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MASTER

SIMCON

A DIGITAL COMPUTER PROGRAM FOR COMPUTING
THERMIONIC CONVERTER PERFORMANCE
CHARACTERISTICS

U.S. ATOMIC ENERGY COMMISSION
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NUCLEAR THERMIONIC POWER OPERATION
NUCLEAR TECHNOLOGY DEPARTMENT
GENERAL ELECTRIC
PLEASANTON, CALIFORNIA

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SIMCON
A DIGITAL COMPUTER PROGRAM FOR COMPUTING
THERMIONIC CONVERTER PERFORMANCE CHARACTERISTICS

D. R. Wilkins

February 1, 1968

For
U. S. Atomic Energy Commission
Contract No. AT(04-3)-189
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NUCLEAR THERMIONIC POWER OPERATION
Nuclear Technology Department
General Electric Company
Pleasanton, California

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SIMCON

A Digital Computer Program for Computing
Thermionic Converter Performance Characteristics

I. INTRODUCTION

A theoretical description of thermionic converter operation is an important tool for interpreting experimental converter measurements, evaluating schemes for converter improvement, developing converter diagnostic techniques, and designing power plants employing thermionic converters. The equations which describe important phenomena in the thermionic converter are nonlinear, however, and numerical results are obtained only through iterative calculations.

This report describes the theoretical foundation and provides a user's manual for a digital computer program (SIMCON - SIMulated CONverter) which performs the above calculations and provides a numerical description of the electrical, thermal and physical operating characteristics of cesium thermionic converters. The program user must supply certain data which characterizes the converter electrodes and interelectrode gas; and the converter operating point defined by the electrode and vapor reservoir temperatures, interelectrode spacing, and output current density. Using this input, the program obtains iterative solutions of the nonlinear equations which describe governing phenomena within the converter. The program returns the converter output voltage and intrinsic efficiency, and many other converter operating parameters of practical interest (e.g., electrode work functions, sheath and neutral plasma voltage drops, etc.).

The SIMCON program has numerous practical applications relating both to basic studies of thermionic converter physics, and to the engineering design and analysis of thermionic converters and power plants.

II. THEORY

A. BACKGROUND

This section contains a theoretical description of the electrical and thermal performance characteristics of cesium thermionic converters. Preliminary reports on this work have been presented.^(1,2)

The development of a useful analytic description of thermionic converter performance requires that consideration be given to electrode surface and interelectrode volume phenomena, and to Schottky effects which occur at the electrode surface-interelectrode volume interfaces. Phenomena in each of these areas have been separately considered in previous theoretical studies.⁽³⁻¹⁸⁾ For example, in studies of surface phenomena, Rasor and Warner,⁽³⁾ Levine and Gyftopoulos⁽⁴⁾ and Steiner and Gyftopoulos⁽⁵⁾ have derived expressions for the work function of a substrate metal partially coated by an adsorbate film. With regard to volume phenomena, Hansen and Warner,^(6,7,8) Houston,⁽⁹⁾ Shavit and Hatsopoulos,⁽¹⁰⁾ and Wilkins and Gyftopoulos⁽¹¹⁾ have studied thermionic converters operating in the extinguished mode, while Rasor,⁽¹²⁾ Hansen,⁽¹³⁾ Warner,⁽¹⁴⁾ Wilkins and Gyftopoulos,⁽¹⁵⁾ and Ingold⁽¹⁶⁾ have studied converters operating in the ignited mode. Schottky effects at the emitter and collector electrodes in thermionic converters have previously been studied by Hansen.^(17,18) These studies have served to isolate and to clarify many of the individual physical processes which are significant in thermionic converter operation. No previous attempt has been made, however, to combine and extend the previous analysis to provide a single, unified theoretical description of thermionic converter operation.

In the present section, a unified theoretical description of thermionic converter performance characteristics is developed. Significant features of the analysis are that it: (a) simultaneously accounts for the influence of surface and volume phenomena and Schottky effects on converter operation, (b) contains no restrictions concerning the "mode" of converter operation,

(c) deals with the plasma sheath polarities self-consistently, (d) is applicable to converters with elevated collector temperatures, and (e) provides for the determination of converter efficiency.

B. VOLUME PHYSICS

1. Plasma Analysis

The neutral interelectrode plasma in a thermionic converter may be analyzed by using a previously derived set of transport differential equations.⁽¹⁹⁾ Since plasma electron temperatures are nearly uniform,⁽²⁰⁾ these equations may be written in the well-known approximate form:

$$(dJ_e/dx) = (dJ_i/dx) = e(\nu_i n - \beta_r n^3) \quad (1)$$

$$J_e = -e\mu_e [\theta_e (dn/dx) + nE]; \quad J_i = -e\mu_i [\theta_i (dn/dx) - nE] \quad (2)$$

where J_α , μ_α , and θ_α are the current density, mobility, and voltage equivalent temperature for the electrons ($\alpha=e$) and ions ($\alpha=i$), respectively; n is the charged particle density; E is the electric field; ν_i is the ionization frequency; and β_r is the three-body recombination coefficient.

The source terms in Equation (1) describe a multistage volume ionization recombination process.⁽¹⁵⁾ The three-body recombination coefficient for Cs is assumed to have a form similar to that for hydrogen,⁽²¹⁾ since both atoms have a valence of unity and similar binding energies for the higher excited states. In particular, it is assumed that

$$\beta_r = k_\beta / \theta_e^{9/2} \text{ cm}^6/\text{sec} \quad (3)$$

where θ_e is in electron volts; and k_β is a constant which is inferred from experimental data (see Section IV). In thermodynamic equilibrium, the ionization frequency ν_i is related to the recombination coefficient β_r through the

Saha equation; namely,

$$v_i/\beta_r = n_s^2 \equiv (2\pi m_e \theta_e/h^2)^{3/2} n_g \exp(-V_i/\theta_e) \quad (4)$$

where n_s is the Saha equilibrium charged particle density; m_α is the mass of species α ; h is Planck's constant; n_g is the interelectrode gas density; and V_i is the ionization potential of the interelectrode gas. It is assumed that Equation (4) is also applicable for nonequilibrium plasmas.

The electron and ion mobilities in Equations (2) are given by the relations:⁽¹⁹⁾

$$\mu_\alpha = e/(m_\alpha \hat{v}_\alpha); \quad \alpha = e, i \quad (5)$$

$$\hat{v}_e \simeq (3/4) \sqrt{2\pi e \theta_e / m_e} n_g \sigma_{eg} + \frac{1.75 e^4 n \ln \Lambda}{64 \sqrt{2\pi m_e} \epsilon_o^2 (e \theta_e)^{3/2}} \quad (6)$$

$$\hat{v}_i \simeq (3/4) \sqrt{\pi e \theta_i / m_i} n_g \sigma_{ig} \quad (7)$$

where $\hat{v}_\alpha$ is the effective momentum transfer collision frequency for species α ; $\sigma_{\alpha g}$ is the effective hard sphere collision cross section for species α ; and ϵ_o is the permittivity of free space. The first term in the expression for the electron collision frequency, Equation (6), accounts for electron-neutral collisions, while the second term accounts for both electron-ion and electron-electron collisions.⁽¹⁹⁾

Equations (1) and (2) can be combined to yield a differential equation for the charged particle density by eliminating the electric field and neglecting spatial variations in the particle mobilities. Thus, one obtains the well-known ambipolar diffusion equation which, in dimensionless form, is

$$d^2 y / d\xi^2 = -2\gamma^2 y(1-y^2); \quad y = n/n_s; \quad \xi = x/d; \quad \gamma^2 = v_i d^2 / (2D_a) \quad (8)$$

where D_a is the ambipolar diffusion coefficient; and d is the interelectrode spacing. Equation (8) can be integrated once to yield the relation

$$p(\xi) \equiv dy/d\xi = \pm \sqrt{K-y^2(2-y^2)} \quad (9)$$

where K is an integration constant. The solutions of Equation (9) are Jacobi elliptic functions.⁽²²⁾ For $K \leq 1$ the general solution may be written as

$$y(\xi) = \sqrt{2m/(1+m)} \operatorname{sn}(u;m); u = \sqrt{2/(1+m)} \gamma(\xi + \xi_0); m = (1 - \sqrt{1-K})/(1 + \sqrt{1-K}) \quad (10a)$$

where $\operatorname{sn}(u;m)$ is the elliptic sine function⁽²²⁾ of argument u and parameter m , and ξ_0 is an integration constant. For $K \geq 1$ the charged particle density profile takes the alternate form

$$y(\xi) = (1/\sqrt{2m-1}) \operatorname{sc}(u;m) \operatorname{dn}(u;m); u = (1/\sqrt{2m-1}) \gamma(\xi + \xi_0); m = (1 + \sqrt{K})/2\sqrt{K} \quad (10b)$$

where $\operatorname{sc}(u;m)$ and $\operatorname{dn}(u;m)$ are elliptic functions⁽²²⁾ of argument u and parameter m .

For near equilibrium plasmas (large γ) asymptotic expansions of the elliptic functions⁽²²⁾ may be employed to simplify Equations (10). These expansions are not presented here, but are used in SIMCØN to provide faster and more reliable convergence of the iterative computations for the special case of a near equilibrium plasma.

The electric field profile in a neutral plasma follows directly from the charged particle density profile. More specifically, Equations (1) and (2) may be combined to yield the relation

$$E = -\frac{\mu_e \theta_e e^{-\mu_i \theta_i}}{(\mu_e + \mu_i) d} \frac{1}{y} \frac{dy}{d\xi} - \frac{J}{e(\mu_e + \mu_i) n_s y} \quad (11)$$

where $J = J_e - J_i$ is the output current density. Equation (11) may be integrated by making use of Equations (10) and integration formulae presented by Abramowitz and Stegun,⁽²²⁾ to yield the electrostatic potential profile in the interelectrode plasma.

The plasma electron temperature, which is assumed to be uniform, must satisfy the energy balance equation.

$$q_{e1} - q_{e0} = J V_P - (J_{i1} - J_{i0}) V_i \quad (12)$$

where q_e is the electron kinetic energy flux; V_P is the voltage drop across the interelectrode plasma; and the subscripts "0" and "1" are used to denote a quantity evaluated at the emitter and collector edges of the plasma, respectively.

The integration constants in Equations (10) and the electron kinetic energy fluxes q_{e0} and q_{e1} (Equation 12) are evaluated from analyses of the electrostatic sheaths as discussed in the following paragraphs.

2. Sheath Analyses

The electrostatic sheaths at the plasma-electrode interfaces are analyzed by writing electron and ion current and electron kinetic energy flux balances across the sheaths; i. e., between each electrode and the adjacent edge of the interelectrode plasma. The exact form of these balances for a given sheath depends upon its polarity, which is referred to here as accelerating or retarding if the sheath accelerates or retards, respectively, an electron traveling toward the collector. Electron and ion current and electron kinetic energy flux balances are given in Table I for accelerating and retarding emitter and collector sheaths. In this table, J_m and I_m are the electron and ion emission current densities, respectively, for the emitter ($m=E$) and collector ($m=C$); V_m are the sheath voltage drops at the emitter ($m=E$) and collector ($m=C$) edges of the plasma; and J_r and I_r are the electron and ion random current densities, respectively, in the interelectrode plasma. Note that the sheath voltage drops are treated as negative and positive for accelerating and retarding sheaths, respectively. The equations presented in Table I contain terms, derived from first order transport theory, to account for the non-Maxwellian anisotropy of the plasma electron and ion distribution functions.

These terms become appreciable if the net particle currents near the sheaths are comparable to the corresponding random currents.

TABLE I. Emitter and Collector Sheath Analyses

ACCELERATING EMITTER SHEATH ($V_E \leq 0$)	ACCELERATING COLLECTOR SHEATH ($V_C \leq 0$)
1. $J_{e0} = J_E - (J_{r0} - 1/2 J_{e0}) \exp(V_E/\theta_e)$	1. $J_{el} = (J_{r1} + 1/2 J_{el}) - J_C \exp(V_C/\theta_C)$
2. $J_{i0} = I_E \exp(V_E/\theta_E) - (I_{r0} - 1/2 J_{i0})$	2. $J_{il} = (I_{r1} + 1/2 J_{il}) \exp(V_C/\theta_C) - I_C$
3. $q_{e0} = 2 J_E (\theta_E - \theta_e) + J_{e0} (2 \theta_e - V_E)$	3. $q_{el} = 2 J_{r1} (\theta_e - \theta_C) + J_{el} (\theta_e + \theta_C)$
RETARDING EMITTER SHEATH ($V_E \geq 0$)	RETARDING COLLECTOR SHEATH ($V_C \geq 0$)
1. $J_{e0} = J_E \exp(-V_E/\theta_E) - (J_{r0} - 1/2 J_{e0})$	1. $J_{el} = (J_{r1} + 1/2 J_{el}) \exp(-V_C/\theta_e) - J_C$
2. $J_{i0} = I_E - (I_{r0} - 1/2 J_{i0}) \exp(-V_E/\theta_E)$	2. $J_{il} = (I_{r1} + 1/2 J_{il}) - I_C \exp(-V_C/\theta_C)$
3. $q_{e0} = 2 J_{r0} (\theta_E - \theta_e) + J_{e0} (\theta_E + \theta_e)$	3. $q_{el} = 2 J_C (\theta_e - \theta_C) + J_{el} (2 \theta_e + V_C)$

Notes

1. Electron Current Balance
2. Ion Current Balance
3. Electron Kinetic Energy Flux Balance

The electron and ion emission current densities from the emitter and collector (Table I) are governed by surface phenomena and Schottky effects. The theoretical determination of these quantities is discussed in the following paragraphs.

