Positron Spectrum from Fission
Antineutrinos on Protons

Frank A. Nezrick
1965

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POSITRON SPECTRUM FROM FISSION ANTINEUTRINOS ON PROTONS

A Thesis Submitted to
Case Institute of Technology
In Partial Fulfillment of the Requirements
for the Degree of
Doctor of Philosophy

RELEASED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

by

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1965

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ABSTRACT

The $P(\bar{\nu}_e, \beta^+)n$ reaction was observed in a 3.2 liter Gadolinium loaded decalin based liquid scintillator target placed between two cylindrical NaI crystals 29 cm in diameter and 7.6 cm. deep. The crystals were set to see 0.511 MeV β^+ annihilation radiation. A prompt triple coincidence of all three detectors followed by the delayed capture of the neutron is the signature for the reaction. This detector was operated within a 27 ton lead shield at one of the Savannah River Reactors. The data were obtained during 2500 hours of operation at the rate of 0.20 events per hour, with a signal to background ratio of approximately 3/1. The observed signal was demonstrated to be due to reactor antineutrinos. The positron spectrum obtained and hence the $\bar{\nu}_e$ spectrum from fission is consistent with prediction except for a larger observed component above 4.0 MeV.

ACKNOWLEDGMENTS

The author is indebted to Professor F. Reines who suggested and guided this work. Thanks are due to Professor T. L. Jenkins and Mr. B. M. Shoffner for valuable discussions and to Mr. A. A. Hruschka for help in many phases of design and construction. A particularly useful comment with regard to the energy dependence of the efficiency was made by Dr. Chudakov of the Lebedev Institute.

Thanks are expressed for the hospitality of the E. I. du Pont de Nemours Company, which operates the Savannah River Plant of the United States Atomic Energy Commission. Thanks are also expressed to the Atomic Energy Commission for their support. Finally the author gratefully acknowledges his wife, Natalie, for her understanding and support so freely given throughout this work.

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I INTRODUCTION

Previous investigations^{1,2} of the reaction

using reactor antineutrinos gave as results that (a) the neutrino can be detected away from its site of creation and (b) the reaction cross section is consistent with the two-component neutrino theory. However, the positron kinetic energy spectrum and hence the antineutrino spectrum from fission fragments have been poorly determined because this early work was designed primarily to detect the reaction and obtain its cross section. In the previous experiments, a liquid scintillation detector gave a delayed coincidence pulse pair, the first pulse of which was proportional to the sum of the positron kinetic energy and the energy of the positron annihilation gamma rays. These energies were superimposed because the positron slows down and annihilates within the resolution time of the system and the entire phenomenon occurred in one detector. A poorly determined positron spectrum was obtained from this arrangement because of the uncertainty in the leakage of the annihilation gamma rays from the detector and its poor energy resolution ($\pm 25\%$ for 1 Mev deposited).

The detector designed and operated in the present experiment avoids these uncertainties by separately measuring the positron kinetic energy and its annihilation gamma rays. Figure 1 is a

schematic of the detector. The proton target is a relatively thin liquid scintillator loaded with Gadolinium to capture the product neutron and provide several energetic gamma rays. A NaI crystal is located on each side of the target detector. Figure 1 illustrates reaction (1) as detected by this system. The positron rapidly slows down ($\sim 10^{-10}$ sec) within the resolution of the electronics (~ 1.5 μ sec) and annihilates so that a prompt triple coincidence is observed between all three detectors. The energy pulses from the two NaI crystals are individually 0.51 Mev and the energy pulse from the liquid scintillator is equal to the positron kinetic energy. The neutron thermalizes in the hydrogenous scintillator and is captured by the Gadolinium which produces on the average four gamma rays, each with a mean energy of ~ 2 Mev. These gamma rays can interact in one, two, or three of the detectors. Reaction (1), then, has for a signature a prompt triple coincidence produced by the positron followed by a delayed pulse (or coincidence pulses) produced by the neutron capture gamma rays. In this detection scheme, the background is greatly reduced by requiring that the NaI pulses in the prompt triple coincidence fall within a narrow energy window centered at the annihilation gamma ray energy.

The NaI crystal³ diameter of 29 cm was fixed by the present state of the art of crystal growing. Since the NaI crystals are used to detect 0.51 Mev gamma rays (absorption mean free path of 6.8 cm in NaI), a thickness of 7.5 cm was considered sufficient.

The liquid scintillator thickness was chosen with a view to maximizing the signal rate and the probability of totally absorbing

the positron, subject to the requirement that the annihilation gamma rays be transmitted without interacting. Appendix I shows that the optimum thickness is from 6 to 10 cm. Details of the detector elements are given in Section II-C.

Decalin was chosen as the scintillator solvent because it has the highest proton density of available scintillators (Table 1). The decalin was decolored and loaded with 2.0 g/l p-terphenyl as the primary scintillator solute, 4.0 g/l alpha-naphthyl-phenyloxazole as the wave shifter solute and 24 g/l of Gadolinium 2-ethylhexate⁴ to capture the neutron. The liquid scintillator was contained in a right circular lucite cylinder which was viewed on its vertical wall by 13 CBS 7817 photomultiplier tubes.⁵

In order to reduce the gamma ray background from the reactor, it was necessary to enclose the detector in a lead house which ranged in thickness from 12 to 16 inches around the detectors and weighed 30 tons (Section II-B). In addition it was necessary to place a paraffin-borax neutron shield around the detector elements inside the lead house. The total shield reduced the reactor-up background seen by one of the NaI crystals by a factor ranging from 4,400 at 1 Mev to 44,000 at 6.5 Mev so that the reactor-up spectrum in the shield was approximately 10% higher than the reactor-down spectrum for gamma ray energies below 3.0 Mev (Section III).

The data were in the form of photographed oscilloscope traces. Since there were three detectors which had doubly differentiated amplifier outputs and only one oscilloscope beam, the pulses were coded to enable identification of the associated detector. The liquid

scintillator pulses were retained as doubly differentiated pulses while the pulses from the NaI amplifiers were diode biased so that only the positive half and negative half were retained for NaI 1 and NaI 2 respectively. Since both the prompt and delayed signals can be threefold coincidences, the amplifier outputs were "separated in time" by driving their pulses through different length delay lines prior to display on the oscilloscope. A typical display is illustrated in Fig. 2. In the figure, h_1 , h_3 and h_5 (where $h_3 = h_5 = 0.51$ Mev) form the threefold prompt coincidence signal, while h_2 , h_4 and h_6 (where only one need be non-zero) form the coincident delayed signal produced by the neutron capturing at a time t_c from its birth. The oscilloscope display was photographed every time a proper prompt signal occurred or approximately at the rate of 20 per hour. The film was later scanned for delayed pulses which met the criteria chosen for neutron capture. Such delayed coincidences occurred at the approximate rate of 0.25 per hour. The electronic system is described in Section II-D.

The data collected during 2484 hours of reactor-up time and 356.8 hours of reactor-down time satisfied the following criteria:

a. The positron kinetic energy, h_1 , must be greater than 0.50 Mev. This threshold is necessary because of the rapidly rising background below 0.50 Mev.

b. The oscilloscope is triggered only when the prompt signal is characteristic of a positron annihilation; i.e., the NaI crystals must observe gamma rays in the vicinity of 0.51 Mev. Therefore, $h_3 = h_5 = 0.51$ Mev $\pm$ the crystal resolution. However, most of the

oscilloscope triggers were not due to positrons since, as operated, the system did not discriminate sufficiently against correlated energy losses which met the trigger requirements.

c. The sum of the energy deposits in the delayed pulse must be greater than 0.75 Mev.

d. The neutron must capture within the time gate of $2.8 \leq t_c < 100$ microseconds.

e. Oscilloscope traces are rejected if more than one neutron is observed or if any pulse occurs before the prompt signal.

Requirements c and d served to distinguish neutrino-produced triggers. The delayed coincidence data were grouped into two time slots with widths of 50 microseconds. Since the liquid scintillator was measured to have a mean life of 8.0 microseconds for neutron capture, most of the neutrons were captured within the first time slot ($0 < t_c \leq 50$ microseconds). The second time slot gave the accidental delayed coincidence background. Subtracting the rate in the second time slot from the rate in the first gives the correlated reactor-up (100% power) signal. A similar determination with the reactor down gives the correlated reactor-down (0% power) signal. The correlated reactor-associated signal rate is the difference between the correlated reactor-up and reactor-down rates.

To complete the experiment, tests were performed (Sec. VI) to verify that:

- a. The reactor-associated signal is not due to neutrons.
- b. The reactor-associated signal is correlated and hence is not due to the increase in the accidental background.

c. The second pulse results from neutron capture.

d. The first pulse is characteristic of a positron.

On the basis of these tests, it was concluded that the observed signal⁶ is due to reaction (1).

In a determination of the positron, and hence $\bar{\nu}_e$, spectrum, it is necessary to know the variation of detection efficiency with $\bar{\nu}_e$ energy. The positron detection efficiency was both calculated and measured. The neutron detection efficiency was calculated by Monte Carlo techniques. The overall detection efficiency is the product of these two.

The reactor-associated positron spectrum was obtained (Sec. VIII) by folding the detector resolution and efficiency with a two-parameter function which has the general shape of the expected spectrum. The parameters were determined by a least squares fitting to the data of the expected spectrum distorted by the detector response. The unnormalized fission antineutrino spectrum was obtained from the unfolded positron spectrum by taking the energy dependence of the cross section as given by phase-space arguments.

The prompt triple coincidence background is analyzed (Sec. IX) and methods are proposed for its reduction. If this background is sufficiently reduced, it may be possible to perform inverse beta decay experiments without recourse to the neutron with its small detection efficiency. The virtue of such an experiment would be higher signal rates than in the present arrangement.

II EXPERIMENTAL ARRANGEMENT

A. Introduction

A preliminary indication of the feasibility of the present approach was obtained by measurements made with a mockup in an existing experimental area⁷ at one of the Savannah River Plant reactors of the United States Atomic Energy Commission. The mockup was composed of two NaI crystals⁸ 5-inch diameter by 4-inch thick and a plastic scintillator⁹ 3-inch diameter by 2-inch thick observed by two phototubes placed radially at right angles to each other.

The signal-to-background ratio estimated from this mockup warranted the construction of the full-size detection system and a special lead house in a more permanently available experimental area. A survey of the background in the new area revealed that a 30-ton lead house was required (Section B). The weight of this shield necessitated the construction of a special platform the details of which are given in Appendix II.

After the lead house was constructed, a survey revealed the need for a neutron shield within it. The interior neutron shield was chosen because of space limitations. The neutron shield was composed of two concentric blankets around the detectors: an exterior three-inch thick blanket of paraffin to moderate and an inner three-inch thick blanket of commercial grade borax to capture the neutrons.

The detector elements were mounted within the lead house in a simple but convenient manner and were accessible through an electrically operated 4-ton lead door.

The preamplifiers, high voltage distribution box and the two-way communication system were mounted on top of the house. The experimental area and electronics area were connected by 300 feet of double shielded signal, high voltage and intercommunication cables carried in a shielded cable tray. The electronics (Section D) are of conventional (~ 0.2 microsecond rise time) design. The double differentiated amplifier outputs were coded so the pulses could be identified with the detectors of their origin. The three detector pulses were then displayed on an oscilloscope and photographed.

B. Lead House

To determine the thickness of the lead shield needed in the new experimental area to reduce the background to acceptable levels, a survey was made with a 5-inch diameter, 4-inch thick NaI crystal. The gamma ray background spectrum was measured with the reactor at full power from 0.20 Mev to 8.0 Mev as a function of the thickness of a lead shield placed around the crystal. The effects of varying the shield thickness from 0 to 12 inches on the high-energy spectrum from 1.0 to 8.0 Mev is shown on Figure 3 along with the spectrum measured inside the old lead shield. Figure 4 shows the low-energy spectrum from 0.20 to 0.80 Mev for a shield thickness of 12 inches compared with the spectrum measured in the old lead shield and in the low background Case laboratory located in the Morton Company

Painesville Ohio salt mine. The salt mine is the least radioactive experimental area encountered by the Case neutrino group and therefore suggests a limit for the obtainable background.

Figure 3 suggests that there may be neutrons in the area which give rise to background via capture gamma rays since the background does not decrease sufficiently rapidly for increasing shield thicknesses greater than 8 inches. It was therefore decided to build a 12- to 16-inch thick lead house plus a neutron shield to reduce the background.

The available width of the new experimental area was 48 inches. The minimum volume needed for the detector assembly was 24 inches square by 30 inches high. The lead house was therefore constructed as shown in Figure 5. The shield walls are 12 and 16 inches thick and the door is designed so no straight line cracks exist through the shield leading to the detector cavity.

A special platform was designed to bear the lead shield and to provide tracks on which the door carriage could roll. The platform was covered with 1/2-inch thick steel plate to provide a smooth foundation for the construction of the shield. The I-beam door tracks were covered with 1/2-inch thick strips of cold rolled steel to provide a smooth rolling surface (Appendix II).

The lead shield was constructed of virgin lead bricks 2" x 4" x 8". The supports for the top of the detector cavity and the door overhang were 1/2-inch thick steel boiler plates. The door is pulled open up an inclined plane (1-1/4-inch rise in seven feet) by an electric hoist and is allowed to roll closed under gravity.

(Figs. 6,7). The signal and high voltage cables enter through the top of the shield via a 1-inch by 1-inch stepped hole.

An aluminum liner was placed in the detector cavity. This liner, shown in Figure 8, has three sets of slides on which the three detectors were placed. The neutron shield was stacked within the aluminum liner in two concentric 3-inch thick shells. The outer shell, the neutron moderator, was paraffin; and the inner shell, the neutron absorber, was borax. Figure 9 shows the neutron shield partially assembled around NaI 2. Ideally, the neutrons should be captured exterior to the gamma shield rather than inside it as is done here. Space limitations prevented the use of such an external neutron shield.

C. Detector Elements

1. NaI Crystals

The crystal detectors, shown in Figure 8, are NaI(Tl) crystals 11-1/2 inches in diameter by 3 inches thick of the Harshaw matched window line housed in 0.019 inches of stainless steel. Each crystal assembly is viewed by 7 mumetal shielded RCA 3-inch diameter photomultiplier tubes with low background tube base assemblies. The high-voltage divider chains for these photomultiplier tubes is given in Figure 11. The photomultiplier tubes were balanced in the multiple tube systems as described in Appendix III.

