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A STATISTICAL DETERMINATION OF THE REDUCED PROMPT
NEUTRON GENERATION TIME, $\Lambda/\bar{\beta}$, IN THE SPERT IV REACTOR

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by

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ABSTRACT

This paper describes an experimental determination of the reduced prompt neutron generation time Λ/β in the Spert IV reactor utilizing a technique based on the statistical behavior of the neutron population. A mathematical model including the effects of delayed neutrons which relates the statistic "variance-to-mean ratio of the count rate as a function of counting time" to integral reactor parameters is derived. This model is used in conjunction with subcritical counting measurements from the Spert IV reactor to provide a value of Λ/β that is in good agreement with an independent measurement obtained from super prompt-critical power excursion experiments.

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I. INTRODUCTION

One of the fundamental problems in the field of nuclear reactor technology is determination of the value of parameters that occur in the equations used to represent a given system. In the areas of reactor control and safety, a knowledge of the dynamic nuclear reactor parameters is particularly important as an aid in determining if the reaction can be controlled in a safe and useful manner.

Determination of dynamic nuclear parameters is often accomplished by introducing a perturbation in either the reactivity or neutron source level of the system. Examples of the reactivity perturbations customarily used are the sinusoidal, step, and ramp insertions of reactivity. Pulsed neutron techniques are examples of perturbations in the source level. In these analyses, the response of the reactor to the perturbation is measured and the dynamic parameters are inferred from the results. It is also possible to determine dynamic parameters without introducing a perturbation by making observations of the inherent fluctuations which occur in the neutron population. In this case the neutron population is statistically sampled as a function of time and analyzed in terms of an analytical model.

The primary purpose of this investigation is to determine the reduced prompt neutron generation time Λ/β in the Spert IV reactor based on a statistical analysis of neutron counting data. Numerous techniques have been formulated in the past based on the premise that because the statistical behavior of the neutron population in a multiplying medium is intrinsic to the system, a study of the neutron population statistics will yield information on fundamental parameters of the system. Many of these techniques have certain advantages and disadvantages that make their application suitable only in specific situations. The particular statistic to be described in this paper is the variance-to-mean ratio of the counting rate as a function of counting time. This statistic was chosen because it requires less special experimental equipment and will provide information from a wider variation of reactor types than will some other statistical techniques.

One of the first published works to give a theoretical description of the statistical fluctuations which occur in a nuclear reactor system was that of de Hoffmann [1] in 1944. He showed that the variance-to-mean ratio of the count rate was a convenient measure of the fluctuations in count rate observed by a neutron detector placed in the near vicinity of a multiplying medium and a formula was derived which related the variance-to-mean ratio as a function of counting time to fundamental system parameters. This derivation was based on essentially physical arguments and took into account only those fluctuations due to prompt neutrons and neglected any effects due to delayed neutrons. This work by de Hoffmann, even though it contained certain simplifying assumptions, formed the foundation of all later investigations of variance-to-mean ratio.

A number of other authors [2 through 11] have published accounts of both theoretical and experimental investigations of the variance-to-mean ratio. Several have utilized the methods of probability-generating functions to derive generalized expressions which describe the statistical behavior of the neutron population in a nuclear reactor. These derivations are somewhat more rigorous than de Hoffmann's, of single most importance being the inclusion of delayed neutron effects. In various types of reactors, both the simplified and more general

theories have been used in conjunction with experimental data for the measurement of prompt neutron lifetime, dispersion in the number of neutrons emitted per fission, and effective delayed neutron fraction.

The content of this paper may be briefly summarized as follows: A mathematical model is developed which describes the statistical fluctuations of the neutron population as observed by a neutron detector in terms of the variance-to-mean ratio of the counting rate as a function of counting time. This derivation is based on arguments similar to those of de Hoffmann's except that the effects of delayed neutrons have been accounted for. Several families of curves are generated using an IBM 7090 computer program which show how reactivity and generation time affect variance-to-mean ratio as a function of gate time. These curves indicate that, under certain conditions, a significant difference can exist between a model which includes delayed neutrons and one which neglects delayed neutrons.

Experimental work has been carried out on the Spert IV reactor with the primary purpose to determine Λ/β . The raw counting data were recorded on magnetic tape and statistically sampled at a later time, thus reducing reactor operation time. Data were taken at four subcritical values of reactivity ranging from about -0.75% to about -0.13% . A regulating rod with a small reactivity worth per unit length was used for the accurate measurement of reactivity.

The raw counting data were statistically sampled by playing the signal from the magnetic tape back through a gated scaler at gate times ranging from 0.0010 to 2.0000 sec. Approximately 800 gates were taken at each gate time and automatically recorded on punched paper tape. The unbiased variance-to-mean ratio was computed for each gate time directly from the paper tape by an IBM 1620 computer.

The method utilized to determine Λ/β from the data is similar to that used by Tachon [9], in that the contribution of delayed neutrons to the experimental variance-to-mean measurements is essentially subtracted out. The modified data then represent an artificial system which essentially includes only the contribution due to prompt neutrons. A least-squares fit to the modified data then allows prediction of the root of largest magnitude of the inhour equation, which contains the Λ/β information.

In order to make practical use of this fitting technique, and thus eliminate tedious hand calculations, a generalized program was written for the IBM 7090 computer. This program subtracts out the delayed neutron effects by means of an iterative technique and predicts a least-squares value of the root of largest magnitude of the inhour equation.

Data for each of the four reactivities were processed through this program and a value of $\Lambda/\beta = (8.07 \pm 0.17) \times 10^{-3}$ sec obtained for the Spert IV reactor. The general theory developed is shown to fit the experimental variance-to-mean data quite well over the ranges of reactivity and gate time investigated using this value of Λ/β . In addition, an independent measurement of Λ/β from dynamic tests in the Spert IV reactor [12] has yielded a value, $\Lambda/\beta = (7.93 \pm 0.12) \times 10^{-3}$ sec, which is in good agreement with that measured by the statistics technique.

II. MATHEMATICAL DESCRIPTION OF REACTOR STATISTICAL BEHAVIOR

A mathematical model describing the statistical behavior of a neutron-multiplying medium in terms of the parameters that describe the system is developed in this chapter. The primary purpose of this analysis is directed at derivation of an expression that will permit determination of the reduced prompt neutron generation time $\Lambda/\bar{\beta}$ from experimental measurements of the statistic, "variance-to-mean ratio of the counting rate as a function of counting time", for typical reactor systems where the space-independent kinetics equations are applicable. The derivation is similar to that employed by de Hoffmann [1] in that it is essentially based on "physical-mathematical" concepts rather than more sophisticated mathematical concepts such as probability-generating functions. Delayed neutron effects, however, are included in this derivation whereas de Hoffmann considered only prompt neutrons. Derivations by other authors [2, 7, 8, 9, 10, 11] also have included delayed neutrons and are in some respects more rigorous than that developed here, but because of their generality are not as amenable to application for determination of reactor parameters. Following the derivation, the effect of changes in parameters appearing in the model is discussed.

1. AVERAGE BEHAVIOR OF A MULTIPLYING MEDIUM

Before considering a description of the statistical behavior of the neutron population in a multiplying medium, a few of the fundamental relationships associated with the average behavior of the neutron population will be reviewed. These concepts provide a basis upon which much of the statistical derivation will be grounded. The experimental work was conducted on a light-water-moderated, highly enriched U-235 system, so the discussion of average behavior will be described in terms of constants typical of such a system. The relationships are, however, in general applicable to other types of systems by making appropriate changes in the constants.

Fission of U-235 results in the emission of $\bar{\nu}$ neutrons, most of which are emitted promptly and a small fraction of which are delayed in time. If $\bar{\nu}$ is defined as the average number of neutrons emitted per fission and β as the average fraction of those neutrons which are delayed, then the fraction emitted promptly is $(1-\beta)$. Measurements by Keepin [13] have shown that delayed neutrons may be categorized into several groups, each group characterized by an average decay constant λ_i and average energy of emission E_i . Each of the individual fractions associated with the i th delayed neutron group may be denoted β_i , where by definition $\beta = \sum_i \beta_i$. Table I lists the latest published delayed neutron parameter values by Keepin [13] for fast-fission of U-235, including the estimated standard deviations. The relative abundance values are defined by $a_i = \beta_i/\beta$ with $\beta = 0.0065$ for U-235.

Attention is now turned to the definition of several parameters directly related to a neutron chain reactor. The effective reproduction factor k may be defined as the ratio of the number of neutrons born in the $(n + 1)$ st generation to the number of neutrons born in the n th generation. When this ratio is exactly

TABLE I
U-235 FAST-FISSION DELAYED NEUTRON PARAMETERS

Group Index i	Decay Constant λ_i (sec ⁻¹)	Relative Abundance a_i
1	0.0127 $\pm$ 0.0002	0.038 $\pm$ 0.003
2	0.0317 $\pm$ 0.0008	0.213 $\pm$ 0.005
3	0.115 $\pm$ 0.003	0.188 $\pm$ 0.016
4	0.311 $\pm$ 0.008	0.407 $\pm$ 0.007
5	1.40 $\pm$ 0.081	0.128 $\pm$ 0.008
6	3.87 $\pm$ 0.369	0.026 $\pm$ 0.003

one the reactor is said to be critical. In general, the average neutron population of a non-critical reactor, if not zero, will vary with time. An exception is a subcritical reactor having an extraneous source of neutrons, in which case the average neutron population will, after a time, achieve some constant value. The absolute reactivity ρ of a system may be defined as $(k-1)/k$ which is observed to be zero for a critical reactor. A unit of reactivity which casts the reactor kinetics equations into measureable quantities is the dollar, which may be defined as the amount of excess reactivity needed to make a given reactor critical on prompt neutrons alone. From this definition it can be shown that the reactivity in dollars r is given by,

$$r(\$) = \frac{\rho}{\beta} = \frac{k - 1}{k\bar{\beta}} \quad (1)$$

The dollar unit of reactivity r will be used in most cases throughout this paper. The term $\bar{\beta}$ is defined as the effective fraction of delayed neutrons. Since delayed neutrons are born at lower energies than prompt neutrons, they are less likely to escape from a finite-sized reactor during thermalization than are prompt neutrons. A delayed neutron thus has a higher probability of producing a fission than does a prompt neutron. This can be expressed by assigning relative importance or effectiveness factors γ_i to each group of delayed neutrons. One may then define,

$$\bar{\beta} = \sum_i \gamma_i \beta_i = \bar{\gamma} \beta \quad (2)$$

$$\bar{\gamma} = \sum_i \gamma_i \frac{\beta_i}{\beta} = \sum_i \gamma_i a_i \quad (3)$$

where $\bar{\gamma}$ is an average effectiveness factor determined by a weighted average of the individual γ_i values. In order to differentiate between relative abundance a_i and effective relative abundance $(\gamma_i/\bar{\gamma}) a_i$ one may define,

$$f_i = \frac{\gamma_i}{\bar{\gamma}} a_i = \frac{\gamma_i \beta_i}{\bar{\gamma} \beta} = \frac{\bar{\beta}_i}{\bar{\beta}} \quad (4)$$

where the f_i are now effective relative abundance values unique to a given finite system. A detailed evaluation of the effectiveness factors for a given system is usually based on a calculational approach using perturbation methods. In general one expects greater effectiveness factors for small systems relative to large systems due to the higher non-leakage probability of delayed neutrons compared to prompt neutrons in a small system.

