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**TURBULENT HEAT TRANSFER
IN A ROD BUNDLE
WITH LIQUID METAL COOLANT**

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U.S. ATOMIC ENERGY COMMISSION
CONTRACT AT(04-3)-189
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BREEDER REACTOR DEVELOPMENT OPERATION
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TURBULENT HEAT TRANSFER
IN A
ROD BUNDLE WITH LIQUID METAL COOLANT

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ABSTRACT

The coupled heat-transfer equations in the fuel, gap, cladding, and coolant have been solved for a triangular-spaced, close-packed fuel rod bundle with liquid metal coolant. The model includes temperature-dependent material properties, turbulent velocity distribution in the coolant, and contributions to coolant heat transfer by turbulent mixing.

The model has been programmed as a digital computer code and used in a parametric study to determine the heat-transfer characteristics for this geometry as a function of Peclet number, pitch-to-diameter ratio, and linear pin power.

1. INTRODUCTION

1.1 PROBLEM DESCRIPTION

The work reported here was performed under Task G (Core Neutronics, Thermal Hydraulics, and Safety Analysis) of the Fast Ceramic Reactor Development Program* as a part of the methods development needed to provide basic analytical and design tools for the Liquid Metal Fast Breeder Reactor (LMFBR) core design. Results of this effort are applied to investigations of the effects of mechanical and hydraulic design variations, nonuniform spacing and heat flux, and fuel rod distortion on local temperature and flow distribution.

The thermal conditions which will exist in most advanced liquid metal cooled fast reactors are large heat fluxes and high power densities. Fuel elements are usually arranged in a triangular array with low pitch-to-diameter ratios to optimize nuclear safety and economics. These circumstances demand a reliable knowledge of local temperature distributions and heat transfer coefficients in the core, since net energy costs tend to become optimum at high coolant and cladding temperatures.

The heat-transfer process in the coolant channel of a rod bundle employing a fluid with a low Prandtl number is strongly influenced and, in some cases, dominated by molecular conduction in the coolant. Since geometry, through the conduction and fluid flow processes, will have a distinct effect on the temperature distribution, it is necessary to consider the exact, circumferentially asymmetric shape of the coolant channel for close-packed rod bundles. Of particular interest are the effects of circumferential variations of the temperature distribution and heat-transfer coefficient which become significant as the pitch-to-diameter ratio of the bundle is reduced. Such variations may give rise to large thermal stresses in the fuel pin and cause distortion or even failure of the cladding.

Since fluid flow in a fast reactor core is highly forced, the liquid metal coolant is in the turbulent flow regime. It is important to examine, therefore, the magnitude of turbulent heat transfer in the liquid metal compared to molecular heat transfer. This comparison will provide a definition of the flow regimes where turbulent heat transfer can be neglected, and an analysis by slug flow is appropriate. The slug flow model greatly simplifies the mathematics and computation involved.

1.2 METHOD OF SOLUTION

In previous analyses of the problem, questionable assumptions have been used regarding the surface heat flux distribution or the turbulence of the flow. Nijsing, Gargantini, and Eifler¹ used radially averaged sections; their assumptions on coolant properties were not valid for liquid metals. Magee and Tromel² employed a slug flow model. Dwyer³ assumed a uniform heat flux around the cladding periphery and derived a fluid velocity profile from correlations for other geometries. Buleev⁴ presented a very detailed analysis of the turbulent flow and heat-transfer mechanisms, but did not couple heat transfer in the fluid with that in the fuel, gap, and cladding.

The present analysis of the heat-transfer problem integrates the coupled equations in the fuel, gap, cladding, and coolant. The gap, cladding, and coolant employ a nodal model which is coupled to an approximate analytic solution in the fuel. The velocity field is calculated from the model of Ibragimov,⁵ which considers the effect of the irregular flow geometry on the turbulent structure of the flow. The eddy diffusivity of heat is assumed to be related to the eddy diffusivity of momentum through the correlation of Dwyer,⁶ and thus becomes a point function in the coolant channel and describes turbulent heat transfer in both the radial and circumferential directions.

The complete velocity and temperature solution has been programmed as the VELVET-II computer code. This code is a second generation version of the original VELVET⁷ work. The original VELVET model employed a velocity calculation based on a mixing length model, which subsequent study (Section 2) has shown to be inadequate for describing the complex turbulent structure of the flow in this geometry. Therefore, a new hydrodynamic model, described in Section 2, was incorporated into the calculation.

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1.3 SUMMARY AND CONCLUSIONS

The VELVET-II code was used in an analysis of heat transfer to liquid metal coolant flowing parallel to closely packed, unsupported rods on a triangular pitch. The analysis is for the turbulent flow regime and includes the effects of the velocity field and turbulent diffusivity.

The average heat-transfer results can be correlated by the equation

$$Nu = A(P/D) + B(P/D) [\Psi Pe]^{0.8} \quad (1)$$

where

$$A(P/D) = -126.5 + 193(P/D) - 66(P/D)^2 \quad (2)$$

$$B(P/D) = 0.0245 \quad \text{for } 1.2 \leq P/D \leq 1.5 \quad (3)$$

$$= 0.0118 (P/D)^4 \quad \text{for } 1.1 \leq P/D \leq 1.2 \quad (4)$$

This equation is valid for the ranges $1.1 \leq P/D \leq 1.5$ and $10^2 \leq Pe \leq 10^4$. The original results from which the equation is derived are shown in Section 4. The variation of $\Psi = \epsilon_H/\epsilon_M$ with P/D and Peclet number is shown in Section 4.

The cladding outside surface temperature distribution is described by the following equations:

at 15 kw/ft linear power

$$T_{wo}(\phi) - \bar{T}_{wo} = 2.28(P/D)^{-8.7} [21 - 0.010 Pe + 2.73(10^{-6})Pe^2] \cos 6\phi \quad (5)$$

for arbitrary power level

$$\frac{T_{wo}(\phi) - \bar{T}_{wo}}{(q''' R_F^2/k_c)} = \frac{T_{wo}(\phi) - \bar{T}_{wo}}{(q'_L/\pi k_c)} = 0.00526(P/D)^{-8.7} [21 - 0.010 Pe + 2.73(10^{-6})Pe^2] \cos 6\phi \quad (6)$$

which are valid for the ranges $1.1 \leq P/D \leq 1.5$ and $10^2 \leq Pe \leq 10^4$.

As implied by the above correlations the primary variables affecting rod-bundle heat transfer are the pitch-to-diameter ratio and the Peclet number.* The lower the P/D , the poorer the thermal performance. The reduction in heat transfer is particularly severe for P/D values of 1.15 or less. For each P/D , the Nusselt number has the "dog-leg" variation with Peclet number characteristic of essentially all liquid metal flows. At Peclet numbers below about 250, in the range of negligible thermal diffusion, the Nusselt number is constant. At higher Peclet numbers, the Nusselt number varies approximately as $Pe^{0.8}$. This increase in Nusselt number is the result of the growing importance of turbulent diffusion as the Peclet number is increased.

The fuel-to-cladding conductance was found to have negligible effect on circumferential temperature variations for fast reactor fuel bundles with P/D of 1.05 or greater. Therefore, it is concluded that no significant thermal variations occur in the cladding and coolant from fuel swelling before the time the swollen fuel begins to strain the cladding.

The results of this study are in good agreement with those of the published single-region (coolant only) analyses of Dwyer³ and Borishanski, et al.⁸ Although a boundary condition of circumferentially uniform surface heat flux was used in the latter studies to simplify the problem, this difference appears to be of little importance. The average heat transfer results of the present study fall between those of the cited references except at low P/D where the results in this report are about 10% higher. The calculated circumferential temperature distribution are also very similar.

*The Peclet number, frequently used in liquid metal heat-transfer correlations, is defined as the product of the Reynolds number times the Prandtl number. Thus, for a specific liquid metal at a fixed temperature, variations in Peclet number correspond directly to variations in Reynolds number.

Reasonable agreement between calculated and measured Nusselt numbers exists for open bundles ($P/D \geq 1.5$) at all Peclet numbers or for closely packed bundles with Peclet numbers above 300. However, a discrepancy, which increases with decreasing Peclet number, is observed at Peclet numbers for close packed bundles. This discrepancy is attributed to poor experimental data resulting from oxide contamination and, for mercury, surface non-wetting. The most recent and carefully designed experimental investigation, that of Pashek,⁹ agrees best with the theoretical predictions. Because of the experimental difficulties and the simplicity of the theoretical models, it is believed that the analytic correlations are as representative of true heat-transfer conditions at low P/D as are the experimental data now available.

The applicability of VELVET-II is limited since it is an analysis for unsupported rods in the interior of fuel bundles. The possible thermal effects of rod spacing devices, e.g., grids and wire wrap were not considered in this analysis, nor were the effects of unbalanced channel areas, as they occur at the bundle edge.

Future work in thermal analysis of fuel bundles should be concentrated on the effects not modelled in VELVET-II. The additional turbulence level induced by various grid designs must be defined for intensity and rate of decay with distance from the grid. The directional effects of wire wraps must be evaluated in terms of increased pressure drop, distorted velocity profiles, and reduced circumferential temperature distributions. In addition, the local cladding temperature anomaly at the wire wrap or grid contact must be analyzed.

2. FLOW DISTRIBUTION MODEL AND RESULTS

Determination of the velocity distribution in the interior flow channels of a fuel bundle is of prime importance in the thermal design of sodium-cooled fast breeder reactors. Without an understanding of the flow pattern, an accurate prediction of the heat-transfer performance and the temperature distribution is impossible.

In this section a summary of the velocity field calculations employed in the VELVET-II code is presented. This portion of the model provides local coolant velocities and momentum diffusivities which are used as inputs to the thermal model. In the following paragraphs, basic modeling assumptions are discussed, resultant velocity profiles are presented, and comparisons are made with available experimental data.

2.1 BASIC DEFINITIONS AND ASSUMPTIONS

To isolate the turbulent flow mechanisms from any complicating factors, such as edge and entrance effects, an infinite rod bundle is assumed in both the axial and radial directions (Figure 2-1a). In this configuration, each fuel rod has associated with it a unit cell for momentum transfer. By additional symmetry arguments, the fundamental unit of interest can be reduced to the triangular section of fuel rod and coolant shown in Figure 2-1b.

The following velocity field assumptions are made:

1. The flow is fully developed, single-phase, steady, and turbulent;
2. The coolant is incompressible;
3. No momentum flux occurs across the boundaries of the channel;
4. Temperature differentials in the coolant are not severe so that material properties may be evaluated at the bulk fluid temperature and considered constant across the flow cross section.

Since there is no momentum flux across the boundaries, and the flow is incompressible, the velocity components in the radial and circumferential directions are zero:

$$u_r = u_\phi = 0 \quad (7)^*$$

This statement implies, from continuity, that the velocity component in the axial direction (u_z) is constant.

$$u_z = u_z(r, \phi) \quad (8)$$

From the r and ϕ components of the momentum equation,

$$p = p(r, \phi) \quad (9)$$

Therefore, the axial pressure gradient, $(\partial p / \partial z)$, is a constant across the flow cross section.

The governing flow equation thus becomes the z component of the momentum equation,

$$\frac{\partial p}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} [r \tau_r] + \frac{1}{r} \frac{\partial \tau_\phi}{\partial \phi} \quad (10)$$

*See page 96 for identification nomenclature.

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Figure 2-1 Fuel Pin Geometry

The quantities τ_r and τ_ϕ are the radial and circumferential turbulent shear stresses, respectively. Also of usefulness are the eddy diffusivities of momentum in the radial and circumferential directions (ϵ_{Mr} and $\epsilon_{M\phi}$). These values are customarily defined in relationship to the shear stresses and velocity gradients in turbulent flow:

$$\tau_r = \frac{\rho_c}{g_0} (\epsilon_{Mr} + \nu_c) \frac{\partial u}{\partial r} \quad (11)$$

$$\tau_\phi = \frac{\rho_c}{g_0 r} (\epsilon_{M\phi} + \nu_c) \frac{\partial u}{\partial \phi} \quad (12)$$

2.2 ANALYSIS

2.2.1 THEORETICAL MODELS OF CIRCUMFERENTIAL MOMENTUM TRANSFER

Almost all investigations of turbulent momentum diffusion have dealt with either internal tube flow or flow over a flat plate. In either case, the flow patterns are two-dimensional. In such cases, consideration of transfer can be limited to that associated with the radial eddy diffusivity, ϵ_{Mr} , which defines the velocity distribution as a function of distance from the surface.

For geometries which do not have simple angular symmetry, however, circumferential momentum transfer becomes important. These geometries generate a complex turbulence structure. An example is flow in a triangular passage. Circumferential momentum transfer is also significant for closely packed rod bundles (say for P/D less than 1.35).

Several types of simplifying assumptions have been used to analyze flow in complex geometries. The earliest and simplest approach was to neglect any large-scale circumferential eddying and to assume equality of the eddy diffusivities of momentum (i.e., $\epsilon_{Mr} = \epsilon_{M\phi}$). This assumption was employed by Deissler and Taylor¹⁰ in 1956 to analyze flow parallel to rod bundle arrays. For a channel cell bounded by a portion of the wetted perimeter and by normals to the isotachs (equal-velocity lines), they wrote the force-balance equation in the form

$$\tau_w \Delta l = - \frac{dp}{dz} \Delta A \quad (13)$$

where Δl is the wetted perimeter length considered and ΔA is the flow area bounded by Δl and the normals to the isotachs. Since in this analysis a turbulence structure identical to that for tube flow was assumed, it is valid only for channels of slowly varying shape. Calculations based on this method deviate considerably from experimental data in channels of rapidly varying shapes. In particular, the wall shear stress variation is overpredicted because the smoothing effect of large circumferential turbulence is neglected.

A similar assumption was implicit in the recent rod bundle analysis of Dwyer.³ His flow field calculations were essentially identical to those of Deissler and Taylor.¹⁰

A more sophisticated approach is to maintain a difference between ϵ_{Mr} and $\epsilon_{M\phi}$ to account for the complex turbulence structure. However, very little information exists on the magnitude of circumferential eddy diffusivity.

Some experiments in circular tubes, notably those of Sanborn,¹¹ indicate that $\epsilon_{M\phi}$ is essentially independent of radial distance and is equal to the maximum value of the radial diffusivity. Following this lead, Rapier and Redman¹² assumed that the circumferential eddy diffusivity for rod bundle flow could be approximated as:

$$\epsilon_{M\phi}(\phi) = \frac{u^* \hat{y}}{10} \quad (14)$$

where $u^*(\phi) = \text{Local shear velocity, } \sqrt{g_0 \tau_w / \rho}$;
 $\hat{y}(\phi) = \text{normal distance from rod surface to line of maximum velocity};$
 $\tau_w(\phi) = \text{local wall shear stress.}$

This value of $\bar{\epsilon}_{M\phi}$ is equal to the maximum value of ϵ_{Mr} along the normal at the angle ϕ .

Elder¹³ measured the dispersion of marked fluid in turbulent flow in pipes and concluded that the average effective diffusivity for circumferential mass transfer could be expressed as:

$$\epsilon_{M\phi} = 0.0115 \nu_c Re^{7/8} \quad (15)$$

Nijssing, et al.,¹ analyzed the flow in a triangular array of parallel rods by an integral method which required an average value of $\epsilon_{M\phi}$ at each angular position. Based on experimental measurements of the peripheral velocity distribution, Nijssing concluded that Elder's equation applies locally if a local mean Reynolds number is used. Thus,

$$\epsilon_{M\phi} = 0.0115 \nu_c \left[\frac{\bar{u}_\phi}{\bar{u}} \frac{\Delta d_H}{d_H} Re \right]^{7/8} \quad (16)$$

where $\bar{u}_\phi$ = Local mean velocity along a normal to the clad at position ϕ ;
 $\bar{u}$ = Bulk mean velocity for total flow channel;
 Δd_H = Hydraulic diameter of local flow area section (strip normal to surface);
 d_H = Hydraulic diameter of total flow channel;
 Re = Reynolds number for total flow channel.

The maximum measured peripheral variation of the mean velocity at 1.15 P/D was only $\pm 10\%$. With a larger P/D ratio, the variation is even less and Nijssing's correlation reduces to that of Elder.

This assumption was also used by Cook¹⁴ in his recent analysis of velocity distributions in interior, edge, and corner channels of rod bundles. At 1.1 P/D and 10^5 Reynolds number, Cook predicted a maximum variation in $\bar{u}_\phi/\bar{u}$ from 0.87 to 1.08 or approximately $\pm 10\%$.

Buleev⁴ has recognized that the structure (i.e., size, energy and frequency distributions) of turbulent eddies in a flow channel is strongly affected by the channel geometry. He has developed highly sophisticated analytic methods to calculate the structure of turbulence. However, his formulations are very complex and the required calculations are laborious and time-consuming.

A simplified, yet accurate, approximation is presented by Ibragimov, et al.⁵ This semi-empirical analysis directly relates the channel shape to the wall shear stress profile, the velocity distribution and the over-all friction factor. Eddy diffusivities are not calculated.

The analysis of Ibragimov will be discussed in detail as it has been directly incorporated into this study and is the basis of the velocity field predictions.

2.2.2 SEMI-EMPERICAL ANALYSIS OF IBRAGIMOV

Ibragimov⁵ suggests that the total circumferential transfer of momentum is the sum of two processes: a gradient (or diffusion) transfer of momentum caused by molecular friction and small-scale turbulent eddies, and a convective transfer of momentum caused by the large-scale motion of eddies (i.e., secondary flow). Convective transfer becomes important for channels with sharply varying cross-sectional shapes where exchange between eddies of considerably different velocities may occur.

Whereas the diffusive transfer is determined by local flow characteristics, the convective transfer depends mainly on the over-all geometric features of the channel. In complex channels, the effect of large-scale eddies on the velocity field will be most significant in the direction in which the velocity varies slowly (i.e., circumferentially along the channel perimeter). Normal to the perimeter surface, however, convective transfer will be unimportant because the velocity gradient and the diffusive transfer are high.

On the basis of physical reasoning, Ibragimov postulates that the wall shear stress distribution is a function of the change in the shape of the channel and the relative dimensions of the channel:

$$\frac{\tau_w(\phi)}{\bar{\tau}_w} = f\left(\frac{\hat{y}(\phi)}{\hat{y}_{av}}, \frac{A_c}{\hat{y}_{av}^2}\right) \quad (17)$$

where $\hat{y}$ = Normal distance from surface to line of maximum velocity;
 $\hat{y}_{av}$ = Average normal distance;
 A_c = Flow channel area.

The form of the dependence is interred from theoretical considerations, while the coefficients are fixed from experimental shear stress data taken on three channels of widely different geometry (triangular channel with $P/D = 1.41$, right-angled isosceles triangle, and square):

$$\frac{\tau_w(\phi)}{\bar{\tau}_w} = C \left[1 - \exp\left(-\frac{7.7}{X^{0.8}} \frac{\hat{y}(\phi)}{\hat{y}_{av}}\right) \right] \quad (18)$$

where

$$X = \frac{A_c}{\hat{y}_{av}^2} \quad (19)$$

and

$$\frac{1}{C} = \frac{1}{\phi_{\max}} \int_0^{\phi_{\max}} \left[1 - \exp\left(-\frac{7.7}{X^{0.8}} \frac{\hat{y}(\phi)}{\hat{y}_{av}}\right) \right] d\phi \quad (20)$$

In a companion paper¹⁵, Ibragimov derives an expression for the channel friction factor compared to that of a circular tube with the same hydraulic diameter:

$$\frac{f}{f_{ct}} = [0.58 + 0.42 \exp(-0.021 K^3)] \left[1 + 0.1 \left(\frac{\hat{y}_{av}}{R_c} + 1 \right)^{4/3} \right] \quad (21)$$

where

$$K = X^{0.25} \left(\frac{\hat{y}_{\max} - \hat{y}_{\min}}{\hat{y}_{av}} \right); \quad (22)$$

and

R_c = Radius of curvature of surface.

Knowledge of the wall shear stress distribution permits the entire velocity field to be defined provided some sort of a universal turbulent profile can be assumed. The obvious possibility is the Von Karman profile derived from circular tube data:

$$u^+ = y^+ \quad (y^+ \leq 5) \quad (23)$$

$$u^+ = -3.5 + 5.0 \ln y^+ \quad (5 < y^+ \leq 30) \quad (24)$$

$$u^+ = 5.5 + 2.5 \ln y^+ \quad (y^+ > 30) \quad (25)$$

Fortunately, experimental measurements of velocity profiles in rod bundles are consistent with these equations. Levchenko, et al.,¹⁶ carefully measured velocities in a tricuspid channel formed by close-packed rods ($P/D = 1.0$) and found excellent agreement with the Von Karman profile along all normals checked ($\phi = 7.5$ to 30°). Nijsing, et al.,¹ obtained velocity measurements in channels with P/D from 1.05 to 1.15 and concluded that "the velocity distribution normal to the rod surface obeys, at each peripheral position, the same laws as those valid for round tubes," although they did not show a direct comparison to the Von Karman profile.

2.3 COMPARISON OF RESULTS WITH EXPERIMENTAL DATA

Velocity field predictions have been compared to the experimental data of Nijsing, et al.,¹ Levchenko,¹⁶ and Hunsbedt, et al.,¹⁷

The experimental test area of Nijsing, et al., is reproduced in Figure 2-2. Since the basic 30° unit of symmetry is multiply present, like angles are discriminated by a value of n . When compared to an infinite, spacerless array, the effect of spacers will be to redistribute the flow toward the center of the test area. Peak effect should be at the n values 5, 15, 24, and 34 corresponding to $\phi = 30^\circ$.

Experimental datum points for P/D ratios of 1.05, 1.10, and 1.15 (all at a Reynolds number of approximately 30,000) are shown with predictions of this study in Figures 2-3, 2-4, and 2-5. In each case, the agreement is excellent. Even the 30° peaking of the environmental data is noticeable.

Levchenko, et al.,¹⁶ investigated flow in a closed channel simulating a channel of a tight-packed ($P/D = 1.0$) bundle of rods. The local velocity field was carefully measured, and from the velocity profiles near the wall, the wall shear stress distribution was calculated.

These results were used by Ibragimov as checks on his analysis. Results of this study are identical to those of Ibragimov. The calculated friction factor ratio (f/f_{cd}) is 0.66 compared to the experimental 0.64. For comparison, Deissler and Taylor¹⁰ predict 0.56 for this geometry. The agreement of the calculated and experimental wall shear stress distributions is shown in Figure 2-6. The more exaggerated distribution predicted by Deissler and Taylor is also shown.

Comparisons have also been made with the velocity profiles measured at General Electric under P.A. 10, Task F* and reported by Hunsbedt, et al.¹⁷ In general, the agreement is not so good as with the data of Nijsing or Levchenko but appears attributable to experimental uncertainties.

Results for the relatively wide spacing of 1.3 P/D are shown in Figures 2-7 and 2-8. Measurements were made at the cusp position (0°) and at the midchannel location (30°). The experimental and calculated radial profiles have very similar shapes but are displaced relative to each other. This displacement is probably due to uncertainties in determining the total flow in the test channel. The average velocity was calculated by a numerical integration of the local velocities over the channel cross section. Thus, the average flow in the test channel could be uncertain within several per cent. Best agreement with the calculated profiles is achieved by decreasing the experimental average velocity 2.5% and thus raising the data 2.5% (or, alternately, by reducing the calculated profiles 2.5%).

Somewhat larger discrepancies exist at 1.1 P/D as shown in Figure 2.9. Velocity measurements were made in two opposing channels (180° apart across a rod). One channel was of 1.1 P/D while the other was somewhat oversized at 1.108 P/D . The large difference in the skewness of the velocity profiles measured in the two channels is an experimental anomaly and cannot be explained by the insignificant difference in channel P/D . However, it will be noted that the analytic predictions, when corrected for a 3.5% discrepancy in average flow, are in good agreement with the averages of the data from the two channels.

