\} \quad (F-8)$$

* At this point, the program has a sign error.

and

$$W_1 = \frac{W \cdot N_1 \cdot F_1}{\left(\sum_{i=1}^{N_1} F_i \right) \left(\sum_{i=1}^{N_1} dm_i \right)} \quad [00100700]. \quad (F-8)$$

W is q(n); it is calculated in subroutine GOLDN.

When $NOK \leq 1$ (see Ref. 1, p. 33),

$$k_f = \left(A_1 + \frac{B_1}{T} + C_1 T^3 \right) \left(\frac{1-P}{1+\beta P} \right)^* \quad [001013300] \quad (F-9)$$

and

$$\frac{dk_f}{dT} = \left(\frac{-B_1}{T^2} + 3 C_1 T^2 \right) \left(\frac{1-P}{1+\beta P} \right) \quad [00101400]. \quad (F-10)$$

When $NOK \geq 2$,

$$k_f = A_2 + \frac{1}{\left(B_2 - \frac{C_2 \rho}{\rho_o} \right) T} \quad [00101700] \quad (F-11)$$

and

$$\frac{dk_f}{dT} = \frac{-1}{\left(B_2 - \frac{C_2 \rho}{\rho_o} \right) T^2} \quad [00101800]. \quad (F-12)$$

P and ρ_o are the porosity and the theoretical density of the fuel, respectively, and A_1 , B_1 , C_1 , B_2 , C_2 and β are constants defined by this subroutine.

The average temperature ($\bar{T}$) and thermal expansion ($\bar{\alpha T}$) of the fuel is determined by

$$\bar{T} = \frac{\int T dm}{\int dm} \quad (F-13)$$

and

$$\bar{\alpha T} = \frac{\int \alpha T dm}{\int dm}, \text{ respectively.} \quad (F-14)$$

* P should be the value at room temperature. The program used the value at elevated temperature.

Equations (F-13) and (F-14) are numerically obtained in [00102100], [00102300], [00104500], and [00104600].

The migration velocity of the fuel can be found as

$$v = C_1 T^{-3/2} e^{-C_2/T} \frac{dT}{dr}, \quad (F-15)$$

where C_1 and C_2 are constants depending on the fuel properties. The natural log of v is calculated in [00105800]. It should be noted that the migration velocity calculated is in $\text{g-in.}^{-2}\text{-hr}^{-1}$. The mass transfer due to fuel migration is

$$CM(I) = 2\pi r(I)v \quad [00106100, 00106200], \quad (F-16)$$

where $I = 1, 2, 3$ denotes the columnar, equiaxed, and undisturbed regions of the fuel, respectively.

3. Constants Defined by Subroutine

For determining thermal conductivity

$$A_1 = 0.018 \text{ W-cm}^{-1}\text{-}^\circ\text{C}^{-1}$$

$$A_2 = 0.011 \text{ W-cm}^{-1}\text{-}^\circ\text{C}^{-1}$$

$$B_1 = 54.0 \text{ W-cm}^{-1}$$

$$B_2 = 0.485 \text{ cm-W}^{-1}$$

$$C_1 = 1.75 \times 10^{-2} \text{ W-cm}^{-1}\text{-}^\circ\text{C}^{-1}$$

$$C_2 = 0.4465 \text{ cm-W}^{-1}$$

$$\beta = 1$$

$$\rho_o = 11.0 \text{ g-cm}^{-3}$$

$$\rho \text{ (columnar)} = 10.89 \text{ g-cm}^{-3}$$

$$\rho \text{ (equiaxed)} = 10.62 \text{ g-cm}^{-3}$$

$$\rho \text{ (undisturbed)} = 10.58 \text{ g-cm}^{-3}$$

For determining migration velocity

$$C_1 \text{ (columnar-equiaxed)} = 2 \times 10^{13} \text{ g-in.}^{-1}\text{-hr}^{-1}\text{-}^\circ\text{C}^{-0.5}$$

$$C_1 \text{ (equiaxed-undisturbed)} = 3.3 \times 10^{13} \text{ g-in.}^{-1}\text{-hr}^{-1}\text{-}^\circ\text{C}^{-0.5}$$

$$C_2 \text{ (columnar-equiaxed)} = 7 \times 10^4 \text{ } ^\circ\text{C}$$

$$C_2 \text{ (equiaxed-undisturbed)} = 5 \times 10^4 \text{ } ^\circ\text{C}$$

G. Subroutine PLUTO

1. Purpose

This subroutine determines the redistribution of plutonium in the mixed fuel.

2. Equations and Solutions

The flux of plutonium (J^{Pu}) can be determined as

$$J^{\text{Pu}} = \rho_f W^{\text{Pu}} W^{\text{U}} D' \nabla T - \rho_f D \nabla W^{\text{Pu}}, \quad (\text{G-1})$$

$$D = D_0 e^{-Q/RT}, \quad (\text{G-2})$$

and

$$D' = \frac{D\alpha^*}{T}, \quad (\text{G-3})$$

where ρ_f , the fuel density, is input from the MAIN program, ∇T is the temperature gradient calculated in subroutine FLTMP, W^{Pu} and W^{U} are the wt % of plutonium and uranium, respectively, D_0 is the thermal diffusion coefficient ($0.046 \text{ cm}^2\text{-sec}^{-1}$), Q is the activation energy ($1 \times 10^5 \text{ cal-mole}^{-1}$), α^* is the thermal diffusion factor (11.0), R is the universal gas constant, and T is the fuel temperature obtained in subroutine FLTMP. The value of W^{Pu} is approximated by the initial plutonium concentration, which is the input of the program. When an approximate-finite-difference technique is used at nodal point K ,

$$\nabla W_K^{\text{Pu}} = \frac{2(W_K^{\text{Pu}} - W_{K-1}^{\text{Pu}})}{r_{K+1} - r_{K-1}} \quad (\text{G-4})$$

and

$$W_K^{\text{U}} = 1 - W_K^{\text{Pu}}. \quad (\text{G-5})$$

By substitution of Eqs. (G-2) to (G-5) into Eq. (G-1), the flux of plutonium around the circumference is found as

$$J_A^{Pu} = 2\pi r_K J_K^{Pu} \quad [00167800-00168000]. \quad (G-6)$$

The time derivative of the plutonium concentration, which determines the plutonium redistribution, is found as

$$\frac{dW^{Pu}}{dt} = - \frac{1}{\rho_f} \text{Div } J^{Pu}. \quad (G-7)$$

The numerical solution of Eq. (G-7) is given in [00168100-00169500] of this subroutine.

H. Subroutine GASOUT

1. Purpose

This subroutine determines the gas release of the fuel element.

2. Equations and Solutions

The total fission-gas production (G_p) during time Δt can be expressed as

$$G_p = n_a \cdot V_f \cdot F_d \cdot D_f / A_n \quad [00111600], \quad (H-1)$$

where

$$D_f = n_f \cdot POW \cdot \Delta t \quad [00110900],$$

n_a is the atoms of fission gas created per fission, V_f is the fuel volume, F_d is calculated in subroutine FLTMP, A_n is Avogadro's number, n_f is the number of fissions per kW-hr/12, POW is the power generated, and Δt is the time interval.

It is known that the fractional gas-release rates (G_r) for each of the three zones is increased if the swelling of the zone as a result of

the accumulation of fission gas rises above a specified limit (e.g., 25% volume expansion). The modification takes the form

$$G_r^m(i) = G_r(i) - \left[\left(\frac{\Delta V}{V} \right)_{i_{\text{gas}}} - \left(\frac{\Delta V}{V} \right)_{i_{\text{CRIT}}} \right] 100 \cdot [G_r(i) - G_r(\text{columnar})]^* \\ [00112200], \quad (\text{H-2})$$

where $(\Delta V/V)_{i_{\text{gas}}}$ is the fractional volume increase due to fission-gas swelling in the ith zone and $(\Delta V/V)_{i_{\text{CRIT}}}$ is the critical fractional increase due to fission-gas swelling in the ith zone.

The differential equation for gas release is

$$\frac{dG_T}{dt} = -F \cdot G_T + \dot{G}_P, \quad (\text{H-3})$$

where G_T is the gas release, $\dot{G}_P$ is the gas-production rate, and

$$F = G_r^m(i). \quad (\text{H-4})$$

Equation (H-3) can be solved as

$$G_T = \frac{G_P}{F\Delta t} (1 - e^{-F\Delta t}) + G_o e^{-F\Delta t} \quad [00113300], \quad (\text{H-5})$$

where G_o is the amount of gas release in the previous time step. The initial amount of gas release is defined in subroutine BEGIN.

3. Constants Defined by Subroutine

$$\begin{aligned} A_n &= 6.02 \times 10^{23} \\ G_r(\text{columnar}) &= 0.005 \\ G_r(\text{equiaxed}) &= 0.00011 \\ G_r(\text{undisturbed}) &= 0.00005 \\ n_a &= 0.27 \\ n_f &= 9.3 \times 10^{15} \text{ fissions-ft-kW}^{-1}\text{-hr}^{-1}\text{-in.}^{-1} \end{aligned}$$

* A sign error is in Eq. (21) of Ref. 1.

I. Subroutine PLENP

1. Purpose

This subroutine determines the plenum pressure in the fuel element.

