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MASTER

Transient Dipole Fields in Two Semi-Infinite Media*

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An approximate solution is presented for the electromagnetic fields from a vertical electric dipole above a plane conducting semi-infinite medium. The Laplace transform of the Hertz vectors in each of the two media are developed and expanded considering the lower medium to be a good but imperfect conductor. The solution follows but extends the classical development by Sommerfeld and subsequently work by Bhattacharyya, Van der Pol, Wait and others.

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The electromagnetic pulse (emp) resulting from a nuclear bomb explosion appears to be roughly describable, in part, as radiation from a pulsed vertical electric dipole. Consequently in an attempt to understand the fields from explosions over various media and for extrapolation of the field, we have been motivated to investigate the idealized problem of determining the transient electromagnetic field from a vertical electric dipole in two semi-infinite media, the solution of which might serve as a guide in analyzing more realistic problems.

In recent years, there have been many theoretical studies on this problem of transient dipole field in two semi-infinite media.^{1, 2, 3} Most of these works were associated either with geophysical exploration or with radio propagation. The method of solution most often used was still essentially the same as first developed by A. Sommerfeld, except that Laplace transformed rather than Fourier transformed Maxwell equations, with respect to time, t ,* have been used. However, the inversion of the resulting solution in Laplace transformed domain is surprisingly difficult. Consequently, various approximation methods have been used, often at the expense of displacement current. The solutions, so obtained, hence are not suitable for discussing the early transient behavior of the EM field. Furthermore, due to practicality and individual preference only fields in the medium where the source is present have been given by the previous works. The only exceptions were those papers where the dipole was put at the interface between the media or where conductivity in both media

* For an account of Sommerfeld's original work on radio propagation see reference 1.

were assumed to be zero. A series of papers published by Pekeris and Alterman,⁴ De Hoop and Frankena,⁵ and Frankena⁶ used a different method with the dubious advantage that by setting $\sigma = 0$ an exact solution can be obtained, taking into account the full effect of displacement current. However for all realistic situations at least one medium has non-vanishing conductivity. For non-vanishing conductivity this method yields transforms which are at least as difficult to invert as the first mentioned method and has the further disadvantage of not being easily interpreted so as to make reasonable approximations. In this presentation we shall derive a useful approximate solution for both media and the solution shall be valid for early time.

Vertical Electric Dipole above a Plane, Conducting Earth

The sphere is to be described by two homogeneous semi-infinite media, where $\mu = \mu_0$, $\epsilon = \epsilon_0$, $\sigma = 0$ for the upper medium and $\mu = \mu_0$, $\epsilon = \epsilon_1$, $\sigma = \sigma_1$ for the lower medium. Cylindrical coordinates (ρ, θ, z) are chosen such that the boundary separating the media is at $z = 0$ and the vertical electric dipole is placed to $z = h > 0$ on the z -axis.

Following Sommerfeld, the EM field components can be expressed in terms of a Hertz vector of a single component, $\vec{\Pi} = \Pi_z \mathbf{e}_z$. After taking the Laplace transform of the Maxwell equations with respect to time, the resulting partial differential equations become with the solutions in the s -domain,

$$\tilde{\Pi}_z(s) = \tilde{\Pi}_0(s) + \tilde{\Pi}_1(s) \quad \text{for } z \geq 0$$

$$\tilde{\Pi}_z'(s) = \tilde{\Pi}_2(s) \quad \text{for } z \leq 0$$

with $\tilde{\Pi}_0(s)$, $\tilde{\Pi}_1(s)$, $\tilde{\Pi}_2(s)$ given by

$$\tilde{\Pi}_0(s) = A(s) \frac{e^{-\gamma_0 r}}{r} \quad (1)$$

$$\tilde{\Pi}_1(s) = A(s) \left\{ \frac{e^{-\gamma_0 r'}}{r'} - 2 \int_0^\infty J_0(\lambda \rho) e^{-U_0(z+h)} \frac{U_1}{n^2 U_0 + U_1} \frac{\lambda d\lambda}{U_0} \right\} \quad (2)$$

$$\tilde{\Pi}_2(s) = A(s) \left\{ 2 \int_0^\infty J_0(\lambda \rho) e^{U_1 z - U_0 h} \frac{\lambda d\lambda}{n^2 U_0 + U_1} \right\} \quad (3)$$

where $\gamma_1^2 = \mu_0 \epsilon_1 s^2 + \mu_0 \sigma_1 s$, $U_1^2 = \gamma_1^2 + \lambda^2$ with $i = 0, 1$