The mean prompt neutron generation time Λ or the mean prompt neutron lifetime ℓ are used in describing the average kinetic behavior of a reactor. Conceptually, one may consider Λ as the mean time from birth to regeneration of prompt neutrons and ℓ as the mean time from birth to death of prompt neutrons. Generation time is thus related to a production probability whereas lifetime is related to a destruction probability. The question then arises as to which parameter is more appropriate for use in a given experimental analysis or theoretical development. Lewins [14] has pointed out that for an experiment in which the departure from criticality is achieved by movement of "black" absorbers or control rods, hence changing the neutron leakage, Λ and not ℓ , is a constant during the change. If, however, a perturbation is made by means of a "gray" fuel-follower control rod, thus altering the macroscopic fission cross section, then ℓ , but not Λ , would be constant. The change in the magnitude of either Λ or ℓ is the order of the change in k ; thus, for experiments conducted near critical, $\Lambda \approx \ell$. Since the experimental work to be described in this paper was conducted on a reactor controlled by movement of "black" control rods, Λ should be constant and the theory will be developed in terms of Λ rather than ℓ .

In order to describe the statistical behavior of the neutron population, a description of the mean kinetic behavior of a multiplying system as a function of time is needed. This description is provided by the conventional reactor kinetics equations, which, as derived from time-dependent transport theory by Henry [15] for the case of a stationary fuel neutron chain reactor, may be written in the form,

$$\frac{dn}{dt} = \frac{\rho - \bar{\beta}}{\Lambda} n + \sum_{i=1}^m \lambda_i C_i + S_0 \quad (5)$$

$$\frac{dC_i}{dt} = \frac{\bar{\beta}_i}{\Lambda} n - \lambda_i C_i \quad (6)$$

where,

n = neutron density

- C_i = concentration of the i^{th} group of delayed neutron emitters
 λ_i = decay constant of the i^{th} group of delayed neutron emitters
 S_0 = source level (neutrons/cm³ - sec)
 Λ = mean prompt neutron generation time
 k = effective neutron reproduction factor for finite geometry
 $\rho = (k-1)/k$
 $\bar{\beta}$ = effective delayed neutron fraction
 $\bar{\beta}_i$ = effective yield of the i^{th} delayed neutron group
 m = number of delayed neutron groups

A general solution of Equations (5) and (6) for the neutron density explicitly as a function of time (space and time assumed separable) for a subcritical system is given in Appendix A. In general the solution may be written in terms of a source term plus a fission term, or,

$$n(t) = n(t)_s + n(t)_f \quad (7)$$

where the fission term is,

$$n(t)_f = n_0 \sum_{j=1}^{m+1} A_j e^{\alpha_j t} \quad (8)$$

with

$$A_j = \frac{r}{r - \sum_{i=1}^m \frac{f_i \alpha_j^2}{(\alpha_j + \lambda_i)^2}} \quad (9)$$

The $\alpha_j \equiv \frac{1}{T}$ are the roots of the familiar inhour equation which, as derived in Appendix A, may be written,

$$r = \frac{\Lambda}{\bar{\beta}} \alpha + \sum_{i=1}^m \frac{f_i \alpha}{(\alpha + \lambda_i)} \quad (10)$$

where for a subcritical system, the roots are all real and negative. The source term is of the form

$$n(t)_s = S_o \left[\sum_{j=1}^{m+1} A'_j e^{\alpha_j t} - A_o \right] \quad (11)$$

where

$$A'_j = \frac{\Lambda/\bar{\beta}}{r - \sum_{i=1}^m \frac{f_i \alpha_j^2}{(\alpha_j + \lambda_i)^2}} \quad (12)$$

$$A_o = \frac{\Lambda/\bar{\beta}}{r} \quad (13)$$

For a subcritical system, the time-dependent portion of the source term approaches zero as t becomes sufficiently large. The neutron density due to a source term then approaches the constant value

$$n(t)_s = -S_o A_o \quad (14)$$

Substituting for A_o and expressing in terms of k , one obtains

$$n(t)_s = \frac{-S_o \Lambda k}{k-1} = \frac{S_o \Lambda k}{1-k} \quad (15)$$

With this expression for the steady-state neutron density in a subcritical system, the mean neutron production rate may be defined as

$$F = \frac{n(t)_s}{\Lambda k} = \frac{S_o}{1-k} \quad (16)$$

when a source of S_o neutrons per unit time is multiplied by the system. The mean number of neutrons produced by fission per unit time, or mean neutron generation rate, may be defined as

$$G = \frac{n(t)_s}{\Lambda} = \frac{S_o k}{1-k} = Fk \quad (17)$$

The generation rate is simply the production rate excluding the extraneous source neutrons and also may be obtained by subtracting the source neutrons from the total production rate F , or,

$$\begin{aligned}
G &= F - S_o \\
&= \frac{S_o}{1-k} - S_o \\
&= \frac{S_o k}{1-k} \\
&= Fk
\end{aligned} \tag{18}$$

which is identical with Equation (17).

2. DERIVATION OF A STATISTICAL MATHEMATICAL MODEL

This section is devoted to the derivation of a model which can be used to describe the statistical nature of the neutron population in a multiplying medium. The system to be described is that of a subcritical neutron chain reactor in an equilibrium state sustained by a primary source of neutrons per unit time. The neutron population of such a system, as observed by a neutron detector placed in or near the multiplying medium, will fluctuate about some mean value. These fluctuations are a result of the discrete and statistical nature of the individual neutron production and emission processes. If the processes were independent or random in nature, the fluctuations would be describable by a Poisson distribution. For example, the detection of neutrons emitted by a neutron source in the absence of a multiplying medium is in general a random process, ie, a neutron detected at one time is not related to a neutron detected at another time. Due to the nature of the fission process in a multiplying medium, however, a correlation may exist between a neutron detected at one time and a neutron detected at another time. For example, a source neutron may generate a fission from which ν neutrons (prompt and delayed) are emitted. These ν neutrons may generate other fissions, thus perpetuating a neutron chain. The neutrons of such a chain are thus related to a common ancestor and a correlation may be said to exist when two or more neutrons of the same chain are detected. The number of neutrons emitted per fission as well as the neutron chain length are variable from one case to another, resulting in fluctuations that show some degree of correlation in the neutron population. As pointed out by de Hoffmann[1], a convenient measure of the fluctuations in the neutron population as observed by a neutron detector is given by the variance-to-mean ratio of the count rate.

A typical reference experiment consists of counting the number of neutrons registered by a neutron detector over a fixed counting or gate time. This process is repeated a number of times at the same gate time and the resulting distribution analyzed for the ratio of variance to mean. The mean, or first moment of the count rate, is simply the average count rate. The variance, or second moment of the count rate, is the average of the sum of the squares of the deviations about the mean. For a Poisson or random distribution, the variance-to-mean ratio is equal to one. In the case of a multiplying medium, the variance-to-mean ratio of the count rate will in general be greater than one as a result of the correlated fluctuations in the neutron population. Since the time constants associated with

the correlations are related to the mean prompt neutron generation time and the delayed neutron half-lives, the magnitude of the variance-to-mean ratio will be a function of counting time.

With these ideas in mind, a model may be derived to relate the measurable fluctuations in neutron population to parameters of the system. The use of probabilities is required to describe the discrete neutron processes. The elements of the probabilities will for the most part be expressed in terms of average parameters of the system. An approach which would result in expressions of a more general nature than presented in this paper would use the methods of probability-generating functions.

It is assumed the space-independent kinetics equations are applicable; ie, a neutron born at one place in the system is assumed to produce the same subsequent effect on the system as one born at another place. Also, a single neutron velocity is assumed; however, the relative importance of each delayed neutron group compared to prompt neutrons is taken into account. The neutron detector is assumed to absorb the neutrons detected and to create a negligible perturbation of the system. Use of a fission counter would modify the arguments slightly since this type of detection perpetuates the neutron chain.

The mean counting rate of the detector is proportional to the mean rate of neutron loss from the system which at steady state is equal to the mean rate of production, or

$$\frac{\overline{dc}}{dt} = FE \quad (19)$$

where the constant of proportionality E is the detector efficiency defined as the mean number of neutrons detected per neutron lost from the system and F is the steady-state neutron production (loss) rate including the contribution of the source. The mean number of counts collected over a counting or gate time T is then

$$\bar{c} = F E T \quad (20)$$

As indicated previously, the nature of the fission process results in neutron population fluctuations which are in part correlated. The problem is then to develop an analytical expression which describes the correlations.

Consider two arbitrary times, t_1 and t_2 , where $t_1 < t_2$. The expected number of counts occurring in pairs, one count in the interval $(t_1, t_1 + dt_1)$, and one count in the interval $(t_2, t_2 + dt_2)$ will now be calculated. Two different types of pairs of counts, "accidental" and "coupled", will be postulated. An accidental pair is defined as one in which the two neutrons which comprise the pair are independent, ie, an accidental pair may be comprised of a count at $(t_1, t_1 + dt_1)$ caused by a neutron from one chain, and a count at $(t_2, t_2 + dt_2)$ caused by a neutron from another chain. A coupled pair is defined as one in which the two neutrons which comprise the pair are related. A coupled pair is thus due to a count at $(t_1, t_1 + dt_1)$ caused by a neutron from one chain, and a count at $(t_2, t_2 + dt_2)$ caused by a neutron having a common ancestor with the neutron counted at $(t_1, t_1 + dt_1)$. A coupled pair thus arises from the

same chain and may be traced back to a common fission. The total number of expected pairs in $(t_1, t_1 + dt_1)$ and $(t_2, t_2 + dt_2)$ may be expressed as the sum of the accidental and coupled pairs.

The expected number of accidental pairs may be expressed as the product of two independent probabilities: (a) the probability of counting a neutron in the interval $(t_1, t_1 + dt_1)$, or $F E dt_1$, and (b) the probability of counting a neutron in the interval $(t_2, t_2 + dt_2)$, or $F E dt_2$. The product gives

$$\text{expected number of accidental pairs} = F^2 E^2 dt_1 dt_2 \quad (21)$$

Note that the production rate includes the source neutrons since they can contribute to the number of accidental pairs.

An expression will now be derived for the expected number of coupled pairs occurring in $(t_1, t_1 + dt_1)$ and $(t_2, t_2 + dt_2)$. Since a coupled pair is traceable to a common fission, assume the common fission occurred in the interval $(t, t + dt)$, where $-\infty < t < t_1$. The probability that one coupled pair will be detected in the intervals $(t_1, t_1 + dt_1)$ and $(t_2, t_2 + dt_2)$ may be expressed as the product of the following four independent probabilities.

- (1) The probability that a fission will occur in the interval $(t, t + dt)$. This probability may be expressed in terms of the neutron production rate by fission ($G = Fk$) and the average number of neutrons produced per fission, giving

$$\frac{Fk}{\bar{\nu}} dt$$

- (2) The probability that this fission will produce ν neutrons, or

$$P_\nu$$

- (3) The probability that a neutron descendant from this fission will be detected in the interval $(t_1, t_1 + dt_1)$.

