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STABILITY ANALYSIS OF THE RFD-2
FLIGHT TEST VEHICLE

F. L. Paris, 9326
Sandia Laboratory, Albuquerque

June 1965

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ABSTRACT

The theoretical and experimental determinations of the aerodynamic coefficients of the RFD-2 re-entry vehicle are presented. The resulting coefficients are incorporated into an aerodynamic analysis of the re-entry trajectory where the changes of configuration and the resultant changes in the physical and aerodynamic parameters of the vehicle are considered. Radar and camera-tracking data were obtained from the flight test which took place October 9, 1964. These experimental trajectory data correlated with the calculated trajectory within 400 feet of distance or 1 second of flight time throughout the re-entry flight.

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LIST OF SYMBOLS

- A - reference area (square feet)
- C_D - drag coefficient, referenced to base area
- $\overline{C}_f$ - local friction factor referenced to surface area
- C_{D_f} - drag coefficient due to friction, referenced to base area
- $C_{D_f}^*$ - drag coefficient due to friction, corrected for Mach number, referenced to base area
- C_L - lift coefficient, referenced to base area
- C_M - pitching moment coefficient, referenced to base area and base diameter
- $C_{M_q} + C_{M_\alpha}$ - dynamic stability derivative $\frac{\partial C_M}{\partial \left(\frac{q\ell}{2V}\right)} + \frac{\partial C_M}{\partial \left(\frac{\alpha\ell}{2V}\right)}$, referenced to base area and base diameter
- C_N - normal force coefficient, referenced to base area
- d, D - reference length (feet)
- g - gravitational constant (32.2 feet/second²)
- I - pitch moment of inertia (slug-feet²)
- ℓ - reference length (feet)
- m - mass (slugs)
- M - Mach number, pitching moment (foot-pounds)
- N - normal force (pounds)
- R_{e_x} - Reynolds number per unit length (per foot)
- q - dynamic pressure, $1/2\rho V^2$ (pounds/foot²)
- s - flight path length (feet)
- T - time (seconds)
- V - velocity (feet/second)
- X_{cp} - distance to center of pressure (feet), from station 73.750
- X, Y - cone length (feet)
- y - altitude (feet)

LIST OF SYMBOLS (cont)

Greek

- α - angle of attack (degrees or radians)
- β - exponent in atmosphere equation $\rho = \rho_0 e^{-\beta y}$ (per foot)
- Γ - flight path angle referred to local horizon (degrees)
- ρ - air density (slugs/foot³)
- σ - radius of gyration, $\sqrt{\frac{I}{m}}$ (feet)
- ω - pitch frequency (radians per second)

Subscripts

- α - derivative with respect to angle of attack
- o - sea level conditions
- E - re-entry condition (at 400,000 feet)

Superscripts

- ' - differentiation with respect to s
- .

STABILITY ANALYSIS OF THE RFD-2 FLIGHT TEST VEHICLE

Introduction

This report presents the aerodynamic analysis of the re-entry vehicle employed in the RFD-2 flight test which took place October 9, 1964. This test was a continuation of the Aerospace Nuclear Safety (ANS) program¹ sponsored by the Division of Reactor Development of the Atomic Energy Commission. The principal purpose of the test was to establish disassembly characteristics, during flight, of a SNAP 19 type of isotopic generator in a simulated orbital re-entry trajectory to check the validity of predictions resulting from current theoretical techniques. The generator case was constructed in such a fashion that disassembly and destruction of the generator in a definite sequence were predicted. This disassembly of the generator resulted in changes of the physical and aerodynamic parameters of the re-entry vehicle which were taken into account in the aerodynamic analysis.

Theoretical Aerodynamic Coefficients

In order for preliminary studies to be conducted concerning the trajectory, aerodynamic heating and loads, and accelerations of the vehicle, it was necessary to estimate the vehicle aerodynamic coefficients. The techniques employed for the estimation of the aerodynamic coefficients are approximations, but the results were required for preliminary studies and time did not permit the use of sophisticated techniques.

