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TOLERANCES FOR QUADRUPOLES IN A SUPERCONDUCTING STORAGE ACCELERATOR

G. Parzen

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Abstract

Tolerances for the magnetic field errors in the quadrupoles of a superconducting storage accelerator are proposed. Systematic errors in the magnetic field and random errors in the magnetic field are considered.

I. Introduction

The paper discusses the tolerances for the magnetic field in the quadrupoles in a superconducting storage accelerator like ISABELLE.

The quadrupoles in ISABELLE have a much smaller total length in each cell than the bending magnets by a factor of about 10. Because of this, the effects of errors in the magnetic field in quadrupoles tend to be less important than the effects of errors in the bending magnet field.

A suggested set of tolerances on the systematic multipoles present, which are due to effects which are the same from quadrupole to quadrupole like saturation, magnetization and end effects, are

$$\begin{aligned} b_5 &\leq 0.264 \times 10^{-3}/\text{in.}^4 \\ b_9 &\leq 0.037 \times 10^{-3}/\text{in.}^8 \\ b_{13} &\leq 0.004 \times 10^{-3}/\text{in.}^{12} \end{aligned} \quad (1.1a)$$

where the b_n are defined by writing the median plane magnetic field in the quadrupole as

$$B = B_1(x + b_5x^5 + b_9x^9 + b_{13}x^{13} \dots)$$

B_1 is the gradient present in the quadrupole.

Equation (1.1) implies tolerances on the gradient variation $\Delta B'/B_1$, and on B''/B_0 of roughly

$$\begin{aligned} \Delta B'/B_1 &< 1 \times 10^{-3} \\ B''/B_0 &< 3 \times 10^{-3}/\text{in.}^2 \end{aligned} \quad (1.1b)$$

where B_0 is the dipole magnetic field, and $\Delta B'$ is the variation in the gradient from $x = 0$ to $x = 1$ in. All results are for a magnet with a 3-in. aperture.

The tolerances are largely based on what appears doable with cosine quadrupoles, and probably could be relaxed if necessary.

Assuming a tolerance on the random errors in the current block positions as 0.0025 in., then the random multipoles that will be present are

$$\begin{aligned}
 \Delta B_0/B_1 &= 1.04 \times 10^{-3} \text{ in.} \\
 \Delta B_1/B_1 &= 1.08 \times 10^{-3} \\
 \Delta B_2/B_1 &= 0.885 \times 10^{-3}/\text{in.} \\
 \Delta B_3/B_1 &= 0.618 \times 10^{-3}/\text{in.}^2 \\
 \Delta B_4/B_1 &= 0.413 \times 10^{-3}/\text{in.}^3 \\
 \Delta B_5/B_1 &= 0.264 \times 10^{-3}/\text{in.}^4 \\
 \Delta B_6/B_1 &= 0.165 \times 10^{-3}/\text{in.}^5 \\
 \Delta B_7/B_1 &= 0.101 \times 10^{-3}/\text{in.}^6 \\
 \Delta B_8/B_1 &= 0.059 \times 10^{-3}/\text{in.}^7 \\
 \Delta B_9/B_1 &= 0.037 \times 10^{-3}/\text{in.}^8 \\
 \Delta B_{10}/B_1 &= 0.020 \times 10^{-3}/\text{in.}^9
 \end{aligned} \tag{1.3}$$

where the ΔB_n are defined by writing the error in the median plane field as

$$\Delta B = \sum_{n=0}^{\infty} \Delta B_n x^n \tag{1.4}$$

The suggested tolerance of $\epsilon = 0.0025$ in. is again something that appears doable. With this tolerance the effect of the random multipoles in the quadrupoles is much smaller than the effect of the random multipoles in the bending magnets because of the much smaller length of the quadrupoles.

II. Random Error Effects

This section considers the tolerances on the random errors in the quadrupoles, which are field errors that vary randomly from quadrupole to quadrupole, and the strength of the multipoles generated by the random errors. The primary source of these random errors in the field appears to be random errors in the position of the current blocks which make up the current distribution in a cosine quadrupole.

