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ATOMICS INTERNATIONAL <small>A Division of North American Aviation, Inc.</small>		NAA-SR TDR NO. 11417	APPROVALS <i>D. W. Staub</i>
TECHNICAL DATA RECORD		PAGE 1 OF 16	
AUTHOR A. K. Smith	DEPT. & GROUP NO. 722-261	DATE 6-7-65	
		GO NO. 7561	
TITLE Performance of A Thermoelectric Converter with Leakage Current		S/A NO. 1241	TWR
		SECURITY CLASSIFICATION	
		(CHECK ONE BOX ONLY)	(CHECK ONE BOX ONLY)
		AEC DOD	
PROGRAM SNAP 10A		SUBACCOUNT TITLE Engineering & Development	UNCL. ☒ ☐ RESTRICTED DATA ☐
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			SECRET ☐ ☐
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STATEMENT OF PROBLEM Derive expressions for the determination of the thermoelectric converter electrical performances from flight test data. Using available "on line" data, extrapolate the performance of the SNAP 10A FS-4 and estimate the effect of converter leakage current on system operation.			
ABSTRACT: Expressions relating converter insulation resistance to converter internal resistance, load voltage, and leakage current are derived. Analysis of "on line" flight test data indicates that converter conductance has been increasing at a linear and essentially constant rate of 2.04 millimhos per day subsequent to 651 hours after launch.			
The performance of the system has been calculated on the basis of the following:			
1. Converter insulation conductance continues to increase at a constant rate equal to that experienced to date.			
2. Converter open circuit voltage & internal resistance data may be extrapolated on the basis of FS-3 system performance.			
The results of this analysis are graphically presented. It is concluded that:			
1. Power loss due to leakage is expected to be 38 watts at 90 days from launch.			
2. System power output is expected to be 473 watts at 90 days from launch.			
3. Leakage current is estimated to reach 1.89 amperes at 90 days from launch.			
4. Minimum converter return current is expected to be reached about 90 days after launch.			
5. Extended operation beyond 90 days should be realized at present Agena Load levels. However, output power would be less than 500 watts beyond 60 days of operation.			
6. Flight instrumentation of leaky converter system power output must recognize the effect of converter leakage current on output power.			

I. Converter Insulation Resistance Determination from Flight Test Data

In the SNAP 10A Flight Test System direct measurement of converter insulation resistance cannot be accomplished because the converter is connected to the vehicle ground point. Converter leakage current is monitored by a differential current transducer which is connected to measure the difference between converter return and converter output current. This differential current is the converter leakage current. The amount of leakage current that flows is determined by the converter isolation resistance to ground and by the converter voltage distribution. If it is assumed that the converter generated voltage, isolation resistance, and internal resistance are uniformly distributed, then the relationship between converter leakage current and insulation resistance can be developed. This relationship is developed in Appendix I following method 1 of reference 2. Equation (A19) of Appendix I shows that leakage current is a function of isolation resistance, internal resistance, and output voltage. This relationship is:

$$I_L = \frac{V_n}{R_i} \left\{ \sqrt{\frac{R_i}{R_g}} \left[\frac{\cosh \sqrt{\frac{R_i}{R_g}} - 1}{\sinh \sqrt{\frac{R_i}{R_g}}} \right] \right\} \quad (1)$$

If the resistance to ground is considerably greater than that of the internal resistance, then:

$$\cosh \sqrt{\frac{R_i}{R_g}} \sim 1 + \frac{\left[\sqrt{\frac{R_i}{R_g}} \right]^2}{2!} \quad (2)$$

$$\text{OR } \cosh \sqrt{\frac{R_i}{R_g}} \sim 1 + \frac{R_i}{2 R_g} \quad (3)$$

$$\text{AND } \sinh \sqrt{\frac{R_i}{R_g}} \sim \sqrt{\frac{R_i}{R_g}} \quad (4)$$

Using these approximations it is clear that:

$$I_L = \frac{V_n}{2 R_g} \quad (5)$$

This approximation, however, leads to calculated resistances which are lower than the actual, when the isolation resistance becomes of the same order of magnitude as the internal resistance. Since leakage current, load voltage, and internal resistance are all known from flight test data they may be used to determine isolation resistance from the exact equation.