2. Equations and Solutions

If the volume of the gap between the fuel and the cladding is neglected, the total volume (V_P) of gas in the element is

$$V_P = V_{pl} + \sum_{j=1}^{NF} V_{void}(j) = \pi r_{c1}^2 (L_c - L_f) + \sum_{j=1}^{NF} \pi r_{1a}^2 L_j, \quad (I-1)$$

where V_{pl} is the plenum volume, $V_{void}(j)$ is the volume of the central void in axial section j , L_c is the cladding length, L_f is the fuel length, r_{1a} is the inside radius of the columnar region of the fuel, and L_j is the length of fuel in each section.

The volume average temperature of the gas is

$$T_{av} = \frac{V_P}{\frac{V_{pl}}{T_{pl}} + \sum_{j=1}^{NE} \frac{V_{void}(j)}{T_{void}(j)}} \quad [00265500], \quad (I-2)$$

where the plenum temperature (T_{pl}) is assumed to be the same as the outlet coolant temperature from the MAIN program,* and $T_{void}(j)$ is the temperature in the fuel-void region, which is calculated in subroutine FLTMP.

The plenum pressure (P_{pl}) is obtained using the perfect gas law:

$$P_{pl} = \frac{nRT_{av}}{V_P} \quad [00265600], \quad (I-3)$$

where n is the number of gas moles in the fuel element, which is calculated in subroutine GASOUT, and $R = 73.592 \text{ lb-in.-mole}^{-1}\text{-}^\circ\text{K}^{-1}$, the universal gas constant.

*The assumption that T_{pl} is the same as the outlet coolant temperature may not be true in reality.

J. Subroutine XLAME

1. Purpose

This subroutine determines the material properties of the fuel and the cladding.

2. Equations and Solutions

The Young's modulus (E_f) and the Poisson's ratio (ν_f) of the fuel can be expressed as

$$E_f = C_1(1 + C_2P)(1 - T/C_3) \quad [00288900] \quad (J-1)$$

and

$$\nu_f = \nu_o(1 + C_4P)(C_3 - T) + 0.5(T - T_o)/(C_3 - T_o) \\ [00289000, 00289100], \quad (J-2)$$

where C_1 - C_4 are constants defined in this subroutine, T is the existing fuel temperature, ν_o is the Poisson's ratio at room temperature, and T_o is the room temperature. The shear modulus (G_f) of the fuel is defined by

$$G_f = \frac{E_f}{2(1 + \nu_f)} \quad [00289200]. \quad (J-3)$$

The shear modulus (G_c) and Poisson's ratio (ν_c) of the cladding are given by

$$G_c = A_3 + A_4T \quad [00290000] \quad (J-4)$$

and

$$\nu_c = A_7 + A_8T \quad [00290100], \quad (J-5)$$

where A_3 , A_4 , and A_7 are constants defined in this subroutine, and

$$A_8 = -0.2 A_4/A_3 \quad [00287700]. \quad (J-6)$$

3. Constants Defined by Subroutine

$$\begin{aligned} A_3 &= 1.14 \times 10^7 \text{ psi} \\ A_4 &= -2.69 \times 10^3 \text{ lb-in.}^{-2}\text{-}^\circ\text{F}^{-1} \\ A_7 &= 0.3 \\ C_1 &= 3.59 \times 10^7 \text{ psi} \\ C_2 &= -2.35 \\ C_3 &= 22,200^\circ\text{F} \\ C_4 &= -0.46 \end{aligned}$$

K. Subroutine SWELL

1. Purpose

This subroutine determines the swelling strain and average pressure of the fuel.

2. Equations and Solutions

The volume increase of the fuel due to swelling is obtained by multiplying the solid volume created per fission ($0.7 \times 10^{-24} \text{ in.}^3$) by the number of fissions produced in each section. The swelling strain (e_s) is calculated in [00209900]. In the columnar and equiaxed regions, the swelling rates due to gaseous fission products are

$$\left(\frac{\Delta V}{V} \right)_{\text{gas}} = \frac{A_2}{P_1 + P_1'}, \quad (\text{K-1})$$

where

$$A_2 = \frac{n_1 R \bar{T}_1}{V_1} \quad [00210400].$$

ΔV_1 , V_1 , P_1 , P_1' , n_1 , and $\bar{T}_1$ are defined on p. 21* of Ref. 1. The value n_1 is calculated in subroutine GASOUT, and $\bar{T}_1$ is calculated in subroutine FLTMP. P_1' is estimated to be 500, 1,000, and 1,000,000 psi in the columnar, equiaxed, and undisturbed regions, respectively. To account for the

*The definition of P_1 on p. 21 of Ref. 1 has a sign error.

incompressible part of fission gas, 2.38 in.³/mole of gas produced is added to the undisturbed region in [00211300].

When the three relations in Eq. (26) on p. 18 of Ref. 1 are added together, and constant volume is assumed for the plastic region,

$$\left. \begin{aligned} EB &= \frac{1}{3E} (\sigma_r + \sigma_\theta + \sigma_z)(1 - 2\nu_1) + \frac{SWS6}{3} + \frac{1}{3} \left(\frac{\Delta V}{V} \right)_{i_{gas}}, \\ EB &= \frac{1}{3} (\epsilon_r^T + \epsilon_\theta^T + \epsilon_z^T) - \overline{\alpha T_1}, \\ \text{and} \\ SWS6 &= \epsilon_r^S + \epsilon_\theta^S + \epsilon_z^S = e_s, \end{aligned} \right\} \quad (K-2)$$

where

$$\begin{aligned} E &= \text{Young's modulus,} \\ \sigma_{r,\theta,z} &= \text{components of stress,} \\ \nu &= \text{Poisson's ratio,} \\ \epsilon_{r,\theta,z}^T &= \text{components of total strain,} \end{aligned}$$

and

$$\epsilon_{r,\theta,z}^S = \text{components of swelling strain.}$$

EB is given in the MAIN program.

If the effect of hot-pressing on the fuel swelling is taken into account, the strain rate of the fuel is decreased by

$$\left. \begin{aligned} (\Delta \epsilon_1)^{hp} &= -CK(P_0 + P_1) \left[RHF - \left(\frac{\Delta V}{V} \right)_{i_{gas}} \right]^*, \\ \text{where} \\ CK &= \frac{C}{2T_1} \exp(-Q_{rK}/\overline{T_1}) \Delta t \quad [00209700], \\ \text{and} \\ RHF &= 1 - \frac{\rho_1}{\rho_0} \quad [00209800]. \end{aligned} \right\} \quad (K-3)$$

P_1 , which equals $-\frac{1}{3} (\sigma_\theta + \sigma_r + \sigma_z)$, is calculated in the MAIN program by calling subroutine STRESS, P_0 is the hydrostatic pressure at the previous time interval, and C and Q_{rK} are numerical constants defined in this subroutine.

* On p. 51 of Ref. 1, ΔT (temperature difference) should be replaced by Δt (time difference) in the equation that expresses the hot-pressing rate of the fuel.

We define a new variable, x , which is

$$x = P_1 + P_1'.$$

So,

$$P_1 = x - P_1' \quad [00212900]. \quad (K-4)$$

If the hot-pressing of Eq. (K-3) is included in Eq. (K-2), we find that

$$\begin{aligned} EB = & -\frac{P_1}{3E} (1 - 2v_1) + \frac{SWS6}{3} + \frac{1}{3} \left(\frac{\Delta V}{V} \right)_{1_{\text{gas}}} \\ & - \frac{CK}{3} (P_0 + P_1) \left[RHF - \left(\frac{\Delta V}{V} \right)_{1_{\text{gas}}} \right]. \end{aligned} \quad (K-5)$$

By substituting Eqs. (K-1) and (K-4) into Eq. (K-5), we find that

$$EB = \frac{-P_1}{BM} + \frac{SWS6}{3} + \frac{A_2}{3x} - \frac{CKNU}{3} (x + P_0 - P_1'), \quad (K-6)$$

where

$$BM = \frac{3E}{1 - 2v_1},$$

and

$$CKNU = CK \left[RHF - \frac{A_2}{P_0 + P_1'} \right] \quad [00211400]. \quad (K-7)$$

Equation (K-6) can be written as

$$\frac{x^2}{BN} - \left[\left(\frac{SWS7}{3} - EB \right) + \frac{P_1'}{BN} \right] x - \frac{A_2}{3} = 0, \quad (K-8)$$

where

$$BN = \frac{1}{\left(\frac{1}{BM} + \frac{CKNU}{3} \right)} \quad [00211900], \quad (K-9)$$

and

$$SWS7 = SWS6 - CKNU \cdot P_0 \quad [00211700]. \quad (K-10)$$

Rearranging Eq. (K-8), we find that

$$x^2 - 2C_1x - C_2 = 0, \quad (K-11)$$

where

$$C_1 = \frac{1}{2} P_1' + \left(\frac{SWS7}{3} - EB \right) BN, \quad (K-12)$$

and

$$C_2 = \frac{1}{3} (A_2 \cdot BN).$$

Equation (K-11) can be solved as

$$\left. \begin{aligned} x &= C_1 + \sqrt{C_1^2 + C_2} \quad [00212600] \\ x &= C_1 - \sqrt{C_1^2 + C_2} \quad [00212800]. \end{aligned} \right\} \quad (K-13)$$

The pressure P_1 therefore can be found from Eq. (K-4). The fuel swelling (FSN) can be expressed by

$$\left. \begin{aligned} FSN &= EB + \frac{P_1}{BM} \quad [00213800] \\ &= \epsilon_r^S + \Delta \epsilon_r^S \\ &= \epsilon_\theta^S + \Delta \epsilon_r^S \\ &= \epsilon_z^S + \Delta \epsilon_z^S. \end{aligned} \right\} \quad (K-14)$$

The swelling-strain increments are therefore

$$\left. \begin{aligned} \Delta \epsilon_r^S &= FSN - \epsilon_r^S \quad [00213800] \\ \Delta \epsilon_\theta^S &= FSN - \epsilon_\theta^S \quad [00213900] \\ \Delta \epsilon_z^S &= FSN - \epsilon_z^S \quad [00214000]. \end{aligned} \right\} \quad (K-15)$$

3. Constants Defined by Subroutine

$$C = 5.56 \times 10^5 \text{ } ^\circ\text{K-cm}^2\text{-sec}^{-1}\text{-dyne}^{-1}$$

$$Q_{rK} = 43,800^\circ\text{K}$$

L. Subroutine HKNS

1. Purpose

This subroutine determines the swelling strain of the cladding.

