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**A DESIGN STUDY OF A 14,000-AMPERE INJECTION SYSTEM
FOR A FUSION REACTOR**

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A DESIGN STUDY OF A 14,000-AMPERE INJECTION SYSTEM
FOR A FUSION REACTOR*

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ABSTRACT. A design study is presented which indicates that the injection and trapping of steady-state beams of the order of 14,000 A into a fusion reactor should be feasible under present technology using negative ions (D^- , T^- and/or He^-) injected along a magnetic guide field. Each component of the proposed 14,000-A system is based upon technology proven at various laboratories by design studies and by operating apparatus. The only required assumptions are the increase of areas of existing sources and beams and the scientific feasibility of the reactor itself. The major components are the following:

- (1) A wide-area source of negative ions.
- (2) An acceleration system operating under modest requirements of beam density and optics.
- (3) A magnetic guide field in which the beam density is increased by a factor of 100 or more.
- (4) The reactor plasma in which the negative ions are subjected to final acceleration, neutral atom production and trapping, with a trapped fraction up to 84%.

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The techniques can be applied to either a mirror fusion reactor or to a toroidal reactor with the guide field built into a diverter. It appears that the remaining questions on reactor injection are associated with economics and power balance, but not with feasibility.

1. INTRODUCTION

The injected beam requirement for a 1000-MW mirror fusion reactor with direct energy recovery has been estimated in the range of 2000 to 14,000 A, depending upon the injection energy and other factors. Forsen's estimate¹ of 14,000 A probably represents an upper limit for this requirement, since apparently a relatively low injection energy (about 200 keV) was chosen to maximize the D-T fusion yield rather than to optimize the overall efficiency and economics by a higher energy.

The purpose of this paper is to show that such a requirement is within the reach of present technology, using negative ions injected along a magnetic guide field into the reactor. This statement is not yet true for conventional neutral injection systems which require extrapolations beyond present technology with respect to injector power density, gas loading, and other practical problems. Charged-ion injection has been neglected in several recent steady-state injection studies² because of the lack of a credible trapping mechanism. Such a mechanism will be explained in the appropriate section of this paper.

The credibility of each component of the 14,000-A system will be established by showing that its performance is attainable using techniques

developed and proven at various laboratories by design studies and by operating apparatus. The only required assumptions are the increase of areas of existing sources and beams and the scientific feasibility of the reactor itself. The major components are the following:

1. A wide-area source of negative ions (D^- , T^- , or ${}^3He^-$).
2. An acceleration system operating under modest requirements of beam density, optics and voltage.
3. A magnetic guide field in which the beam density is increased by a factor of 100 or more as the negative ions are guided from the low-density acceleration system to the high-density reactor.
4. The reactor plasma in which the negative ions are subjected to final acceleration, neutral atom production and trapping.

2. HIGH CURRENT SOURCES OF NEGATIVE IONS

Intense beams of negative ions have been produced by electron capture in gas or vapor cells³ and by direct extraction from calutron-type ion sources.⁴ As a model for the 14,000-A system we shall use the alkali metal vapor technique which is suitable for a large-area application.

Most relevant for the immediate purpose is a test reported by Osher and Hamilton⁵ based upon Osher's computations,⁶ and a patent by Post et al. for using the alkali metal vapor technique to produce D^- to be accelerated and transformed to D^0 for neutral injection. The same technique had been used for negative ion production for other purposes at many other laboratories.

In this test, a 200-mA beam of 1.5-keV D^+ ions was transformed to D^- by electron capture in a cesium vapor cell with 25% efficiency. The D^+ beam was emitted by a multiple-aperture, low-energy source (MATS)⁷ with an output of 50 mA/cm² averaged over a 30-cm² overall source area. Only a portion of the MATS beam was used in the test; however, the entire beam will be acceptable for the 14,000-A system—including its 8-eV transverse ion energy. The conversion efficiency has been independently measured by Schlachter et al.⁸ with results $(21 \pm 4\%)$ agreeing within the accuracy of the measurements. Compatible results have also been reported by Gruebler et al.⁹

From these data we can conclude that a wide-area source of 1.5 keV D^- with an output of 12.5 mA/cm² is now available. There are reasons to believe the output can be increased by an order of magnitude, but this is not necessary for the 14,000-A system. The required source area is 200 m² to provide 25,000 amperes of D^- and T^- and thereby allow for a 70% accelerator transparency and 80% trapping efficiency. Construction of 1.5-m-diam sources and extraction grids has been proven feasible for electrostatic propulsion.¹⁰

Negative helium ions can be produced by a similar process by electron capture in alkali metal vapor by metastable helium atoms.¹¹ The maximum reported efficiency is only 1.2%, using 3-keV primary $^4He^+$ ions (equivalent to 2.25-keV $^3He^+$ ions). We can therefore expect to produce 0.6 mA/cm² He^- from a wide area source, which is adequate for a high energy He-D reactor. Because of the high energy and the low He content in an optimized reactor plasma, the He beam requirement is only about 500 A.