Table I shows the variation of leakage current with time as determined from "on line" data. The indicated current is computed from the relation:

$$I_{IND} = 0.0403 e \quad (6)$$

where: I_{IND} is the indicated leakage current in amperes
 e is the T/M signal in millivolts

Indicated current must be corrected using calibration data developed by "end-to-end" testing of the entire instrument at the launch pad. The relationship between indicated current and actual leakage current as developed in reference 1 is:

$$I_L = 0.908 I_{IND} + 0.172 \quad (7)$$

where: I_L is the leakage current in amperes
 I_{IND} is the indicated "on-line" current in amperes.

Leakage current appears to be increasing at a linear rate of about 30 milliamperes per day from an inferred zero leakage at 651 hours after launch.

Using the corrected leakage current values from Table I and the exact expression (Equation A19) Appendix I) together with the actual values of converter internal resistance and load voltage, converter isolation resistance, isolation conductance, and rate of change of isolation conductance can be determined. These calculations are shown in Table II, entitled "Converter Isolation Resistance Calculations".

Inspection of Table II shows that the average rate of change of converter leakage conductance as determined from "on-line" data has been essentially constant at:

$$\frac{\partial G_G}{\partial T} = 2.04 \pm 0.07 \text{ millimhos/day} \quad (8)$$

Starting from an inferred zero conductance 651 hours after launch. These data indicate that converter leakage conductance may be expected to follow the relationship:

$$G_g = 2.04 \left(\frac{t - 651}{24} \right) \text{ millimhos} \quad (9)$$

Figure 1 shows the converter isolation (leakage) conductance vs time from launch. The solid curve indicates the results of analysis of flight test data and the dashed curve indicates the expected variation extrapolated by equation (9) to later periods of flight.

Figure 1 also shows the converter isolation resistance which is simply the reciprocal of the converter isolation conductance.

II. Performance of the SNAP 10A, FS-4 System Operating Under the Condition of Linearly Increasing Leakage Conductance.

If it is assumed that converter leakage conductance will continue to increase linearly with time at the same constant rate experienced to date, then system performance may be calculated to determine the effect of leakage current on system performance. Calculations of system performance are shown in Table III. These calculations are based on the following relationships:

1. Converter leakage conductance varies in accordance with equation (9) as shown in figure 1.
2. Converter internal resistance varies as shown in figure 1. This curve is based on actual flight test data extrapolated on the basis of FS-3 ground test experience.
3. Open circuit voltage varies as shown in figure 1. This curve is based on calculated performance of FS-4 from flight test data extrapolated on the basis of FS-3 ground test experience.
4. Load voltage is maintained constant at 30 volts by voltage regulator action.
5. Converter generated voltage, internal resistance, and isolation resistance are uniformly distributed.
6. Converter leakage current is calculated from equation (A19) of Appendix I using the dimensionless α relationship shown in figure (A1) of Appendix I.