The crystals were mounted on 23-inch by 23-inch aluminum trays as shown in Figure 8. All attempts were made to provide a shock-proof mounting to protect the crystals. A rubber washer was used

between the crystal mounting rings and the trays. Also the bolts securing the crystals to the trays were completely separated from the trays by rubber insulators (Fig. 12).

Since every photomultiplier tube has an individual high voltage fine adjustment to balance the gains, a common high voltage was connected to all the tubes on each detector. The outputs of all seven tubes on each detector were fed to a common cable. The signal and high-voltage yokes which connected the seven tubes to a common cable were designed to produce a minimum amount of connector noise and also to consume a minimum amount of space within the shield. The signal and high-voltage yokes on each crystal use one BNC signal T and one BNC high-voltage T connector on each tube. A high-voltage unshielded cable is soldered connecting the center conductors of adjacent T connectors. Over this cable is placed an insulator and two layers of braided shielding. The two layers of braid are soldered to the outside of the T connectors and the yoke is made moisture proof by shrinking a piece of plastic tubing over the braid. The results are the yokes shown on the crystals in Figure 8.

2. Liquid Scintillation Detector

This detector is 3.7 liters of decalin based liquid scintillator contained in a right circular lucite cylinder and viewed on its lateral wall by 13, 2-inch diameter photomultiplier tubes. As discussed in the introduction, the scintillator is decalin based with 2.0 g/l p-terphenyl as the primary solute, 4.0 g/l alpha-naphthyl-phenyloxazole as the wave shifter solute and 24 g/l of Gadolinium 2-ethylhexate which is the neutron absorber. The lucite

cylindrical container is shown in Figure 13.

The side wall of the cylinder and end covers are respectively $3/8$ -inch and $1/16$ -inch thick. There are 13 flats of width $2-1/8$ inches cut equally spaced around the periphery of the detector to accommodate the photomultiplier tubes. It was found at a later date that the scintillator attacked the lucite and became less transparent. To prevent this deterioration of the scintillator, the entire inside surface of the lucite detector was covered with pieces of $1/2$ mil mylar film. The detector was filled through a $1/2$ -inch diameter hole in the center of the top cover.

The liquid scintillator detector was rigidly fixed in the aluminum housing shown in Figure 10. This housing is an aluminum cylinder 14 inches in diameter by $3-1/4$ inches high mounted on an aluminum tray 12 inches long by 23 inches wide to fit on the slides of the aluminum shield insert as shown in Figure 8. Thirteen mumetal shields are epoxied to the detector housing equally spaced around the lateral wall. The liquid scintillation detector is held in the housing by three lucite guides fixed to the housing wall which slide into slots milled in the side wall of the detector cylinder.

A felt pad was cemented inside each mumetal shield so that a 2-inch diameter phototube was held firmly centered in the shield. The CBS 7817 phototubes were coupled to the polished flats on the detector wall with Dow Corning QC 2-0057 coupling grease. The tubes were spring loaded to exert 3 pounds of force on the detector flats thus keeping them firmly seated and insuring good optical coupling. The inside of the aluminum detector housing including the mumetal

shields were painted white¹⁰ to enhance photon collection. Dust covers, made of 1 mil mylar sheets painted white and mounted on metal rings, were inserted in the top and bottom of the aluminum housing. These dust covers, in addition to preventing foreign matter from collecting on the detector and altering its total internal reflection properties were also found to help smooth out the dependence of pulse height on the radial position of the scintillation.

The width of the detector cavity in the lead house and hence the shielding wall thickness is determined by the diameter of the liquid scintillator detector assembly. To minimize this diameter, the high-voltage dropping chains were constructed on bakelite cards and mounted on cut-down tube sockets so they were perpendicular to the axis of the photomultiplier tubes (Figs. 8, 10). The high voltage dropping chains used are shown in Figure 14.

D. Electronics

A schematic diagram of the detector and associated electronics is shown in Figure 15. The scintillation light produced in the detectors is converted into electrical impulses by the photomultiplier tubes mounted on each detector. The photomultiplier tube outputs are matched to the 100 ohm characteristic impedance of the signal cable, RG 71U, by the preamplifier. The preamplifier outputs drive 300-foot cables which deliver the signals to the electronics area for further amplification by doubly differentiated amplifiers. The amplifier outputs are fed into pulse height selectors, whose triggered

outputs are tested for the prompt triple coincidence which is characteristic of a signal. Upon receiving a triple coincidence, the coincidence unit triggers the oscilloscope and the camera control circuit, i.e., the camera shutter is opened, the oscilloscope sweeps 10 cm at 20 microseconds/cm, the shutter closes, the film advances and the frame number and time are photographed on the top of the next frame.

During the process of determining whether the prompt signal meets the selection criteria, the pulses from the three amplifiers pass through delay lines, are coded, are mixed together on a single line and are stored in an additional delay line. If the selection criteria are satisfied, then the information stored in the delay line is displayed on the oscilloscope and photographed. As seen from the oscilloscope display (Fig. 16), the coding of the pulses is achieved by diode biasing the amplifier outputs so that NaI 1 only has a negative pulse and NaI 2 has only a positive pulse while the pulse from the liquid scintillator has both positive and negative excursions. The camera control circuit activates the pre-set stop circuit which automatically cuts off the camera power after a pre-set number of photographs have been taken. This prevents the camera from running out of film and, because of a peculiarity of its design, overheating.

Since the two NaI crystal detectors had different intrinsic gains, it was necessary to operate them at two different high-voltages, a condition which was met by using one high-voltage supply and a distribution circuit. This circuit was mounted on top of the lead

house along with a meter which monitored the high voltage. A glance at the monitor sufficed to tell if the high voltage were turned on or off before working inside the lead shield where exposed high voltage lines existed.

The AC power in the electronics racks was divided into an isolated filtered line and an unfiltered line. All of the electrically noisy gear were operated off the unfiltered line, e.g. camera, preset stop circuit, and rack blowers. All other electronic circuits used the isolated filtered power. Appendix IV lists the electronic circuits used and Appendix V contains details of all custom designed circuits.

The data in the form of oscilloscope traces were photographed using 100-foot rolls of 35 mm Kodak Royal X Pan Recording film.¹¹ Each roll of film contained 1725 pictures which represented an accumulation of about 65 hours of reactor-up data. The film¹² developed in Kodak DK 60a developer for 10 minutes at 68° F with constant agitation gave good but not excellent results. The quality of the photographs was limited because the 3 kilovolt CRT of the oscilloscope could not produce a well-focused intense trace at a writing speed of 20 microseconds per cm. The ultra-fast film required was very grainy and had a noticeable fog density. The film was scanned and analyzed on a Recordak film reader.

III BACKGROUND OBSERVED BY DETECTORS

The background in the experimental area was measured with one of the NaI detectors without shielding with the reactor at full power. This background spectrum, given in Figure 17 (spectrum a), resulted primarily from the reaction $O^{16}(n,p)N^{16}$ in the reactor cooling water¹³ which produces 6.2 and 7.1 Mev gamma rays with a branching ratio of 15 to 1. To reduce this background, the lead shield was built. The background measured within this shield is given in Figure 17 by spectrum b and spectrum d for the reactor at full-power and zero-power respectively. The full-power spectrum measured within the shield was greater than calculated from the unshielded crystal spectrum and the lead house thickness. This suggested that there were neutrons in the area which being relatively unaffected by the lead shield captured within the shield producing gamma rays.

The addition of the paraffin-borax neutron shield, which absorbed the neutrons by non-radiative capture, reduced the spectrum within the lead house to spectrum c in Figure 17. The reduction in the full-power spectrum verified the presence of neutrons. The remaining difference between the reactor-up and the reactor-down spectra suggests that a better and more complete shield could be built. Also, since the reactor-off spectrum was decreased by

inserting the neutron shield, we conclude that cosmic ray neutrons are an additional source of background.

The background below 3.5 Mev within the shield is produced primarily by the natural radioactivity of the detectors and shield. The peaks in the background spectrum result from the K^{40} in the photomultiplier tube glass and socket, the thorium series (ThC , ThC'') and the uranium series (RaC).¹⁴ These peaks and their sources are listed in Table II.

The background observed by the liquid scintillation detector within the lead and paraffin-borax shield with the reactor at full power is given in Figure 18. A tabulation of the integral spectra for the NaI detector within the complete shield with the reactor up is given in Table III. The background rates within the shield for the various detectors is given in Table IV.

IV DETECTOR PROPERTIES

Monoenergetic gamma rays or electrons when totally absorbed by a scintillation detector produce signals whose pulse height spectrum approaches a Gaussian shape. The peak location and the resolution, measured as the full width at half maximum (FWHM) divided by the peak location, is in general a function of the position and magnitude of the energy deposited within the detector. In the following two sections, these variations will be investigated for gamma rays absorbed in the NaI crystals and electrons absorbed in the organic detector.

A. NaI Crystal Properties

Since the target in this experiment is located between the NaI crystals and the gamma rays are produced within the target, it is only necessary because of cylindrical symmetry to investigate the variations of peak location and resolution resulting from a gamma source placed between the crystals at various radial positions from the crystal axis. The prime function of the NaI crystals is to detect annihilation gamma rays with a good energy resolution which allows a very narrow window discriminator to be used and hence discriminates against backgrounds not due to positrons. To determine the minimum useful width of the window discriminator, the

variations in photo-peak position and resolution were measured for a Cu^{64} positron source. This source was placed at various radial positions, the spectra measured and the variations determined.

Table V shows the position of the photo peak and its FWHM for both crystals for several radial positions of the source. Since the peak position and the FWHM were measured on a pulse height analyzer, the uncertainty in determining these values is one analyzer channel. It is seen that no variations were observed within statistical fluctuations. The resolutions for 0.51 Mev gamma rays absorbed by NaI 1 and NaI 2 are respectively $9.3 \pm 0.3 \%$ and $10.4 \pm 0.3 \%$.

The energy calibration of the crystals using the photo peaks of Cu^{64} (0.51 Mev), Cs^{137} (0.662 Mev) and Y^{88} (0.90 Mev, 1.85 Mev and their sum of 2.76 Mev) showed that the crystal outputs were proportional to the energy deposited in the crystals.

B. Properties of the Organic Detector

The resolution function and energy calibration of the liquid scintillation detector were obtained so the true positron spectrum could be deduced from the observed positron spectrum. The energy dependence of the resolution function (as determined by the dependence of the FWHM resolution for the electron absorption peak on the electron energy) was obtained from pulse height analysis of the detector response to an electron source. The source used was Bi^{207} which electron captures to excited states of Pb^{207} . The decay scheme¹⁵ is shown in Figure 19. The 0.569 Mev and 1.064 Mev gamma rays from the excited levels of Pb^{207} can produce K conversion electrons of

0.477 Mev and 0.972 Mev in cascade with 1.064 Mev and 0.569 Mev gamma rays respectively. The 0.477 Mev or the 0.972 Mev conversion line spectra were selectively obtained by immersing the Bi^{207} source into the liquid scintillator and by requiring a coincidence between the liquid scintillator and one of the NaI crystals, where the crystal was set to see energy deposits of 1.064 Mev or 0.569 Mev respectively. The discriminator on the crystal output supplied the coincidence gate pulse for the pulse height analyzer which analyzed the liquid scintillator output pulses.

Since the K conversion coefficients are 0.113 and 0.018 for the 1.064 Mev and 0.569 Mev gamma rays respectively, it was possible for the unconverted gamma rays to interact in the organic detector and hence produce false electron signals whose coincidence with the cascade gamma ray seen by the NaI would distort the line spectrum accumulated on the pulse height analyzer. To compensate for this effect, an equal live time background subtraction was made with an 80 mil thick high Z shield covering the source. Since this shield was thicker than the 3.54 Mev electron range, it absorbed the electrons leaving only the unconverted gamma rays to produce the coincidence signal thus subtracting off the false electron signals. Using this procedure, the organic detector's response to the two electron lines were found to be approximately Gaussian in shape.

The electron source was constructed by placing the Bi^{207} between two pieces of mylar tape which were then thermally fused together. This produced a source which was optically transparent and which degraded the electrons by only 0.005 Mev. The K conversion

electrons detected by the scintillator were therefore degraded to 0.472 Mev and 0.967 Mev. In measuring the FWHM, it was necessary to use twice the lower half width rather than the actual FWHM because of the distortion produced by the L conversion electrons of 0.554 Mev and 1.049 Mev.

The properties of these electron spectra observed by the liquid scintillator are given in Table VI for several source positions. The peak position and FWHM measured for several radial positions were weighed by their corresponding detector volumes and a composite absorption spectrum was constructed for each source. The composite spectra, which approximate the detector response to the electron sources averaged over the detector volume, are approximately Gaussian in shape and have resolutions of $43.3 \pm 3.6\%$ and $31.3 \pm 1.6\%$ respectively for the 0.472 Mev and 0.967 Mev electrons. There is a correction to the resolution measurements because they were made with a liquid scintillation detector amplifier gain of X16 while the experiment was performed with an amplifier gain of X8. By observing on the pulse height analyzer the pulser spread produced by the electronic noise at amplifier gains of X8 and X16, it was found that the FWHM spread of the pulser spectrum which was less than one channel at X8 was increased to 3.0 ± 0.1 channels at X16 within the range of 0.50 Mev to 4.0 Mev. Making this correction on the FWHM gives $33.4 \pm 3.4\%$ and $25.4 \pm 1.6\%$ for the scintillator resolutions at 0.472 Mev and 0.967 Mev respectively. These resolutions are consistent with those expected from photon statistics.¹⁶ Since the resolution of the liquid scintillator is limited by photon statistics,

it can be written as proportional to $E^{-1/2}$ or the full width, W , which is the FWHM in Mev for an electron of kinetic energy E can be written as $W = K\sqrt{E}$ where K is a constant. Using the above data we find $K = 0.246 \pm 0.016$.

The energy calibration of the liquid scintillation detector was made using the Compton recoil peak produced by the gamma rays from Cs^{137} , the two K conversion lines from Bi^{207} and the end point (3.54 Mev) of the K^{42} electron spectrum. The Compton recoil peak from Cs^{137} has an energy of 0.43 Mev and a pulser setting of $0.47 \pm 0.01 \times 1/5$ matched the peak location. The Bi^{207} K conversion peaks as measured above correspond to pulser settings of $(0.55 \pm 0.01) \times 1/5$ and $(1.15 \pm 0.02) \times 1/5$ for the 0.472 Mev and 0.967 Mev lines respectively.