2. Equations and Solutions

The subroutine provides several options for prediction of the cladding swelling. If $\text{NOH} < 0$, an empirical model developed by Westinghouse and the Battelle Pacific Northwest Laboratory is employed. It gives the swelling strain of the cladding as

$$e_{cs} = \frac{B_0(\phi t)^n}{300} 10^{(B_1/T + B_2/T^2)} \quad [00129900, 00130000], \quad (L-1)$$

where B_0 , B_1 , B_2 , and n are constants defined in this subroutine, ϕt is fast fluence (input from the MAIN program), and T is the temperature, in $^\circ\text{K}$, obtained in subroutine CLTMP.

For options with $\text{NOH} \geq 0$, the theoretical models of Harkness and Li are used. The equation and the computer program are very complicated, and we will not discuss them here. However, it is suspected that when $\text{NOH} = 1$, disagreements may exist between the computer program and Ref. 2, and the user should be careful if the model in Ref. 2 is chosen.

3. Constants Defined by Subroutine

$$B_0 = 4.9 \times 10^{-49}$$

$$B_1 = 1.55 \times 10^4$$

$$B_2 = -5.99 \times 10^6$$

$$n = 1.71$$

M. Subroutines EQUIEP and EQUISI

1. Purpose

These subroutines determine the equivalent creep-strain increment ($\Delta\epsilon_p$) and equivalent stress (σ_e) of the fuel element.

2. Equations and Solutions

The equivalent creep-strain increment is defined by

$$\Delta\epsilon_p = \frac{\sqrt{2}}{3} \left[\left(\Delta\epsilon_r^p - \Delta\epsilon_\theta^p \right)^2 + \left(\Delta\epsilon_r^p - \Delta\epsilon_z^p \right)^2 + \left(\Delta\epsilon_\theta^p - \Delta\epsilon_z^p \right)^2 \right]^{1/2}$$

[00248800]. (M-1)

The quantities $\Delta\epsilon_r^p$, $\Delta\epsilon_\theta^p$, and $\Delta\epsilon_z^p$, the components of the creep-strain increment, are calculated in subroutines CREEF and CREEC for the fuel and the cladding, respectively, and stored in the MAIN program.

The equivalent stress is defined as

$$\sigma_e = \frac{\sqrt{2}}{2} \left[\left(\sigma_r - \sigma_\theta \right)^2 + \left(\sigma_r - \sigma_z \right)^2 + \left(\sigma_\theta - \sigma_z \right)^2 \right]^{1/2}$$

[00249500]. (M-2)

The quantities σ_r , σ_θ and σ_z are calculated in subroutine STRESS and stored in the MAIN program.

N. Subroutine CREEC

1. Purpose

This subroutine determines the equivalent stress and the creep-strain increment of the cladding.

2. Equations and Solutions

The equivalent creep-strain increment of the cladding can be written as

$$\Delta \epsilon_p = f(\sigma_e) = \left(A \sigma_e^m e^{-Q/Kt} + B \sigma_e^n \phi \right) \Delta t, \quad (N-1)$$

where Q is the average activation energy for thermal creep, K is Boltzmann's constant, T is absolute temperature, and A , B , m , and n are either constants or fluence- and shear-modulus-dependent quantities, depending on the mathematical model employed.

The relationship between the equivalent stress and equivalent creep strain can be written as

$$\sigma_e = \frac{G}{3} \epsilon_p, \quad (N-2)$$

where G is shear modulus.

If the subscript i denotes the number of the iteration, then the equivalent strain after the i th iteration is

$$(\epsilon_p)_{i+1} = \frac{3}{G} (\sigma_e)_i + \frac{3}{G} \Delta(\sigma_e)_i + f[(\sigma_e)_i] + f'[(\sigma_e)_i] \Delta(\sigma_e)_i. \quad (N-3)$$

From equation (N-3),

$$\Delta(\sigma_e)_i = \frac{(\epsilon_p)_{i+1} - f[(\sigma_e)_i] - \frac{3}{G} (\sigma_e)_i}{f'[(\sigma_e)_i] + \frac{3}{G}}, \quad (N-4)$$

where

$$f'(\sigma_e) = \left(Am \sigma_e^{m-1} e^{-Q/KT} + n B \sigma_e^{n-1} \phi \right) \Delta t. \quad (N-5)$$

With iteration,

$$(\sigma_e)_{i+1} = (\sigma_e)_i + \Delta(\sigma_e)_i = \frac{(\epsilon_p)_{i+1} - f[(\sigma_e)_i] + (\sigma_e)_i f'[(\sigma_e)_i]}{f'[(\sigma_e)_i] + \frac{3(\sigma_e)_i}{G}}$$

The convergence criterion of Eq. (N-6) is satisfied if

$$\frac{(\epsilon_p)_{i+1} - (\epsilon_p)_i}{(\epsilon_p)_{i+1}} < 10^{-4} \text{ and } (\sigma_e)_{i+1} - (\sigma_e)_i < 0.5 \text{ psi.}$$

3. Constants Defined by Subroutine

$$Q = 95,000 \text{ cal-mole}^{-1}$$

$$R = 1.034 \text{ cal-R}^{-1}\text{-mole}^{-1}$$

For NOH = 0:

$$A = 2.7 \times 10^{-11}$$

$$B = 4.655 \times 10^{34}$$

$$m = 7.0$$

$$n = 3.0$$

For NOH > 0:

$$A = K_4 L_5 G^{-6}$$

$$B = K_5 L_5 J_2 G^{-2} \phi^{-1}$$

$$m = 7$$

$$n = 2$$

Here,

$$G = \frac{3}{8} [2.6 \times 10^7 - 15,000(T-293)] \quad [00081300], \quad (N-7)$$

and $K_4 = 1.125 \times 10^{35}$, $K_5 = 23.0$, and L_5 and J_2 are calculated in subroutine HKNS.

For NOH = -1:

$$A = 2.7 \times 10^{-11}$$

$$B = C_e + (A_e/B_e) \exp(-\phi t/B_e)$$

$$m = 7.0$$

$$n = 1.0$$

Here, $A_e = 1.0 \times 10^{-8}$, $B_e = 2.2 \times 10^{20}$, and $C_e = 4.3 \times 10^{-30}$.

For $\text{NOH} = -2$, the values are the same as for $\text{NOH} = -1$, except that A_e and C_e are assumed inversely proportional to absolute temperature, and B_e proportional to that temperature.*

0. Subroutine CREEF

1. Purpose

This subroutine determines the equivalent stress and the creep-strain increment of the fuel.

2. Equations and Solutions

The creep-strain increment of fuel can be expressed as

$$\Delta \epsilon_p = A \exp(-Q/KT) \sigma_e^m + [A_1/d^2 \exp(-Q_1/KT) + B] \sigma_e^n \Delta t, \quad (0-1)$$

where

$$\left. \begin{aligned} A &= (1.376 \times 10^{-4})/(-90.5 + D), \\ A_1 &= 9.726 \times 10^6/(-87.7 + D), \end{aligned} \right\} \quad (0-2)$$

and the quantities Q , R , m , Q_1 , B , n , d (the grain diameter), and D (the fuel density in percent) are defined by this subroutine.

The method of solution and the convergence criteria in this subroutine are the same as described in subroutine CREEC.

3. Constants Defined by Subroutine

$$\begin{aligned} B &= 8 \times 10^{-24} \\ d &= 10 \mu \\ D &= 92.0\% \\ m &= 4.5 \end{aligned}$$

*On p. 49 of Ref. 1, the 910°C should be 910°R .

$$n = 1$$

$$Q = 132,000 \text{ cal-mole}^{-1}$$

$$Q_1 = 90,000 \text{ cal-mole}^{-1}$$

$$R = 1.1034 \text{ cal-}^\circ\text{R}^{-1}\text{-mole}^{-1}$$

F. Subroutines AMQUAN, ABDE, DID2, and ANQUAN

1. Purpose

These subroutines calculate the coefficients used in determining the radial displacements at the inner and the outer surfaces of a cylindrical shell (such as the cladding, capsule, or fuel).