Drag

An estimate of the aerodynamic drag of the re-entry vehicle was obtained in the following manner. The contributions of skin friction of each of the body sections (see Figures 1 and 2), forebody, and base effects to the total drag were calculated and summed in the lower Mach number regimes (subsonic and supersonic), while at hypersonic conditions the Newtonian value was assumed to be valid.

As the skin friction is dependent upon Reynolds number, a trajectory had to be assumed to determine the relation between the Mach and Reynolds numbers. The RFD-1 trajectory was chosen for a first estimate, as the vehicle and re-entry conditions were similar to those expected for the RFD-2 flight. The skin friction calculations were based on turbulent flow, as the poor aerodynamic contour of the generator would induce a high turbulence level in the boundary layer. With a turbulent boundary layer, the average friction coefficients of a cone and flat plate differ by as little as 2 percent if the local Reynolds numbers are identical.² This allowed the average friction coefficients of each of the body sections (where the

generator and the rounded nose fairing were approximated by a cylinder and a truncated cone, respectively) to be calculated from the flat plate Karman-Schoenherr equation,²

$$\sqrt{\bar{C}_f} \log_{10} \left(R_{e_x} \bar{C}_f \right) = 0.242,$$

after the average Reynolds number of each section was estimated for a wide range of flight conditions. The resulting coefficients were then corrected for the initial conditions (nonzero boundary layer thickness) and effective area of each section and were referenced to the maximum cross-sectional area of the vehicle. The following Mach number corrections³ were then applied to obtain the final frictional drag coefficients:

$$C_{D_f}^* = C_{D_f} \left(\frac{1}{1+0.08M^2} \right) \quad (M < 1)$$

$$C_{D_f}^* = C_{D_f} \left(\frac{1}{1+0.2M^2} \right)^{0.467} \quad (M > 1).$$

The estimate of the base pressure was crude, since no adequate theory concerning the base pressure exists when the boundary layer is turbulent. However, a comparison of data⁴ (Figs 9-30) which apply to turbulent boundary layers on thin bodies of revolution with laminar theory⁵ results in good agreement, indicating that either reference may give an adequate approximation. The laminar base pressure coefficient - Mach number relation⁴ was used to calculate the base drag coefficients.

The forebody drag was calculated initially by assuming that the flow had expanded to the free stream static pressure on the cylindrical section, and then a combination of Prandtl-Meyer expansions and conical shocks was employed to determine the pressures over the remainder of the body.

The Newtonian drag coefficient was calculated from the equations given by Bustamante,¹ (Table I) and a viscous interaction correction¹ (Fig 49) was then applied to the Newtonian drag value.

The final individual and total drag coefficients are given in Figure 3.

Stability Derivatives

The stability derivatives were estimated in the supersonic Mach number range from conical theory. Since conical theory cannot be applied to complicated geometries, the generator and rounded adapter section on the nose of the re-entry vehicle were approximated by a cylinder and a truncated cone, respectively. The coefficients for the truncated cones were then determined by taking the complete cone values⁶ and subtracting similar cones, as shown in Figure 4. For cone B in Figure 4, the normal force is expressed as,

$$N_B = C_{N_\alpha} \alpha q(D)^2 \frac{\pi}{4}.$$

For cone C,

$$N_C = C_{N_\alpha} \propto q(d)^2 \frac{\pi}{4}.$$

Then, for the truncated cone A,

$$N_A = N_B - N_C = C_{N_\alpha} \propto q \frac{\pi}{4} (D^2 - d^2) = C_{N_{\alpha_A}} \propto q \frac{\pi}{4} D^2,$$

or

$$C_{N_{\alpha_A}} = C_{N_\alpha} \left[1 - \left(\frac{d}{D} \right)^2 \right].$$

In a similar manner,

$$M_B = C_{M_\alpha} \propto q \frac{\pi}{4} D^2 D = 2/3 \times N_B$$

$$M_C = C_{M_\alpha} \propto q \frac{\pi}{4} d^2 d = 2/3 \times N_C$$

and

$$M_A = M_B - M_C = 2/3 C_{N_\alpha} \propto q \frac{\pi}{4} D^2 X - d^2 Y = C_{M_{\alpha_A}} \propto q \frac{\pi}{4} D^2 D.$$