The K^{42} source was made identical to the Bi^{207} source except for the use of 2.0 mg of separated K^{41} .¹⁷ The mylar clad K^{41} sample was irradiated¹⁸ in the thermal beam at one of the Savannah River Plant test reactors to produce an activity an activity of ~ 0.1 μ curie. The source was placed inside the liquid scintillation detector and its spectrum¹⁹ was measured at a radial position of 3.0 inches. The detector response was found to be nearly independent of the source position, presumably because of the longer electron ranges. The pulser value for the electron spectrum end point was obtained by using a Kurie plot of the form²⁰

$$\left[\frac{N(W)}{\alpha F(-Z, W) W \sqrt{W^2 - 1}} \right]^{1/2} = W - W_0$$

where W , N , and $F(-Z, W)$ are respectively the electron energy in mc^2

units, the number of electrons observed in the interval dW at W and the coulomb factor. The shape factor α of the form

$$\alpha = L_0(W_0 - W)^2 + 9L_1$$

where L_0 and L_1 have been tabulated.²¹ Using the electron spectrum after taking into account the energy resolution of the system, the Kurie plot gave an end point of $(4.18 \pm 0.05) \times 1/5$ pulser units.

The linearity of the system is graphically represented in Figure 20. The precision with which the 3.54 Mev calibration fits the energy calibration curve verifies the correctness of the experimentally obtained full-width resolution function from 2.0 to 3.5 Mev.

V OPERATIONAL CALIBRATIONS OF THE DETECTOR

Twenty-five hundred hours of data were collected for this experiment during a period of seven months. During this period it was found to be necessary to "tune" the system and calibrate the detectors every 60 hours. The "tuning" consisted of adjusting the position and width of the NaI window discriminators and the position of the integral discriminator level for the liquid scintillation system.

Because of temperature dependent drifts in the NaI window discriminator circuits, the windows were set from 1/10 to 1/10 of the amplitude of the Gaussian photo-peak produced by a 0.51 Mev gamma ray absorbed by the crystals. With the windows set this wide, 97.5% of the annihilation gamma rays absorbed by one crystal fell within the window and the window drifts in 60 hours produced a variation in this amount of less than $\pm 1.2\%$. The detectors were calibrated by placing a Cs^{137} source on the crystal axis between NaI 1 and the organic detector (Fig. 1). A precision pulser was used to set the NaI window discriminators. The source in this position produced in the liquid scintillator a spectrum whose peak fell at 0.43 Mev as calculated from the energy of the internal conversion line of a Cs^{137} source immersed in the liquid scintillator. This 0.43 Mev peak was used as the secondary standard to calibrate the liquid

scintillator every 60 hours because (1) it did not have to be immersed into the liquid scintillator hence reducing the possibility of contamination and (2) the organic detector and the NaI detectors can all be calibrated with the source in only one position. Using this 0.43 Mev calibration, the base line of the liquid scintillator integral discriminator was set to 0.35 Mev.

The data were collected by photographing oscilloscope traces (Fig. 2) when the signals from the three detectors triggered the discriminators and formed a prompt triple coincidence. Because of the wide NaI discriminator windows, the rate at which photographs were taken was 28 per hour of which 99% were background. With this rate of film consumption, a 100-foot roll of film containing 1750 photographs was used every 60 hours. At the beginning and end of each roll of film, the system was calibrated and "tuned" and energy calibration pulses were placed on the film. A time calibration consisting of pulses from a 100 KCPS crystal oscillator was also put on the film.

As a check on the window discriminator positions and the organic detector integral discriminator position, the individual gate rates resulting from the natural background were measured and compared with previously measured rates. The double coincidence rates between the two NaI crystals and the NaI crystal and organic detector were likewise measured and checked for consistency.

The stability of the liquid scintillation detector system was determined by measuring the stability of the μ -meson thru peak produced in the detector. A separate monitoring amplifier (Fig. 15)

was used to amplify the pulses and feed the pulse height analyzer.

This additional amplifier was needed because the normal amplifier had a dynamic range of 0 to 10 Mev while the thru peak had an energy of ~ 13 Mev. By using the two amplifiers on the liquid scintillation system, the stability test could be conducted concurrently with the collection of data. The thru peak spectrum was accumulated continuously for seven days with a destructive printing of the spectrum from the pulse height analyzer every 12 hours. A seven-day stability test was conducted once a month and the overall gain of the liquid scintillation detector system was found to be stable to about 2% over a seven-day period.

VI DEMONSTRATION THAT SIGNAL WAS INDUCED BY REACTOR ANTINEUTRINOS

The data collected consisted of oscilloscope traces as shown in Figure 2. The class of events containing the signal consisted of a prompt triple coincidence plus a delayed pulse whose time delay could be from 2.8 to 100 μ sec. was divided into two time slots of 50 μ sec width each. Because of the Gadolinium concentration in the liquid scintillator, the delayed pulses produced by the neutrons capturing were predominantly (99%) contained in the first time slot. The accidental delayed signals predominated in the second time slot. The correlated signal rate is the rate in the first time slot minus the rate in the second time slot. The rate of the reactor-associated signal (RA signal) is then the difference between the reactor-up and reactor-down correlated rates.

To demonstrate that the RA signal was produced by reactor antineutrinos undergoing reaction (1), it was verified that the signal is not produced by the background; i.e.,

1. the RA signal is correlated and hence is not due to the increase in the accidental background;
 2. the RA signal is not due to neutrons;
- and the RA signal has the characteristics expected from reaction (1), i.e.,
3. the second pulse results from neutron capture in the

organic detector;

4. the first pulse is consistent with positrons.

We now discuss these tests in detail.

A. Reactor-Associated Signal Not Produced by the Background

Gamma rays can mockup the signal by having one gamma produce the prompt triple coincidence and an accidental gamma produce the delayed pulse. However, the capture time distribution for these accidental signals is flat indicating a lack of correlation.

Reactor neutrons can produce a correlated reactor-associated signal. A neutron can mockup reaction (1) by scattering in one or more of the detectors producing part of the prompt triple coincidence and then thermalizing and being captured by the Gadolinium. Such a neutron must be more energetic than ~ 1.0 Mev.²² This is satisfied by the neutrons produced from U^{235} fission because they have an energy spectrum proportional to $e^{-E} \sinh(2E)^{1/2}$ between 0.075 and 15 Mev and a mean energy of 1.75 Mev.²³ If we consider the case where the neutron produces the liquid scintillation detector signal and the gamma ray background triggers the two NaI detectors, then the prompt triple coincidence rate can be calculated. The accidental coincidence rate between the NaI crystals is 1/min. If we attribute 50% of the reactor-associated liquid scintillation detector rate to neutron scatterings (an extremely liberal estimate) the neutron induced liquid scintillation detector rate is 150/min. Using a coincidence resolution time of 2.0 μ sec the prompt triple coincidence rate is 6×10^{-4} /hr. Since the neutron capture probability is much less than

100%, this background would account for less than 1/4% of the observed RA signal. Similar calculations show that the only conceivable non-negligible background is that produced by a neutron which provides all three of the prompt triple coincidence signals and then captures in the detector.

To test the efficiency for a neutron producing a prompt triple coincidence and then capturing, a Pu/Be neutron source was placed near the detectors and the increase in RA signal was investigated. The decay schemes of the constituents of the neutron source²⁴ are given in Figure 21. The neutrons produced have an energy spectrum from 0 to 12 Mev with a mean energy of 4.2 Mev. It is important for this test that the source neutrons are more likely to produce a prompt triple coincidence than are reactor neutrons because the source neutrons have a higher mean energy and 60% of the source neutrons have coincident gamma rays.

When the reactor was off, the source was placed inside the lead house centrally located on the back of the door and in the plane of the detector. A summary of the detector rates is given in Table IV. Three thousand photographs were collected in 31.89 hours. The associated source rate, i.e., the rate captured in time slot No. 1, minus the rate in time slot No. 2 is 0.69 ± 0.20 per hour. If one assumes that as an upper limit the reactor-associated triple coincidence rate without source is entirely due to a reactor neutron background and that the efficiency for observing the neutron after it produces the triple coincidence is the same for the source and the reactor neutrons, then the RA signal rate due to reactor neutrons

NRA, can be written as

$$\text{NRA signal} < \frac{(\text{source associated signal})(\text{RA triple coincidence rate})}{(\text{source associated triple coincidence rate})}$$

The source-associated signal rate is $(0.69 \pm 0.20/\text{hr.}) - (0.035 \pm 0.02/\text{hr.})$

$= 0.66 \pm 0.20/\text{hr.}$ where $0.035 \pm 0.02/\text{hr.}$ is the reactor-down delayed

coincidence rate. The NRA signal rate is then $< 0.153 \pm 0.047/\text{hr.}$

However, approximately 90% of the reactor-up triple coincidence rate is due to non-neutron induced mechanisms (Sec. IX-A) such as pair production in the liquid scintillation detector and feed-through electrons triggering both the NaI and the liquid scintillation detectors. This reduces the NRA signal rate to $0.015 \pm 0.005/\text{hr.}$

which is only 6% of the observed RA signal rate. The NRA signal rate is seen to be even less important when the relative efficiencies for a reactor neutron and a source neutron to produce the triple coincidence are considered. The conclusion is that the RA signal, except for a few per cent at most, is not produced by the reactor-associated background.

B. Reactor-Associated Signal Shows Characteristics Expected from the Reaction

If the RA signal is produced by reaction (1), then the organic detector pulse in the prompt triple coincidence should be consistent with the expected positron spectrum and the capture-time spectrum for the second pulse should be consistent with the mean-neutron capture time of the liquid scintillation detector.

To obtain a neutron capture-time spectrum for the liquid

scintillation detector, a Pu/Be source (Fig. 21) was used. The source was placed within the organic detector on the axis and midway between the crystals. The neutron capture times were measured by triggering a 100 μ sec oscilloscope sweep each time one of the NaI crystals absorbed a 4.43 Mev gamma ray which announced the birth of 75% of the source neutrons. By calibrating the oscilloscope sweep with a crystal oscillator, the capture times are determined from the sweep distance to the delayed pulses from the detectors. The result of the analysis of 10,200 oscilloscope traces is the capture-time spectrum shown in Figure 22. This distribution has the characteristic shape expected from neutrons capturing in the Gadolinium loaded scintillator.

The capture-time spectrum obtained from 2500 hours of collecting data is presented as a histogram in Figure 22. The RA capture-time spectrum corrected for background has a slope which is proportional to the mean-capture time in the scintillator and is consistent with the slope of the neutron source spectrum. Also shown in Figure 22 is the capture-time spectrum calculated by a Monte Carlo method (Appendix VI) for 20 Kev neutrons produced uniformly in the target. The mean-capture times from all three spectra are consistent. It is therefore concluded that the second pulse of the RA signal is neutron produced.

To associate the first pulse of the RA signal with the positrons expected from reaction (1), the following support is given. First, it has been shown that the background could not produce the RA signal observed. Second, the energy spectrum of the organic detector

pulses which should be the positron kinetic energy spectrum is not consistent with the organic detector background spectrum as shown in Figure 23. Third, the organic detector spectrum is not inconsistent with the shape expected from the known dependence on energy for the interaction cross section and a monotonic decreasing fission antineutrino spectrum (Sec. VII-B).

VII DETECTION EFFICIENCY AS A FUNCTION OF ENERGY

A. Variation of Cross Section with Energy

Neglecting the small anisotropy, the cross section can be written²⁵

$$\sigma(E) = (1.17 \pm 0.02) \times 10^{-44} \left\{ \frac{E - 1.29}{0.26} \right\} [(E - 1.29)^2 - 0.26]^{\frac{1}{2}} \text{cm}^2,$$

where E is in Mev units. This cross section which is valid for $E < 15$ Mev is derived from the "Second Golden Rule" where the energy independent matrix element, assuming a two-component neutrino theory, is evaluated from the neutron half life²⁶ of 17.7 ± 0.3 min and the energy dependence is determined by phase-space arguments.

The best expression for the incident $\bar{\nu}_e$ energy spectrum was determined by least-square fitting the data of Carter, et al²⁷ for the $\bar{\nu}_e$ spectrum calculated from the beta spectrum from U^{235} fission for a nuclear charge of 45. The expression obtained for the number of $\bar{\nu}_e$ per fission per Mev which is valid for $1.0 \text{ Mev} \leq E < 8.5 \text{ Mev}$ is

$$f(E) = 5.01 \exp (-0.505 E - 0.0544 E^2) .$$

B. $\bar{\eta}$ - Average Detection Efficiency as a Function of Energy

To calculate the efficiency for detecting reaction (1) events, consider the target divided into thin slabs parallel to the faces

of the organic detector and at perpendicular distances x_i from the center of the detector. The average detection efficiency can then be written as $\bar{\eta}(E) = \sum_i \eta_{\beta^+}(x_i, E) \eta_n(x_i)$ where

$\eta_{\beta^+}(x_i, E)$ = probability of detecting a positron of energy E which is produced in slab x_i

$\eta_n(x_i)$ = probability of detecting a neutron which is produced in slab x_i

1. Positron Detection Efficiency

The positron detection efficiency is the probability that the detectors will observe a prompt triple coincidence when a positron is produced in the target and annihilates. Since the triple coincidence requires energy deposits of at least 0.50 Mev in the organic detector and 0.51 Mev in each of the NaI crystals, the efficiency can be written

$$\eta_{\beta^+}(x_i, E) = P_1(x_i, E) P_2 P_3 + P_4(x_i, E) P_5(x_i, E) P_3$$

where

$P_1(x_i, E)$ = probability that the positron will slow down and annihilate within the organic detector

P_2 = probability that the kinetic energy of the positron which annihilates in the organic detector is in excess of 0.50 Mev

P_3 = probability that both annihilation gamma rays will escape from the organic detector without interacting and be totally absorbed by the NaI crystals

$P_4(x_i, E)$ = probability that the positrons which escape from the organic detector annihilate in the inert material

between the target and the NaI crystals

$P_5(x_1, E)$ = probability that the electrons deposit more than 0.50 Mev in the organic detector before escaping.