C. SURFACE PHYSICS

The field-free, electron work function ϕ_m of a substrate metal (m) partially coated by an adsorbate film depends upon the substrate and adsorbate

properties, the substrate temperature T_m , and the adsorbate reservoir temperature T_r . In other words, for a specified substrate and adsorbate; $\phi_m = \phi_m(T_m, T_r)$. The theoretical prediction of this relation for thermionic emitters ($m=E$) and collectors ($m=C$) is the objective of thermionic converter surface physics studies.

In the present analysis the recently developed Steiner-Gyftopoulos formalism⁽⁵⁾ is employed to determine the field-free, emitter and collector work functions. This formalism is an extension of the earlier work of Levine and Gyftopoulos⁽⁴⁾ and is based upon principles of molecular chemistry and statistical mechanics. The pertinent equations are briefly reviewed in the following paragraphs.

According to the method of Steiner and Gyftopoulos,⁽⁵⁾ the work function of a cesiated electrode results from a three-part analysis. First, the electron work function and atom and ion desorption energies are determined as functions of surface coverage. Second, the atom and ion emission rates are derived in terms of the corresponding desorption energies. Finally, the surface "equation of state" is employed to yield the surface coverage and, in turn, the characteristic energies and emission rates for the several particle species.

1. Work Function and Desorption Energies

The electron work function ϕ_m and atom and ion desorption energies (ϕ_{gm} and ϕ_{im} , respectively) are given by the relations:⁽⁵⁾

$$\phi_m = \phi_{sm} - (\phi_{sm} - \phi_{gm}) [1 - G(\Theta_m)] - a_m f_m \Theta_m / (1 + b_m \Theta_m^{3/2}) \quad (13)$$

$$\phi_{gm} = D_{om} \sqrt{1 - f_m^2} + (\phi_{sm} - \phi_{gm}) G(\Theta_m) f_m - \left(\frac{1}{2}\right) D_{lm} f_m^2 \quad (14)$$

$$\phi_{im} = \phi_{gm} + V_i - \phi_m \quad (15)$$

$$a_m = \frac{0.905 \times 10^{-6} [(R_{sm} + R_g)^2 - 1/(2\sigma_{sm})]^{1/2} \sigma_{gm}}{1 + R_g^3 / (R_{sm} + R_g)^3}$$

$$b_m = 9 R_g^3 \sigma_{gm}^{3/2}$$

$$D_{om} = \frac{2S_{sm}S_g}{S_{sm}^2 + S_g^2} \sqrt{H_{sm}H_g}$$

where, for electrode m , ϕ_{sm} and ϕ_{gm} are work functions at zero and one-monolayer coverage, respectively; Θ_m is the coverage in monolayers; f_m is fraction of an electronic charge which is transferred between the substrate and adsorbate valence orbitals; R_{sm} and R_g are the metallic radii of the substrate and adsorbate atoms, respectively; σ_{sm} and σ_{gm} are the surface densities of substrate atoms and adsorption sites, respectively; S_{sm} and S_g are the bond strengths of the substrate and adsorbate, respectively; H_{sm} and H_g are the heats of sublimation of the substrate and adsorbate, respectively; D_{lm} is a characteristic energy of the adsorption system; and $G(\Theta) \equiv 1 - 3\Theta^2 + 2\Theta^3$. * The numerical constant in the expression a_m has the units (eV cm). The fractional charge transfer f_m is chosen so as to maximize the atom desorption energy given by Equation (14).

2. Atom and Ion Emission Rates

Principles of statistical mechanics may be employed to derive expressions for the atom and ion emission rates (E_{gm} and E_{im} , respectively). Following Levine and Gyftopoulos, ⁽⁴⁾ one obtains the relations:

$$E_{gm} = \omega_g \nu_m \sigma_{gm} \Theta_m \exp[\Delta S_m / k] \exp[-\phi_{gm} / \theta_m] \quad (16)$$

$$E_{im} = \omega_i \nu_m \sigma_{gm} \Theta_m \exp[\Delta S_m / k] \exp[-\phi_{im} / \theta_m] \quad (17)$$

$$\nu_m = \nu_m' \exp\left[\left(\Theta_m / \nu_m'\right) \left(\partial \nu_m' / \partial \Theta_m\right)\right];$$

*The $G(\Theta)$ function employed here stems from the work of Levine and Gyftopoulos. ⁽⁴⁾ Steiner and Gyftopoulos ⁽⁵⁾ have suggested an alternate expression for $G(\Theta)$ which yields nearly identical numerical results.

$$\nu_m' = \frac{1}{2\pi} \left\{ \frac{\phi_{gm}(m_g + 4m_{sm})}{2m_g m_{sm} [(R_{sm} + R_g)^2 - 1/(2\sigma_{sm})]} \right\}^{1/2}$$

$$\Delta S_m = k \left[\ln \chi_m + \left(\frac{\Theta}{\chi_m} \right) \left(\partial \chi_m / \partial \Theta_m \right) \right]; \quad \chi_m = \frac{1}{(1 - \sqrt{\Theta_m}) \sqrt{1 - \Theta_m}}$$

where m_{sm} is the mass of the substrate atoms; ν_m is the characteristic frequency of oscillation of adsorbed particles; and ΔS_m is identified as a configurational entropy charge associated with the emission processes.

3. Surface Equation of State

The surface equation of state is obtained by balancing arrival and departure rates of heavy particles at the electrode surface.⁽⁴⁾ More specifically:

$$A_{gm} + A_{im} = E_{gm} + E_{im} \quad (18)$$

where A_{gm} and A_{im} are the arrival rates of atoms and ions, respectively, at the surface of electrode m .

A simplified form of Equation (18) is valid provided: (a) the surface ionization probability is small compared to unity, and (b) the neutral plasma adjacent to the electrode is weakly ionized. These conditions are almost always fulfilled under practical thermionic converter operating conditions, and permit one to neglect the ionic contributions to the arrival and emission rates at the electrode surface. The surface equation of state then becomes:

$$A_{gm} = p_g / \sqrt{2\pi e m_g \Theta_m} \doteq E_{gm} \quad (18a)$$

Equation (18a) is employed in the present analysis.

For specified electrode and cesium reservoir temperatures, Equations (13-17 and 18a) may be solved using an iterative procedure to yield the field-free cesiated electrode work function and other electrode properties of practical interest.

4. Comments on Electrode Characterization

One implication of the Steiner-Gyftopoulos formalism is that a thermionic converter electrode is characterized by the quantities ϕ_{sm} , ϕ_{gm} , σ_{sm} , σ_{gm} , H_{sm} , H_g , R_{sm} , R_g , S_{sm} , S_g , and D_{lm} . The parameters R_{sm} , R_g , S_{sm} , and S_g are properties of the substrate and adsorbate atoms, and may be viewed as independent parameters. There is reason to believe, however, that the remaining parameters are not independent, but are all related to the atomic properties of the substrate and adsorbate, and to the electrode crystallographic arrangement. Such relations are only partially understood at the present time,⁽⁵⁾ and no attempt to account for them is made in this study.

The electrode characterization scheme described above presents difficulties in practice since certain of the required parameters are inaccessible for direct measurement, and yet significantly influence electrode performance. The eventual solution to this problem will involve reformulating the surface physics equations in terms of basic, independent, measurable quantities such as substrate and adsorbate atomic properties and the electrode crystallographic arrangement. In the interim, satisfactory results can be obtained by inferring the unmeasured parameters from indirect experimental data such as thermionic converter output current characteristics. This procedure has been successfully implemented in practice and typical results are presented in Section VI of this report.

D. SCHOTTKY EFFECTS

The field-free work functions do not, in general, prevail at the emitter and collector electrodes in a thermionic converter. For example, when the

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FIGURE 1. Influence of electrostatic sheath on work function of thermionic converter electrode

$$E_m = k_s \left| V_m \right| / L_m \quad (20)$$

where L_m is the electron Debye length at the edge of the plasma adjacent to the sheath; and k_s is a dimensionless constant of order unity. The plasma electron Debye length $L(x)$ at position x is evaluated from the local charged particle density $n(x)$ and electron temperature θ_e according to the relation

$$L = \sqrt{(\epsilon_0 \theta_e) / (en)}. \quad (21)$$

Equations (19, 20) and (21) are employed in the present analysis to describe Schottky effects at the emitter ($m=E$) and collector ($m=C$) electrodes in thermionic converters. The field-free work functions ϕ_m are computed from surface physics considerations; Equations (19-21) then yield the effective work functions ϕ'_m , where ϕ'_m is greater or less than ϕ_m depending upon the polarity of the sheath adjacent to the electrode.

Once the effective work function ϕ'_m is determined, emission current densities of electrons and ions from a thermionic converter electrode $m(m=E, C)$ follow directly from the Richardson and Saha-Langmuir equations, respectively. In particular:

$$J_m = A T_m^2 \exp(-\phi'_m / \theta_m); \quad \theta_m = k T_m / e \quad (22)$$

$$I_m = e p_g (2\pi m_g e \theta_m)^{-1/2} (1/2) \exp[-(V_i - \phi'_m) / \theta_m] \quad (23)$$

where A is the Richardson constant. Equation (23) is an approximate form of the Saha-Langmuir equation which is valid provided the surface ionization probability is small compared to unity.

E. THERMIONIC CONVERTER PERFORMANCE

1. Output Current Characteristics

The thermionic converter output voltage V for a specified output

current density J is obtained by summing the voltage drops between the emitter and collector Fermi levels. Thus,

$$V = \phi_E' + V_E - V_P + V_C - \phi_C' \quad (24)$$

The several terms in this relation follow directly from the preceding analyses.

2. Intrinsic Efficiency Characteristics

The intrinsic efficiency η_i of a thermionic converter is defined here as the efficiency in the absence of electrode, lead, and structural support losses. This efficiency is independent of detailed converter design considerations and is given by

$$\eta_i = P / (Q_e + Q_i + Q_r + Q_g) \quad (25)$$

where $P = JV$ is the output power density; and Q_e , Q_i , Q_r and Q_g are the heat fluxes removed from the emitter due to electrons, ions, radiation, and gas conduction, respectively. The first three of these quantities are given by

$$Q_e = J_{e0} (\phi_E' + V_E) + q_{e0} \quad (26)$$

$$Q_i = -J_{i0} (\phi_E' + V_E) + J_{i0} V_i \quad (27)$$

$$Q_r = \sigma \left[\frac{T_E^4}{1/\epsilon_E(T_E) + 1/\epsilon_C(\sqrt{T_E T_C}) - 1} - \frac{T_C^4}{1/\epsilon_E(\sqrt{T_E T_C}) + 1/\epsilon_C(T_C) - 1} \right] \quad (28)$$

where σ is the Stefan-Boltzmann constant; and $\epsilon_E(T_E)$ and $\epsilon_C(T_C)$ are the temperature dependent emitter and collector emissivities, respectively.

These later quantities are approximated by the linear relations:

$$\epsilon_m = A_m + B_m T_m; \quad m = E, C \quad (29)$$

where the intercept A_m and temperature coefficient B_m are determined from experimental data. An expression for the interelectrode gas conduction term Q_g for Cs has been given by Kitrilakis et al.⁽²³⁾ Specifically:

$$Q_g = \frac{1.10 \times 10^{-6}(T_E^{3/2} - T_C^{3/2})}{d + 1.15 \times 10^{-5}(T_E + T_C)/p_g} \quad (30)$$

where T_E and T_C are in $^{\circ}\text{K}$, d is in cm, and p_g is in torr. Note that Equation (25) is readily modified to compute device efficiencies for specific converter designs by including terms to account for electrode, lead and structural support losses.

III. SIMCON PROGRAM DESCRIPTION

The SIMCON digital computer program has been written to solve the non-linear equations developed in Section II. Important features of this program are discussed in this section. Detailed instructions for use of the program appear in subsequent sections.

A. PROGRAM STRUCTURE AND INTERFACING REQUIREMENTS

The subroutine structure of the SIMCON program is shown in Figure 2. The main program calls four subroutines: TAPENO, SYSERR, SURFAS, and PLASMA. The PLASMA subroutine in turn calls three additional subroutines: EBC, CBC, and ELPSIN. The functions of the various routines in SIMCON are described below. Logical flow diagrams for the SIMCON main program, and for the SURFAS, PLASMA, EBC and CBC subroutines appear in Appendix A, and a complete program listing appears in Appendix B.