The conversion efficiency for D^- , T^- , and He^- can be improved by multiple vapor cells.⁶ However, it has not yet been proved whether multiple cells will also improve the current density, and therefore we will consider multiple cells as not yet strictly within our existing technology.

The source parameters discussed above are summarized in the first section of Table 1.

3. MULTIPLE-APERTURE ACCELERATOR

The 17,500-A accelerator will be preceded by a grid, biased at a few hundred volts to prevent thermal electrons formed in the alkali vapor cell from entering the high-voltage accelerator. The potential profile of Fig. 1 shows the electron-suppression grid relative to the potentials of the low-voltage ion source and the high-voltage accelerator. Grids such as these have been tested by Moir et al.¹² in beam energy recovery systems. They have been proven successful in suppressing electrons from entry into a high-voltage gap equivalent to the high-voltage accelerator, and have survived while being directly exposed to calutron beams of voltages (20 kV) and power densities (200 W/cm^2) much higher than those of the low-energy negative ion beam to be produced in the vapor cell. If a few electrons somehow are able to penetrate through the biased grids, their energy can be recovered after they are accelerated and magnetically separated from the negative ion beam.

The low-voltage grids will not affect the 1.5-keV negative ions produced in the vapor cell. The grid lifetime will be determined by sputtering rather than heating because of the low beam energy and current density. The grid lifetime can be long because there is no need for a transparency greater than 90%.

A considerable body of experience has been acquired on high-current, multiple-aperture injectors for controlled fusion since the first publication on this topic appeared in 1967.¹³ Multiple apertures had earlier been developed for propulsion and other technologies, but were optimized for other purposes and therefore were overlooked within the controlled fusion program except for some unpublished exploratory work. The oversight, documented for example by Kelley's 1967 review,¹⁴ was world-wide and therefore not the fault of any individual.

The modest densities of beam current (12.5 mA/cm^2) and beam power (1.72 kW/cm^2) required for the 17,500-A accelerator represent little problem at 200 keV. Many examples can be quoted of current and power densities in pulsed systems far higher than these requirements. In continuous systems it appears that the immediate limitations are due to electrode heating caused by stray beam particles and electrons. This limitation will probably be raised by improvements in the optics. Examples can be found of already-achieved continuous beams with the required densities; therefore the 17,500-A accelerator does not necessarily require improvements in the optics beyond present technology.

High-current, single-aperture systems such as the injector for DCX-II¹⁵ have continuously operated with beam densities far above the requirements. It has not yet been proven whether many such injectors can be stacked together in a close array at the energies of DCX-II. However, the required beam power density has been achieved with multiple apertures by a 50 kV source¹⁶ continuously operated at 70 mA/cm^2 ; no attempt has been made to improve upon this for continuous operation since 1968. In the 50 keV

test most of the stray beam was eliminated by an intermediate accel-decel stage at a lower voltage. Therefore, direct cooling of electrodes was not necessary. The test result was a beam power density of 3.5 kW/cm^2 with a beam divergence of less than 1° . This result has essentially been confirmed at other laboratories using lower voltages.

Because of the low current density and high accelerator voltage, accelerator aperture dimensions may be relatively large ($\geq 1 \text{ cm}$) in respect to space charge limitations. Rigidity and water-cooling of accel-decel grids are therefore compatible with high transparency.

We shall specify an allowable beam divergence after acceleration of $\pm 1^\circ$, which is acceptable for injection into the guide field. This corresponds to a transverse ion energy of 60 eV or more. The required beam brightness as defined by Kelley¹⁴ is not greater than $5.2 \times 10^7 \text{ mA/MeV-cm}^2\text{-rad}^2$, which is one or two orders of magnitude lower than the brightnesses attained by many existing beams.^{7, 13, 14, 15, 16} The low requirement for brightness is due to the very large phase space acceptable for injection into the guide field.