R_i R_g G_g

LINE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	PROJECT	SHEET
	TIME FROM LAUNCH T DAYS	TIME FROM LAUNCH t HOURS	CONVERTER ISOLATION CONDUCTANCE G _G MILLIMHOS	CONVERTER ISOLATION RESISTANCE R _G OHMS	CONVERTER INTERNAL RESISTANCE R _i OHMS	CONVERTER RESISTANCE RATIO $\beta = \frac{R_i}{R_G}$ PER UNIT	CONVERTER VOLTAGE RATIO $\alpha = \frac{I_L R_i}{V_n}$ PER UNIT	CONVERTER LEAKAGE CURRENT $I_L = \alpha \frac{V_n}{R_i}$ AMPERES	CONVERTER OPEN CIRCUIT VOLTAGE E _{oc} VOLTS	CONVERTER SHORT CIRCUIT CURRENT $I_s = \frac{E_{oc}}{R_i}$ AMPERES	CONVERTER LOAD TERM I _x = $\frac{V_n \sqrt{3}}{R_i \tanh \beta}$ AMPERES	CONVERTER OUTPUT CURRENT I _L = I _s - I _x AMPERES	CONVERTER OUTPUT POWER P = V _n I _L WATTS	ZERO LEAKAGE OUTPUT CURRENT I _o = $\frac{E_{oc} - V_n}{R_i}$ AMPERES	ZERO LEAKAGE OUTPUT POWER P _o = V _n I _o WATTS	OUTPUT POWER REDUCTION DUE TO LEAKAGE ΔP = P _o - P WATTS	CONVERTER RETURN CURRENT I _R = I _L + I _o AMPERES					
1	0	0																			TABLE III PERFORMANCE OF LEAKY CONVERTER ATOMICS INTERNATIONAL A Division of North American Aviation, Inc.	10FS4
2	10	240																				
3	20	480			1.76				63.4					18.95	586.5							
4	30	720	5.87	170.00	1.78	0.0143			62.8	35.20	16.83	18.37	551.1	18.40	552.0	0.9						
5	40	960	26.25	38.10	1.80	0.0473	0.0235	0.392	62.5	34.70	16.90	17.80	534.0	18.02	540.6	6.6	18.19					
6	50	1200	46.50	21.50	1.82	0.0848	0.0425	0.702	62.2	34.10	16.95	17.15	514.5	17.65	529.5	15.0	17.85					
7	60	1440	67.60	14.90	1.83	0.1223	0.0600	0.985	62.0	33.90	17.18	16.72	501.6	17.49	524.7	23.1	17.71					
8	70	1680	87.30	10.40	1.84	0.1608	0.0790	1.288	61.9	33.60	17.21	16.39	491.7	17.32	519.6	27.9	17.68					
9	80	1920	107.60	9.30	1.85	0.1989	0.0970	1.571	61.8	33.40	17.28	16.12	483.6	17.21	516.3	32.7	17.69					
10	90	2160	128.00	7.82	1.86	0.2380	0.1170	1.888	61.7	33.10	17.35	15.75	472.5	17.00	510.0	37.5	17.64					
11	100	2400	148.20	6.73	1.86	0.2760	0.1360	2.190	61.7	33.10	17.60	15.50	465.0	17.00	510.0	45.0	17.69					
12	110	2640	168.90	5.95	1.86	0.3142	0.1520	2.448	61.7	33.10	17.80	15.30	459.0	17.00	510.0	51.0	17.75					
13	120	2880	189.20	5.29	1.87	0.3530	0.1690	2.721	61.6	33.00	17.89	15.11	453.3	16.86	505.8	52.5	17.83					
14	130	3120	210.00	4.76	1.87	0.3920	0.1880	3.030	61.6	33.00	18.02	14.98	449.4	16.86	505.8	56.4	18.01					
15	140	3360	230.00	4.35	1.87	0.4300	0.2050	3.300	61.6	33.00	18.28	14.72	441.6	16.86	505.8	64.2	18.02					
16	150	3600	250.20	3.99	1.87	0.4680	0.2210	3.560	61.6	33.00	18.48	14.52	435.6	16.86	505.8	70.2	18.08					
17	160	3840	270.20	3.70	1.87	0.5050	0.2390	3.846	61.6	33.00	18.68	14.32	429.6	16.86	505.8	76.2	18.17					
18	170	4080	291.00	3.44	1.87	0.5430	0.2530	4.073	61.6	33.00	18.86	14.14	424.2	16.86	505.8	81.6	18.21					
19	180	4320	311.8	3.22	1.87	0.5820	0.2720	4.380	61.6	33.00	19.00	14.00	420.0	16.86	505.8	85.8	18.38					
20	190	4560	332.0	3.02	1.88	0.6250	0.2950	4.710	61.5	32.70	19.20	13.50	405.0	16.75	502.5	97.5	18.21					
21	200	4800	351.8	2.84	1.88	0.6620	0.3100	4.950	61.5	32.70	19.28	13.42	402.6	16.75	502.5	99.9	18.37				ORIGINATED BY A.K. SMITH CALCULATED BY CHECKED BY DATE MAY 17, 1965	
22	210	5040	372.3	2.68	1.88	0.7020	0.3290	5.250	61.5	32.70	19.52	13.18	395.4	16.75	502.5	107.1	18.43					
23	220	5280	393.2	2.54	1.88	0.7410	0.3430	5.480	61.5	32.70	19.70	13.00	390.0	16.75	502.5	112.5	18.48					
24	230	5520	414.0	2.42	1.88	0.7800	0.3610	5.760	61.5	32.70	19.92	12.78	383.4	16.75	502.5	119.1	18.54					
25	240	5760	434.5	2.31	1.88	0.8160	0.3770	6.020	61.5	32.70	20.08	12.62	378.6	16.75	502.5	123.9	18.64					
26	250	6000	453.5	2.21	1.88	0.8555	0.3930	6.280	61.5	32.70	20.23	12.47	374.1	16.75	502.5	128.4	18.75					
27	260	6240	474.0	2.12	1.88	0.8900	0.4090	6.500	61.5	32.70	20.42	12.28	368.4	16.75	502.5	134.1	18.78					
28																						
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7. Converter output current is calculated from equation (A14) of Appendix I. The two terms of equation (A14) are defined as converter short circuit current (I_s) and converter load term (I_x) in Table III.
8. The remaining relationships are defined in Table III as used.