1. SIMCON Main Program

The role of the SIMCON main program is threefold: (a) to accept input data from the user, (b) to guide a computational subroutines through the desired calculations, and (c) to control the return of output to the user.

2. TAPENO Subroutine

The TAPENO subroutine assigns numbers to the input and output tapes. This subroutine must be supplied by the user and must be consistent with the call statement which is of the form CALL TAPENO (KIN, KOUT), where KIN and KOUT are the input and output tape numbers, respectively.

3. SYSERR Subroutine

The SYSERR subroutine permits the user to retain control of the SIMCON program in the event of a computational fault which would normally

FIGURE 2 SUBROUTINE STRUCTURE OF THE SIMCON PROGRAM

terminate a run (e.g., an attempt to evaluate the square root or logarithm of a negative number). The call to `SYSERR` appears only near the beginning of the `SIMCON` main program and has the form: `CALL SYSERR (IND)`. The index `IND` is returned as zero following a normal call to `SYSERR`. If a computational fault occurs in any of the `SIMCON` subroutines, however, program control is transferred to the `SYSERR` subroutine and `IND` is returned as a non-zero integer. Hence, by monitoring the value of `IND` upon return from `SYSERR`, one is able to advance to the next case following a computational fault without having the run terminated.

The internal logic of the `SYSERR` subroutine is unique to the GE-635 computer, and would not function on other machines. For this reason a dummy `SYSERR` subroutine (which always returns `IND = 0`) is included with distributed versions of the `SIMCON` program. It is recommended, however, that users of `SIMCON` replace the dummy `SYSERR` subroutine with one which provides the capability of recovering from normally fatal computational faults.

4. SURFAS Subroutine

The `SURFAS` subroutine computes the field-free work functions and charged particle emission current densities of the emitter and collector electrodes.

5. PLASMA Subroutine

The `PLASMA` subroutine performs computations related to phenomena which occur in the neutral interelectrode plasma region of a thermionic converter. Charged particle densities, currents, and temperatures, and the neutral plasma voltage drop are computed in this subroutine. Near equilibrium plasmas are treated as a special case (see Appendix A).

6. EBC and CBC Subroutines

The EBC (Emitter Boundary Conditions) and CBC (Collector Boundary Conditions) subroutines are called by PLASMA to permit self-consistent sheath voltage drops and field-modified work functions to be computed for each set of converter operating conditions. The EBC and CBC subroutines have completely analogous logical structure (see Appendix A).

7. ELPSIN Subroutine

The ELPSIN subroutine evaluates the elliptic sine function of argument u and parameter m . The numerical procedure is based on the use of ascending and descending Landen Transformations as described by Abramowitz and Stegun.⁽²²⁾ Alternate elliptic functions are evaluated from the elliptic sine function.

B. CONVERGENCE PROPERTIES

The SIMCON digital computer program has been extensively tested for emitter temperatures between 1700 and 2300°K, collector temperatures between 800 and 1200°K, cesium reservoir temperatures between 550 and 670°K, and interelectrode spacings between 6 and 20 mils. Converged solutions have been obtained for better than 99% of more than 1500 operating points, indicating a very favorable convergence probability for the SIMCON program.

In spite of its good convergence record, the SIMCON program does occasionally fail to obtain a converged solution for a prescribed set of converter operating conditions. The most common faults lead to: (a) a negative number as the argument of the square root or logarithm function, or (b) excessive numbers of iterations within certain computational loops of the PLASMA subroutine. In the former case, program control is regained through the previously described SYSERR subroutine. In the latter case, the computation is terminated when built-in iteration limits are exceeded. In both cases, an error message is printed and a new case is initiated.

C. OPERATING CONDITION RESTRICTIONS

Two, normally unimportant, restrictions exist concerning the converter operating conditions which can be handled by the SIMCON digital computer program. In particular, the output current density must be positive, and the collector temperature must be sufficiently low that ion emission from the collector is small. These restrictions are not introduced directly into the equations (Section II) which form the basis of SIMCON, but result instead from the logical scheme through which these equations are solved. Consequently, these restrictions do not introduce inaccuracies into computed SIMCON results; they do, however, limit the operating domain for which converged results can be obtained. The collector temperature restriction has presented no limitations in practice for collector temperatures as high as 1500°K .

D. CORE STORAGE AND RUNNING TIME

The SIMCON program requires 16K of allocated core storage on a GE-635 computer. Running time on this machine is approximately 10 seconds to compute 10 points on a single output current characteristic.

E. NUMERICAL ACCURACY

Output voltages computed by the SIMCON program are expected to be converged to within approximately ± 0.01 V. Numerical inaccuracies in the electron and ion cooling heat fluxes from the emitter have not been extensively investigated but are believed to be less than 0.05 W/cm^2 (absolute) or 0.5% (relative), whichever is greatest.

IV. SIMCON INPUT DATA FORMAT

This section describes the format for the input data which the user must supply to initiate a SIMCON run. The card formats for independent and dependent cases are discussed separately.

A. INDEPENDENT CASES

The data for an independent SIMCON case consists of the converter operating conditions, properties of the converter electrodes and interelectrode vapor, and control parameters which ensure the desired output. The first case of any sequence must be independent. The data for an independent case is contained on seven or more cards in the following format.

Card 1 - Converter Operating Conditions

Columns	Parameter	Units	Format
1-8	Emitter Temperature, T_E	$^{\circ}\text{K}$	F8.3
9-16	Collector Temperature, T_C	$^{\circ}\text{K}$	F8.3
17-24	Cesium Reservoir Temperature, T_r	$^{\circ}\text{K}$	F8.3
25-32	Interelectrode Spacing, d	mils	F8.3
33-40	Case Parameter, I	none	I8
	$I = 1$ for independent cases		
	$I = 2$ or 3 for dependent cases		
41-80	Blank	-	-

Card 2 - Emitter Properties ($m = E$)

Columns	Parameter	Units	Format
1-8	Atomic Weight, A_{sm}	none	F8.3
9-16	Zero Coverage Work Function, ϕ_{sm}	eV	F8.3
17-24	Full Coverage Work Function, ϕ_{gm}	eV	F8.3
25-32	Heat of Sublimation, H_{sm}	eV	F8.3
33-40	Metallic Radius, R_{sm}	$\text{\AA}$	F8.3
41-48	Bond Strength, S_{sm}	none	F8.3

49-56	Substrate Atom Density, σ_{sm}	$\text{\AA}^{-2}$	F8.3
57-64	Adsorption Site Density, σ_{gm}	$\text{\AA}^{-2}$	F8.3
65-72	Characteristic Energy, D_{1m}	eV	F8.3
73-80	Blank	-	-

Card 3 - Collector Properties (m = C)

The third card contains the properties of the collector electrode in a format identical to that employed for the emitter electrode (Card 2).

Card 4 - Vapor Properties

Columns	Parameter	Units	Format
1-8	Atomic Weight, A_g	none	F8.3
9-16	Heat of Sublimation, H_g	eV	F8.3
17-24	Metallic Radius, R_g	$\text{\AA}$	F8.3
25-32	Bond Strength, S_g	none	F8.3
33-40	Ionization Potential, V_i	eV	F8.3
41-48	Electron-Neutral Cross Section, σ_{eg}	$\text{\AA}^2$	F8.3
49-56	Ion-Neutral Cross Section, σ_{ig}	$\text{\AA}^2$	F8.3
57-64	Recombination Coefficient Constant, k_{β}	$\text{cm}^6(\text{eV})^{9/2}/\text{sec}$	E8.3
65-72	Schottky Effect Constant, k_s	none	F8.3
73-80	Blank	-	-

Card 5 - Thermal Radiation Parameters

Columns	Parameter	Units	Format
1-8	Emitter Emissivity Intercept, A_E	none	F8.3
9-16	Emitter Emissivity Temperature Coefficient, B_E	$^{\circ}\text{K}^{-1}$	E8.3
17-24	Collector Emissivity Intercept, A_C	none	F8.3
25-32	Collector Emissivity Temperature Coefficient, B_C	$^{\circ}\text{K}^{-1}$	E8.3
33-80	Blank	-	-

Card 6 - Computation Control Card

Columns	Parameter	Units	Format
1-8	Integer Number, N, of output current densities for which results are desired. N must be less than or equal to 50.	none	I8
9-16	Minimum Output Voltage, V_m , for which results are desired. If a voltage less than or equal to V_m is computed, results for higher output current densities will not be obtained.	volts	F8.3
17-24	Print Parameter IP. This parameter controls the content and format of the printed output as discussed in Section V. Permissible values are IP = 1, 2 or 3.	none	I8
25-80	Blank	-	-

Cards 7-11 - Output Current Densities

The output current densities for which results are desired are supplied in successive eight-character fields, ten fields per card, beginning on Card 7 and continuing on up to four additional cards until N values ($N \leq 50$) have been read in. These output current densities should be arranged in increasing order. The format for Cards 7-11 is as follows.

Columns	Parameter	Units	Format
1-80	Output Current Densities for which results are desired	amps/cm ²	10F8.3

B. DEPENDENT CASES

Dependent cases may be executed by the SIMCON program following an initial independent case. Two dependent case options are provided and are selected by assigning the value I = 2 or I = 3 to the Case Parameter I on Card 1; which is always the first input data card for both independent and dependent cases.

1. Dependent Case Option I = 2

If the Case Parameter I on Card 1 is set equal to 2, only Cards 2-5 are required to complete the input data. The information normally provided on Cards 6-11 is retained from the previous case. Cards 2-5 contain properties of the converter electrodes and interelectrode gas. Consequently, this dependent case option is useful for parametric studies in which the influence of different materials properties on converter performance is to be deduced.

2. Dependent Case Option I = 3

If the Case Parameter I on Card 1 is set equal to 3, no additional data is required to complete the input data. The information normally provided in Cards 2-11 is retained from the previous case. Since Card 1 contains the converter operating conditions, this dependent case option is useful for computing the performance characteristics of a thermionic converter for a range of operating conditions.

V. SIMCON OUTPUT

A. OUTPUT OPTIONS

The printed output from a SIMCON run consists of a complete edit of the input data plus the computed results requested by the user through one of the output options. Three such options, which differ with regard to the amount of information transmitted to the user, are provided. Selection of a desired output option is made through specification of the Print Parameter IP on Input Card 6.

1. Output Option IP = 1

For $IP = 1$, the SIMCON output consists of the zero-field electrode work functions and emission current densities, and the electrical and thermal performance characteristics of the converter. The latter include (for each operating point) the output current density, output voltage, output power density, intrinsic efficiency, and the several components of the heat flux leaving the emitter. This output option is most convenient when the primary interest is in the performance aspects of converter operation.

2. Output Option IP = 2

The SIMCON output for $IP = 2$ includes that described above for $IP = 1$. In addition, it contains (for each operating point) the field-modified electrode work functions and emission current densities; and the prevailing voltage drops across the emitter and collector sheaths, and neutral inter-electrode plasma. A quantitative understanding of the converter motive diagram is gained from this latter information.

3. Output Option IP = 3

The choice $IP = 3$ yields the SIMCON output for $IP = 2$; plus additional, more detailed information concerning the state of the interelectrode

plasma. For each operating point, the latter consists of the:

- (1) average neutral particle density in the plasma,
- (2) number of electron-neutral and ion-neutral mean free paths across plasma,
- (3) ionization parameter α (see Equation 8),
- (4) plasma electron temperature,
- (5) Saha equilibrium charged particle density (see Equation 4),
- (6) plasma Coulomb resistivity,
- (7) net ion current densities at the emitter and collector edges of the plasma,
- (8) integration constant K and related elliptic function parameter m (see Equations 10), and
- (9) dimensionless charged particle densities and density gradients at the emitter and collector edges of the plasma.

Detailed analysis of these plasma properties provides considerable insight into the basic physics of thermionic converter operation.

B. OUTPUT FORMAT

The SIMCON output format is self-explanatory provided the nomenclature and units are understood. A glossary of the same appears in Appendix C.

VI. ILLUSTRATIVE APPLICATION OF SIMCON

Application of the SIMCON program to the computation of thermionic converter performance characteristics is illustrated in this section. The specific example involves use of the SIMCON program to numerically simulate the performance characteristics of a W-emitter, Mo-collector, variable spacing converter.⁽²⁴⁾ A comparison of computed and measured current-voltage characteristics for this converter is presented.

A. CONVERTER PROPERTIES

The data employed to characterize the converter electrodes and inter-electrode vapor for the converter under study is presented in Table 2. Unmeasured parameters were either estimated from independent sources, or chosen through an iterative process to yield a good numerical simulation of the measured current-voltage characteristics⁽²⁴⁾ for a broad range of converter operating conditions. Table 2 also contains, for comparison, values of related electrode and interelectrode vapor properties reported in other studies. It is apparent that the values employed here for these properties are in satisfactory agreement with the available, but sometimes limited and inapplicable, independent data. It should be recognized, however, that small variations in certain of the properties of Table 2 can lead to significant variations in the computed results.