The results given in previous papers^{1,2} for the multipoles generated by errors in the positions of the coils may be applied to a magnet whose unperturbed field is

$$B = B_m r^m \quad (2.1)$$

and one finds for the rms multipoles generated in either B_z or B_r

$$\frac{\Delta B_\ell}{B_m} = \sqrt{\frac{2(m+1)}{N_b}} \frac{\ell+1}{R^{\ell+1}} R^m \epsilon \quad (2.2)$$

where ϵ is the rms position error in the coil, R is the average radius of the coil and N_b is the total number of current blocks in the magnet. $m = 0$ for a dipole, $m = 1$ for a quadrupole, and $m = 2$ for a sextupole.

The ΔB_ℓ are defined by expanding the error field in the median plane in either B_z or B_r as

$$\Delta B = \sum_{\ell=0}^{\infty} \Delta B_\ell r^\ell \quad (2.3)$$

For a quadrupole one finds

$$\frac{\Delta B_\ell}{B_1} = \sqrt{\frac{4}{N_b}} \frac{\ell+1}{R^{\ell+1}} R \epsilon \quad (2.4)$$

where B_1 is the unperturbed gradient in the quadrupole.

For a quadrupole having 3 current blocks per octant with an average radius of $R = 1.875$ in., one finds for $\epsilon = 0.0025$ in. rms error in the current block positions:

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1. G. Parzen, BNL Accelerator Dept. Informal Report AADD-187 (1972).
 2. M. Month and G. Parzen, Particle Accelerators 2, 227 (1971)

$$\begin{aligned}
 \Delta B_0/B_1 &= 1.04 \times 10^{-3} \text{ in.} \\
 \Delta B_1/B_1 &= 1.08 \times 10^{-3} \\
 \Delta B_2/B_1 &= 0.885 \times 10^{-3}/\text{in.} \\
 \Delta B_3/B_1 &= 0.618 \times 10^{-3}/\text{in.}^2 \\
 \Delta B_4/B_1 &= 0.413 \times 10^{-3}/\text{in.}^3 \\
 \Delta B_5/B_1 &= 0.264 \times 10^{-3}/\text{in.}^4 \\
 \Delta B_6/B_1 &= 0.165 \times 10^{-3}/\text{in.}^5 \\
 \Delta B_7/B_1 &= 0.101 \times 10^{-3}/\text{in.}^6 \\
 \Delta B_8/B_1 &= 0.059 \times 10^{-3}/\text{in.}^7 \\
 \Delta B_9/B_1 &= 0.037 \times 10^{-3}/\text{in.}^8 \\
 \Delta B_{10}/B_1 &= 0.020 \times 10^{-3}/\text{in.}^9
 \end{aligned} \tag{2.5}$$

The coil position error of $\epsilon = 0.0025$ in. is the position error allowed for the cosine dipole and is roughly what seems to be doable.

In order to compare the effect of the multipoles generated by errors in the quadrupole coils with the effect of the multipoles generated by errors in the dipole coils, one needs to compare the azimuthal harmonics introduced by the two errors. The azimuthal harmonics associated with the above multipoles may be estimated by dividing the above ΔB_l by $\sqrt{M}$, M being the number of quadrupoles around the ring and $M \approx 100$. One finds that

$$\begin{aligned}
 \Delta B_{0,n}/B_0 &= 0.00579 \times 10^{-3} \\
 \Delta B_{1,n}/B_0 &= 0.00601 \times 10^{-3}/\text{in.} \\
 \Delta B_{2,n}/B_0 &= 0.00492 \times 10^{-3}/\text{in.}^2 \\
 \Delta B_{3,n}/B_0 &= 0.00344 \times 10^{-3}/\text{in.}^3 \\
 \Delta B_{4,n}/B_0 &= 0.00229 \times 10^{-3}/\text{in.}^4 \\
 \Delta B_{5,n}/B_0 &= 0.00147 \times 10^{-3}/\text{in.}^5 \\
 \Delta B_{6,n}/B_0 &= 0.00092 \times 10^{-3}/\text{in.}^6 \\
 \Delta B_{7,n}/B_0 &= 0.00056 \times 10^{-3}/\text{in.}^7 \\
 \Delta B_{8,n}/B_0 &= 0.00032 \times 10^{-3}/\text{in.}^8 \\
 \Delta B_{9,n}/B_0 &= 0.00020 \times 10^{-3}/\text{in.}^9 \\
 \Delta B_{10,n}/B_0 &= 0.00011 \times 10^{-3}/\text{in.}^{10}
 \end{aligned} \tag{2.6}$$