With the background indicated above, it can be concluded that the modest requirements of the 17,500-A accelerator are clearly within the reach of existing technology. Multiple slots seem to be more suitable than circular apertures in order to avoid thermal distortion and to provide for water cooling. Accelerator specifications for three possible reactor injection systems are listed in Part 2 of Table 1.

4. MAGNETIC GUIDE FIELD

As a model for the magnetic guide field to conduct the negative ions from the low-density accelerator to the high-density reactor, we will use a field designed by Post¹⁷ as an expander for an energy recovery system (Figs. 2 and 3). Post's guide field is shaped like a fan, extending over 180° or more, at each end of a projected mirror fusion reactor. The field is radial to guide the plasma lost from the reactor to an energy recovery system at the perimeter of the expander. For the system designed by Post, the energy recovery system requires 628 m^2 —a tube of flux 1 m thick defined by the perimeters of the two expanders, each about 100 m in radius. Post's dimensions, chosen for a high-energy reactor, are adequate for the injection guide field throughout the range of reactor energies (200-1000 keV), although they may be inadequate for energy recovery for a low-energy system.

We propose to extend Post's expander guide field by 10% to 30% as shown in Fig. 2 to provide the guide field for the injection system. The injection system may be divided into four sections, with a total source area of about 100 to 200 m^2 . After magnetic compression by a factor of 100/1 or more, the injected beam area will be 1 to 2 m^2 as it enters the magnetic bottle where B_{max} is about 150 kG. The third section of Table 1 shows some possible guide field specifications for the three reactor examples.

To establish the credibility of the injection guide field it is necessary to consider the structural, thermal and vacuum feasibility, the cost, the stability of flow, the plasma escaping from the reactor into the expander,

beam optics, beam space charge, and background plasma. The first three items on this list (feasibility, cost, and stability) have been adequately considered by Post and by a Direct Energy Recovery Study Group.¹⁸ The injector vacuum problem is tractable because there is no neutralizer in the system and because ample room is available for pumping the gas from the source and from the beam. The internal supports of the chamber must not be exposed to the injected beam even at 3.5 kW/cm^2 , but this is no problem.

Stability analysis has shown that the flow in the expander guide field is quite stable in respect to instabilities such as the ion-acoustic and the universal drift instabilities.¹⁸ This is because of the low beam density and the brief transit time for the single transit through the expander.

Presumably, the flow through the injector will be stable for the same reasons.

The required beam particle density (10^{10} cm^{-3}) and compression ratio (about 100/1) have already been attained on a miniature scale by a combination of electrostatic focusing and magnetic guidance of a 2.5-mA H^- beam produced by a calutron-type source for injection into a cyclotron.¹⁹ No evidence of beam instability or blow-up was reported, although the 13-keV H^- beam was much more sensitive to such effects than the high energy beams required for reactor injection.

Further testing is advisable to make sure of the stability of a beam in a full-size guide field. However, no evidence now exists to indicate that difficulties of this nature would be encountered under reactor conditions.

It is fortunate that the edges of the guide field are most suitable for injection and trapping, while least suitable for energy recovery. If the mirror field is slightly increased at the injection sections, the scattered ions will not

enter the injection system but will go out the opposite end, or will precess along drift surfaces until they escape into the recovery system.

The beam optics in the guide field have been numerically computed by Post and are consistent with adiabatic behavior, except for the non-adiabatic transition between the 500- to 1500-G field at the perimeter and the zero field outside the guide field. One of Post's computed trajectories is shown in Fig. 3. The only optical requirement after the ion has entered the guide field is imposed by the maximum pitch angle θ_m desired for injection into the reactor, i.e.,

$$\frac{\sin \theta_1}{\sin \theta_m} = (B_1/B_{\max})^{1/2}.$$

We shall soon find that because of a limitation at the neck of the magnetic bottle it is prudent but probably not essential to require the pitch angle θ_m to be less than 10° where the beam enters the reactor under conditions of maximum density. Therefore, we shall require that the compression ratio $B_{\max}/B_1$ be not more than 100 and that the pitch angle θ_1 be not greater than 1° as the beam enters the guide field.

The bottleneck limitation does not apply to the He^- injection system because of the lower He^- density available at the source. Therefore in Table 1 we have indicated a 300/1 compression ratio and $\theta_m = 18^\circ$ for He^- .