2. Equations and Solutions

The radial displacement at the outside surface of a hollow cylinder is

$$u_o = \frac{b_1}{r_o} + b_2 r_o + AM11, \quad (P-1)$$

where r_o is the outside radius of the cylinder,

$$b_1 = (AM22)p_i + (AM33)p_o + AM44,$$

$$b_2 = (AM55)p_i + (AM66)p_o + AM77 - \nu \epsilon_z^T,$$

$$\left. \begin{aligned} AM11 = & \frac{\nu}{r_o(1-\nu)} \int_{r_i}^{r_o} r(e_s)dr + \frac{1+\nu}{r_o(1-\nu)} \int_{r_i}^{r_o} r\alpha T dr \\ & + \frac{r_o(1-2\nu)}{4G(1-\nu)} [A(r_o) + D(r_o) - B(r_o) - E(r_o)] \\ & [00231400, 00231500, 00231700, 00231800], \end{aligned} \right\} \quad (P-2)$$

$$AM22 = \frac{r_1^2 r_o^2}{2G(r_o^2 - r_1^2)} \quad [00231900],$$

$$AM33 = -AM22 \quad [00232000],$$

$$AM44 = \frac{r_o^2 r_1^2}{2G(r_o^2 - r_1^2)} (d_1 + d_2) \quad [00232200],$$

$$AM55 = \frac{1 - 2\nu}{2G} \left(\frac{r_o^2}{r_o^2 - r_1^2} - 1 \right) \quad [00232400],$$

$$AM66 = \frac{2\nu - 1}{2G} \left(\frac{r_o^2}{r_o^2 - r_1^2} \right) \quad [00232300],$$

$$AM77 = \frac{1 - 2\nu}{2G} \left(\frac{r_o^2}{r_o^2 - r_1^2} \right) (d_1 + d_2) \quad [00232500],$$

$$\begin{aligned} A(r_o) &= \frac{2G}{r_o^2} \int_{r_1}^{r_o} r (\epsilon_r^p + \epsilon_\theta^p) dr \\ &= \frac{G(r_o^2 - r_1^2) (\epsilon_r^p + \epsilon_\theta^p)}{r_o^2} \quad [00229900], \end{aligned}$$

$$\begin{aligned} B(r_o) &= 2G \int_{r_1}^{r_o} \frac{1}{r} (\epsilon_\theta^p - \epsilon_r^p) dr \\ &= 2G (\epsilon_\theta^p - \epsilon_r^p) \log(r_o/r_1) \quad [00230000], \end{aligned}$$

$$\begin{aligned} D(r_o) &= \frac{2G}{r_o^2} \int_{r_1}^{r_o} r (\epsilon_r^s + \epsilon_\theta^s) dr \\ &= \frac{G(r_o^2 - r_1^2) (\epsilon_r^s + \epsilon_\theta^s)}{r_o^2} \quad [00230200], \end{aligned}$$

(P-2)

$$\begin{aligned}
 E(r_0) &= 2G \int_{r_1}^{r_0} \frac{1}{r} (\epsilon_\theta^s - \epsilon_r^s) dr \\
 &= 2G(\epsilon_\theta^s - \epsilon_r^s) \log(r_0/r_1) \quad [00230300], \\
 d_1 &= \frac{1}{2(1-\nu)} \left[(1-2\nu)A|_{r=r_0} + B|_{r=r_0} \right] \\
 &\quad + \frac{2G}{r_0} \frac{1}{2} \left(\frac{1+\nu}{1-\nu} \right) \int_{r_1}^{r_0} arTdr \quad [00243400], \\
 \text{and} \\
 d_2 &= \frac{1}{2(1-\nu)} \left[(1-2\nu)D|_{r=r_0} + E|_{r=r_0} \right] \\
 &\quad + \frac{2G}{r_0} \frac{1}{2} \left(\frac{\nu}{1-\nu} \right) \int_{r_1}^{r_0} re_s dr \quad [00243500].
 \end{aligned}$$

(P-2)

The quantities p_i and p_o are the internal and external pressures, respectively, and r_i and r_o are the inside and outside radii, respectively. AM11, AM22, AM33, AM44, AM55, AM66, and AM77 are calculated in subroutine AMQUAN; $A(r_o)$, $B(r_o)$, $D(r_o)$, and $E(r_o)$ are calculated in subroutine ABDE; and d_1 and d_2 are calculated in subroutine DID2. The variables r_o , r_i , G , and ν are input from the MAIN program.

The radial displacements at the inner and the outer surfaces can also be written as

$$u_i = -(AN1A)p_i - (AN2A)p_o - (AN3A)\epsilon_z^T - AN4A \quad (P-3)$$

and

$$u_o = -(AN1B)p_i - (AN2B)p_o - (AN3B)\epsilon_z^T - AN4B, \text{ respectively.} \quad (P-4)$$

The values of AN1A, AN2A, AN3A, AN4A, AN1B, AN2B, AN3B, and AN4B are calculated in subroutine ANQUAN. They are:

$$AN1A = \frac{AM22}{r_1} - (AM55)r_1 \quad [00233300],$$

$$AN2A = -\frac{AM33}{r_1} - (AM66)r_1 \quad [00233400],$$

$$AN3A = Gr_1 \quad [00233500],$$

$$AN4A = -\frac{AM44}{r_1} - (AM77)r_1 \quad [00233600],$$

$$AN1B = -\frac{AM22}{r_o} - (AM55)r_o \quad [00233800],$$

$$AN2B = -\frac{AM33}{r_o} - (AM66)r_o \quad [00233900],$$

$$AN3B = GR_o \quad [00234000],$$

and

$$AN4B = -\frac{AM44}{r_o} - (AM77)r_o - AM11 \quad [00234100].$$

(P-5)

In addition, subroutine ABDE also calculates the following quantities:

$$C(r_o) = \frac{2G}{r_o^2} \int_{r_1}^{r_o} r(\epsilon_\theta^p - \epsilon_r^p) dr$$

$$= \frac{G(r_o^2 - r_1^2)(\epsilon_\theta^p - \epsilon_r^p)}{r_o^2} \quad [00230100]$$

(P-6)

and

$$F(r_o) = \frac{2G}{r_o^2} \int_{r_1}^{r_o} r(\epsilon_\theta^s - \epsilon_r^s) dr$$

$$= \frac{G(r_o^2 - r_1^2)(\epsilon_\theta^s - \epsilon_r^s)}{r_o^2} \quad [00230400].$$

(P-7)

Q. Subroutines XHELP1 and XHELP2

1. Purpose

These subroutines calculate the coefficients used in determining the external pressure of the columnar and the equiaxed regions of the fuel.

2. Equations and Solutions

Let the subscripts 1, 2, and 3 denote the columnar, equiaxed, and undisturbed regions, respectively, and let suffix a denote the inside radius and suffix b the outside radius of the region. The continuity conditions for the fuel cylinder are

$$u_{1b} = u_{2a}; u_{2b} = u_{3a}. \quad (Q-1)$$

By substituting Eq. (P-3) into Eq. (Q-1), we find that

$$\left. \begin{aligned} (XX1)p_{1b} - (XX2)p_{2b} + (XX4)\epsilon_z^T + XX5 &= 0 \\ (XX6)p_{1b} + (XX7)p_{2b} - (XX8)p_{3b} + (XX9)\epsilon_z^T + XX10 &= 0 \end{aligned} \right\} \quad (Q-2)$$

and

It was noted (in the preceding section, P) that the variables AN1A, AN2A, AN3A, AN4A, AN1B, AN2B, AN3B, and AN4B in subroutine ANQUAN are functions of r_1 , r_o , G , and v . If we write $AN1A = AN1A(r_1, r_o, G, v)$, $AN2A = AN2A(r_1, r_o, G, v)$. . . , then XX1 to XX10 in Eq. (Q-2) can be written as

$$\begin{aligned} XX1 = & \frac{r_{1a}^2 r_{1b}}{2G_1(r_{1b}^2 - r_{1a}^2)} + \frac{(1 - 2v_1) r_{1b}^3}{2G_1(r_{1b}^2 - r_{1a}^2)} + \frac{r_{2a} r_{2b}^2}{2G_2(r_{2b}^2 - r_{2a}^2)} \\ & - \frac{r_{2a} (1 - 2v_2)}{2G_2} + \frac{(1 - 2v_2) r_{2b}^2 r_{2a}^2}{2G_2(r_{2b}^2 - r_{2a}^2)} \end{aligned}$$

$$= \text{AN1B} (r_{1a}, r_{1b}, G_1, v_1) - \text{AN1A} (r_{2a}, r_{2b}, G_2, v_2)$$

$$[00283100], \quad (\text{Q-3})$$

$$\text{XX2} = \frac{r_{2a} r_{2b}^2}{2G_2(r_{2b}^2 - r_{2a}^2)} + \frac{(1 - 2v_2) r_{2b}^2 r_{2a}}{2G_2(r_{2b}^2 - r_{2a}^2)}$$

$$= \text{AN2A} (r_{2a}, r_{2b}, G_2, v_2) \quad [00283200], \quad (\text{Q-4})$$

$$\text{XX4} = v_1 r_{1b} - v_2 r_{1b} = \text{AN3B}(r_{1b}, v_1) - \text{AN3A}(r_{2a}, v_2)$$

$$[00283300], \quad (\text{Q-5})$$

$$\begin{aligned} \text{XX5} = & - \frac{r_{1a}^2 r_{1b}}{2G_1(r_{1b}^2 - r_{1a}^2)} (d_1 + d_2) - \frac{(1 - 2v_1) r_{1b}^3}{2G_1(r_{1b}^2 - r_{1a}^2)} (d_1 + d_2) \\ & - \frac{v_1}{r_{1b}(1 - v_1)} \int_{r_{1a}}^{r_{1b}} r e_s dr - \frac{1 + v_1}{r_1(1 - v_1)} \int_{r_{1a}}^{r_{1b}} r \alpha T dr \\ & - \frac{r_{1b}(1 - 2v_1)}{4G_1(1 - v_1)} [A(r_{1b}) + D(r_{1b}) - B(r_{1b}) - E(r_{1b})] \\ & + \frac{r_{2a}^2 r_{2b}}{2G_2(r_{2b}^2 - r_{2a}^2)} (d_1 + d_2) + \frac{(1 - 2v_2) r_{2a} r_{2b}^2}{2G_2(r_{2b}^2 - r_{2a}^2)} (d_1 + d_2) \\ & + p_{1a} \left[\frac{-r_{1a}^2 r_{1b}}{2G_1(r_{1b}^2 - r_{1a}^2)} + \frac{r_{1b}(1 - 2v_1)}{2G_1} - \frac{(1 - 2v_1) r_{1b}^3}{2G_1(r_{1b}^2 - r_{1a}^2)} \right] \end{aligned}$$

