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ON THE FEASIBILITY OF COLLIDING BEAM EXPERIMENTS
WITH LONGITUDINALLY POLARIZED ELECTRONS

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ABSTRACT

Various methods for obtaining longitudinal polarization of electrons circulating in a high energy storage ring are analysed. One apparently practical method is found. Such longitudinal polarization could be used to detect directly the parity violating effects of the weak interactions in proposed high energy electron-proton colliding beam machines.

A technique for observing collisions between longitudinally polarized electrons and unpolarized protons circulating in two intersecting storage rings has recently been suggested by M. Schwartz⁽¹⁾. Since any dependence of the collision cross section on the electron's longitudinal polarization would imply parity violation, such a technique could be used to study the Weak Interactions at the very high center of mass energies available in colliding beam machines. In this note we discuss Schwartz's method for obtaining longitudinal polarization of the electrons and suggest an alternative scheme.

There is now both theoretical argument⁽²⁾ and experimental evidence⁽³⁾ that electrons circulating in a storage ring acquire a net transverse polarization anti-parallel to the magnetic field. This polarization develops with a time constant⁽⁴⁾

$$\tau_{\perp} = \frac{8}{5\sqrt{3}} \frac{m c R}{\hbar} \frac{1}{\gamma^5} \frac{R^2}{r_e c} \quad (1)$$

where R is the radius of the ring (assumed circular); m , $E = \gamma m c^2$, and r_e are the mass, energy and classical radius of the electron. If we consider a 4 GeV electron ring with the parameters proposed for the SPEAR project⁽⁵⁾ (effective radius⁽⁶⁾

$$R_{\text{eff}} = 18 \text{ m}, \quad \gamma = 7830)$$

$$\tau_{\perp} \sim 10 \text{ min.} \quad (2)$$

Schwartz proposes that a pulsed solenoid be used in a straight section of the electron ring to produce a magnetic field parallel to the electron momentum. The strength of this field is so adjusted that on passing through the solenoid, the electron's spin is rotated through 90° into the plane of the storage ring. The magnet is switched off

after all the electrons have passed through it. Since the electron's spin is now in the plane of the orbit, the angle θ between the spin and the momentum will change continuously⁽⁷⁾

$$\theta = \frac{g-2}{2} \frac{e}{mc} \int_0^t B dt = \frac{g-2}{2} \int_0^N \gamma 2\pi dN \quad (3)$$

where $(g-2)/2 \sim \alpha/2\pi$ is the anomalous part of the electron's gyromagnetic ratio, t is the time elapsed since the electron's spin was flipped into the horizontal plane and N the corresponding number of turns (integer plus fraction) executed by the electron around the ring. The electrons travel in bunches in phase with the applied rf field and each e-p collision can be attributed to an electron in an identifiable bunch. Thus N for an electron at the time of scattering can be known quite precisely so that the direction of its polarization might be determined from Eq. (3).

If this proposal is to be useful the electrons in a given bunch must remain polarized for a considerable time after the initial spin flip. Clearly the original vertical polarization will be restored by synchrotron radiation with a time constant $\tau_{\perp}$. However, the horizontal component of polarization for electrons in a given bunch may decrease much more rapidly because of slight variations in γ from electron to electron within a bunch. As can be seen from Eq. (3), even small fluctuations in γ may cause large variations in θ if N is large, and hence significant depolarization. We will discuss three different mechanisms causing a variation in γ among electrons in a given bunch.