The probable number of neutrons present at time t_1 may be determined from the solution of the space-independent kinetics equations. Utilizing Equation (8), which describes the time behavior of fission neutrons, the probable number of neutrons present at t_1 , as a result of the ν neutrons produced by the fission at $(t, t + dt)$, is

$$\sum_{j=1}^{m+1} A_j e^{\alpha_j(t_1 - t)}$$

The probability that one of the neutrons present at t_1 is lost during the interval $(t_1, t_1 + dt_1)$ is given by the probable number

present at t_1 times the probability of loss in the interval $(t_1, t_1 + dt_1)$, or

$$\nu \sum_{j=1}^{m+1} A_j e^{-\alpha_j (t_1 - t)} \frac{dt_1}{\tau_j}$$

where the loss probability is expressed as dt_1/τ_j . The quantity τ_j is defined as the mean lifetime of a neutron in the system or the average time from birth to death. (Birth in this case is taken to mean the time of fission t . The mean lifetime of a delayed neutron is thus much greater than the mean lifetime of a prompt neutron due to the inherent time delay before the delayed neutron appears in the system.) The concept of mean lifetime is discussed in Appendix B and it is shown that the mean lifetime appropriate for this derivation is $\tau_j = \rho/\alpha_j$. Having an expression which defines the probability that one of the neutrons present at t_1 is lost during the interval $(t_1, t_1 + dt_1)$, it remains only to define the probability that one of the neutrons lost is detected in the interval $(t_1, t_1 + dt_1)$.

Since the detector has an efficiency E (defined previously as the mean number of neutrons detected per neutron lost from the system), this probability becomes

$$E \nu \sum_{j=1}^{m+1} A_j e^{-\alpha_j (t_1 - t)} \frac{dt_1}{\tau_j}$$

(4) The probability that another neutron descendant from the common fission at $(t, t + dt)$ will be detected in the interval $(t_2, t_2 + dt_2)$. Assuming the detector to be an absorber, then one of the original ν neutrons was lost from the system through detection in the interval $(t_1, t_1 + dt_1)$.

Applying the same arguments as used above in 3, this probability becomes

$$E (\nu-1) \sum_{k=1}^{m+1} A_k e^{-\alpha_k (t_2 - t)} \frac{dt_2}{\tau_k}$$

where the subscripts have been changed for clarity of notation.

The product of the above four independent probabilities is the probability that one coupled pair will be detected in the intervals $(t_1, t_1 + dt_1)$ and $(t_2, t_2 + dt_2)$ from the ν neutrons emitted from a common fission in the interval $(t, t + dt)$, or

$$\frac{F E^2 k P_v v(v-1)}{\bar{v}} \sum_{j,k=1}^{m+1} \frac{A_j A_k}{\tau_j \tau_k} e^{\alpha_j (t_1 - t)} e^{\alpha_k (t_2 - t)} dt_1 dt_2 dt$$

The total expected number of coupled pairs may be obtained by integrating this probability over t from $-\infty$ to t_1 and then taking the sum over the distribution of neutrons emitted per fission, or

$$\text{expected number of coupled pairs} = \sum_v \left[\int_{-\infty}^{t_1} \frac{F E^2 k P_v v(v-1)}{\bar{v}} \sum_{j,k=1}^{m+1} \frac{A_j A_k}{\tau_j \tau_k} e^{\alpha_j (t_1 - t)} e^{\alpha_k (t_2 - t)} dt_1 dt_2 dt \right] \quad (22)$$

Integration (recognizing that all α 's are negative for a subcritical system), followed by summing, yields

$$\text{expected number of coupled pairs} = F E^2 k \left[\frac{\bar{v}^2 - \bar{v}}{\bar{v}} \right]$$

$$\sum_{j,k=1}^{m+1} \frac{A_j A_k}{-(\alpha_j + \alpha_k) \tau_j \tau_k} e^{\alpha_k (t_2 - t_1)} dt_1 dt_2 \quad (23)$$

where the sum over v , $\sum_v P_v v(v-1)$, yields the factor $(\bar{v}^2 - \bar{v})$ for a single fissile species. Summing over j allows Equation (23) to be written in the form

$$\text{expected number of coupled pairs} = - F E^2 k \left[\frac{\bar{v}^2 - \bar{v}}{\bar{v}} \right]$$

$$\sum_{k=1}^{m+1} A_k B_k e^{\alpha_k (t_2 - t_1)} dt_1 dt_2 \quad (24)$$

where

$$B_k = \sum_{j=1}^{m+1} \frac{A_j}{(\alpha_j + \alpha_k) \bar{\tau}_j \tau_k} \quad (25)$$

The total expected number of pairs is the sum of the accidental pairs and the coupled pairs, or

$$\begin{aligned} \text{expected number of pairs} &= F^2 E^2 dt_1 dt_2 - F E^2 k \left[\frac{\bar{v}^2 - \bar{v}}{\bar{v}} \right] \\ &\sum_{k=1}^{m+1} A_k B_k e^{\alpha_k (t_2 - t_1)} dt_1 dt_2 \end{aligned} \quad (26)$$

The expected number of pairs registered in a counting or gate time T may be obtained by integration of Equation (26) from 0 to t_2 for t_1 , and from 0 to T for t_2 , or

$$\begin{aligned} \text{expected number of pairs in } T &= \int_{t_1=0}^{t_1=t_2} \int_{t_2=0}^{t_2=T} \left[F^2 E^2 - F E^2 k \right. \\ &\left. \left(\frac{\bar{v}^2 - \bar{v}}{\bar{v}} \right) \sum_{k=1}^{m+1} A_k B_k e^{\alpha_k (t_2 - t_1)} \right] dt_1 dt_2 \\ &= \frac{F^2 E^2 T^2}{2} + F E^2 T k \left[\frac{\bar{v}^2 - \bar{v}}{\bar{v}} \right] \sum_{k=1}^{m+1} \frac{A_k B_k}{\alpha_k} \left[1 - \frac{1 - e^{-\alpha_k T}}{-\alpha_k T} \right] \end{aligned} \quad (27)$$

The expected number of pairs also can be calculated from the number of counts c accumulated during the gate time T . The number of combinations of c counts taken two at a time is $c(c-1)/2$, so the expected number of pairs in T is $\bar{c}(\bar{c}-1)/2$, which is an identity with Equation (27). Setting these two expressions equal and noting that $FET = \bar{c}$ from Equation (20) gives

$$\frac{\bar{c}^2 - \bar{c}}{2} = \frac{\bar{c}^2}{2} + \bar{c} E k \left[\frac{\bar{v}^2 - \bar{v}}{\bar{v}} \right] \sum_{k=1}^{m+1} \frac{A_k B_k}{\alpha_k} \left[1 - \frac{1 - e^{-\alpha_k T}}{-\alpha_k T} \right] \quad (28)$$

Multiplying both sides by $2/\bar{c}$ and rearranging gives

(29)

$$\frac{\text{Var}}{\text{Mean}} (T) = \frac{\bar{c}^2 - \bar{c}^2}{\bar{c}} = 1 + 2 E k \left[\frac{\bar{v}^2 - \bar{v}}{\bar{v}} \right] \sum_{k=1}^{m+1} \frac{A_k B_k}{\alpha_k} \left[1 - \frac{1 - e^{\alpha_k T}}{-\alpha_k T} \right]$$

where the quantity $(\bar{c}^2 - \bar{c}^2)/\bar{c}$ is recognized as the variance-to-mean ratio of the count rate as a function of counting time and denoted $\frac{\text{Var}}{\text{Mean}} (T)$.

Equation (29) represents a model which can be used in conjunction with experimental measurements of variance-to-mean ratio as a function of counting time to determine Λ/β for the particular system in question. The use of this model to determine Λ/β for the Spert IV reactor is described in Section IV.

The model derived is in terms of population statistics which means the number of samples must approach infinity for a time estimate. For a finite number of samples the calculated variance must be multiplied by $n/(n-1)$, where n is the number of samples, in order to obtain the unbiased variance[18].

3. PARAMETRIC STUDY OF MATHEMATICAL MODEL

This section presents several families of curves which show the effect of changes in r and Λ on the behavior of the model derived in the previous section. Equation (29), written in the form,

$$\left[\frac{\text{Var}}{\text{Mean}} (T) - 1 \right] = 2 E k \left[\frac{\bar{v}^2 - \bar{v}}{\bar{v}} \right] \sum_{k=1}^{m+1} \frac{A_k B_k}{\alpha_k} \left[1 - \frac{1 - e^{\alpha_k T}}{-\alpha_k T} \right] \quad (30)$$

will be the subject of discussion. The delayed neutron parameters of Table I are used (f_i assumed equal to a_i) along with $\beta = 0.008$ in all the calculations. The roots of the inhour equation α_k appropriate to each case were calculated using an IBM 7090 computer program [$m + 1$ solutions of Equation (8)]. Parametric variations of Equation (30) were then generated using another IBM 7090 computer program.

Figures 1 and 2 show $\left[\frac{\text{Var}}{\text{Mean}} (T) - 1 \right] / 2 E k \left[\frac{\bar{v}^2 - \bar{v}}{\bar{v}} \right]$ plotted as a function of T for $\Lambda = 100 \mu\text{sec}$. (All curves in this section are intended to show only shape changes, the magnitudes are arbitrary and the factor $2 E k \left[\frac{\bar{v}^2 - \bar{v}}{\bar{v}} \right]$ is thus only a scaling constant.) Figure 1 is for $r = -0.60\%$ and Figure 2 is for $r = -0.20\%$. A family of eight curves is shown in each case, $k=1$ through

FIG. 1 VARIANCE-TO-MEAN AS A FUNCTION OF GATE TIME FOR $R = -0.60\$$.

FIG. 2 VARIANCE-TO-MEAN AS A FUNCTION OF GATE TIME FOR $R = -0.20\$$.

7 representing each individual component and "summation" representing the sum of the 7 components. The component represented by $k = 7$ corresponds to the root α_7 of largest magnitude which is dependent for the most part on prompt neutron behavior. This curve is typical of a model which neglects the delayed neutron contribution. The components $k = 1$ through 6 are representative of the delayed neutron contribution and correspond to the roots α_1 through α_6 in increasing magnitude. These curves show the general behavior of the model and also that the contribution of delayed neutrons becomes greater as the magnitude of r decreases (or as criticality is approached).

Figure 3 shows $\left[\frac{\text{Var}}{\text{Mean}} - 1 \right] / 2 E k \left[\frac{\bar{v}^2 - \bar{v}}{\bar{v}} \right]$ vs T for a constant value

of $r = -0.20\%$ and for $\Lambda = 5, 100, \text{ and } 1000 \mu\text{sec}$. In this case only the summation term and component corresponding to α_7 are shown for each generation time. These curves show that the relative delayed neutron contribution increases as generation time is increased, due primarily to a decrease in the magnitude of α_7 relative to α_1 through α_6 as Λ increases.

FIG. 3 EFFECT OF GENERATION TIME ON VARIANCE-TO-MEAN AS A FUNCTION OF GATE TIME

In summary, these parametric studies show that the use of a model which neglects the contribution of delayed neutrons (resulting in curves corresponding to $k = 7$) will not be a good representation of a physical system with a long generation time and/or operation near critical. The model which includes the delayed neutron contribution must be utilized for experimental work with systems that have a relatively long prompt neutron generation time or where data are taken near critical.