$$= \text{AN4B} (r_{1a}, r_{1b}, G_1, v_1) - \text{AN4A} (r_{2a}, r_{2b}, G_2, v_2)$$

$$+ \text{AN1B} (r_{1a}, r_{1b}, G_1, v_1) p_{1a} \quad [00283400], \quad (\text{Q-6})$$

$$\begin{aligned} \text{XX6} &= \frac{-r_{2a}^2 r_{2b}}{2G_2(r_{2b}^2 - r_{2a}^2)} + \frac{r_{2a}(1 - 2v_2)}{2G_2} - \frac{r_{2b}^3(1 - 2v_2)}{2G_2(r_{2b}^2 - r_{2a}^2)} \\ &= \text{AN1B}(r_{2a}, r_{2b}, G_2, v_2) \quad [00284700], \end{aligned} \quad (\text{Q-7})$$

$$\begin{aligned} \text{XX7} &= -\frac{r_{2a}^2 r_{2b}}{2G_2(r_{2b}^2 - r_{2a}^2)} + \frac{r_{2b}^3(1 - 2v_2)}{2G_2(r_{2b}^2 - r_{2a}^2)} + \frac{r_{3a} r_{3b}^2}{2G_3(r_{3b}^2 - r_{3a}^2)} \\ &\quad - \frac{r_{3a}(1 - 2v_3)}{2G_3} + \frac{r_{3a} r_{3b}^2(1 - 2v_3)}{2G_3(r_{3b}^2 - r_{3a}^2)} \\ &= \text{AN2B}(r_{2a}, r_{2b}, G_2, v_2) - \text{AN1A}(r_{3a}, r_{3b}, G_3, v_3) \\ &\quad [00284800], \end{aligned} \quad (\text{Q-8})$$

$$\begin{aligned} \text{XX8} &= \frac{r_{3a} r_{3b}^2}{2G_3(1 - 2v_3)} + \frac{r_{3b}^2 r_{3a}(1 - 2v_2)}{2G_3(r_{3b}^2 - r_{3a}^2)} \\ &= \text{AN2A}(r_{3a}, r_{3b}, G_3, v_3) \quad [00284900], \end{aligned} \quad (\text{Q-9})$$

$$\begin{aligned} \text{XX9} &= r_{3a}(v_2 - v_3) = \text{AN3B}(r_{2b}, v_2) - \text{AN3A}(r_{3a}, v_3) \\ &\quad [0028500], \end{aligned} \quad (\text{Q-10})$$

and

$$\begin{aligned} \text{XX10} &= \frac{r_{2a}^2 r_{2b}}{2G_2(r_{2b}^2 - r_{2a}^2)} (d_1 + d_2) - \frac{(1 - 2v_2)r_{2b}^3}{2G_2(r_{2b}^2 - r_{2a}^2)} (d_1 + d_2) \\ &\quad - \frac{v_2}{r_{2b}(1 - v_2)} \int_{r_{2a}}^{r_{2b}} r e_s dr - \frac{1 + v_2}{r_{2b}(1 - v_2)} \int_{r_{2a}}^{r_{2b}} r o_T dr \\ &\quad - \frac{r_{2b}(1 - 2v_2)}{4G_2(1 - v_2)} [A(r_{2b}) + D(r_{2b}) - B(r_{2b}) - E(r_{2b})] \end{aligned}$$

$$\begin{aligned}
 & + \frac{r_{3a} r_{3b}}{2G_3(r_{3b}^2 - r_{3a}^2)} (d_1 + d_2) + \frac{r_{3b}^2 r_{3a} (1 - 2v_3)}{2G_3(r_{3b}^2 - r_{3a}^2)} (d_1 + d_2) \\
 & = AN4B (r_{2a}, r_{2b}, G_2, v_2) - AN4A (r_{3a}, r_{3b}, G_3, v_3) \\
 & [00285100]. \tag{Q-11}
 \end{aligned}$$

Equations (Q-3) to (Q-11) are calculated in subroutine XHELP1, in which p_{1a} is the plenum pressure obtained (as P_{p1}) in subroutine PLENP. Since there is no gap between the different regions of the fuel, $r_{1b} = r_{2a}$ and $r_{2b} = r_{3a}$.

The quantities p_{1b} and p_{2b} can be obtained in terms of ϵ_z^T and p_{3b} by solving the simultaneous equations labeled Eq. (Q-2); the solution gives

$$\left. \begin{aligned}
 p_{1b} &= PA0 + (PAZ)\epsilon_z^T + (PAFC)p_{3b} \\
 p_{2b} &= PBO + (PBZ)\epsilon_z^T + (PBFC)p_{3b}
 \end{aligned} \right\} \tag{Q-12}$$

PA0, PAZ, PAFC, PBO, PBZ, and PBFC are calculated in subroutine XHELP2; they are

$$\left. \begin{aligned}
 PA0 &= -(XX7 \cdot XX1 + XX2 \cdot XX10)/DD \quad [00285700], \\
 PAZ &= -(XX7 \cdot XX4 + XX2 \cdot XX9)/DD \quad [00285800], \\
 PAFC &= XX2 \cdot XX8/DD \quad [00285900], \\
 PBO &= (XX6 \cdot XX5 - XX1 \cdot XX10)/DD \quad [00286000], \\
 PBZ &= (XX6 \cdot XX4 - XX1 \cdot XX9)/DD \quad [00286100], \\
 PBFC &= XX1 \cdot XX8/DD \quad [00286200], \\
 DD &= XX7 \cdot XX1 + XX2 \cdot XX6 \quad [00286300].
 \end{aligned} \right\} \tag{Q-13}$$

and

where

R. Subroutines PHELPF and PHELPM

1. Purpose

These subroutines calculate the coefficients used to determine the average fuel axial strain over the three fuel regions.

2. Equations and Solutions

If Eqs. (51) to (56) in Ref. 1 are substituted into Eq. (48c) in that reference and used to find the average stress over the area of each region of the fuel, we obtain

$$\begin{aligned} \bar{\sigma}_z = & 2G (1 + \nu) \bar{\epsilon}_z^T - (\bar{\epsilon}_z^P + \bar{\epsilon}_z^S) - \left(\frac{1 + \nu}{1 - \nu} \right) \bar{\alpha T} \\ & + \frac{2\nu}{1 - 2\nu} (AM55 \cdot p_1 + AM66 \cdot p_o + AM77) \\ & + \frac{\nu}{1 - \nu} (\bar{\epsilon}_r^P + \bar{\epsilon}_r^S - \bar{e}_u) - \frac{\bar{B} + \bar{E}}{2G}. \end{aligned} \quad (R-1)$$

AM55, AM66, and AM77 are calculated in subroutine AMQUAN, $\bar{\epsilon}_z^P$ and $\bar{\epsilon}_r^P$ in subroutine CREEF, $\bar{\epsilon}_z^S$, $\bar{\epsilon}_r^S$, and $\bar{e}_u$ in subroutine SWELL, and $\bar{\alpha T}$ in subroutine FLTMP. (A bar over the quantity denotes the average value.)

$$\begin{aligned} \bar{B} + \bar{E} = & \frac{2}{r_o^2 - r_1^2} \int_{r_1}^{r_o} [B(r) + E(r)] dr \\ = & \frac{r_o^2}{r_o^2 - r_1^2} [B(r_o) + E(r_o) - C(r_o) - F(r_o)] \end{aligned}$$

$$[00256900, 00257000].$$

(R-2)

The expressions of $B(r_o)$, $E(r_o)$, $C(r_o)$, and $F(r_o)$ are calculated in subroutine ABDE.

The average axial stress over the three regions of the fuel ($\bar{\sigma}_z^*$) is

$$\bar{\sigma}_z^* = \frac{\sum_{m=1}^3 \bar{\sigma}_z^m A_m}{\sum_{m=1}^3 A_m}, \quad (R-3)$$

where A_m is the cross-sectional area of each fuel region.