The first term in the efficiency equation is due to the positrons annihilating within the organic detector. The second term is necessary because the positrons which escape from the target can annihilate in the inert material between the detectors and produce a proper triple coincidence. However, the positron which escapes with sufficient energy to penetrate the inert material and deposits its remaining kinetic energy in a crystal before annihilating, cannot be detected as a triple coincidence because the energy deposited in one NaI crystal will be in excess of the 0.51 Mev required. Each of the above probabilities will now be determined.

a. $P_1(x_1, E)$ Determination

For a given positron energy E (Mev) the effective positron range, $r(\text{gm/cm}^2)$, is given²⁸ by

$$\begin{aligned} r(\text{gm/cm}^2) &= 0.412 E(1.265 - 0.094 \ln E) && \text{for } E \leq 2.5 \text{ Mev} \\ &= 0.530 E - 0.106 && \text{for } E > 2.5 \text{ Mev.} \end{aligned}$$

Using a density of 0.88 gm/cm^3 for the liquid scintillator, the positron ranges can be determined in centimeters. If a positron of range r is produced at random in the slab x_1 and the positrons are produced with an isotropic angular distribution in the laboratory system, then the probability that the positron will be contained within the target is approximately the fractional area of the sphere

of radius r which lies inside the target. For the organic detector of radius $Q = 13.0$ cm and thickness 7.0 cm, one obtains, including edge effects

$$P_1(x_i, E) = 0.75 + 0.25(1 - r/Q) \quad \text{for } x_i \leq 3.50 - r$$

$$= \left\{ 1 + \frac{3.5 - x_i}{r} \right\} \left[0.375 + 0.125(1 - r/Q)^2 \right] \quad \text{for } x_i > 3.5 - r.$$

b. P_2 Determination

The probability that the kinetic energy of the positron which annihilates within the organic detector is in excess of 0.50 Mev is determined from the predicted positron spectrum. Using the theoretical cross section for reaction (1) and the best available $\bar{\nu}_e$ flux spectrum expected from U^{235} fission obtained by earlier workers,²⁷ the predicted positron spectrum, $N(E)$, except for normalization is given by

$$N(E) = (E + 0.51) \left[(E + 0.51)^2 - 0.26 \right]^{1/2} \exp \left\{ -0.505(E + 1.80) - 0.0544(E + 1.80)^2 \right\}$$

where E is now the positron kinetic energy in Mev. The fraction of the total area of the spectrum below 0.50 Mev is 3.6% ; therefore,

$$P_2 = 0.96.$$

c. P_3 Determination

The probability that both annihilation gamma rays will not interact in the organic detector but will be completely absorbed by the NaI crystals was calculated and measured. The model used in calculating P_3 is shown in Figure 24 where N_1 and N_2 are the maximum

path lengths available in the crystals and R_1 and R_2 are the path lengths in the organic detector for γ_1 and γ_2 traveling in the direction shown. The expression for P_3 is the product of the solid angle factors for the γ rays to strike the crystals, the probability that neither γ will interact in the distances R_1 and R_2 and the probability of total absorption in N_1 and N_2 ; i.e.,

$$P_3 = \frac{1}{4\pi} \int \frac{\bar{r} \cdot \hat{n}}{r^3} da \left(\exp\left(-\frac{R_1}{\lambda_1} - \frac{R_2}{\lambda_1}\right) \right) \left(1 - \exp\left(-\frac{N_1}{\lambda_2}\right) \right) \left(1 - \exp\left(-\frac{N_2}{\lambda_2}\right) \right)$$

where $\bar{r}$ is the flight vector from the annihilation site to the unit of area da on the surface with unit normal $\hat{n}$ of the NaI crystal, λ_1 is the attenuation mean-free path in the liquid scintillator and λ_2 is the absorption mean-free path in NaI for 0.51 Mev gamma rays.

Using an isotropic angular distribution of annihilation gamma rays P_3 was calculated as a function of y and found to be $P_3 = 0.20 \pm 0.02$ for $0 \leq y < 13.0$ cm.

An experimental determination of P_3 was made using a calibrated ^{64}Cu positron source. The source sealed between two pieces of mylar tape was placed within the organic detector and the coincidence rate was measured between the two NaI crystals with the NaI window discriminators tightly set on the annihilation peaks. The source position was varied radially and parallel to the detector axis of symmetry. The coincidence rate normalized to the source strength gave $P_3 = 0.24 \pm 0.02$ independently of the source position within the organic detector.

d. $P_4(x_i, E)$ Determination

The inert material between the organic detector and the NaI crystals includes the lucite cover of the target, the stainless steel housing of the NaI crystals and the MgO packing around the NaI crystals. The total thickness of inert material was 0.944 gm/cm^2 . Using the positron range formula (Sec. VII-B.1,a), this corresponds to an energy loss of 1.80 Mev or a distance traveled in the organic detector of 0.97 cm. $P_4(x_i, E)$ can now be written as

$$P_4(x_i, E) = P_4^a P_4^b$$

where P_4^a = probability that the positron escapes through the top or bottom of the target

P_4^b = probability that the positron which escapes from the target will not penetrate the inert material.

P_4^a includes only positrons which escape through the top and bottom of the target since escapes through the side result in annihilations which cannot be detected by both crystals. Using the same method and notation as before, we obtain

$$\begin{aligned} P_4^a &= 0 && \text{for } x_i \leq 3.5 - r \\ &= 0.50 \left[1 - (3.5 - x_i)/r \right] && \text{for } x_i > 3.5 - r \\ \text{and } P_4^b &= 1.0 && \text{for } E \leq 1.80 \text{ Mev} \\ &= 1.0 && \text{for } E > 1.80 \text{ Mev, } x_i \leq 4.47 - r \\ &= 1 - \frac{1 - (4.47 - x_i)/r}{1 - (3.5 - x_i)/r} && \text{for } E > 1.80 \text{ Mev, } x_i > 4.47 - r \end{aligned}$$

from which $P_4(x_i, E)$ is easily calculated.

e. $P_5(x_i, E)$ Determination

$P_5(x_i, E)$ is the probability that a positron which escapes from the target before annihilating deposited at least 0.50 Mev in the target. Using Figure 25 where r_1 and r are the ranges for 0.50 Mev and E energy electrons respectively, it is apparent that

$$P_5(x_i, E) = 1 - \frac{1 - (3.5 - x_i)/r_1}{1 - (3.5 - x_i)/r} \quad \text{for } x_i > 3.5 - r_1$$

$$= 1.0 \quad \text{for } x_i \leq 3.5 - r_1.$$

2. Neutron Detection Efficiency

The neutron detection efficiency, $\eta_n(x_i)$ is the probability of a neutron created in slab x_i to thermalize and capture within the detector system. The requirements imposed on the neutron capture signal are that the capture time t_c must be between $2.8 \leq t_c < 50$ μsec after the prompt triple coincidence signal and that the sum of the energy deposits in the three detectors for the neutron-capture signal must exceed 0.75 Mev.

This efficiency could in principle be determined by using a 20 Kev neutron source which had an associated gamma ray. This source could be placed within the detector and by triggering on the associated gamma ray, one could observe the number of neutrons captured, the capture-time spectrum and the capture gamma ray energy spectrum. However, a suitable low energy neutron source could not be found. The most promising of the available sources was a $\text{Sb}^{124} - \text{Be}$ photoneutron source²⁹ which produced 20 Kev neutrons in coincidence

with 0.603 Mev gamma rays. Because of the low photoneutron cross section, however, there would be only one photoneutron produced for every 3.3×10^4 gamma rays passing through the Be converter. Such a low neutron yield per gamma ray would produce an accidental background to neutron capture signal ratio of about 10/1 which would make a 10% determination of $\eta_n(x_i)$ a formidable task.

Since $\eta_n(x_i)$ could not be measured experimentally, it was calculated using

$$\eta_n(x_i) = P_6(x_i) \cdot P_7 \cdot P_8$$

where

$P_6(x_i)$ = probability that the neutrons produced in slab x_i will be captured within the detection system

P_7 = probability that the neutron will capture within the time gate of $2.8 \mu\text{sec} \leq t_c < 50 \mu\text{sec}$

P_8 = probability that the capture gamma rays will produce a total energy deposited in the three detectors in excess of 0.75 Mev.

a. $P_6(x_i)$ Determination

By making a detailed Monte Carlo calculation $P_6(x_i)$ was determined for neutrons produced uniformly throughout the volume of the target. In the calculation, neutrons were followed from their birth position in slab x_i through successive scatterings until they either were captured in the detection system or escaped from it.

The neutrons were allowed to scatter between any of the three detectors and they could capture in any of the detectors. The details of this calculation are given in Appendix VI. The calculated values of $P_6(x_i)$, presented in Table VII, were found to be independent of the neutron energy in the range expected from reaction (1), i.e., between 2.0 Kev and 20 Kev.

b. P_7 Determination

In the Monte Carlo calculation (Appendix VI) the capture-time spectrum was obtained for neutrons born in slab x_i and capturing in the system. The shape of the capture-time spectrum was independent of x_i and was in good agreement with the spectrum obtained in the experiment. The probability that the capturing neutron will be in the time gate of $2.8 \mu\text{sec} \leq t_c \leq 50 \mu\text{sec}$ obtained from the calculated spectrum, Figure 22, is $P_7 = 0.62 \pm 0.01$.

c. P_8 Determination

When a neutron is captured by Gadolinium, it produces on the average four gamma rays with mean energies of ~ 2 Mev.³⁰ The probability of one of these gamma rays striking a NaI crystal is ~ 0.65 averaged over the target volume. The probability of the gamma ray interacting in the detectors and depositing more than 0.75 Mev is ~ 0.85 . If only one gamma ray were given off in the radiative capture, then $P_8 \approx 0.55$. However, since four gamma rays, on the average, are emitted, and we require that they make a total energy deposit in excess of 0.50 Mev, P_8 is on the order of $P_8 \approx 0.95$.

for uncorrelated gamma rays. Since complete correlation would give $P_8 = 0.65$, it seems reasonable to favor the higher figure. In an absolute determination of efficiency, it would be necessary to look further into this question.

3. Evaluation of the Average Detection Efficiency

Collecting the numerical results from the previous sections:

$$\begin{aligned} \eta(x_i, E) = \eta_{\beta^+}(x_i, E) \eta_n(x_i) = P_1(x_i, E) P_6(x_i) (0.13 \pm 0.01) \\ + P_4(x_i, E) P_5(x_i, E) P_6(x_i) (0.14 \pm 0.01). \end{aligned}$$

To determine $\bar{\eta}$ it was necessary to evaluate $\eta(x_i, E)$ for each x_i slab from 0.175 cm to 3.325 cm in steps of 0.35 cm for each positron energy from 0.75 Mev to 8.25 Mev. A sample of the results for $E = 2.75$ Mev is presented in Table VIII. The $\eta(x_i, E)$ values were summed over the x_i slabs for fixed values of E . The energy dependence of the efficiency is shown in Figure 26.

VIII REDUCTION OF DATA

In the 2500 hours of reactor-up data, 928 events satisfied the selection criterion for reaction (1). These events were separated into two classes depending on whether their delayed pulses occurred in time slot 1 or 2. The liquid scintillation detector first pulse spectra associated with these time slots are given in Figure 27. The correlated positron spectrum is the difference between these two spectra and is given in Figure 28.

The corresponding reactor-down (357 hours) spectra and the capture-time spectrum are given in Figure 29. The correlated reactor-down spectrum represents $< 10\%$ of the reactor-up signal rate and is, in consequence, poorly determined. We therefore neglect this correction to the positron spectrum except to note that the meager data are grouped at the lower energies.

By assuming a two-parameter expression for the "true" positron spectrum (Appendix VII), the unfolded spectrum was determined by minimizing the sum of the squared residuals between the observed positron spectrum and the resolution modified "true" positron spectrum. The unfolded positron spectrum so obtained is also presented in Figure 28.

The least square fitting to the observed positron spectrum (Appendix VII) was performed by using the "true spectrum" as the

product of the cross section which is treated as a known function of the energy, and an expression for the $\bar{\nu}_e$ flux which contains two parameters. The unnormalized $\bar{\nu}_e$ spectrum obtained from this best fit of the data for the two parameters is

$$e^{-1.325 E + 0.0396 E^2}$$

The $\bar{\nu}_e$ spectrum for U^{235} fission previously determined from the fission beta spectrum (Sec. VIII-B) is given by

$$f(E) = 5.01 e^{-0.505E - 0.50544 E^2}.$$

A comparison of these two spectra (Fig. 30) indicates a higher energy tail than previously inferred from direct measurement of the beta particles from U^{235} fission.

The consequences of this higher energy tail relate to both further reactor antineutrino experiments and the fission process itself. In particular, though it is not a large fraction of the $\bar{\nu}_e$ spectrum from fission this tail is of importance to the reaction $\bar{\nu}_e + d$ because of its high threshold (4.0 Mev). Further, the energies under discussion (~ 10 Mev) represent nuclides of such short half lives that the conventional methods of chemical separation and beta analysis are not sufficiently sensitive. Further study of these interesting questions is contemplated.

IX SYSTEM IMPROVEMENTS

A. Background Analysis

The primary improvement that can be made in this experiment is to reduce the prompt triple coincidence background. This background, which is $\sim 25/\text{hr}^*$, cannot be produced by accidental triple coincidences or the combination of a double Compton scattering of a gamma ray in two detectors and an accidental in the third detector. Two mechanisms which can produce this background are (1) pair production in the liquid scintillation detector and (2) electrons which are driven between adjacent detectors. Each of these mechanisms will now be dismissed.

1. Pair Production Background

The prompt triple coincidence background can be produced by high energy gamma rays which produce pairs in the liquid scintillation detector. The pairs slow down producing the liquid scintillation pulse while the NaI crystals are triggered by the positron annihilation gamma rays. The incident gamma ray must have an energy greater than 1.5 Mev because of the selection criterion imposed on the prompt coincidence pulses. The gamma flux above 1.5 Mev incident on the liquid scintillation detector

*This rate is for the system with the Gd loading. A rate of 18/hr, measured in the absence of Gd, is relevant to the situation in which the neutron capture is uninteresting.

was determined from the spectrum observed by the NaI detector scaled by their respective surface areas, to be 320/min. The rate of pairs produced in the liquid scintillation detector can be written as

$$N_p = \sum_i F_\gamma(\Delta E_i) \sigma_p(\Delta E_i) D_c V$$

where $F_\gamma(\Delta E_i)$ is the incident gamma ray flux in the interval ΔE_i at E_i , $\sigma_p(\Delta E_i)$ is the carbon pair cross section,³¹ D_c is the detector carbon density and V is the detector volume.