Figure 2 presents the results of the performance calculations and graphically illustrates the effect of leakage current on system performance.

REFERENCES

1. NAA-SR-TDR 11418 FS-4 Differential Current Transducer Performance Evaluation, A. K. Smith, dated May 14, 1965
2. IL - Estimation of Converter Isolation Resistance - P. Jalichandra and T. J. Boyle to H. M. Dieckamp and W.T.Morgan, dated March 3, 1965.

FIGURE 2 CONVERTER CURRENT & POWER OUTPUT
VS. TIME FROM LAUNCH
AT CONSTANT OUTPUT VOLTAGE OF 20 VOLTS

APPENDIX I. DERIVATION OF PERFORMANCE
EQUATIONS FOR A LEAKY CONVERTER

$$\frac{dV}{d\delta} = E - IR \quad (1)$$

$$\frac{dI}{d\delta} = -\frac{V}{r} \quad (2)$$

ASSUMING THE INTERNAL emf E IS
CONSTANT,

$$\frac{d^2V}{d\delta^2} = \frac{V}{r} R \quad (3)$$

THE GENERAL SOLUTION FOR (3) IS:

$$V = C_1 e^{2\delta} + C_2 e^{-2\delta} \quad (4)$$

$$\text{WHERE: } 2 = \sqrt{\frac{R}{r}} \quad (5)$$

$$\text{DEFINE: } R_G = \frac{r}{n} \quad (6)$$

$$R_i = nR \quad (7)$$

$$E_{oc} = nE \quad (8)$$

$$\text{THEN: } 2 = \frac{1}{n} \sqrt{\frac{R_i}{R_G}} \quad (9)$$

$$\text{AT } \delta = 0, V = 0 \text{ AND AT} \quad (10)$$

$$\delta = n, \frac{dV}{d\delta} = E - I_L R \quad (11)$$

$$\text{SO THAT: } V(\delta) = \sqrt{\frac{R_G}{R_i}} \frac{[E_{oc} - I_L R_i] \sinh \frac{\delta}{n} \sqrt{\frac{R_i}{R_G}}}{\cosh \sqrt{\frac{R_i}{R_G}}} \quad (12)$$

At $\delta = n$

$$V_n = \sqrt{\frac{R_G}{R_i}} [E_{oc} - I_L R_i] \tanh \sqrt{\frac{R_i}{R_G}} \quad (13)$$

SOLVING EQUATION (13) FOR LOAD CURRENT:

$$I_L = \frac{E_{oc}}{R_i} - \frac{V_n}{R_i} \left[\frac{\sqrt{\frac{R_i}{R_G}}}{\tanh \sqrt{\frac{R_i}{R_G}}} \right] \quad (14)$$

EQUATION (2) MAY BE REWRITTEN AS

$$\frac{dI}{d\delta} = -\frac{V}{r} = -\frac{V}{n R_G} \quad (15)$$

SUBSTITUTING EQUATION (12) IN EQUATION (2)

GIVES THE RESULT:

$$\frac{dI}{d\delta} = -\frac{\sqrt{\frac{R_G}{R_i}} [E_{oc} - I_L R_i]}{n R_G} \left[\frac{\sinh \sqrt{\frac{R_i}{R_G}} \frac{\delta}{n}}{\cosh \sqrt{\frac{R_i}{R_G}}} \right] \quad (16)$$

LEAKAGE CURRENT (I_L) IS DEFINED AS:

$$I_L = - \int_0^n \left[\frac{dI}{d\delta} \right] d\delta \quad (17)$$

SUBSTITUTING EQUATION (16) IN EQUATION (17)