B. SAMPLE CALCULATION1. Converter Operating Conditions

The converter operating conditions chosen for an illustrative example include emitter, collector and cesium reservoir temperatures of 2000°K, 1000°K, and 600°K, respectively; and an interelectrode spacing of 10 mils. Numerical results are desired for output current densities of 0, 0.5, 1, 2, 4, 6, 8, 10, 12, 15, 20, 25, 30, and 35 amps/cm²; provided the

TABLE 2. Thermionic Converter Parameters

Parameter	This Study	Reported	Reference
<u>Emitter (W)</u>			
Atomic Weight (amu)	184	-	-
Work Function, Zero Coverage (eV)	4.75	(a)	-
Work Function, Full Coverage (eV)	1.95	1.96 (b)	25
Heat of Sublimation (eV)	8.61	8.61	4
Metallic Radius (Å)	1.37	1.37	26
Bond Strength	2.62	2.62	4
Substrate Atom Density (Å ⁻²)	0.192	0.192 (c)	4
Adsorption Site Density (Å ⁻²)	0.048	0.048 (d)	27
D ₁ Energy	1.564	1.07-1.71 (d)	5
Emissivity Intercept	0.020	0.020 (e)	28
Emissivity Temperature Coeff. (°K ⁻¹)	0.117 x 10 ⁻³	0.117 x 10 ⁻³ (e)	28
<u>Collector (Mo)</u>			
Atomic Weight (amu)	96	-	-
Work Function, Zero Coverage (eV)	4.38	4.38	4
Work Function, Full Coverage (eV)	1.95	1.96 (b)	25
Heat Sublimation (eV)	6.77	6.77	4
Metallic Radius (Å)	1.36	1.36	26
Bond Strength q	2.31	2.31	4
Substrate Atom Density (Å ⁻²)	0.192	(f)	-
Adsorption Site Density (Å ⁻²)	0.048	(f)	-
D ₁ Energy	1.065	(f)	-
Emissivity Intercept	0.0	0.0 (e)	28
Emissivity Temperature Coeff. (°K ⁻¹)	0.103 x 10 ⁻³	0.103 x 10 ⁻³ (e)	28
<u>Vapor (Cs)</u>			
Atomic Weight (amu)	133	-	-
Heat of Sublimation (eV)	0.73	0.73	4
Metallic Radius (Å)	2.66	2.60	5

TABLE 2. (Continued)

Bond Strength (dimensionless)	1.00	1.00	4
Ionization Potential (eV)	3.89	3.87	4
Electron-Neutral Cross Section ($\text{\AA}^2$)	400	400	29
Ion-Neutral Cross Section ($\text{\AA}^2$)	570	570 (g)	30
$k_\beta [\text{cm}^6(\text{eV})^{9/2}/\text{sec}]$	2.8×10^{-26}	$5.6 \times 10^{-27} \text{ (h)}$	21
k_s	1.0	-	-

Notes

- (a) No bare work function was reported for the emitter being considered. The emitter was believed to have a slightly preferred (110) orientation, however, and hence its work function should be higher than that for polycrystalline tungsten (4.57 eV).
- (b) The work function of Cs is reported to be 1.96 eV.
- (c) For body-centered cubic crystals it is assumed⁽⁴⁾ that one adsorption site exists for each four substrate atoms.
- (d) Reported for an unspecified W surface.
- (e) Inferred from measured emissivity data.
- (f) No data reported.
- (g) Inferred from ion current measurements in thermionic converters.
- (h) Computed value for hydrogen; not cesium.

output voltage remains positive. Results are not required for negative output voltages.

2. Input Deck Setup

Figure 3 shows a listing of the input data cards (see Section IV) required for an independent SIMCON case based on the data of Table 2 and the above converter operating conditions. The compressed output option (IP = 1) is selected.

3. SIMCON Output

Figure 4 shows the converter properties edit for the sample problem. This edit appears at the beginning of the SIMCON output for the first (independent) case, and for any subsequent case in which any of the converter properties are altered.

Figure 5 contains the computed electrical and thermal performance results returned by the SIMCON program for the sample problem. Note that a negative output voltage was obtained for an output current density of 25 amps/cm²; results for higher output current densities were therefore not computed.

Figures 6 and 7 show the computed SIMCON output obtained by re-running the sample problem for a single output current density of 10 amps/cm², and using the expanded output options IP = 2 and IP = 3, respectively. Analogous results are obtained for each output current density under these latter options.

C. COMPARISON OF COMPUTED AND MEASURED RESULTS

The solid curves in Figures 8-10 are experimental current-voltage characteristics obtained by TECO⁽²⁴⁾ for a thermionic converter with a polycrystalline, heat-treated W emitter, and a polycrystalline Mo collector.


```

CARD 1
2000.0 1000.0 600.0 10.0 1

CARD 2
184.0 4.75 1.95 8.61 1.37 2.62 0.192 0.048 1.564

CARD 3
96.0 4.38 1.95 6.77 1.36 2.31 0.192 0.048 1.065

CARD 4
133.0 0.73 2.66 1.0 3.89 400.0 570.0 2.8E-26 1.0

CARD 5
0.025 1.17E-4 0.0 1.03E-4

CARD 6
15 0.0 1

CARD 7
0.0 0.5 1.0 2.0 3.0 4.0 6.0 8.0 10.0 12.0

CARD 8
15.0 20.0 25.0 30.0 35.0

```

Figure 3. SIMCON Input Data Cards

EMITTER PROPERTIES								
AT WT	B W FUN	C W FUN	SUB HEAT	RADIUS	BOND STR	SUB DEN	ADS DEN	D1 ENERGY
184.000	4.750	1.950	8.610	1.370	2.620	0.192	0.048	1.564
COLLECTOR PROPERTIES								
AT WT	B W FUN	C W FUN	SUB HEAT	RADIUS	BOND STR	SUB DEN	ADS DEN	D1 ENERGY
96.000	4.380	1.950	6.770	1.360	2.310	0.192	0.048	1.065
VAPOR PROPERTIES								
AT WT	SUB HEAT	RADIUS	BOND STR	ION POT	E-N XSEC	I-N XSEC	REC CON	SCHOTTKY
133.000	0.730	2.660	1.000	3.89	400.000	570.00	0.28 E 25	1.000
RADIATION PARAMETERS								
EM EMIS	EM T COEF	CO EMIS	CO T COEF					
0.020	0.117E-03	0.	0.103E-03					

CESR-2100

Figure 4. Converter Properties Edit

THERMIONIC CONVERTER OPERATING CONDITIONS									
EM TEMP	CO TEMP	RES TEMP	SPACING	WFO	JEO	IEO	WFO	JCO	ICO
2000.000	1000.000	600.000	10.000	3.060	9.414	0.156374	1.749	0.184	0.000000
OUTPUT CURRENT CHARACTERISTIC									
CURRENT	VOLTAGE	POWER	EFF	F-COOL	I-COOL	CS COND	TH RAD	HEAT IN	
0.	1.946	0.	0.	0.083	0.006	1.833	8.853	10.775	
0.500	1.931	0.816	0.065	1.901	0.011	1.833	8.853	12.597	
1.000	1.935	1.435	0.101	3.556	0.009	1.833	8.853	14.251	
2.000	1.946	2.492	0.143	6.748	-0.001	1.833	8.853	17.432	
3.000	1.185	3.555	0.172	9.987	-0.018	1.833	8.853	20.655	
4.000	1.145	4.580	0.192	13.235	-0.043	1.833	8.853	23.877	
6.000	1.167	6.391	0.211	19.667	-0.132	1.833	8.853	30.221	
8.000	0.975	7.801	0.215	25.970	-0.294	1.833	8.853	36.361	
10.000	0.875	8.754	0.207	32.125	-0.554	1.833	8.853	42.256	
12.000	0.766	9.198	0.192	38.124	-0.933	1.833	8.853	47.877	
15.000	0.577	8.658	0.156	46.748	-1.819	1.833	8.853	55.614	
20.000	0.279	4.182	0.065	60.083	-4.404	1.833	8.853	66.364	
25.000	-3.161	-4.025	-0.054	72.422	-8.443	1.833	8.853	74.665	

Figure 5. SIMCON Output for IP=1

THERMIONIC CONVERTER OPERATING CONDITIONS

EM TEMP	CO TEMP	RES TEMP	SPACING	WFEO	JE0	IE0	WFCO	JCO	ICO
2000.000	1000.000	600.000	10.000	3.060	9.414	0.156374	1.749	0.184	0.000000

CONVERTER PARAMETERS FOR J = 10.000

VOLTAGE AND EMISSION CURRENT SUMMARY

EM W FUN	EM SHEATH	PL VOLT	CO SHEATH	CO W FUN	JE	JC	IE	IC
3.005	-0.721	-0.179	0.476	1.706	12.951	0.305	0.113666	0.000000

ELECTRICAL AND THERMAL PERFORMANCE SUMMARY

CURRENT	VOLTAGE	POWER	EFF	E-COOL	I-COOL	CS COND	TH RAD	HEAT IN
10.000	0.875	8.754	0.207	32.125	-0.554	1.833	8.853	42.256

Figure 6. SIMCON Output for IP=2

THERMIONIC CONVERTER OPERATING CONDITIONS

EM TEMP	CO TEMP	RES TEMP	SPACING	FEO	JEO	IFO	WFCO	JCO	ICO
2000.000	1000.000	600.000	10.000	3.06	9.414	0.156374	1.749	0.184	0.000000

CONVERTER PARAMETERS FOR J = 10.000

CHARGED PARTICLE DENSITY PROFILE

INT CON X	MODULUS M	Y(X=0)	P(X=0)	Y(X=D)	P(X=D)
1.000	1.000	0.501	4.828	0.444	-5.207

PLASMA PROPERTIES

CS DEN	E-MFP	I-MFP	GAMMA	EL TEMP	SAHA DEN	COUL RES	JI(X=0)	JI(X=D)
0.231E 17	15.745	13.952	6.456	2379.242	0.193E 15	0.262	-0.345209	0.331366

VOLTAGE AND EMISSION CURRENT SUMMARY

EM W FUN	EL SHEATH	PL VOLT	CO SHEATH	CO X FUN	JE	JC	IE	IC
5.025	-0.721	-0.179	0.476	1.705	14.951	0.305	0.113666	0.000000

ELECTRICAL AND THERMAL PERFORMANCE SUMMARY

CURRENT	VOLTAGE	POWER	EFF	E-COOL	I-COOL	CS COND	TH RAD	HEAT IN
10.000	0.875	8.754	0.207	32.125	-0.554	1.833	8.853	42.256

Figure 7. SIMCON Output for IP=3

The characteristics encompass a broad range of converter operating conditions, and are divided into three groups to exhibit the effects of independent variations in emitter temperature (Figure 8), cesium reservoir temperature (Figure 9), and interelectrode spacing (Figure 10).

The dashed curves in Figures 8-10 are theoretical output current characteristics computed for the indicated operating conditions using the SIMCON program and the data of Table 2. The results are in excellent qualitative and good quantitative agreement with the corresponding experimental measurements for the entire range of converter operating conditions. Even better agreement could be anticipated, moreover, through further refinement of the input data of Table 2.

FIGURE 8. COMPARISON OF COMPUTED AND MEASURED OUTPUT CURRENT CHARACTERISTICS FOR SEVERAL EMITTER TEMPERATURES

FIGURE 9. COMPARISON OF COMPUTED AND MEASURED OUTPUT CURRENT CHARACTERISTICS FOR SEVERAL CESIUM RESERVOIR TEMPERATURES

FIGURE 10. COMPARISON OF COMPUTED AND MEASURED OUTPUT CURRENT CHARACTERISTICS FOR SEVERAL INTER-ELECTRODE SPACINGS

VII. SUMMARY AND CONCLUSIONS

The theoretical foundation and use of a digital computer program for computing thermionic converter performance characteristics is described. The analyses and program simultaneously account for the influence of surface phenomena, volume phenomena, and Schottky effects on converter operation; and yield output current characteristics which are in good agreement with experimental data for a very broad range of converter operating conditions. The program has practical applications for converter physics studies, converter performance evaluations, and thermionic power plant design.

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APPENDIX A
LOGICAL FLOW DIAGRAMS
FOR THE
MAJOR SIMCON SUBROUTINES

FIGURE A-1 LOGICAL FLOW DIAGRAM FOR SIMCON MAIN PROGRAM

FIGURE A-2 LOGICAL FLOW DIAGRAM FOR SURFAS SUBROUTINE

FIGURE A-3 LOGICAL FLOW DIAGRAM FOR PLASMA SUBROUTINE

FIGURE A-4 LOGICAL FLOW DIAGRAM FOR EBC AND CBC SUBROUTINES.