Using the incident gamma spectral shape determined by the NaI crystals gives $N_p = 28/\text{hr}$. The probability that the annihilation gamma rays will escape the liquid scintillation detector without interacting and then be totally absorbed by the NaI detectors (Sec. VII-D) is 24%. Therefore, the triple coincidence background rate resulting from pair production is $\sim 6.5/\text{hr}$.

The pair production background can be reduced by lowering the number of gamma rays above 1.5 Mev incident on the liquid scintillation detector. This could be accomplished in two ways. First, the neutron shield should be located exterior to the lead shield so that the latter will attenuate the neutron-capture gamma rays. Second, a more effective lead shield can be constructed. Because of space limitations, the side walls of the lead shield were restricted to a thickness of 12 inches. A more effective gamma shield could be obtained by using for the side

walls Kennertium W-2,³² a tungsten alloy, which is 60% more effective than lead as a gamma shield.

2. Feed-Through Electrons

The prompt triple coincidence background in this case is produced by a gamma ray which Compton scatters in two detectors, driving one of the electrons into the third detector. This mechanism is schematically represented in Figure 31. The electrons in passing through the inert matter between the detectors will lose 1.80 Mev. The driven electron must possess a minimum initial energy of 2.80 Mev. The gamma threshold for producing the driven electron and the coincident second scattering is ~ 4.2 Mev. The gamma ray threshold for producing the various events of Figure 31 were determined from the relationship connecting the gamma energy and the average kinetic energy of the Compton electron.³³

Using the NaI background spectrum as the flux on the detectors, the rate of driven events was calculated from the product of gamma flux, Compton cross section and total number of targets. The total background rate due to the four possible types of events is on the order of 19/hr, which accounts for approximately 75% of the observed triple coincidence background when the liquid scintillation detector contains Gd. This background could be eliminated by inserting thin plastic scintillator sheets between the detectors which would anticoincidence the driven electron-

produced events.

B. Virtues of Reducing the Prompt Triple Coincidence Background

If the triple coincidence background can be sufficiently reduced, many advantages can be realized. With a high signal-to-background ratio for the prompt pulse, a larger fraction of the photographs taken would contain delayed events resulting in simpler data handling and reduce film consumption.

If the background can be reduced sufficiently, it may be possible to distinguish inverse beta decay events by the positron signal alone. If this is the case, the energies of the NaI crystal pulses for a prompt coincidence would determine a point in the E_1, E_2 space where E_1 and E_2 are the energies of the NaI 1 and NaI 2 pulses respectively. The positron-induced reactions would produce a peak in this space. The advantage of being able to detect inverse beta decay events by observing the positron pulse alone are:

- a. A higher signal rate, since it is not reduced by the neutron detection efficiency.
- b. The detection efficiency is just the positron efficiency and hence is more easily determined.
- c. Inverse beta decay reactions can be investigated in which no neutron is produced, e.g., $\bar{\nu}_e + \text{He}^3$.

APPENDIX I OPTIMUM THICKNESS OF THE LIQUID SCINTILLATION DETECTOR

The liquid scintillator thickness has three effects on the observed prompt triple coincidence rate, TCR, for reaction (1).

1. For a fixed detector radius, the number of targets and therefore the signal rate is linearly dependent on the detector thickness.
2. The probability of containing positrons of energy E in the liquid scintillation detector and hence observing the positrons' full kinetic energy is from the geometry

$$0.75 + 0.25 \cdot \left[\frac{(R - r)^2}{R^2 t} (t - 2r) \right]$$

where R and t are the detector radius and thickness respectively.

r is the distance from the birth position to the annihilation position of the positron of energy E.

3. The probability that the two annihilation gamma rays will penetrate the liquid scintillator without interacting can be calculated by using the following approximation. Assume the detector system to be a slab of liquid scintillator of thickness t and infinite lateral extent placed between two half planes of NaI. The approximated detector is shown in Fig. A-1. This approximation is good to about $\pm 10\%$ for the central point of the detector used in the experiment. The optimum thickness determined from this model will be a good approximation to the optimum thickness of the actual detector for non-central positions

because the probability of the annihilation gamma rays escaping from the target without interacting is only appreciable for small θ therefore making edge effects in the actual detector small corrections.

It is seen from Fig. A-1 that the two annihilation gamma rays produced at the point P and traveling the distances ℓ_1 and ℓ_2 have the same probability of a single interaction as does one annihilation gamma ray produced at P_1 and required to travel the distance $\ell = \ell_1 + \ell_2$, and this probability is the same for a birth point P anywhere along the line ℓ .

We can write the probability of both annihilation gamma rays escaping the liquid scintillation detector for all birth positions between the NaI crystals and averaged over all solid angles as

$$\frac{1}{2\pi} \int_0^1 e^{-\ell/\lambda} 2\pi d(\cos\theta)$$

where $\ell = t/\cos\theta$ and λ is the interaction mean-free path for 0.51 Mev gamma rays in the liquid scintillator.

The prompt triple coincidence rate (TCR) can now be written as

$$TCR = Kt \left\{ 0.75 + 0.25 \frac{(R - r)^2}{R^2 t} (t - 2r) \right\} \int_0^1 e^{-t/\lambda \cos\theta} d(\cos\theta)$$

where K is a constant. Evaluating TCR as a function of t for the case of $R = 17.8$ cm, $r = 0.6$ cm (1.75 Mev positron) and $\lambda = 12$ cm gives the desired dependence of the triple coincidence rate on the detector thickness shown on Fig. A-2.

APPENDIX II LEAD HOUSE PLATFORM

To support the lead house and to provide rails for the door, a platform was designed³⁴ to hold 50 tons. The platform is built of 6WF31 beams on 21-1/4" centers grouted 6 inches deep in the concrete walls (Figs. A-3 and A-4). The beams are covered with a 1/2" thick steel plate (Fig. A-5). The door rails are 17-pound 8" x 5-1/4" beams which extend from the main platform to the wall in which they are grouted. The rails are covered with strips of 1/2" thick cold rolled steel. The rails were designed to support 8 tons.

The lead door is built on the door carriage shown in Fig. A-6. The carriage is 23 inches long to provide sufficient distance between the front and back wheels for a stable door platform. The carriage ran on 4-1/2" diameter hard steel, flanged, fixed-axle wheels. These wheels were individually rated to carry a nominal load of 1100 pounds. Fig. 7 shows the door carriage and door rails as constructed.

APPENDIX III
OPERATING INSTRUCTIONS FOR MULTIPLE PHOTOTUBE ARRAYS
WITH LOW BACKGROUND BASES

In multiple tube systems, the tubes must be adjusted so that each tube has the same output pulse height. Such balancing is necessary for optimum performance of the unit. Since tubes may exhibit fatigue, it is necessary to check and realign the balance condition among the tubes periodically. For ease of balancing, each low background base is equipped with a rheostat connected potentiometer, P_2 , by means of which the overall tube voltage may be adjusted (Fig. 11). The rest of the network is quite straightforward. A potentiometer, P_1 , is included to allow adjustment of the focusing electrode potential for optimum electron collection efficiency. Each low background base has a separate high voltage and signal connector and screw driver adjustments for P_2 and P_1 . Signal cables from each base should be joined at the grid of a single cathode follower preamplifier.

The balancing procedure is as follows:

1. Place a Cs^{137} or other monoenergetic gamma ray source on the axis of the crystal. This source should be sufficiently close to the crystal to give workable counting rates, but not so close as to cause fatigue or gain changes in the phototubes. About 30,000 counts per minute in the photopeak is acceptable.

2. During the balancing procedure, it is necessary that all signal cables between phototubes and the common cathode follower be connected. This insures that the phototube anode capacitance remains constant during the alignment procedure.
3. With all signal cables connected, examine the output of each tube separately with an oscilloscope or pulse height analyzer. This may be done by connecting the high voltage cable only to the tube being examined. The signal from each tube should now be adjusted for maximum pulse height by varying both the gain and focus potentiometers, P_1 and P_2 . Note the pulse height of the tube with the lowest output signal.
4. Adjust the gain controls, P_1 , of the higher output tubes until all pulse heights are matched to the pulse height of the lowest gain unit found in Step 3. The balance should be brought to within 2% for optimum performance.
5. Apply voltage to all tubes and observe the summation pulse height. If properly balanced, and if the crystal response is independent of photomultiplier tube position, then the final pulse height should be just N times the individual pulse heights where N is the number of tubes involved.

Note that the tube array is now balanced only for the supply voltage used in the above procedure. Since phototubes have widely differing gain vs. voltage characteristics, operation at a different high voltage will necessitate rebalancing of the array.

APPENDIX IV ELECTRONICS USED IN EXPERIMENT

Photomultiplier tubes on NaI detector	-	RCA-8054
Photomultiplier tubes used on organic detector	-	CBS-7817
Preamplifiers	-	Franklin Model 359
Amplifiers ³⁵	-	Franklin Model 358
Pulse height selectors	-	Franklin Model 20
Triple coincidence unit	-	custom made ³⁶
Scaler	-	TMC Model SG-3A
Scope trigger and camera control circuit	-	custom made ³⁷
Pre-set stop circuit	-	custom made
Camera	-	Beattie Coleman Model DK-5 with Model CS-24 magazines
Mixer and coding circuit	-	custom made
Oscilloscope	-	Tektronix Model RM-15
Pulse generator	-	Franklin Model 370
High voltage supply	-	John Fluke Model 405
High voltage supply	-	John Fluke Model 412A
AC Filter	-	custom made ³⁶
AC Isolation transformer	-	UTC RC77

Delay lines

- Columbia Technical Corporation
Model HH-2500

Signal cable

- RG-71U

High voltage cable

- RG-59U

APPENDIX V
DESCRIPTION OF CUSTOM-BUILT ELECTRONICS

1. Mixer and Coder

This circuit accepts three doubly differentiated signals, codes them and mixes them onto a common line while keeping the three input lines isolated from each other (Fig. A-7). Signal No. 1 is inverted, diode biased to cut off the positive half of the pulse and put on the grid of pentode No. 1. Signal No. 2 is diode biased to cut off the negative half of the pulse and is put on the grid of pentode No. 2. Signal No. 3 is put directly onto the grid of pentode No. 3. The three pentode plate signals were added together, inverted and driven through a cathode follower. The gain of all three channels was constant for input signals up to 8.5 volts.

2. AC Line Filter

A filter (Fig. A-8) was used to reject high frequency noise from the 60 cps 110-volt power line. This network is a transmission line designed ³⁶ to carry a 30-ampere load and has a cut-off frequency of approximately 2000 cps.

3. Preset Stop Circuit

The preset stop circuit (Fig. A-9) is designed around a Sodeco register type 2TCeF4PE. This register which one presets to a given

value diminishes its reading by one each time the register is activated and opens a relay when it reaches zero. In the experiment, the AC pulse from the camera control circuit is also used to close a relay which activates the register. When the register reaches zero, it opens another relay which cuts off the AC power to the camera and to the electric timer which records the total run time. The timer reads up to 1000 hours in units of 0.01 hours.

4. Triple Coincidence Unit

This coincidence unit (Figs. A-10 and A-11) was designed³⁶ to accept negative pulses of 10 volts amplitude and varying widths. The resolution time of the circuit is ~ 2.0 μsec . The logic of this circuit is composed of commercial digital modules³⁸. The input signals trigger multivibrators (MV) whose positive output pulses have a 2 μsec width. The negative outputs from these MV's are buffered through a NAND gate and are available to be monitored. The positive MV outputs are put into a three-input NAND gate which is the coincidence logic. If all three inputs are positive, then the output of the NAND gate triggers a MV (Fig. A12) which gives a 1 μsec width output pulse of +18 volts amplitude.

APPENDIX VI

MONTE CARLO CALCULATION OF NEUTRON SLOWING DOWN, DIFFUSION AND CAPTURE

In order to determine the neutron capture time distribution as well as the variation of overall detection efficiency with positron kinetic energy, it was necessary to calculate the neutron slowing down diffusion and capture process for various locations in the liquid scintillation $\bar{\nu}_e$ target. The point here is that the neutron detection efficiency is a function of location as is the positron detection efficiency which is, in addition, a function of the positron's kinetic energy. Consequently, a determination of the overall detection efficiency requires a calculation for the neutron. The Monte Carlo technique⁴⁰ was employed in this calculation.

A neutron was introduced into the system at appropriate weighted radial and "z" coordinates and allowed to collide with the nuclei of the scintillator scattering or capturing according to the conventional prescriptions related to the relevant cross sections.

These cross sections were obtained from the AEC neutron cross section compilation except for the H_2 scattering cross sections which were taken from the work of Melkonian and Jones.⁴¹ The most current I^{127} radiative neutron capture cross section was also used.⁴²

Neutrons of energy 2 and 20 Kev were introduced into the system since these represent the energy limits characteristic of equation (1). The results were found to be energy independent in this region.

A tally was kept of neutrons which remained in the system and their capture times as well as those which left the system and were scattered back in by the NaI crystals.

The capture versus time spectrum shown in Figure 22, represents the fates of 12,000 initial neutrons. This capture versus time spectrum only reflects the radiative capture of neutrons by hydrogen and Gadolinium in the liquid scintillation detector and by iodine and sodium in the crystal detectors.

The spatial variation of the neutron capture distribution for this sample is shown in Table VII.

The flow sheet for the calculation which was performed⁴³ on the Case 1107 computer is shown schematically in Figure A-13.

The details of the Monte Carlo calculation, while not described, can be obtained from a reproduction of the Algol program which is presented in Figure A-14.