It can be concluded that the calculational method gives velocity distributions which are not in disagreement with the test data, and the mismatch appears to be principally the result of experimental uncertainties. This conclusion is reinforced by the excellent agreement of the calculations with the more accurate measurements of Nijsing and Levchenko.

In summary, it appears that the most adequate method of predicting velocity distributions for turbulent flow in rod bundles is to calculate the average friction factor and wall shear stress variation by the method of Ibragimov. The velocity field is then constructed by assuming a universal profile along each normal to the surface. The method has the additional computational advantage of being noniterative and, in fact, is simple enough for hand calculation.

*U.S.AEC Contract A1(04-3)-189

Figure 2-2 Geometrical Position of the Radial Velocity Profiles Measured by Eifler and Nijsing

GEAP-10052

Figure 2-3 Velocity Correlation at $P/D = 1.05$

Figure 2-4 Velocity Correlation at $P/D = 1.10$

GEAR-10052

Figure 2-5 Velocity Correlation at $P/D = 1.15$

Figure 2-6 Predicted Wall Shear Stress Distributions for $P/D=1.0$

Figure 2-7 Comparison with GE Velocity Data $P/D = 1.3$ at 30°

Figure 2-8 Comparison with GE Velocity Data $P/D = 1.3$ at 0°

Figure 2-9 Comparison with GE Velocity Data at P/D = 1.1

2.4 EVALUATION OF MOMENTUM DIFFUSIVITY TERMS

With Ibragimov's method of calculation, the velocity field is determined without explicit reference to the eddy diffusivities of momentum. However, these terms are required to calculate the thermal diffusivities and the temperature field.

The radial momentum diffusivities can be calculated from the radial shear stress distribution and the universal velocity profile. Consider a flow area strip of differential width taken normal to the rod surface (Figure 2-10). The outer radius of the strip is $r_0/\cos\phi$ and the inner radius is r_w . The radial shear stress variation is defined by continuity of momentum:

$$\frac{\partial p}{\partial z} = \frac{(\text{Perimeter}) \tau_w}{(\text{Flow Area})} = \frac{2r_w \Delta\phi \tau_w}{\Delta\phi (r_0)^2 - \Delta\phi r_w^2} = \frac{2r \Delta\phi \tau_r}{\Delta\phi (r_0)^2 - \Delta\phi r^2} \quad (26)$$

or

$$\tau_r = \tau_w \frac{r_w}{r} \left[\frac{r_0^2 - r^2 \cos^2\phi}{r_0^2 - r_w^2 \cos^2\phi} \right] \quad (27)$$

Figure 2-10 Differential Flow Area for Shear Stress Calculation

This distribution is very similar to the radial shear stress variation within a circular tube in that it specifies an almost linear decrease in shear stress varying from τ_w at the rod surface to zero at the line of symmetry of the flow channel (which corresponds to the circular tube centerline).

The radial momentum diffusivities can then be evaluated from Equation (11) and the Von Karman universal velocity profile. Thus,

$$\epsilon_{Mr} = \frac{u^* r_w (r - r_w)}{2.5} \left[\frac{r_0^2 - r^2 \cos^2 \phi}{r_0^2 - r_w^2 \cos^2 \phi} \right] - \nu_c \quad \text{for } y^+ > 30 \quad (28)$$

$$\epsilon_{Mr} = \frac{u^* r_w (r - r_w)}{5.0} \left[\frac{r_0^2 - r^2 \cos^2 \phi}{r_0^2 - r_w^2 \cos^2 \phi} \right] - \nu_c \quad \text{for } 5 < y^+ \leq 30 \quad (29)$$

and

$$\epsilon_{Mr} = 0 \quad \text{for } y^+ \leq 5 \quad (30)$$

The circumferential momentum diffusivity is much more difficult to define. It would appear that, with ϵ_{Mr} and τ_w defined, there is sufficient information to calculate directly the circumferential shear stress and the circumferential diffusivity. Indeed, the circumferential shear stress can be determined, but the proper relationship between it and the circumferential diffusivity is unclear.

Substitution of the radial shear stress expression into the basic momentum equation gives:

$$\frac{\partial \tau_\phi}{\partial \phi} = r \left[\frac{\partial p}{\partial z} + \frac{2 \tau_w r_w}{\left(\frac{r_0}{\cos \phi} \right)^2 - r_w^2} \right] \quad (31)$$

which can be directly integrated with respect to ϕ to yield the circumferential shear stress. The appropriate boundary conditions are that τ_ϕ equal zero at ϕ equal to 0 and 30° for all values of r .

Calculated circumferential shear stress distributions for P/D of 1.1, 1.2, and 1.3 at 10^5 Reynolds numbers are given in Figures 2-11, 2-12, and 2-13, respectively. The radial shear stress distributions at $\phi = 0$ and 30° are shown for comparison in each case. As would be expected, the relative (and absolute) magnitude of the circumferential shear stress decreases as the bundle spacing is increased.

Figure 2-11 Shear Stress Distribution for $P/D = 1.1$

Figure 2-12 Shear Stress Distribution for $P/D = 1.2$

Figure 2-13 Shear Stress Distribution for $P/D = 1.3$

Calculation of the circumferential momentum diffusivity requires an equation relating the diffusivity to the shear stress. The usual equation is:

$$\tau_\phi = \frac{\rho_c}{g_0 r} (\epsilon_{M\phi} + \nu_c) \frac{\partial u}{\partial \phi} \quad (12)$$

However, this equation is not adequate for flow channels with complex shapes. As suggested by Ibragimov, the total circumferential transfer of momentum in these cases is the sum of a diffusive transfer and a convective transfer (secondary flow). Ibragimov postulates that the correct form of the above equation is, therefore,

$$\tau_\phi = \frac{\rho_c}{g_0 r} (\epsilon_{M\phi} + \nu_c) \frac{\partial u}{\partial \phi} + \frac{\rho_c}{g_0} \overline{u'V'_\phi} \quad (32)$$

where V'_ϕ is the pulsation velocity of large eddies in the ϕ direction. Equation 32 implies that $\epsilon_{M\phi}$ cannot be determined directly from τ_ϕ .

Calculations were made of an "effective" circumferential diffusivity from the equation

$$\epsilon_{M\phi, \text{eff}} = \frac{r g_0 \tau_\phi}{\rho_c (\partial u / \partial \phi)} - \nu_c \quad (33)$$

A summary of the results is presented in Figure 2-14. At a given Reynolds number, the circumferential diffusivity increases with increasing P/D . This variation is in contrast to the radial diffusivity which decreases with increasing P/D . The increase is due to the very small circumferential velocity derivative at large P/D .

Because the large-scale eddying motion will have relatively little effect on heat-transfer behavior of the high-conductivity liquid metal coolant, the common assumption of equal radial and circumferential diffusivities at a point is used in the thermal calculations. That is,

$$\epsilon_{M\phi}(r, \phi) = \epsilon_{Mr}(r, \phi) \quad (34)$$

and, correspondingly,

$$\epsilon_{H\phi}(r, \phi) = \epsilon_{Hr}(r, \phi) \quad (35)$$

In this assumption the possible effect of secondary (convective) flow on the temperature field is ignored. While this effect will be significantly less than that for the velocity calculations, its neglect implies that circumferential temperature gradients will be somewhat overestimated.

Further discussion of this effect and a comparison of Nusselt numbers and temperature profiles calculated from alternate circumferential diffusivity models are presented in Appendix A.

Figure 2-14. Effective Circumferential Momentum Diffusivity for P/D = 1.1 and 1.3

3. TEMPERATURE DISTRIBUTION MODEL

The thermal analysis separates the basic repeating unit of the fuel bundle into two parts: (1) fuel material and (2) gas gap, cladding, and coolant channel. A Fourier series representation of temperatures is used for the fuel; the fuel surface heat flux is specified as a boundary condition and the corresponding temperature distribution is calculated. The gap, cladding, and coolant are simulated in a nodal model. The computer analysis is iterated between the two simulations until the fuel surface temperature and heat flux distributions are matched.

3.1 ASSUMPTIONS AND GOVERNING ENERGY EQUATIONS

The following basic assumptions are employed in the thermal model.

1. Fully developed turbulent flow and heat transfer in the steady state.
2. No heat flux across the boundaries of the geometric model.
3. No axial conduction in any of the materials.
4. No circumferential heat transfer in the gap.
5. The fuel is operating under steady-state conditions with a uniform radial and axial power density.
6. A fraction of the total power is caused by gamma heating in the fuel, cladding, and coolant.

Note that the model has retained material properties as a function of temperature and will include a velocity profile in the coolant.

The temperature distribution in the fuel and cladding is described by Fourier's heat conduction equation for steady state in two dimensions.

$$\vec{\nabla} \cdot (k \vec{\nabla} T) + \dot{q}''' = 0 \quad (36)$$

In Equation (30), $\dot{q}'''$ is the volumetric heat generation rate, which, in the fuel and cladding, respectively, is

$$\dot{q}'''_F = \frac{3414}{A_F} \dot{q}'_L \left[1 - \gamma_H \left(1 - \frac{\rho_F A_F}{m_T} \right) \right] \quad (37a)$$

$$\dot{q}'''_w = \frac{3414}{A_F} \dot{q}'_L \gamma_H \frac{(\rho_w A_w)}{m_T} \quad (37b)$$

where γ_H is the fraction of the power that is due to gamma heating. Note that it is mass weighted between the fuel, cladding, and coolant. $\dot{q}'_L$ is the linear heat generation rate for the fuel pin in kilowatts per foot.

The temperature drop across the gap is determined by the gap heat-transfer coefficient, h_G , and the surface heat flux of the fuel.

$$(\Delta T)_{GAP} = \frac{\dot{q}''_{FS}(\phi)}{h_G} \quad (38)$$

The coolant temperature distribution is described by the energy equation for steady, two-dimensional turbulent flow.

$$u \frac{\partial T}{\partial z} = \frac{\dot{q}'''_c}{\rho_c C_{pc}} + (\alpha_c + \epsilon_{Hr}) \frac{\partial^2 T}{\partial r^2} + \frac{(\alpha_c + \epsilon_{Hr})}{r} \frac{\partial T}{\partial r} + \frac{(\alpha_c + \epsilon_{Hr})}{r^2} \frac{\partial^2 T}{\partial \phi^2} \quad (39)$$

The functions ϵ_{Hr} and $\epsilon_{H\phi}$ are the eddy diffusivities of heat in the radial and circumferential directions, respectively. They are defined by the heat fluxes and temperature gradients in the fluid.

$$\dot{q}''_r = \rho_c C_{pc} (\alpha_c + \epsilon_{Hr}) \frac{\partial T}{\partial r} \quad (40a)$$

$$\dot{q}''_{\phi} = \rho_c C_{pc} (\alpha_c + \epsilon_{H\phi}) \frac{1}{r} \frac{\partial T}{\partial \phi} \quad (40b)$$

The heat generation in the fluid is due to gamma heating and is given as

$$\dot{q}'''_c = \frac{3414}{A_F} \dot{q}'_L \gamma_H \frac{\rho_c A_c}{m_T} \quad (40c)$$

For fully developed, heat-transfer conditions and uniform heat flux per unit length, the axial temperature gradient is constant [see, for example, Kays¹⁸]. Therefore, a heat balance on the channel cross section yields

Total Heat Generated = Heat Carried Away by Coolant

$$\begin{aligned} \frac{3414}{12} \dot{q}'_L &= \rho_c C_{pc} \bar{u} \frac{\partial T}{\partial z} A_c \\ \frac{\partial T}{\partial z} &= \frac{3414 \dot{q}'_L}{12 \rho_c C_{pc} \bar{u} A_c} \end{aligned} \quad (41)$$

These differential equations are coupled through the conditions of equal temperatures and heat fluxes at the material interfaces, and are subject to the following boundary conditions:

$$T(R_F, \pi/12) = T_{FS} \quad (42a)$$

Fuel surface temperature at $\phi = 15^\circ$ is a specified value*

$$\left. \frac{\partial T}{\partial \phi} \right|_{\phi=0} = 0 \quad (42b)$$

$$\left. \frac{\partial T}{\partial \phi} \right|_{\phi=\pi/6} = 0 \quad (42c)$$

$$\left[\cos \phi \frac{\partial T}{\partial r} - \frac{1}{r} \sin \phi \frac{\partial T}{\partial \phi} \right]_{r=R_0/\cos \phi} = 0 \quad (42d)$$

*The problem has been formulated as a Neuman problem, i.e., as a boundary value problem with the value of the derivatives specified on the boundary. Therefore, to locate these gradients on the temperature scale, a temperature must be specified at some point in the domain of the solution.

The actual solution incorporated in this analysis does not solve these equations in this form, but an examination of the governing differential equations has indicated the important heat-transfer processes that must be included in the working formulation of the problem.

3.2 FUEL TEMPERATURE

The Fourier series representation of the fuel temperature distribution employed in VELVET-II will be developed in this section. The series solution is that for a circular region with the surface gradient (heat flux) distribution specified as a boundary condition. The major complication in the analysis is the strong temperature dependence of the fuel conductivity.

Expanding Equation (36) for the fuel in (r, ϕ) geometry, and assuming the conductivity is temperature dependent,

$$k_F \frac{\partial^2 T}{\partial r^2} + \frac{k_F}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial k_F}{\partial r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial k_F}{\partial \phi} \frac{\partial T}{\partial \phi} + \dot{q}'''_F = 0 \quad (43)$$

Now examining the terms that represent the temperature dependence of the conductivity,

$$\frac{\partial k_F}{\partial r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial k_F}{\partial \phi} \frac{\partial T}{\partial \phi} = \frac{\partial k_F}{\partial T} \left[\left(\frac{\partial T}{\partial r} \right)^2 + \left(\frac{1}{r} \frac{\partial T}{\partial \phi} \right)^2 \right]$$

Since the peripheral heat flux in the fuel is much less than the radial heat flux,

$$\frac{\partial k_F}{\partial r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial k_F}{\partial \phi} \frac{\partial T}{\partial \phi} \approx \frac{\partial k_F}{\partial r} \frac{\partial T}{\partial r} \quad (44)$$

Thus, Equation (43) becomes

$$k_F(r) \nabla^2 T + \frac{\partial k_F}{\partial r} \frac{\partial T}{\partial r} + \dot{q}'''_F = 0 \quad (45)$$

Since Equation (44) represents a second order contribution to Equation (45), this term is approximated by a known function.

By using a model of no angular heat flux and constant conductivity, the equation for the first approximation to the temperature is

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T_o}{\partial r} \right) + \frac{\dot{q}'''_F}{k_{Fo}} = 0$$

whose solution is

$$T_o(r) = A_o + B_o \ln(r) - \frac{\dot{q}'''_F r^2}{4 k_{Fo}} \quad (46)$$

where

$$A_o = T_{FS} + \frac{\dot{q}'''_F R_F^2}{2 k_{Fo}} - \frac{\dot{q}'''_F R_v^2}{2 k_{Fo}} \ln(R_F) \quad (47a)$$

and

$$B_o = \frac{\dot{q}'''_F R_v^2}{2 k_{Fo}} \quad (47b)$$

It is assumed the conductivity can be approximated with a quadratic in temperature of the form

$$k_F(T) = a_0 + a_1 T + a_2 T^2 \quad (48)$$

and by approximating the temperature in Equation (48) with Equation (46), the conductivity as a function of position becomes

$$k_F(r) \cong b_1 + b_2 r^2 + b_3 r^4 + b_4 \ln(r) + b_5 [\ln(r)]^2 + b_6 r^2 \ln(r) \quad (49)$$

where

$$b_1 = a_0 + a_1 A_0 + a_2 A_0^2 \quad (50a)$$

$$b_2 = - \left[\frac{a_1 \dot{q}'''_F}{4 k_{Fo}} + \frac{A_0 \dot{q}'''_F a_2}{2 k_{Fo}} \right] \quad (50b)$$

$$b_3 = \frac{\dot{q}'''_F^2 a_2}{16 k_{Fo}^2} \quad (50c)$$

$$b_4 = a_1 B_0 + 2 a_2 A_0 B_0 \quad (50d)$$

$$b_5 = a_2 B_0^2 \quad (50e)$$

$$b_6 = - \frac{a_2 B_0 \dot{q}'''_F}{2 k_{Fo}} \quad (50f)$$

The nonlinear term in Equation (45) is now a known function of r .

$$\frac{\partial k_F}{\partial r} \frac{\partial T}{\partial r} \cong \frac{\partial}{\partial r} \left[k_F(T_0) \right] \frac{\partial T_0(r)}{\partial r} = k_F^*(r) \quad (51)$$

Equation (45) is now written as

$$k_F(r) \nabla^2 T = - \dot{q}'''_F - k_F^*(r) \quad (52)$$

The solution to this equation is a homogeneous solution plus a particular integral

$$T(r, \phi) = T_H(r, \phi) + T_p(r, \phi) \quad (53)$$

The homogeneous equation is

$$k_F(r) \nabla^2 T = 0$$

Since $k_F(r)$ is nonzero over the range of interest,

$$\nabla^2 T = 0$$

This is the Laplace equation whose solution in two dimensions in cylindrical geometry is¹⁷

$$T(r, \phi) = \hat{A}_0 + \hat{B}_0 \ln(r) + \sum_{n=1}^{\infty} \left\{ \left[\hat{A}_n r^n + \hat{B}_n r^{-n} \right] \left[\hat{C}_n \sin(n\phi) + \hat{D}_n \cos(n\phi) \right] \right\} \quad (54)$$

It is now necessary to find a particular integral of

$$k_F(r) \nabla^2 T = - \dot{q}'''_F - k_F^*(r)$$

Since the nonhomogeneous term is only a function of r , assume

$$T = T(r)$$

Thus

$$k_F(r) \frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) = -\dot{q}'''_F - k_F^*(r) \quad (55)$$

To facilitate the integration, approximate $k_F^*(r)$ with Equation (49), therefore Equation (55) becomes

$$k_F(r) \frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) \approx -\dot{q}'''_F - \frac{dk_F}{dr} \frac{dT}{dr}$$

or

$$\frac{1}{r} \frac{d}{dr} \left(r k_F \frac{dT}{dr} \right) \approx -\dot{q}'''_F \quad (56)$$

By integrating Equation (56) directly,

$$r k_F(r) \frac{dT}{dr} = -\frac{\dot{q}'''_F r^2}{2} + C_1$$

and performing the second integration, from R_F to r ,

$$dT = -\frac{\dot{q}'''_F r dr}{2k_F(r)} + C_1 \frac{dr}{r k_F(r)}$$

$$\int_{T_{FS}}^T d\xi = -\frac{\dot{q}'''_F}{2} \int_{R_F}^r \frac{\xi d\xi}{k_F(\xi)} + C_1 \int_{R_F}^r \frac{d\xi}{\xi k_F(\xi)}$$

$$T - T_{FS} = -\frac{\dot{q}'''_F}{2} I_{p1}(r) + C_1 I_{p2}(r)$$

where

$$I_{p1}(r) = \int_{R_F}^r \frac{\xi d\xi}{k_F(\xi)} \quad (57a)$$

and

$$I_{p2}(r) = \int_{R_F}^r \frac{d\xi}{\xi k_F(\xi)} \quad (57b)$$

so that the particular solution is

$$T(r) = T_{FS} - \frac{\dot{q}'''_F}{2} I_{p1}(r) + C_1 I_{p2}(r) \quad (58)$$

After substituting Equations (54) and (58) into Equation (53), the complete solution to Equation (43) is

$$T(r, \phi) = \hat{A}_0 + \hat{B}_0 \ln(r) + T_{FS} - \frac{\dot{q}'''_F}{2} I_{p1}(r) + C_1 I_{p2}(r) + \sum_{n=1}^{\infty} \left\{ \left[\hat{A}_n r^n + \hat{B}_n r^{-n} \right] \left[\hat{C}_n \sin(n\phi) + \hat{D}_n \cos(n\phi) \right] \right\} \quad (59)$$

Equation (59) is subject to the boundary conditions given by Equations (42a) to (42c) and

$$\left. \frac{\partial T}{\partial r} \right|_{r=R_v} = 0 \quad (60)$$

No heat flux at the central void surface

$$-k_F(R_F) \left. \frac{\partial T}{\partial r} \right|_{r=R_F} = \dot{q}''_{FS}(\phi) \quad (61)$$

The fuel surface is subject to a heat flux which will be determined when the fuel solution is coupled to that in the gap, cladding, and coolant.

It now remains to use the boundary conditions to determine the constants* in the solution, Equation (59).

Use of the boundary conditions at the circumferential extremities, Equations (42b) and (42c), yields

$$\hat{C}_n = 0 \quad (62)$$

$$n = 1, 6, 12, 18 \dots \quad (63)$$

Then equation (59) can be written as

$$T(r, \phi) = \hat{A}_0 + \hat{B}_0 \ln r + T_{FS} - \frac{\dot{q}'''_F}{2} I_{p1}(r) + C_1 I_{p2}(r) + \sum_{n=1}^{\infty} \left\{ \left[\hat{A}_{6n} r^{6n} + \hat{B}_{6n} r^{-6n} \right] \cos(6n\phi) \right\} \quad (64)$$

From Equations (42a), (60) and (61), A_0, B_0 and C_1 are determined.

$$\hat{A}_0 = -\hat{B}_0 \ln R_F \quad (65)$$

$$\hat{B}_0 = \frac{\dot{q}'''_F R_v^2}{2 k_F(R_v)} - \frac{C_1}{k_F(R_v)} \quad (66)$$

$$C_1 = \frac{\dot{q}'''_F R_F^2 k_F(R_v) - 2 \dot{q}_{FS} R_F k_F(R_v) - \dot{q}'''_F R_v^2 k_F(R_F)}{2 [k_F(R_F) + k_F(R_v)]} \quad (67)$$

where $\dot{q}''_{FS}$ = average fuel surface heat flux

*The method of applying the boundary conditions and solving for the constants is illustrated in detail in Appendix B.

Also by using Equations (60) and (61) and employing the orthogonality of the cosine functions, the coefficients A_{6n} and B_{6n} are:

$$\hat{A}_{6n} = \frac{-2R_F^{-6n+1}}{n\pi k_F(R_F) \left[1 - \left(\frac{R_v}{R_F}\right)^{12n}\right]} \int_0^{\pi/6} \dot{q}''_{FS}(\phi) \cos(6n\phi) d\phi \quad (68)$$

and

$$\hat{B}_{6n} = \hat{A}_{6n} R_v^{12n} \quad (69)$$

The solution is now complete for the constants in Equation (59). The final form of the fuel temperature solution employed in VELVET-II is

$$T(r, \phi) = A_0^* + B_0^* \ln\left(\frac{r}{R_F}\right) - \frac{\dot{q}'''_F}{2} I_{p1}(r) + C_1 I_{p2}(r) + \sum_{n=1}^{\infty} \left\{ \left[A_{6n}^* \left(\frac{r}{R_F}\right)^{6n} + B_{6n}^* \left(\frac{R_v}{r}\right)^{6n} \right] \cos(6n\phi) \right\} \quad (70)$$

where

$$A_{6n}^* = \frac{-2R_F}{n\pi k_F(R_F) \left[1 - \left(\frac{R_v}{R_F}\right)^{12n}\right]} \int_0^{\pi/6} \dot{q}''_{FS}(\phi) \cos(6n\phi) d\phi \quad (71a)$$

$$B_{6n}^* = A_{6n}^* \left(\frac{R_v}{R_F}\right)^{6n} \quad (71b)$$

$$B_0^* = \frac{\dot{q}'''_F R_v^2}{2k_F(R_v)} - \frac{C_1}{k_F(R_v)} \quad (72a)$$

$$A_0^* = T_{FS} - \sum_{n=1}^{\infty} \left\{ \left[A_{6n}^* + B_{6n}^* \left(\frac{R_v}{R_F}\right)^{6n} \right] \cos \frac{n\pi}{2} \right\} \quad (72b)$$

$$C_1 = \text{See Equation (67)}$$

3.3 GAP, CLADDING, AND COOLANT TEMPERATURE

The differential equations for heat transfer in the gap, cladding, and coolant are represented in nodal form within VELVET-II. The required finite difference equations are developed in this section.