Then

$$C_{M_{\alpha_A}} = 2/3 C_{N_\alpha} \left[\frac{X}{D} - \frac{d^2 Y}{D^3} \right].$$

The center of pressure of the truncated cone is then (measured from the apex of the total cone B),

$$\frac{x_{cp}}{D} = \frac{C_{M_{\alpha A}}}{C_{N_{\alpha A}}} = 2/3 \left[\left(x_D - \frac{d^2 Y}{D} \right) / \left(D^2 - d^2 \right) \right].$$

In the hypersonic region, the stability derivatives were evaluated by Newtonian theory. The equations for the individual body components are developed in other papers¹ and are reproduced here in Table I.

There is a decrease in the stabilizing effect of the afterbody flare at the higher Mach numbers, as there is a loss of dynamic pressure due to the increasing shock strength. The forces and moments contributed by the flare were accordingly corrected.

The calculated values of normal force coefficients and static stability derivatives are presented in Figures 5 and 6, respectively.

Experimental Aerodynamic Coefficients

The preceding theoretical coefficients are approximations and were employed only for a preliminary trajectory study. In order to conduct an accurate trajectory analysis, it was necessary to obtain the aerodynamic coefficients experimentally. For this purpose, a series of force and dynamic wind tunnel tests were conducted in the Mach number range of from 0.5 to 22.⁷⁻¹¹ The facilities used and the type of tests conducted are outlined in Table II.

As it is seldom possible to match all flight parameters in existing test facilities, especially at the higher Mach numbers, the majority of the data was taken at reduced Reynolds numbers. Figures 7 and 8 show both the flight Mach - Reynolds numbers relation as obtained from the preliminary trajectory study and the corresponding wind tunnel test points. These illustrations show that in the Mach number range of 0.5 to 1.25, the test Reynolds number was below that of actual flight, causing the boundary layer to be laminar over a longer section of the body during the wind tunnel tests. (This ignores the tunnel turbulence effect on transition.) In the Mach number range of 1.6 to 4.6, transition of the boundary layer would occur earlier on the body during the wind tunnel tests. Above a Mach number of 5.3, the situation would revert to that of the lower Mach numbers. Although large discrepancies existed between the Mach - Reynolds number relations of the tests and actual flight, no Reynolds number corrections were applied to the data because effects upon the boundary layer of the vehicle blunt nose, aerodynamic heating, and mass addition through ablation are not predictable. Also, the data indicate (see Figures 9 through 13) that gross changes in Reynolds number (at Mach numbers of 1.5 and 4.5) have no discernible effect upon the aerodynamic coefficients.