APPENDIX VII RESOLUTION UNFOLDING AND LEAST SQUARE FITTING OF THE POSITRON SPECTRUM

The problem of calculating the distortion of a spectrum due to the resolution function of a detector has been formulated.^{38,39} In the present case we are concerned with the inverse problem in which the distorted spectrum (the observed positron spectrum) and the resolution function of the liquid scintillation detector are known and it is desired to obtain the true positron spectrum. The distortion of a spectrum by a resolution function can be written as

$$M(E) = \int_0^{E_{max}} N(E') L(E, E') dE'$$

where

$M(E) dE$ = number of events observed with energy between E and $E + dE$

$N(E) dE$ = number of events with true energy between E and $E + dE$

$L(E, E')$ = the probability of observing between E and $E + dE$ a particle whose true energy is E' .

The resolution function of the liquid scintillation detector was determined (Section IV, B) to be approximately a Gaussian with an energy dependent half-width given by $FWHM = K \sqrt{E}$.

Therefore,

$$L(E, E') = \frac{1}{K\sqrt{\pi E}} e^{-\frac{(E' - E)^2}{K^2 E}}$$

The true spectrum which is assumed not to be grossly different in shape from the predicted spectrum (Section VII, A) can be written in an un-normalized form as

$$N(E) = e^{bE + cE^2} (E - 1.29) \{ (E - 1.29)^2 - 0.26 \}^{1/2}$$

where

$$b = -0.505 + \delta b$$

$$c = -0.0544 + \delta c$$

E = Neutrino energy in Mev.

The parameters δb and δc can vary the location of the peak and the shape of the spectrum.

Since the resolution unfolding is done by a machine calculation it is expedient to perform the least squares fitting of the data and resolution unfolding together. The least squares fitting is performed by minimizing the quantity, $\mathcal{L}$,

$$\mathcal{L} = \sum_i \left[M(E_i) - \int_0^{E_{max}} N(E') L(E_i, E') dE' \right]^2.$$

It has been determined in previous calculations that there is little advantage in taking weighted residuals, therefore this was not done here. Taking into account the finite window width (0.50 Mev) of the positron spectrum and the energy dependence of the detection efficiency, $\mathcal{L}$ can be written as

$$L = \sum_i \left[M(E_i) - \frac{\chi}{\chi'} \int_0^{10.8} e^{bE' + cE'^2} P_i(E - 1.29) [(E - 1.29)^2 - 0.26]^{1/2} \right. \\ \left. \int_{E_i - 0.25}^{E_i + 0.25} \frac{e^{-\frac{(E' - E_i)^2}{K^2(E' - 1.8)}}}{K \sqrt{\pi(E' - 1.8)}} dE' dE_i \right]^2$$

where

$$\chi = \sum_i M(E_i)$$

$$\chi' = \sum_i \int_0^{10.8} e^{bE' + cE'^2} (E - 1.29) [(E - 1.29)^2 - 0.26]^{1/2} \\ \int_{E_i - 0.25}^{E_i + 0.25} \frac{e^{-\frac{(E' - E_i)^2}{K^2(E' - 1.8)}}}{K \sqrt{\pi(E' - 1.8)}} dE' dE_i$$

P_i = total detection efficiency for energy E_i .

The $\frac{\chi}{\chi'}$ term is necessary for renormalization of the true spectrum. Since the unfolded spectrum is assumed not to differ greatly from the predicted spectrum the minimum value of L was found by searching the $\delta b, \delta c$ space in the vicinity of $\delta b = \delta c = 0$.

List of References

1. F. Reines and C. L. Cowan Jr., Phys. Rev. 92, 830 (1953).
2. F. Reines and C. L. Cowan Jr., Phys. Rev. 113, 273 (1959).
3. These NaI (Tl) crystals are from the "Matched Window Line" of the Harshaw Chemical Company, Cleveland Ohio, and contained low background tube bases.
4. The decalin solvent was loaded with 24 g/l Gadolinium 2-ethylhexate by Pilot Chemicals Inc., Watertown, Mass.
5. There is a variation in minimum photocathode areas of two-inch diameter photomultiplier tubes depending on the manufacturer. The minimum photocathode areas for Dumont 6292 and CBS 7817 are respectively 1.76 in² and 2.39 in². 13 CBS tubes are therefore equivalent in photocathode area to 17.7 Dumont 6292 tubes.
6. An analysis is underway to relate the observed $\bar{\nu}_e$ signal to the absolute cross section for reaction (1) averaged over the fission $\bar{\nu}_e$ spectrum. As of this writing there remains some questions regarding the absolute value of the neutron detection efficiency.
7. This experimental area is the same one used in the previous reactor $\bar{\nu}_e$ experiments cited above 1, 2.
8. These NaI (Tl) crystals are from the "Matched Window Line" of the Harshaw Chemical Company.
9. A Pilot B Plastic Scintillator polished on all surfaces was used.
10. To obtain a high reflectivity placite 7133 paint supplied by Wisconsin Protective Coating Company.
11. Kodak Royal X Pan Recording Film was used because it was the fastest (ASA of 1250) 35 mm film supplied on 00 foot rolls.
12. A less foggy background is produced if the film is developed in D 76

13. T. Rockwell, "Reactor Shielding Design Manual", (McGraw-Hill Book Co., 1956).
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15. R. A. Ricci, Physica 23, 693 (1957).
16. A 0.976 Mev electron absorbed by the liquid scintillator produces approximately 6.51×10^3 photons. Assuming no absorption by the liquid scintillator and no reflections from the walls, 9.35% of the photons will fall on the photocathode producing with a 10% photocathode efficiency 60 photoelectrons. The FWHM resolution expected then from photon statistics would be $\sim 26\% (\sim \frac{2}{\sqrt{60}})$.
17. Two mg of separated K^{41} were purchased from Oak Ridge National Laboratory.
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22. Since the relative efficiency for proton to deposit energy in a liquid scintillator is $\sim \frac{1}{2}$ that for electrons a 0.50 Mev "electron" signal must be produced by a proton of ~ 1.0 Mev.
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32. Kennametal Inc., Dept. N. R., Latrobe, Pennsylvania
33. R. D. Evans, pg. 688.
34. The platform was designed by Mr. A. A. Hruschka.
35. The amplifiers are described in detail in the Oak Ridge National Laboratory Report No. 3348.
36. The circuit was designed by Mr. B. M. Shoffner.
37. This circuit was designed by L. V. East and is described in his thesis (1965) Case Institute of Technology, Cleveland, Ohio.
38. G. E. Owen and H. Primakoff, Rev. Sci. Instr. 21, 447 (1950).
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41. E. Melkonian, Phys. Rev. 76, 1750 (1949); W. B. Jones, Jr., Phys. Rev. 74, 364 (1948).
42. Y. P. Popov and F. L. Shapiro, JETP 15 683 (1962).
43. Mrs. R. Alexander was most helpful in coding and running this calculation.

TABLE I
COMPARISON OF SCINTILLATORS

Type	Formula	Molecular Weight	H/C	Density	Proton Density $\times 10^{22}/\text{cm}^3$
Toluene	C_7H_8	92	1.14	0.87	4.55
Decalin	$\text{C}_{10}\text{H}_{18}$	138	1.80	0.88	6.90
Pilot B	$\text{C}_{10}\text{H}_{11}$	131	1.10	1.02	5.16
Pilot	$\text{C}_{11}\text{H}_{14}$	146	1.27	1.02	5.76
Specials	$\text{C}_{10}\text{H}_{16}\text{O}_2$	168	1.60	0.89	5.73

TABLE II

GAMMA RAY LINES OBSERVED IN THE BACKGROUND SPECTRUM AND THEIR SOURCES

Observed Line Energy (Mev)	Source and Energy (Mev)
0.603	ThC" (0.58), RaC (0.60)
0.934	RaC (0.933)
1.14	RaC (1.12)
1.47	K ⁴⁰ (1.47)
1.77	RaC (1.76), ThC (1.80)
2.20	RaC (2.20), ThC (2.20)
2.60	ThC" (2.62)
3.24	ThC" (0.58 + 2.62 = 3.20)

TABLE III

INTEGRAL BACKGROUND SPECTRUM WITH REACTOR-UP

Energy Threshold (Mev)	NaI Detector (min ⁻¹)
0.50	1425
0.75	1005
1.0	774
1.5	304
2.0	175
2.5	106
3.0	67.6
3.5	49.6
4.0	39.4
4.5	30.5
5.0	22.2
5.5	16.1
6.0	10.3
6.5	5.70
7.0	1.72

TABLE IV
DETECTOR RATES IN NEUTRON SOURCE INVESTIGATION

Detector	0% Power w/o source	0% Power w source	100% Power w/o source
NaI 1 window	210/min	565/min	250/min
NaI 2 window	200/min	580/min	250/min
organic detector > .40 Mev	1100/min	2315/min	1300/min
NaI 1 - NaI 2 coin	---	4.4±.9/min	1.0/min
triple coin	5.6±0.3/hr	103±1.8/hr	28.2 ±0.7/hr
NaI 1 or NaI 2 > 0.75 Mev	850/min	3,200/min	1000/min
organic detector > 0.75 Mev	650	1,490/min	760/min

TABLE V

PHOTO-PEAK PROPERTIES FOR 0.51 MEV GAMMA RAYS IN NaI DETECTORS

Radial Position (in)	NaI 1		NaI 2	
	Peak Channel	FWHM (Channels)	Peak Channel	FWHM (Channels)
0.50	178.5 $\pm$ 1	16.5 $\pm$.5	155 $\pm$ 1	15.5 $\pm$.5
2.00	178.5 $\pm$ 1	16.5 $\pm$.5	154.5 $\pm$ 1	16.0 $\pm$.5
3.25	177 $\pm$ 1	17.0 $\pm$.5	154 $\pm$ 1	16.0 $\pm$.5
4.00	176.5 $\pm$ 1	16.5 $\pm$.5	154 $\pm$ 1	16.0 $\pm$.5
4.75	176.5 $\pm$ 1	16.5 $\pm$.5	154 $\pm$ 1	16.5 $\pm$.5

TABLE VI

SPECTRUM PROPERTIES FOR ELECTRONS ABSORBED IN THE
LIQUID SCINTILLATION DETECTOR

Electron Energy = 0.472 Mev			Electron Energy = 0.967 Mev		
Radial Position (in)	Peak Channel	FWHM (Channels)	Radial Position (in)	Peak Channel	FWHM (Channels)
0	28 $\pm$ 1	13	0	55.5 $\pm$ 1	19
2	30.5 $\pm$ 1	14	2	57 $\pm$ 1	19
3.5	29 $\pm$ 1	13	3.5	58.5 $\pm$ 1	19
4.75	34 $\pm$ 1	13	4.75	67 $\pm$ 1	19

TABLE VII

$P_6(X_i)$ CALCULATED BY MONTE CARLO METHOD

$X_i(\text{cm})$	$P_6(X_i)$
0.175	0.645±0.23
0.525	0.637
0.875	0.622
1.225	0.620
1.575	0.612
1.925	0.605
2.275	0.580
2.625	0.538
2.975	0.502
3.325	0.408±.019

TABLE VIII

NUMERICAL CALCULATIONS OF DETECTION EFFICIENCIES FOR $E = 2.75$ Mev

X_i	P_1	P_4	P_5	$0.13 P_1 P_6$	$0.14 P_4 P_5 P_6$	$(X_i, 2.75)$
.1749	9.705.-02	0.000. 00	0.999. 00	7.512.-03	0.000. 00	7.512.-03
.5249	9.705.-02	0.000. 00	0.999. 00	7.419.-03	0.000. 00	7.419.-03
.8749	9.705.-02	0.000. 00	0.999. 00	7.244.-03	0.000. 00	7.244.-03
1.2249	9.705.-02	0.000. 00	0.999. 00	7.221.-03	0.000. 00	7.221.-03
1.5749	9.705.-02	0.000. 00	0.999. 00	7.127.-03	0.000. 00	7.127.-03
1.9249	9.705.-02	0.000. 00	0.999. 00	7.046.-03	0.000. 00	7.046.-03
2.2749	8.504.-02	9.967.-03	0.999. 00	5.919.-03	7.515.-04	6.670.-03
2.6249	7.424.-02	2.140.-02	0.999. 00	4.793.-03	1.497.-03	6.290.-03
2.9749	6.343.-02	3.169.-02	0.999. 00	3.821.-03	2.068.-03	5.890.-03
3.3249	5.263.-02	3.169.-02	8.118.-01	2.576.-03	1.364.-03	3.941.-03
Sum	8.577.-01	9.477.-02	9.811. 00	6.068.-02	5.682.-03	6.636.-02

TABLE IX

ENERGY DEPENDENCE OF EFFICIENCY TERMS AVERAGED OVER DETECTOR VOLUME

E	P_1	P_4	P_5	$0.13 P_1 P_6$	$0.14 P_4 P_5 P_6$	$\gamma(E)$
0.75	0.971	.0228	.963	.0677	.0007	.0684
1.25	0.944	.0432	.977	.0662	.0019	.0681
1.75	0.915	.0644	.979	.0644	.0033	.0677
2.25	0.888	.0824	.980	.0627	.0046	.0673
2.75	0.858	.0948	.981	.0607	.0057	.0664
3.25	0.828	.102	.981	.0586	.0065	.0651
3.75	0.798	.107	.982	.0566	.0070	.0636
4.25	0.768	.111	.982	.0544	.0075	.0619
4.75	0.738	.114	.982	.0523	.0079	.0602
5.25	0.708	.116	.982	.0501	.0082	.0583
5.75	0.678	.118	.982	.0479	.0085	.0564
6.25	0.652	.120	.982	.0461	.0087	.0548
6.75	0.630	.119	.982	.0445	.0087	.0532
7.25	0.611	.114	.983	.0431	.0084	.0515
7.75	0.593	0.107	.983	.0418	.0079	.0497
8.25	0.578	0.100	.983	.0407	.0074	.0481

Figure 1

Schematic diagram of the antineutrino detection scheme.

Figure 2

Typical oscilloscope trace showing a prompt and delayed signal.

Figure 3

Background reduction vs lead house thickness in new experimental area.

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Figure 4

Comparison of low energy background spectra.

Top View

Side View

End View

Figure 5

Lead shield dimensions.
(all dimensions in inches)

Figure 6

Arrangement for electrically opening shield door.

Figure 7

The door of the lead house in open position. The door is pulled open by an electric hoist mounted on the ceiling.

Figure 8

The arrangement of the detectors inside the lead house. The advantage of mounting the photo-tube voltage dividers on flat cards for the organic detector is evident.

Figure 9

The detectors as seen inside the lead house with the door open. The neutron shield around NaI 2 is shown partially disassembled. The preamplifiers, high voltage distribution box and high voltage monitor are mounted on top of the lead house.