Post has shown that the transverse impulse applied during the nonadiabatic transition as the ion enters the guide field can be computed by Busch's theorem. Using Cartesian coordinates as shown in Figs. 2 and 3 where the

beam velocity is parallel to the z axis and the plane of the fan-shaped guide field is the x-z plane, Busch's theorem takes the following form²⁰:

$$m\delta \dot{x} = q B_1 y$$

where y is the distance from the plane of symmetry to the ion trajectory, and $m\delta \dot{x}$ is the transverse impulse received.

It is necessary to correct the optics for this transverse impulse if $\delta x/v > \sin 1^\circ$. The correction angle is a function of the coordinate y, and attains a maximum value of 47° for the extreme case (200 keV D^+ , $B_1 = 1500$ G; $y = 0.5$ m). This correction can be accomplished by shaping the multiple-slit accelerator as a linear function of y. The correction need be precise only within 1° . Alternatively, by shaping the flux bundle at the perimeter it is possible to reduce the dimension y and thereby reduce the required correction. It may be possible to avoid the transition by immersing the accelerator within the 1500-G guide field.

Space charge neutralization is required of any intense charged beam to avoid blow-up. This is normally provided by a background plasma produced by ionization of the low-density background gas. The background plasma inherently has a residual positive ambipolar potential of a few volts or hundreds of volts because the background electrons are more mobile than the background ions. The positive ambipolar potential is required to equalize the loss rates of ions and electrons from the background plasma. The result generally observed is self-focusing of a beam of negative ions or electrons because of the positive ambipolar potential of the background plasma. Beam blow-up is a common problem for positive ions,¹⁵ but seldom occurs with negative ions.¹⁹

We now come to the bottleneck limitation—a fundamental limitation on beam density within which the system must be designed. If the background plasma in the guide field is too dense, the negative ions will prematurely lose their extra electrons by collisions with the background plasma. This problem is most acute at the bottleneck where the magnetic field, beam density and background plasma density all attain their maximum values. To avoid losing a large fraction of the beam, it is prudent to include three precautions in the system design:

1. The vacuum conditions must be controlled such that the background plasma density is not appreciably higher than the beam density. For example, if beam density is 1 A/cm^2 at 200 kV, the background plasma density must be not much more than $1.4 \times 10^{10} \text{ cm}^{-3}$, equivalent to a gas pressure of $4.3 \times 10^{-7} \text{ Torr}$ if fully ionized.

2. If the tolerable electron detachment is 2%, the beam density and background plasma density must be such that

$$n_{\text{background}} \sigma L < 0.02$$

where

$$j \leq n_{\text{background}} qv \leq \frac{0.02 qv}{\sigma L}$$

and where L is the path length through the bottleneck where the problem is acute. Using Post's magnetic field profile, shown by Fig. 3, $L \leq 5 \text{ m}$ in the 150-kG bottleneck region. Elsewhere in the guide field, the densities are reduced by a factor of 25 or more.

The cross section for electron detachment from H^- by electron impact has been measured by Dance et al.²¹ and has been found to vary from

1.0 to $2.5 \times 10^{-15} \text{ cm}^2$ for relative velocities in the range of interest for reactor injection. The worst condition is with the lowest energy for which v/σ is minimum and the current requirement is maximum. We find that j at B_{max} is limited to approximately 1 A/cm^2 under worst conditions. The parameters of Table 1 indicate that this limitation is compatible with all requirements for reactor injection, and is insignificant at higher energies.

3. The beam is not lost by electron detachment in the bottleneck if the pitch angle is small enough so the resulting D^0 enters the reactor. If the pitch angle is less than 10° almost all the neutrals produced within 5 m will enter the reactor. The net loss will be much smaller than 2% because of this extra precaution.

5. ACCELERATION, NEUTRAL ATOM PRODUCTION, AND TRAPPING BY THE REACTOR PLASMA

The concept of using the reactor plasma for final acceleration, neutral atom production and trapping of the injected beam is apparently new. This technique overcomes the usual objections to charged-particle injection based on Liouville's theorem, and also changes the pitch angle as required for trapping.

Charged-particle injection into steady-state controlled fusion devices has been neglected since the DCX experiments were completed. Injection perpendicular to the magnetic field suffers from the difficulty that the injected ion penetrates only one gyro-diameter into the magnetic field. Injection of positive molecular ions along field lines was considered, but was rejected

without testing or publication when it was realized that charged products of dissociation or ionization would have the same pitch angle as the injected ions and would therefore be lost after a single transit. This objection does not apply to neutrals derived from negative ions.