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APPENDIX B

SIMCON PROGRAM LISTING

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CSIMCON      SIMCON MAIN PROGRAM
      DIMENSION @CUR(50)
      COMMON /A/ AG,HG,RG,SG,VIG,VALG,CSEG,CSIG
      COMMON /B/ KB,KR,KS,GAMCR
      COMMON /C/ WFE@,JE@,IE@,WFC@,JC@,IC@,WFE,JE,IE,WFC,JC,IC,J
      COMMON /D/ K@UT,IP,      I@NV,TE,TC,PG,D,V,QEL,QION
      REAL KB,KR,KS
      REAL JE@,IE@,JC@,IC@,JE,IE,JC,IC,J
      CALL TAPEN@ (KIN,K@UT)
      VALG = 1.0
      GAMCR = 5.0
      KR = 1.0
      CALL SYSERR(IND)
      IF(IND.NE.0) WRITE(K@UT,110) J
      IF(IND.NE.0) GO TO 61
5 CONTINUE
CC
C      READ INPUT DATA
CC
      READ(KIN,80) TE,TC,TR,D,I
      IF(I.EQ.3) GO TO 30
10 READ(KIN,81) AE,F@E,FCE,HE,RE,SE,SDE,ADE,CE
      READ(KIN,81) AC,F@C,FCC,HC,RC,SC,SDC,ADC,CC
      READ(KIN,88) AG,HG,RG,SG,VIG,CSEG,CSIG,KB,KS
      READ(KIN,97) A1,B1,A2,B2
      IF(I.EQ.2) GO TO 15
      READ(KIN,82) NM,VM,IP
      READ(KIN,81) (@CUR(N),N=1,NM)
CC
C      WRITE @UT CONVERTER PROPERTIES
CC
15 WRITE(K@UT,84)
      WRITE(K@UT,85)
      WRITE(K@UT,86) AE,F@E,FCE,HE,RE,SE,SDE,ADE,CE
20 WRITE(K@UT,87)
      WRITE(K@UT,85)
      WRITE(K@UT,86) AC,F@C,FCC,HC,RC,SC,SDC,ADC,CC
25 WRITE(K@UT,89)
      WRITE(K@UT,90)
      WRITE(K@UT,91) AG,HG,RG,SG,VIG,CSEG,CSIG,KB,KS
      WRITE(K@UT,104)
      WRITE(K@UT,105)

```

```

      WRITE(KOUT,99) A1,B1,A2,B2
30  CONTINUE
CC
C    COMPUTE THERMAL RADIATION AND CS CONDUCTION
CC
      PG = 2.45E+8*EXP(-8910.0/TR)/SQRT(TR)
      T1 = TE
      T2 = TC
      T3 = SQRT(T1*T2)
      E1 = 1.0/(1.0/(A1+B1*T1)+1.0/(A2+B2*T3))-1.0)
      E2 = 1.0/(1.0/(A1+B1*T3)+1.0/(A2+B2*T2))-1.0)
      QR = 5.67*(E1*T1**4 - E2*T2**4)*1.0E-12
      QCS = 1.1E-6*(TE**1.5-TC**1.5)/(D/400.0+1.15E-5*(TE+TC)/PG)
CC
C    EVALUATE ZERO-FIELD ELECTRODE PERFORMANCE
CC
36  CALL SURFAS (AE,FE,FCE,HE,PE,SF,SDF,ADE,CE,TE,PG,WFE@,JE@,IE@,DE)
41  CALL SURFAS (AC,FC,FCC,HC,RC,LC,SUC,ADC,CC,TC,PG,WFC@,JC@,IC@,DC)
50  WRITE(KOUT,92)
      WRITE(KOUT,93)
      WRITE(KOUT,94) TE,TC,TR,D,WFE@,JE@,IE@,WFC@,JC@,IC@
CC
C    SET OUTPUT CURRENT DENSITY
CC
55  IF(IP,FQ,1) WRITE(KOUT,95)
      IF(IP,TQ,1) WRITE(KOUT,96)
      IF(IP,FQ,1) WRITE(KOUT,106)
60  N = 0
61  N = N + 1
      IF(N.GT.NM) GO TO 5
      J = @CUR(N)
65  IF(IP,NE,1) WRITE(KOUT,102) J
CC
C    PERFORM VOLUME PHYSICS AND SCHOTTKY EFFECT COMPUTATIONS
CC
      CALL PLASMA
      IF(IC@NV,EQ,0) WRITE(KOUT,110) J
      IF(IC@NV,EQ,0) GO TO 61
      IF(V,LT,VM) N=NM
CC
C    EVALUATE AND WRITE OUT ELECTRICAL AND THERMAL PERFORMANCE

```

CC

```

      P = J*V
      QIN = QEL + QION + QR + QCS
      EFF = P/QIN
      IF(IP.NE.1)WRITE(KOUT,103)
      IF(IP.NE.1)WRITE(KOUT,96)
      IF(IP.NE.1) WRITE(KOUT,106)
      WRITE(KOUT,98) J,V,P,EFF,QEL,QION,QCS,QR,QIN
      GO TO 61
80  FORMAT(4F8.3,4I8)
81  FORMAT(10F8.3)
82  FORMAT(18,F8.3,I8)
84  FORMAT(1H1,50X,18HEMITTER PROPERTIES)
85  FORMAT(108H0      AT WT      B W FUN      C W FUN      SUB HEAT      R
      1ADIUS      BOND STR      SUB DEN      ADS DEN      DI ENERGY)
86  FORMAT(1HC,9F12.3)
87  FORMAT(1H0,49X,20HCOLLECTOR PROPERTIES)
88  FORMAT(7F8.3,E8.3,F8.3)
89  FORMAT(1H0,51X,16HVAPOR PROPERTIES)
90  FORMAT(108H0      AT WT      SUB HEAT      RADIUS      BOND STR      IO
      1N POT      E-N XSEC      I-N XSEC      REC CON      SCHOTTKY)
91  FORMAT(1H0,7F12.3,E12.3,F12.3)
92  FORMAT(1H1,40X,41HTHERMIONIC CONVERTER OPERATING CONDITIONS)
93  FORMAT(120H0      EM TEMP      CO TEMP      RES TEMP      SPACING
      1 WFE0      JE0      IE0      WFC0      JC0      IC0)
94  FORMAT(1H0,6F12.3,F12.6,2F12.3,F12.6)
95  FORMAT(1H0,45X,29HOUTPUT CURRENT CHARACTERISTIC)
96  FORMAT(108HC      CURRENT      VOLTAGE      POWER      EFF      E
      1-COOL      1-COOL      CS COND      TH RAD      HEAT IN)
97  FORMAT(F8.3,E8.3,F8.3,E8.3)
98  FORMAT(12F12.3)
99  FORMAT(1HC,F12.3,E12.3,F12.3,F12.3)
102  FORMAT(////1H4, 29HCONVERTER PARAMETERS FOR J = ,F7.3)
103  FORMAT(1H0, 42HELECTRICAL AND THERMAL PERFORMANCE SUMMARY)
104  FORMAT(1H0, 50X,20HRADIATION PARAMETERS)
105  FORMAT( 48H0      EM EMIS      EM T COEF      CO EMIS      CO T COEF)
106  FORMAT(1H0)
110  FORMAT(1HC,29HSOLUTION UNCONVERGED FOR J = ,F8.3, 7H A/SQCM/)
      END

```

```
CSYSERR      ERROR RECOVERY SUBROUTINE  
SUBROUTINE SYSERR(IND)  
  IND = 0  
  RETURN  
END
```

```

CSURFAS      SURFACE PHYSICS SUBROUTINE
SUBROUTINE  URFAS(AS,FOS,FOS,HG,RS,SS,SSS,ASS,C,TS,PG,FO,EES,EIS,C
15)
COMMON/AF/ AC,HG,PG,SG,VIS,VALG,CSEG,CSIC
CC
C      EVALUATE ADSORPTION SYSTEM PARAMETERS
CC
      SWC = 2.0
      SDS = 100.0*SSS
      ADS = 100.0*ASS
      R = RG + RS
      COSP = SQRT(1.0 - 50.0/(SDS*R**2.0))
      ALF = RG**3 + 1.65*RS**3
      C1 = 0.905*R*ADS*COSP/(1.0+ALF/R**3)
      C2 = 9.0E-3*ALF*ADS**1.5
      C3 = C
      S = 2.0/(SC/SS + SS/SG)
      D0 = SQRT(HS*HG)*S
      APS = 0.55E+4*PG/SQRT(AG*TS)
CC
C      BEGIN SURFACE COVERAGE ITERATION
CC
      NCS = 0
      CS = 0.5
      DELCS = 0.5
10  CONTINUE
      NCS = NCS + 1
      IF(NCS.GT.15) GO TO 45
      G = 1.0 - 3.0*CS*CS + 2.0*CS*CS*CS
      DG = -6.0*CS*(1.0-CS)
CC
C      BEGIN FRACTIONAL CHARGE TRANSFER ITERATION
CC
      NF = 0
      F=0.5
12  CONTINUE
      NF = NF + 1
      IF(NF.GT.15) GO TO 45
      FOF = D0*F/SQRT(1.0-F**2) + C3*F - (FOS-FOG)*G
      DFOF = D0/(1.0-F**2)**1.5 + C3
      FN = F - FOF/DFOF

```

```

      IF(FN.LE.0.0) FN = 0.5*F
      IF(FN.GE.1.0) FN = 0.5*(1.0+F)
      IF(ABS(FN-F).LE.0.01) GO TO 15
      F = FN
      GO TO 12

```

CC

C

END FRACTIONAL CHARGE TRANSFER ITERATION

CC

```

15 FAS = D0*SQRT(1.0-F**2) - 0.5*C3*F**2 + F*(F0S-F0G)*G
   DFAS = F*(F0S-F0C)*DG
   FS = F0G + (F0S-F0C)*G - F*C1*CS/(1.0+C2*CS**1.5)
   FREQ1 = 2.5E-6*SQRT(FAS)*SQRT((AG+4.0*AS)/(2.0*AS*AS))/(R*C0SB)
   U = 0.5*CS*DFAS/FAS
   FREQ = FREQ1*EXP(U)
   ENT = EXP(0.5*(CS/(1.0-CS)+SQRT(CS)/(1.0-SQRT(CS))))/(SQRT(1.0-CS)
1*(1.0-SQRT(CS)))
   ERS = 1.0E+14*FREQ*SWG*ADS*CS*ENT*EXP(-11600.0*FAS/TS)
   IF(ABS((ERS-ARS)/ARS)-0.005)40,20,20
20 IF(ERS-ARS)25,40,30
25 DELCS = DELCS/2.0
   CS = CS + DELCS
   GO TO 10
30 DELCS = DELCS/2.0
   CS = CS - DELCS
   GO TO 10

```

CC

C

END SURFACE COVERAGE ITERATION

CC

C

COMPUTE ELECTRON AND ION EMISSION CURRENT DENSITIES

CC

```

40 EES = 120.0*TS**2*EXP(-11600.0*FS/TS)
   EIS = 2.598E+1*PG*EXP(11600.0*(FS-VIG)/TS)/SQRT(AG*TS/11600.0)
45 RETURN
   END

```



```

CPLASMA      PLASMA PHYSICS SUBROUTINE
SUBROUTINE PLASMA
COMMON /A/ AG,HG,PG,SG,VIG,VALG,CSES,CSIC
COMMON /B/ KB,KR,KS,GAMCR
COMMON /C/ WFE,JF,IE,WFC,JC,IC,WFE,JE,IE,WFC,JC,IC,J
COMMON /D/ KOUT,IP,ICNV,TF,TC,PG,D,V,QEL,QION
COMMON /E/ RE,RI,PA,VTE,VTC,VTL,AA,BB,VE,VC,JRQ,IRQ,YIQ,YOQ,NE,NC
COMMON /F/ JEM,IEM
REAL KB,KR,KS
REAL JE,IE,JC,IC,JE,IE,JC,IC,J
REAL JRQ,IRQ
REAL K,M,MP,J11,J12,JL1,JL2,JRQ,KL
REAL JEM,IEM
C(I(X1)) = 1.3862944 + (1.0-X1)*(0.1119723+0.0725296*(1.0-X1)) -
1(0.50+(1.0-X1)*(0.1213478+0.0288729*(1.0-X1)))*ALOG(1.0-X1)
ICNV = 0
VTE = TE/11600.0
VTC = TC/11600.0
VTC = 0.5*(VTE+VTC)
VTA = (JF/(JE+JC))*VTE + (JC/(JE+JC))*VTC
PE = 1.25E-4*PG*D*CSH/VTC
RI = 0.884E-4*PG*D*CSIC/VTC
DGAS = 1.94E+10*PG/(TE+TC)
JEM = JE
IEM = IE
IF((J/JEM).LT.0.99) GO TO 5
JEM = 1.05*J
IEM = IE*JE/JEM
5 CONTINUE
YIR = 0.0