(i) Let us first consider variations in γ arising from the oscillations of the phase ϕ of the electron with respect to the rf field. If we let ϕ_0 and γ_0 be the equilibrium

values of ϕ and γ , then in the approximation $\beta \simeq 1$ (made throughout this paper),

$$\frac{d \Delta \phi}{dt} = \eta c \frac{2\pi}{\lambda} \left[\frac{\Delta \gamma}{\gamma_0} + \alpha_2 \left(\frac{\Delta \gamma}{\gamma_0} \right)^2 + \dots \right] \quad (4)$$

$$\begin{aligned} \frac{d \Delta \gamma}{dt} &= \frac{e \mathcal{E}}{mc} (\sin \phi - \sin \phi_0) \\ &= \frac{e \mathcal{E}}{mc} \cos \phi_0 \left[\Delta \phi - \frac{1}{2} \tan \phi_0 (\Delta \phi)^2 + \dots \right] \end{aligned} \quad (5)$$

where $\mathcal{E}$ is the magnitude of the rf accelerating field, $\Delta \gamma = \gamma - \gamma_0$,

$\Delta \phi = \phi - \phi_0$, λ is the rf wavelength, α_2 and η are constants. If the quadratic terms are neglected, these equations imply

$$\frac{d^2 \Delta \gamma}{dt^2} = \left[\frac{\eta}{\gamma_0} \frac{2\pi}{\lambda} \frac{e \mathcal{E}}{m} \cos \phi_0 \right] \Delta \gamma \quad (6)$$

with solution

$$\gamma(t) = \gamma_0 + \hat{\Delta \gamma} \cos \omega_1(t + \delta) \quad (7)$$

for

$$\omega_1^2 = -\frac{\eta}{\gamma} \frac{2\pi}{\lambda} \frac{e \mathcal{E}}{m} \cos \phi_0. \quad (8)$$

Thus, if the amplitude of energy oscillations $\hat{\Delta \gamma}$ is known, Eq. (7) and Eq. (3) give the variation in polarization direction from this cause. However, we may carry out the integral in (3) with the aid of Eq. (4) directly, yielding a deviation in $\theta(t)$ from its equilibrium value of

$$\begin{aligned} \theta_1(t) &= \theta_1(0) + \frac{g-2}{2} \frac{2\pi c}{C} \int_0^t \Delta \gamma(t') dt' \\ &= \theta_1(0) + \frac{g-2}{2} \frac{\lambda \gamma_0}{C \eta} (\Delta \phi(t) - \Delta \phi(0)) \end{aligned} \quad (9)$$

where C is the ring's circumference. The longitudinal particle displacement is just $\Delta\phi \lambda/2\pi$. Using the SPEAR parameters⁽⁴⁾ scaled to 4 GeV ($\eta = 0.038$, root-mean-square bunch length 6.2 cm, $C = 234$ m)

$$(\theta_1)_{\text{rms}} = .4 \text{ radian} \quad (10)$$

which corresponds to a rather modest depolarization.

(ii) A second variation in γ which should be considered arises because the mean value $\bar{\gamma}$ of γ possessed by an electron undergoing phase oscillations depends slightly on the magnitude $\hat{\Delta\gamma}$ of those oscillations. This effect is predicted by the quadratic terms in Eqs (4) and (5) which imply that

$$\bar{\gamma} = \gamma_0 - \frac{\alpha_2}{2} \frac{(\hat{\Delta\gamma})^2}{\gamma_0} \quad (11)$$

Thus $\bar{\gamma}$ will vary among the electrons in a bunch because of the spread in $\hat{\Delta\gamma}$. However, the uniform oscillation predicted by Eqs (4) and (5) will be altered by the random mixing among electron trajectories induced by synchrotron radiation with a time scale

$$\tau_R = \frac{E}{\Delta E} \frac{C}{c} \quad (12)$$

where ΔE is the energy loss per turn for a single electron. For our SPEAR example ($\Delta E = 1.78$ MeV)

$$\tau_R = 2 \times 10^{-3} \text{ sec.} \quad (13)$$

Hence a particular electron undergoing phase oscillations with an amplitude $\hat{\Delta\gamma}$ will maintain that value for a time of the order of τ_R . In that time its polarization angle will deviate from the average by an amount

$$\Delta\theta_2 = \tau_R \frac{g-2}{2} \frac{2\pi c}{C} \frac{\alpha_2}{2} \frac{(\hat{\Delta}\gamma)^2}{\gamma_0} \quad (14)$$