Since $\bar{\sigma}_z^* = -p(z)$,¹ if Eq. (R-3) above and Eq. (59) in Ref. 1 are substituted into Eq. (R-1), it is found that

$$(P1)\bar{\epsilon}_z^T = (P2)F + (P3)p_{3b} + P4, \quad (R-4)$$

where

$$\left. \begin{aligned} P1 &= \sum_{m=1}^3 (PP1)_m G_m A_m / \sum_{m=1}^3 A_m \quad [00262700], \\ P2 &= \sum_{m=1}^3 (PP2)_m G_m A_m / \sum_{m=1}^3 A_m \quad [00262800], \\ P3 &= \sum_{m=1}^3 (PP3)_m G_m A_m / \sum_{m=1}^3 A_m \quad [00262900], \\ P4 &= \sum_{m=1}^3 (PP4)_m G_m A_m / \sum_{m=1}^3 A_m \quad [00263000], \\ (PP1)_m &= 1 + \nu_m + \frac{2\nu_m}{1 - 2\nu_m} (AM55 \cdot PAZ + AM66 \cdot PBZ) \\ &\quad [00255900], \\ (PP2)_m &= (z - L_{p1}) \{ 2G_m \pi L_f (r_{mb}^2 - r_{ma}^2) \} \quad [00256000], \\ (PP3)_m &= - \frac{2\nu_m}{1 - 2\nu_m} (AM55 \cdot PAFC + AM66 \cdot PBFC) \quad [00256100], \end{aligned} \right\} \quad (R-5)$$

and

$$\left. \begin{aligned} (PP4)_m = & - [p_{1a} + \rho_f g (L_f - z)] / \left[2G_m + \bar{\epsilon}_z^p + \bar{\epsilon}_z^s + \frac{1 + \nu_m}{1 - \nu_m} \bar{\alpha T} \right. \\ & - \frac{2\nu_m}{1 - 2\nu_m} (AM55 \cdot PAZ + AM66 \cdot PBZ + AM77) \\ & \left. - \frac{\nu_m}{1 - \nu_m} (\bar{\epsilon}_r^p + \bar{\epsilon}_r^s - \bar{e}_s - \frac{\bar{B} + \bar{E}}{2G_m}) \quad [00256200]. \right\} \quad (R-5) \end{aligned}$$

F is the axial sliding force between fuel and cladding, L_{p1} is plenum length, and g is force of gravity. P1, P2, P3, and P4 are calculated in subroutine PHELPM, PP1, PP2, PP3, and PP4 in subroutine PHELPF, and PAZ, PBZ, PAFC, and PBFC in subroutine XHELP2. The subscript m denotes the fuel region: 1 = columnar region, 2 = equiaxed region, and 3 = undisturbed region.

S. Subroutines PHELPC and PHELP4

1. Purpose

These subroutines calculate the coefficients used to determine the average axial strain of the cladding for $0 \leq z \leq L_f$.*

2. Equations and Solutions

The average axial strain of the cladding given in Eq. (63) of Ref. 1 can be expressed as

$$(PP5) \bar{\epsilon}_z^T = (PP6)F + (PP7)P_{3b} + PP8, \quad (S-1)$$

where

$$\left. \begin{aligned} PP5 = & 1 + \nu_4 \quad [00250600], \\ PP6 = & \frac{1}{2 G_4 (r_{4b}^2 - r_{4a}^2)} \quad [00250700], \end{aligned} \right\} \quad (S-2)$$

* A term of $\bar{\epsilon}_z^s$ should be added to the righthand side of Eq. (63) in Ref. 1, and $\frac{\bar{B} + \bar{E}}{2G}$ should be replaced by $\frac{\bar{B} + \bar{E}}{2G}$ in Eq. (60) of Ref. 1.

$$PP7 = -2v_4 \cdot AM55 / (1 - 2v_4) \quad [00250800],$$

$$PP8 = PP6(S26 + S27) + S28 + S29 + S31 \quad [00252100],$$

$$S26 = -p_T - \pi(r_{4b}^2 p_{4b} - r_{4a}^2 p_{p1}) \quad [00252000],$$

$$S27 = -\rho_4 g \pi (r_{4b}^2 - r_{4a}^2) (L_c - z) \quad [00252100],$$

$$S28 = \bar{\epsilon}_z^p + \bar{\epsilon}_z^s + \frac{1 + v_4}{1 - v_4} \bar{\alpha T} \quad [00252200],$$

$$S29 = \frac{-2v_4}{1 - 2v_4} (AM66 \cdot p_{4b} + AM77) \quad [00252300],$$

$$S31 = \frac{v_4}{1 - v_4} \left(\bar{\epsilon}_s - \bar{\epsilon}_r^p \quad \bar{\epsilon}_r^s + \frac{\bar{B} + \bar{E}}{2G_4} \right) \quad [00252400].$$

(S-2)

and

Subscript 4 denotes the cladding, and p_T is the axial load applied to top of the fuel element.

Eq. (S-2) is given in subroutine PHELP4 for various values of z . For $z = L_f/2$, all relationships shown in Eq. (S-2) remain the same except S27, which is given in subroutine PHELPC in the form

$$S27 = \rho_4 g \pi \left(r_{4b}^2 - r_{4a}^2 \right) \left(L_c - \frac{L_f}{2} \right) \quad [00254700]. \quad (S-3)$$

T. Subroutines VZJRW, PVHELP, and VHELP2

1. Purpose

These subroutines determine the radial displacement and pressure of the fuel element.

2. Equations and Solutions

Equations (Q-2), (R-4), and (S-1) can be rewritten as

$$\left. \begin{aligned}
 (XX1)p_{1b} - (XX2)p_{2b} + (XX4)\epsilon_{zf}^{-T} + XX5 &= 0 \\
 (XX6)p_{1b} + (XX7)p_{2b} - (XX8)p_{3b} + (XX9)\epsilon_{zf}^{-T} + XX10 &= 0 \\
 (P1)\epsilon_{zf}^{-T} - (P2)F - (P3)p_{3b} - P4 &= 0 \\
 (PP5)\epsilon_{zc}^{-T} - (PP6)F - (PP7)p_{3b} - PP8 &= 0.
 \end{aligned} \right\} \quad (T-1)$$

The quantities ϵ_{zf}^{-T} , ϵ_{zc}^{-T} , p_{1b} , and p_{2b} can be determined by Eq. (T-1), using two boundary conditions: (a) when the gap between fuel and cladding is open, and (b) when the gap is closed.

(a) With the gap open, $(u_{3b} - u_{4a}) < (r_{4a} - r_{3b})$, i.e.,

$$p_{3b} = p_{1a} = p_{p1}; F = 0. \quad (T-2)$$

If Eq. (T-2) is substituted into Eq. (T-1), the unknowns ϵ_{zf}^{-T} , ϵ_{zc}^{-T} , p_{1b} , and p_{2b} can be expressed in terms of the plenum pressure p_{p1} as follows:

$$\left. \begin{aligned}
 \epsilon_{zf}^{-T} &= (P3 \cdot p_{p1} + P4)/P1 \quad [00218700], \\
 \epsilon_{zc}^{-T} &= (PP7 \cdot p_{p1} + PP8)/PP5 \quad [00219000], \\
 p_{2b} &= \frac{PP6(P4 \cdot \epsilon_{zf}^{-T} + PP5) - P9 \cdot P1 \cdot \epsilon_{zf}^{-T} - (P10 - P8 \cdot p_{p1})}{PP7 \cdot P1 + P2 \cdot PP6} \quad (T-3) \\
 &\quad [00219500, 00219600],
 \end{aligned} \right\}$$

and

$$p_{1b} = (P2 \cdot p_{2b} - P4 \cdot \epsilon_{zf}^{-T} - PP5)/P1 \quad [00219700].$$

The quantity p_{p1} is determined in subroutine PLENP, XX1-XX10 are calculated in subroutine XHELP1, and P1, P2, P3, P4, PP5, PP6, PP7, and PP8 are calculated in subroutines PHELPM, PHELPC, and PHELP4.

The radial displacements on the outside surface of the fuel can be obtained by substituting r_{3a} for r_1 and r_{3b} for r_0 in Eqs. (P-4) and (P-5). The radial displacements on the outside surface of the cladding can

be obtained by substituting r_{4a} for r_1 and r_{4b} for r_0 in those equations. These displacements are calculated in [00221300] and [00222900], respectively. The radial displacement on the inside surface of the cladding is calculated in [00222700] by substituting r_{4a} for r_1 and r_{4b} for r_0 in Eqs. (P-3) and (P-5).

(b) With the gap closed, $(u_{3b} - u_{4a}) \geq (r_{4a} - r_{4b})$, i.e.,

$$u_{3b} = u_{4a}. \quad (T-4)$$

By substituting Eqs. (P-3) and (F-4) into Eq. (R-4), we obtain

$$\begin{aligned} &XX11 \cdot p_{2b} + XX12 \cdot p_{3b} + XX13 \cdot \frac{-T}{\epsilon_{zf}} + XX14 \\ &\quad \cdot \frac{-T}{\epsilon_{zc}} + XX15 = 0, \end{aligned} \quad (T-5)$$

where

$$\left. \begin{aligned} XX11 &= AN1B(r_{3a}, r_{3b}, G_3, \nu_3) \quad [00223600], \\ XX12 &= AN2B(r_{3a}, r_{3b}, G_3, \nu_3) - AN1A(r_{4a}, r_{4b}, G_4, \nu_4) \\ &\quad [00223800], \\ XX13 &= AN3B(r_{3b}, \nu_3) \quad [00223700], \\ XX14 &= -AN3A(r_{4a}, \nu_4) \quad [00223900], \\ \text{and} \\ XX15 &= -AN2A(r_{4a}, r_{4b}, G_c, \nu_c)p_{4b} + AN4B(r_{3a}, r_{3b}, G_3, \nu_3) \\ &\quad - AN4A(r_{4a}, r_{4b}, G_4, \nu_4) - r_{3b} + r_{4a}. \end{aligned} \right\} \quad (T-6)$$

The value p_{4b} is input from the MAIN program, and AN1A to AN4B are calculated in subroutine ANQUAN.