III. EXPERIMENTAL EQUIPMENT

The experimental measurements were carried out utilizing the Spert IV facility and a portion of the operational-type neutron-detecting equipment routinely used for start-up of the reactor. Several pieces of additional counting equipment also were used in conjunction with a magnetic tape recorder for collection, storage, and playback of the counting data. This chapter describes the pertinent features of the reactor and instrumentation.

1. BRIEF DESCRIPTION OF SPERT IV REACTOR

Spert IV is a swimming pool reactor designed for investigation of reactor safety problems with various types of light-water-moderated and -cooled cores. The core in place for the statistics measurements, designated the Spert IV D-12/25 core, was a highly enriched, plate-type core composed of an array of 25 fuel assemblies containing a total of 3.75 kg of U-235. An isometric drawing of the operational core is shown in Figure 4. Each fuel assembly was comprised of a 3-in.-square aluminum retaining can, a lower end box, two grooved side plates, a lifting bail, and 12 removable fuel plates. Each fuel plate consisted of 14 grams of U-235 alloyed with aluminum melting stock to produce a "meat section" 0.020 in. thick, 2.45 in. wide, and 24 in. long. This fuel section was clad with type 6061 aluminum to produce a plate 2.704 in. wide, 25-1/8 in. long, and 0.060 in. thick, overall.

Normal reactor control was accomplished by the use of four, double-blade, aluminum-boron (Binal) control rods containing 7 wt% natural boron. These rods, operated as a bank, fit into guide slots in the standard fuel assemblies, each control rod blade replacing two fuel plates. An aluminum follower blade acts as a guide and flux peak suppressor when the control rod blade is withdrawn.

In addition to the four operational control rods, a stainless steel "regulating rod" was designed and fabricated in order to measure subcritical reactivity levels for the statistics experiment. This rod was inserted in the central fuel assembly in place of the transient rod and was driven by the transient rod drive mechanism.

A complete description of the Spert IV facility and of the core and its static characteristics is given in reports by Heffner^[16] and Crocker^[17].

2. FUNCTIONAL DESCRIPTION OF COUNTING INSTRUMENTATION AND DATA COLLECTION SYSTEM

The material presented in this section will describe the general features of the counting instrumentation and data collection system. The reasons governing the choice of the particular system are discussed in terms of theoretical as well as practical considerations. A more detailed description of the individual instrumentation components is given in Appendix C.

FIG. 4 ISOMETRIC DRAWING OF SPERT IV LATTICE AND STRUCTURE.

Several basic criteria must be satisfied in order to collect and reduce a sufficient amount of data to determine an accurate value of Λ/β for a nuclear reactor system using the variance-to-mean technique.

Those criteria considered to be of importance are: (a) the neutron detector efficiency should be large enough to detect as many correlations as possible and yet not overly perturb the system (an efficiency the order of 10^{-4} to 10^{-3} is considered the right order of magnitude); (b) the noise level of the instrumentation needs to be reduced to a minimum so that extraneous counts are not introduced into the data; (c) the counting system should be designed such that counting losses are negligible at the maximum count rates encountered; (d) the data collection system should be capable of rapidly collecting and storing a large

volume of data in a form readily amenable to analysis; and (e) reactor operating time needed for data collection should be minimized.

With the above criteria in mind, the data collection system represented in the block diagram of Figure 5 was designed. The neutron detectors, pre-amplifiers, and linear amplifiers were a part of the conventional counting equipment routinely used for start-up of the Spert IV reactor. For this experiment, the neutron detectors (standard B-10-lined pulse chambers) were located around the periphery of the reactor core as shown in Figure 6. Since the

FIG. 5 BLOCK DIAGRAM OF DATA COLLECTION SYSTEM.

FIG. 6 PLAN VIEW OF SPERT IV OPERATIONAL CORE.

reactor core is the source of neutron chains and hence of correlated events, the neutron detectors were placed as near the core as possible. Three detectors were used to provide a chamber efficiency about a factor of three over that possible with one detector. The preamplifiers and linear amplifiers conditioned the pulse signals for transmission to the control center which was located about 3000 ft from the reactor building. The mixer summed the three input signals and its output was recorded on magnetic tape. The magnetic tape provided temporary storage of the raw counting signal, thus minimizing reactor operating time. A 100-kc timing signal was recorded simultaneously on an adjacent channel of the magnetic tape in order to eliminate time base uncertainties during playback of the raw counting signal. The portion of the experiment involving reactor operation was completed once counting data at various subcritical reactivities were recorded on magnetic tape.

The remainder of the equipment shown in Figure 5 was used to statistically sample the pre-recorded magnetic tape data. The gated scaler was similar in design and operation to a conventional scaler with the exception that the counting or gate time could be varied from 0.0001 to 9.9999 sec in 0.0001-sec increments. Utilizing the 100-kc signal from the magnetic tape as a time base, the gated scaler accumulated counts from the data channel of the magnetic tape over a preset gate time. The accumulated count and the gate time were then automatically transferred to a one-word (10-digit) memory of the punch control unit. This memory was electronically scanned and the contents recorded on paper tape by the paper tape punch. Once a word was accepted by the punch control unit, the gated scaler automatically reset and began accumulating counts once again over the preset gate time. The gate counter was a simple scaler which kept a running total of the number of words recorded on the paper tape for each fixed gate time. The sequence was manually stopped after a predetermined number of samples were collected at a fixed gate time. A new gate time was then set on the gated scaler and the sequence restarted. This sampling technique allowed rapid collection of data in a form (punched paper tape) readily acceptable to an IBM 1620 computer for the calculation of variance-to-mean ratio.

Prior to the collection of any experimental data, numerous checkout procedures were used to assure that the counting and data collection system were operating properly. These procedures were, for the most part, routine techniques normally used for checkout of operational counting equipment. Special care was taken to minimize the occurrence of spurious counts in the system since extraneous noise could invalidate the experimental measurements. The neutron source strength and position relative to the core, as indicated in Figure 6, were chosen such that the maximum count rate encountered during the experiment did not exceed 10,000 counts per sec, a level well within the limitations of the system. A final check on the noise level of the complete system was accomplished by conducting a "purity" check. This consisted of exposing the neutron detectors to a random neutron source (one in which the neutron emission is describable by a Poisson distribution) in the absence of a multiplying medium, and recording the counting signal on magnetic tape. The magnetic tape data were then played back through the gated scaler and recorded on punched paper tape for several gate times. An analysis of these data showed that within the statistical uncertainty, the variance-to-mean ratio was equal to one, a result indicative of a Poisson distribution. If any correlated noise had been present in the system, the variance-to-mean ratio would have in general been greater than one.

IV. EXPERIMENTAL MEASUREMENTS AND DATA ANALYSIS

This section discusses the method by which the experimental data were obtained, reduced, and analyzed. In order to utilize the model developed in Chapter II for determination of $\Lambda/\bar{\beta}$ on a given system, it is necessary to determine the variance-to-mean ratio of the count rate as a function of counting time at a constant, known value of subcritical reactivity. This experimental information was obtained by sampling four sets of counting data (designated runs 1 through 4), each taken at a different value of subcritical reactivity. The reduced data were then related to the model to determine a value of $\Lambda/\bar{\beta}$ for the Spert IV reactor. Details of the measurements and analyses are described in the following paragraphs.

1. REACTIVITY MEASUREMENT

The Spert IV reactor, as described in Section III, is normally controlled by four double-blade control rods operated as a bank. These rods, in the region of cold, clean critical, have a total reactivity worth of about 1.00\$/in. The relatively high worth of these control rods makes an accurate calibration difficult, since a small change in control rod position causes an appreciable change in the reactivity of the system. Consequently, the inherent uncertainty in control rod position can result in a relatively large uncertainty in the measurement of reactivity per unit length.

An accurate measurement of the system reactivity was required for this experiment since the uncertainty in the value of $\Lambda/\bar{\beta}$ obtained is strongly dependent on the uncertainty in the measurement of reactivity. This requirement was satisfied by fabricating a stainless steel regulating control rod to replace the transient rod. The regulating rod had a maximum reactivity worth of about 0.18\$/in.; thus small changes in reactivity required relatively large movements of the rod. Hence, the uncertainty in measurement of reactivity due to the uncertainty in rod position was minimized.

The regulating rod was calibrated by the conventional rod bump-period technique using the other four control rods as poison shims. Since the regulating rod replaced the normal transient rod, the transient rod drive system was used, thus allowing control of this rod independent of the other four control rods. The calibration procedure was as follows: The regulating rod was withdrawn to a selected position, as indicated by the control rod position indicators. The reactor was then brought to criticality by withdrawing and positioning the other four control rods. The regulating rod was then inserted a sufficient amount to reduce the reactor power several decades and then withdrawn to a new position, higher than the initial critical position (usually 1 to 2 in.). This caused the reactor to go on a positive period (the order of 20 to 40 sec) which, after becoming stable, was determined by analyzing the power history recorded on a previously calibrated strip-chart recorder. The reactivity worth of this rod bump was then determined by the inhour equation which, in the long-period region, is only dependent on the delayed neutron parameters given in Table I (f_i assumed equal to a_i). The reactivity worth per inch was then calculated and defined as the rod worth at the average regulating rod position [average position being defined as $(x_i + x_f)/2$ where x_i was the initial or critical position and x_f was the final

or bump position]. This procedure was repeated at selected rod positions until sufficient data were obtained to define accurately the rod worth as a function of rod position. The data are shown in Figure 7, differential rod worth being

FIG. 7 DIFFERENTIAL REGULATING ROD WORTH CURVE.

plotted as a function of rod position. The curve drawn through the data was obtained by fitting the experimental data to an IBM 650 least-squares trigonometric fitting program which is normally used for obtaining a calibration curve for the Advanced Reactivity Measurement Facility (ARMF) regulating rod. The trigonometric fitting function was of the form

$$\rho = a_0 + \sum_{k=1}^n a_k \sin k\theta + \sum_{k=1}^n a_{k+n} \cos k\theta \quad (31)$$

The best fit to the data was obtained for $n = 4$, which corresponded to the fundamental mode plus three higher-order harmonics.

Counting data were taken at four subcritical values of reactivity. For each run the regulating rod was initially withdrawn to the 14.00-in. mark and the four control rods withdrawn and positioned until criticality was achieved. The regulating rod was then inserted to a subcritical position and the reactivity of the system obtained by integration of the differential rod worth curve with a planimeter. The subcritical reactivity values determined for each run are tabulated in Table II.

TABLE II
SUBCRITICAL REACTIVITY VALUES

<u>Run</u>	<u>Critical Reg. Rod Pos. (in.)</u>	<u>Subcritical Reg. Rod Pos. (in.)</u>	<u>Reactivity (\$)</u>
1	14.00	11.00	-0.4894 $\pm$ 0.0030
2	14.00	12.20	-0.2804 $\pm$ 0.0018
3	14.00	13.10	-0.1334 $\pm$ 0.0009
4	14.00	9.50	-0.7530 $\pm$ 0.0045

The uncertainties associated with each reactivity value represent the estimated standard deviation. These are based on a constant estimated standard deviation of $\pm$ \$0.001/inch in differential rod worth.