In the SPEAR example, taking $\alpha_2 \sim 3$ and the average $(\hat{\Delta}\gamma/\gamma_0)^2$ about 10^{-6} , we find

$$\Delta\theta_2 \sim .2 \quad (15)$$

again a small depolarization. However, if this variation in the direction of polarization is added randomly over a large time t , a significant net r. m. s. deviation will occur

$$(\theta_2)_{\text{rms}} = \Delta\theta_2 \sqrt{t/\tau_R} \quad (16)$$

This implies complete depolarization after $\sim .5$ sec for the SPEAR case. Such a short depolarization time undoubtedly makes this scheme impractical. Nevertheless, for completeness, one further depolarizing mechanism will be analysed.

(iii) Finally, we discuss variation in γ caused by variations in the amplitude of betatron oscillations. First consider the effects of vertical betatron oscillations. We assume that the vertical oscillation and the associated horizontal components of magnetic field do not appreciably rotate the electron spin out of the horizontal plane. (8) However, the orbit of an electron undergoing vertical betatron oscillations with magnitude $\hat{\Delta}z$ and frequency ω_β will be longer than the equilibrium orbit of the same radius by a fraction $\frac{1}{4}\psi^2$ where $\psi = \hat{\Delta}z \omega_\beta/c$. If this electron is to stay in phase with the electric field, its γ (and consequently the radius of its orbit) must be altered by the amount required by Eq. (4)

$$\Delta\gamma = \frac{1}{\eta} \gamma \frac{1}{4}\psi^2 \quad (17)$$

The betatron oscillation angle ψ for a particular electron varies because of synchrotron radiation with the time constant τ_R . During this time the electron's polarization direction will change from the average by an amount

$$\Delta \theta_3 \approx \frac{g-2}{2} 2\pi \frac{c}{C} \tau_R \frac{\gamma}{\eta} \frac{1}{4} \psi^2. \quad (18)$$

For the SPEAR machine with $\psi = 4 \times 10^{-5}$ we find

$$\Delta \theta_3 \approx 1.4 \times 10^{-3} \text{ rad.} \quad (19)$$

Over a time t these fluctuations in polarization angle will accumulate, yielding an r.m.s. deviation

$$(\theta_3)_{\text{rms}} = \Delta \theta_3 \sqrt{t/\tau_R} \quad (20)$$

and complete depolarization in $\sim 10^4$ sec for our SPEAR example. The effect of radial betatron oscillation is quite similar.

In addition to the variation in γ , given by eq (17), that is needed to compensate the increased circumference of an orbit with non-zero betatron oscillation angle ψ , one must also include the change in the average value of the vertical magnetic field (and thus in γ) seen by an electron undergoing betatron oscillations. This variation in the vertical field arises from the vertical or radial non-linearity of the vertical magnetic field and is also quadratic in ψ . These two effects are of comparable size and in fact cancel exactly for radial oscillations in a circular machine with a vertical magnetic field which varies as a power of the radius.

Now let us turn to a second method⁽⁹⁾, perhaps more practical than that just discussed, which provides continuous longitudinal polarization. This is accomplished

by allowing the electron spins to remain perpendicular to the magnetic field during only a small portion of each revolution. One possible scheme adds to the usual electron storage ring a series of four magnets: one solenoid, two bending magnets and then a final solenoid arranged as in the plan view of Fig. 1. The first solenoid, with field parallel to the electron momentum bends the vertical electron spin into the horizontal plane of the ring. The second magnet bends the momentum through an angle $\theta_0 = \frac{1}{\gamma} \pi / (g - 2)$ so that the spin precesses through $\pi/2 + \theta_0$ becoming longitudinal. Finally the spin is rotated back through an angle of $-\pi/2 - \theta_0$ and then returned to its original vertical direction by the last solenoid. Throughout this process the beam remains in the horizontal plane of the ring.

