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Coriolis Anti-Pairing Transition

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ABSTRACT

The pairing plus quadrupole-quadrupole interaction is studied for a single j level by a sum rule method ("current algebra") in the regime where the "nucleus" is deformed. The ground state rotational band is shown to undergo the Coriolis anti-pairing transition.

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The coriolis anti-pairing phase transition at high angular momentum seems definitely to have been observed in the rare earth region by several experimental groups. [1-3] This effect, first predicted more than ten years ago, [4] has been the subject of several studies based on schematic models [5-7]; quite recently, realistic models have also been studied, utilizing plausible extensions of the cranking method [8,9] and, for the s-d shell, variation after projection. [10]

In this note we present a picture of the coriolis anti-pairing effect which is quite distinct from any previously given. Our approach is algebraic and in the same theoretical spirit as a previous treatment of the spherical to deformed transition. [11] All results have been obtained for a single kind of nucleon moving in a single-j level and it remains for us to show that these same methods can be extended to a more realistic description of the nucleus. Since it is possible to give a simple physical picture of our results, we shall do so before entering into the technical results. For this discussion we shall, however, require a few definitions.

We consider particles confined to a single-j level within the framework of the conventional pairing plus quadrupole-quadrupole interaction Hamiltonian,

$$H = - G \Omega A_0^{(0)\dagger} A_0^{(0)} - \frac{1}{2} \times \sum_q (-)^q B_q^{(2)} B_{-q}^{(2)} \quad (1)$$

The operators which enter are special examples of the general pairing and multiple operators,

$$A_q^{(k)\dagger} = (A_q^{(k)})^\dagger = \frac{1}{\sqrt{2}} \sum_m \begin{bmatrix} j & j & k \\ m & q-m & q \end{bmatrix} a_m^\dagger a_{q-m}^\dagger, \quad (2)$$

$$B_q^{(k)} = \frac{1}{\sqrt{2k+1}} \sum_m \begin{bmatrix} j & j & k \\ m & q-m & q \end{bmatrix} (-)^{j-m+q} a_m^\dagger a_{m-q}. \quad (3)$$

In our previous work [11] we have given an algebraic justification of the compatibility of pairing and quadrupole deformation correlations near the ground state of a deformed nucleus. As we now go to states of higher angular momentum, we find that it is algebraically consistent to keep the shape of the nucleus unaltered. This means that the famous rigid rotator relations for the matrix elements of the operator B_q^2 as given in Eq. (9) below remain approximately valid at high angular momentum. This rigidity of shape is moreover completely consistent with a change in the nature of the superfluidity inside the nucleus. As we feed angular momentum into the system, the monopole pairing correlations must be broken up and replaced by pairing correlations which carry a net angular momentum. In our model, and this represents an approximation, we assume that the pairs carry at most angular momentum 2, so that large angular momenta are achieved by coupling many individual pairs, rather than having a few particles align maximally. With this picture we are able to reproduce qualitatively the kind of behavior found in the experiments.

Turning now to the technical means used to achieve our solution, the problem divides into two parts. The first is a problem in ground state correlations and is identical to that which was solved in our previous work. It determines a set of starting matrix elements which includes the intrinsic quadrupole moment of the band. At the second stage we find that matrix elements characteristic of the higher states of the band are fully determined from kinematical considerations alone as soon as we assume that we can ignore centrifugal stretching and that all other relevant algebraic relations involving the pairing and multipole operators are saturated within the ground state band.

Turning finally to some details, we define reduced matrix elements of the pairing and of the multipole operators within the ground state band by means of the conventional equations

$$\langle IM | B_q^{(k)} | I'M' \rangle = [2I+1]^{-\frac{1}{2}} \begin{bmatrix} I' & k & I \\ M' & q & M \end{bmatrix} B_k(I, I') , \quad (4)$$

$$\langle IM | A_q^{(k)\dagger} | I'M' \rangle = (2I+1)^{-\frac{1}{2}} \begin{bmatrix} I' & k & I \\ M & q & M \end{bmatrix} A_k(I, I') . \quad (5)$$

The fact that in the last of these equations the bands refer to neighboring even nuclei has been suppressed in the notation. Indeed, throughout the ensuing argument we shall assume that all matrix elements are smooth in N , the number of particles, and that $N \gg 1$. This assumption is a simplification rather than a necessity and does not change any of our qualitative conclusions.