$n^2 = \frac{\gamma_1^2}{\gamma_0^2} = \frac{1}{\epsilon_0} (\epsilon_1 + \frac{\sigma_1}{s})$, $A(s) = \frac{I_0 \tilde{f}(s)}{4\pi \epsilon_0 s}$

$r^2 = \rho^2 + (z-h)^2$, $r'^2 = \rho^2 + (z+h)^2$

s is the Laplace transform variable.

$\tilde{f}(s)$ is the Laplace transform of the dipole source current function.

$J_0(x)$ is the Bessel function of the first kind of order zero.

By the method of reference 7 $\tilde{\Pi}_1(s)$ and $\tilde{\Pi}_2(s)$ may be put into the following form:

$$\tilde{\Pi}_1(s) = A(s) \left\{ \frac{e^{-\gamma_0 R}}{R} - \frac{1}{\pi} \int_0^\infty \rho' d\rho' \int_0^\infty d\xi \int_0^{2\pi} d\varphi \frac{e^{-\gamma_0 R}}{R} \frac{\partial^2}{\partial \xi^2} \frac{e^{-\gamma_1 R'}}{R'} \right\} \quad (4)$$

$$\tilde{\Pi}_2(s) = A(s) \left\{ \frac{\gamma_0^2}{\pi} \int_0^\infty \rho' d\rho' \int_0^\infty d\xi \int_0^{2\pi} d\varphi ab f(\rho', \xi, \varphi) g(\rho', \xi, \varphi) \right\} \quad (5)$$

where $R'^2 = \rho'^2 + \xi^2$, $R^2 = \rho'^2 - 2\rho\rho' \cos\varphi + \rho^2 + (z+h+n^2\xi)^2$

$a = h + \gamma_1^2 \xi$, $b = |z| + \gamma_0^2 \xi$

$$f(\rho', \xi, \varphi) = \frac{e^{-\gamma_0 [\rho'^2 + a^2]^{\frac{1}{2}}}}{[\rho'^2 + a^2]^{\frac{3}{2}}} (1 + \gamma_0 [\rho'^2 + a^2]^{\frac{1}{2}})$$

$$g(\rho', \xi, \varphi) = \frac{e^{-\gamma_1 [\rho'^2 - 2\rho\rho' \cos\varphi + \rho^2 + b^2]^{\frac{1}{2}}}}{[\rho'^2 - 2\rho\rho' \cos\varphi + \rho^2 + b^2]^{\frac{3}{2}}} (1 + \gamma_1 [\rho'^2 - 2\rho\rho' \cos\varphi + \rho^2 + b^2]^{\frac{1}{2}})$$

Equations (4) and (5) cannot be integrated exactly. To determine an approximation, first recall that γ_1 is a function of the Laplace transform variable, s, and eventually it will be necessary to take the

Laplace inverse transform of $\tilde{\Pi}$, i.e.,

$$\Pi(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \tilde{\Pi}(s) e^{st} ds$$

The path of integration must be to the right of all singularities of $\tilde{\Pi}(s)$. The real part of γ_1 is positive and could be as large as desired, so long as it remains finite. Thus the integrand of equations (4) and (5) will converge very fast and significant contributions to the integral come from small values of ρ' . It is thus permissible to neglect ρ' as compared to ρ ; the φ and ρ' integration can then be carried out immediately.