CC
C      BEGIN ELECTRON TEMPERATURE ITERATION
CC
N1 = 0
N2 = 0
N9 = 0
VTL = 0.0862
VTLU = 0.3448
VTL = VTA + (0.22-VTA)*J/JEM
10 N2 = N2 + 1
IF(N2.GT.15) GO TO 325

```

12 CONTINUE

N3 = 0
N4 = 0
N5 = 0
N6 = 0
N7 = 0
N8 = 0
N9 = 0
NE = 0

15 CONTINUE

PA = PI*(VIG/(VTL+VIG))
TFL = 11600.0*VTL
JRQA = 4.23E+6*SQR(PG/VIG)*VTL**1.2E*EXP(-0.5*VIG/VTL)
XX = 2.15E-4*JRQA*VIG/(SQR(VTL)*PG)
JPG = JRQA*(SQR(1.0+XX**2)-XX)
JRG = 2.334E-2*SQR(VIG/(A3*VTL))*JRG
JSAHA = 0.377E+12*JRG/SQR(VTL)
CLN = ALG(2.54E+4*VTL**1.7/SQR(JRG))
REI = 5.23E-3*CLN**R/VTL**1.5
IF(N2.GT.1.0R.N9.GT.1.0) G@ 1@ 30

CC

C REFINED INITIAL ELECTRON TEMPERATURE ESTIMATE

CC

20 N1 = N1 + 1

IF(N1.GT.15) G@ T@ 325
IF((0.5*J/JRG).LT.1.0) G@ T@ 25
VTL = VTL + 0.017
G@ T@ 15

25 DD = ALG(((1.0-0.5*J/JRG)/(1.0+0.5*J/JRG))*((JC+J)/(JEM-J)))

F = 2.0*(JEM+JC)*(VTL-VTA)+4.0*JRG*VIG - J*(DD*VTL + RE*VTL*J/JRG
1+ J*REI*(D/400.0))

IF(F.LT.0.0) VTLL = VTL

IF(F.GT.0.0) VTLU = VTL

DF = 2.0*(JEM+JC) + 4.0*(VIG/VTL)**2*JRG - DD*J + RE*(VIG/VTL-1.0
1)*J**2/JRG + 1.5*J*(J*REI*(D/400.0)/VTL)

VTLN = VTL - F/DF

IF(VTLN.LE.VTLL.0R.VTLN.GE.VTLU) VTLN = 0.5*(VTLL+VTLU)

IF(ABS((VTL-VTLN)/VTL).LE.0.01) G@ T@ 28

VTL = VTLN

G@ T@ 15

28 CONTINUE

VTLL = 0.0862

```

      VTLU = 0.3448
CC
C      CHECK FOR NEAR EQUILIBRIUM PLASMA
CC
30  GAM = 5.91E+18*JRQ*D*SQRT(PG*CSIG/(VTG*(VTG+VTL)*VTL**5.5))*KB
      Y1Q = (JC@+0.5*J-IC@)/(JRQ-IRQ)
      Y@Q = (JE@-0.5*J-IE@)/(JRQ-IRQ)
      AA = 1.414*(VTL/VTG)*(CSEG/CSIG)
      BB = JRQ*RE!*D/(4J0.0*RE*VTL)
      Y1M = 0.5*(J-IC@)/JRQ
      IF(Y1M.LT.0.0) Y1M = 0.0
      Y1L = Y1M
      Y1U = 1.0
      IF(GAM.GT.GAMCR) GO TO 2J0
CC
C      CHECK FOR MINIMUM VALUE OF K
CC
      Z1L = AA*(1.0+BB*Y1L)
      P1L = RA*(Z1L*J/JRQ-IC@/IRQ)
      KL = (P1L/GAM)**2 + Y1L**2*(2.0-Y1L**2)
      IF(KL.LT.1.0) GO TO 35
      GO TO 115
CC
C      DENSITY PROFILE FOR  $\gamma$  EQUAL TO 1
CC
35  CONTINUE
      P1 = GAM*(1.0-Y1L)*(1.0+Y1L)
      DP1L = P1 - P1L
      DP1@ = DP1L
      Y1P = Y1L
      Y1 = Y1Q
      IF(Y1.LE.Y1L.@R.Y1.GE.Y1U) Y1 = 0.5*(Y1L+Y1U)
40  N3 = N3 + 1
      IF(N3.GT.20) GO TO 325
      P1 = GAM*(1.0-Y1)*(1.0+Y1)
      CALL CBC(Y1,P12)
      IF(NC.CT.15) GO TO 325
      K = (P12/GAM)**2 + Y1**2*(2.0-Y1**2)
      DP1 = P1 - P12
      IF(DP1.GT.0.0) Y1L = Y1
      IF(DP1.LT.0.0) Y1U = Y1

```

```

      Y1N = Y1 - DP1*(Y1-Y1P)/(DP1-DP1P)
      IF(Y1N.LE.Y1L.OR.Y1N.GE.Y1U) Y1N = Y1 - DP1*(Y1-Y1L)/(DP1-DP1L)
      IF(Y1N.LE.Y1L.OR.Y1N.GE.Y1U) Y1N = 0.5*(Y1L+Y1U)
      IF(ABS(K-1.0).GE.1.0E-4) GO TO 45
      IF(ABS((Y1N-Y1)/Y1).GE.0.001) GO TO 45
      IF(Y1.LE.Y1Q.AND.ABS((Y1N-Y1)/(Y1Q-Y1M)).GE.0.05) GO TO 45
      GO TO 50
45  Y1P = Y1
      DP1P = DP1
      Y1 = Y1N
      GO TO 40
50  U@ = 0.5*ALOG((1.0+Y1)/(1.0-Y1))
      Y@ = TANH(GAM+J@)
      P@ = GAM*(1.0-Y@)*(1.0+Y@)
      CALL EBC(Y@,P@,Z)
      IF(NE.JT.15) GO TO 325
      DP@ = P@ - P@2
      IF(DP@.GE.0.0) Y1L = 0.5*(J-IC@)/JRC
      IF(DP@.GE.0.0) Y1U = Y1
      IF(DP@.LT.0.0) Y1L = Y1
      IF(DP@.LT.0.0) Y1U = 1.0
      DP@P = DP@
      Y1P = Y1
      Y@P = Y@
      MP = 1.0
      IF(DP@.GE.0.0) GO TO 60
      IF(DP@.LT.0.0) GO TO 120
CC
C      DENSITY PROFILE AND Y INTEGRAL FOR K LESS THAN 1
CC
60  CONTINUE
      Y1 = Y1L
      P1 = P1L
      K = KL
      N4 = 1
      GO TO 65
61  Y1 = 0.5*(Y1L+Y1U)
62  N4 = N4 + 1
      IF(N4.GT.25) GO TO 325
      CALL CBC(Y1,P1)
      IF(NC.GT.15) GO TO 325
      K = (P1/GAM)**2 + Y1**2*(2.0-Y1**2)

```

```

      IF(K.LT.1.0) GO TO 65
      Y1U = Y1
      Y1 = 0.5*(Y1+Y1P)
      GO TO 62
65  M = (1.0-SQRT(1.0-K))/(1.0+SQRT(1.0-K))
      E1 = CE1(M)
      DU = SQRT(2.0/(1.0+M))*GAM
      IF(DU.GT.2.0*E1) GO TO 105
      YM = SQRT(2.0*M/(1.0+M))
      W1 = Y1/YM
      U1 = #1
      N7 = 0
80  N7 = N7 + 1
      IF(N7.GT.15) GO TO 325
      W1C = SN(U1,M)
      DU1 = (W1-W1C)/SQRT((1.0-W1C**2)*(1.0-M*W1C**2))
      IF(ABS(DU1/U1).LE.0.01) GO TO 85
      U1 = U1 + DU1
      GO TO 80
85  U@ = U1 + DU
      IF(U@.GT.2.0*E1) GO TO 105
      ESU@ = SN(U@,M)
      IF(ESU@.GE.1.0) ESU@ = 0.99999
      W@ = ESU@
      Y@ = YM*W@
      IF(U@.GE.E1) P@ = - GAM*SQRT(1-Y@**2*(2.0-Y@**2))
      IF(U@.LT.E1) P@ = + GAM*SQRT(1-Y@**2*(2.0-Y@**2))
      CALL EBC(Y@,P@2)
      IF(NE.GT.15) GO TO 325
      DP@ = P@ - P@2
      IF(N4.EQ.1.AND.DP@.GT.0.0) GO TO 290
      IF(N4.LE.3) GO TO 90
      IF(ABS((M-MP)/M).GE.0.01 .OR. ABS((M-MP)/(1.0-M)).GE.0.01) GO TO 90
      IF(Y1.LE.Y10.AND.ABS((Y1-Y1P)/(Y1-Y1M)).GE.0.05) GO TO 90
      GO TO 110
90  IF(DP@.GT.0.0) Y1U = Y1
      IF(DP@.LT.0.0) Y1L = Y1
      IF(N4.EQ.1) Y1N = Y1C
      IF(N4.GT.1) Y1N = Y1 - ((Y1-Y1P)/(DP@-DP@P))*DP@
      IF(Y1N.LE.Y1L.OR.Y1N.GE.Y1U) Y1N = 0.5*(Y1L+Y1U)
      Y1P = Y1

```

```

      DP@P = DP@
      MP = M
      Y1 = Y1N
      GO TO 62
105 Y1L = Y1
      MP = M
      GO TO 61
110 ESU1 = W1
      ECU1 = SQRT(1.0-W1**2)
      EDU1 = SQRT(1.0-M*W1**2)
      IF(U@.LE.EI) ECU@ = +SQRT(1.0-W@**2)
      IF(U@.GT.EI) ECU@ = -SQRT(1.0-W@**2)
      EDU@ = SQRT(1.0-M*W@**2)
      YINV = (1.0+M)/2.0*GAM*SQRT(M))*AL@G(((EDU@-ECU@)/(EDU1-ECU1))*
1(ESU1/ESJ@))
      Y1B = Y1
      GO TO 250
CC
C      DENSITY PROFILE AND Y INTEGRAL FOR K GREATER THAN 1
CC
115 CONTINUE
      Y1U = 1.1E+6
      Y1 = Y1L
      P1 = P1L
      K = KL
      GO TO 125
120 CONTINUE
      Y1U = 1.1E+6
      IF(Y1G.GT.Y1L) Y1 = Y1G
      IF(Y1G.LE.Y1L) Y1 = 1.0
122 N4 = N4 + 1
      IF(N4.GT.25) GO TO 325
      CALL CFC(Y1,P1)
      IF(NC.GT.15) GO TO 325
      K = (P1/GAM)**2 + Y1**2*(2.0-Y1**2)
      IF(K.GT.1.0) GO TO 125
      Y1L = Y1
      IF(Y1U.LT.1.0E+6) Y1 = 0.5*(Y1L+Y1U)
      IF(Y1U.GT.1.0E+6) Y1 = 2.0*Y1
      GO TO 122
125 M = (1.0+SQRT(K))/(2.0*SQRT(K))
      EI = CEI(M)

```

```

DU = GAM/SQRT(2.0*M-1.0)
IF(DU.GT.EI) G@ T@ 170
YM = 1.0/SQRT(2.0*M-1.0)
W1 = Y1/YM
U1 = W1
N8 = 0
140 N8 = N8 + 1
IF(N8.GT.15) G@ T@ 325
ESU1 = SN(U1,M)
ECU1 = SQRT(1.0-ESU1**2)
EDU1 = SQRT(1.0-M*ESU1**2)
W1C = ESU1*EDU1/ECU1
DU1 = (ECU1**2/(EDU1**2-M*(ESU1*ECU1)**2))*(W1-W1C)
IF(ABS(DU1/U1).LE.0.01) G@ T@ 150
U1 = U1 + DU1
G@ T@ 140
150 U@ = U1 + DU
IF(U@.GT.EI) G@ T@ 170
ESU@ = SN(U@,M)
IF(ESU@.GE.1.0) ESU@ = 0.99999
ECU@ = SQRT(1.0-ESU@**2)
EDU@ = SQRT(1.0-M*ESU@**2)
W@ = ESU@*EDU@/ECU@
Y@ = YM*U@
P@ = GAM*SQRT(K - Y@**2*(2.0-Y@**2))
160 CALL EBC(Y@,P@2)
IF(N@.GT.15) G@ T@ 325
DP@ = P@ - P@2
IF(N4.EQ.0.AND.DP@.GT.0.0) G@ T@ 290
IF(N4.LE.3) G@ T@ 163
IF(ABS((Y1-Y1P)/Y1).GE.0.01.OR.ABS((Y@-Y@P)/Y@).GE.0.01) G@T@163
IF(Y1.LE.Y1Q.AND.ABS((Y1-Y1P)/(Y1Q-Y1M)).GE.0.05) G@ T@ 163
IF(ABS(P@).GT.1.0.AND.ABS(DP@/P@).GE.0.01) G@ T@ 163
IF(ABS(P@).LE.1.0.AND.ABS(DP@).GE.0.01) G@ T@ 163
G@ T@ 175
163 IF(DP@.GT.0.0) Y1U = Y1
IF(DP@.LT.0.0) Y1L = Y1
IF(N4.EQ.0.AND.Y1Q.GT.Y1L) Y1N = Y1Q
IF(N4.EQ.0.AND.Y1Q.LE.Y1L) Y1N = 1.0
IF(N4.EQ.0) G@ T@ 165
IF(Y1U.LT.1.0E+6) G@ T@ 164

```