In terms of the quantities defined in Eqs. (4) and (5), the expectation value of the energy in the state of angular momentum I , takes the form

$$(2I+1) W(I) = - G \Omega A_0(I, I)^2 - \frac{1}{2} \times [B_2(I, I-2)^2 + B_2(I, I)^2 + B_2(I, I+2)^2] . \quad (6)$$

This expression is correct if sum rules are saturated within a single band - the ground state band. This assumption is the sine qua non of all our considerations.

In the general formulation of the current algebraic method, the dynamics of the system is expressed, first of all, by the requirement that the expressions (6) be stationary with respect to the parameters exhibited therein. A second general dynamical condition that the Hamiltonian be diagonal is here automatically satisfied by the rotational invariance of the Hamiltonian and the restriction to a single band.

The variations are to be carried out subject to kinematical constraints which follow from the algebra of the operators, and from the requirement that the system consist of fermions. As an example, from the commutator

$$[B_q^{(k)}, B_{q'}^{(k')}] = \sum_{k''} (-)^{2j-k''} [1 - (-)^{k+k'+k''}] \sqrt{2k''+1} \begin{bmatrix} k & k' & k'' \\ q & q' & q+q' \end{bmatrix} \begin{Bmatrix} k & k' & k'' \\ j & j & j \end{Bmatrix} B_{q+q'}^{(k'')} , \quad (7)$$

applied to $k = k' = 2$, it has been convincingly argued that in the region of the deformed nuclei, the right hand side of equation (7) may safely be ignored. [12] From this argument, there follows the approximate sum rule

$$\sum_{I''} \left\{ \begin{matrix} 2 & 2 & f \\ I & I'' & I' \end{matrix} \right\} B_2(I, I'') B_2(I'', I') = 0, \quad f = 1, 3, \quad (8)$$

which has as solution the famous rigid rotator relation

$$B_2(I, I') = B_2(02) \sqrt{2I+1} \begin{bmatrix} I & 2 & I' \\ 0 & 0 & 0 \end{bmatrix}. \quad (9)$$

From the remaining commutation relations among the operators of angular momentum 2 we derive further sum rules, first two sum rules involving diagonal matrix elements,

$$\begin{aligned} [A_2(I, I+2) - A_2(I+2, I)] B_2(I, I+2) + (10/\Omega)^{\frac{1}{2}} (2I+1)^{\frac{1}{2}} A_0(I, I) \\ = B_2(I, I-2) [A_2(I-2, I) - A_2(I, I-2)], \end{aligned} \quad (10)$$

$$A_2(I, I+2)^2 - A_2(I+2, I)^2 = 5(2I+1)(N-\Omega-2)/\Omega, \quad (11)$$

and then from the same source that yielded equation (10) an off-diagonal matrix element

$$\begin{aligned} & \left\{ \begin{matrix} 2 & 2 & 2 \\ I & I & I-2 \end{matrix} \right\} \left\{ 2A_2(I, I) B_2(I, I-2) - B_2(I, I) [A_2(I, I-2) + A_2(I-2, I)] \right\} \\ & = \left\{ \begin{matrix} 2 & 2 & 2 \\ I-2 & I & I-2 \end{matrix} \right\} \left\{ 2A_2(I-2, I-2) B_2(I, I-2) - B_2(I-2, I-2) [A_2(I, I-2) + A_2(I-2, I)] \right\} \\ & + 2 \left\{ \begin{matrix} 2 & 2 & 2 \\ j & j & j \end{matrix} \right\} [A_2(I-2, I) - A_2(I, I-2)]. \end{aligned} \quad (12)$$

We emphasize that equations (10) - (12) involve no approximation beyond the restriction to the single band and the number non-

conservation. Finally, and by contrast to the above, the nature of the representation space, that is the Pauli principle, is expressed by the approximate Casimir operator, whose diagonal matrix elements are given by the expression

$$\begin{aligned}
 & 2A_0(I, I)^2 + 2A_2(I, I+2)^2 + 2A_2(I, I)^2 \\
 & + 5B_2(I, I+2)^2 + 5B_2(I, I)^2 \\
 & = (2I+1) \left\{ N - (N^2/2\Omega) + 3(N/\Omega) - [3I(I+1)/j(j+1)(2j+1)] \right\} \\
 & - 2A_2(I, I-2)^2 - 5B_2(I, I-2)^2 .
 \end{aligned} \tag{13}$$

This equation, which is exact in the limit of good seniority, has been shown to remain qualitatively valid, also within the deformed region by direct numerical comparison with the exact solution. [13] This comparison has been made only near the bottom of the band. In our view the extrapolation of this relation to arbitrarily large angular momentum remains the most questionable assumption in our solution.