By substituting Eq. (T-5) into the first two equations of (T-1) and eliminating p_{1b} and p_{2b} , we find that

$$P9 \cdot \epsilon_{zf}^{-T} + P10 \cdot \epsilon_{zc}^{-T} + P11 \cdot p_{3b} + P12 = 0, \quad (T-7)$$

where

$$\left. \begin{aligned} P9 &= XX11 \cdot V1 + XX13 \quad [00267200], \\ P10 &= XX14 \quad [00267300], \\ P11 &= XX11 \cdot V2 + XX12 \quad [00267400], \\ P12 &= XX11 \cdot V3 + XX15 \quad [00267500], \\ V1 &= (XX6 \cdot XX4 - XX1 \cdot XX9)/DX \quad [00266700], \\ V2 &= XX8 \cdot XX1/DX \quad [00266900], \\ V3 &= (XX6 \cdot XX5 - XX1 \cdot XX10)/DX \quad [00267100], \\ DX &= XX2 \cdot XX6 + XX1 \cdot XX7 \quad [00266800]. \end{aligned} \right\} \quad (T-8)$$

and

where

Equation (T-8) is calculated in subroutine PVHELP, and the pressures p_{1b} and p_{2b} are found in terms of p_{3b} :

$$p_{1b} = (XX2 \cdot p_{2b} - XX4 \cdot \epsilon_{zf}^{-T} - XX5)/XX1 \quad (T-9)$$

and

$$p_{2b} = V1 \cdot \epsilon_{zf}^{-T} + V2 \cdot p_{3b} + V3. \quad (T-10)$$

If the fuel and the cladding are stuck together,

$$\Delta \epsilon_{zf}^T = \Delta \epsilon_{zc}^T, \quad (T-11)$$

and

$$\left. \begin{aligned} \Delta \epsilon_{zf}^T &= \epsilon_{zf}^T - (\epsilon_{zf}^P + \epsilon_{zf}^S) \\ \Delta \epsilon_{zc}^T &= \epsilon_{zc}^T - (\epsilon_{zc}^P + \epsilon_{zc}^S) \end{aligned} \right\} \quad (T-12)$$

By substituting Eqs. (T-11) and (T-12) into Eq. (T-7) and the last two relationships of Eq. (T-1), we obtain

$$\left. \begin{aligned} P1 \cdot \Delta \epsilon_{zf}^T - P2 \cdot F - P3 \cdot p_{3b} - PP4 &= 0, \\ P5 \cdot \Delta \epsilon_{zf}^T - P6 \cdot F - P7 \cdot p_{3b} - PP8 &= 0, \\ (P9 + P10) \Delta \epsilon_{zf}^T + P11 \cdot p_{3b} + PP12 &= 0, \end{aligned} \right\} \quad (T-13)$$

and

where

$$\left. \begin{aligned} PP4 &= P4 - P1(\epsilon_{zf}^P + \epsilon_{zf}^S) \quad [00267600], \\ PP8 &= P8 - P5(\epsilon_{zc}^P + \epsilon_{zc}^S) \quad [00267700], \end{aligned} \right\} \quad (T-14)$$

and

$$PP12 = P12 + P9(\epsilon_{zc}^P + \epsilon_{zf}^S) + P10(\epsilon_{zc}^P + \epsilon_{zc}^S) \quad [00267800].$$

The values for $\Delta \epsilon_{zf}^T$, F , and p_{3b} can be found by solving Eq. (T-13):

$$\left. \begin{aligned} \Delta \epsilon_{zf}^T &= \Delta \epsilon_{zc}^T = V6/V7 \quad [00224700], \\ F &= V4 \cdot \Delta \epsilon_{zf}^T + V5 \quad [00224800], \end{aligned} \right\} \quad (T-15)$$

and

$$p_{3b} = [-(P9 + P10) \Delta \epsilon_{zf}^T + PP12]/P11 \quad [00224900].$$

V4, V5, V6, and V7 are calculated in subroutine PVHELP as the following:

$$\left. \begin{aligned}
 V4 &= [P5 \cdot P11 + P7(P9 + P10)]/P11 \quad [00267600], \\
 V5 &= (P7 \cdot PP12 - P11 \cdot PP8)/(P11 \cdot P6) \quad [00268000], \\
 V6 &= P2 \cdot V5 - (P3 \cdot PP12 - PP4 \cdot P11)/P11 \quad [00268100], \\
 V7 &= P1 - P2 \cdot V4 + P3 (P9 + P10)/P11 \quad [00268200].
 \end{aligned} \right\} \quad (T-16)$$

and

By substituting the value of p_{3b} into Eqs. (T-9) and (T-10), we obtain p_{1b} and p_{2b} . The frictional force between the fuel and cladding is

$$F = 2\mu p_{3b} \pi r_{3b} L_f, \quad (T-17)$$

where μ is the friction coefficient, which is defined in subroutine VZJRW. (The value of μ is 0.8 when there is no relative motion between fuel and cladding and is 0.7 when there is such motion.) If $F > F_c$, "slip" occurs and

$$F = 2\mu p_{3b} \pi r_{3b} L_f. \quad (T-18)$$

When Eq. (T-18) is substituted into Eq. (T-13), we find that

$$\left. \begin{aligned}
 P1 \cdot \Delta \epsilon_{zf}^T - (P2 \cdot V8 + P3)p_{3b} - PP4 &= 0, \\
 P5 \cdot \Delta \epsilon_{zc}^T - (P6 \cdot V8 + P7)p_{3b} - PP8 &= 0, \\
 P9 \cdot \Delta \epsilon_{zf}^T + P10 \cdot \Delta \epsilon_{zf}^T + P11 \cdot p_{3b} + PP12 &= 0.
 \end{aligned} \right\} \quad (T-19)$$

and

By solving Eq. (T-19), we find that

$$\left. \begin{aligned}
 \Delta \epsilon_{zf}^T &= \frac{-(P10 \cdot V12 + P11 \cdot V10 + PP12)}{(P9 + P10 \cdot V11 + P11 \cdot V9)} \quad [00226500], \\
 \Delta \epsilon_{zc}^T &= V11 \cdot \Delta \epsilon_{zf}^T + V12 \quad [00226600],
 \end{aligned} \right\} \quad (T-20)$$

and

$$P_{3b} = V9 \cdot \Delta \epsilon_{zf}^T + V10 \quad [00227100]. \quad \left. \vphantom{P_{3b}} \right\} \quad (T-20)$$

The expressions of V9, V10, V11, and V12 are calculated in subroutine VHELP2.
They are:

$$\left. \begin{aligned} V9 &= P1/DX1 \quad [00279800], \\ V10 &= -P4/DX1 \quad [00279900], \\ V11 &= V9(PP6 \cdot V8 + PP7)/P5 \quad [00280000], \\ V12 &= PP8/PP5 + V11 \cdot V10/V9 \quad [00280100], \end{aligned} \right\} \quad (T-21)$$

and

$$DX1 = P2 \cdot V8 + P3 \quad [00279700].$$

U. Subroutine DISPL

1. Purpose

This subroutine determines the radial displacements in each section of the fuel element.

2. Equations and Solutions

From Eqs. (P-3) and (P-4), the radial displacements of the inside and outside surfaces of the columnar region of the fuel are

$$\left. \begin{aligned} u_{1a} &= -AN1A(r_{1a}, r_{1b}, G_1, v_1)p_{1a} - AN2A(r_{1a}, r_{1b}, G_1, v_1)p_{1b} \\ &\quad - AN3A(r_{1a}, r_{1b}, G_1, v_1)\epsilon_{zf}^T - AN4A(r_{1a}, r_{1b}, G_1, v_1) \end{aligned} \right\} \quad (U-1)$$

[00248000]

and

$$\left. \begin{aligned} u_{1b} = & -AN2B(r_{1a}, r_{1b}, G_1, v_1)p_{1a} - AN2B(r_{1a}, r_{1b}, G_1, v_1)p_{1b} \\ & - AN3B(r_{1a}, r_{1b}, G_1, v_1)\epsilon_{zf}^T - AN4B(r_{1a}, r_{1b}, G_1, v_1) \end{aligned} \right\} (U-1)$$

[00247900];

the radial displacements of the inside and outside surfaces of the equiaxed region of the fuel are

$$u_{2a} = u_{1b}$$

and

$$\begin{aligned} u_{2b} = & -AN1B(r_{2a}, r_{2b}, G_2, v_2)p_{2a} - AN2B(r_{2a}, r_{2b}, G_2, v_2)p_{2b} \\ & - AN3B(r_{2a}, r_{2b}, G_2, v_2)\epsilon_{zf}^T - AN4B(r_{2a}, r_{2b}, G_2, v_2) \end{aligned}$$

[00247500]; (U-2)

the radial displacements of the inside and outside surfaces of the undisturbed region of the fuel are

$$u_{3a} = u_{2b}$$

and

$$\begin{aligned} u_{3b} = & -AN1B(r_{3a}, r_{3b}, G_3, v_3)p_{3a} - AN2B(r_{3a}, r_{3b}, G_3, v_3)p_{3b} \\ & - AN3B(r_{3a}, r_{3b}, G_3, v_3)\epsilon_{zf}^T - AN4B(r_{3a}, r_{3b}, G_3, v_3) \end{aligned}$$

[0024700]; (U-3)

and the radial displacements of the inside and outside surfaces of the cladding are

$$u_{4a} = -AN1A(r_{4a}, r_{4b}, G_4, v_4)p_{4a} - AN2A(r_{4a}, r_{4b}, G_4, v_4)p_{4b} \\ - AN3A(r_{4a}, r_{4b}, G_4, v_4)\epsilon_{zc}^T - AN4A(r_{4a}, r_{4b}, G_4, v_4) \\ [00246600]$$

and

$$u_{4b} = -AN1B(r_{4a}, r_{4b}, G_4, v_4)p_{4a} - AN2B(r_{4a}, r_{4b}, G_4, v_4)p_{4b} \\ - AN3B(r_{4a}, r_{4b}, G_4, v_4)\epsilon_{zc}^T - AN4B(r_{4a}, r_{4b}, G_4, v_4) \\ [00246500].$$

(U-4)

AN1A through AN4B are calculated in subroutine ANQUAN, p_{1a} is the plenum pressure calculated in subroutine PLENP, p_{1b} ($=p_{2a}$), p_{2b} ($=p_{3a}$), and p_{3b} ($=p_{4a}$) are calculated in subroutine VZJRW, and p_{4b} is sodium pressure from the MAIN program.