A mirror-contained plasma has a positive ambipolar potential because the electrons are scattered more rapidly into the loss cone than the ions. The ambipolar potentials of the reactor examples computed by Futch et al.²² are approximately one-third of the injection energy. Most of this potential gradient is applied at the outer boundary of the reactor plasma. Therefore, the negative ion receives its final acceleration as it enters the plasma. The feature was pointed out by Perona.

A Fokker-Planck calculation will be required to determine the effect of the final acceleration upon the reactor Q . It seems that any mechanism that recycles energy from electrons back into the reacting ions will improve the Q , even including the effect of the extra electron introduced into the reactor. At most, the improvement in Q and the reduction of requirements for beam voltage and power will be 20% to 30%. In a later paper, several effects of injected beams upon reactor containment will be discussed.

Riviere has pointed out that the cross section for electron loss by D^- is much greater than that for D^0 because the outer electron is bound by only 0.75 eV.² Consequently, the first electron will be collisionally stripped after a relatively short penetration of the reactor plasma, with virtually 100% probability of neutral atom production. The resultant neutral atom will penetrate much more deeply into the plasma before losing the second electron.

For trapping, it is necessary but not sufficient to overcome Liouville's theorem by a nonreversible process such as removal of the two electrons.

It is also necessary to change the pitch angle such that $\theta_- < \theta_{LC} < \theta_+$, where θ_- is the pitch angle of the negative ion injected through the loss cone, θ_{LC} is the loss cone angle, and θ_+ is the pitch angle of the trapped positive ion.

The magnetic field direction is highly nonuniform in the reactor plasma. Therefore, the change in pitch angle can be accomplished by choosing an injection path, as shown in Fig. 4, such that the direction of $\vec{B}$ changes as the atom travels through the plasma. The probability of trapping can be enhanced by choosing a strongly curved field line for injection. For this reason, we have chosen to inject the negative ions through a guide field leading into the edge of the reactor where the curvature is maximum, as seen in Fig. 4.

The probability of trapping will also depend upon the plasma density profile, since this will determine where the second electron is removed. The density profile will tend to adjust itself in a favorable way, since the density will be high where the trapping is most effective. Some control of the density profile is possible by shaping the magnetic field. In the Appendix, we have computed the trapping fraction expected by injection of negative ions into a Yin Yang reactor, with strongly curved field lines and under realistic ranges of plasma thickness and effective mirror ratio. In this example the trapping fraction varies from 30% to 84%, depending upon parameters. Even higher trapping fractions could be attained if the density profile were more strongly peaked in the center of the plasma.

The untrapped beam will be ionized and go out the ends into the energy recovery system if the plasma is thick enough. The untrapped beam will be spread throughout the energy recovery system at an average density of less than 1 mA/cm^2 .

6. CONCLUSIONS

Using a magnetic guide field for injection of negative ions, it appears to be possible to inject and trap beams of the order of 14,000 A, using technology now available, at energies in the range of mirror fusion reactors. Injection of negative ions rather than neutrals simplifies or eliminates problems associated with gas loading, magnetic shielding, beam optics, beam density and reactor Q. Probably the optimum injection energy will be somewhat higher than the energy of maximum D-T fusion yield. The remaining questions on reactor injection are associated with economics, efficiency and power balance, but apparently not with feasibility. The same techniques can be used for injection into toroidal reactors, using a guide field built into a diverter structure.

For applications below 100 keV, injection of positive molecular ions also deserves consideration for neutral production within the reactor plasma.

APPENDIX

TRAPPING FRACTION ATTAINABLE BY NEGATIVE ION INJECTION

In this Appendix, we will compute the trapping fraction attainable by injection of negative ions along field lines into a fusion reactor, to be converted to neutrals by collisional electron detachment and subsequently to be trapped by collisional ionization. The precise value of this trapping fraction is important for computations involving the efficiency and economics of a reactor system, but is not essential merely to demonstrate feasibility since the source area can, in principle, simply be scaled as required by the trapping fraction. We will speak in terms of D^- , D^0 , and D^+ , but the analysis is also valid for injection of other negative ions.