For describing heat transfer in the radial direction, consider two radially adjacent nodes as shown in Figure 3-1.

Figure 3-1 Adjacent Radial Nodes

The radial heat flow between nodal temperatures T_1 and T_2 is

$$\dot{q}'_R = Y_{12} (T_1 - T_2) \quad (73)$$

where the nodal admittance, Y_{12} , is

$$Y_{12} = \frac{\Delta\phi}{\frac{\ln(\hat{r}/r_1)}{k'_{R1}} + \frac{\ln(r_2/\hat{r})}{k'_{R2}}} \quad (74)$$

and the effective radial conductivity for the n -th node, k'_{Rn} , is

$$k'_{Rn} = k_c(T_n) + \rho_c C_{pc} \epsilon_{Hr} \text{ in the coolant} \quad (75a)$$

$$k'_{Rn} = k_w(T_n) \text{ in the cladding} \quad (75b)$$

Now examining heat transfer in the circumferential direction, consider two circumferentially adjacent nodes as shown in Figure 3-2.

The circumferential heat flow between nodal temperature T_1 and T_2 is

$$\dot{q}'_\phi = Y_{12} (T_1 - T_2)$$

where the nodal admittance, Y_{12} , is

$$Y_{12} = \frac{\Delta r / r}{\frac{\Delta\phi_1}{k'_{\phi 1}} + \frac{\Delta\phi_2}{k'_{\phi 2}}} \quad (76)$$

and the effective circumferential conductivity is

$$k'_{\phi n} = k_c(T_n) + \rho_c C_{pc} \epsilon_{H\phi} \text{ in the coolant} \quad (77a)$$

$$k'_{\phi n} = k_w(T_n) \text{ in the cladding} \quad (77b)$$

Figure 3-2 Adjacent Circumferential Nodes

The equations governing the temperature distribution in the gap, cladding and coolant are determined by considering a heat balance on a node and its adjacent nodes, as illustrated in Figure 3-3.

Figure 3-3 Nodal Heat Balance

By taking a heat balance on the central node at (r_k, ϕ_j) then heat conducted into node - heat conducted out of node = heat carried away by convection - heat generated in the node.

In terms of the heat-transfer equations, this physical relationship becomes

$$Y_1(T_{k-1,j} - T_{k,j}) + Y_3(T_{k+1,j} - T_{k,j}) + Y_4(T_{k,j-1} - T_{k,j}) + Y_2(T_{k,j+1} - T_{k,j}) = (\rho_{k,j} C_{pk,j} u_{k,j} \frac{\partial T}{\partial z} - \dot{q}'''_{k,j}) A_{k,j} \quad (78)$$

The convection term is zero in the cladding.

The boundary conditions must also be formulated as finite difference equations to correspond to the form of the equations for the nodal temperatures (Equation 78).

Consider a node whose surface constitutes part of the boundary at $\phi = 0$, as illustrated in Figure 3-4.

Figure 3-4 Boundary Node at $\phi = 0$

Use of a first order forward difference for Equation (42b) yields

$$\left. \frac{\partial T}{\partial \phi} \right|_{\phi=0} = 0 = \frac{T_0 - T_1}{(\Delta \phi / 2)}$$

Thus,

$$T_0 = T_1 \quad (79)$$

Similarly, for a boundary node, T_N at $\phi = 30^\circ$, the form of Equation (42c) becomes

$$T_N = T_{N-1} \quad (80)$$

The relationships (79) and (80) are used in the heat balance, Equation (78), to reduce the boundary heat flux to zero. The values for T_0 and T_N are not explicitly calculated until the temperature distribution in the interior nodes has been obtained.

The final difference equation needed for equation (42d) is for a boundary node at $r = R_0/\cos\phi$, as shown in Figure 3-5.

Figure 3-5 Boundary Node at $r = R_0/\cos\phi$

By using first-order differences, Equation (42d) takes the form

$$\cos\phi_j \frac{T_1 - T_2}{\Delta r} - \frac{\sin\phi_j}{r_k} \frac{T_3 - T_2}{\Delta\phi} = 0$$

$$T_1 = T_2 - \frac{\Delta r}{r_k \Delta\phi} \frac{\sin\phi_j}{\cos\phi_j} (T_3 - T_2) \quad (81)$$

3.4 EDDY DIFFUSIVITY OF HEAT

It is assumed that the heat transfer by turbulent mixing is proportional to the momentum transfer by turbulent mixing.

$$\epsilon_H \propto \epsilon_M$$

or

$$\epsilon_H = \Psi \epsilon_M$$

With the correlation of Dwyer⁶, the constant of proportionality in the radial direction is

$$\Psi_r = 1 - \frac{1.82}{\text{Pr} \left(\frac{\epsilon_{Mr}}{\nu_c} \right)_{\max}^{1.4}} \quad (82)$$

In a similar analogy, the constant of proportionality for the circumferential turbulent heat transfer is

$$\Psi_\phi = 1 - \frac{1.82}{Pr \left(\frac{\epsilon_{M\phi}}{\nu_c} \right)_{\max}^{1.4}} \quad (83)$$

Thus, the equations for radial and circumferential eddy diffusivity of heat are

$$\epsilon_{Hr} = \Psi_r \epsilon_{Mr} \quad (84a)$$

$$\epsilon_{H\phi} = \Psi_\phi \epsilon_{M\phi} \quad (84b)$$

The values of momentum diffusivity come from Equations (28), (29), (30), and (34) which determine velocity and momentum diffusivity fields as functions of r and ϕ in the channel of a rod bundle. Thus, since ϵ_{Mr} and $\epsilon_{M\phi}$ are point functions in the coolant channel, the radial and circumferential diffusivities of heat are local effects throughout the flow cross section.

3.5 HEAT-TRANSFER CALCULATIONS

When the temperature distribution has been determined, it is of interest to calculate the convective heat-transfer coefficients for the coolant.

The net heat-transfer coefficient is defined as

$$\bar{h}_c = \frac{\dot{q}''_{wo}}{T_{wo} - T_{CB}} \quad (85)$$

and the local heat-transfer coefficient is

$$h_c(\phi) = \frac{\dot{q}''_{wo}(\phi)}{T_{wo}(\phi) - T_{CB}} \quad (86)$$

The average outside wall temperature is

$$T_{wo} = \frac{\int_0^{\pi/6} T_{wo}(\phi) d\phi}{\pi/6}$$

or, approximately,

$$T_{wo} = \frac{\sum_{j=1}^{M-1} \left[\frac{T_{wo,j+1} + T_{wo,j}}{2} \right] \Delta\phi_j}{\pi/6} \quad (87)$$

where M is the number of nodes on the outside cladding wall surface.

The bulk coolant temperature is defined as

$$T_{CN} = \frac{\int_{A_c} T(r, \phi) u(r, \phi) r dr d\phi}{\int_{A_c} u(r, \phi) r dr d\phi}$$

which in finite difference form becomes

$$T_{CB} = \frac{\sum_j \sum_k T_{j,k} u_{j,k} A_{j,k}}{\sum_j \sum_k u_{j,k} A_{j,k}} \quad (88)$$

where the summation is taken over all the interior nodes in the coolant.

The local heat flux to the coolant is

$$\begin{aligned} \dot{q}''_{wo}(\phi) &= -k_w \left. \frac{\partial T}{\partial r} \right|_{r=R_w} \\ (\dot{q}''_{wo})_j &= k_w \frac{(T_{w1})_j - (T_{wo})_j}{\Delta r} \end{aligned} \quad (89)$$

where $(T_{w1})_j$ is the interior node in the cladding that is radially adjacent to $(T_{wo})_j$. $(\dot{q}''_{wo})_j$ can then be integrated over the interval $(0, \pi/6)$, in the same manner as was done with $(T_{wo})_j$, to find the average heat flux to the coolant, $\bar{q}''_{wo}$.

The Prandtl number is now evaluated at the bulk coolant temperature.

$$Pr = \frac{\bar{\mu}_c \bar{C}_{pc}}{\bar{k}_c} \quad (90)$$

The Nusselt number is calculated from the net heat transfer coefficient for the coolant.

$$Nu = \frac{\bar{h}_c D_H}{\bar{k}_c} \quad (91)$$

and the Peclet number is

$$Pe = Pr Re \quad (92)$$

3.6 CALCULATIONAL PROCEDURE

The model is comprised of an approximate analytic solution in the fuel and a nodal solution in the gap, cladding and coolant. These two solutions must be coupled at the fuel surface through the conditions of equal temperature and heat flux.

The procedure used is to assume, as a first approximation, that the fuel surface temperature is circumferentially uniform. This is used as a boundary condition to solve for the first approximation to the temperature profile in the gap, cladding, and coolant. A nodal grid, such as that typically represented in Figure 3-6, is set up in the cladding and coolant. The equations for the nodal solution are formulated from a nodal heat balance of the form of Equation (78) for each interior node and the appropriate boundary conditions, Equation (79), (80), and (81), are used at the boundary nodes. These equations form a set of simultaneous, linear algebraic equations which are solved for the nodal temperatures by the technique of relaxation by lines.

The velocity distribution and the eddy diffusivity of momentum in the coolant are calculated from the model described in Section 2. Thus, by establishing the velocity field and two components of momentum diffusivity at each coolant node, the equations of Section 3.4 are used to determine the eddy diffusivity of heat and the coolant convection term of Equation (78) can be evaluated. Note the total heat flux from the fuel for the nodal solution is determined by the heat balance used to find the axial temperature gradient in the coolant, Equation (41).

When the first approximation to the nodal solution has been calculated, the temperature drop across the gap is used to calculate the heat flux from the fuel surface and is approximated with an equation of the form*

$$\dot{q}''_{FS}(\phi) = \bar{q}''_{FS} [1 + \Delta Q_1 \cos(6\phi) + \Delta Q_2 \cos(12\phi)] \quad (93)$$

Equation (93) is substituted into Equations (70) and (71a) to find a new fuel surface temperature distribution.

Now, re-evaluating the material properties in the cladding and coolant at the nodal temperatures from the previous iteration, and using the new values of the fuel surface temperature, the next approximation to the nodal temperature profile is calculated.

*This functional form is suggested by the analytic solution for slug flow obtained in Appendix B.

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Figure 3-6 Nodal Grid Structure for Clad and Coolant

By this method of iterating between the heat flux across the gap as determined from the nodal solution and the fuel surface temperature from the analytic model in the fuel, successive approximations to the fuel surface temperature and heat flux are calculated until consecutive iterations converge to within a desired limit.

When the nodal solution is converged and the Fourier coefficients A_{6n}^* and B_{6n}^* are determined, the fuel temperature is calculated from Equation (70) and the coolant heat-transfer data from Section 3.5.

This model has been programmed as the digital computer code VELVET-II which performs the entire velocity and temperature calculation.

4. THERMAL RESULTS AND CONCLUSIONS

4.1 RANGE OF PARAMETERS

The developed computer code was used in a parametric study to determine temperature distributions and heat-transfer data in the fuel, cladding, and coolant as a function of Reynolds number, pitch-to-diameter ratio, and other significant variables.

The materials used in the analysis were $\text{UO}_2\text{-PuO}_2$ fuel, stainless steel cladding, and sodium coolant. The property variations with temperature were assumed to be the following:

$\text{UO}_2\text{-PuO}_2$ fuel

$$k_F (\text{Btu/h-ft} \cdot ^\circ\text{F}) = 4.1609 - 0.0019826T + 3.30242(10^{-7})T^2$$

$$\rho_F (\text{lb/ft}^3) = 690$$

Stainless Steel Clad

$$k_w (\text{Btu/h-ft} \cdot ^\circ\text{F}) = 7.3 + 0.0044T$$

$$\rho_w (\text{lb/ft}^3) = 480$$

Sodium Coolant

$$k_c (\text{Btu/h-ft} \cdot ^\circ\text{F}) = 53.438 - 0.01573T$$

$$\rho_c (\text{lb/ft}^3) = 59.4 - 0.0083T$$

$$C_{pc} (\text{Btu/lb} \cdot ^\circ\text{F}) = 0.346 - 8.0(10^{-5})T + 3.44(10^{-8})T^2$$

$$\mu_c (\text{lb/lh-ft}) = 1.4806 - 0.001536T + 0.56(10^{-6})T^2$$

The bulk sodium temperature was held at approximately 1000°F in all cases ($\text{Pr} = 0.0040$). The nominal value of the gap coefficient was $1500 \text{ Btu/h-ft}^2 \cdot ^\circ\text{F}$. The physical dimensions of the fuel pin were:

$$\text{Radius of central void} = 0$$

$$\text{Outside radius of fuel} = 0.11 \text{ in.}$$

$$\text{Outside radius of cladding} = 0.125 \text{ in.}$$

Calculations were made for the following parametric values:

$$\text{P/D} = 1.01, 1.05, 1.10, 1.15, 1.20, 1.25, 1.30, 1.50$$

$$\text{Re}(1000) = 10, 50, 100, 150, 200, \text{ and for slug flow}$$

The linear power generation rate was assumed to be 15 kW/ft in each case. An additional set of cases was run at a gap coefficient ten times nominal ($15,000 \text{ Btu/h-ft}^2 \cdot ^\circ\text{F}$) to determine the importance of this parameter. For this set, only 1.5×10^5 Reynolds number was investigated.

It should be noted that the thermal calculations to be presented are based on the assumptions of isotropic eddy diffusion

$$\text{i.e., } \epsilon_{Mr} = \epsilon_{M\phi} \text{ and } \epsilon_{Hr} = \epsilon_{H\phi}$$

The thermal effects of alternate circumferential diffusivity assumptions are discussed in Appendix A.

4.2 RESULTS

4.2.1 IMPORTANCE OF TURBULENT DIFFUSION

Figure 4-1 shows the calculated values of $(\epsilon_M/\nu_c)_{\max}$ which were then used in Dwyer's correlation⁶ to determine the magnitude of thermal diffusivity. The parameter $(\epsilon_M/\nu_c)_{\max}$ is determined as part of the velocity field calculations. The result and correlation function between the thermal diffusivity ϵ_H and the momentum diffusivity ϵ_M

Figure 4-1 Maximum Eddy Diffusivity of Momentum

$$\frac{\epsilon_H}{\epsilon_M} = 1 - \frac{1.82}{Pr (\epsilon_M/\nu_c)_{\max}^{1.4}} \quad (94)$$

is shown in Figure 4-2.

Turbulent heat transfer is significant at 1.3 P/D only for values of Peclet numbers greater than 200. As the pitch-to-diameter ratio is decreased, the "threshold" Peclet number is also decreased. Although cases were not run below 300 Peclet number, it would appear that at 1.1 P/D the threshold Peclet number is below 100.

The turbulent diffusion contribution to heat transfer is perhaps more clearly illustrated in Figure 4-3 in which a plot of the ratio $(\epsilon_H/\alpha_c)_{\max}$ is given. A value of zero for this parameter implies that only the molecular conductivity acts to transfer heat within the coolant in a plane perpendicular to the flow direction. When $(\epsilon_H/\alpha_c)_{\max}$ equals 1.0, the effective conductivity is twice the molecular conductivity (i.e., turbulence diffusion is of equal importance as molecular conduction).

The variation of eddy diffusivity over the flow channel cross section is shown in Figure 4-4 for the case of 1.1 P/D and 616 Pe (corresponding to 1.5×10^5 Reynolds number in sodium at 1000°F). The variation is typical for all cases investigated.

4.2.2 AVERAGE HEAT-TRANSFER RESULTS

The average Nusselt number is shown in Figure 4-5 as a function of Peclet number and pitch-to-diameter ratio.

For each P/D, the Nusselt number has the "dog-leg" variation with Peclet number characteristic of essentially all liquid metal flow. At low Peclet numbers, the Nusselt number is constant. This is the regime of negligible thermal diffusion (i.e., $\epsilon_H/\alpha \approx 0$), although the flow is fully turbulent. The relatively sudden drop in Nusselt number when the flow becomes laminar which occurs in the 10 to 40 range of Peclet numbers is not shown.

At higher Peclet numbers, the Nusselt number varies approximately as $Pe^{0.8}$. This rise in Nusselt number is the result of the increasing importance of turbulent diffusion as the Reynolds (or Peclet number) is raised.

The variation with P/D is also significant. The lower the P/D; the poorer are the heat-transfer characteristics of the bundle. The reduction is particularly severe for P/D values of 1.1 or below. For comparison, the Nusselt-Peclet correlation for sodium flow in a circular tube is included in Figure 4-5. It is seen to fall between the 1.1 and 1.15 P/D bundle results, and is very similar in shape to the bundle correlations.

Slug flow solutions have been calculated for P/D = 1.1, 1.15, 1.2, and 1.3. Slug flow implies the use of two simplifying, but incorrect assumptions: (1) uniform velocity everywhere in the channel, and (2) negligible turbulent diffusivity. These two assumptions tend to have counter balancing effects, and the results of liquid metal slug flow analyses are often reasonable approximations. It will be noted that slug flow does not permit a Reynolds number dependence.

In Figure 4-5, the slug flow results are shown at the Reynolds number at which the Nusselt number matches that of the complete VELVET solution. The slug flow results correspond to Peclet numbers in the range of 350 to 450. Slug flow results are conservative for higher Peclet numbers, but somewhat optimistic (up to 20 or 30% higher) for lower Peclet numbers. The adequacy of the slug flow assumptions for calculations of cladding temperature variations will be discussed in the next Section 4.2.3.

The Nusselt number results have been replotted as average heat transfer coefficients and are given in Figure 4-6. Because of the temperature dependence of the thermal conductivity, this plot is valid only for sodium at or near 1000°F. Heat-transfer coefficients at other temperatures can be found by multiplying values given in Figure 4-6 by the ratio $k_c/k_{1000^\circ\text{F}}$. The unusual variation, namely heat-transfer coefficients maximum at 1.1 P/D, is the result of the relative changes of Nusselt number and hydraulic diameter with P/D.

Figure 4-2 Ratio of Eddy Diffusivity of Heat to Eddy Diffusivity of Momentum

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Figure 4-3 Maximum Eddy Diffusivity of Heat

Figure 4-4 Ratio of Eddy-to-Molecular Thermal Diffusivity

Figure 4-5 Nusselt Number Variation

Figure 4-6 Heat Transfer Coefficient

4.2.3 CLADDING TEMPERATURE AND HEAT FLUX DISTRIBUTIONS

The temperature distributions on the outside surface of the cladding are given in Figures 4-7, 4-8, and 4-9. These distributions have the general form

$$T_{WO}(\phi) = \bar{T}_{WO} + A_{WO} \cos(6\phi) \quad (95)$$

with second order contributions from the higher harmonics, $\cos(6n\phi)$.

The coefficient A_{WO} is a function of the Peclet number, P/D , and the average heat flux and increases linearly with heat flux (or linear power). The variation of this coefficient with Peclet number and P/D is illustrated in Figure 4-7. This plot is for an average heat flux of 0.8×10^6 Btu/h-ft² which, for the 0.25-inch-diameter pins considered, corresponds to 15 kW/ft linear power. For P/D greater than 1.2, the cladding temperature does not vary significantly in the circumferential direction. The slug flow predictions are reasonably accurate, although compared to the VELVET-II results, they slightly underpredict the temperature variations for Peclet numbers below 600. The errors are negligible for P/D greater than 1.10.

Figures 4-8 and 4-9 are plots of the complete temperature distributions. Figure 4-8 shows the variation with P/D at a constant Peclet number of 616 (1.5×10^5 Reynolds number). Figure 4-9 shows the variation with Peclet number for a constant P/D of 1.1.

Plots of the circumferential variations of the cladding-to-coolant heat flux are given in Figures 4-10 and 4-11. The same type of dependence on P/D and Peclet number is seen as for the cladding temperature variation. However, the cladding surface heat flux is not well approximated by a simple cosine function for P/D less than 1.1. For P/D greater than approximately 1.3, the heat flux can be assumed to be circumferentially constant for all Peclet numbers.

4.2.4 EFFECT OF GAS GAP CONDUCTANCE

A thermal parameter of potential significance is the fuel-to-cladding gap conductance. As the reactor operates, the fuel undergoes irradiation swelling. This swelling acts to close the gap and increase the conductance. An opposing effect is fission gas release which reduces the gap conductivity. One would anticipate that the greater the conductance becomes, the more closely bound the fuel will be to the cladding, the smaller the cladding temperature variation, and the greater the average convective coefficient. A thermal-hydraulic study of the SNAP-8 reactor core² suggests that the gap conductance has a significant effect on the thermal characteristics and local variations for tightly packed ($P/D = 1.0$) fuel bundles.

To check the importance of the gap conductance for fast reactor fuel pins, a series of cases was run with ten times the nominal conductance. Pitch-to-diameter ratios from 1.05 to 1.50 were considered at a Reynolds number of 1.5×10^5 . Although the largest variations calculated were for the lowest pitch-to-diameter ratios, all values were within 1% of those calculated for the nominal gap conductance. Therefore, it can be concluded that this effect is negligible for bundles with P/D of 1.05 or greater and that no significant thermal changes occur in the cladding and coolant caused by fuel swelling before the time the swollen fuel begins to strain the cladding.

4.3 COMPARISON WITH PREVIOUS INVESTIGATIONS

4.3.1 AVERAGE HEAT-TRANSFER PERFORMANCE

Dwyer³ and Borishanski, et al.,⁸ have analytically investigated heat transfer to liquid metals flowing parallel to closely packed rod bundles for pitch-to-diameter ratios as low as 1.1. Their analyses are extensive and complex, and in general, very similar to the VELVET assumptions and method of solution. However, both employ a single region analysis; that is, they analyze only the coolant. This single-region analysis requires the somewhat unrealistic assumption of uniform cladding heat flux as a boundary condition. As noted by Axford²⁰ this idealization is reasonable at larger P/D 's, but becomes increasingly questionable as P/D approaches 1.0 (the case of touching rods).

The analytic predictions for 1.2 and 1.1 P/D are replotted and shown in Figures 4-12 and 4-13 along with VELVET results. At Peclet numbers below critical (approximately 300), all three analyses predict uniform Nusselt numbers, a consequence of the assumption of negligible eddy conduction. Above the critical Peclet number, the curves begin to slope upward as eddy conduction becomes important.

Figure 4-7 Outside Clad Wall Maximum Temperature Variation

Figure 4-8 Outside Clad Wall Temperature Profiles Variation with Pitch-to-Diameter Ratio

Figure 4-9 Outside Clad Wall Temperature Profiles Variation with Peclet Number

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Figure 4-10 Outside Clad Wall Heat Flux Variation with P/D

Figure 4-11 Outside Clad Wall Heat Flux Distributions Variation with Peclet Number

The results of the present study fall between those of the two single-region analyses except at low P/D . The single-region, constant-surface, heat-flux solutions should give lower limits for Nusselt numbers. The results are consistent with this conclusion. For P/D equal to or greater than 1.2 or Peclet numbers greater than 300, the surface heat-flux variation has no significant effect on the average or local heat-transfer characteristics. It is only at low P/D and Peclet numbers that the heat flux variation is of importance.

At high P/D values, the available experimental data are in reasonable agreement with theoretical results. For example, experimental investigations²¹⁻²⁴ conducted at Brookhaven National Laboratory scatter within $\pm 20\%$ of the theoretical curves for 1.75 P/D over the range $200 \leq Pe \leq 600$. Subbotin, et al.,²⁵ report data at 1.5 P/D which are in excellent agreement with theoretical predictions for Peclet numbers greater than 600. The data are somewhat low for $300 \leq Pe \leq 600$.