$$\tilde{\Pi}_1(s) = A(s) \left\{ \frac{e^{-\gamma_0 r'}}{r'} + 2 \int_0^\infty \frac{e^{-\gamma_0 [\rho^2 + (z+h+n^2\xi)^2]^{\frac{1}{2}}}}{[\rho^2 + (z+h+n^2\xi)^2]^{\frac{1}{2}}} \frac{\partial}{\partial \xi} e^{-\gamma_1 \xi} d\xi \right\} \quad (6)$$

$$\tilde{\Pi}_2(s) = A(s) \left\{ \frac{2}{\gamma_0^2} e^{-\gamma_0 h} \int_0^\infty \left[-\frac{\partial}{\partial \xi} g(\alpha_1 + \xi) \right] e^{-\gamma_1 \xi} \gamma_0 \xi d\xi \right\} \quad (7)$$

where $\alpha_1 = \frac{|z|}{\gamma_0}$, $\alpha_2 = \frac{\rho}{\gamma_0}$

and $g(\alpha_1 + \xi) = \frac{e^{-\gamma_1 \gamma_0^2 [\alpha_2^2 + (\alpha_1 + \xi)^2]^{\frac{1}{2}}}}{[\alpha_2^2 + (\alpha_1 + \xi)^2]^{\frac{1}{2}}}$

In order to get a practical form for $\tilde{\Pi}_1(s)$ and $\tilde{\Pi}_2(s)$ it is necessary to evaluate or approximate the integrals occurring in the above expressions,

$$\Gamma_1 = -\gamma_1 \int_0^\infty \frac{e^{-\gamma_0 [\rho^2 + (z+h+n^2\xi)^2]^{\frac{1}{2}}}}{[\rho^2 + (z+h+n^2\xi)^2]^{\frac{1}{2}}} e^{-\gamma_1 \xi} d\xi$$

$$\Gamma_2 = \frac{e^{-\gamma_1 \gamma_0^2 [\alpha_2^2 + \alpha_1^2]^{\frac{1}{2}}}}{[\alpha_2^2 + \alpha_1^2]^{\frac{1}{2}}} - \gamma_0 \gamma_1 \int_0^\infty g(\alpha_1 + \xi) e^{-\gamma_0 \gamma_1 \xi} d\xi$$

Again due to the quick convergence of the integrand, the following expansions may be used.

$$[\rho^2 + (z + h + n^2 \xi)^2]^{\frac{1}{2}} \approx r' + \frac{(h+z)n^2 \xi}{r'} + \frac{n^4 \xi^2}{2r'}$$

$$[\alpha_2^2 + (\alpha_1 + \xi)^2]^{\frac{1}{2}} \approx [\alpha_2^2 + \alpha_1^2]^{\frac{1}{2}} + \frac{\alpha_1 \xi}{[\alpha_2^2 + \alpha_1^2]^{\frac{1}{2}}} + \frac{\xi^2}{2[\alpha_2^2 + \alpha_1^2]^{\frac{1}{2}}}$$

Thus, to first order of approximation,

$$\Gamma_1 = -\gamma_1 \frac{e^{-\gamma_0 r'}}{r'} \int_0^\infty e^{-\gamma_0} \left[\frac{(h+z)n^2 \xi}{r'} + \frac{n^4 \xi^2}{2r'} + n\xi \right] d\xi$$

$$\Gamma_2 = \frac{e^{-\gamma_1 \gamma_0^2 \beta}}{\beta} - \gamma_0 \gamma_1^2 \frac{e^{-\gamma_1 \gamma_0^2 \beta}}{\beta} \int_0^\infty e^{-\gamma_1 \gamma_0^2} \left[\frac{\alpha_1}{\beta} \xi + \frac{1}{2\beta} \xi^2 + n\xi \right] d\xi$$

where $\beta = [\alpha_2^2 + \alpha_1^2]^{\frac{1}{2}}$

Both of the integrals are of the form,

$$\int_0^\infty e^{-a^2 s^2} e^{-bs} ds = \frac{\pi^{\frac{1}{2}}}{2a} e^{b^2/4a^2} \operatorname{erfc} [b/(2a)]$$