2. COLLECTION AND REDUCTION OF RAW COUNTING DATA

The procedure used to collect and reduce the raw counting data will be discussed in this section. Once the regulating rod had been set at a subcritical operating position, the reactor power was allowed to decay until an equilibrium level was attained. (The count rate was visually observed on a strip chart recorder and usually about 1/2 hour waiting time was needed to attain equilibrium.) The system was then ready to record a set of counting data. The counting signal from the three B-10-lined pulse chambers was fed through the associated preamplifiers and linear amplifiers and into the mixer located in the control room. The mixed signal from these chambers was then recorded on magnetic tape with a 100-kc timing signal recorded simultaneously on another channel. Approximately thirty minutes of data were recorded at a tape speed of 60 in./sec for each reactivity. The magnetic tape thus provided temporary storage of the raw counting signal and reduced the overall reactor operation time. The experimental program with the reactor was completed following the acquisition of four sets of counting data.

In order to reduce the raw counting data to a form that could be analyzed in terms of the model developed in Section II, it was necessary to sample each set of counting data as a function of gate time. Based on a previous estimate of the value of Λ/β for the Spert IV core (deduced from similarity with the

Spert I D core), it was determined that gate times in the range of 0.0010 to 1.0000 sec would provide sufficient information to obtain a statistical value of Λ/β . However, since delayed neutron effects become more pronounced at long gate times, additional data were taken for gate times to 2.0000 sec to provide a more rigorous test of the theory.

The data reduction sequence was as follows: A pre-recorded tape reel for a selected run was placed on the tape transport with appropriate electronics such that the data signal and timing signal were available as independent outputs. These two signals were then fed into the gated scaler. A selected gate time was set on the gated scaler which used the 100-kc timing signal from the magnetic tape as a time base. The gated scaler then accumulated counts from the magnetic tape data channel over the preset gate time. Following the completion of a gate time, the accumulated count and the gate time were automatically transferred to the punch control unit memory. This memory was electronically scanned and the content punched on paper tape by the paper tape punch unit. Once the gated scaler transferred a set of output information to the punch control memory, it automatically reset and started accumulating counts again. Each time a word was punched, an independent scaler advanced one digit, thus keeping a running total of the number of gates. This sequence continued until the desired number of gates at a fixed gate time were recorded. For each gate time, approximately 800 samples were taken. Data were collected for 65 to 70 gate times between 0.0010 and 2.0000 sec for each reactivity.

A total of about 3×10^5 samples at various gate times and reactivities were collected on punched paper tape. A program was written for an IBM 1620 computer in order to reduce this large volume of data. The IBM 1620 accepted the paper tape directly and computed the sample mean, unbiased sample variance, and the ratio of sample variance to sample mean. Since the paper tape punch was a mechanical device and subject to errors, each piece of input data was checked for word length, gate time, and parity errors. Any words containing errors were rejected and not considered in the calculations. The detailed results of these calculations are tabulated in Appendix D.

3. DETERMINATION OF Λ/β

This section discusses the technique utilized to relate the experimental measurements of variance-to-mean ratio to the analytical model in order to determine a unique value of Λ/β for the Spert IV D-12/25 core. The primary consideration was inclusion of the contribution due to delayed neutrons and thus eliminating the systematic error obtained when delayed neutrons are neglected.

The most straightforward method would have been a least-squares fit of the data to Equation (29) derived in Section II, which may be written in the form,

$$\left[\frac{\text{Var}}{\text{Mean}} (T) - 1 \right] = \sum_{k=1}^{m+1} K_k \left[1 - \frac{1 - e^{-\alpha_k T}}{-\alpha_k T} \right] \quad (32)$$

where K_k represents a combination of constant terms. Utilizing as input $\left[\frac{\text{Var}}{\text{Mean}} (T) - 1 \right]$ vs T data, the fitting procedure must predict a total of 14 unique coefficients (7 values each of K and α). The α of largest magnitude (commonly designated as the prompt root of the inhour equation) then allows determination of Λ/β . This technique, however, was rejected primarily because of the scatter in the data and experience of other investigators in attempting fits of this type [10].

Since the only coefficient needed to determine Λ/β is the α of largest magnitude, a method was developed similar to that used by Tachon [9] which essentially "subtracts out" the delayed neutron contribution to the data. The modified data, which are then representative of the prompt neutron behavior of the system, were least squares fit to an equation of the form

$$\left[\frac{\text{Var}}{\text{Mean}} (T) - 1 \right] = K_p \left[1 - \frac{1 - e^{\alpha_p T}}{-\alpha_p T} \right] \quad (33)$$

and values of α_p and K_p determined (the subscripts refer to prompt root and prompt asymptote, respectively). The immediate question that occurs is: If one does not already know the correct value of Λ/β , then how can the data be properly modified? This problem was solved by setting up a general fitting program for the IBM 7090 computer which utilized an iterative technique to modify the experimental data and predict least squares values of α_p and K_p in which the systematic uncertainty due to delayed neutron effects was negligible.

A description of the logic used in the program, with reference to the flow diagram shown in Figure 8, is as follows: The input consisted of the delayed neutron constants (β, λ_i, f_i) , estimates of K_p and α_p , a convergence value ϵ , and experimental $\left[\frac{\text{Var}}{\text{Mean}} (T) - 1 \right]$ vs T data. The first operation was a least-squares fit (LSTS subprogram) of the experimental $\left[\frac{\text{Var}}{\text{Mean}} (T) - 1 \right]$ vs

T data using the α_p and K_p estimates and Equation (33). This resulted in least-squares value of α_p and K_p (each having a systematic error due to the neglect of delayed neutrons). The α_p value and the delayed neutron parameters were then used to generate a solution of the inhour equation (INHOURL Subprogram) for the additional six delayed roots. The seven roots of the inhour equation and the delayed neutron parameters were then used to obtain a solution of the general variance-to-mean equation for each of the input gate times [VARMN Subprogram, using Equation (29), generates curves similar to those in Figures 1 and 2, Section II]. A ratio of the value of the prompt component ($k = 7$) to the summation term was formed at each input gate time and this ratio was multiplied with the appropriate experimental $\left[\frac{\text{Var}}{\text{Mean}} (T) - 1 \right]$ values. This resulted in a modified set of $\left[\frac{\text{Var}}{\text{Mean}} (T) - 1 \right]$ vs T data which were then sent back into the least-squares-fitting portion of the program. The data were required to go through the complete loop at least once. The second and each succeeding time through the loop, a convergence criterion was tested to see if $|\alpha_p(n-1) -$

FIG. 8 FLOW DIAGRAM OF DATA-FITTING PROGRAM.

$\alpha_p(n)/\alpha_p(n) < \epsilon$, where $\alpha_p(n)$ was the last least-squares value of α_p and $\alpha_p(n-1)$ the next to last value of α_p (n being the number of times through the loop.) If this criterion was satisfied, the computation was concluded and the output printed; if the criterion was not satisfied, the program continued through another loop. The choice of ϵ was arbitrary and was chosen such that the final

value of α_p contained at least four unique significant figures (ϵ equal 10^{-4} to 10^{-5} satisfied this requirement).

The output consisted of the final least-squares values of α_p and K_p and their respective standard deviations; the final set of modified $\left[\frac{\text{Var}}{\text{Mean}} (T) - 1 \right]$ vs T data; least-squares values of $\left[\frac{\text{Var}}{\text{Mean}} (T) - 1 \right]$ for each T using the least-squares values α_p and K_p and Equation (33); and eight sets of $\left[\frac{\text{Var}}{\text{Mean}} (T) - 1 \right] / 2 E_k \left[\frac{\bar{v}^2 - \bar{v}}{\bar{v}} \right]$ vs T computed using Equation (29). This program was carefully checked out for convergence and uniqueness of solutions over the range of experimental parameters by using test case data originally generated by the VARMN subprogram.

This approach, even though the description appears a bit complex, reduces the experimental data in such a way that the modified data describe an artificial system containing essentially only prompt neutron effects. The artificial system, however, retains the features needed to yield a value of $\Lambda/\bar{\beta}$ appropriate to the realistic system.

The experimental $\left[\frac{\text{Var}}{\text{Mean}} (T) - 1 \right]$ vs T data were fit over gate times ranging from 0.0010 to 2.0000 sec for each of the four subcritical reactivities. The least squares values of α_p and K_p along with their standard deviations are tabulated in Table III for each case.

TABLE III
LEAST-SQUARES FITTING CONSTANTS

Run	Reactivity (\$)	α_p (sec ⁻¹)	K_p
1	-0.4894 $\pm$ 0.0030	-184.5 $\pm$ 7.2	15.54 $\pm$ 0.11
2	-0.2804 $\pm$ 0.0018	-158.0 $\pm$ 6.2	20.27 $\pm$ 0.15
3	-0.1334 $\pm$ 0.0009	-140.3 $\pm$ 6.7	23.78 $\pm$ 0.23
4	-0.7530 $\pm$ 0.0045	-220.1 $\pm$ 8.5	11.48 $\pm$ 0.09

Figures 9 through 12 show the experimental and modified data plotted as a function of gate time. The line labeled "prompt" represents the least squares fit to the modified data. The line labeled "prompt and delayed" represents the curve predicted by Equation (29) based on the least squares value of α_p and normalized to the asymptote K_p . These plots show that the model, including delayed neutron effects, fits the experimental data quite well over the range of parameters investigated.

FIG. 9 EXPERIMENTAL VARIANCE-TO-MEAN DATA, $R = -0.4894\$$.

FIG. 10 EXPERIMENTAL VARIANCE-TO-MEAN DATA, $R = -0.2804\$$.

FIG. 11 EXPERIMENTAL VARIANCE-TO-MEAN DATA, $R = -0.1334\$$.

FIG. 12 EXPERIMENTAL VARIANCE-TO-MEAN DATA, $R = -0.7530\$$.

The final step of the analysis is determination of Λ/β utilizing the experimental values of α_p and reactivity. The inhour equation with the prompt root inserted is,

$$r = \frac{\Lambda}{\beta} \alpha_p + \Sigma \quad (34)$$

where

$$\Sigma = \sum_{i=1}^m \frac{f_i \alpha_p}{\alpha_p + \lambda_i} \quad (35)$$

Equation (34) may be rewritten as

$$\frac{\Lambda}{\beta} \alpha_p = r - \Sigma \quad (36)$$

or, since α_p and r are always negative for a subcritical system, it is convenient to write Equation (36) in the form

$$\frac{\Lambda}{\beta} |\alpha_p| = |r| + \Sigma \quad (37)$$

The quantity $(|r| + \Sigma)$ was then plotted vs $|\alpha_p|$, as shown in Figure 13, to give a straight line through the origin with slope of Λ/β . From a least-squares linear fit of the data

$$\Lambda/\beta = (8.07 \pm 0.17) \times 10^{-3} \text{ sec}$$

for the Spert IV D-12/25 core where the uncertainty represents the standard deviation from the weighted mean. The uncertainty was calculated by a propagation of error technique considering the standard deviation in $|\alpha_p|$ associated with each point.