Experimental data at low P/D are very limited. The only experimental results for rod bundles with P/D below 1.5 are those of Borishanski and Firsova,²⁶ Subbotin, et al.,²⁵ Borishanski, et al.,⁸, and Pashek⁹.

Borishanski and Firsova²⁶ report data for sodium flowing through a bundle of 1.2 P/D . The test rod, with embedded thermocouples and a copper sheath, was the central rod in a bundle of seven, all of which were heated. The spread of the data is indicated in Figure 4-12. At the highest Peclet numbers obtained (300 to 400), the data are in good agreement with the analytical prediction. But as the Peclet number decreases, the experimental results fall further and further below the predicted Nusselt number.

Figure 4-12 Comparison of Calculated Nusselt Numbers with Previous Investigations

Figure 4-13 Comparison of Calculated Nusselt Numbers with Previous Investigations

Subbotin, et al., measured rod-average heat-transfer coefficients for mercury flow through bundles with P/D ranging from 1.1 to 1.5. Their correlation equation for 1.2 P/D is shown in Figure 4-12 and falls considerably below the analytic predictions and the other experimental data. Interestingly, there is no change in slope or flattening of the curves at low Peclet numbers as would be expected when molecular conduction becomes the dominant mode of heat transfer.

The fourth experimental investigation was reported recently by Pashek⁹ for flow of sodium through a seven-rod 1.2 P/D bundle in the temperature range 180°C to 220°C. Pashek made his measurements in the range of $20 < Pe < 120$ which represents the lower end of the turbulent flow regime and may extend into the transitional regime. Referring again to Figure 4-12, it is seen that Pashek's results agree best of all with the theoretical curves. The sodium in his experiments was continuously cold-trapped at a temperature well below that of the sodium in his test loop. Thus, it would appear that his heat-transfer system was the "cleanest" of the three, and this would seem to explain the better agreement with theory.

Dwyer²⁷ discusses these experimental studies in detail and concludes that all but the latter are in error at low Peclet numbers. He suggests the possibility of sodium contamination by inert gas or oxide particulates in these studies and the lack of complete wetting which produces an extraneous thermal resistance at the wall in the mercury experiments of Subbotin.

This conclusion appears reasonable. It is difficult to obtain good experimental liquid metal heat-transfer data, and especially so at low Peclet numbers. This difficulty is illustrated by the scatter of data points from any one experimental investigation (typically 20 to 30%) and by the discrepancies between investigations. Apparently there are one or more poorly understood experimental problems which become progressively more severe as the Peclet number is reduced. Because of the experimental difficulties and the simplicity of the theoretical models, it is believed that the analytic correlations are as representative of true heat transfer conditions at low P/D as are the experimental data now available.

4.3.2 CLADDING CIRCUMFERENTIAL TEMPERATURE VARIATIONS

A comparison of the cladding circumferential temperature distributions calculated in this study can be made with the analytic predictions of Dwyer³ for uniform surface heat flux. The comparison is shown in Figure 4-14 in which the ratio of the cladding outside surface temperature difference to the average temperature difference between the surface and the flowing coolant is plotted as a function of angular position. Results are shown for 1.1 and 1.2 P/D .

Although the surface heat flux distributions are different for the two analyses, the surface temperature distributions are quite similar. This similarity in surface temperature is in agreement with the conclusion on the effect of gap conductance, in Section 1.3; a tenfold change in gap conductance does not change the circumferential variations appreciably. Also, both studies found the temperature variations are not significantly affected by a tenfold increase in flow rate (in the turbulent range). Apparently, the bundle pitch-to-diameter ratio and the linear power generation level are the only significant parameters determining circumferential temperature variations, at least for pitch-to-diameter ratios of 1.05 or greater.

For P/D ratios greater than approximately 1.5, there is no significant variation in the local wall temperature. Hlavac, et al.,²⁴ found this to be true for a bundle of 1.75 P/D with stainless steel sheathed rods and mercury coolant. For the situation of a completely uniform heat flux, Dwyer³ suggests that noticeable temperature variations begin to appear at about 1.6 P/D . Similar results are indicated by the present study for more realistic heat flux variations. However, these variations do not significantly affect the applicability of an equivalent annulus model to predict the average convective heat-transfer coefficient until P/D falls to about 1.35.

Figure 4-14 Temperature Variation Outside Surface of Cladding

REFERENCES

1. NIJSING, R., GARGANTINI, I., and EIFLER, W., "Fundamental Studies of Fluid Flow and Heat Transfer in Fuel Element Geometries," AEC Report EUR-2193 Parts I and II (1965)
2. MAGEE, P. M., and TROMEL, R. H., "Heat Transfer from Fuel Elements in a Tightly Packed, Liquid-Metal-Cooled Compact Reactor," *Trans. ANS* 9:569 (1966)
3. DWYER, O. E., "Analytical Study of Heat Transfer to Liquid Metal Flowing In-Line through Closely Packed Rod Bundles," *Nucl. Sci. Eng.* 25: 343 (1966)
4. BULEEV, N. I., "Theoretical Model of the Mechanism of Turbulent Exchange in Fluid Flows," Atomic Energy Research Establishment, UK, Translation 957 (1963).
5. IBRAGIMOV, M. KH., et al., "Calculation of the Tangential Stresses at the Wall of a Channel and the Velocity Distribution in a Turbulent Flow of Liquid," *Atomnaya Energiya* 21:101 (1966).
6. DWYER, O. E., "Eddy Transport in Liquid-Metal Heat Transfer," *AIChE Journal* 9:261 (1963).
7. BENDER, D. J., SWITICK, D., and FIELD, J. H., "Turbulent Velocity Distribution in a Rod Bundle," GEAP-5411 (1967)
8. BORISHANSKI, V. M., GOTOVSKI, M. A., and FIRSOVA, E. V., "Heat Exchange When a Liquid Flows Longitudinally Through a Bundle of Rods Arranged in a Triangular Lattice," *Atomnaya Energiya* 22:318 (1966)
9. PASHEK, M., "Study of Local Heat Transfer and Thermal Fields in a Seven-Rod Bundle with Axial Flow of Sodium," Report #1815, Institute of Nuclear Research, Czechoslovakian Academy of Sciences, 1967
10. DEISSLER, R. G., and TAYLOR, M. F., "Analysis of Axial Turbulent Flow and Heat Transfer Through Banks of Rods or Tubes," TID 7529, Reactor Heat Transfer Conf. (Pt. 1) Book 2, 1957, pp 416-461.
11. SANBORN, V. A., "Experimental Evaluation of Momentum Terms in Turbulent Pipe Flow", NASA TN-3266 (1955)
12. RAPIER, A. C., and REDMAN, J. D., "The Calculation of Velocity Distributions in Rod Clusters," TRG Report 838(W), Reactor Development Lab, Windscale, UK, 1964
13. ELDER, J. W., "The Dispersion of Marked Fluid in Turbulent Shear Flow," *J. Fluid Mech.* 5:544 (1959)
14. COOK, K. W., "Velocity Distribution Around Fuel Pins in Sodium-Cooled Fast Reactors with Triangular Fuel Pin Arrays," GEAP-5725 (1968)
15. IBRAGIMOV, M. Kh., et. al. "Calculation of Hydraulic Resistivity Coefficients for Turbulent Fluid Flow in Channels of Noncircular Cross Section," *Atomnaya Energiya* 23:300 (1967)
16. LEVCHENKO, Y. D., SUBBOTIN, V. I., and USHAKOV, P. A., "The Distribution of Coolant Velocities and Wall Stresses in Closely Packed Rods," *Atomnaya Energiya* 22:218 (1967)
17. HUNSBEDT, A., et al., "Velocity Profiles in a Seven-Rod Spacerless Bundle," BRDO Internal Memo (January 30, 1969)
18. KAYS, W. M., *Convective Heat and Mass Transfer*, McGraw-Hill Book Co., Inc., New York, 1966, p. 104
19. HILDEBRAND, F., *Advanced Calculus for Applications*, Prentice-Hall, Inc., Englewood Cliffs, N. J. 1963, p. 485.
20. AXFORD, R. A., "Summary of Theoretical Aspects of Heat Transfer in Clustered Rod Geometries," *Heat Transfer in Rod Bundles*, ASME, 1968
21. KALISH, S., and DWYER, O. E., "Heat Transfer to NaK Flowing Through Unbaffled Rod Bundles," *Intern. J. Heat Mass Transfer* 10:1533 (1967)
22. NIMMO, B., and DWYER, O. E., "Heat Transfer to Mercury Flowing In-Line Through a Rod Bundle," *J. Heat Transfer* 87:312 (1965)
23. MARESCA, M. W., and DWYER, O. E., "Heat Transfer to Mercury Flowing In-Line Through a Bundle of Circular Rods," *J. Heat Transfer* 86:180 (1964)
24. HLAVAC, P. J., et al., "Heat Transfer to Mercury Flowing In-Line Through An Unbaffled Rod Bundle: Experimental Study of the Effect of Rod Displacement (Part 1)," Paper #68-WA/HT-30, ASME, 1968

25. SUBBOTIN, V. I., et al., "Heat Removal From Reactor Fuel Elements Cooled by Liquid Metals," Paper #328, Third U.N. International Conf. on Peaceful Uses of Atomic Energy, Geneva (August 31-September 9, 1964)
26. BORISHANSKI, V. M., and FIRSOVA, E. V., "Heat Exchange in Separated Bundles of Rods with Metallic Sodium Flowing Longitudinally," *Atomnaya Energiya* 16:457 (1964)
27. DWYER, O. E., "Heat Transfer to Liquid Metals Flowing In-Line Through Unbaffled Rod Bundles: A Review," *Heat Transfer in Rod Bundles*, ASME, 1968
28. BOBKOV, V. P., IBRAGIMOV, M. Kh., and SUBBOTIN, V. I., "Calculating the Coefficient of Turbulent Heat Transfer for a Liquid Flowing in a Tube," *Atomnaya Energiya* 24:442 (1968)
29. AXFORD, R. A., "Multiregional Analysis of Temperature Fields and Heat Fluxes in Tube Bundles with Internal, Solid, Nuclear Heat Sources," LA-3167.

APPENDIX A

ALTERNATE EDDY DIFFUSIVITY ASSUMPTIONS

For accurate calculation of the velocity field for coolant flow through a rod bundle, it is necessary to postulate a circumferential momentum transfer greater than the radial transfer. Some investigators, notably Rapier and Redman¹ and Nijssing, et al.,² account for this additional transfer by increasing the circumferential eddy diffusivity of momentum. This implies that the physical effect is one of small-scale, localized eddies.

A more satisfactory approach to the problem of rod bundle flow is provided by Ibragimov, et al.³. They consider the additional circumferential momentum transfer to be the result, not of small, localized eddies, but of large-scale fluid motion induced by the irregular geometry. With this concept, they are able to make very accurate calculations of velocity fields in irregularly shaped channels. Their analysis has been incorporated in this study. The excellent agreement of the calculated velocity distributions with experimental data for rod bundle flow is shown in Section 2.3.

A problem arises in the subsequent calculation of the temperature field. In general, there is an analogy between convective momentum and heat transfer. This analogy is usually expressed, for turbulent flow, as a direct relationship between the eddy diffusivities of momentum and heat (e.g., Dwyer's Ψ correlation for liquid metals). An increased circumferential momentum diffusivity, as postulated by Nijssing and Rapier and Redman, would result in proportionally increased circumferential thermal diffusivity.

However, the physical model of Ibragimov shows this line of reasoning to be incorrect; it suggests that the effective circumferential thermal diffusivity will not be much larger than the radial diffusivity. Because of the high molecular conductivity of liquid metals, fluid particles lose their differential energy content well before they traverse the relatively large eddy distances associated with the large-scale fluid motion. Thus, it can be expected that the effect of the geometrically induced, large-scale fluid motion will be much less on the circumferential heat transfer than on the circumferential momentum transfer.

The standard assumption of equal thermal diffusivities at a point (i.e., $\epsilon_{H\phi} = \epsilon_{Hr}$) has been employed in this study. An idea of the resultant thermal uncertainties can be found by rerunning the thermal calculations for alternative diffusivity assumptions. Three alternates have been considered. The first two were discussed previously in Section 2.

1. Momentum diffusivity model of Rapier and Redman

$$\epsilon_{M\phi}(r, \phi) = \frac{u^*(\phi) \hat{y}(\phi)}{10} \quad (A-1)$$

$$\epsilon_{H\phi}(r, \phi) = \Psi \epsilon_{M\phi}(r, \phi) \text{ from Dwyer's correlation.}$$

2. Momentum diffusivity model of Eifler and Nijssing

$$\epsilon_{M\phi}(r, \phi) = 0.0115 \nu_c \left[\frac{\bar{u} \bar{q} y}{\bar{u} \hat{y}} \text{Re} \right]^{7/8} \quad (A-2)$$

$$\epsilon_{H\phi}(r, \phi) = \Psi \epsilon_{M\phi}(r, \phi) \text{ from Dwyer's correlation.}$$

The third alternate assumption is based on a theoretical expression for the circumferential thermal diffusivity in a circular tube proposed by Bobkov, et al.⁴

$$\epsilon_{H\phi}(r, \phi) = 1 + \left[\frac{0.2}{1.02 - \frac{r}{R}} \right] \epsilon_{Hr}(r, \phi) \quad (A-3)$$

where r is the radial coordinate measured from the tube centerline and R is the tube radius. The comparable expression for flow parallel to a rod bundle would appear to be

$$\epsilon_{H\phi}(r, \phi) = \left[1 + \frac{0.2}{0.02 + \frac{y}{\phi}} \right] \epsilon_{Hr}(r, \phi) \quad (A-4)$$

A summary of the thermal differences resulting from the three alternates and the standard assumptions are presented in Table A-1. The basic conclusion is that the standard assumption of isotropic thermal diffusion [i.e., $\epsilon_{H\phi}(r, \phi) = \epsilon_{Hr}(r, \phi)$] is the most conservative.

The variation of the radial thermal diffusivity is the same for each case; and the thermal differences are entirely due to the differences in the circumferential diffusivities. The model of Rapier and Redman assumes $\epsilon_{H\phi}$ is constant over the turbulent core of the flow fluid at a value equal to the maximum ϵ_{Hr} . Nijsing, et al., assumed $\epsilon_{H\phi}$ has its maximum values along the line of flow symmetry. At $\phi = 30^\circ$, $\epsilon_{H\phi}$ is approximately 2.5 times the maximum ϵ_{Hr} . In Bobkov's model, the peak $\epsilon_{H\phi}$ is found one-third the way from the cladding surface to the line of flow symmetry and is about 1.6 times the maximum ϵ_{Hr} .

A relative measure of the average heat-transfer performance is provided by the magnitude of the average Nusselt number. Nijsing's model, with its very high $\epsilon_{H\phi}$, predicts a very high Nusselt number. The other two models yield Nusselt numbers only a few per cent higher than that for the standard model.

TABLE A-1
THERMAL RESULTS FOR VARIOUS $\epsilon_{H\phi}$ ASSUMPTIONS

All at 1.5×10^5 Re, 15 kW/ft linear power, 0.25-in. dia rods

P/D	ASSUMPTIONS	Nu	Pe	ϵ_{Hr}/α Max	$\epsilon_{H\phi}/\alpha$ Max	Cladding Surface		
						$\Delta T\phi$ (°F)	Max-To-Av Heat Flux	Min-To-Av Heat Flux
1.1	Standard, $\epsilon_{H\phi} = \epsilon_{Hr}$	9.33	617	0.91	0.91	32.5	1.105	0.890
	Rapier and Redman Model	9.56	617	0.91	0.91	29.1	1.088	0.892
	Eifler and Nijsing Model	10.63	617	0.91	2.32	19.0	1.054	0.927
	Bobkov Model	9.70	617	0.91	1.41	27.5	1.082	0.897
1.2	Standard, $\epsilon_{H\phi} = \epsilon_{Hr}$	14.07	614	0.69	0.69	11.57	1.039	0.961
	Rapier and Redman Model	14.17	614	0.69	0.69	10.37	1.034	0.965
	Nijsing Model	14.55	614	0.69	1.75	5.15	1.021	0.981
	Bobkov Model	14.21	614	0.69	1.11	9.7	1.031	0.966
1.3	Standard, $\epsilon_{H\phi} = \epsilon_{Hr}$	15.63	610	0.58	0.58	5.78	1.020	0.981
	Rapier and Redman Model	15.68	610	0.58	0.58	5.11	1.018	0.983
	Nijsing Model	15.86	610	0.58	1.47	2.57	1.011	0.991
	Bobkov Model	15.69	610	0.58	0.95	4.72	1.016	0.984

Models for Circumferential Thermal Diffusivity

1. Standard, $\epsilon_{H\phi} = \epsilon_{Hr}$ (used in text of report)
2. Rapier and Redman

$$\epsilon_{M\phi} = u^* \sqrt{y}/10; \epsilon_{H\phi} = \Psi \epsilon_{M\phi}$$

3. Nijsing

$$\epsilon_{M\phi} = 0.0115 \nu_c \left[\frac{\bar{u}_L y}{\bar{u} \hat{y}} \text{Re} \right]^{7/8}; \epsilon_{H\phi} = \Psi \epsilon_{M\phi}$$

4. Bobkov

$$\epsilon_{H\phi} = \left[1 + \frac{0.2}{0.02 + \frac{y}{\hat{y}}} \right] \epsilon_{Hr}$$

A more severe discrimination between the models is provided by the variation in the outside surface temperature of the cladding. The largest temperature differences are predicted with the standard models. The model of Rapier and Redman predicts variations 10 to 12% less than the standard model; that of Bobkov predicts 15 to 18%. The Nijsing model predicts very low values; temperature variations less than 50% of those with the standard model.

The strongest argument for the retention of the standard diffusivity assumption is the available experimental heat-transfer data which tends to fall below the standard predictions. There is no evidence of Nusselt numbers higher than those based on equal diffusivities as would be required to substantiate the alternate assumptions.

Experimental data do not exist to allow a direct comparison of the calculated cladding temperature variations. Theoretically, it would appear that the standard model with its assumption of equal radial and circumferential diffusivities could somewhat overpredict the circumferential temperature gradients. However, this overprediction cannot be large since it would also imply an increased average Nusselt number in contradiction to the available data.

1. Rapier, A. C., and Redman, J. D., "The Calculation of Velocity Distributions in Rod Clusters," TRG Report 838(W), Reactor Development Laboratory, Windscale, UK, 1964.
2. Nijsing, R., Gargantini, I., and Eifler, W., "Fundamental Studies of Fluid Flow and Heat Transfer in Fuel Element Geometries," AEC Report EUR-2193, Parts I and II, (1965).
3. Ibragimov, M. Kh., et al., "Calculation of the Thermal Tangential Stresses at the Wall of a Channel and Velocity Distribution in a Turbulent Flow of Liquid," Atomnaya Energiya, 21, 101 (1966).
4. Bobkov, V. P., Ibragimov, M. Kh., and Subbotin, V. I., "Calculating the Coefficient of Turbulent Heat Transfer for a Liquid Flowing in a Tube," Atomnaya Energiya, 24, 442 (1968).

APPENDIX B

SLUG FLOW MODEL

B.1 INTRODUCTION

A first approximation for the problem of convective heat transfer with a liquid metal coolant amenable to analytic techniques is the slug flow assumption with constant material properties.

With the slug flow approximation, the coolant is assumed to be at a uniform velocity throughout the flow cross section and turbulent heat transfer is neglected. These two assumptions tend to be reasonable in liquid metal turbulent heat-transfer analysis since the turbulent heat diffusion is often of second order, and the uniform velocity assumption counterbalances the neglect of turbulent heat transport.

The problem is then reduced to determining the temperature distribution in the three coupled regions of fuel, cladding, and coolant. This analysis has previously been performed by Axford,¹ but the solution of Axford neglects the presence of a heat-transfer resistance between the fuel and cladding. The problem, therefore, is to include the fuel-cladding heat-transfer coefficient.

The resultant solution is not actually incorporated into the VELVET-II code, but has been programmed separately. In this way, the solution acts as a useful check on the numerical procedures and coding used in the VELVET-II program, and also provides valuable insight into the mathematical structure of the solution to this type of problem.

B.2 FORMULATION

In addition to slug flow, the following assumptions will be made for the model used in this analysis:

1. Fully developed flow and heat transfer in the steady state,
2. Uniform power density in the fuel,
3. Constant material properties,
4. Radially and axially infinite, equilaterally spaced tube bundle,
5. No circumferential heat transfer in the gap,
6. No axial conduction in any of the materials,
7. No gamma-neutron heating.