Hence $\Gamma_1 = -\gamma_1 \frac{e^{-\gamma_0 r'}}{r'} \frac{\pi^{\frac{1}{2}}}{2x_1} e^{[y_1^2/(4x_1^2)]} [1 - \operatorname{erf}(y_1/2x_1)]$ (8)

$$\Gamma_2 = \frac{\gamma_0^2 e^{-\gamma_1 q}}{q} - \gamma_0^3 \gamma_1^2 \frac{e^{-\gamma_1 q}}{q} \frac{\pi^{\frac{1}{2}}}{2x_2} e^{(y_2^2/2x_2^2)} [1 - \operatorname{erf}(y_2/2x_2)]$$
 (9)

with $q = [\rho^2 + |z|^2]^{\frac{1}{2}}$

$$x_1^2 = \frac{\gamma_0 n^4}{2r'}, \quad y_1 = \gamma_0 \left[\frac{h+z}{r'} n^2 + n \right]$$

$$x_2^2 = \frac{\gamma_1 \gamma_0^4}{2q}, \quad y_2 = \gamma_1 \gamma_0^2 \left[\frac{|z|}{q} + n \right]$$

Using a series expansion of $\operatorname{erf}(z)$ in equations (8) and (9) the inverse transform of the Hertz vectors may be evaluated. It is required that

$$\rho \gamma_1 \gg 1 \quad (8-a)$$

$$n^2 > 1 \quad (8-b)$$

$$r' \gamma_0 < 1 \quad (8-c)$$

$$\text{and } \frac{h+z}{r'} \ll 1 \quad (8-d)$$

$$\text{then } \frac{y_1^2}{4x_1^2} = \frac{r' \gamma_0}{2} \left[\frac{h+z}{r'} + \frac{1}{n} \right]^2 \ll 1$$

$$\text{and } \frac{y_2^2}{4x_2^2} = \frac{\gamma_1 q}{2} \left[\frac{|z|}{q} + n \right]^2 \gg 1$$

An ascending power series for $\text{erf}(y_1/2x_1)$ must be used and a descending power series for $\text{erf}(y_2/2x_2)$. These series are,⁸

$$\text{erf}(z) = \frac{2e^{-z^2}}{\pi^{\frac{1}{2}}} \sum_{m=0}^{\infty} \frac{z^{2m+1}}{(2m+1)!!} \quad \text{for } z < 1$$

$$\text{erf}(z) = 1 - \frac{e^{-z^2}}{z\pi^{\frac{1}{2}}} \sum_{m=0}^{\infty} \frac{(-1)^m \Gamma(m+\frac{1}{2})}{z^{2m}} \quad \text{for } z > 1$$

By using the above expansions for the appropriate case and inverting the resulting Hertz vectors term by term the Hertz vector in time domain may be obtained and the EM field components evaluated. Typically, keeping only the first term of the expansion the expressions become,

$$\tilde{\Pi}_z(s) = A(s) \left\{ \frac{e^{-\gamma_0 r}}{r} + \frac{e^{-\gamma_0 r'}}{r'} \left[1 + \gamma_1 \frac{y_1}{x_1^2} - \gamma_1 \frac{\pi^{\frac{1}{2}}}{x_1} e^{(y_1^2/4x_1^2)} \right] \right\}$$

$$\tilde{\Pi}_z'(s) = A(s) \left\{ 2e^{-\gamma_0 h} \frac{e^{-\gamma_1 q}}{q} \left(1 - \frac{\gamma_0 \gamma_1^2}{\gamma^2} \right) \right\}$$

Discussion of the Solution

Since the solution is not exact, it is necessary to justify the approximations used and study carefully the domain of validity for t , ρ , and z . In deriving equations (6) and (7) a rather physical argument was used. As first pointed out by Van der Pol,⁷ the triple integral in equation (4) represents a volume integration over the semi-infinite

space - $\infty \leq z \leq -h$, $0 \leq \rho < \infty$, $0 \leq \varphi \leq 2\pi$ and the integrand can be interpreted as that part of the EM radiation at (ρ, z) due to the scattering at (ρ', ξ, φ) of the radiation emitted by the image dipole located at $z = -h$. Hence if the lower medium is highly dissipative, as earth usually is, only major contributions can be expected from points (ρ', ξ, φ) near the image dipole; hence the approximation. There does not seem to exist any such simple physical interpretation for the integrand of equation (5). However, by the same dissipative behavior of the medium, the exponential in $g(\rho', \xi, \varphi)$ makes the integral converge very fast, especially if $\rho\gamma_1 \gg 1$ as condition (8-a) assumes. The steps immediately following are based on the same arguments. The crucial step was taken when conditions (8-a), (8-b), (8-c), and (8-d) were assumed. They give the domain of validity of the final solution. Taking each condition in turn:

Condition (8-d) states that within a cone, with vertex at $z = h$ and opening toward $z = \infty$, the accuracy of solution will be low and everywhere else the solution will be accurate. Note that (8-d) poses no limitation on the solution $\tilde{\Pi}'_z(s)$. Condition (8-b) is physically correct, n is the index of refraction for EM waves and certainly has a large real part for even a moderately dissipative medium. The more interesting conditions are (8-a) and (8-c). It is to be noted that they are inequalities in s -domain and must be transformed back to the t -domain. Condition (8-a):

$$\gamma_1 \rho \gg 1$$

or

$$\frac{\rho}{\sqrt{s}} \gg \frac{1}{\sqrt{\mu_0 \epsilon_1}} \frac{1}{s \sqrt{s + \frac{\sigma_1}{\epsilon_1}}}$$

provided the denominator does not vanish.

Taking the Laplace inverse of both sides, since the path of integration avoids all singularities of both sides of the above inequality, the condition, in t -domain is:

$$\rho^2 \gg \frac{\pi t}{\mu_0 \sigma_1} [\text{erf}(\sqrt{\frac{\sigma_1 t}{\epsilon_1}})]^2 \quad (10)$$

Substituting typical values of the parameters for an earth medium,

$$\frac{\sigma_1}{\epsilon_1} = 0.9 \times 10^8 / \text{sec} \quad , \quad \sqrt{\mu_0 \epsilon_1} = 1.7 \times 10^{-8} \text{ sec/m}$$

$$\sqrt{\mu_0 \epsilon_0} = 0.33 \times 10^{-8} \text{ sec/m}, \quad \mu_0 \sigma_1 = 2.5 \times 10^{-8} \text{ sec/m}^2$$

formula (10) becomes

$$\rho^2 \gg 10^8 t [\text{erf}(\sqrt{t} \times 10^4)]^2$$

If further, $t > 10^{-7}$ then $\text{erf}(\sqrt{t} \times 10^4) \approx 1$, hence

$$\rho^2 \gg 10^8 t$$

The approximation thus is valid for near field (~ 10 m) at early times ($\sim 10^{-8}$ sec) and the region of validity is a function of time. This property asserts that the approximation gives good results for the initial pulse received at any point of observation as was desired, i.e., to describe the early transient. Conditions (8-a) and (8-c) are also compatible; it is easily shown that both conditions may be satisfied simultaneously. We have the inequality,

$$\gamma_0 r' < 1 \ll \rho \gamma_1$$

or
$$\frac{r'}{\gamma_1} < \frac{\rho}{\gamma_0}$$

Using the same method as before

$$\frac{\sqrt{\rho^2 + (z+h)^2}}{\sqrt{\mu_0 \epsilon_1}} e^{-(\sigma_1 t / 2 \epsilon_1)} I_0(\sigma_1 t / 2 \epsilon_1) \ll \frac{\rho}{\sqrt{\mu_0 \epsilon_0}}$$

Using the previously assumed values of t , $\sqrt{\mu_0 \epsilon_0}$, $\sqrt{\mu_0 \epsilon_1}$, and $\frac{\sigma_1}{\epsilon_1}$, the inequality becomes,

$$\sqrt{\rho^2 + (z+h)^2} \ll \rho \sqrt{t} \times 10^5 \quad (11)$$

Hence as time increases, the depth at which the approximation is valid also increases. This is a reasonable result, in view of the finite rate of propagation of the signal. Furthermore, inequalities (11) and (10)

are also compatible. As a further evaluation of the accuracy of these solutions the field component must be calculated numerically and compared at the boundary $z = 0$ to assess the degree of fulfillment of the boundary conditions.

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