It should be pointed out that the variation in the birth energy of delayed neutrons has been neglected in the analysis of the data, ie, it was assumed that $\gamma_i = \bar{\gamma}$, which implies that $f_i = a_i$. A determination of the individual effectiveness factors for the Spert IV system has not been done; however, their inclusion in the analysis would result in a small change in the magnitude of the reactivity and α_p values.

FIG. 13 REACTIVITY AS A FUNCTION OF PROMPT ROOT.

V. SUMMARY OF RESULTS AND CONCLUSIONS

This paper has presented a description of an experimental determination of the reduced prompt neutron generation time Λ/β in the Spert IV reactor based on the statistical behavior of the neutron population. A model has been derived which relates the statistic "variance-to-mean ratio of the count rate as a function of counting time" to integral reactor parameters. Parametric studies have been conducted on this model which show that the contribution of delayed neutrons to the variance-to-mean ratio becomes significant for long counting times, for operation near critical, and for systems with a relatively long prompt neutron generation time. These studies also indicate that a systematic error, even under ideal parametric conditions, will be present in experimental results inferred from a model which does not include the contribution of delayed neutrons.

A data-fitting program has been developed for the IBM 7090 computer for analysis of experimental variance-to-mean data. This program essentially subtracts out the contribution of delayed neutrons to the variance-to-mean ratio, thus allowing a determination of Λ/β in which the systematic error due to delayed neutrons is negligible. This program should be useful for the determination of Λ/β in thermal reactors of many different designs. This program has been used in conjunction with experimental measurements to determine a value of $\Lambda/\beta = (8.07 \pm 0.17) \times 10^{-3}$ sec for the Spert IV D-12/25 core. This value is in good agreement with an independent measurement of $\Lambda/\beta = (7.93 \pm 0.12) \times 10^{-3}$ sec, obtained from super-prompt critical power excursion tests in the Spert IV reactor.

The data system used for the collection, storage, and sampling of counting data has demonstrated the importance of automation in an experiment of this type. The use of a magnetic tape recorder for temporary storage of the raw counting data minimized reactor operation time. The collection of data samples on punched paper tape in a form acceptable to an IBM 1620 computer allowed reduction of a large volume of data in an efficient manner.

Conventional neutron detection equipment has been utilized which presents an advantage over perturbation techniques which require installation of special in-pile instrumentation, whereas a disadvantage is the necessity for measuring subcritical reactivities. Also, this technique requires that the background source level of the core be low enough to keep the count rates within the limitations of the counting system, thus in general precluding its use on cores which have been operated extensively at high power.

Several areas which need further investigation in terms of improving and evaluating this statistical technique include: (a) application of method to various reactor types, especially systems in which the prompt neutron generation time is relatively long, (b) an evaluation of the sensitivity of the model to the delayed neutron effectiveness factors, (c) further improvements in data-handling and data-fitting techniques, and (d) application and evaluation of a more general analytical model derived by the use of probability-generating functions.

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APPENDIX A

SOLUTION OF THE REACTOR KINETICS EQUATIONS

The reactor kinetics equations as derived by Henry [15] for a stationary fuel reactor may be written in the form

$$\frac{dn}{dt} = \frac{\rho - \bar{\beta}}{\Lambda} n + \sum_{i=1}^m \lambda_i C_i + S_0 \quad (\text{A-1})$$

$$\frac{dC_i}{dt} = \frac{\bar{\beta}_i}{\Lambda} n - \lambda_i C_i \quad (\text{A-2})$$

where

n = neutron density

C_i = concentration of the i^{th} group of delayed neutron emitters

λ_i = decay constant of the i^{th} group of delayed neutron emitters

S_0 = source strength (neutrons/sec-cm³)

Λ = mean prompt neutron generation time

k = effective neutron reproduction factor for finite geometry

$\rho = (k - 1)/k$

β_i = yield of the i^{th} delayed neutron group

$\beta = \sum_i \beta_i$

$\bar{\beta}$ = effective delayed neutron fraction = $\sum_i \gamma_i \beta_i$

γ_i = effectiveness (in producing fission) of the i^{th} group of delayed neutrons compared with prompt neutrons

$\bar{\beta}_i = \gamma_i \beta_i$

m = number of delayed neutron groups

Now, let $r = \rho/\bar{\beta}$, $C_i = \frac{\bar{\beta}_i}{\Lambda \lambda_i} W_i$, $f_i = \bar{\beta}_i/\bar{\beta}$, where the variables, n , C_i , ρ , r , and W_i are to be regarded as functions of time. (Space and time assumed separable). Then, Equations (A-1) and (A-2) become

$$\frac{dn}{dt} = \frac{\bar{\beta}}{\Lambda} \left[(r - 1) n + \sum_{i=1}^m f_i W_i \right] + S_o \quad (A-3)$$

$$\frac{dW_i}{dt} = \lambda_i (n - W_i) \quad (A-4)$$

For an initial steady-state condition, $r = r_o$, $n = n_o$, $W_i = W_{io}$, and $\dot{n} = \dot{C}_i = 0$, and Equations (A-3) and (A-4) become

$$\frac{\bar{\beta}}{\Lambda} \left[(r_o - 1) n_o + \sum_{i=1}^m f_i W_{io} \right] + S_o = 0 \quad (A-5)$$

$$\lambda_i (n_o - W_{io}) = 0 \quad (A-6)$$

From Equation (A-6)

$$n_o = W_{io} \quad (A-7)$$

Inserting Equation (A-7) into Equation (A-5) gives

$$S_o = - \frac{\bar{\beta}}{\Lambda} r_o n_o \quad (A-8)$$

Now, taking the Laplace transform of Equations (A-3) and (A-4) and letting r represent the steady-state reactivity following a step insertion of reactivity at $t = 0$

$$s n(s) - n_o = \frac{\bar{\beta}}{\Lambda} \left[(r - 1) n(s) + \sum_{i=1}^m f_i W_i(s) \right] + \frac{S_o}{s} \quad (A-9)$$

$$s W_i(s) - W_{io} = \lambda_i [n(s) - W_i(s)] \quad (A-10)$$

From Equation (A-10)

$$W_i(s) = \frac{\lambda_i n(s) + W_{io}}{s + \lambda_i} \quad (A-11)$$

Substituting Equations (A-7), (A-8), and (A-11) into Equations (A-9) and rearranging gives

$$n(s) = \frac{n_o}{s} + \frac{n_o (r - r_o)}{s^2 \left[\frac{\Lambda}{\beta} - \frac{r}{s} + \sum_{i=1}^m \frac{f_i}{s + \lambda_i} \right]} \quad (A-12)$$

Now, let

$$D(s) = \frac{s\Lambda}{\beta} - r + s \sum_{i=1}^m \frac{f_i}{s + \lambda_i} \quad (A-13)$$

then

$$n(s) = \frac{n_o}{s} + \frac{n_o (r - r_o)}{s D(s)} \quad (A-14)$$

Equation (A-14) may be returned to a time-dependent form by means of the inverse transform giving

$$n(t) = n_o + L^{-1} \left[\frac{n_o (r - r_o)}{s D(s)} \right]$$

The inverse transform of the quotient is given by

$$L^{-1} \left[\frac{f(s)}{g(s)} \right] = \sum_j \frac{f(a_k)}{g'(a_k)} e^{a_k t}$$

where $f(s)$ and $g(s)$ are polynomials and the poles of the quotient $f(s)/g(s)$ are located at $s = a_k$. The quotient $f(a_k)/g'(a_k)$ then represents the residues of $f(s)/g(s)$ evaluated at its poles. If one represents $f(s)$ by 1 and $g(s)$ by $s D(s)$, then the zeros of $s D(s)$ are the a_k and

$$n(t) = n_o + n_o (r - r_o) \sum_{i=0}^{m+1} \frac{e^{\alpha_i t}}{\left[\frac{d}{ds} [s D(s)] \right]_{s = \alpha_i}} \quad (A-15)$$

The root of $s D(s) = 0$ corresponding to $i = 0$ is $s = 0$. If one writes Equation (A-15) as

$$n(t) = n_0 + n_0 (r - r_0) \sum_{i=0}^{m+1} \frac{e^{\alpha_i t}}{\left[D(s) + s \frac{d}{ds} D(s) \right]_{s=\alpha_i}}$$

then the contribution of $n(t)$ for $i = 0$ may be evaluated immediately. Utilizing Equation (A-14), it is seen that for $S = 0$, $D(0) = -r$, $s \frac{d}{ds} D(s) = 0$ and that $D(\alpha_i) = 0$ for $i > 0$. The expression for $n(t)$ then becomes

$$\begin{aligned} n(t) &= n_0 - \frac{n_0 (r - r_0)}{r} + n_0 (r - r_0) \sum_{i=1}^{m+1} \frac{e^{\alpha_i t}}{\left[s \frac{d}{ds} D(s) \right]_{s=\alpha_i}} \\ &= \frac{n_0 r_0}{r} + n_0 (r - r_0) \sum_{i=1}^{m+1} \frac{e^{\alpha_i t}}{\left[s \frac{d}{ds} D(s) \right]_{s=\alpha_i}} \end{aligned} \quad (A-16)$$

The remainder of the roots may be calculated by solving Equation (A-15) with $D(s) = 0$ which may be written as

$$r = \frac{s\Lambda}{\bar{\beta}} + s \sum_{i=1}^m \frac{f_i}{s + \lambda_i}$$

Upon substitution of $s = \alpha_1 = \frac{1}{T}$, this becomes,

$$r = \frac{\Lambda}{\bar{\beta}T} + \sum_{i=1}^m \frac{f_i}{1 + \lambda_i T} \quad (A-17)$$

which is the familiar inhour equation with the α_i 's determined by finding its solutions for T. Each of these $m + 1$ solutions for T determines an α_i by the inverse relation $\alpha_i \equiv 1/T$. Utilizing Equation (A-8) and rearranging terms, one may write Equation (A-16) in the form

$$n(t) = S_0 \frac{\Lambda}{\beta} \left[\sum_{i=1}^{m+1} \frac{e^{\alpha_i t}}{\left[s \frac{d}{ds} D(s) \right]_{s=\alpha_i}} - \frac{1}{r} \right] + n_0 r \sum_{i=1}^{m+1} \frac{e^{\alpha_i t}}{\left[s \frac{d}{ds} D(s) \right]_{s=\alpha_i}} \quad (A-18)$$

In this form, $n(t)$ is composed of two distinct portions, ie, one due to the source and one due to fission, or

$$n(t) = \text{source term} + \text{fission term}$$

Equation (A-18) may then be written as

$$n(t) = S_0 \left[\sum_{i=1}^{m+1} A'_i e^{\alpha_i t} - A_0 \right] + n_0 \sum_{i=1}^{m+1} A_i e^{\alpha_i t} \quad (A-19)$$

where

$$A'_i = \frac{\Lambda/\beta}{\left[s \frac{d}{ds} D(s) \right]_{s=\alpha_i}} \quad (A-20)$$

$$A_0 = \frac{\Lambda/\beta}{r} \quad (A-21)$$

$$A_i = \frac{r}{\left[s \frac{d}{ds} D(s) \right]_{s=\alpha_i}} \quad (A-22)$$

These coefficients will now be put in terms of integral reactor properties. Consider first the term $\left[s \frac{d}{ds} D(s) \right]_{s=\alpha_i}$ which becomes

$$\begin{aligned}
\left[s - \frac{d}{ds} D(s) \right]_{s=\alpha_j} &= r - \alpha_j^2 \sum_{i=1}^m \frac{f_i}{(\alpha_j + \lambda_i)^2} \\
&= r - \sum_{i=1}^m \frac{f_i}{(1 + \lambda_i T_j)^2}
\end{aligned} \tag{A-23}$$

where the subscript j refers to the $m + 1$ individual roots of the inhour equation and has been changed to avoid confusion with the delayed group summation. Equations (A-20) through (A-22) then become

$$A_j = \frac{\Lambda/\bar{\beta}}{r - \sum_{i=1}^m \frac{f_i}{(1 + \lambda_i T_j)^2}} \tag{A-24}$$

$$A_o = \frac{\Lambda/\bar{\beta}}{r} \tag{A-25}$$

$$A_j = \frac{r}{r - \sum_{i=1}^m \frac{f_i}{(1 + \lambda_i T_j)^2}} \tag{A-26}$$

Equation (A-19) then defines the neutron density $n(t)$ explicitly as a function of time in terms of integral reactor properties with the reactivity r expressed in dollars.