We shall denote as L_- and L_0 the average penetration distances traveled through the reactor plasma by the negative ion and by the neutral atom before the electrons are removed by collisional detachment and ionization. The integrated plasma thicknesses T_- and T_0 corresponding to L_- and L_0 are

$$T_- = \int_0^{L_-} n(\ell) d\ell = \frac{v_-}{\sum \langle \sigma_- v_{rel} \rangle} \quad (\text{particles/cm}^2)$$

and

$$T_0 = \int_{L_-}^{L_-+L_0} n(\ell) d\ell = \frac{v_0}{\sum \langle \sigma_0 v_{rel} \rangle} \quad (\text{particles/cm}^2). \quad (A-1)$$

In these definitions of plasma attenuation thicknesses, the numerator is the injected speed and the denominator is the sum of the rates of electron detachment, ionization, and charge exchange due to collisions with plasma ions and electrons, averaged according to relative velocities. Riviere²³

has carefully computed T_0 for the penetration of D^0 beams injected into reactor plasmas. Riviere's result is that T_0 ranges from $0.2 \times 10^{16} \text{ cm}^{-2}$ for 100-keV injection energy to $2.0 \times 10^{16} \text{ cm}^{-2}$ for 1000 keV. A good estimate of T_- may be obtained by the assumption that the plasma penetration is inversely proportional to the cross section for electron removal by collisions with plasma electrons:

$$\frac{T_0}{T_-} \approx \frac{\sigma_-}{\sigma_0} \approx 20 \text{ for equal velocities.} \quad (\text{A-2})$$

The cross sections σ_- and σ_0 for electron detachment from H^- and for ionization of H^0 by electron collisions have been measured by Dance et al.²¹ and by Fite and Brackman,²⁴ respectively. The cross section σ_- for electron detachment of D^- is large because of the low binding energy (0.75 eV) of the extra electron.

It is apparent from the definitions (A-1) that the plasma penetration and therefore the trapping fraction will depend upon the density profile $n(\ell)$. This profile will tend to adjust itself in a favorable way, since the density will be greatest where the trapping is most effective. We will not count upon this advantage, but use as an example a realistic model resembling profiles computed by Hamilton and Moir,²⁵ determined by Coulomb scattering in a magnetic mirror field. As a model, we will use

$$\frac{n(\ell)}{n_0} = \frac{1}{2} (1 + \cos \pi \ell / R) . \quad (\text{A-3})$$

Here we have defined $\ell = 0$ where the negative ion enters the reactor plasma at the magnetic mirror; R is the plasma radius, measured from the center of the

plasma to the magnetic mirror. The central plasma density, at $\ell = R$, is denoted as n_0 . This density profile is illustrated by Fig. A-1(b).

In order to write the differential equations expressing the penetration of D^- and D^0 through the plasma, we shall introduce the variable $x(\ell)$, defined as the integrated thickness:

$$x(\ell) = \int_0^\ell n(\ell') d\ell' = \frac{n_0 R}{2\pi} (\pi \ell / R - \sin \pi \ell / R) . \quad (A-4)$$

The differential equations representing the attenuation and production of the three species D^- , D^0 and D^+ as the beams I_- , I_0 and I_+ penetrate the reactor plasma are the following:

$$\frac{dI_-}{dx} = - I_- / T_-$$

$$\frac{dI_0}{dx} = + I_- / T_- - I_0 / T_0$$

$$\frac{dI_+}{dx} = + I_0 / T_0 \quad (A-5)$$

Subject to the initial conditions $I_- = I$ and $\ell = x = I_0 = I_+ = 0$, the solutions are

$$I_- / I = e^{-x/T_-} \quad (A-6)$$

$$\frac{I_0}{I} = \frac{T_0}{(T_0 - T_-)} (e^{-x/T_0} - e^{-x/T_-}) \quad (A-7)$$

and

$$\frac{I_+}{I} = \frac{1}{(T_0 - T_-)} \left[T_0 (1 - e^{-x/T_0}) - T_- (1 - e^{-x/T_-}) \right] . \quad (A-8)$$

These solutions are shown in Fig. (A-1) as functions of ℓ and of $x(\ell)$.

We shall determine the trapping fraction first by determining the interval $\ell_1 < \ell < \ell_2$ in which the pitch angle is suitable for trapping, and then by finding the fraction I_+/I of trapped D^+ produced within this interval.

$$\begin{aligned} \text{Trapping fraction} &= \frac{1}{I} \int_{\ell_1}^{\ell_2} \frac{dI_+}{d\ell} d\ell \\ &= \frac{I_+}{I} (\ell = \ell_2) - \frac{I_+}{I} (\ell = \ell_1) . \end{aligned} \quad (\text{A-9})$$

The trapping fraction is strongly dependent upon ℓ_1 , but is almost independent of ℓ_2 because the beam is almost totally ionized after it has passed through the center of the plasma. For optimized conditions, $\ell_2 \approx 2R$. I_+/I is almost 100% at ℓ_2 , as seen by Eq. (A-8), plotted in Fig. (A-1).