```

      Y1N = 2.0*Y1
      GO TO 165
164 Y1N = Y1 - ((Y1-Y1P)/(DP0-DP0P))*DP0
      IF(Y1N.LT.Y1L.OR.Y1N.GT.Y1U) Y1N = 0.5*(Y1L+Y1U)
165 Y1P = Y1
      Y0P = Y0
      DP0P = DP0
      MP = M
      Y1 = Y1N
      GO TO 122
170 IF(N4.EQ.0) GO TO 290
      Y1U = Y1
      Y1 = 0.5*(Y1L+Y1U)
      GO TO 122
175 YINV = (1.0/(GAM*SQRT(K)))*ALOG((ESU0/ESU1)*(EDU1/EDU0))
      Y1B = Y1
      GO TO 250
CC
C      DENSITY PROFILE AND Y INTEGRAL FOR NEAR-EQUILIBRIUM PLASMA
CC
200 CONTINUE
      K = 1.0
      M = 1.0
      Y1 = Y1B
      IF(Y1.LE.Y1L.OR.Y1.GE.Y1U) Y1 = 0.5*(Y1L+Y1U)
205 N5 = N5 + 1
      IF(N5.GT.15) GO TO 325
      P1 = GAM*(1.0-Y1)*(1.0+Y1)
      CALL CFC(Y1,P1Z)
      IF(NC.GT.15) GO TO 325
      DP1 = P1 - P1Z
      IF(DP1.GT.0.0) Y1L = Y1
      IF(DP1.LT.0.0) Y1U = Y1
      IF(N5.EQ.1) Y1N = Y1 + DP1/(2.0*GAM*Y1)
      IF(N5.GT.1) Y1N = Y1 - ((Y1-Y1P)/(DP1-DP1P))*DP1
      IF(Y1N.LE.Y1L.OR.Y1N.GE.Y1U) Y1N = 0.5*(Y1L+Y1U)
      IF(N5.GT.1.AND.ABS((Y1-Y1N)/Y1).LE.0.01.AND.ABS(DP1/P1).LE.0.01)
100 GO TO 215
      Y1P = Y1
      DP1P = DP1
      Y1 = Y1N
      GO TO 205

```



```

215 X01 = (1.0/GAM)*0.5*ALOG((1.0+Y1)/(1.0-Y1))
      Y1B = Y1
      Y0L = 0.0
      Y0U = 1.0
      Y0 = 1.0
225 N6 = N6 + 1
      IF(N6.GT.15) GO TO 325
      P0 = -GAM*(1.0-Y0)*(1.0+Y0)
      CALL EBC(Y0,P02)
      IF(N6.GT.15) GO TO 325
      IF(P02.LE.0.0) GO TO 235
      IF(N6.EQ.1) GO TO 290
      Y0L = Y0
      Y0 = 0.5*(Y0L+Y0U)
      GO TO 225
235 DP0 = P0 - P02
      IF(DP0.GT.0.0) Y0U = Y0
      IF(DP0.LT.0.0) Y0L = Y0
      IF(N6.EQ.1) Y0N = Y0 - DP0/(2.0*GAM*Y0)
      IF(N6.GT.1) Y0N = Y0 - ((Y0-Y0P)/(DP0-DP0P))*DP0
      IF(Y0N.LE.Y0L.OR.Y0N.GE.Y0U) Y0N = 0.5*(Y0L+Y0U)
      IF(N6.GT.1.AND.ABS((Y0-Y0N)/Y0).LE.0.01.AND.ABS(DP0/P0).LE.0.01)
1GO TO 245
      Y0P = Y0
      DP0P = DP0
      Y0 = Y0N
      GO TO 225
245 X00 = (1.0/GAM)*0.5*ALOG((1.0+Y0)/(1.0-Y0))
248 YINV = 1.0 + X01 + X00 + (1.0/GAM)*ALOG(SQRT((1.0-Y0)*(1.0+Y0)*(1.
10-Y1)*(1.0+Y1)))/(4.0*Y0*Y1))
CC
C      CONCLUDE ELECTRON TEMPERATURE ITERATION (ELECTRON ENERGY BALANCE)
CC
250 RM0B = 3.30E-2*SQRT(VTL/(AG*VTG))*(CSEG/CSIG)
      RM0B1 = RM0B*(1.0+BB*Y1)
      RM0B0 = RM0B*(1.0+B3*Y0)
255 JI1 = -RM0B1*J + (IRQ/RA)*P1
      JI0 = -RM0B0*J + (IRQ/RA)*P0
      JL1 = J + JI1
      JI0 = J + JI0
260 IF(VC.GE.0.0) QC = 2.0*(JL1+JC)*VTL - 2.0*JC*VTC + JI1*(VIG+VC)

```

```

      IF(VC.LT.0.0) QC = 2.0*(JRQ*Y1+0.5*JL1)*VTL - 2.0*(JRQ*Y1-0.5*JL1)
      1*VTC - JL1*VC + JI1*(VIG+VC)
265 IF(VE.GE.0.0) QE = 2.0*(JRQ*Y0+0.5*JL0)*VTE - 2.0*(JRQ*Y0-0.5*JL0)
      1*VTL + JL0*VE + JI0*(VIG-VE)
      IF(VE.LE.0.0) QE = 2.0*JE*VTE - 2.0*(JE-JL0)*VTL + JI0*(VIG-VE)
      VP = -VTL*ALOG(Y1/Y0)-0.5*(JL1+JL0)*(RE*VTL*YINV/JRQ+REI*D/400.0)
270 F = QC - QE + J*(VE+VC+VP)
      V = WFE - WFC + VE + VP + VC
      IF(F.LT.0.0) VTLL = VTL
      IF(F.GT.0.0) VTLU = VTL
      IF(N2.EQ.1.AND.J.GE.JE) GO TO 280
      IF(N2.EQ.1) DF = 2.0*(JE+JC) + 4.0*(VIG/VTL)**2*IRQ - J*ALOG(((1.0
      1-0.5*J/JRQ)/(1.0+0.5*J/JRQ))*((JC+J)/(JE-J))) + RE*VTL*J/JRQ + J*
      2REI*(D/400.0)
      IF(N2.GT.1) DF = (F-FP)/(VTL-VTLP)
280 CONTINUE
      VTLN = VTL - F/DF
      IF(VTLN.LE.VTLL.OR.VTLN.GE.VTLU) VTLN = 0.5*(VTLL+VTLU)
285 IF(N2.LE.3.OR.ABS((VTLN-VTL)/VTL).GE.0.002) GO TO 288
      DELV = ((V-VPRV)/(VTL-VTLP))*(VTLN-VTL)
      IF(ABS(DELV).LE.0.005)GO TO 300
288 VPRV = V
      VTLP = VTL
      FP = F
      VTL = VTLN
      GO TO 10
290 CONTINUE
      N9 = N9 + 1
      IF(N9.GT.15) GO TO 325
      VTLL = VTL
      IF(VTLU.LT.0.344) VTL = 0.5*(VTL+VTLU)
      IF(VTLU.GE.0.344) VTL = VTL + 0.017
      IF(VTL.LE.VTLL.OR.VTL.GE.VTLU) GO TO 325
      GO TO 12
300 CONTINUE
      P0X = -P0
      P1X = -P1
      QEL = QE - JI0*(VIG-VE) + JL0*WFE
      QI0N = JI0*(VIG-VE-WFE)
      ICONV = 1
CC
C PRINT OUTPUT FOR CONVERGED SOLUTION

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CC
  IF(IP.EQ.1) GO TO 360
  IF(IP.EQ.2) GO TO 310
  WRITE(KOUT,330)
  WRITE(KOUT,331)
  WRITE(KOUT,332)K,M,Y0,P0X,Y1,P1X
  WRITE(KOUT,333)
  WRITE(KOUT,334)
  WRITE(KOUT,335)DGAS,RE,RI,GAM ,TEL,DSAHA,RE1,J10,J11
310 CONTINUE
  WRITE(KOUT,336)
  WRITE(KOUT,337)
  WRITE(KOUT,338)WFE,VE,VP,VC,WFC,JE,JC,IE,IC
325 GO TO 360
330 FORMAT(1H0, 32HCHARGED PARTICLE DENSITY PROFILE)
331 FORMAT(1H0, 72H INT CON K MODULUS M Y(X=0) P(X=0)
  1 Y(X=D) P(X=D))
332 FORMAT(1H0,6F12.3)
333 FORMAT(1H0, 17HPLASMA PROPERTIES)
334 FORMAT(1H0,108H CS DEN E-MFP I-MFP GAMMA
  1 EL TEMP SAHA DEN COUL RES J1(X=0) J1(X=D))
335 FORMAT(1H0,E12.3,4F12.3,E12.3,F12.3,2F12.6)
336 FORMAT(1H0, 36HVOLTAGE AND EMISSION CURRENT SUMMARY)
337 FORMAT(1H0,108H EM W FUN EM SHEATH PL VOLT CO SHEATH
  1 CO W FUN JE JC IE IC)
338 FORMAT(1H0,7F12.3,2F12.6)
360 RETURN
END

```

```

CEBC      Emitter Boundary Conditions
SUBROUTINE EBC(Y@,P@)
COMMON /B/ KB,KR,KS,GAMCR
COMMON /C/ WFE@,JE@,IE@,JFC@,JC@,IC@,WFE,JF,IE,JFC,JC,IC,J
COMMON /E/ RE,RI,RA,VTE,VTE@,VTL,AA,BB,VTL,JC,JRQ,IRQ,YIQ,Y@Q,NE,NC
COMMON /F/ JEM,IEM
REAL KB,KR,KS
REAL JE@,IE@,JC@,IC@,JE,IF,JC,IC,J
REAL JRQ,IRQ
REAL LA,IE@
REAL JFM,IEM
WFE = WFE@
JE = JE@
IE = IE@
Z@ = AA*(1.0+BB*Y@)
L@ = 0.487*VTL**1.75/SQRT(JRQ*Y@)
IF(Y@-Y@Q) 10,10,20

CC
C      COMPUTATIONS FOR RETARDING Emitter Sheath
CC
10 NE = 0
R = (JRQ+IRQ)*Y@/JF + 2.0 + 0.5*J/JF
C = (J+IE+2.0*JRQ*Y@)/JF
XX = B*B/C
IF(XX.LT.100.0) G = 0.5*XX*(1.0-SQRT(1.0-4.0/XX))
IF(XX.GE.100.0) G = 1.0
E = (C/B)*G
VE = -VTE*ALOG(E)
15 NE = NE + 1
IF(NE.GT.15) G@ TO 40
DWFE = 0.0076*SQRT(KS*F/L@)
REF = EXP(-DWFE/VTE)
JE = JE@*REF
IE = IE@/REF
P@ = RA*(IE/IRQ-Y@*F)/(1.0-0.5*F) + RA*/@*J/JRQ
F = (JRQ+IRQ)*Y@*F-2.0*JRQ*Y@-(1.0-F*E)*(J-2.0*JL*F)-IF
DF = -(JRQ+IRQ)*Y@*F/VTE - (2.0 - F)*(1.0+0.5*DWFE/VL)*JE*/VTE
1 = 0.5*(J-2.0*JF*F)*F/VTE - 0.5*IE*DWFE/(VTE*TF)
VEN = VE - F/DF
IF(VEN.LE.0.0) VEN = 0.5*VE
IF(NE.LE.1.0,ABS(VF-VEN).GE.0.005) GO TO 18
IF(P@.LE.1.0,AND(ABS(P@-P@P),GT.0.005) GO TO 18

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```

      IF(P0.GT.1.0.AND.ABS((P0-P0P)/P0).GE.0.005) GO TO 18
      WFE = WFE0 + DWFE
      GO TO 40
18  POP = P0
      VE = VEN
      E = EXP(-VE/VTE)
      GO TO 15
CC
C   COMPUTATIONS FOR ACCELERATING EMITTER SHEATH
CC
20  NE = 0
      IF(J.LT.JE0) GO TO 23
      VE = -VTL
      E = EXP(VE/VTL)
      GO TO 25
23  B = ((JQ+IRQ)*Y0-0.5*J+2.0*IE)/IF
      C = (JF-J+2.0*IRQ*Y0)/IC
      XX = B*B/C
      IF(XX.LT.100.0) G = 0.5*XX*(1.0-SQRT(1.0-4.0/XX))
      IF(XX.GE.100.0) C = 1.0
      E = (C/B)*G
      VE = VTL*ALOG(F)
25  NE = NE + 1
      IF(NE.GT.15) GO TO 40
      DWFE = 0.0076*SQRT(-KS*VE/L0)
      ENH = EXP(DWFE/VTE)
      JE = JF0*ENH
      IE = IE0/ENH
      IEP = IE*E**((VTL/VTE))
      P0 = RA*(2.0*IEP/IRQ+Z0*J/JQ-2.0*Y0)
      F = (JQ+IRQ)*Y0*E-2.0*IRQ*Y0+(1.0-0.5*E)*(J+2.0*IEP) - JE
      DF = (JQ+IRQ)*Y0*F/VTL + (2.0-E)*(IEP/VTE)*(1.0-DWFE/VE) - 0.5*(J+
12.0*IEP)*E/VTL - 0.5*JE*DWFE/(VTE*VF)
      VEN = VE - F/DF
      IF(VEN.GE.0.0) VEN = 0.5*VE
      IF(NE.LE.1.0R.ABS(VE -VEN).GE.0.005) GO TO 35
      IF(P0.LE.1.0.AND.ABS(P0-P0P).GE.0.005) GO TO 35
      IF(P0.GT.1.0.AND.ABS((P0-P0P)/P0).GE.0.005) GO TO 35
      GO TO 38
35  POP = P0
      VE = VEN