The temperature distribution in the fuel and cladding is described by Fourier's law of heat conduction, which, under the assumptions on the problem, reduces to

$$\frac{\partial^2 T_F}{\partial r^2} + \frac{1}{r} \frac{\partial T_F}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T_F}{\partial \phi^2} + \frac{\dot{q}'''}{k_F} = 0 \quad (B-1)$$

$$\frac{\partial^2 T_w}{\partial r^2} + \frac{1}{r} \frac{\partial T_w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T_w}{\partial \phi^2} = 0 \quad (B-2)$$

The coolant temperature is described by the energy equation, which reduces to

$$\frac{\partial^2 T_c}{\partial r^2} + \frac{1}{r} \frac{\partial T_c}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T_c}{\partial \phi^2} = \frac{u}{\alpha_c} \frac{\partial T_c}{\partial z} \quad (B-3)$$

From the assumption of a radially and axially infinite tube bundle, the basic geometric area of interest reduces to the triangular section shown in Figure B-1:

Equations (B-1), (B-2), and (B-3) represent a coupled set of simultaneous partial differential equations. The boundary conditions for these equations are:

Figure B-1 Geometry Description for Slug Flow Model

Fuel

$$\left. \frac{\partial T_F}{\partial \phi} \right|_{\phi=0} = 0 \quad (\text{B-4a})$$

$$\left. \frac{\partial T_F}{\partial \phi} \right|_{\phi=\pi/6} = 0 \quad (\text{B-4b})$$

$$\left. \frac{\partial T_F}{\partial r} \right|_{r=R_v} = 0 \quad (\text{B-4c})$$

Fuel-Cladding

$$k_F \left. \frac{\partial T_F}{\partial r} \right|_{r=R_F} = \left(\frac{R_{wI}}{R_F} \right) k_w \left. \frac{\partial T_w}{\partial r} \right|_{r=R_{wI}} \quad (\text{B-5a})$$

$$T_w(R_{wI}, \phi) = T_F(R_F, \phi) + \frac{k_F}{h_G} \left. \frac{\partial T_F}{\partial r} \right|_{r=R_F} \quad (\text{B-5b})$$

Cladding

$$\left. \frac{\partial T_w}{\partial \phi} \right|_{\phi=0} = 0 \quad (\text{B-6a})$$

$$\left. \frac{\partial T_w}{\partial \phi} \right|_{\phi=\pi/6} = 0 \quad (\text{B-6b})$$

Cladding-Coolant

$$T_w(R_{wO}, \phi) = T_c(R_{wO}, \phi) \quad (\text{B-7a})$$

$$k_w \left. \frac{\partial T_w}{\partial r} \right|_{r=R_{wO}} = k_c \left. \frac{\partial T_c}{\partial r} \right|_{r=R_{wO}} \quad (\text{B-7b})$$

Coolant

$$\left. \frac{\partial T_c}{\partial \phi} \right|_{\phi=0} = 0 \quad (\text{B-8a})$$

$$\left. \frac{\partial T_c}{\partial \phi} \right|_{\phi=\pi/6} = 0 \quad (\text{B-8b})$$

$$\left[\cos \phi \frac{\partial T_c}{\partial r} - \frac{\sin \phi}{r} \frac{\partial T_c}{\partial \phi} \right]_{r=R_O/\cos \phi} \quad (\text{B-8c})$$

As now stated, this formulation represents the Neumann problem in which the gradients are unique but not the temperature. To fix these gradients on the temperature scale, a temperature must be specified. For convenience, the temperature at the center of the cladding is specified. Thus, the final boundary condition is

$$T_w(R_{wA}, \pi/12) = T_{wD} \quad (\text{B-9})$$

B.3 GENERAL SOLUTION

Equation (B-2) for the cladding temperature is the Laplace equation in two dimensions in cylindrical coordinates which has the solution²

$$T_w = A_1^w + A_2^w \ln(r) + \sum_{k=1}^{\infty} \left\{ \left[a_k^w r^k + b_k^w r^{-k} \right] \left[c_k^w \cos(k\phi) + d_k^w \sin(k\phi) \right] \right\} \quad (B-10)$$

Now, let

$$T_F^* = T_F + \frac{r^2}{4} \frac{\dot{q}'''}{k_F} \quad (B-11a)$$

and

$$T_c^* = T_c - \frac{r^2}{4} \frac{u}{\alpha_c} \frac{\partial T_c}{\partial z} \quad (B-11b)$$

Equations (B-1) and (B-3) transform to the Laplace equation

$$\frac{\partial^2 T_F^*}{\partial r^2} + \frac{1}{r} \frac{\partial T_F^*}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T_F^*}{\partial \phi^2} = 0 \quad (B-12a)$$

$$\frac{\partial^2 T_c^*}{\partial r^2} + \frac{1}{r} \frac{\partial T_c^*}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T_c^*}{\partial \phi^2} = 0 \quad (B-12b)$$

Thus, the general solutions to Equations (B-1) and B-3) are, respectively,

$$T_F = A_1^F + A_2^F \ln(r) - \frac{\dot{q}r^2}{4k_F} + \sum_{k=1}^{\infty} \left\{ \left[a_k^F r^k + b_k^F r^{-k} \right] \left[c_k^F \cos(k\phi) + d_k^F \sin(k\phi) \right] \right\} \quad (B-13)$$

$$T_c = A_1^c + A_2^c \ln(r) + \frac{Sr^2}{2} + \sum_{k=1}^{\infty} \left\{ \left[a_k^c r^k + b_k^c r^{-k} \right] \left[c_k^c \cos(k\phi) + d_k^c \sin(k\phi) \right] \right\} \quad (B-14)$$

where

$$S = \frac{u}{2\alpha_c} \frac{\partial T_c}{\partial z} \quad (B-15)$$

For a fully developed thermal boundary layer, the axial temperature gradient in the coolant is a constant.³ Write a heat balance,

$$\text{Heat generated by fuel} = \text{heat carried away by coolant.}$$

Thus,

$$\begin{aligned} \frac{\pi(R_F^2 - R_v^2)\dot{q}'''}{12} &= C_{pc} \rho_c u A_c \frac{\partial T_c}{\partial z} \\ u \frac{\partial T_c}{\partial z} &= \frac{\pi(R_F^2 - R_v^2)\dot{q}'''}{12 \rho_c C_{pc} A_c} \end{aligned} \quad (B-16)$$

Substitution of Equation (B-16) into (B-15) yields

$$S = \frac{\pi(R_F^2 - R_v^2)\dot{q}'''}{24 k_c \Delta_c} \quad (B-17)$$

where

$$A_c = \frac{R_o^2}{2\sqrt{3}} - \frac{\pi R_{wo}^2}{12} \quad (B-18)$$

B.4 APPLICATION OF UNCOUPLED BOUNDARY CONDITIONS

First consider Equation (B-13). By differentiating with respect to ϕ ,

$$\frac{\partial T_F}{\partial \phi} = \sum_{k=1}^{\infty} \left\{ \left[a_k^F r^k + b_k^F r^{-k} \right] \left[-c_k^F k \sin(k\phi) + d_k^F k \cos(k\phi) \right] \right\} \quad (B-19)$$

and substituting Equation (B-19) into (B-4a),

$$\sum_{k=1}^{\infty} \left\{ \left[a_k^F r^k + b_k^F r^{-k} \right] \left[k d_k^F \right] \right\} = 0$$

Since each term of the series is independent,

$$\begin{aligned} (a_k^F r^k + b_k^F r^{-k}) (k d_k^F) &= 0 \\ \therefore d_k^F &= 0 \end{aligned} \quad (B-20)$$

Substitution of Equation (B-19) into (B-4b) yields

$$\begin{aligned} \sum_{k=1}^{\infty} \left\{ \left[a_k^F r^k + b_k^F r^{-k} \right] \left[-c_k^F k \sin\left(\frac{k\pi}{6}\right) \right] \right\} &= 0 \\ c_k^F k \sin\left(\frac{k\pi}{6}\right) &= 0 \end{aligned}$$

Since $c_k^F = 0$ implies a null solution, i.e., $T_F = 0$

$$\begin{aligned} \sin \frac{k\pi}{6} &= 0 \\ \therefore k &= 6, 12, 18, \dots, 6j, \dots \end{aligned} \quad (B-21)$$

By absorbing the constant c_k^F into a_k^F and b_k^F , Equation (B-13) now becomes

$$T_F = A_1^F + A_2^F \ln(r) - \frac{\dot{q}''' r^2}{4k_F} + \sum_{j=1}^{\infty} \left\{ \left[a_{6j}^F r^{6j} + b_{6j}^F r^{-6j} \right] \cos(6j\phi) \right\} \quad (B-22)$$

Similarly, by substituting Equation (B-10) into (B-6a) and (B-6b), and Equation (B-14) into (B-8a) and (B-8b), the cladding and coolant temperature equations become

$$T_w = A_1^w + A_2^w \ln(r) + \sum_{j=1}^{\infty} \left\{ \left[a_{6j}^w r^{6j} + b_{6j}^w r^{-6j} \right] \cos(6j\phi) \right\} \quad (B-23)$$

$$T_c = A_1^c + A_2^c \ln(r) + \frac{Sr^2}{2} + \sum_{j=1}^{\infty} \left\{ \left[a_{6j}^c r^{6j} + b_{6j}^c r^{-6j} \right] \cos(6j\phi) \right\} \quad (B-24)$$

Substitution of Equation (B-22) into (B-4c) yields

$$\frac{A_2^F}{R_v} - \frac{\dot{q}''' R_v}{2k_F} + \sum_{j=1}^{\infty} \left\{ \left[a_{6j}^c 6j R_v^{6j-1} - b_{6j}^c 6j R_v^{-6j-1} \right] \cos(6j\phi) \right\} = 0 \quad (B-25)$$

Now, note the following orthogonal properties of the cosine function

$$\int_0^{\pi/6} \cos(6j\phi) d\phi = 0 \quad (\text{B-26a})$$

$$\int_0^{\pi/6} \cos(6j\phi) \cos(6m\phi) d\phi = \frac{\pi}{12} \delta_{jm} \quad (\text{B-26b})$$

where δ_{jm} is the Kronecher delta function. These two relationships will be used throughout the analysis.

Multiply Equation (B-25) by $d\phi$ and integrate $(0, \pi/6)$,

$$\left[\frac{A_2^F}{R_v} - \frac{\dot{q}''' R_v}{2k_F} \right] \frac{\pi}{6} = 0$$

$$\therefore A_2^F = \frac{\dot{q}''' R_v^2}{2k_F} \quad (\text{B-27})$$

Multiply Equation (B-25) by $\cos(6m\phi) d\phi$ and integrate $(0, \pi/6)$,

$$\left[a_{6j}^F 6j R_v^{6j-1} - b_{6j}^F 6j R_v^{-6j-1} \right] \frac{\pi}{12} = 0$$

$$\therefore b_{6j}^F = a_{6j}^F R_v^{12j} \quad (\text{B-28})$$

Making use of Equations (B-27) and (B-28), and let

$$A_{6j}^F = a_{6j}^F R_F^{6j} \quad (\text{B-29})$$

Then Equation (B-22) is written as

$$T_F = A_1^F + \frac{\dot{q}''' R_v^2}{2k_F} \ln(r) - \frac{\dot{q}''' r^2}{4k_F} + \sum_{j=1}^{\infty} \left\{ A_{6j}^F \left[\left(\frac{r}{R_F} \right)^{6j} + \zeta_{6j}^F \left(\frac{R_v}{r} \right)^{6j} \right] \cos(6j\phi) \right\} \quad (\text{B-30})$$

where

$$\zeta_{6j}^F = \left(\frac{R_v}{R_F} \right)^{6j} \quad (\text{B-31})$$

Substitute Equation (B-23) into (B-9) and

$$T_{wB} = A_1^w + A_2^w \ln(R_{wA}) + \sum_{j=1}^{\infty} \left\{ \left[a_{6j}^w R_{wA}^{6j} + b_{6j}^w R_{wA}^{-6j} \right] \cos\left(\frac{j\pi}{12}\right) \right\}$$

$$\therefore A_1^w = T_{wB} - A_2^w \ln(R_{wA}) - \sum_{j=1}^{\infty} (R_{wA}) \quad (\text{B-32})$$

where

$$\sum (R_{wA}) = \sum_{j=1}^{\infty} \left\{ \left[a_{6j}^w R_{wA}^{6j} + b_{6j}^w R_{wA}^{-6j} \right] \cos\left(\frac{j\pi}{2}\right) \right\} \quad (B-33)$$

The final uncoupled boundary condition, Equation (B-8c), will be saved for later in the solution when its application is more convenient.

B.5 APPLICATION OF COUPLED BOUNDARY CONDITIONS

Substitute Equations (B-23) and (B-30) into Equation (B-5a)

$$\begin{aligned} & k_F \left\{ \frac{\dot{q}''' R_v^2}{2k_F R_F} - \frac{\dot{q}''' R_F}{2k_F} + \sum_{j=1}^{\infty} A_{6j} \left[6j \frac{R_F^{6j-1}}{R_F^{6j}} - 6j \xi_{6j}^F \frac{R_v^{6j}}{R_F^{6j+1}} \right] \cos(6j\phi) \right\} \\ &= \frac{R_{wI}}{R_F} k_w \left\{ \frac{A_2^w}{R_{wI}} + \sum_{j=1}^{\infty} \left[a_{6j}^w 6j R_{wI}^{6j-1} - b_{6j}^w 6j R_{wI}^{-6j-1} \right] \cos(6j\phi) \right\} \end{aligned} \quad (B-34)$$

Substitute Equations (B-23) and (B-30) into Equation (B-5b)

$$\begin{aligned} & A_1^w + A_2^w \ln(R_{wI}) + \sum_{j=1}^{\infty} a_{6j}^w R_{wI}^{6j} + b_{6j}^w R_{wI}^{-6j} \cos(6j\phi) \\ &= A_1^F + \frac{\dot{q}''' R_v^2}{2k_F} \ln(R_F) - \frac{\dot{q}''' R_F^2}{4k_F} + \sum_{j=1}^{\infty} \left\{ A_{6j}^F \left[1 + (\xi_{6j}^F)^2 \right] \cos(6j\phi) \right\} \\ &+ \frac{k_F}{h_G} \left\{ \frac{\dot{q}''' R_v^2}{2k_F R_F} - \frac{\dot{q}''' R_F}{2k_F} + \sum_{j=1}^{\infty} A_{6j}^F \left[6j \frac{R_F^{6j-1}}{R_F^{6j}} - 6j \xi_{6j}^F \frac{R_v^{6j}}{R_F^{6j+1}} \right] \cos(6j\phi) \right\} \end{aligned} \quad (B-35)$$

Multiply Equation (B-34) and (B-35) by $d\phi$ and integrate $(0, \pi/6)$

$$\left[\frac{\dot{q}''' R_v^2}{2k_F R_F} - \frac{\dot{q}''' R_F}{2k_F} \right] \frac{\pi}{6} = \left[\frac{R_{wI}}{R_F} \frac{k_w}{k_F} \frac{A_2^w}{R_{wI}} \right] \frac{\pi}{6} \quad (B-36a)$$

$$\left[A_1^w + A_2^w \ln(R_{wI}) \right] \frac{\pi}{6} = \left[A_1^F + \frac{\dot{q}''' R_v^2}{2k_F} \ln(R_F) - \frac{\dot{q}''' R_F^2}{4k_F} + \frac{\dot{q}''' R_v^2}{2h_G R_F} - \frac{\dot{q}''' R_F}{2h_G} \right] \frac{\pi}{6} \quad (B-36b)$$

Use Equations (B-36a), (B-36b), and (B-32)

$$A_2^w = - \frac{\dot{q}''' R_F^2}{2k_w} \left[1 - \left(\frac{R_v}{R_F} \right)^2 \right] \quad (B-37)$$

$$A_1^w = T_{wB} - \sum (R_{wA}) + \frac{\dot{q}''' R_F^2}{2k_w} \left[1 - \left(\frac{R_v}{R_F} \right)^2 \right] \ln(R_{wA}) \quad (B-38)$$

$$\begin{aligned} A_1^F = T_{wB} - \sum (R_{wA}) + \frac{\dot{q}''' R_F^2}{2k_w} \left[1 - \left(\frac{R_v}{R_F} \right)^2 \right] \ln\left(\frac{R_{wA}}{R_{wI}}\right) + \frac{\dot{q}''' R_F}{2h_G} \left[1 - \left(\frac{R_v}{R_F} \right)^2 \right] \\ + \frac{\dot{q}''' R_F^2}{2k_F} - \frac{\dot{q}''' R_v^2}{2k_F} \ln(R_F) \end{aligned} \quad (B-39)$$

Multiply Equations (B-34) and (B-35) by $\cos(6m\phi) d\phi$ and integrate $(0, \pi/6)$,

$$k_F A_{6j}^F \left[\frac{6j}{R_F} - \frac{6j}{R_F} (\zeta_{6j}^F)^2 \right] \frac{\pi}{6} = \frac{R_{wl}}{R_F} k_w \left[a_{6j}^w 6j R_{wl}^{6j-1} - b_{6j}^w R_{wl}^{-6j-1} \right] \pi/12 \quad (B-40a)$$

$$\left[a_{6j}^w R_{wl}^{6j} + b_{6j}^w R_{wl}^{-6j} \right] \frac{\pi}{12} = A_{6j}^F \left[1 + (\zeta_{6j}^F)^2 \right] \frac{\pi}{12} + \frac{k_F}{h_G} A_{6j}^F \left[\frac{6j}{R_F} - \frac{6j}{R_F} (\zeta_{6j}^F)^2 \right] \frac{\pi}{12} \quad (B-40b)$$

Solve Equations (B-40a) and (B-40b) as simultaneous equations for a_{6j}^w and b_{6j}^w ,

$$a_{6j}^w = R_{wl}^{-6j} A_{6j}^F \gamma_{6j}^w \quad (B-41)$$

where

$$\gamma_{6j}^w = \frac{1}{2} \left\{ \left[1 + \frac{k_F}{k_w} + \frac{6j k_F}{R_F h_G} \right] + \left[1 - \frac{k_F}{k_w} - \frac{6j k_F}{R_F h_G} \right] (\zeta_{6j}^F)^2 \right\} \quad (B-42)$$

and

$$b_{6j}^w = R_{wl}^{6j} A_{6j}^F \zeta_{6j}^w \quad (B-43)$$

where

$$\zeta_{6j}^w = \frac{1}{2} \left\{ \left[1 - \frac{k_F}{k_w} + \frac{6j k_F}{R_F h_G} \right] + \left[1 + \frac{k_F}{k_w} - \frac{6j k_F}{R_F h_G} \right] (\zeta_{6j}^F)^2 \right\} \quad (B-44)$$

Thus, the cladding temperature Equation (B-23) may now be written as

$$T_w = A_1^w + A_2^w \ln(r) + \sum_{j=1}^{\infty} \left\{ A_{6j}^F \left[\gamma_{6j}^w \left(\frac{r}{R_{wl}} \right)^{6j} + \zeta_{6j}^w \left(\frac{R_{wl}}{r} \right)^{6j} \right] \cos(6j\phi) \right\} \quad (B-45)$$

Substitute Equations (B-24) and (B-45) into Equation (B-7a),

$$\begin{aligned} & A_1^w + A_2^w \ln(R_{wo}) + \sum_{j=1}^{\infty} \left\{ A_{6j}^F \left[\gamma_{6j}^w \left(\frac{R_{wo}}{R_{wj}} \right)^{6j} + \zeta_{6j}^w \left(\frac{R_{wl}}{R_{wo}} \right)^{6j} \right] \cos(6j\phi) \right\} \\ &= A_1^c + A_2^c \ln(R_{wo}) + \frac{S R_{wo}^2}{2} + \sum_{j=1}^{\infty} \left\{ \left[a_{6j}^c R_{wo}^{6j} + b_{6j}^c R_{wo}^{-6j} \right] \cos(6j\phi) \right\} \end{aligned} \quad (B-46)$$

Substitute Equations (B-24) and (B-45) into Equation (B-7b),

$$\begin{aligned} & k_w \left\{ \frac{A_2^w}{R_{wo}} + \sum_{j=1}^{\infty} A_{6j}^F \left[\frac{R_{wo}^{6j-1}}{R_{wl}^{6j}} 6j \gamma_{6j}^w - \frac{R_{wl}^{6j}}{R_{wo}^{6j+1}} 6j \zeta_{6j}^w \right] \cos(6j\phi) \right\} \\ &= k_c \left\{ \frac{A_2^c}{R_{wo}} + S R_{wo} + \sum_{j=1}^{\infty} \left[a_{6j}^c 6j R_{wo}^{6j-1} - b_{6j}^c 6j R_{wo}^{-6j-1} \right] \cos(6j\phi) \right\} \end{aligned} \quad (B-47)$$

Multiply (B-46) and (B-47) by $d\phi$ and integrate $(0, \pi/6)$

$$\left[A_1^w + A_2^w \ln(R_{wo}) \right] \frac{\pi}{6} = \left[A_1^c + A_2^c \ln(R_{wo}) + \frac{S R_{wo}^2}{2} \right] \frac{\pi}{6} \quad (B-48a)$$

$$\frac{k_w A_2^w}{R_{wo}} \frac{\pi}{6} = \left[\frac{k_c A_2^c}{R_{wo}} + S k_c R_{wo} \right] \frac{\pi}{6} \quad (B-48b)$$

Solve Equations (B-48a) and (B-48b) for A_1^c and A_2^c , and use the known expressions for the other constants,

$$A_1^c = T_{wb} - \sum R_{wA} - \frac{\dot{q}''' R_F^2}{2k_w} \left[1 - \left(\frac{R_v}{R_F} \right)^2 \right] \ln \left(\frac{R_{wo}}{R_{wA}} \right) - \frac{S R_{wo}^2}{2} + \frac{\dot{q}''' R_F^2}{2k_c} \left[1 - \left(\frac{R_v}{R_F} \right)^2 \right] \ln(R_{wo}) + S R_{wo}^2 \ln(R_{wo}) \quad (B-49)$$

$$A_2^c = - \frac{\dot{q}''' R_F^2}{2k_c} \left[1 - \left(\frac{R_v}{R_F} \right)^2 \right] - S R_{wo}^2 \quad (B-50)$$

Multiply Equations (B-46) and (B-47) by $\cos(6m\phi) d\phi$ and integrate $(0, \pi/6)$

$$A_{6j}^F \left[\gamma_{6j}^w \left(\frac{R_{wo}}{R_{wI}} \right)^{6j} - \zeta_{6j}^w \left(\frac{R_{wI}}{R_{wo}} \right)^{6j} \right] = a_{6j}^c R_{wo}^{6j} + b_{6j}^c R_{wo}^{-6j} \quad (B-51a)$$

$$\frac{k_w A_{6j}^F}{R_{wo}} \left[\gamma_{6j}^w \left(\frac{R_{wo}}{R_{wI}} \right)^{6j} - \zeta_{6j}^w \left(\frac{R_{wI}}{R_{wo}} \right)^{6j} \right] = \frac{k_c a_{6j}^c}{R_{wo}} - \frac{k_c b_{6j}^c}{R_{wo}} \quad (B-51b)$$

Solve Equations (B-51a) and (B-51b) for a_{6j}^c and b_{6j}^c ,

$$a_{6j}^c = A_{6j}^F R_{wo}^{-6j} \gamma_{6j}^c \quad (B-52)$$

where

$$\gamma_{6j}^c = \frac{1}{2} \left\{ \gamma_{6j}^w \left(\frac{R_{wo}}{R_{wI}} \right)^{6j} \left(1 + \frac{k_w}{k_c} \right) + \zeta_{6j}^w \left(\frac{R_{wI}}{R_{wo}} \right)^{6j} \left(1 - \frac{k_w}{k_c} \right) \right\} \quad (B-53)$$

and

$$b_{6j}^c = A_{6j}^F R_{wo}^{-6j} \zeta_{6j}^c \quad (B-54)$$

where

$$\zeta_{6j}^c = \frac{1}{2} \left\{ \gamma_{6j}^w \left(\frac{R_{wo}}{R_{wI}} \right)^{6j} \left(1 - \frac{k_w}{k_c} \right) + \zeta_{6j}^w \left(\frac{R_{wI}}{R_{wo}} \right)^{6j} \left(1 + \frac{k_w}{k_c} \right) \right\} \quad (B-55)$$

Thus, the coolant temperature becomes

$$T_c = A_1^c + A_2^c \ln(r) + \frac{S r^2}{2} + \sum_{j=1}^{\infty} \left\{ A_{6j}^F \left[\gamma_{6j}^c \left(\frac{r}{R_{wo}} \right)^{6j} + \zeta_{6j}^c \left(\frac{R_{wo}}{r} \right)^{6j} \right] \cos(6j\phi) \right\} \quad (B-56)$$

The final boundary condition, Equation (B-8c), is used to determine the Fourier constants A_{6j}^F . Thus, substitution of Equation (B-56) into Equation (B-8c) yields

$$\left\{ \cos \phi \left[\frac{A_2^c}{r} + S r + \sum_{j=1}^{\infty} A_{6j}^F \left(\gamma_{6j}^c \frac{r^{6j-1}}{R_{wo}^{6j}} 6j - \zeta_{6j}^c \frac{R_{wo}^{6j}}{r^{6j+1}} 6j \right) \cos(6j\phi) \right] - \frac{\sin \phi}{r} \left[\sum_{j=1}^{\infty} A_{6j}^F \left(\gamma_{6j}^c \frac{r^{6j}}{R_{wo}^{6j}} + \zeta_{6j}^c \frac{R_{wo}^{6j}}{r^{6j}} \right) \right] \left[-6j \sin(6j\phi) \right] \right\}_{r=R_0/\cos \phi} = 0$$

Evaluate this relationship at the indicated boundary,

$$\sum_{j=1}^{\infty} A_{6j}^F 6j \left\{ \gamma_{6j}^c \left[\frac{R_0^{6j-1}}{R_{wo}^{6j}} \cos \phi \cos(6j\phi) \cos^{6j+1} \phi + \frac{R_0^{6j-1}}{R_{wo}^{6j}} \sin \phi \cos^{6j+1} \phi \sin(6j\phi) \right] - \zeta_{6j}^c \left[\frac{R_{wo}^{6j}}{R_0^{6j+1}} \cos^{6j+1} \phi \cos \phi \cos(6j\phi) - \frac{R_{wo}^{6j}}{R_0^{6j+1}} \cos^{6j+1} \phi \sin(6j\phi) \right] \right\} = -\frac{A_2^c}{R_0} \cos 2\phi - S R_0 \quad (B-57)$$

By using the relationship of the cosine of the sum or difference of two angles, Equation (B-57) takes the form

$$\sum_{j=1}^{\infty} A_{6j}^F 6j \left\{ \gamma_{6j}^c \left(\frac{R_0}{R_{wo}} \right)^{6j} \cos[(6j-1)\phi] \cos^{6j} \phi - \zeta_{6j}^c \left(\frac{R_{wo}}{R_0} \right)^{6j} \cos[(6j+1)\phi] \cos^{6j} \phi \right\} = \frac{\dot{q}''' R_F^2}{2k_c} \left[1 - \left(\frac{R_v}{R_F} \right)^2 \right] \cos \phi + S \left[R_{wo}^2 \cos \phi - \frac{R_0^2}{\cos \phi} \right] \quad (B-58)$$

Equation (B-58) cannot be satisfied exactly at all points on the line $r = R_0/\cos \phi$, but it can be satisfied at N points, i.e.,

$$\phi_n = \frac{(n-1)\pi}{(N-1)6} \quad n = 1, 2, 3, \dots, N \quad (B-59)$$

Note, that as N becomes infinitely large, it forms the line $r = R_0/\cos \phi$; thus there is an infinite sum to satisfy the boundary condition. Therefore, the procedure is to use a finite number of points and sum the series until it converges within a desired limit.¹

Substitute Equation (B-59) into (B-58),

$$\sum_{j=1}^{\infty} A_{6j}^F 6j \left\{ \gamma_{6j}^c \left(\frac{R_0}{R_{wo}} \right)^{6j} \cos \left[\frac{(6j-1)(n-1)\pi}{(N-1)6} \right] \cos^{6j} \left[\frac{(n-1)\pi}{(N-1)6} \right] - \zeta_{6j}^c \left(\frac{R_{wo}}{R_0} \right)^{6j} \cos \left[\frac{(6j+1)(n-1)\pi}{(N-1)6} \right] \cos^{6j} \left[\frac{(n-1)\pi}{(N-1)6} \right] \right\} = \frac{\dot{q}''' R_F^2}{2k_c} \left[1 - \left(\frac{R_v}{R_F} \right)^2 \right] \cos \left[\frac{(n-1)\pi}{(N-1)6} \right] + S \left\{ R_{wo}^2 \cos \left[\frac{(n-1)\pi}{(N-1)6} \right] - R_0^2 \cos^{-1} \left[\frac{(n-1)\pi}{(N-1)6} \right] \right\}$$

Equation (B-60) is a set of N simultaneous linear equations for the Fourier constants A_{6j}^F of the form

$$X_{n, 6j} A_{6j}^F = C_n \quad n = 1, 2, \dots, N \quad (B-61)$$

where

$$X_{n,6j} = 6j\gamma_{6j}^c \left(\frac{R_0}{R_{w0}}\right)^{6j} \cos \left[\frac{(6j-1)(n-1)\pi}{(N-1)6} \right] \cos^{6j} \left[\frac{(n-1)\pi}{(N-1)6} \right] \\ - 6j\zeta_{6j}^c \left(\frac{R_{w0}}{R_0}\right)^{6j} \cos \left[\frac{(6j+1)(n-1)\pi}{(N-1)6} \right] \cos^{6j} \left[\frac{(n-1)\pi}{(N-1)6} \right] \quad (B-62)$$

and

$$C_n = \left\{ \frac{\dot{q}''' R_F^2}{2k_c} \left[1 - \left(\frac{R_v}{R_F}\right)^2 \right] + S R_{w0}^2 \right\} \cos \left[\frac{(n-1)\pi}{(N-1)6} \right] - S R_0^2 \cos^{-1} \left[\frac{(n-1)\pi}{(N-1)6} \right] \quad (B-63)$$

A calculation of the coefficients A_{6j}^F for a fast reactor fuel pin with typical materials, dimensions, and power density yielded the following values as a function of pitch-to-diameter ratio, (R_0/R_{w0}) .