Figure 10
The organic detector shown with the top dust cover removed. The thirteen photo-tubes are spring loaded and are held in alignment by mu-metal shields.

Figure 11

Voltage divider for NaI photomultiplier tubes.

Figure 12

Shock-proof mounting for NaI crystals.

Material - Preshrunken Plexiglas
All dimensions in Inches

Figure 13

Liquid scintillator container.

Figure 14

Voltage divider for organic detector photomultiplier tubes.

Figure 15

Schematic diagram of detector and associated electronics.

Figure 16

Oscilloscope display of prompt signal.

Figure 17

Background seen inside lead shield by 11.5" x 3" NaI crystal.

Figure 18

Reactor-up spectrum observed by liquid scintillation detector.

Figure 19

Decay scheme of Bi^{207} .

Figure 20

Energy calibration of the liquid scintillation detector.

Figure 21

The decay scheme of the constituents of a Pu/Be source.
All values in Mev.

Figure 22
Capture time spectra for neutrons in the liquid scintillation detector.

Figure 23
Comparison of the positron and background spectra.

Figure 24

Model used for P_3 calculation.

Figure 25

Model used for $P_5(x_i, E)$ calculation.

Figure 26

Total detection efficiency dependence on energy.

Figure 27
Reactor-up energy spectra.

Figure 28

Observed and resolution unfolded positron spectrum.

Figure 29

Reactor-down energy spectra and capture time spectrum.

Figure 30.

Comparison of the observed fission $\bar{\nu}_e$ spectrum with the spectrum determined from fission betas.

Figure 31
Modes for driven electron background.

Figure A-1

Approximate model of the detector.

Figure A-2

Effect of liquid scintillator thickness on prompt triple coincidence rate.

A - Beams 8WF31 Load 50 ton
 B - Beams 8" x 5 1/4" 171b
 Load 8 ton

Figure A-3

I beam layout for shield platform.

Details of grouting in baseplates.

Figure A-4

Details of shield platform.

1/2" Steel Plate Tack Welded to Beams.

Figure A-5

Cover plate for shield platform.

All 6" channels
 A - 1/2" Steel Plate
 B - 3/8" Steel Plate
 C - 3/8" Steel Plate

Figure A-6

Carriage for shield door.

Figure A-7

Schematic diagram of mixer and coder.

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Figure A-8

High frequency AC-line filter.

Figure A-9

Preset stop control circuit.

I counts backward and opens switch S when it reaches zero.

All Diodes 1N4009
All Transistors 2N697

Figure A-12

Multivibrator with output of +15 volts, 1 μsec wide

Flow diagram for the Macie Carlo calculation of the neutron detection efficiency.

```

ALGOL      MARCH 21,1965      INTERFACE      FEBRUARY 15,1965
1      BOOLEAN NAI1, NAI2 $
2      REAL RAD1, RAD2, RAD3 $
3      INTEGER TABINX $
4      INTEGER ARRAY SPREAD(1..10, 1..4, 1..4) $
5      LOCAL LABEL WHERE, QUAD, SETCNT $
6      BOOLEAN OUTSIDE, LSLOSS $
7      REAL NSUBH, NSUBC, NSUBGD, MSUBN, MSUBP, MSUBC, SIGCS, SIGGDA,
8          SIGHA, SIGHSET, SIGHSEN, T, TMAX, ASUBH, SSUBH, TSUBH, ASUB,
9          ASUBC, SSUBC, TSUBC, RBAR, ASUBTH, LAMBDA1T, LAMBDAST, LAMBDAAT,
10         SIGT, LAMBDA10, LAMBDA50, LAMBDA40, SIGE0, LAMBDA1, LAMBDA5,
11         LAMBDA4, SIG, LAMBDA2, LAMBDA3, LAMBDA4, RADIUS, HEIGHT, E, ESUBT,
12         K, R, S, RR, COSPHI, SINPHI, X, Y, Z, RHO, U, V, W, ZETA, LUPR,
13         DELTA, ANSW1, ANSW2, ANSW3, ANSW4, SIGPR, LAMBDA5I, ASUBI,
14         ASUBNA, NSUBI, NSUBNA,
15         ZPRIME, XPRIME, YPRIME, MU, A, B, C, D, ALPHA, RESULT, EIN,ANS$
16     REAL ARRAY EVAL1 (1..18 ), EVAL2 (1..10 ), EVAL3 (1..6 ),
17         CRSEC1 (1..18), CRSEC2 (1..10), CRSEC3 (1..6) $
18     INTEGER ARRAY CAPTURE (1..32, 1..10), PARTLOSS (1..32, 1..10),
19         CUTOFF (1..10), TOTPARTL (1..10), TOTALCAP (1..10),
20         TOTALLSLOSS (1..10), PARTLOSSLS (1..32, 1..10),
21         TOTALLSCAP (1..10), TOTALNACAP (1..10), NACAPTURE (1..32, 1..10),
22         NOBRTHS (1..10), SQ(1..4), SR(1..4)$
23     INTEGER I, J, N, NOPART, G, EX, GG, SUMT$
24     LOCAL LABEL OUT1, LOSS, L3, L4, L6, L7, PATH, SCAT, L8, LET,
25         NEWLAMBDA, IFCAP $
26     FORMAT GVAL('G OF ',"I2,X2,"Q1",X8,"Q2",X8,"Q3",X8,"Q4",X6,
27         'S(Q)", A2.1),

```

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Fig. A-14

Algol program for Monte Carlo Calculation.

```

28      F3(X3, 'R1', 5(I5, X5), A1, X3, 'R2', 5(I5, X5), A1, X3, 'R3',
29      5(I5, X5), A1, X3, 'R4', 5(I5, X5), A1),
30      F4(X1, 'S(R)', 4(I5, X5), A1)
31      DIST('DISTRIBUTION OF BIRTH POINTS', A2, 0) $
32      FORMAT T1(X11, 'G', X5, 'NO. OF BIRTHS', A1, 1),
33      T2(X11, 'G', X6, 'NO. OF PART.', A1, 1),
34      T3(E1, 'TOTAL NUMBER OF NEUTRONS= ', I7, A1, 0),
35      T4('NUMBER OF BIRTHS IN SLABS', A3, 1),
36      T5('TIME CUTOFF SCALERS', A3, 1),
37      T6('PARTICLE LOSS FROM SYSTEM', A3, 1),
38      T7('TOTAL CAPTURE SCALERS', A3, 1),
39      T8(X6, 'T', X6, 'G=1', X4, 'G=2', X4, 'G=3', X4, 'G=4', X4, 'G=5', X4,
40      'G=6', X4, 'G=7', X4, 'G=8', X4, 'G=9', X3, 'G=10', A1, 1),
41      T9('PARTICLE LOSS FROM L.S.', A3, 1),
42      T10('CAPTURE BY SODIUM IODIDE', A3, 1),
43      T11('CAPTURE WITHIN L.S.', A3, 1),
44      T12('TOTAL PARTICLE LOSS FROM SYSTEM', A3, 1),
45      T13('TOTAL PARTICLE LOSS FROM L.S.', A3, 1),
46      T14('TOTAL CAPTURE BY SODIUM IODIDE', A3, 1),
47      T15('TOTAL CAPTURE WITHIN L.S.', A3, 1),
48      F2(X5, I2, X2, I1(I7), A1),      F1(X10, I2, X11, I7, A1) $
49      REAL PROCEDURE ADJUST (EX) $
50      VALUE EX $ INTEGER EX $
51      BEGIN      IF W GTR 0.0 THEN GG = 1 ELSE GG = 0 $
52      DELTAL = ((Z - HEIGHT*GG)/W)*(((SIGPR/SIG)**EX) - 1) $
53      Z = Z + W*DELTAL $
54      X = X + U*DELTAL $
55      Y = Y + V*DELTAL $
56      RR = X*X + Y*Y $
57      END ADJUST $

```

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Fig. A-14 Cont.

```

58 REAL PROCEDURE ANGLE (A, B)$
59 REAL A, B$ BEGIN REAL DENOM$
60 TOOBIG.. R = RANDOM(1) $ S = RANDOM(1) $
61 DENOM = R * R + S * S $
62 IF DENOM GTR 1.0 THEN GO TO TOOBIG$
63 A = (R * R - S * S) / DENOM $
64 B = 2 * R * S / DENOM $
65 R = RANDOM(1) $
66 IF R - 0.5 LSS 0.0 THEN B = - B $
67 END ANGLE $
68 INTEGER PROCEDURE ADDARRAY (SCALER) $
69 INTEGER ARRAY SCALER $
70 BEGIN T = (T * 1.08+6 $
71 FOR J = (1, 1, 32) DO
72 IF ENTIER (T) LSS J THEN BEGIN
73 SCALER (J,G) = SCALER (J,G) + 1 $
74 GO TO DONE $ END $
75 DONE.. END ADDARRAY $
76 REAL PROCEDURE ECAL (SSUBH, TSUBH, ASUB) $
77 REAL SSUBH, TSUBH, ASUB $
78 BEGIN R = RANDOM (1) $ MU = 2.0*R - 1.0 $
79 E = E * (SSUBH + TSUBH * MU) $
80 A = (1.0 + ASUB * MU) / SQRT (1.0 + ASUB * ASUB + 2.0 * ASUB*MU)$
81 END ECAL $
82 REAL PROCEDURE TABLESEARCH (ENERGY, CRSEC, N, P, ANSWER) $
83 VALUE P $
84 REAL ARRAY ENERGY, CRSEC $ INTEGER N, P $ REAL ANSWER $
85 BEGIN I = N $

```

Fig. A-14 Cont.

```

86 LOOP.. IF ENERGY (I) GEQ E AND E GEQ ENERGY (I - 1) THEN
87     BEGIN ANSWER = ((E - ENERGY (I - 1))/(ENERGY (I) - ENERGY (I - 1)))
88     *(CRSEC (I) - CRSEC (I - 1)) + CRSEC (I - 1) $
89 GO TO OUT $ END $
90     IF E GTR ENERGY (I) THEN I = I + 2**P ELSE
91     I = I - 2**P $ P = P - 1 $ IF P LSS - 1 THEN GO TO ERR $
92     IF I GTR 1 THEN GO TO LOOP $
93 ENDPT.. I = I + 2**P $ P = P - 1 $ IF P LSS -1 THEN GO
94 TO ERR$ IF I GTR 1 THEN GO TO LOOP ELSE GO TO ENDPT $
95 ERR.. WRITE ('UNSUCCESSFUL TABLESEARCH WITH E OF', E,
96 CRSEC (I), CRSEC (N)) $
97 OUT.. END TABLESEARCH $
98 INTEGER PROCEDURE PRTARRAY (PARTLOSS) $
99 INTEGER ARRAY PARTLOSS $
100 BEGIN
101     WRITE (T8) $
102 FOR J = (1, 1, 32) DO BEGIN
103     SUMT = 0 $
104 FOR I = (1, 1, 10) DO SUMT = SUMT + PARTLOSS (J, I) $
105 WRITE (F2, J, FOR I = (1, 1, 10) DO PARTLOSS (J, I), SUMT) $ END $
106 END PRTARRAY $
107 INTEGER PROCEDURE PRTSUM (PARTLOSS, TOTPARTL) $
108 INTEGER ARRAY PARTLOSS, TOTPARTL $
109 BEGIN
110     WRITE (T2) $
111 FOR I = (1, 1, 10) DO BEGIN
112 FOR J = (1, 1, 32) DO
113     TOTPARTL (I) = TOTPARTL (I) + PARTLOSS (J, I) $
114     WRITE (F1, I, TOTPARTL (I)) $ END $
115 END PRTSUM $

```

Fig. A-14 Cont.


```

116 NSUBN = NSUBP = 1.0 $ NSUBC = 12.0 $ RADIUS = 13.0 $
117 RAD3 = 11.25 $ RAD2 = 9.20 $ RAD1 = 6.50 $
118 HEIGHT = 7.0 $ ESUBT = .025 $ ASUBH = 1.0 $
119 FOR I = (1, 1, 10) DO BEGIN
120     FOR J = (1, 1, 32) DO BEGIN
121         PARTLOSS (J,I) = CAPTURE (J,I) = 0 $
122         PARTLOSSLS(J,I) = NACAPTURE(J,I) = 0 $ ENDS$
123         CUTOFF (I) = 0 $ TOTPARTL (I) = TOTALCAP (I) = 0 $
124         TOTALLSLOSS(I) = TOTALLSCAP(I) = TOTALNACAP(I) = 0 $
125         NOBRTHS (I) = 0 $ END $
126     FOR G = (1, 1, 10) DO FOR I = (1, 1, 4) DO FOR J = (1, 1, 4) DO
127         SPREAD(G, I, J) = 0 $
128         READ, (N, EIN) $
129         READ(EVAL1,CRSEC1,EVAL2,CRSEC2,EVAL3,CRSEC3 ) $
130         NOPART = 0 $
131         NSUBH = 6.918+22 $ NSUBC = 3.848+22 $
132         NSUBI = 1.47822 $ NSUBNA = 1.47822 $
133         NSUBGD = 2.242+19 $ TMAX = 33.08-6 $
134         K = .7248-6 $ SIGCS = 4.88-24 $
135         SIGGDA = 4.78-20 $ SIGHA = .338-24 $
136         SIGHSET = 55.08-24 $ SIGHSEN = 21.08-24 $
137         ASUBI = 126.9 $ ASUBNA = 22.9 $
138         ASUBC = NSUBC / NSUBN $
139         RBAR = ( (ASUBC - 1.0) / (ASUBC + 1.0) ) **2 $
140         SSUBC = 0.5 * (1.0 + RBAR) $
141         TSUBC = 0.5 * (1.0 - RBAR) $
142         LAMBDAIT = NSUBH * SIGHSET $
143         LAMBDAST = LAMBDAIT + NSUBC * SIGCS $
144         LAMEDAAT = NSUBH * SIGHA + NSUBGD * SIGGDA $
145         SIGT = LAMEDAST + LAMEDAAT $