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B-21

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      E = EXP(VE/VTL)
      GO TO 25
38  CONTINUE
      WFE = WFE@ - DWFE
40  RETURN
      END
```

GESR-2109

```

CCBC      COLLECTOR BOUNDARY CONDITIONS
SUBROUTINE CBC(Y1,P1)
COMMON /B/ KB,KR,KS,GAMCR
COMMON /C/ WFE@,JF@,IE@,WFC@,JC@,IC@,WFE,JE,IE,WFC,JC,IC,J
COMMON /E/ RE,R1,RA,VTE,VTC,VTL,AA,BB,VE,VC,JRQ,IRQ,Y1Q,Y@Q,NE,NC
REAL KB,KR,KS
REAL JF@,IE@,JC@,IC@,JF,IE,JC,IC,J
REAL JRQ,IRQ
REAL L1,ICP
WFC = WFC@
JC = JC@
IC = IC@
Z1 = AA*(1.0+BB*Y1)
L1 = 0.487*VTL**0.75/SQRT(JRQ*Y1)
IF(Y1,NE,Y1Q) GO TO 8
VC = 0.
P1 = 2.*RA*(Y1-IC/IRQ) + RA*Z1*J/JRQ
GO TO 40
8 CONTINUE
IF(Y1-Y1Q) 10,10,20
CC
C      COMPUTATIONS FOR ACCELERATING COLLECTOR SHEATH
CC
10 NC = 0
B = (JRQ+IRQ)*Y1/JC - 0.5*J/JC + 2.0
C = 2.0*JRQ*Y1/JC + IC/JC - J/JC
XX = B*B/C
IF(XX.LT.100.0) G = 0.5*XX*(1.0-SQRT(1.0-4.0/XX))
IF(XX.GE.100.0) G = 1.0
E = (C/B)*G
VC = VTC*ALOG(E)
15 NC = NC + 1
IF(NC.GT.15) GO TO 40
DWFC = 0.0076*SQRT(-KS*VC/L1)
RED = EXP(-DWFC/VTC)
JC = JC@*RED
IC = IC@/RED
P1 = RA*(Y1*E-IC/IRQ)/(1.0-0.5*E) + RA*Z1*J/JRQ
F = (JRQ+IRQ)*Y1*E - 2.*JRQ*Y1 + (1.-.5*E)*(J+2.*JC*E)-IC
DF = (JRQ+IRQ)*Y1*E/VTC + (2.-E)*(1.-.5*DWFC/VC)*JC*E/VTC
1- .5*(J+2.*JC*E)*E/VTC - .5*IC*DWFC/(VTC*VC)
VCN = VC - F/DF

```

```

      IF(VCN.GE.0.0) VCN = 0.5*VC
      IF(NC.LE.1.0R.ABS(VC-VCN).GE.0.005) GO TO 18
      IF(P1.LE.1.0.AND.ABS(P1-P1P).GE.0.005) GO TO 18
      IF(P1.GT.1.0.AND.ABS((P1-P1P)/P1).GE.0.005) GO TO 18
      WFC = WFC@ + DWFC
      GO TO 40
18  P1P = P1
      VC = VCN
      E = EXP(VC/VTC)
      GO TO 15
CC
C   COMPUTATIONS FOR RETARDING COLLECTOR SHEATH
CC
20  NC = 0
      E = (2.0*IRQ*Y1+JC+J-2.0*IC)/((JRQ+IRQ)*Y1+0.5*J-IC)
      VC = -VTL*ALOG(E)
25  NC = NC + 1
      IF(NC.GT.15) GO TO 40
      DWFC = 0.0076*SQRT(KS*VC/L1)
      ENH = EXP(DWFC/VTC)
      JC = JC@*ENH
      IC = IC@/ENH
      ICP = IC*EXP(-VC/VTC)
      P1 = RA*(2.0*Y1+Z1*J/JRQ-2.0*ICP/IRQ)
      F = Y1*((JRQ+IRQ)*E-2.0*IRQ)-(1.0-0.5*E)*(J-2.0*ICP) - JC
      DF = - (JRQ+IRQ)*Y1*E/VTL - JC*DWFC/(2.0*VC*VTC) - (2.0-E)*(ICP/VTL
1C)*(1.0+0.5*DWFC/VC) - 0.5*(J-2.0*ICP)*E/VTL
      VCN = VC - F/DF
      IF(VCN.LE.0.0) VCN = 0.5*VC
      IF(NC.LE.1.0R.ABS(VC-VCN).GE.0.005) GO TO 35
      IF(P1.LE.1.0.AND.ABS(P1-P1P).GE.0.005) GO TO 35
      IF(P1.GT.1.0.AND.ABS((P1-P1P)/P1).GE.0.005) GO TO 35
      GO TO 38
35  P1P = P1
      VC = VCN
      E = EXP(-VC/VTL)
      GO TO 25
38  CONTINUE
      WFC = WFC@ - DWFC
40  RETURN
      END

```



```

CELP SIN      ELLIPTIC SINE FUNCTION
FUNCTION SN(XIN,C)
  DIMENSION P(10),P1(10),V(10),Y(10),Z(10)
  CEI(X1) = 1.3862944 + (1.0-X1)*(0.1119723+0.0725296*(1.0-X1)) -
1(0.50+(1.0-X1)*(0.1213478+0.0289729*(1.0-X1)))*ALOG(1.0-X1)
  EI = CEI(C)
  IF(XIN.GT.EI)X=2.0*EI-XIN
  IF(XIN.LE.EI)X=XIN
  IF (C-0.5)5,5,15
CC
C      COMPUTATIONS FOR C LESS THAN 0.5
CC
5  J = 1
  P(J) = C
  P1(J) = 1.0 - C
  V(J) = X
  GO TO 9
7  J = J + 1
  P(J) = ((1.0-SQRT(P1(J-1)))/(1.0+SQRT(P1(J-1))))**2.0
  P1(J) = 1.0 - P(J)
  V(J) = V(J-1)/(1.0+SQRT(P(J)))
9  IF(P(J)-0.0001)10,7,7
10 JMAX = J
  Y(JMAX) = SIN(V(JMAX))-0.25*P(JMAX)*(V(JMAX)-SIN(V(JMAX)))*COS(V(JM
1AX)))*COS(V(JMAX))
  IF(JMAX-1)13,13,11
11 M = JMAX - 1
  DO 12 L = 1,M
  N = JMAX - L
12 Y(N) = ((1.0+SQRT(P(N+1)))*Y(N+1))/(1.0+SQRT(P(N+1))*Y(N+1)**2.0)
13 CONTINUE
  SN = Y(1)
  GO TO 25
CC
C      COMPUTATIONS FOR C GREATER THAN 0.5
CC
15 J = 1
  P(J) = C
  P1(J) = 1.0 - C
  V(J) = X
  GO TO 21
17 J = J + 1

```

```

      P(J)= 4.0*SQRT(P(J-1))/(1.0+SQRT(P(J-1)))*2.0
      P1(J) = ((1.0-SQRT(P(J-1)))/(1.0+SQRT(P(J-1)))*2.0
20  V(J) = 0.5*V(J-1)*(1.0+SQRT(P(J-1)))
21  IF(P1(J)-0.0001)23,17,17
23  JMAX = J
      HTN = TANH(V(JMAX))
      HSC= SQRT(1.0-HTN**2.0)
      HSN=HTN/HSC
      HCN= 1.0/HSC
      Z(JMAX) = HSC + 0.25*P1(JMAX)*(HSN*HCN+V(JMAX))*HTN*HSC
      IF(JMAX-1)40,40,30
30  M = JMAX -1
      DO 35 L = 1,M
      N = JMAX - L
35  Z(N) = (1.0-SQRT(P1(N+1)))*(Z(N+1)**2.0+SQRT(P1(N+1)))/(P(N+1)*Z(N
      1+1))
40  CONTINUE
      SN = SQRT((1.0-Z(1)**2.0)/C)
25  RETURN
      END

```

APPENDIX C
SIMCON OUTPUT NOMENCLATURE
AND UNITS

ADS DEN - electrode adsorption site density ($\text{\AA}^{-2}$)
 AT WT - atomic weight (amu)
 B W FUN - electrode work function at zero coverage (eV)
 BOND STR - bond strength (dimensionless)
 C W FUN - electrode work function at one monolayer coverage (eV)
 CO EMIS - collector emissivity intercept (dimensionless)
 CO SHEATH - collector sheath voltage drop (eV)
 CO T COEF - collector emissivity temperature coefficient ($^{\circ}\text{K}^{-1}$)
 CO TEMP - collector temperature ($^{\circ}\text{K}$)
 COUL RES - plasma Coulomb resistivity ($\Omega \text{ cm}$)
 CO W FUN - field modified collector work function (eV)
 CS COND - cesium conduction heat flux (w/cm^2)
 CS DEN - cesium vapor density (cm^{-3})
 CURRENT - output current density (A/cm^2)
 D1 ENERGY - electrode adsorption system characteristic energy (eV)
 E-COOL - electron cooling heat flux at emitter (w/cm^2)
 EFF - intrinsic efficiency (neglecting electrode, lead and structural support losses)
 EL TEMP - plasma electron temperature ($^{\circ}\text{K}$)
 EM EMIS - emitter emissivity intercept (dimensionless)
 E-MFP - number of electron-cesium mean free paths across plasma (dimensionless)
 EM SHEATH - emitter sheath voltage drop (eV)
 EM T COEF - emitter emissivity temperature coefficient ($^{\circ}\text{K}^{-1}$)
 EM TEMP - emitter temperature ($^{\circ}\text{K}$)
 EM W FUN - field modified emitter work function (eV)

E-N XSEC - electron-caesium cross section ($\text{\AA}^2$)
 GAMMA - ionization parameter γ (dimensionless)
 HEAT IN - total heat flux at emitter (w/cm^2)
 IC - field modified ion emission from collector (A/cm^2)
 ICO - zero field ion emission from collector (A/cm^2)
 I-COOL - ion cooling heat flux at emitter (w/cm^2)
 IE - field modified ion emission from emitter (A/cm^2)
 IEO - zero field ion emission from emitter (A/cm^2)
 I-MFP - number of ion-caesium mean free paths across plasma (dimensionless)
 I-N XSEC - ion-caesium cross section ($\text{\AA}^2$)
 INT CON K - integration constant K (dimensionless)
 ION POT - ionization potential (eV)
 JC - field modified electron emission from collector (A/cm^2)
 JCO - zero field electron emission from collector (A/cm^2)
 JE - field modified electron emission from emitter (A/cm^2)
 JEO - zero field electron emission from emitter (A/cm^2)
 JI (X=D) - ion current density at collector (A/cm^2)
 JI (X=O) - ion current density at emitter (A/cm^2)
 MODULUS M - elliptic function modulus M (dimensionless)
 P(X=D) - charged particle dimensionless density gradient at collector
 edge of plasma
 P(X=O) - charged particle dimensionless density gradient at emitter
 edge of plasma
 PL VOLT - interelectrode plasma voltage drop (eV)
 POWER - output power density at electrodes (w/cm^2)
 RADIUS - metallic radius ($\text{\AA}$)

REC CON - recombination coefficient constant $\left[\text{cm}^6 (\text{eV})^{9/2} / \text{sec} \right]$

RES TEMP - cesium reservoir temperature ($^{\circ}\text{K}$)

SAHA DEN - Saha equilibrium charged particle density (cm^{-3})

SCHOTTKY - Schottky effect constant (dimensionless)

SPACING - interelectrode spacing (mils)

SUB DEN - electrode substrate atom density ($\text{\AA}^{-2}$)

SUB HEAT - heat of sublimation (eV)

TH RAD - thermal radiation heat flux (w/cm^2)

VOLTAGE - output voltage at electrodes (V)

WFCO - zero field collector work function (eV)

WFEO - zero field emitter work function (eV)

Y(X=D) - charged particle dimensionless density at collector
edge of plasma

Y(X=O) - charged particle dimensionless density at emitter
edge of plasma