A_{6j}	1.05	R_0/R_{w0} 1.10	1.20
j = 1	3.69976	1.69386	0.60526
j = 2	7.26340×10^{-2}	1.45796×10^{-2}	1.43784×10^{-3}
j = 3	2.34903×10^{-3}	2.64841×10^{-4}	1.16246×10^{-5}
j = 4	8.09980×10^{-5}	3.47571×10^{-6}	-3.42341×10^{-8}
j = 5	3.87688×10^{-6}	2.14274×10^{-7}	9.23203×10^{-9}
j = 6	7.78819×10^{-8}	-1.60398×10^{-8}	-7.13779×10^{-10}
j = 7	2.28946×10^{-8}	2.58341×10^{-9}	6.17460×10^{-11}
j = 8	-4.40664×10^{-9}	-6.01758×10^{-10}	-9.28107×10^{-12}
j = 9	-8.37291×10^{-11}	-1.24441×10^{-11}	-4.68466×10^{-14}

Since these coefficients multiply a number less than unity [see Equation (B-64)], the series converges quite readily with a reasonable number of terms. Also note that as the pitch-to-diameter ratio increases, the convergence increases.

B.6 TEMPERATURE SUMMARY

By using the previously established results, the temperature equations are now written in the following form.

FUEL

$$T_F = A_1^{F*} + A_2^{F*} \ln \left(\frac{r}{R_F} \right) + \frac{\dot{q}''' R_F^2}{4k_F} \left[1 - \left(\frac{r}{R_F} \right)^2 \right] + \sum_{j=1}^{\infty} \left\{ A_{6j}^F \left[\left(\frac{r}{R_F} \right)^{6j} + \left(\frac{R_v}{r} \right)^{6j} \gamma_{6j}^F \right] \cos(6j\phi) \right\} \quad (B-64)$$

where

$$A_1^{F*} = T_{wB} - \sum (R_{wA}) + \frac{\dot{q}''' R_F^2}{2k_w} \left[1 - \left(\frac{R_v}{R_F} \right)^2 \right] \ln \left(\frac{R_{wA}}{R_{wI}} \right) + \frac{\dot{q}''' R_F}{2h_G} \left[1 - \left(\frac{R_v}{R_F} \right)^2 \right] \quad (B-65)$$

$$A_2^{F*} = \frac{\dot{q}''' R_v^2}{2k_F} \quad (B-66)$$

$$\sum (R_{wA}) = \sum_{j=1}^{\infty} \left\{ A_{6j}^F \left[\gamma_{6j}^w \left(\frac{R_{wA}}{R_{wI}} \right)^{6j} + \zeta_{6j}^w \left(\frac{R_{wI}}{R_{wA}} \right)^{6j} \right] \cos \left(\frac{j\pi}{2} \right) \right\} \quad (B-67)$$

A_{6j}^F - See Equations (B-61), (62), and (B-63)

γ_{6j}^w - See Equation (B-71)

ζ_{6j}^w - See Equation (B-72)

CLADDING

$$T_w = A_1^{w*} - A_2^{w*} \ln \left(\frac{r}{R_{wA}} \right) + \sum_{j=1}^{\infty} \left\{ A_{6j}^F \left[\gamma_{6j}^w \left(\frac{r}{R_{wI}} \right)^{6j} + \zeta_{6j}^w \left(\frac{R_{wI}}{r} \right)^{6j} \right] \cos(6j\phi) \right\} \quad (B-68)$$

where

$$A_1^{w*} = T_{wB} - \sum (R_{wA}) \quad (B-69)$$

$$A_2^{w*} = \frac{\dot{q}''' R_F^2}{2k_w} \left[1 - \left(\frac{R_v}{R_F} \right)^2 \right] \quad (B-70)$$

$$\gamma_{6j}^w = \frac{1}{2} \left[1 + \frac{k_F}{k_w} + \frac{6jk_F}{R_F h_G} \right] + \frac{1}{2} \left(\frac{R_v}{R_F} \right)^{12j} \left[1 - \frac{k_F}{k_w} - \frac{6jk_F}{R_F h_G} \right] \quad (B-71)$$

$$\zeta_{6j}^w = \frac{1}{2} \left[1 - \frac{k_F}{k_w} + \frac{6jk_F}{R_F h_G} \right] + \frac{1}{2} \left(\frac{R_v}{R_F} \right)^{12j} \left[1 + \frac{k_F}{k_w} - \frac{6jk_F}{R_F h_G} \right] \quad (B-72)$$

COOLANT

$$T_c = A_1^{c*} + A_2^{c*} \ln \left(\frac{r}{R_{wo}} \right) + \frac{S R_{wo}^2}{2} \left[\left(\frac{r}{R_{wo}} \right)^2 - 1 \right] + \sum_{j=1}^{\infty} \left\{ A_{6j}^F \left[\gamma_{6j}^c \left(\frac{r}{R_{wo}} \right)^{6j} + \zeta_{6j}^c \left(\frac{R_{wo}}{r} \right)^{6j} \right] \cos(6j\phi) \right\} \quad (B-73)$$

where

$$A_1^{c*} = T_{wB} - \sum (R_{wA}) - \frac{\dot{q}''' R_F^2}{2k_w} \left[1 - \left(\frac{R_v}{R_F} \right)^2 \right] \ln \left(\frac{R_{wo}}{R_{wA}} \right) \quad (B-74)$$

$$A_2^{c*} = \frac{\dot{q}''' R_F^2}{2k_c} \left[1 - \left(\frac{R_v}{R_F} \right)^2 \right] S R_{wo}^2 \quad (B-75)$$

$$\gamma_{6j}^c = \frac{1}{2} \gamma_{6j}^w \left(\frac{R_{wo}}{R_{wI}} \right)^{6j} \left(1 + \frac{k_w}{k_c} \right) + \frac{1}{2} \zeta_{6j}^c \left(\frac{R_{wI}}{R_{wo}} \right)^{6j} \left(1 - \frac{k_w}{k_c} \right) \quad (B-76)$$

$$\zeta_{6j}^c = \frac{1}{2} \gamma_{6j}^w \left(\frac{R_{wo}}{R_{wI}} \right)^{6j} \left(1 - \frac{k_w}{k_c} \right) + \frac{1}{2} \zeta_{6j}^c \left(\frac{R_{wI}}{R_{wo}} \right)^{6j} \left(1 + \frac{k_w}{k_c} \right) \quad (B-77)$$

SLUG FLOW NOMENCLATURE

VARIABLE	DESCRIPTION
A_c	Cross-sectional area of coolant channel
A_1^c, A_2^c, S	Constants in coolant temperature solution
A_1^F, A_2^F	Constants in fuel temperature solution
A_1^w, A_2^w	Constants in cladding temperature solution
$a_k^c, b_k^c, c_k^c, d_k^c$	Fourier coefficients in coolant temperature solution
$a_k^F, b_k^F, c_k^F, d_k^F$	Fourier coefficients in fuel temperature solution
$a_k^w, b_k^w, c_k^w, d_k^w$	Fourier coefficients in cladding temperature solution
h_G	Gap heat-transfer coefficient
k_c	Coolant conductivity
k_F	Fuel conductivity
k_w	Cladding conductivity
$\dot{q}'''$	Volumetric heat generation rate in the fuel
R_F	Outside radius of fuel pellet
R_0	One-half the pitch of the rods
R_v	Radius of the void surface
R_{wA}	Radius of the centerline of the cladding
R_{wI}	Inside radius of the cladding
R_{wo}	Outside radius of the cladding
T_c	Coolant temperature
T_c^*	Coolant temperature transform variable
T_F	Fuel temperature
T_F^*	Fuel temperature transform variable
T_w	Cladding temperature
T_{wB}	Cladding temperature at the center, i.e., at the point $(R_{wA}/\pi 12)$.
u	Velocity of the coolant slug
α_c	Molecular diffusivity of heat of the coolant
C_{pc}	Specific heat of the coolant
ρ_c	Density of the coolant

NOTE: Other variables are defined as they appear in the text.

1. Axford, R. A., "Multiregional Analysis of Temperature Fields and Heat Fluxes in Tube Bundles with Internal, Solid, Nuclear Heat Sources," LA-3167
2. Hunsbedt, A., et al., "Velocity Profiles in a Seven Rod Spacerless Bundle," BRDO Internal Memorandum, January 30, 1969.
3. Levchenko, Y. S., Subbotin, V. I., and Ushakov, P. A., "The Distribution of Coolant Velocities and Wall Stresses in Closely Packed Rods," Atomnaya Energiya, 22, 218(1967)

APPENDIX C

THE VELVET-II CODE

C.1 INTRODUCTION

The velocity and temperature distribution models for turbulent flow of a liquid metal in a close-packed rod bundle, described in Sections 2 and 3, have been programmed in FORTRAN IV as the VELVET-II code.

The code performs the following general functions:

1. Establishes nodal structure.
2. Calculates velocity profiles in the coolant.
3. Calculates eddy diffusivity of momentum in the coolant by one of four available models.
4. Edits velocity profiles and flow field results (upon user option calculation may be terminated at this point).
5. Correlates eddy diffusivity of heat-to-eddy diffusivity of momentum by an internal model or by a user-specified value.
6. Calculates temperature profiles in the fuel, cladding, and coolant with velocity distribution and eddy diffusivity effects or with a slug flow model.
7. Performs analysis of the heat transfer from cladding to coolant and calculates appropriate heat-transfer coefficients.
8. Edits temperature and eddy diffusivity profiles and heat-transfer results.

C.2 CODE DESCRIPTION

A flow chart of VELVET-II is shown in Figure C-1. The method of calculation is shown below in the sequence it occurs in the code.

1. Subroutine CASE is called and the common block is zeroed appropriate to either an independent or dependent case.
2. Input is read and printed out.
3. Subroutine VGEO is called to set up the nodal geometry, as shown in Figure 3-6 (with a similar structure in the fuel.) The number of radial and circumferential nodes are specified as input. If a slug flow analysis is being performed, the calculation goes from here directly to the velocity edit and then to the temperature solution.
4. Subroutine VUNIV is called to calculate a wall shear stress distribution by Equation (18) and velocity by Equations (23) through (25). Eddy diffusivity of momentum in the radial direction is calculated from Equations (28) through (30), and the circumferential component is calculated from one of four models (choice specified by user) in accordance with Equations (34), (A-1), (A-2) or (A-4).
5. Subroutine VED1 is called. This subroutine is an edit of the wall shear stress, velocity distribution and eddy diffusivity of momentum calculations.
6. If desired, calculation may be terminated at this point.
7. Subroutine VET1 is called. It calculates a first approximation to the nodal temperatures in the cladding and coolant, assuming uniform fuel surface temperature, uniform fuel surface heat flux, and no circumferential heat transfer.
8. At this point, an iterative procedure is entered for calculation of the nodal temperature distributions in the cladding and coolant. This procedure is composed of an inner and outer iteration cycle.

The outer iteration is performed on the fuel surface temperature and fuel surface heat flux (unknown quantities) which are determined by matching the nodal solution for the cladding and coolant with the analytical model for the fuel at the fuel surface.

The inner iteration is on the simultaneous solution of the cladding and coolant nodal temperatures, having assumed a particular fuel surface temperature distribution. Since these nodal temperatures represent a large system of simultaneous algebraic equations, the solution of the system of equations is obtained by an approximate, iterative method. The technique employed is that of relaxation by lines. This technique is executed by solving the temperatures, on any one radial line, simultaneously and exactly, using the most recent approximation to the circumferentially adjacent nodal temperatures. Each radial line of nodes is successively solved until convergence is realized.

9. The mechanics of the inner and outer iterations are as follows:

- a. Subroutine VET2 is called. This first calculates the eddy diffusivity of heat by Equations (84a) and (84b), with the correlation values either calculated from an internal model, Equations (82) and (83), or by user specified values. The nodal admittance factors, Equations (74) and (76), are calculated using the most recent values of temperature to evaluate the material properties. Also, at this time, the right-hand side of the heat balance in Equation (78) is calculated.
 - b. Subroutine VET3 is called. This incorporates the boundary conditions into the nodal heat balance for the appropriate nodes, i.e., those nodes adjacent to a boundary. In this way, it is possible to solve exclusively for the internal nodes and also enforce the boundary conditions.
 - c. Having set up the simultaneous equations for the nodal temperatures, their solution is accomplished (the inner iteration) with subroutines VSOL and VREL, which perform the solution by relaxation by lines. Subroutine VSOL rearranges the heat balance in Equation (78), on each radial line of nodes, by transferring the circumferentially adjacent temperatures to the right-hand side of the equation using the most recent approximations to these temperatures. Subroutine VREL is then called to solve the equations on a given radial line (which now has a tri-diagonal coefficient matrix) by elimination and back substitution. Subroutine VSOL sweeps through the entire temperature field twice, then checks for convergence. This procedure is repeated until convergence (of the inner iteration) is realized, and control is returned to the executive package, VELVET-II.
 - d. When the simultaneous equations for the temperatures have converged, subroutine VET4 is called to calculate a new fuel surface temperature distribution. If the fuel surface temperature has not converged, a new fuel surface temperature distribution is calculated, and control returned to point (9a). If the fuel surface temperature has converged, the outer iteration is converged and the calculation proceeds.
10. Remaining in subroutine VET4, the fuel temperature distribution is calculated via Equation (70). Boundary temperatures in the clad and coolant are calculated.
 11. Subroutine VED2 is called, which calculates the heat transfer data of section (3.5) and then performs a complete editing of the temperature distribution, heat transfer, and eddy diffusivity of heat calculations.
 12. A new case is called and control returned to point (1) above or execution is terminated.

C.3 INPUT DESCRIPTION

The input to VELVET-II is entered in a free-form format. A sample input, for an independent case followed by a dependent case, is shown in Figure C-2. A sample output for the independent case of Figure C-2 is shown in Figure C-3.

The following is a description of the input variables for VELVET-II.

ROVI	Void Radius (in)
AI	Fuel Radius (in)
AI1	Inside Clad Radius (in)
BI	Outside Clad Radius (in)
CI	One-half the Rod Pitch (in)
RHOF	Fuel Density (lbm/ft ³)
RHOW	Clad Density (lbm/ft ³)
GC	Gap Heat Transfer Coefficient (Btu/hr.ft ² .°F)
ZKF	First Approximation to Fuel Conductivity (Btu/hr.ft.°F)
ZKW	First Approximation to Clad Conductivity (Btu/hr.ft.°F)
ZKC	First Approximation to Coolant Conductivity (Btu/hr.ft.°F)
QL	Linear Heat Generation Rate (Kw/ft)
UAVGI	Average Coolant Velocity (ft/sec)
TFS	Fuel Surface Temperature (°F) at the Angular Midpoint, i.e., $\phi = 15^\circ$.
GRH	Fraction of Heat Generation due to Neutron-Gamma Heating ($0 \leq \text{GRH} \leq 1$)
PSIR	(Input if IPOP = 1) Ratio of eddy diffusivity of heat to eddy diffusivity of momentum in the radial direction.
PSIP	(Input if IPOP = 1) Ratio of eddy diffusivity of heat to eddy diffusivity of momentum in the circumferential direction.
NF	Number of radial divisions in the fuel (Max = 10)
NW	Number of radial divisions in the clad (Max = 5)
NC	Number of radial divisions in the coolant between the outer edge of the buffer layer and one-half the pitch, i.e., the outer coolant boundary at $\phi = 0$. (Max = 10)
NT	Number of circumferential divisions (The same in all regions) (Max = 10)
IEMP = 1	Calculate circumferential eddy diffusivity of momentum as equal to the radial component.
= 2	Calculate circumferential eddy diffusivity of momentum via Rapiet and Redman's model (Equation A-1)
= 3	Calculate circumferential eddy diffusivity of momentum via Eifler and Nijsing's model (Equation A-2)
= 4	Calculate circumferential eddy diffusivity of momentum via Bobkov's model (Equation A-4)
ISLUG = 0	Calculate velocity and temperature profiles
= 1	Calculate temperature profiles with slug flow approximation to the coolant distribution.
= 2	Calculate only the velocity profile and skip the temperature calculation. If this option is used, the input for the temperature must still be entered since it is used to calculate the approximate bulk coolant temperature and evaluate the coolant material properties.

IPOP = 0	Calculate the ratio of eddy diffusivity of heat to eddy diffusivity of momentum by the correlation specified in the code (Equations 82 and 83)
= 1	Use the input values of the ratio of eddy diffusivity of heat to eddy diffusivity of momentum (see variables PSIR and PSIP)
EPSI	Maximum change between iterations to define the convergence of the solution for the simultaneous equations for the nodal temperatures (recommend 0.01 °F)
EPSIT	Maximum change between iterations to define the convergence of the solution for the fuel surface temperature (recommend 0.01 °F)
IENT	Maximum number of iterations on the simultaneous equations for the nodal temperatures (recommend 250).
ICNVRG	Maximum number of iterations on the fuel surface temperature (recommend 25)
DENC(J) J=1, 2, 3	Three values for quadratic fit of coolant density to temperature. (lbm/ft ³ (°F) ^{J-1})
SHC(J) J=1, 2, 3	Three values for quadratic fit of coolant specific heat to temperature (Btu/lbm (°F) ^J)
TCC(J) J=1, 2, 3	Three values for quadratic fit of coolant conductivity to temperature (Btu/hr ft (°F) ^J)
TCW(J) J=1, 2, 3	Three values for quadratic fit of clad conductivity to temperature (Btu/hr ft (°F) ^J)
TCF(J) J=1, 2, 3	Three values for quadratic fit of fuel conductivity to temperature (Btu/hr ft (°F) ^J)
VISC(J) J=1, 2, 3	Three values for quadratic fit of coolant absolute viscosity to temperature (lbm/hr ft (°F) ^{J-1})

Figure C-1 VELVET-II Flow Chart

Figure C-1 VELVET-II Flow Chart (Continued)

Figure C-1 VELVET-II Flow Chart. (Continued)

Figure C-1 VELVET-II Flow Chart (Continued)

)VELV12 *

)INPUT

RUVI	=	0.0,		
AI	=	0.105,		
AII	=	0.110,		
BI	=	0.125,		
CI	=	0.1375,		
RHOI	=	690.0,		
RHOW	=	480.0,		
GC	=	1500.0,		
ZKF	=	1.5,		
ZKW	=	12.0,		
ZKC	=	40.0,		
QL	=	15.0,		
UAVGI	=	20.0,		
TFS	=	1710.0,		
GRH	=	0.0,		
NF	=	10,		
NW	=	3,		
NC	=	6,		
NT	=	10,		
IEMP	=	1,		
ISLUG	=	0,		
IPOP	=	0,		
EPSI	=	0.01,		
EPSIT	=	0.01,		
TEST	=	250,		
ICNVRG	=	25,		
DENC(1)	=	59.4,	-8.30E-3,	0.0,
SHC(1)	=	0.346,	-8.00E-5,	3.44E-8,
VISC(1)	=	1.481,	-1.54E-3,	5.60E-7,
TCC(1)	=	53.44,	-1.57E-2,	0.0,
TCW(1)	=	7.3,	4.40E-3,	0.0,
TCF(1)	=	4.161,	-1.98E-3,	3.30E-7,

)VELV12 *

)INPUT

ISLUG = 1,

)LAST *

Figure C-2 Sample Input

START OF CASE JVELV

063569 1127.7

NAMELIST		INPUT	
AI	0.10500000E 03	AI	0.11000000E 00
ROVI	0.	RHOI	0.69000000E 03
ZKW	0.12000000E 02	ZKC	0.40000000E 02
OL	0.15000000E 02	GRH	0.
PSIP	0.	TFS	0.17100000E 04
CI	0.13750000E 00	ZKF	0.15000000E 01
UAVGI	0.20000000E 02	PSIR	0.
NC	6	NF	10
ISLUG	0	IPOP	0
EPSI	0.10000000E-01		
ICNVRG	25		
EPSIT	0.10000000E-03		
IESI	250		
DENC	(1)		
1	0.59400000E 02	-0.83000000E-02	0.
SHC	(1)		
1	0.34600000E 01	-0.80000000E-04	0.34400000E-07
TCC	(1)		
1	0.53440000E 02	-0.15700000E-01	0.
TCW	(1)		
1	0.73000000E 01	0.44000000E-02	0.
VISC	(1)		
1	0.14810000E 01	-0.15400000E-02	0.56000000E-06
TCF	(1)		
1	0.41610000E 01	-0.19800000E-02	0.33000000E-05
END	NAMELIST	INPUT	

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Figure C-3 Sample Output (Continued)

***** VELOCITY AND EDDY DIFFUSIVITY CALCULATION *****

INPUT AVG VEL (FT/SEC)= 20.000 PITCH TO DIAM RATIO= 1.10000 HYDRAULIC DIAM (IN)= 0.08355
 EQUIV ANNULUS WIDTH (IN) ..= 0.01939 REYNOLDS NUMBER= 50732 FRICTION FACTOR= 0.02290
 DP/DZ (LBF/SQ.IN/IN)=-0.60536 AVG WALL SHEAR (LBF/SQ.IN) = 0.01265 CALC AVG VEL (FT/SEC)= 20.859

ABS. VISCOSITY(LBM/HR.FT) = 0.50589 BULK COOLANT TEMP (F)= 988.53 DENSITY (LBM/CU.FT)= 51.195
 KIN. VISCOSITY(SQ.FT/HR) = 0.00988

CIRCUMFERENTIAL EDDY DIFFUSIVITY OF MOMENTUM CALCULATED AS EQUAL TO THE RADIAL COMPONENT

ANGULAR POSITION (DEGREES) = 0.