APPENDIX B

DISCUSSION OF MEAN LIFETIME

The model derived in Section II describes the statistical behavior of the neutron population in a nuclear reactor in terms of integral reactor parameters. The concept of mean lifetime of a neutron in the system was introduced in order to define the probability that a neutron is lost from the system in a specified time interval. This section discusses the concept of mean lifetime in the context used in this paper and shows that a unique mean lifetime may be associated with each decaying component of the neutron density due to fission in a subcritical system. The arguments are first developed in terms of a system having only prompt neutrons and then extended to the general case including delayed neutrons.

Consider a subcritical multiplying medium into which n_0 neutrons are introduced as a result of fission in the interval $(t_0, t_0 + dt_0)$. Assume a simplified system in which fission produces only prompt neutrons, ie, delayed neutrons do not exist. The time rate of change of the neutron population for $t > t_0$ is then

$$\frac{dn}{dt} = -\alpha n_0 \quad (B-1)$$

where α is the probability of loss per unit time weighted with some probability that a neutron lost will be replaced. The probability of replacement is simply the prompt reproduction factor k_p . If delayed neutrons are neglected in the space-independent kinetics equations, then

$$\alpha = \frac{1 - k_p}{\ell} \quad (B-2)$$

where $1 - k_p$ is the probability of non-replacement and ℓ is the mean prompt neutron lifetime or the average time from birth to death where birth is defined as the time of fission and death is defined as either productive capture, non-productive capture, or leakage. In terms of mean prompt neutron generation time, Equation (B-2) becomes

$$\alpha = \frac{(1 - k_p)/k_p}{\Lambda} \quad (B-3)$$

Consider now the general case of a subcritical multiplying medium including delayed neutrons. If n_0 fission neutrons are present at time t_0 , then the time rate of change of the neutrons for $t > t_0$ is described by Equations (5) and (6). There are $m + 1$ values of α which are needed to describe the decay process. These $m + 1$ values are the roots of Equation (10), the inhour equation. Equation (10) may be written in the form

$$\alpha_j = \frac{(k - 1)/k}{\Lambda + \sum_{i=1}^m \frac{\bar{\beta}_i}{\alpha_j + \lambda_i}} \quad (\text{B-4})$$

which is similar to the form of Equation (B-3). A mean lifetime τ_j may then be defined for each component of the neutron density, or

$$\tau_j = \Lambda + \sum_{i=1}^m \frac{\bar{\beta}_i}{\alpha_j + \lambda_i} \quad (\text{B-5})$$

Equation (B-4) then becomes

$$\alpha_j = \frac{(k - 1)/k}{\tau_j} \quad (\text{B-6})$$

or, rearranging and expressing the reactivity in dollars

$$\tau_j = \frac{r \bar{\beta}}{\alpha_j} \quad (\text{B-7})$$

The form of Equation (B-7) is convenient for calculations when using the general variance-to-mean expression given by Equation (29).

APPENDIX C

DETAILS OF EXPERIMENTAL EQUIPMENT

A functional description of the data collection system was given in Section III. This section will present the specifications and operating characteristics of the individual components of the counting and data collection system.

1. Neutron Detectors

Three boron-lined neutron detectors operated as proportional counters were used. The manufacturer's specifications and operating characteristics are listed below:

Manufacturer	Reuter-Stokes Co.
Model	RSN-17A
Maximum diameter	1-1/32 in.
Maximum overall length	15-3/8 in.
Sensitive length	11-7/16 in.
Shell material	Aluminum, 0.032 in. thick
Anode	0.001-in.-diam tungsten
Connector	Magnesium
Insulation	Alumina ceramic
Neutron-sensitive material	Boron enriched to > 92% B-10
Coating thickness	0.4 mg/cm ²
Fill gas	CO ₂ , 1 cm Hg; Argon, 19 cm Hg
Thermal neutron sensitivity	4 cps per n/cm ² -sec
Thermal neutron flux range	0.25 to 2.5 x 10 ⁴ n/cm ² -sec
Operating voltage	≈850 volts
Output pulse magnitude	≈10 millivolts
Pulse rise time	≈0.2 μsec

2. Preamplifiers

The preamplifiers were transistorized current-sensitive devices with the input impedance matched to the coaxial cable impedance of about 50 ohms. These preamplifiers were fabricated by Spert Electronics personnel and

are routinely used for reactor start-up counting equipment. The following characteristics have been determined:

Pulse rise time	≈ 75 nanosec
Pulse width	≈ 0.5 μ sec
Resolution time	≈ 0.7 μ sec
Average neutron pulse output	-100 mv peak

3. Linear Amplifiers

Two types of linear amplifiers were used. One was a modified Baird-Atomic and the other two were Hammer amplifiers. The manufacturer's specifications are as follows:

Manufacturer	Baird-Atomic Inc.
Model	204B
Gain (max)	6400
Rise time	≈ 0.2 μ sec
Bandwidth	≈ 1 mc
Output (max)	+ 100 volts
Manufacturer	Hammer Electronics Inc.
Model	N-371
Gain (max)	6000
Rise time	≈ 0.3 sec
Bandwidth	≈ 1 mc
Output (max)	+ 110 volts

4. Mixer

The purpose of the mixer was to sum the three input counting signals. A 4 x 4 matrix allowed any combination of four inputs to be summed. The mixer was built by Spert Electronics personnel and utilized standard diode mixing techniques.

5. Magnetic Tape Recorder

The magnetic tape recorder used to record the raw counting signal utilized direct record and playback electronics and the following specifications are appropriate to these electronics. A 100-kc timing channel was recorded simultaneously with the counting data for use as a time base, thus eliminating any time base uncertainties due to variations in tape speed.

Manufacturer	Ampex Corp.
Model	FR-614
No. of channels	14
Tape speed	7.5, 15, 30, or 60 ips
Tape width	1 in.
Tape length	5000 ft., 1.5-mil thickness
Drive	Capstan, friction drive
Bandwidth	250 kc max
Pulse rise time	2.5 μ sec max
Noise level	$\approx$ 40 millivolts
Signal level	$\approx$ 6 volts
Time displacement error (total static and dynamic between any two adjacent tracks on the same head)	< 3 μ sec
Type of heads	Standard IRIG

6. Gated Scaler

The gated scaler was designed and fabricated utilizing transistorized, plug-in elements by Spert Electronics personnel. The scaler includes an internal time base as well as provision for use with an external time base. The counting or gate time was manually variable to five significant figures with gate time and accumulated counts visually displayed. The following specifications are characteristic of the gated scaler.

Display	One-plane digital
Display time	10 μ sec min
Count rate capability (evenly spaced pulses)	3.5×10^5 counts/sec max
Counting range	1 to 10^5 counts per gate time
Gate times	10^{-4} to 10 sec in 10^{-4} sec increments
Time base frequency	100 kc/sec
Time base stability	1 part in 10^5
Input requirement	1 to 12 volts

Output

Binary coded decimal

7. Punch Control Unit

The punch control unit also was designed and fabricated utilizing transistorized plug-in elements by Spert Electronics personnel for use in conjunction with the gated scaler. It was designed to accept data from the gated scaler, store the data in a one-word memory, and electronically scan the memory for storage on punched paper tape. An inhibit circuit prevented the acceptance of a new word into the memory until the last word has been scanned. Several of the characteristics are as follows:

Display	One-plane digital
Scan time	16-2/3 msec per digit
Scan	Variable, 1 to 10 digits
Input	Binary coded decimal
Output	Binary coded decimal

8. Paper Tape Punch

The paper tape punch was used to store the counting data samples and was controlled by the punch control unit. The output format was binary coded decimal such that the data could be automatically converted to cards or fed directly into the IBM 1620 computer. The paper tape punch had the following characteristics.

Manufacturer	Tally Register Corp.
Model	420 PR
Paper tape width	1 in.
Tape handling	Reels
No. of channels	5 to 8
Punch rate (max)	60 characters/sec
Output format	Dependent on punch control unit

APPENDIX D

TABULATION OF EXPERIMENTAL COUNTING DATA

TABLE D-I

EXPERIMENTAL COUNTING DATA FOR $r = -0.4894$

<u>Gate Time (Sec)</u>	<u>Number of Samples</u>	<u>Mean</u>	<u>$\left[\frac{\text{Var}}{\text{Mean}} (T) - 1 \right]$</u>
0.0012	800	3.3	1.429
0.0015	800	4.4	2.080
0.0017	800	5.1	2.320
0.0020	800	5.8	2.356
0.0025	800	7.2	2.870
0.0035	800	10.6	4.311
0.0040	800	11.6	4.551
0.0045	800	13.4	5.125
0.0055	800	15.6	5.859
0.0060	800	17.0	5.964
0.0065	800	19.7	6.846
0.0075	800	22.6	8.067
0.0080	800	24.1	7.367
0.0090	800	26.4	7.658
0.0100	800	29.1	8.164
0.0115	800	33.1	10.06
0.0150	800	42.5	10.39
0.0170	800	48.4	10.92
0.0185	799	53.1	10.68
0.0225	800	67.6	11.96
0.0250	800	73.4	12.93
0.0300	800	84.4	12.39
0.0350	800	103.6	13.91
0.0400	800	114.2	12.14
0.0450	800	129.7	13.78
0.0500	800	143.0	14.15
0.0550	800	158.6	13.25
0.0600	800	175.0	14.55
0.0700	799	200.1	15.23
0.0750	800	222.4	15.07
0.0800	800	225.6	14.55
0.0900	800	270.5	15.66
0.1000	800	286.1	14.42
0.1150	800	341.1	15.81
0.1300	800	374.9	16.33