In computing $\Delta B_{\ell,n}/B_0$ one divides by $\sqrt{M} = 10$, and then by 1.8 which is B_0/B_1 , where B_0 is the dipole field and B_1 is the quadrupole gradient, and then by 10 since the azimuthal length of the dipoles is roughly 10 times the length of the quadrupoles.

One sees that with the assumed coil error of $\epsilon = 0.0025$ in., the azimuthal harmonics introduced by the random errors in the quadrupoles are about 10 times smaller than those introduced by the random errors in the dipoles³. Thus the imperfection nonlinear resonances will be largely driven by the dipole random errors.

III. Systematic Error Effects

v-spread in beam

A consideration that is important in setting the tolerances for bending magnets³ is the maximum allowable Δv spread in the beam due to momentum spread $\Delta p/p$. The allowable Δv is estimated³ as $\Delta v \leq 0.05$. Assuming that the momentum spread in the beam is about $\Delta p/p \sim 5 \times 10^{-3}$ corresponding to a beam size of $\Delta x \approx 0.7$ cm or 0.3 in., then the requirement that the maximum allowable Δv in the beam be $\Delta v < 0.05$ can be rephrased as requiring a v -variation over the aperture of $\Delta v < \pm 0.15$ over $x = \pm 1$ in. The corresponding limitation on the gradient variation across the aperture can be computed using the formula⁴ for the Δv due to a change in gradient $\Delta B'$

$$\Delta v = \frac{1}{2} \frac{R}{\rho} \frac{1}{L_c} \int_0^{L_c} dl \frac{R \Delta B'}{B_0} \beta, \quad (3.1)$$

where L_c is the length of a cell, the integration is over a cell, and B_0 is the dipole magnetic field.

Equation (3.1) can be approximated by

$$\Delta v = \frac{1}{2} \left(\frac{R}{\rho} \right) \frac{R \Delta B'}{B_0} \left(\frac{\beta_{\max} - \beta_{\min}}{2R} \right) \frac{2l_Q}{L_c}, \quad (3.2)$$

where $\Delta B'$ is the gradient change in the quadrupole, l_Q is the quadrupole length,

3. G. Parzen, BNL Accelerator Dept. Informal Report AADD-194 (1972).

4. E. D. Courant and H. S. Snyder, Ann. Phys. 3, 1 (1958).

and it is assumed that the quadrupoles are located where β takes on its maximum and minimum values. We find for $\Delta v = 0.15$, that

$$\frac{\Delta B'}{B_1} = 7.0 \times 10^{-3}, \quad (3.3a)$$

where B_1 is the quadrupole gradient, and where we use for ISABELLE

$$(\beta_{\max} - \beta_{\min})/2R = 1/14.7, R/\rho = 1.3, B_0/B_1 = 1.8, 2\ell_Q/L_c = 0.1, \text{ and } R = 8700 \text{ in.}$$

The gradient variation of $\Delta B'/B_1 = 7 \times 10^{-3}$ implies the restriction on the multipoles of

$$\begin{aligned} b_5 &< 1.4 \times 10^{-3}/\text{in.}^4 \\ b_9 &< 0.82 \times 10^{-3}/\text{in.}^8 \\ b_{13} &< 0.54 \times 10^{-3}/\text{in.}^{12} \end{aligned} \quad (3.3b)$$