LOCAL WALL SHEAR STRESS (LBF/SQ.IN)= 0.01151
 LOCAL WALL SHEAR NORMALIZED TO AVG= 0.90997
 RADIAL DISTANCE TO OUTER BOUNDARY (IN) = 0.01250

DIST FROM WALL (IN) = 0.8024E-04 0.5620E-03 0.1927E-02 0.3849E-02 0.5772E-02 0.7695E-02 0.9618E-02 0.1154E-01
 DIST FROM WALL / EQV. RAD. = 0.4138E-02 0.2899E-01 0.9937E-01 0.1985E 00 0.2977E 00 0.3969E 00 0.4961E 00 0.5953E 00
 COOLANT VELOCITY (FT/SEC) = 0.2539E 01 0.1101E 02 0.1605E 02 0.1782E 02 0.1885E 02 0.1959E 02 0.2016E 02 0.2062E 02
 VELOCITY NORMALIZED TO AVG = 0.1217E 00 0.5280E 00 0.7696E 00 0.8543E 00 0.9038E 00 0.9390E 00 0.9663E 00 0.9886E 00
 PERIPHERAL DIFF/ VISCOSITY = 0. 0.2327E 01 0.1920E 02 0.3203E 02 0.3751E 02 0.3567E 02 0.2649E 02 0.9979E 01
 RADIAL DIFF / VISCOSITY = 0. 0.2327E 01 0.1920E 02 0.3203E 02 0.3751E 02 0.3567E 02 0.2649E 02 0.9979E 01

ANGULAR POSITION (DEGREES) = 1.50

LOCAL WALL SHEAR STRESS (LBF/SQ.IN)= 0.01152
 LOCAL WALL SHEAR NORMALIZED TO AVG= 0.91121
 RADIAL DISTANCE TO OUTER BOUNDARY (IN) = 0.01255

DIST FROM WALL (IN) = 0.8024E-04 0.5620E-03 0.1927E-02 0.3849E-02 0.5772E-02 0.7695E-02 0.9618E-02 0.1154E-01
 DIST FROM WALL / EQV. RAD. = 0.4138E-02 0.2899E-01 0.9937E-01 0.1985E 00 0.2977E 00 0.3969E 00 0.4961E 00 0.5953E 00
 COOLANT VELOCITY (FT/SEC) = 0.2542E 01 0.1102E 02 0.1606E 02 0.1783E 02 0.1887E 02 0.1960E 02 0.2017E 02 0.2064E 02
 VELOCITY NORMALIZED TO AVG = 0.1219E 00 0.5285E 00 0.7702E 00 0.8549E 00 0.9045E 00 0.9397E 00 0.9670E 00 0.9894E 00
 PERIPHERAL DIFF/ VISCOSITY = 0. 0.2330E 01 0.1923E 02 0.3210E 02 0.3767E 02 0.3592E 02 0.2686E 02 0.1048E 02
 RADIAL DIFF / VISCOSITY = 0. 0.2330E 01 0.1923E 02 0.3210E 02 0.3767E 02 0.3592E 02 0.2686E 02 0.1048E 02

Figure C-3 Sample Output (Continued)

0.

ANGULAR POSITION (DEGREES) = 4.50

LOCAL WALL SHEAR STRESS (LBF/SQ.IN)= 0.01165
 LOCAL WALL SHEAR NORMALIZED TO AVG= 0.92091
 RADIAL DISTANCE TO OUTER BOUNDARY (IN) = 0.01293

DIST FROM WALL (IN) = 0.8024E-04 0.5620E-03 0.1927E-02 0.3849E-02 0.5772E-02 0.7695E-02 0.9618E-02 0.1154E-01
 0.1271E-01 0.1293E-01
 DIST FROM WALL / EQV. RAD. = 0.4138E-02 0.2899E-01 0.9937E-01 0.1985E 00 0.2977E 00 0.3969E 00 0.4961E 00 0.5953E 00
 0.6557E 00 0.6863E 00
 COOLANT VELOCITY (FT/SEC) = 0.2569E 01 0.1111E 02 0.1616E 02 0.1794E 02 0.1898E 02 0.1972E 02 0.2029E 02 0.2076E 02
 0.2101E 02 0.2105E 02
 VELOCITY NORMALIZED TO AVG = 0.1232E 00 0.5326E 00 0.7749E 00 0.8601E 00 0.9100E 00 0.9454E 00 0.9728E 00 0.9953E 00
 0.1007E 01 0.1009E 01
 PERIPHERAL DIFF / VISCOSITY = 0. 0.2352E 01 0.1945E 02 0.3271E 02 0.3884E 02 0.3783E 02 0.2969E 02 0.1442E 02
 0.1606E 01 0.
 RADIAL DIFF / VISCOSITY = 0. 0.2352E 01 0.1945E 02 0.3271E 02 0.3884E 02 0.3783E 02 0.2969E 02 0.1442E 02
 0.1606E 01 0.

ANGULAR POSITION (DEGREES) = 7.50

LOCAL WALL SHEAR STRESS (LBF/SQ.IN)= 0.01187
 LOCAL WALL SHEAR NORMALIZED TO AVG= 0.93895
 RADIAL DISTANCE TO OUTER BOUNDARY (IN) = 0.01369

DIST FROM WALL (IN) = 0.8024E-04 0.5620E-03 0.1927E-02 0.3849E-02 0.5772E-02 0.7695E-02 0.9618E-02 0.1154E-01
 0.1271E-01 0.1331E-01 0.1369E-01
 DIST FROM WALL / EQV. RAD. = 0.4138E-02 0.2899E-01 0.9937E-01 0.1985E 00 0.2977E 00 0.3969E 00 0.4961E 00 0.5953E 00
 0.6557E 00 0.6863E 00 0.7059E 00
 COOLANT VELOCITY (FT/SEC) = 0.2620E 01 0.1127E 02 0.1635E 02 0.1814E 02 0.1919E 02 0.1994E 02 0.2051E 02 0.2099E 02
 0.2124E 02 0.2136E 02 0.2143E 02
 VELOCITY NORMALIZED TO AVG = 0.1256E 00 0.5402E 00 0.7837E 00 0.8697E 00 0.9201E 00 0.9558E 00 0.9835E 00 0.1006E 01
 0.1016E 01 0.1024E 01 0.1027E 01
 PERIPHERAL DIFF / VISCOSITY = 0. 0.2393E 01 0.1985E 02 0.3384E 02 0.4103E 02 0.4142E 02 0.3500E 02 0.2178E 02
 0.1039E 02 0.3656E 01 0.
 RADIAL DIFF / VISCOSITY = 0. 0.2393E 01 0.1985E 02 0.3384E 02 0.4103E 02 0.4142E 02 0.3500E 02 0.2178E 02
 0.1039E 02 0.3656E 01 0.

ANGULAR POSITION (DEGREES) = 10.50

LOCAL WALL SHEAR STRESS (LBF/SQ.IN)= 0.01218
 LOCAL WALL SHEAR NORMALIZED TO AVG= 0.96286
 RADIAL DISTANCE TO OUTER BOUNDARY (IN) = 0.01484

DIST FROM WALL (IN) = 0.8024E-04 0.5620E-03 0.1927E-02 0.3849E-02 0.5772E-02 0.7695E-02 0.9618E-02 0.1154E-01
 0.1271E-01 0.1331E-01 0.1426E-01 0.1484E-01
 DIST FROM WALL / EQV. RAD. = 0.4138E-02 0.2899E-01 0.9937E-01 0.1985E 00 0.2977E 00 0.3969E 00 0.4961E 00 0.5953E 00
 0.6557E 00 0.6863E 00 0.7358E 00 0.7655E 00
 COOLANT VELOCITY (FT/SEC) = 0.2686E 01 0.1148E 02 0.1659E 02 0.1840E 02 0.1947E 02 0.2022E 02 0.2081E 02 0.2129E 02

Figure C-3 Sample Output (Continued)

0.2154E 02 0.2166E 02 0.2184E 02 0.2205E 02
 VELOCITY NORMALIZED TO AVG = 0.1288E 00 0.5502E 00 0.7952E 00 0.8823E 00 0.9333E 00 0.9695E 00 0.9975E 00 0.1020E 01
 0.1033E 01 0.1038E 01 0.1047E 01 0.1052E 01
 PERIPHERAL DIFF / VISCOSITY = 0. 0.2448E 01 0.2038E 02 0.3536E 02 0.4398E 02 0.4625E 02 0.4217E 02 0.3173E 02
 0.2226E 02 0.1656E 02 0.6070E 01 0.
 RADIAL DIFF / VISCOSITY = 0. 0.2448E 01 0.2038E 02 0.3536E 02 0.4398E 02 0.4625E 02 0.4217E 02 0.3173E 02
 0.2226E 02 0.1656E 02 0.6070E 01 0.
 * * * * *

ANGULAR POSITION (DEGREES) = 13.50

LOCAL WALL SHEAR STRESS (LBF/SQ.IN)= 0.01251
 LOCAL WALL SHEAR NORMALIZED TO AVG= 0.98962
 RADIAL DISTANCE TO OUTER BOUNDARY (IN) = 0.01641

DIST FROM WALL (IN) = 0.8024E-04 0.5620E-03 0.1927E-02 0.3849E-02 0.5772E-02 0.7695E-02 0.9618E-02 0.1154E-01
 0.1271E-01 0.1331E-01 0.1426E-01 0.1563E-01 0.1641E-01
 DIST FROM WALL / EQV. RAD. = 0.4138E-02 0.2899E-01 0.9937E-01 0.1985E 00 0.2977E 00 0.3969E 00 0.4961E 00 0.5953E 00
 0.6557E 00 0.6863E 00 0.7358E 00 0.8059E 00 0.8462E 00
 COOLANT VELOCITY (FT/SEC) = 0.2761E 01 0.1171E 02 0.1685E 02 0.1869E 02 0.1977E 02 0.2054E 02 0.2113E 02 0.2162E 02
 0.2187E 02 0.2199E 02 0.2218E 02 0.2242E 02 0.2255E 02
 VELOCITY NORMALIZED TO AVG = 0.1324E 00 0.5613E 00 0.8079E 00 0.8962E 00 0.9479E 00 0.9846E 00 0.1013E 01 0.1036E 01
 0.1049E 01 0.1054E 01 0.1063E 01 0.1075E 01 0.1081E 01
 PERIPHERAL DIFF / VISCOSITY = 0. 0.2508E 01 0.2098E 02 0.3709E 02 0.4737E 02 0.5183E 02 0.5045E 02 0.4325E 02
 0.3601E 02 0.3151E 02 0.2308E 02 0.8620E 01 0.
 RADIAL DIFF / VISCOSITY = 0. 0.2508E 01 0.2098E 02 0.3709E 02 0.4737E 02 0.5183E 02 0.5045E 02 0.4325E 02
 0.3601E 02 0.3151E 02 0.2308E 02 0.8620E 01 0.
 * * * * *

ANGULAR POSITION (DEGREES) = 16.50

LOCAL WALL SHEAR STRESS (LBF/SQ.IN)= 0.01285
 LOCAL WALL SHEAR NORMALIZED TO AVG= 1.01623
 RADIAL DISTANCE TO OUTER BOUNDARY (IN) = 0.01841

DIST FROM WALL (IN) = 0.8024E-04 0.5620E-03 0.1927E-02 0.3849E-02 0.5772E-02 0.7695E-02 0.9618E-02 0.1154E-01
 0.1271E-01 0.1331E-01 0.1426E-01 0.1563E-01 0.1741E-01 0.1841E-01
 DIST FROM WALL / EQV. RAD. = 0.4138E-02 0.2899E-01 0.9937E-01 0.1985E 00 0.2977E 00 0.3969E 00 0.4961E 00 0.5953E 00
 0.6557E 00 0.6863E 00 0.7358E 00 0.8059E 00 0.8979E 00 0.9493E 00
 COOLANT VELOCITY (FT/SEC) = 0.2835E 01 0.1194E 02 0.1711E 02 0.1898E 02 0.2007E 02 0.2085E 02 0.2145E 02 0.2194E 02
 0.2220E 02 0.2232E 02 0.2251E 02 0.2276E 02 0.2305E 02 0.2320E 02
 VELOCITY NORMALIZED TO AVG = 0.1359E 00 0.5722E 00 0.8204E 00 0.9099E 00 0.9623E 00 0.9995E 00 0.1028E 01 0.1052E 01
 0.1064E 01 0.1070E 01 0.1079E 01 0.1091E 01 0.1105E 01 0.1112E 01
 PERIPHERAL DIFF / VISCOSITY = 0. 0.2569E 01 0.2160E 02 0.3888E 02 0.5091E 02 0.5766E 02 0.5916E 02 0.5539E 02
 0.5051E 02 0.4730E 02 0.4104E 02 0.2992E 02 0.1135E 02 0.
 RADIAL DIFF / VISCOSITY = 0. 0.2569E 01 0.2160E 02 0.3888E 02 0.5091E 02 0.5766E 02 0.5916E 02 0.5539E 02
 0.5051E 02 0.4730E 02 0.4104E 02 0.2992E 02 0.1135E 02 0.
 * * * * *

ANGULAR POSITION (DEGREES) = 19.50

LOCAL WALL SHEAR STRESS (LBF/SQ.IN)= 0.01315
 LOCAL WALL SHEAR NORMALIZED TO AVG= 1.04018
 RADIAL DISTANCE TO OUTER BOUNDARY (IN) = 0.02087

Figure C-3 Sample Output (Continued)

```

DIST FROM WALL (IN)      = 0.8024E-04 0.5620E-03 0.1927E-02 0.3849E-02 0.5772E-02 0.7695E-02 0.9618E-02 0.1154E-01
                           0.1271E-01 0.1331E-01 0.1426E-01 0.1563E-01 0.1741E-01 0.1964E-01 0.2087E-01
DIST FROM WALL / EQV. RAD. = 0.4138E-02 0.2899E-01 0.9937E-01 0.1985E 00 0.2977E 00 0.3969E 00 0.4961E 00 0.5953E 00
                           0.6557E 00 0.6863E 00 0.7358E 00 0.8059E 00 0.8979E 00 0.1013E 01 0.1076E 01
COOLANT VELOCITY (FT/SEC) = 0.2902E 01 0.1214E 02 0.1734E 02 0.1923E 02 0.2034E 02 0.2112E 02 0.2173E 02 0.2223E 02
                           0.2249E 02 0.2262E 02 0.2281E 02 0.2306E 02 0.2335E 02 0.2368E 02 0.2385E 02
VELOCITY NORMALIZED TO AVG = 0.1391E 00 0.5820E 00 0.8315E 00 0.9221E 00 0.9751E 00 0.1013E 01 0.1042E 01 0.1066E 01
                           0.1078E 01 0.1084E 01 0.1093E 01 0.1105E 01 0.1119E 01 0.1135E 01 0.1143E 01
PERIPHERAL DIFF/ VISCOSITY = 0. 0.2624E 01 0.2218E 02 0.4061E 02 0.5434E 02 0.6338E 02 0.6772E 02 0.6737E 02
                           0.6484E 02 0.6290E 02 0.5882E 02 0.5102E 02 0.3724E 02 0.1431E 02 0.
RADIAL DIFF / VISCOSITY    = 0. 0.2624E 01 0.2218E 02 0.4061E 02 0.5434E 02 0.6338E 02 0.6772E 02 0.6737E 02
                           0.6484E 02 0.6290E 02 0.5882E 02 0.5102E 02 0.3724E 02 0.1431E 02 0.
                           * * * * *

```

ANGULAR POSITION (DEGREES) = 22.50

```

LOCAL WALL SHEAR STRESS (LBF/SQ.IN) ....= 0.31340
LOCAL WALL SHEAR NORMALIZED TO AVG .....= 1.35987
RADIAL DISTANCE TO OUTER BOUNDARY (IN) = 0.32383

```

```

DIST FROM WALL (IN)      = 0.8024E-04 0.5620E-03 0.1927E-02 0.3849E-02 0.5772E-02 0.7695E-02 0.9618E-02 0.1154E-01
                           0.1271E-01 0.1331E-01 0.1426E-01 0.1563E-01 0.1741E-01 0.1964E-01 0.2235E-01 0.2383E-01
DIST FROM WALL / EQV. RAD. = 0.4138E-02 0.2899E-01 0.9937E-01 0.1985E 00 0.2977E 00 0.3969E 00 0.4961E 00 0.5953E 00
                           0.6557E 00 0.6863E 00 0.7358E 00 0.8059E 00 0.8979E 00 0.1013E 01 0.1153E 01 0.1229E 01
COOLANT VELOCITY (FT/SEC) = 0.2957E 01 0.1231E 02 0.1753E 02 0.1944E 02 0.2056E 02 0.2135E 02 0.2196E 02 0.2246E 02
                           0.2273E 02 0.2286E 02 0.2305E 02 0.2330E 02 0.2350E 02 0.2393E 02 0.2429E 02 0.2446E 02
VELOCITY NORMALIZED TO AVG = 0.1410E 00 0.5899E 00 0.8406E 00 0.9320E 00 0.9855E 00 0.1023E 01 0.1053E 01 0.1077E 01
                           0.1090E 01 0.1096E 01 0.1105E 01 0.1117E 01 0.1131E 01 0.1147E 01 0.1164E 01 0.1173E 01
PERIPHERAL DIFF/ VISCOSITY = 0. 0.2671E 01 0.2269E 02 0.4218E 02 0.5752E 02 0.6871E 02 0.7574E 02 0.7863E 02
                           0.7835E 02 0.7762E 02 0.7560E 02 0.7097E 02 0.6176E 02 0.4520E 02 0.1753E 02 0.
RADIAL DIFF / VISCOSITY    = 0. 0.2671E 01 0.2269E 02 0.4218E 02 0.5752E 02 0.6871E 02 0.7574E 02 0.7863E 02
                           0.7835E 02 0.7762E 02 0.7560E 02 0.7097E 02 0.6176E 02 0.4520E 02 0.1753E 02 0.
                           * * * * *

```

ANGULAR POSITION (DEGREES) = 25.50

```

LOCAL WALL SHEAR STRESS (LBF/SQ.IN) ....= 0.01359
LOCAL WALL SHEAR NORMALIZED TO AVG .....= 1.07470
RADIAL DISTANCE TO OUTER BOUNDARY (IN) = 0.02734

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```

DIST FROM WALL (IN)      = 0.8024E-04 0.5620E-03 0.1927E-02 0.3849E-02 0.5772E-02 0.7695E-02 0.9618E-02 0.1154E-01
                           0.1271E-01 0.1331E-01 0.1426E-01 0.1563E-01 0.1741E-01 0.1964E-01 0.2235E-01 0.2559E-01
                           0.2734E-01
DIST FROM WALL / EQV. RAD. = 0.4138E-02 0.2899E-01 0.9937E-01 0.1985E 00 0.2977E 00 0.3969E 00 0.4961E 00 0.5953E 00
                           0.6557E 00 0.6863E 00 0.7358E 00 0.8059E 00 0.8979E 00 0.1013E 01 0.1153E 01 0.1320E 01
                           0.1410E 01
COOLANT VELOCITY (FT/SEC) = 0.2998E 01 0.1243E 02 0.1768E 02 0.1960E 02 0.2072E 02 0.2152E 02 0.2213E 02 0.2264E 02
                           0.2291E 02 0.2304E 02 0.2323E 02 0.2348E 02 0.2378E 02 0.2412E 02 0.2447E 02 0.2485E 02
                           0.2503E 02

```

Figure C-3 Sample Output (Continued)

VELOCITY NORMALIZED TO AVG = 0.1437E 00 0.5959E 00 0.8474E 00 0.9394E 00 0.9933E 00 0.1032E 01 0.1061E 01 0.1085E 01
 0.1098E 01 0.1104E 01 0.1114E 01 0.1126E 01 0.1140E 01 0.1156E 01 0.1173E 01 0.1191E 01
 0.1200E 01
 PERIPHERAL DIFF/ VISCOSITY = 0. 0.2708E 01 0.2313E 02 0.4356E 02 0.6035E 02 0.7349E 02 0.8300E 02 0.8885E 02
 0.9164E 02 0.9102E 02 0.9091E 02 0.8920E 02 0.8420E 02 0.7353E 02 0.5394E 02 0.2105E 02
 0.
 RADIAL DIFF / VISCOSITY = 0. 0.2708E 01 0.2313E 02 0.4356E 02 0.6035E 02 0.7349E 02 0.8300E 02 0.8885E 02
 0.9064E 02 0.9102E 02 0.9091E 02 0.8920E 02 0.8420E 02 0.7353E 02 0.5394E 02 0.2105E 02
 0.

ANGULAR POSITION (DEGREES) = 28.50

LOCAL WALL SHEAR STRESS (LBF/SQ.IN)= 0.01372
 LOCAL WALL SHEAR NORMALIZED TO AVG= 1.08489
 RADIAL DISTANCE TO OUTER BOUNDARY (IN) = 0.03146

DIST FROM WALL (IN) = 0.8024E-04 0.5620E-03 0.1927E-02 0.3849E-02 0.5772E-02 0.7695E-02 0.9618E-02 0.1154E-01
 0.1271E-01 0.1331E-01 0.1426E-01 0.1563E-01 0.1741E-01 0.1964E-01 0.2235E-01 0.2559E-01
 0.2941E-01 0.3146E-01
 DIST FROM WALL / EQV. RAD. = 0.4138E-02 0.2899E-01 0.9937E-01 0.1985E 00 0.2977E 00 0.3969E 00 0.4961E 00 0.5953E 00
 0.6557E 00 0.6863E 00 0.7358E 00 0.8059E 00 0.8979E 00 0.1013E 01 0.1153E 01 0.1320E 01
 0.1517E 01 0.1623E 01
 COOLANT VELOCITY (FT/SEC) = 0.3027E 01 0.1251E 02 0.1777E 02 0.1970E 02 0.2083E 02 0.2163E 02 0.2225E 02 0.2276E 02
 0.2303E 02 0.2316E 02 0.2335E 02 0.2361E 02 0.2391E 02 0.2424E 02 0.2460E 02 0.2498E 02
 0.2537E 02 0.2556E 02
 VELOCITY NORMALIZED TO AVG = 0.1451E 00 0.6000E 00 0.8520E 00 0.9445E 00 0.9986E 00 0.1037E 01 0.1067E 01 0.1091E 01
 0.1104E 01 0.1110E 01 0.1120E 01 0.1132E 01 0.1146E 01 0.1162E 01 0.1180E 01 0.1198E 01
 0.1216E 01 0.1225E 01
 PERIPHERAL DIFF/ VISCOSITY = 0. 0.2736E 01 0.2348E 02 0.4473E 02 0.6280E 02 0.7769E 02 0.8939E 02 0.9791E 02
 0.1015E 03 0.1029E 03 0.1045E 03 0.1055E 03 0.1043E 03 0.9889E 02 0.8659E 02 0.6362E 02
 0.2495E 02 0.
 RADIAL DIFF / VISCOSITY = 0. 0.2736E 01 0.2348E 02 0.4473E 02 0.6280E 02 0.7769E 02 0.8939E 02 0.9791E 02
 0.1015E 03 0.1029E 03 0.1045E 03 0.1055E 03 0.1043E 03 0.9889E 02 0.8659E 02 0.6362E 02
 0.2495E 02 0.