V. Subroutine STRAIN

1. Purpose

This subroutine determines the average radial and circumferential strains of each fuel region and of the cladding.

2. Equations and Solutions

The average radial and circumferential strains of the fuel element are defined by

$$\bar{\epsilon}_r = \frac{\int_{r_1}^{r_0} \frac{\partial u}{\partial r} dr}{\int_{r_1}^{r_0} r dr}$$

(V-1)

and

$$\bar{\epsilon}_\theta = \frac{\int_{r_1}^{r_0} u dr}{\int_{r_1}^{r_0} r dr}, \quad (V-2)$$

where r_0 and r_1 denote the outside and inside radii of each fuel region and of the cladding. By substituting Eq. (51) of Ref. 1 (the expression of u) into Eqs. (V-1) and (V-2) and noting that

$$\int_{r_1}^{r_0} r dr = \frac{1}{2}(r_0^2 - r_1^2), \quad (V-3)$$

$$\int_{r_1}^{r_0} \frac{1}{r_1} \int_{r_1}^r r_2 dr_2 dr_1 = \frac{1}{4} [(r_0^2 - r_1^2) - 2r_1^2 \ln(r_0/r_1)], \quad (V-4)$$

and

$$\int_{r_1}^{r_0} r_1 \int_{r_1}^r \frac{1}{r_2} dr_1 dr_2 = \frac{1}{4} [(r_1^2 - r_0^2) + 2r_0^2 \ln(r_0/r_1)], \quad (V-5)$$

we obtain the following expressions of $\bar{\epsilon}_r$ and $\bar{\epsilon}_\theta$:

$$\left. \begin{aligned} \bar{\epsilon}_\theta = & C1(AM22 \cdot p_1 + AM33 \cdot p_0 + AM44) + AM55 \cdot p_1 \\ & + AM66 \cdot p_0 - v_1 \epsilon_z^T + AM77 + C2 \left[\frac{v_1}{1 - v_1} \epsilon_s \right. \\ & + \frac{1 + v_1}{1 - v_1} \bar{\alpha T} + \frac{1 - 2v_1}{2(1 - v_1)} (\epsilon_r^p + \epsilon_\theta^p + \epsilon_r^s + \epsilon_\theta^s) \\ & \left. + \frac{1 - 2v_1}{1 - v_1} C3(-\epsilon_r^p + \epsilon_\theta^p + \epsilon_r^s + \epsilon_\theta^s) \right] \quad [00274700, 00274800], \\ & \\ \bar{\epsilon}_r = & -\bar{\epsilon}_\theta + 2(r_0 u_0 - u_1 r_1)/(r_0^2 - r_1^2) \quad [00274900], \end{aligned} \right\} \quad (V-6)$$

and

in which p_i and p_o are the internal and the external pressures of each shell, and u_i and u_o are the radial displacements at the inside and the outside surfaces of each shell. AM11 through AM77 are calculated in subroutine AMQUAN, ϵ_r^p and ϵ_θ^p are calculated in subroutines CREEF and CREEC, ϵ_r^s , ϵ_z^s , and ϵ_θ^s are calculated in subroutines SWELL and HKNS, αT is calculated in subroutines FLTMP and CLTMP, ν_i ($i=1,2,3,4$), which is the Poisson's ratio in each region, is calculated in subroutine XLAME, and

$$\left. \begin{aligned} \epsilon_s &= \epsilon_r^s + \epsilon_\theta^s + \epsilon_z^s \quad [00274600], \\ C1 &= 2 \ln(r_o/r_i) / (r_o^2 - r_i^2) \quad [00274000], \\ C2 &= \frac{1}{2} \left[1 - 2r_i^2 \ln(r_o/r_i) / (r_o^2 - r_i^2) \right] \quad [00274100], \\ \text{and} \\ C3 &= \frac{1}{2} \left[1 - 2r_o^2 \ln(r_o/r_i) / (r_o^2 - r_i^2) \right] \quad [00274200]. \end{aligned} \right\} \quad (V-7)$$

W. Subroutine STRESS

1. Purpose

This subroutine determines the stresses at each axial section of the fuel element.

2. Equations and Solutions

The stresses of a fuel element, as given in Eqs. (48a), (48b), and (48c) of Ref. 1, can be written as

$$\left. \begin{aligned} \sigma_r &= 2G(\epsilon_r^p + CC5 - \epsilon_r^p - \epsilon_r^s) \quad [00278400], \\ \sigma_\theta &= 2G(\epsilon_\theta^p + CC5 - \epsilon_\theta^p - \epsilon_\theta^s) \quad [00278500], \\ \text{and} \\ \sigma_z &= 2G(\epsilon_z^p + CC5 - \epsilon_z^p - \epsilon_z^s) \quad [00278600], \end{aligned} \right\} \quad (W-1)$$

with

$$\left. \begin{aligned} CC5 &= CC3 \cdot CC2 - \alpha T \quad [00278300], \\ CC3 &= \frac{\nu}{1 - 2\nu} \quad [00278200], \\ CC2 &= \epsilon_r^T + \epsilon_\theta^T + \epsilon_z^T - 3\alpha T - CC1 \quad [00278000], \\ \text{and} \\ CC1 &= \epsilon_r^S + \epsilon_\theta^S + \epsilon_z^S \quad [00277900]. \end{aligned} \right\} \quad (W-2)$$

The strains and the temperature are input from the MAIN program as calculated by calling subroutine STRAIN.

The average stress $\bar{\sigma}$ is defined as

$$\bar{\sigma} = \frac{1}{3} (\sigma_r + \sigma_\theta + \sigma_z) = - \frac{2(1 + \nu)G}{3(1 - 2\nu)} CC2 \quad [0027900]. \quad (W-3)$$

X. Subroutines COMPO and CONVG

1. Purpose

These subroutines calculate the strain and stress errors at the end of each time interval and decide whether the time interval used in the previous calculation should be reduced.

2. Equations and Solutions

The plastic-strain increment can be expressed by the Prandtl-Ruess equation

$$d\epsilon_{ij}^P = \frac{1}{c} \sigma'_{ij}, \quad (X-1)$$

where $d\epsilon_{ij}^P$ and σ'_{ij} , the plastic-strain increment and the deviatoric stress, respectively, are written in tensor form. When the increment constant (c) is expressed in terms of equivalent strain increment ($d\epsilon_e^P$) and equivalent stress (σ_e), it is found that

$$c = \frac{\sigma_e}{1.5 d\epsilon_e^P}, \quad (X-2)$$

where σ_e and $d\epsilon_e^P$ were solved in subroutines CREEC and CREEF. The deviatoric stress can then be found as

$$(\sigma'_{ij})_A = cd\epsilon_{ij}^P \quad [00070070-00070090], \quad (X-3)$$

where $d\epsilon_{ij}^P$ was determined in the MAIN program, and the subscript A indicates the assumed value of the deviatoric stress.

By using Eqs. (48a)-(48c) of Ref. 1 and the definition of deviatoric stress, we find that

$$\left. \begin{aligned} (\sigma'_r)_c &= 2G(\epsilon_r^T - \epsilon_r^P - EB) \quad [00070040], \\ (\sigma'_\theta)_c &= 2G(\epsilon_\theta^T - \epsilon_\theta^P - EB) \quad [00070050], \\ \text{and} \\ (\sigma'_z)_c &= 2G(\epsilon_z^T - \epsilon_z^P - EB) \quad [00070060], \end{aligned} \right\} \quad (X-4)$$

where

$$EB = \frac{1}{2}(\epsilon_r^T + \epsilon_\theta^T + \epsilon_z^T).$$

The strains in Eq. (X-4) were calculated in subroutine STRAIN and in the MAIN program. A subscript c denotes the calculated value of the deviatoric stress. The square of the error is given by

$$\begin{aligned} SD2 &= [(\sigma'_r)_c - (\sigma'_r)_A]^2 + [(\sigma'_\theta)_c - (\sigma'_\theta)_A]^2 \\ &\quad + [(\sigma'_z)_c - (\sigma'_z)_A]^2 \end{aligned} \quad (X-5)$$

or

$$\begin{aligned} SD2 &= [(\sigma'_r)_c - (\sigma'_r)_A]^2 + [(\sigma'_\theta)_c - (\sigma'_\theta)_A]^2 \\ &\quad + [(\sigma'_z)_c - (\sigma'_z)_A]^2 + 3(P_{oc} - P_{oA})^2 \quad [00070130], \end{aligned} \quad (X-6)$$

where $P_o = -\frac{1}{3}(\sigma_\theta + \sigma_r + \sigma_z)$, and the error of stress is defined as

$$SD1 = (1.5 SD2)^{\frac{1}{2}} \quad [00070140]. \quad (X-7)$$

The weight sum of the error is defined by

$$SDP1 = [SD2 \cdot W(K) / (2G)^2]^{\frac{1}{2}} \quad [00070360], \quad (X-8)$$

where $W(K)$ is the area weight function calculated in [00032600-00033000] of the MAIN program and in [00069690 and 00069710] of this subroutine.

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