Ideally, after passing through the whole device the spin is returned exactly to the vertical direction. The solenoids will remain energized and the process will be repeated each time the electrons go round the ring. By reversing the solenoid currents the longitudinal polarization of the particles in the interaction region can be reversed if desired.

In practice the solenoid current will not be exactly correct and furthermore the postulated total $(g - 2)$ precession of zero degrees in the horizontal plane may not be accurate since it requires the beam to be returned to precisely its original direction by the second bending magnet. The overall effect of these errors is that the spin is not returned quite to its original direction, but in passing from A to B (Fig. 1) it suffers a small net rotation through an angle ξ_1 (say) about some arbitrary axis $\hat{n}_1$. If this were cumulative over many turns the polarization would be lost.

However, as the particle completes its turn around the machine from B to A any horizontal component of the spin will be subject to a $(g - 2)$ precession about

the vertical axis, say through an angle $\xi_2 + 2\pi n$, where $-\pi < \xi_2 < \pi$ and n is an integer. Combining these two rotations, the result is equivalent to a simple rotation about another axis. If $\xi_1 \ll 1$ and $\xi_2 \gg \xi_1$ this axis will be nearly vertical. Thus if the direction $\hat{n}_1$ and the angle ξ_1 are constant from one orbit to the next, the small errors in the magnets will cause the spin to precess about a fixed, almost vertical axis and there will be no cumulative depolarization, provided the energy is adjusted so that $\gamma(g-2)/2$ is not close to an integer.

Of course, for a given electron the small rotation through the angle ξ_1 about the direction $\hat{n}_1$ caused by errors in the magnets will change slightly from one orbit to the next. Let $\xi_1(l)$ and $\hat{n}_1(l)$ describe the small rotation undergone by a particular electron spin on its l^{th} traversal from A to B. This variation in ξ_1 and $\hat{n}_1$ comes from changes in the electron's trajectory from one orbit to the next caused by: i) classical oscillations of orbit radius and energy, and ii) random fluctuations produced by the quantum nature of synchrotron radiation. In order that the variation in ξ_1 and $\hat{n}_1$ arising from the source i) not have a cumulative effect on polarization, the machine parameters must be so chosen that the series $\xi_1(l)$, $\hat{n}_1(l)$ does not contain a term of the form $\cos [2\pi \gamma(g-2) l/2 + \phi]$. That is, the change in ξ_1 and $\hat{n}_1$ from one orbit to the next must not be synchronous with the $g-2$ precession undergone by the electron on its return trip from B to A.

The frequency spectrum of the contribution to $\xi_1(l)$ and $\hat{n}_1(l)$ arising from fluctuations in synchrotron radiation cannot be so easily controlled. In fact, they will lead to a "random walk" of the spin away from its intended direction. Let us assume that the parameters ξ_1 and $\hat{n}_1$ vary among the electrons in a bunch with an r.m.s. deviation δ from the average. Then, if the spread in the trajectory parameters among

the electrons in a bunch is caused by the random fluctuations in synchrotron radiation, we may expect a given electron trajectory to vary over that range in a radiation damping time τ_R . Thus, in a time t the net depolarization produced by these random variations in ξ_1 and $\hat{n}_1$ will be of the order of

$$\delta \sqrt{t/\tau_R} \quad (21)$$

If we require that depolarization occur after a time larger than the initial polarizing time $\tau_\perp$, so that a steady state polarization is maintained, we must have

$$\delta < \pi \sqrt{\tau_R/\tau_\perp} \quad (22)$$

which for our SPEAR example requires

$$\delta < 5 \times 10^{-3} \quad (23)$$

This is probably a practical requirement for the arrangement shown in Fig. 1 (10).