From Eqs. (9) - (13) we now notice the following remarkable feature: In consequence of Eq. (9) the remaining Eqs. (10) - (13) completely determine the four quantities $A_0(I, I)$, $A_2(I, I)$, $A_2(I, I+2)$ and $A_2(I+2, I)$ as soon as the same quantities with I replaced by $I-2$ are known. As emphasized above, all the equations now under discussion are kinematical.

We naturally seek to start this iterative determination at

the bottom of the ladder, namely for $I = 0$. But in this case, Eq. (12) drops out of the set, and, moreover, we have an additional dynamical quantity, the intrinsic quadrupole moment, $B_2(02)$ to determine. That is, we have only three kinematical conditions for the four quantities $A_0(0,0)$, $A_2(0,2)$, $A_2(2,0)$ and $B_2(0,2)$. As a fourth condition we apply the variational requirement to Eq. (6) for $I = 0$. This is precisely the problem which we have already solved in our discussion of spherical to deformed phase transitions. [11] Thus our numerical scheme is completely defined and we are prepared to present the results.

Fig. 1 shows the behavior of the pairing correlation amplitude as a function of angular momentum for the largest configuration which we studied, namely $\Omega = 200$, $N = 104$, and for a value of the coupling strength which represents a well-deformed nucleus. For this system, the transition occurs at an angular momentum equal to 32. By keeping the pairing correlation fixed at the vanishing value, solutions of our equations could be obtained for still higher values of the angular momentum. Table 1 lists the critical value, I_c , of the angular momentum found for this case and also for the other configurations studied.

Fig. 2 which refers to the very same case as Fig. 1, represents the now enshrined plot of the variable moment of inertia as a function of the square of the angular velocity. From the

parameters chosen we obtain an exaggerated version of the back-bending seen in some experimental cases. From Fig. 3 which represents (for a different configuration) a survey of the same behavior as a function of the relative strength of the pairing to quadrupole force parameters, we see that the backbending disappears for sufficiently deformed nuclei. The credibility of the results expressed by Figs. 2 and 3 depends, of course, on the calculation of (small) energy differences. For this purpose we used the equation of motion for the operator $B_q^{(2)}$ which can be proven, with some rigor, to be accurate practically up to I_c and certainly including the region of "back-bending". As it stands, our theory gives the rigid limit incorrectly because of the omission of the contribution of both higher bands and higher multipoles. These questions will be discussed in a separate paper.

In summary we have produced a model of a coriolis anti-pairing transition which, though over-simple in many respects, has the virtue of being angular momentum conserving throughout. A method for extending these calculations to a multi-level system can be outlined, but remains to be carried into practice.

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Table 1

Critical angular momentum, I_c at which the pairing correlation $A^0(I, I) = 0$ for the different configurations.

Ω, N	I_c
200, 104	32
150, 76	24
100, 50	16
50, 24	8
30, 16	6

Figure Captions

- Fig. 1. The pairing correlation matrix element $[A_0(I,I)/\sqrt{2I+1}]$ as a function of angular momentum I for the configuration $\Omega = 200$, $N = 104$, and coupling strength ratio $\beta = (5G\Omega/\chi) = 0.0002$. Heavy dots represent points calculated.
- Fig. 2. The moment of inertia $(\mathcal{J}/\mathcal{J}_0)$ in dimensionless units as a function of ω^2 , the square of the angular velocity for the same system as in Fig. 1.
- Fig. 3. A survey of the behavior of $(\mathcal{J}/\mathcal{J}_0)$ vs. ω^2 for the configuration $\Omega = 50$, $N = 24$, showing change in behavior as a function of coupling strength ratio.