<u>Gate Time (Sec)</u>	<u>Number of Samples</u>	<u>Mean</u>	<u>$\frac{Var}{Mean} (T) - 1$</u>
0.1500	800	430.8	16.31
0.1700	800	489.1	14.53
0.1850	800	525.9	15.61
0.2000	800	570.1	16.32
0.2250	800	646.1	15.87
0.2500	800	720.6	16.69
0.2750	800	826.2	15.47
0.3000	800	865.9	16.07
0.3500	800	1032.	18.70
0.4000	800	1176.	18.20
0.4500	800	1325.	17.51
0.5000	800	1468.	18.06
0.5500	800	1622.	18.26
0.6000	800	1777.	19.50
0.6500	800	1927.	17.76
0.7000	800	2092.	19.58
0.7500	800	2165.	19.69
0.8000	800	2395.	20.03
0.9000	800	2606.	20.36
1.0000	800	2935.	22.01
1.1500	799	3401.	22.04
1.3000	800	3870.	20.88
1.5000	800	4457.	21.18
1.7000	800	5095.	23.44
1.8500	800	5509.	26.56
2.0000	800	5890.	24.12

TABLE D-II

EXPERIMENTAL COUNTING DATA FOR $r = - 0.2804\%$

<u>Gate Time (Sec)</u>	<u>Number of Samples</u>	<u>Mean</u>	<u>$\frac{Var}{Mean} (T) - 1$</u>
0.0010	799	5.1	1.454
0.0012	785	5.7	1.176
0.0015	800	7.9	2.334
0.0017	800	8.1	2.567
0.0020	800	10.6	2.690
0.0025	800	12.4	3.514
0.0030	800	15.2	4.035
0.0040	800	21.1	5.130
0.0045	800	21.5	5.952
0.0050	800	27.0	6.346

<u>Gate Time (Sec)</u>	<u>Number of Samples</u>	<u>Mean</u>	<u>[Var Mean (T) - 1]</u>
0.0055	800	27.0	6.456
0.0070	800	35.3	7.500
0.0075	800	36.4	8.638
0.0080	800	42.7	9.303
0.0090	800	44.1	9.882
0.0100	799	53.6	10.59
0.0115	800	57.3	10.78
0.0150	800	73.8	12.87
0.0170	800	85.9	12.40
0.0200	800	103.9	14.93
0.0250	800	130.8	16.19
0.0275	800	132.3	14.80
0.0300	800	154.3	15.77
0.0350	800	175.3	17.63
0.0400	784	209.9	18.15
0.0550	800	267.8	18.83
0.0600	800	303.4	19.08
0.0650	800	324.1	19.69
0.0750	800	357.4	18.71
0.0800	800	422.8	20.69
0.0900	800	430.8	19.93
0.1000	800	511.4	19.23
0.1150	800	577.3	19.67
0.1300	799	690.1	21.53
0.1500	800	749.6	20.92
0.1700	800	898.6	22.05
0.1850	800	892.4	20.24
0.2000	800	1021.	22.36
0.2250	800	1123.	22.45
0.2500	800	1324.	23.14
0.2750	799	1313.	20.55
0.3000	800	1592.	23.79
0.3500	800	1728.	21.94
0.4000	800	2063.	24.47
0.4500	800	2175.	23.16
0.5000	800	2625.	26.82
0.5500	799	2664.	23.25
0.6000	799	3138.	27.69
0.6500	800	3160.	23.99
0.7000	785	3510.	27.05
0.7500	800	3654.	25.13
0.8000	800	4021.	28.76
0.9000	799	4382.	27.18
1.0000	800	4921.	28.65
1.1500	798	5649.	29.07
1.3000	800	6373.	32.99
1.5000	798	7413.	32.43
1.7000	799	8446.	35.99
1.8500	800	9127.	35.10
2.0000	799	9891.	35.68

TABLE D-III

EXPERIMENTAL COUNTING DATA FOR $r = -0.1334$

<u>Gate Time</u> <u>(Sec)</u>	<u>Number of</u> <u>Samples</u>	<u>Mean</u>	<u>$\frac{\text{Var}}{\text{Mean}}$ (T) - 1</u>
0.0010	800	10.0	1.583
0.0012	800	12.0	1.567
0.0015	800	15.9	2.416
0.0017	800	17.5	2.543
0.0020	799	20.4	3.078
0.0025	800	25.2	3.595
0.0030	800	29.8	4.967
0.0035	800	34.7	5.280
0.0040	800	41.6	5.566
0.0045	800	48.3	6.424
0.0050	800	51.9	6.094
0.0055	790	58.8	6.867
0.0060	800	62.2	7.457
0.0065	800	69.8	7.945
0.0070	800	75.1	8.869
0.0075	800	80.6	8.887
0.0080	800	84.0	10.39
0.0090	800	92.6	10.49
0.0100	800	101.4	11.37
0.0130	800	132.9	12.45
0.0150	800	147.8	13.81
0.0170	800	181.2	14.42
0.0185	800	194.5	16.12
0.0200	785	204.2	17.58
0.0225	800	233.8	15.90
0.0250	800	264.5	18.65
0.0275	800	292.1	18.31
0.0300	800	309.1	17.73
0.0350	800	362.7	22.75
0.0400	800	404.6	19.30
0.0500	800	520.6	20.29
0.0550	800	536.3	23.91
0.0600	800	637.2	20.58
0.0700	798	719.2	22.78
0.0750	800	796.5	21.33
0.0800	800	851.4	23.75
0.1000	800	1034.	25.18
0.1150	800	1193.	23.04
0.1300	800	1299.	22.78
0.1500	800	1160.	22.43
0.1700	800	1782.	24.15
0.1850	800	1846.	26.44
0.2000	800	2054.	25.32
0.2250	793	2404.	26.74

<u>Gate Time</u> <u>(Sec)</u>	<u>Number of</u> <u>Samples</u>	<u>Mean</u>	<u>$\left[\frac{\text{Var}}{\text{Mean}} (T) - 1 \right]$</u>
0.2500	792	2549.	27.10
0.2750	798	2903.	26.87
0.3000	792	3097.	30.35
0.3500	795	3503.	28.00
0.4000	798	4136.	30.34
0.4500	797	4758.	29.54
0.5000	800	5099.	32.57
0.5500	795	5521.	33.57
0.6000	796	6184.	33.93
0.6500	800	6835.	32.02
0.7000	794	7226.	32.76
0.7500	774	7862.	33.98
0.8000	794	8183.	36.86
0.9000	779	9320.	40.81
1.0000	791	10410.	40.77
1.1500	800	12820.	47.74
1.3000	800	14700.	47.05
1.5000	800	16740.	42.39
1.8500	800	20230.	53.13
2.0000	800	21390.	59.60

TABLE D-IV

EXPERIMENTAL COUNTING DATA FOR $r = -0.7530$

<u>Gate Time</u> <u>(Sec)</u>	<u>Number of</u> <u>Samples</u>	<u>Mean</u>	<u>$\left[\frac{\text{Var}}{\text{Mean}} (T) - 1 \right]$</u>
0.0010	800	1.6	1.317
0.0012	800	2.1	1.329
0.0015	800	2.8	1.547
0.0017	800	3.1	1.910
0.0020	800	3.6	2.666
0.0030	800	5.4	3.262
0.0035	800	6.9	3.568
0.0040	800	7.5	4.402
0.0045	800	3.2	3.813
0.0050	800	9.0	4.540
0.0055	800	10.2	4.713
0.0060	799	10.9	5.164
0.0065	800	11.5	4.850
0.0075	800	14.4	6.015
0.0090	800	16.6	6.043
0.0100	800	18.2	6.999

<u>Gate Time</u> <u>(Sec)</u>	<u>Number of</u> <u>Samples</u>	<u>Mean</u>	<u>[Var</u> <u>Mean (T) - 1]</u>
0.0115	800	21.5	7.546
0.0130	800	24.4	7.460
0.0150	800	27.8	7.683
0.0170	800	31.1	8.459
0.0185	800	34.8	8.859
0.0225	800	42.5	9.533
0.0275	799	50.4	8.948
0.0350	800	65.1	10.26
0.0400	800	73.7	10.14
0.0500	800	92.1	11.32
0.0600	800	112.4	9.978
0.0650	798	117.8	10.94
0.0700	799	126.2	11.54
0.0900	800	162.9	10.73
0.1000	800	179.3	11.99
0.1150	800	215.2	12.07
0.1300	800	240.7	11.01
0.1500	800	277.0	12.32
0.1700	799	312.0	12.09
0.1850	800	335.5	12.22
0.2000	800	366.4	11.72
0.2250	799	407.7	12.39
0.2500	800	451.5	12.88
0.2750	797	496.0	11.81
0.3000	798	540.5	11.99
0.3500	800	647.8	12.44
0.4000	800	719.3	13.84
0.5000	799	914.9	12.17
0.5500	800	991.2	12.91
0.6000	799	1084.	13.00
0.6500	800	1194.	13.53
0.9000	800	1634.	12.84
1.0000	800	1801.	13.59

NOMENCLATURE

NOMENCLATURE

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$A_j = j^{\text{th}}$ fraction of total fission neutron density defined by Equation (9)

$a_i =$ fraction of total delayed neutron yield in the i^{th} group

$\alpha_j = j^{\text{th}}$ root of inhour equation

$\alpha_p =$ root of largest magnitude of inhour equation, referred to as prompt root

$$B_k = \sum_j \frac{A_j}{(\alpha_j + \alpha_k) \tau_j \tau_k} \quad (\text{Equation 25})$$

$\beta =$ total delayed neutron yield fraction

$\beta_i =$ yield of the i^{th} group of delayed neutrons

$\bar{\beta} =$ total effective delayed neutron yield fraction

$C_i =$ density of delayed neutron precursors of the i^{th} group

$c =$ number of counts collected in time interval T

$\bar{c} =$ mean number of counts collected over several samples

$\overline{c^2} =$ dispersion in the number of counts collected over several samples

$E =$ neutron detector efficiency

$\epsilon =$ convergence criterion in data-fitting program

$F =$ mean neutron production rate including source

$f_i =$ effective yield of the i^{th} group of delayed neutrons

$G =$ mean neutron production rate excluding source

$\gamma_i =$ effectiveness (in producing fission) of the i^{th} group of delayed neutrons compared to prompt neutrons

$\bar{\gamma} =$ average effectiveness (in producing fission) of delayed neutrons compared to prompt neutrons

$K_k =$ asymptotic coefficients defined by Equation (32)

$K_p =$ asymptotic coefficients defined by Equation (33)

$k =$ effective total reproduction factor

$k_p =$ effective prompt reproduction factor

L^{-1} = denotes inverse Laplace transform

ℓ = mean prompt neutron lifetime

Λ = mean prompt neutron generation time

λ_i = delayed neutron precursor decay constant of the i^{th} group

m = number of delayed neutron groups

n = number of counting samples taken at fixed counting time

n_0 = initial neutron density

$n(t)$ = neutron density at time t

ν = number of neutrons emitted per fission

$\bar{\nu}$ = mean number of neutrons emitted per fission

$\overline{\nu^2}$ = dispersion in the number of neutrons emitted per fission

ρ = total reactivity in dollars

ρ = total reactivity in absolute units

s = Laplace transform variable

S_0 = neutron source density

$$\Sigma = \sum_i \frac{f_i \alpha_p}{\alpha_p + \lambda_i} \quad (\text{Equation 34})$$

T = counting or gate time

t = time

τ_j = mean lifetime of a neutron from j^{th} component of neutron density

$\frac{\text{Var}}{\text{Mean}} (T)$ = variance-to-mean ratio of the count rate at a fixed counting time T

i, j, k, p = frequently used subscripts

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