It is interesting to note that the more natural arrangement of magnets in which the third and fourth are duplicates, respectively, of the second and first (so that the electron beam is bent through an angle of $2\theta_0$) satisfies the condition (23) only marginally. For such a configuration, shown in figure 2, the electron spin is rotated about the vertical by $2\theta_0 + \pi$ on passing from A to B. If the two bending magnets are constructed so that the momentum of electrons with different γ is bent through the same angle, then the random fluctuations in the phase oscillation amplitude $\hat{\Delta}\gamma$ will give rise to a value of δ

$$\delta = \pi \frac{(\hat{\Delta}\gamma)_{\text{rms}}}{\gamma} \quad (24)$$

which for our SPEAR example is 3×10^{-3} , uncomfortably close to the upper limit (23).

A third arrangement, shown in figure 3, bends the electron beam vertically, four times, through the angle θ_0 . Such a configuration can probably satisfy the requirement (22) and may be more easily realized than the system of figure 1.

In summary, we have analysed two schemes for producing longitudinal polarization of the electrons in a high energy storage ring. In the first method a net polarization is produced in the plane of the ring. The polarization $g - 2$ precesses, producing an oscillating longitudinal polarization for the electrons. However, this polarization appears to be rapidly lost because of the quantum fluctuations in the emitted synchrotron radiation. The second proposal maintains the natural vertical polarization throughout most of the ring, using a system of magnets to make the polarization longitudinal for a small region. The latter scheme appears to be feasible.

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Footnotes

1. M. Schwartz, talk delivered at the e-p Colliding Beams Conference, UCLA, March 1972.
2. A. A. Sokolov, I. M. Temov, Synchrotron Radiation, Pergamon Press (1968).
These authors predict a transverse polarization of 96% for electrons circulating in a uniform magnetic field.
3. D. Potax, "Status Report on ACO", Orsay Report 65-71, September 15, 1971, presented at the 8th International Conference on High Energy Accelerators (CERN).
4. A. A. Sokolov, I. M. Temov, op. cit. To an accuracy of 1.4% this formula can be written $\tau_{\perp} = 100 R_{\text{eff}}^3 / E^5$ sec, where R_{eff} is in meters and E in GeV.
5. 8th International Conference on High Energy Accelerators, CERN, 1971, p. 146.
6. In order to apply Eq. (1) to a machine in which the radius of curvature ρ varies around the ring, we must replace R by an effective radius R_{eff} . This effective radius is chosen so that $R_{\text{eff}}^{-3} = \langle \rho^{-3} \rangle$ where $\langle \rangle$ indicates an average around the ring. For a ring made up of straight sections and curved sections with a single radius of curvature R_{bending} ,

$$R_{\text{eff}}^3 = R_{\text{bending}}^2 R_{\text{gross}}$$
 where $2\pi R_{\text{gross}}$ is the circumference of the ring.
 For the SPEAR machine $R_{\text{bending}} = 12.7$ m and $R_{\text{gross}} = 37.3$ m.
7. Throughout this paper we use Gaussian units unless otherwise stated.
8. Since the net effect of the total magnetic field on an electron's spin can be represented as a single rotation, any trajectory which causes a horizontal spin to acquire a vertical component would cause a deviation, at least as

large, of the original vertical polarization from the vertical direction. For a discussion of depolarization of the original vertical spin orientation, see K. W. Robinson, "Depolarization in a Storage Ring" (CEAL TM 197) 5 July 1972.

9. A method similar to this one has also been considered by M. Schwartz (private communication).

Figure Captions

- Figure 1 Plan view of an arrangement of magnets intended to rotate the direction of electron spin polarization from transverse to longitudinal and then back again to transverse. The double arrow indicates the direction of spin polarization in the plane of the electron orbit while the cross signifies polarization perpendicular to the plane of the paper.
- Figure 2 Plan view of a second magnet configuration in which the electron spin undergoes a net rotation through $\pi + 2\theta_0$ about the vertical axis on passing from A to B.
- Figure 3 Side view of an arrangement of four bending magnets which deflect the electron beam in a vertical plan and return it to its original direction and altitude. Each bend is through an angle θ_0 . There is no net effect on the electron spin on passing from A to B.

Figure 1

Figure 2

Figure 3