AND INTEGRATING OVER THE ENTIRE CONVERTER
GIVES THE RESULT:

$$I_L = \left[\frac{E_{oc} - I_L R_i}{R_i} \right] \left[\frac{\cosh \sqrt{\frac{R_i}{R_G}} - 1}{\cosh \sqrt{\frac{R_i}{R_G}}} \right] \quad (18)$$

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SUBSTITUTING EQUATION (14) FOR (I_L) INTO EQUATION (18) GIVES THE RESULT THAT:

$$I_g = \frac{V_n}{R_i} \left\{ \sqrt{\frac{R_i}{R_g}} \left[\frac{\cosh \sqrt{\frac{R_i}{R_g}} - 1}{\sinh \sqrt{\frac{R_i}{R_g}}} \right] \right\} \quad (19)$$

LET: $\alpha = \left(\frac{I_g R_i}{V_n} \right)$ BE DEFINED AS THE CONVERTER

DIMENSIONLESS VOLTAGE RATIO, AND $\beta = \left(\frac{R_i}{R_g} \right)$

AS THE CONVERTER RESISTANCE RATIO. EQUATION (19) CAN BE USED TO DETERMINE (α) AS A FUNCTION OF (β) . THIS RELATIONSHIP IS SHOWN IN FIGURE 1.

WHERE (V_n) IS A FUNCTION OF (I_L) AS IS THE CASE FOR A CONSTANT RESISTANCE LOAD (R_L) (V_n) IN EQUATION (19) MAY NOT BE KNOWN.

UNDER THESE CONDITIONS:

$$V_n = I_L R_L \quad (20)$$

SUBSTITUTING EQUATION (20) IN EQUATION (14) AND SOLVING FOR I_L GIVES THE RESULT:

$$I_L = \frac{E_{oc}}{R_i \left[1 + \left(\frac{R_L}{R_i} \right) \frac{\sqrt{\frac{R_i}{R_g}}}{\tanh \sqrt{\frac{R_i}{R_g}}} \right]} \quad (21)$$

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AND:

$$V_n = E_{oc} \left[\frac{\left(\frac{R_L}{R_i} \right)}{1 + \left(\frac{R_L}{R_i} \right) \frac{\sqrt{R_i/R_G}}{\tanh \sqrt{R_i/R_G}}} \right] \quad (22)$$

SUBSTITUTING EQUATION (21) INTO EQUATION (18)
GIVES THE RELATIONSHIP:

$$I_L = \frac{E_{oc}}{R_i} \left[\frac{\left(\frac{R_L}{R_i} \right)}{1 + \left(\frac{R_L}{R_i} \right) \frac{\sqrt{R_i/R_G}}{\tanh \sqrt{R_i/R_G}}} \right] \left[\frac{\sqrt{R_i/R_G} \left(\cosh \sqrt{R_i/R_G} - 1 \right)}{\sinh \sqrt{R_i/R_G}} \right] \quad (23)$$

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NOMENCLATURE

a	=	Dimensionless Constant
n	=	Number of Converter Elements
r	=	Resistance per Unit Element - Ohms
t	=	Time from Launch - Hours
E	=	Generated Voltage per Unit Element - Volts
E_o	=	Open Circuit Voltage - Volts
G_o	=	Converter Leakage Conductance - mhos
I	=	Converter Current - amperes
I_L	=	Load Current - amperes
I_s	=	Converter Leakage Current - amperes
I_o	=	Zero Leakage Converter Current - amperes
I_R	=	Converter Return Current - amperes
I_s	=	Converter Short Circuit Current - amperes
I_x	=	Converter Load Term - amperes
P	=	Output Power (with leakage) - watts
P_o	=	Zero Leakage Output Power - watts
ΔP	=	Output Power Reduction Due to Leakage - watts
R	=	Resistance Per Unit Element - ohms
R_o	=	Converter Resistance to Ground - ohms
R_i	=	Converter Internal Resistance - ohms
R_L	=	Load Resistance - ohms
T	=	Time to Launch - days
V	=	Net Converter Voltage Per Unit Element - volts
V_n	=	Load Voltage - volts
X	=	Non Dimensional Distance
α	=	Converter Voltage Ratio
β	=	Converter Resistance Ratio