The distance ℓ_1 is the sum of a and b , which are the penetrations by the D^- and D^0 respectively before trapping begins. A relatively short penetration distance a is first required to detach the first electron and thereby convert the injected D^- to D^0 . On the average, $a = L_-$, which is defined by (A-1). This can be determined from (A-4) and (A-6) by the condition that the D^- beam is attenuated by a factor of e at $\ell = a$, i.e.,

$$x(a) = T_- = \frac{n_0 R}{2\pi} (\pi a/R - \sin \pi a/R) . \quad (\text{A-10})$$

We shall define the dimensionless reactor plasma thickness, $n_0 R/T_0 \approx n_0 R/20T_-$. The ratio a/R can be determined for any value of $n_0 R/T_0$ by solving the transcendental Eq. (A-10) or by reading ℓ/R directly from Fig. A-1, such that $I_-/I = 1/e$.

The additional distance b is the distance the neutral atom must travel through the plasma such that its pitch angle θ_B becomes larger than the loss cone angle θ_{LC} . If the average radius of curvature of the magnetic flux lines is ρ in the region of interest, the change in pitch angle is

$$\theta_B - \theta_A = b/\rho$$

and the condition for trapping is

$$\theta_B = \theta_A + b/\rho > \theta_{LC} = \sin^{-1} R_{eff}^{-1/2} \quad (A-11)$$

where the effective mirror ratio R_{eff} at the point of ionization takes into account the magnetic field ratio, the plasma potential, and the plasma diamagnetism.

In Eqs. (A-11), we are defining θ_A and θ_B as angles lying in the plane defined by the curved field line, as indicated in Fig. 4. θ_A is the projection of θ_- on this plane, where θ_- is the pitch angle of the injected D^- and γ is the phase angle of its gyration at the instant of electron detachment; i.e., $\sin \theta_A = \sin \theta_- \cos \gamma$.

For a feasibility estimate, we can neglect θ_A in Eq. (A-11) if θ_- is small. The average value of θ_A is zero because of the random variation of γ . This approximation is not valid for large values of θ_- because of the asymmetries of the magnetic field.

The distance b required to increase the pitch angle for trapping is therefore $b \geq \rho \theta_{LC}$. Obviously, conditions will be optimum if the radius of curvature ρ is small and if R_{eff} is large.

In the strongly curved Yin-Yang magnetic field designed by Moir and Post [26], shown by Fig. 4, $\rho \leq R/2$ in the region suitable for injection. This

statement is also true in respect to the magnetic fields of diverters through which charged particles may be injected into toroidal containment devices. We shall therefore set $\rho \sim R/2$ as an example, and compute the trapping fraction with various values of R_{eff} and the normalized reactor plasma thickness, $n_0 R/T_0$. The result is shown by Fig. A-2 for values of R_{eff} from 2 to 10 and for values of $n_0 R/T_0$ from 0 to 8.

To compute the trapping fraction we have used (A-9), where I_+/I is given by (A-8) $\ell_2 = 2R$, and $\ell_1 = a + b$. The average D^- penetration a is given by (A-10), and the required D^0 penetration is $b \geq \frac{R}{2} \theta_{LC}$.

To avoid excessive wall bombardment by un-ionized D^0 , we must require $n_0 R/T_0 \geq 4$ for this density profile. This requirement is consistent with other requirements for reactor plasma density.

We find that the trapping fraction is high, 30% to 84%, within this range of parameters. Even better trapping efficiency would be obtained if the density profile were more strongly peaked in the center of the reactor.