```

Fig. A-14 Cont.

```

146      ASUBTH = MSUBP / (SQRT (21.0/55.0) * (MSUBN + MSUBP) - MSUBP) $
147      LAMBDA10 = NSUBH * SIGHSEN $
148      LAMBDA00 = 0.0 $
149      LAMBDA50 = LAMBDA10 + NSUBC * SIGCS $
150      SIGF0 = LAMBDA50 $
151  RESET.. E = EIN $
152      T = 0.0 $ ASUBH = 1.0 $
153      OUTSIDE = FALSE $
154      NAI1 = NAI2 = FALSE $
155      LSLOSS = FALSE $
156      LAMBDA1 = LAMBDA10 $
157      LAMBDA5 = LAMBDA50 $
158      LAMBDA0 = LAMBDA00 $
159      SIG = SIGF0 $
160      NOPART = NOPART + 1 $
161      IF NOPART GTR N THEN GO TO ENDPRG $
162      R = RANDOM(5) $ S = RANDOM(5) $
163      RR = RADIUS * MAX (R, S) $
164      ANGLE (COSPHI, SINPHI) $
165      X = RR * COSPHI $ Y = RR * SINPHI $
166      S = RANDOM(1) $ Z = HEIGHT * S $
167      R = RANDOM(1) $
168      W = 2.0 * R - 1.0 $
169      RHO = SQRT (1.0 - W * W) $
170      ANGLE (COSPHI, SINPHI) $
171      U = RHO * COSPHI $ V = RHO * SINPHI $
172      ZETA = ABS (HEIGHT / 2.0 - Z) $
173      FOR I = (1, 1, 10) DO
174      IF (ZETA - (HEIGHT / 20.0) * I) LEQ 0.0 THEN

```

Fig. A-14 Cont.

```

175      BEGIN G = I $ NOBRTHS (G) = NOBRTHS (G) + 1 $
176      GO TO HERE $ END $
177      HERE.. IF RR GTR RAD3 AND RR LEQ RADIUS THEN BEGIN
178      I = 4 $ GO TO QUAD $ END $
179      IF RR GTR RAD2 AND RR LEQ RAD3 THEN BEGIN I = 3 $ GO TO QUAD $ END $
180      IF RR GTR RAD1 AND RR LEQ RAD2 THEN I = 2 ELSE I = 1 $
181      QUAD.. IF X GEQ 0.0 AND Y GEQ 0.0 THEN BEGIN
182      J = 1 $ GO TO SETCNT $ END $
183      IF X LSS 0.0 AND Y GEQ 0.0 THEN BEGIN
184      J = 2 $ GO TO SETCNT $ END $
185      IF X LSS 0.0 AND Y LSS 0.0 THEN
186      J = 3 ELSE J = 4 $
187      SETCNT.. SPREAD(G, I, J) = SPREAD(G, I, J) + 1 $
188      OUT1.. R = RANDOM(1) $
189      L = - LN (R) / SIG $
190      T = T + K * L / SORT (E) $
191      IF T GEQ TMAX THEN BEGIN
192      CUTOFF (G) = CUTOFF (G) + 1 $
193      GO TO RESET $ END $
194      Z = Z + W * L $
195      X = X + U * L $ Y = Y + V * L $
196      RR = X * X + Y * Y $
197      IF Z LEQ 0.0 OR Z GEQ HEIGHT OR RR - RADIUS*RADIUS GTR 0.0
198      THEN GO TO LOSS $
199      IF NOT (NAI1 OR NAI2) THEN GO TO IFCAP $
200      LSLAMBDA..
201      IF (1.0 - E) LEQ 0.0 THEN BEGIN
202      LAMBDA1 = LAMBDA10 $ LAMBDA5 = LAMBDA50 $ ASUBH = 1.0 $
203      LAMBDA6 = LAMBDA60 $ SIG = SIG0 $

```

Fig. A-14 Cont.

```

204 ADJUST (1) $ GO TO WHERE $ END $
205 IF ESUBT - E GEQ 0.0 THEN BEGIN E = ESUBT $
206 LAMBDA1 = LAMBDA1T $ LAMBDA5 = LAMBDA5T $
207 LAMBDA6 = LAMBDA6T $ SIG = SIGT $ ASUBH = ASUBTH $
208 ADJUST (1) $ GO TO WHERE $ END $
209 OUTSIDE = TRUE $ GO TO L8 $
210 WHERE.. T = T + K*DELTA/SORT(E) $
211 IF Z GEQ 0.0 AND Z LEQ HEIGHT AND
212 RR - RADIUS*RADIUS LEQ 0.0 THEN GO TO IFCAP $
213 NAI1 = NAI2 = FALSE $ GO TO NEWLAMBDA $
214 LOSS.. IF NOT (LSLOSS) THEN BEGIN
215 ADDARRAY (PARTLOSS) $ LSLOSS = TRUE $
216 T = T*1.08-6 $ END $
217 IF Z GEQ 0.0 AND Z LEQ HEIGHT AND RR GEQ RADIUS*RADIUS
218 THEN GO TO SYSLOSS $
219 IF NOT (NAI1 OR NAI2) THEN GO TO NEWLAMBDA $
220 IF NAI2 AND 0.0 GEQ Z AND Z GEQ -7.62
221 AND RR LEQ 14.6*14.6 THEN GO TO PATH $
222 IF NAI1 AND HEIGHT LEQ Z AND Z LEQ 14.62 AND RR LEQ 14.6*14.6
223 THEN GO TO PATH ELSE GO TO LSLAMBDA $
224 SYSLOSS.. BEGIN
225 ADDARRAY (PARTLOSS) $ GO TO RESET $ END $
226 PATH.. P = RANDOM (1) $
227 IF R - LAMBDA5/(LAMBDA5 + LAMBDA6) LSS 0 THEN GO TO SCAT $
228 ADDARRAY (NACAPTURE) $ GO TO RESET $
229 SCAT.. IF E EQ ESUBT THEN GO TO LET $
230 R = RANDOM (1) $
231 IF R - LAMBDA5T/LAMBDA5 LEQ 0.0 THEN ASUB = ASUBI ELSE ASUB = ASUBNA $
232 RBAR = ((ASUB - 1.0)/(ASUB + 1.0))**2 $
233 SSUBH = 0.5*(1.0 + RBAR) $ TSUBH = 0.5*(1.0 - RBAR) $
234 ECAL (SSUBH, TSUBH, ASUB) $ GO TO L6 $

```

Fig. A-14 Cont.

```

235 NEWLAMBDA.. IF E LSS 200.0 THEN ANSW1 = 4.28-24 ELSE ANSW1 = 7.48-24 $
236 IF ESUBT - E GEO 0.0 THEN E = ESUBT $
237 TABLESEARCH(EVAL1,CRSEC1,18.4,ANSW2) $
238 IF E LSS 10 THEN ANSW3 = (1.068-24)/SQRT (E)
239 ELSE BEGIN IF E GTR 100.0
240 THEN ANSW3 = (1.208-22)/SQRT (E)
241 ELSE TABLESEARCH(EVAL2,CRSEC2,10.3,ANSW3)$
242 ENDS IF E LSS 300.0 THEN ANSW4 = 8.748-26/SQRT(E) ELSE
243 BEGIN IF E GTR 4000.0 THEN ANSW4 = 0.0 ELSE
244 TABLESEARCH(EVAL3,CRSEC3,6.2,ANSW4) $ END $
245 LAMBDAI = NSUBI*ANSW1 $
246 LAMBDAI = LAMBDAI + NSUBNA*ANSW2 $
247 LAMBDAI = LAMBDAI + NSUBNA*ANSW3 + NSUBNA*ANSW4 $
248 SIGPR = LAMBDAI + LAMBDAI $
249 IF (NAI1 OR NAI2) THEN BEGIN SIG = SIGPR $
250 GO TO OUT1 $ END $
251 ADJUST (-1) $
252 IF RR GTR 14.6*14.6 THEN GO TO SYSLOSS $
253 IF 0.0 GEO Z AND Z GEO -7.62 THEN NAI2 = TRUE $
254 IF HEIGHT LEQ Z AND Z LEQ 14.62 THEN NAI1 = TRUE $
255 IF NAI1 OR NAI2 THEN BEGIN
256 T = T + K*DELTAL/SQRT(E) $ GO TO PATH $
257 END ELSE GO TO SYSLOSS $
258 LET.. ANGLE (COSPHI, SINPHI) $
259 R = RANDOM (1) $ W = 2*R - 1.0 $
260 RHO = SQRT (1.0 - W*W) $
261 U = RHO*COSPHI $ V = RHO*SINPHI $
262 GO TO NEWLAMBDA $

```

Fig. A-14 Cont.

```

263   IFCAP.. R = RANDOM (1)  $
264       IF (R - LAMBDA5 / (LAMBDA5 + LAMBDA6))GTR 0.0 THEN BEGIN
265           ADDARRAY (CAPTURE) $ GO TO RESET $ END $
266       IF ESUBT - E EQL 0.0 THEN GO TO L3 $
267           R = RANDOM(1) $
268       IF R - LAMBDA1 / LAMBDA5 LEQ 0.0 THEN GO TO L4 $
269       ECAL (SSUBC, TSUBC, ASUBC) $
270           GO TO L6 $
271   L4.. IF ASUBH EQL 1.0 THEN BEGIN
272       R = RANDOM(1) $ S = RANDOM(1) $
273       A = MAX (R, S) $
274       E = E * A * A $
275           GO TO L6 $ END ELSE BEGIN
276       RBAR = ( (ASUBH - 1.0) / (ASUBH + 1.0) ) **2 $
277       SSUBH = 0.5 * (1.0 + RBAR) $
278       TSUBH = 0.5 * (1.0 - RBAR) $
279       ECAL (SSUBH, TSUBH, ASUBH) $ END $
280   L6.. B = SQRT (1.0 - A * A) $
281       ANGLE (C, D) $
282       IF 1.0 - ABS(W) LEQ 0.0001 THEN BEGIN
283           U = R * C $ V = B * D $ W = A * W $ END
284       ELSE BEGIN
285           RESULT = SQRT (1.0 - W * W) $
286           UPR = (B * C * W * U - B * D * V) / RESULT + A * U $
287           V = (B * C * W * V + B * D * U) / RESULT + A * V $
288           U = UPR $
289           W = - B * C * RESULT + A * W $ END $
290   IF NAI1 OR NAI2 THEN GO TO NEWLAMBDA $
291       IF (1.0 - E) LEQ 0.0 THEN GO TO OUT1 $
292       IF ESUBT - E GEQ 0.0 THEN BEGIN

```

Fig. A-14 Cont.

```

293      E = ESUBT $ GO TO L7 $ END $
294 L8.. IF E LEQ 0.220 THEN
295      RESULT = (1.628-23 ) * E ** (-0.331) ELSE
296      RESULT = (2.058-23 ) * E ** (-0.172) $
297      LAMBDA1 = NSUBH * RESULT $
298      LAMBDA2 = NSUBH * SIGHA / SQRT (40.0 * E) $
299      LAMBDA3 = NSUBC * SIGCS $
300      ANS = (4.18-23 ) * (E + 0.30) ** (-6.25) $
301      LAMBDA4 = NSUBGD * ANS $
302      LAMBDA5 = LAMBDA1 + LAMBDA3 $
303      LAMBDA6 = LAMBDA2 + LAMBDA4 $
304      SIG = LAMBDA5 + LAMBDA6 $
305      ALPHA = SQRT (21.08-24 / RESULT) $
306      ASUBH = NSUBP / (ALPHA * (MSUBN + MSUBP) - MSUBP) $
307 IF OUTSIDE THEN BEGIN OUTSIDE = FALSE $
308 ADJUST (1) $ GO TO WHERE $ END $
309      GO TO OUT1 $
310 L3.. E = ESUBT $
311      ANGLE (COSPHI, SINPHI) $
312      R = RANDOM(1) $
313      W = 2 * R - 1.0 $
314      RHO = SQRT (1.0 - W * W) $
315      U = RHO * COSPHI $ V = RHO * SINPHI $
316 L7.. LAMBDA1 = LAMBDA1T $
317      LAMBDA5 = LAMBDA5T $
318      LAMBDA6 = LAMBDA6T $
319      SIG = SIGT $
320      ASUBH = ASUBHT $

```

Fig. A-14 Cont.

```

321         GO TO OUT1 $
322     ENDPRG..
323     WRITE(T3, N) $
324     WRITE(DIST) $
325     FOR G = (1, 1, 10) DO BEGIN
326         WRITE(GVAL, G) $
327         FOR I = (1, 1, 4) DO         SQ(I) = SR(I) = 0         $
328     FOR I = (1, 1, 4) DO BEGIN
329         FOR J = (1, 1, 4) DO BEGIN
330             SQ(I) = SPREAD (G, I, J) + SQ (I) $
331             SR(J) = SPREAD (G, I, J) + SR(J) $ END $     END$
332     WRITE (F3, FOR I = (1, 1, 4) DO (FOR J = (1, 1, 4) DO SPREAD (G, I, J),
333         SQ(I))) $
334     WRITE (F4, FOR J = (1, 1, 4) DO SR(J)) $ END $
335     WRITE(T4) $
336     WRITE(T1) $
337     FOR I = (1, 1, 10) DO
338         WRITE(F1, I, NOBRTHS(I)) $
339     WRITE(T5) $
340     WRITE(T2) $
341     FOR I = (1, 1, 10) DO
342         WRITE(F1, I, CUTOFF(I)) $
343     WRITE (T6) $
344     PRTARRAY (PARTLOSS) $
345     WRITE (T12) $
346     PRTSUM (PARTLOSS, TOTPARTL) $
347     WRITE (T9) $
348     PRTARRAY (PARTLOSSLS) $

```

Fig. A-14 Cont.


```

349 WRITE (T13) $
350     PRTSUM (PARTLOSSLS, TOTALLSLOSS) $
351 WRITE (T10) $
352     PRTARRAY (NACAPTURE) $
353 WRITE (T14) $
354     PRTSUM (NACAPTURE, TOTALNACAP) $
355 WRITE (T11) $
356     PRTARRAY (CAPTURE) $
357 WRITE (T15) $
358     PRTSUM (CAPTURE, TOTALLSCAP) $
359 WRITE (T7) $
360 WRITE (T2) $
361 FOR I = (1, 1, 10) DO BEGIN
362     TOTALCAP (I) = TOTALLSCAP (I) + TOTALNACAP (I) $
363     WRITE (F1, I, TOTALCAP (I)) $ END $
364 FINISH $

```

Fig. A-14 Cont.