B'' considerations

The tolerances, Eqs. (3.3), are based on the maximum allowable Δv in the beam. A more stringent tolerance can be obtained by requiring that the effective nonlinearity as represented by B''/B_0 shall be no larger in the quadrupole than in the bending magnets. The tolerance suggested³ for the systematic B''/B_0 in the bending magnets is $B''/B_0 < 0.5 \times 10^{-3}$. This would lead to requiring $B''/B_0 < 5 \times 10^{-3}$ due to the factor 10 in the relative length of the quadrupoles and dipoles in a cell. This leads to the tolerances

$$\begin{aligned} b_5 &< 0.45 \times 10^{-3}/\text{in.}^4 \\ b_9 &< 0.12 \times 10^{-3}/\text{in.}^9 \\ b_{13} &< 0.058 \times 10^{-3}/\text{in.}^{12}, \end{aligned} \quad (3.4a)$$

and

$$\begin{aligned} B''/B_0 &< 5 \times 10^{-3}/\text{in.}^2 \\ \Delta B'_1/B_1 &< 2.2 \times 10^{-3} \end{aligned} \quad (3.4b)$$

Achievable tolerances

One can find a set of tolerances based on what appears doable with the cosine magnet. A limitation on how far one can reduce the unwanted multipoles, assuming one has correction coils that can correct any systematic unwanted

multipoles that are present, is the random multipoles that are present due to random errors in the coil position. We will assume here that one can reduce the multipoles present using the correction coils to a level about equal to the random multipoles that are present. One finds then for the achievable tolerances using the results of Section II

$$\begin{aligned} b_5 &< 0.264 \times 10^{-3}/\text{in.}^4 \\ b_9 &< 0.037 \times 10^{-3}/\text{in.}^8 \\ b_{13} &< 0.004 \times 10^{-3}/\text{in.}^{12} \end{aligned} \quad (3.5a)$$

Equations (3.5a) imply a tolerance on the gradient variation $\Delta B'/B_1$ from $x = 0$ to $x = 1$ in., and on B''/B_0 of roughly

$$\begin{aligned} \Delta B'_1/B_1 &< 1 \times 10^{-3} \\ B''/B_0 &< 2 \times 10^{-3}/\text{in.}^2 \end{aligned} \quad (3.5b)$$

It is suggested that Eqs. (3.5) be regarded as the tolerances to be met for the quadrupoles in ISABELLE.

Systematic errors present

Sources of systematic errors in the magnetic field include iron saturation effects and superconductor magnetization effects. Calculations were done using the GRACY magnetic program to estimate these two effects for the cosine quadrupole suggested by Dahl, Morgan, and Sampson⁵ which has 3 current blocks per octant with a ribbon distribution of 13, 10 and 6 turns and an 8-cm bore.

Due to iron saturation effects, the multipoles present when the gradient is 25 kG/in. are

$$\begin{aligned} b_5 &= 0.36 \times 10^{-3}/\text{in.}^4 \\ b_9 &= -0.004 \times 10^{-3}/\text{in.}^8 \\ b_{13} &= -0.000 \times 10^{-3}/\text{in.}^{12} \end{aligned}$$

According to the tolerances suggested as given by Eq. (3.5), a b_5 correction coil might be necessary.

5. P.J. Dahl, G.H. Morgan, and W.B. Sampson, BNL Accelerator Dept. Informal Report CRISP 72-59 (1972).

Due to superconductor magnetization effects, the multipoles present when the gradient is 3 kG/in. are

$$b_5 = -0.005 \times 10^{-3}/\text{in.}^4$$

$$b_9 = 0.001 \times 10^{-3}/\text{in.}^8$$

$$b_{13} = 0.000 \times 10^{-3}/\text{in.}^9 .$$

The superconductor magnetization effect is small and would not appear to require any correction coils.

Another source of systematic errors is the end effects due to the finite length of the quadrupole. The size of this effect depends on the design of the coils,⁶ and is thus not known at present. This effect should also be considered in determining which correction coils may be needed.

6. F. E. Mills and G. H. Morgan (to be published).