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Table 1

INJECTED BEAMS FOR THREE POSSIBLE MIRROR FUSION REACTORS

	D-T or D-D Reactor		D-He Reactor	
	1.5 D ⁺ or 2.25 T ⁺		D Beam	He Beam
1. Negative Ion Source				
Primary low-energy beam energy (keV)	1.5 D ⁺ or 2.25 T ⁺		1.5 D ⁺ and 2.25 He ⁺	
Source current density (mA/cm ²)	50		25	50
Efficiency for conversion to negative ions	25%		25%	1.2%
Negative ion source densities ^a (mA/cm ²)	12.5		6.25	0.6
2. Accelerator				
Examples of injection energy ^b (keV)	200	400	800	800
Examples of total trapped current required (A)	14,000	6000	1500 and	500
Total injected current ^c (A)	17,500	7500	1875	625
Source area required ^d (m ²)	200	86	43	150
Accelerator power density (kW/cm ²)	1.72	3.5	3.5	0.33
3. Guide Field				
Magnetic compression ratio, B _{max} /B ₁ (B _{max} = 150 kG)	100/1	100/1	100/1	300/1
Beam area at B _{max} (m ²)	2.0	0.86	0.43	0.5
Current density at B _{max} (mA/cm ²)	875	875	437	125
Maximum pitch angle within guide field:				
Entering θ_1	±1°	±1°	±1°	±1°
Leaving θ_m	±10°	±10°	±10°	±18°
Magnetic flux required for injection guide fields (Webers)	30	13	8.5	7.5

^aTo be multiplied by the accelerator transparency (about 70%) and the trapping fraction (about 80%).

^bNot including final acceleration.

^cEquals trapped current/80% trapping efficiency.

^dEquals $\frac{\text{Total injected current}}{\text{Source current density} \times 70\% \text{ accelerator transparency}}$.

FIGURE CAPTIONS

- Fig. 1** Potential profile of low-voltage source of negative ions, relative to potential of vapor cell. Applied potentials are the following:
1. Plasma source, D^+ , T^+ or He^+ ; +1.5 kV for D^+ .
 2. Low voltage accel-decel extraction grids, +1.5, -0.5, and 0 kV.
 3. Alkali vapor cell; $D^+ \rightarrow D^-$. The actual potential is negative high voltage, but is shown here as 0 kV.
 4. Electron suppression grids, -0.5 kV; to prevent thermal electrons from entering the high-voltage accelerator.
 5. High voltage accelerator. First accel-decel stage perhaps +20 to +100 kV.
- Fig. 2** Magnetic guide fields for injection and energy recovery.
- Fig. 3** Calculated particle trajectory through expander, reproduced from Ref. 14. Notice that the bottleneck region (1) where $B = 150$ kG and the beam density is maximum is less than 5 meters long. Most of the neutrals produced in this region with pitch angles less than 10° will enter the reactor.
- Fig. 4** Negative ion injection and trapping in a reactor with a Yin-Yang magnetic field. Here are shown the penetration distances a and b for the injected D^- and D^0 ; the pitch angles θ_A and θ_B ; the plasma radius R ; and the field line radius of curvature $\rho = R/2$. The shape of the magnetic field lines is reproduced from Ref. 22.

Fig. A-1 Attenuation and production of injected D^- , D_0 , and D^+ . In this example the relative thickness of the reactor plasma is $n_0 R/T_0 = 4$.

(a) I_-/I , I_0/I , and I_+/I as computed by Eqs. (A-6), (A-7), and (A-8) as functions of the relative thickness x/T_0 , assuming $T_0 = 20 T_-$.

(b) I_-/I , I_0/I and I_+/I as functions of the relative penetration distance ℓ/R , assuming the density profile, $n(\ell)/n_0 = \frac{1}{2}(1 + \cos \pi \ell/R)$. The definition $x(\ell) = \int_0^\ell n(\ell') d\ell'$ is used to transform coordinates from x/T_0 to ℓ/R . Also shown are the penetration distances a and b required for trapping for this example.

Fig. A-2 Trapping fraction computed from Eq. (A-9) as a function of reactor plasma thickness $n_0 R/T_0$ and the effective mirror ratio R_{eff} . To avoid excessive bombardment of reactor walls by un-ionized D^0 , we must require $n_0 R/T_0 \geq 4$. For the range of parameters indicated, the trapping fraction varies from 30% to 84%.

Hamilton - Fig. 1

Negative ion source assembly

1×50 meter.

Total of 4 required for area of 200 m^2 .

Energy recovery system

1×314 meters 2 required

Injection guide field
100 meters long.

Total of 4 required.

Expander guide
field total of 2
required (one at each
end of reactor.)

Reactor
150 kG

z

x

Scale — meters

Hamilton - Fig. 2

— Particle trajectory
 - - - Magnetic field line (schematic)
 Particle: 500 keV deuteron

- ① $B_z \approx 150$ K gauss
- ② 6 K gauss
- ③ 450 gauss
- ④ 600 gauss
- ⑤ 70 gauss

Hamilton - Fig. 3

Hamilton - Fig. 4

Hamilton - Fig. A-1(a)

Hamilton - Fig. A-1(b)

Hamilton - Fig. A-2