ANGULAR POSITION (DEGREES) = 30.00

LOCAL WALL SHEAR STRESS (LBF/SQ.IN)= 0.01376
 LOCAL WALL SHEAR NORMALIZED TO AVG= 1.08848
 RADIAL DISTANCE TO OUTER BOUNDARY (IN) = 0.03377

DIST FROM WALL (IN) = 0.8024E-04 0.5620E-03 0.1927E-02 0.3849E-02 0.5772E-02 0.7695E-02 0.9618E-02 0.1154E-01
 0.1271E-01 0.1331E-01 0.1426E-01 0.1563E-01 0.1741E-01 0.1964E-01 0.2235E-01 0.2559E-01
 0.2941E-01 0.3146E-01
 DIST FROM WALL / EQV. RAD. = 0.4138E-02 0.2899E-01 0.9937E-01 0.1985E 00 0.2977E 00 0.3969E 00 0.4961E 00 0.5953E 00
 0.6557E 00 0.6863E 00 0.7358E 00 0.8059E 00 0.8979E 00 0.1013E 01 0.1153E 01 0.1320E 01
 0.1517E 01 0.1742E 01
 COOLANT VELOCITY (FT/SEC) = 0.3037E 01 0.1254E 02 0.1781E 02 0.1974E 02 0.2087E 02 0.2167E 02 0.2229E 02 0.2280E 02
 0.2307E 02 0.2320E 02 0.2339E 02 0.2365E 02 0.2395E 02 0.2429E 02 0.2465E 02 0.2503E 02
 0.2541E 02 0.2580E 02
 VELOCITY NORMALIZED TO AVG = 0.1458E 00 0.6014E 00 0.8537E 00 0.9463E 00 0.1001E 01 0.1039E 01 0.1069E 01 0.1093E 01
 0.1106E 01 0.1112E 01 0.1122E 01 0.1134E 01 0.1148E 01 0.1164E 01 0.1182E 01 0.1200E 01
 0.1218E 01 0.1237E 01
 PERIPHERAL DIFF/ VISCOSITY = 0. 0.2746E 01 0.2363E 02 0.4524E 02 0.6389E 02 0.7956E 02 0.9227E 02 0.1020E 03

Figure C-3 Sample Output (Continued)

	0.1065E 03	0.1083E 03	0.1107E 03	0.1128E 03	0.1134E 03	0.1104E 03	0.1015E 03	0.8305E 02
	0.5051E 02	0.						
RADIAL DIFF / VISCOSITY	0.	0.2745E 01	0.2363E 02	0.4524E 02	0.6389E 02	0.7956E 02	0.9227E 02	0.1020E 03
	0.1065E 03	0.1083E 03	0.1107E 03	0.1128E 03	0.1134E 03	0.1104E 03	0.1015E 03	0.8305E 02
	0.5051E 02	0.						

.....

Figure C-3 Sample Output (Continued)

***** TEMPERATURE DISTRIBUTION AND HEAT TRANSFER DATA *****

HEAT GENERATION DATA

POWER DENSITY (BTU/HR/FT**3) FUEL = 0.2129E 09 CLAD = 0. COOLANT = 0.
 NORMALIZED TO TOTAL PIN POWER 0.1000E 01 0.
 AXIAL COOLANT TEMP GRADIENT (F/FI) = 0.3896E 03 LINEAR POWER DENSITY (KW/FT) = 15.0000

AVERAGE MATERIAL TEMPERATURES (F)

FUEL = 3213.47 CLAD = 1050.77 COOLANT = 988.45

AVERAGE COOLANT PROPERTIES

CONDUCTIVITY (BTU/HR/FT/F) = 37.9214 DENSITY (LBM/FT**3) = 51.20 SPECIFIC HEAT (BTU/LBM/F) = 0.30053
 MOLEC. DIFF. OF HEAT (FT**2/HR) = 2.46469 EHR/EHR = 0.332 EMP/EMP = 0.332

DIMENSIONLESS GROUPS

NUSSELT NUMBER = 0.7242E 01 PRANDTL NUMBER = 0.4009E-02 REYNOLDS NUMBER = 0.5073E 05
 PECLET NUMBER = 0.2034E 03

CLAD TO COOLANT HEAT FLUX (BTU/HR/FT**2)
 AVERAGE VALUE = 0.78432E 06

ANGLE (DEG) =	0.	1.50	4.50	7.50	10.50	13.50
	16.50	19.50	22.50	25.50	28.50	30.00
LOCAL VALUE =	0.68305E 06	0.68488E 06	0.69587E 06	0.71630E 06	0.74342E 06	0.77368E 06
	0.80361E 06	0.83041E 06	0.85212E 06	0.86743E 06	0.87553E 06	0.87687E 06
NORM TO AVG =	0.87087E 00	0.87321E 00	0.88723E 00	0.91327E 00	0.94784E 00	0.98642E 00
	0.10246E 01	0.10588E 01	0.10864E 01	0.11060E 01	0.11163E 01	0.11180E 01

HEAT TRANSFER COEFFICIENTS (BTU/HR/FT**2/F)

AVG OUTSIDE CLAD TEMP (F) = 1008.33
 AVG HT TRAN COEFF = 0.39441E 05

ANGLE (DEG) =	0.	1.50	4.50	7.50	10.50	13.50
	16.50	19.50	22.50	25.50	28.50	30.00
LOCAL VALUE =	0.17818E 05	0.18011E 05	0.19241E 05	0.21925E 05	0.26625E 05	0.34481E 05
	0.47869E 05	0.72093E 05	0.11985E 06	0.22004E 06	0.38342E 06	0.43690E 06
NORM TO AVG =	0.45177E 00	0.45667E 00	0.48784E 00	0.55590E 00	0.67508E 00	0.87425E 00
	0.12137E 01	0.18279E 01	0.30388E 01	0.55791E 01	0.97215E 01	0.11078E 02

***** FUEL TEMPERATURE DISTRIBUTION *****

ANGLE =	0.	1.50	4.50	7.50	10.50	13.50	16.50	19.50	22.50	25.50	28.50	30.00
RADIUS												

Figure C-3 Sample Output (Continued)

(IN)													
0.	4678.54	4678.54	4678.54	4678.54	4678.54	4678.54	4678.54	4678.54	4678.54	4678.54	4678.54	4678.54	4678.54
0.007000	4668.79	4668.79	4668.79	4668.79	4668.79	4668.79	4668.79	4668.79	4668.79	4668.79	4668.79	4668.79	4668.79
0.016333	4624.72	4624.72	4624.72	4624.72	4624.72	4624.72	4624.72	4624.72	4624.72	4624.72	4624.72	4624.72	4624.72
0.026600	4531.85	4531.85	4531.85	4531.85	4531.85	4531.85	4531.85	4531.85	4531.85	4531.85	4531.85	4531.85	4531.85
0.037000	4383.09	4383.09	4383.09	4383.09	4383.08	4383.08	4383.08	4383.08	4383.07	4383.07	4383.07	4383.06	4383.06
0.047444	4167.05	4167.04	4167.04	4167.03	4167.01	4166.99	4166.98	4166.96	4156.94	4166.93	4166.92	4166.92	4166.92
0.057909	3869.92	3869.92	3869.90	3869.86	3869.81	3869.75	3869.69	3869.63	3865.57	3869.54	3869.52	3869.52	3869.52
0.068385	3481.06	3481.06	3481.00	3480.93	3480.77	3480.60	3480.43	3480.27	3480.13	3480.03	3479.98	3479.97	3479.97
0.078867	3005.29	3005.27	3005.15	3004.91	3004.59	3004.20	3003.80	3003.42	3003.16	3002.86	3002.74	3002.72	3002.72
0.089353	2476.66	2476.63	2476.36	2475.85	2475.17	2474.36	2473.51	2472.70	2472.02	2471.53	2471.27	2471.23	2471.23
0.099842	1954.25	1954.18	1953.66	1952.63	1951.32	1949.74	1948.09	1946.53	1945.20	1944.24	1943.74	1943.68	1943.68
0.105000	1717.24	1717.15	1716.44	1715.13	1713.26	1711.12	1708.88	1706.77	1704.95	1703.70	1703.03	1702.94	1702.94

***** CLAD TEMPERATURE DISTRIBUTION *****
 TEMP REFERENCED TO MAT. AVG APPEARS UNDER ACTUAL TEMPERATURE IN DEGREES (F)

ANGLE =	0.	1.50	4.50	7.50	10.50	13.50	16.50	19.50	22.50	25.50	28.50	30.00
RADIUS (IN)												
0.110000	1110.11 59.34	1109.90 59.12	1108.62 57.85	1106.23 55.43	1102.51 52.13	1099.07 48.30	1095.08 44.30	1091.32 40.55	1088.16 37.38	1085.87 35.10	1084.68 33.90	1084.48 33.71
0.112519	1094.33 43.56	1094.12 43.34	1092.82 42.04	1090.55 39.59	1087.00 36.23	1083.10 32.33	1079.04 28.26	1075.22 24.44	1072.00 21.22	1069.67 18.90	1068.46 17.68	1068.26 17.48
0.117518	1065.82 15.05	1065.58 14.00	1064.14 13.36	1061.41 10.04	1057.70 6.95	1053.39 2.62	1048.92 -1.86	1044.72 -6.06	1041.18 -9.59	1038.64 -12.14	1037.30 -13.47	1037.08 -13.69
0.122517	1039.17 -11.61	1038.69 -11.69	1037.20 -13.57	1034.03 -16.74	1029.71 -21.06	1024.72 -26.06	1019.55 -31.22	1014.72 -36.05	1010.67 -40.10	1007.76 -43.02	1006.23 -44.54	1005.98 -44.79
0.125000	1026.78 -23.99	1026.47 -24.30	1024.62 -26.16	1021.12 -29.65	1016.37 -34.40	1010.89 -39.89	1005.24 -45.54	999.97 -50.81	995.56 -55.22	992.39 -58.38	990.73 -60.04	990.46 -60.32

***** COOLANT TEMPERATURE DISTRIBUTION *****
 TEMP REFERENCED TO MAT. AVG APPEARS UNDER ACTUAL TEMPERATURE IN DEGREES (F)

ANGLE =	0.	1.50	4.50	7.50	10.50	13.50	16.50	19.50	22.50	25.50	28.50	30.00
DIST FROM WALL (IN)												

Figure C-3 Sample Output (Continued)

0.	1026.78 38.33	1026.47 38.02	1024.62 36.17	1021.12 32.67	1016.37 27.92	1010.89 22.44	1005.24 16.79	999.97 11.52	995.56 7.11	992.39 3.94	990.73 2.28	990.46 2.01
0.000080	1026.66 38.21	1026.35 37.90	1024.49 36.04	1020.99 32.54	1016.23 27.79	1010.75 22.50	1005.09 16.64	999.82 11.37	995.41 6.96	992.24 3.79	990.58 2.13	990.30 1.85
0.000562	1025.91 37.47	1025.60 37.15	1023.73 35.29	1020.22 31.77	1015.44 26.99	1009.92 21.48	1004.24 15.80	998.95 10.50	994.52 6.07	991.33 2.89	989.67 1.22	989.39 0.94
0.001927	1023.96 35.51	1023.64 35.19	1021.74 33.29	1018.17 29.72	1013.31 24.86	1007.71 19.26	1001.95 13.50	996.58 8.13	992.09 3.64	988.86 0.41	987.17 -1.28	986.89 -1.56
0.003849	1021.67 33.22	1021.34 32.90	1019.39 30.94	1015.70 27.25	1010.70 22.25	1004.94 16.50	999.02 10.58	993.52 5.07	988.92 0.47	985.62 -2.83	983.89 -4.56	983.60 -4.85
0.005772	1019.86 31.41	1019.52 31.07	1017.49 29.04	1013.67 25.22	1008.49 20.04	1002.54 14.09	996.43 7.98	990.76 2.31	986.03 -2.42	982.64 -5.81	980.87 -7.58	980.57 -7.88
0.007695	1018.50 30.05	1018.15 29.70	1016.03 27.58	1012.05 23.60	1006.66 18.21	1000.48 12.03	994.15 5.70	988.29 -0.16	983.41 -5.04	979.91 -8.53	978.09 -10.36	977.78 -10.66
0.009618	1017.59 29.15	1017.22 28.78	1015.00 26.55	1010.84 22.39	1005.26 16.75	998.75 10.30	992.16 3.72	986.08 -2.37	981.03 -7.42	977.42 -11.02	975.54 -12.91	975.23 -13.22
0.011541	1017.14 28.70	1016.75 28.31	1014.41 25.96	1010.03 21.58	1004.10 15.66	997.34 8.89	990.46 2.02	984.13 -4.32	978.89 -9.56	975.15 -13.29	973.21 -15.24	972.89 -15.56
0.012547		1016.75 28.30										
0.012713			1014.33 25.88	1009.79 21.34	1003.66 15.21	996.68 8.23	989.60 1.15	983.10 -5.35	977.72 -10.73	973.90 -14.55	971.91 -16.54	971.57 -16.87
0.012925			1014.32 25.87									
0.013306				1009.70 21.25	1003.46 15.01	996.37 7.92	989.18 0.73	982.59 -5.86	977.14 -11.30	973.28 -15.17	971.26 -17.18	970.93 -17.52
0.013686				1009.66 21.21								
0.014265					1003.21 14.77	995.92 7.48	988.56 0.11	981.81 -6.63	976.26 -12.19	972.31 -16.13	970.27 -18.18	969.92 -18.52
0.014842					1003.11 14.66							
0.015626						995.43 6.99	987.79 -0.66	980.82 -7.63	975.09 -13.36	971.03 -17.42	968.93 -19.52	968.58 -19.87
0.016407						995.24 6.79						
0.017409							987.00 -1.45	979.69 -8.76	973.72 -14.73	969.50 -18.95	967.32 -21.13	966.95 -21.49
0.018405							986.71 -1.74					

Figure C-3 Sample Output (Continued)

0.01964	978.58 -9.87	972.25 -16.20	967.80 -20.65	965.51 -22.93	965.13 -23.32
0.020867	978.21 -10.23				
0.022353		970.85 -17.60	966.08 -22.37	963.64 -24.81	963.23 -25.22
0.023829		970.47 -17.98			
0.025591			964.51 -23.94	961.84 -26.61	961.39 -27.06
0.027340			964.23 -24.22		
0.029409				960.34 -28.11	960.34 -28.11
0.031460				960.09 -28.36	
0.032619					959.99 -28.46

EDDY DIFFUSIVITY OF HEAT/MOLECULAR DIFFUSIVITY OF HEAT
CIRCUMFERENTIAL VALUE APPEARS UNDER RADIAL VALUE

ANGLE =	0.	1.50	4.50	7.50	10.50	13.50	16.50	19.50	22.50	25.50	28.50	30.00
DIST FROM WALL (IN)												
0.000080	0. 0.	0. 0.	0. 0.	0. 0.	0. 0.	0. 0.	0. 0.	0. 0.	0. 0.	0. 0.	0. 0.	0. 0.
0.000	0. 0.	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0. 0.
0.001927	0. 0.	0.03 0.03	0.03 0.03	0.03 0.03	0.03 0.03	0.03 0.03	0.03 0.03	0.03 0.03	0.03 0.03	0.03 0.03	0.03 0.03	0. 0.
0.003849	0. 0.	0.04 0.04	0.04 0.04	0.05 0.05	0.05 0.05	0.05 0.05	0.05 0.05	0.05 0.05	0.06 0.06	0.06 0.06	0.06 0.06	0. 0.
0.005772	0. 0.	0.05 0.05	0.05 0.05	0.05 0.05	0.06 0.06	0.06 0.06	0.07 0.07	0.07 0.07	0.08 0.08	0.08 0.08	0.08 0.08	0. 0.
0.007695	0. 0.	0.05 0.05	0.05 0.05	0.06 0.06	0.06 0.06	0.07 0.07	0.08 0.08	0.08 0.08	0.09 0.09	0.10 0.10	0.10 0.10	0. 0.
0.009618	0. 0.	0.04 0.04	0.04 0.04	0.05 0.05	0.06 0.06	0.07 0.07	0.08 0.08	0.09 0.09	0.10 0.10	0.11 0.11	0.12 0.12	0. 0.
0.011541	0. 0.	0.01 0.01	0.02 0.02	0.03 0.03	0.04 0.04	0.06 0.06	0.07 0.07	0.09 0.09	0.10 0.10	0.12 0.12	0.13 0.13	0. 0.

Figure C-3 Sample Output (Continued)

0.012547	0.										
	0.										
0.012713	0.00	0.01	0.03	0.05	0.07	0.09	0.10	0.12	0.14	0.	
	0.00	0.01	0.03	0.05	0.07	0.09	0.10	0.12	0.14	0.	
0.012925	0.										
	0.										
0.013306	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.		
	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.		
0.013686	0.										
	0.										
0.014265		0.01	0.03	0.05	0.08	0.10	0.12	0.14	0.		
		0.01	0.03	0.05	0.08	0.10	0.12	0.14	0.		
0.014842		0.									
		0.									
0.015626		0.01	0.04	0.07	0.09	0.12	0.14	0.			
		0.01	0.04	0.07	0.09	0.12	0.14	0.			
0.016407		0.									
		0.									
0.017409			0.02	0.05	0.08	0.11	0.14	0.			
			0.02	0.05	0.08	0.11	0.14	0.			
0.018405			0.								
			0.								
0.019640				0.02	0.06	0.10	0.13	0.			
				0.02	0.06	0.10	0.13	0.			
0.020867				0.							
				0.							
0.022353					0.02	0.07	0.12	0.			
					0.02	0.07	0.12	0.			
0.023829						0.					
						0.					
0.025591							0.03	0.08	0.		
							0.03	0.08	0.		
0.027340							0.				
							0.				
0.029409								0.03	0.		
								0.03	0.		

Figure C-3 Sample Output (Continued)

NOMENCLATURE

A_c	Cross-sectional area of the coolant	ft ²
A_F	Cross-sectional area of the fuel	ft ²
A_w	Cross-sectional area of the cladding	ft ²
C_{pc}	Coolant specific heat	Btu/lbm-°F
C_1	Constant in fuel temperature solution (See Equation 67)	Btu/h-ft
D	Fuel pin diameter	ft
d_H	Hydraulic diameter	ft
f	Friction factor for rod bundle	
f_{ct}	Friction factor for circular tube	
g_0	Gravitational constant	lbm-ft/lbf-hr ²
$\bar{h}_c$	Average cladding-coolant heat transfer coefficient	Btu/hr-ft ² -°F
$h_c(\phi)$	Local cladding-coolant heat transfer coefficient	Btu/h-ft ² -°F
h_G	Fuel-cladding heat transfer coefficient	Btu/h-ft ² -°F
I_{p_1}, I_{p_2}	Integrals in fuel temperature solution (See Equations 57a and 57b)	
k	Thermal conductivity	Btu/h-ft-°F
k_c	Coolant thermal conductivity	Btu/h-ft-°F
k_F	Fuel thermal conductivity	Btu/h-ft-°F
k_w	Cladding thermal conductivity	Btu/h-ft-°F
k'_r	Effective conductivity in the radial direction	Btu/h-ft-°F
k'_ϕ	Effective conductivity in the circumferential direction	Btu/hr-ft-°F
m_F	$= (\rho_F A_F + \rho_w A_w + \rho_c A_c)$	lbm/ft
Nu	Nusselt Number	
p	Pressure	lbf/ft ²
Pc	Peclet Number	
Pr	Prandtl Number	
P	Rod pitch	ft
P/D	Pitch-to-diameter ratio	

$\bar{q}''_{Fs}$	Average fuel surface heat flux	Btu/hr ft ²
$q''_{Fs}(\phi)$	Local fuel surface heat flux	Btu/hr ft ²
$\dot{q}'_r$	Radial heat rate/foot	Btu/hr ft
$\dot{q}'_\phi$	Circumferential heat rate/foot	Btu/hr ft
$\bar{q}''_{wo}$	Average cladding-coolant heat flux	Btu/hr ft ²
$q''_{wo}(\phi)$	Local cladding-coolant heat flux	Btu/hr ft ²
$\dot{q}'''$	Volumetric heat generation rate	Btu/hr ft ³
$\dot{q}'''_c$	Coolant heat generation rate	Btu/hr ft ³
$\dot{q}'''_F$	Fuel heat generation rate	Btu/hr ft ³
$\dot{q}'''_w$	Cladding heat generation rate	Btu/hr ft ³
$\dot{q}'_L$	Linear heat generation rate for pin	kw/ft
R_c	Radius of curvature of surface	ft
R_F	Radius of the fuel pellet	ft
R_G	Inside cladding wall radius	ft
R_v	Fuel center void radius	ft
R_w	Outside cladding wall radius	ft
R_o	One-half the pitch (P/2)	ft
r	Radial coordinate	ft
Re	Reynolds Number	
$T(r, \phi)$	Local temperature	°F
T_{CB}	Bulk coolant temperature	°F
$\bar{T}_{wo}$	Average outside cladding wall temperature	°F
$T_{wo}(\phi)$	Local outside cladding wall temperature	°F
$u(r, \phi)$	Local coolant velocity	ft/h
$\bar{u}$	Average coolant velocity	ft/h
$\bar{u}_\perp$	Average coolant velocity along a normal to the cladding surface	ft/h
u^*	Shear velocity, $(\tau_{wo}/\rho_c)^{1/2}$	ft/h
u^+	Nondimensional velocity, u/u^*	
Y	Thermal Admittance	Btu/h-ft-°F

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y	Distance from outside cladding wall along a normal to the cladding surface	ft
y^+	Nondimensional distance, $u^* y / \nu_c$	
$\hat{y}$	Normal distance from cladding outer surface to line of symmetry	ft
$\hat{y}_{av}$	$\hat{y}$ averaged over the cladding perimeter	ft
z	Axial coordinate	ft
α_c	Coolant molecular diffusivity of heat	ft ² /h
γ_H	Fraction of power generated by gamma-neutron heating	
ϵ_{Hr}	Radial eddy diffusivity of heat	ft ² /h
$\epsilon_{H\phi}$	Circumferential eddy diffusivity of heat	ft ² /h
ϵ_{Mr}	Radial eddy diffusivity of momentum	ft ² /h
$\epsilon_{M\phi}$	Circumferential eddy diffusivity of momentum	ft ² /h
$\bar{\epsilon}_{M\phi}$	$\epsilon_{M\phi}$ averaged along a normal to the cladding surface	ft ² /h
ρ_c	Coolant density	lbm/ft ³
ρ_F	Fuel density	lbm/ft ³
ρ_w	Cladding density	lbm/ft ³
ϕ	Angular coordinate	radians
Ψ_r	Ratio of radial eddy diffusivity of heat to radial eddy diffusivity of momentum	
Ψ_ϕ	Ratio of circumferential eddy diffusivity of heat to circumferential eddy diffusivity of momentum	
$\bar{\tau}_r$	Radial shear stress in the coolant	lbf/ft ²
τ_ϕ	Circumferential shear stress in the coolant	lbf/ft ²
τ_w	Radial shear stress at the cladding surface	lbf/ft ²
Δd_H	Hydraulic diameter of local flow area section taken normal to cladding surface	ft
μ	Absolute viscosity	lbm/h-ft
ν	Kinematic viscosity	ft ² /h

SUBSCRIPTS

c	Coolant	w	Cladding
F	Fuel	wI	Cladding inner surface
FS	Fuel surface	wO	Cladding outer surface
v	Void	r	Radial component
		ϕ	Circumferential component

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