TABLE I

Newtonian Aerodynamic Expressions

SHAPE	NEWTONIAN AERODYNAMIC EXPRESSIONS
<p>NOTE: $\alpha \leq \pm 15^\circ$</p>	$C_N = \frac{\pi C_{Ps}}{A} R^2 \sin 2\alpha \left\{ \frac{a}{3R} + \frac{1}{4} \right\}; \text{ FOR A GENERAL } a$ $C_{N\alpha} = \frac{2\pi C_{Ps}}{A} R^2 \left\{ \frac{a}{3R} + \frac{1}{4} \right\};$ $C_M = \frac{\pi R^3 \sin 2\alpha}{AL} C_{Ps} \left\{ \frac{(a/R)^2}{3} + \frac{\pi}{16} a/R + \frac{a/R}{3} + \frac{1}{4} \right\} \text{ FOR GENERAL } a$ $C_{M\alpha} = \frac{-\pi R^3}{AL} C_{Ps} \left[\frac{1}{2} + \left(\frac{\pi}{8} + \frac{2}{3} \right) \frac{a}{R} + \frac{2}{3} \left(\frac{a}{R} \right)^2 \right]$ $C_A = \frac{2\pi C_{Ps}}{A} R^2 \left\{ \frac{2}{3} \frac{(r_0 - R)}{R} + \frac{1}{4} \right\} + \frac{\pi C_{Ps}}{A} (r_0 - R)^2$ $C_{Mq} = \frac{-2 C_{Ps}}{R^2 r_0^2} \int_{\theta_1}^{\theta_2} (a + R \cos \theta) \tan \theta [R - R \sin \theta + (a + R \cos \theta) \tan \theta]^2 R \cos \theta d\theta$ $C_D = C_N \sin \alpha + C_A \cos \alpha$
<p>NOTE C_M ABOUT FLARE NOSE (STA. 0)</p> <p>FLARE SHIELDED BY CYLINDER OF DIAMETER $2 r_F$ $\alpha \leq \theta_F$</p>	$C_N = \frac{\pi C_{Ps}}{4A} \frac{\sin 2\theta_F}{\tan \theta_F} \sin 2\alpha \{ r_b^2 - r_F^2 \} \text{ FOR GENERAL } \alpha$ $C_{N\alpha} = \frac{\pi C_{Ps}}{2A} \frac{\sin 2\theta_F}{\tan \theta_F} \{ r_b^2 - r_F^2 \}$ $C_{M\alpha} = \frac{-\pi C_{Ps}}{AL} \left\{ \frac{2}{3} \frac{\cos^3 \theta_F}{\sin \theta_F} \left[r_b^3 - \frac{3}{2} r_b^2 r_F + \frac{1}{2} r_F^3 \right] - \frac{2}{3} \cos \theta_F \sin \theta_F [r_b^3 - r_F^3] \right\}$ $C_M = -\frac{\pi C_{Ps}}{AL} \frac{\sin 2\alpha}{3} \frac{\cos^3 \theta_F}{\sin \theta_F} \left[r_b^3 - \frac{3}{2} r_b^2 r_F + \frac{1}{2} r_F^3 \right] - \frac{\pi C_{Ps}}{AL} \frac{\sin 2\alpha}{3} \cos \theta_F \sin \theta_F [r_b^3 - r_F^3]$ $C_A = \frac{\pi C_{Ps}}{A} \sin^2 \theta_F \{ r_b^2 - r_F^2 \}$ $C_D = C_N \sin \alpha + C_A \cos \alpha$ $C_{Mq} = \frac{2 C_{Ps}}{L^2 r_b^2 (\tan^2 \theta + 1)} \left\{ \frac{1}{\tan^2 \theta} \left[\frac{1}{4} (r_b^4 - r_F^4) - \frac{2}{3} r_F (r_b^3 - r_F^3) + \frac{r_F}{2} (r_b^2 - r_F^2) \right] + \frac{1}{2} (r_b^4 - r_F^4) - \frac{2}{3} r_F (r_b^3 - r_F^3) + \frac{\tan^2 \theta}{4} [r_b^4 - r_F^4] \right\}$

TABLE I (cont)

NOTE:

$$\alpha \leq \pm 15^\circ$$

$$C_N = \frac{8}{3} \frac{C_{Ps}}{\pi} \frac{l}{d} \sin^2 \alpha$$

$$C_M = -C_N \left(\frac{l/2}{l} \right) = -\frac{C_N}{2}$$

$$C_A = \frac{\pi d^2}{4A} C_{Ps}$$

$$C_D = \frac{8}{3} \frac{C_{Ps}}{\pi} \frac{l}{d} \sin^3 \alpha + \frac{\pi d^2}{4A} C_{Ps} \cos \alpha$$

NOTE:

$$\alpha \leq \pm 15^\circ$$

$$C_N = C_{Ps} \sin^2 \alpha$$

$$C_M = -\frac{C_N}{2}$$

$$C_D = 2 \sin^3 \alpha$$

TABLE II
Wind Tunnel Test Conditions

Facility	Mach No.	Reynolds number per unit length	Diameter of Model D (ft)	Reynolds number based on model diameter	Type
		$Re/L(ft^{-1}) \times 10^{-6}$		$R_{eD} \times 10^{-6}$	
Cornell Aeronautical Laboratories Transonic Tunnel (Reference 10)	0.5	1.87	0.759	1.430	(F + D)*
	0.6	2.17	0.759	1.643	(F + D)
	0.7	2.42	0.759	1.84	(F + D)
	0.8	2.65	0.759	2.05	(F + D)
	0.9	1.87	0.759	1.41	(F + D)
	0.95	1.89	0.759	1.43	(F + D)
	1.0	1.95	0.759	1.48	(F + D)
	1.05	1.94	0.759	1.47	(F + D)
	1.1	1.98	0.759	1.50	(F + D)
	1.15	1.98	0.759	1.50	(F + D)
	1.2	2.00	0.759	1.56	(F + D)
	1.25	2.03	0.759	1.55	(F + D)
Ling-Temco-Vought High-Speed Tunnel (Reference 11)	1.61	8.61	0.759	6.54	(F + D)
	2.01	9.08	0.759	6.89	(F + D)
	2.99	11.35	0.759	8.62	(F + D)
	3.98	14.23	0.759	10.8	(F + D)
	4.59	17.16	0.759	13.05	(F + D)
Sandia Corporation Hypersonic Tunnel (Reference 12)	5.3	0.630	0.2868	0.1806	(F)
	7.6	1.387	0.2868	0.3975	(F)
Ling-Temco-Vought Hypervelocity Tunnel (Reference 13)	14.4	1.3	0.1420	0.1844	(F + D)
	17.3	1.1	0.1420	0.1562	(D)
	20.4	0.37	0.1420	0.0525	(D)
Cornell Aeronautical Laboratories Shock Tunnel (Reference 14)	12.0	0.05	2.84	0.01183	(F)
	12.3	0.500	2.84	0.1183	(F)
	17.0	0.108	2.84	0.02558	(F)
	22.3	0.050	2.84	0.01183	(F)

Note: * F - Force Test
D - Dynamic Test

Stability Analysis

A necessary prerequisite to the stability analysis was a definition of the trajectory to be considered. This entailed an iteration, as the disassembly of the generator, aerodynamic heating, and trajectory were interrelated.

The experimental coefficients of Figures 9 through 13 were incorporated^{*} into an aerodynamic heating analysis from which the disassembly of the generator was predicted. The resulting trajectory and the time-dependent aerodynamic and physical parameters are given in Figures 14 through 21. The solid lines of Figures 14 and 15 show the trajectory employed for the stability analysis. The ticks on these curves show the point in the trajectory where the fourth-stage motor (which was housed in the rear section of the re-entry vehicle) was ejected and the subsequent points where the generator changes were predicted to occur. Figure 16 shows the variation of the vehicle physical parameters as a function of time after re-entry at 400,000 feet altitude. In Figures 17 to 21, as in 14 and 15, the solid line represents the trajectory under consideration.

Shortly before the scheduled test date, changes in the propulsion system required a modification of the theoretical trajectory, as shown by the dashed lines in Figures 14 and 15. This resulted in corrections to the time-dependent aerodynamic coefficients, as given by the dashed lines in Figures 17 through 21. As time did not permit the entire analysis to be repeated, the critical periods in the trajectory were inspected. The individual terms of the convergence criterion were inspected and found to be only slightly changed. The rearward motion of the center of gravity (and the decreased static stability), as shown in Figure 20, was compensated by a corresponding rearward shift of the center of pressure due to the reduced Mach numbers. Figures 18 and 20 illustrate that the RFD-2 re-entry vehicle has ample static stability, but it is seen from Figure 21 that the vehicle is dynamically unstable for a period of 35 seconds. This dynamic instability is compounded by the low dynamic pressure early in the trajectory, which decreases the stability below that indicated in Figures 18 and 20. Various theoretical techniques exist for evaluating the seriousness of the above instabilities, but they are restricted to pure pitching (or planar) motion and do not account for the true three-dimensional motion of the vehicle. However, these theories are useful in defining problem areas which can then be inspected more accurately on a rigid-body 6-degree-of-freedom digital computer program.

Amplitude Envelope

Insight into the dynamic motion of a vehicle is gained by assuming an initial misalignment of the vehicle axis with the velocity vector during re-entry and determining the consequent maximum oscillation envelope of the vehicle. The expression for the maximum oscillation of the vehicle is given¹² as,

$$\frac{\alpha}{\alpha_E} = \frac{1}{\sqrt{\pi}} k_2^{-1/4} e^{\left[k_1 e^{-\beta y} + 1/4 \beta y \right]},$$

^{*}By D. A. Byrne of the Aero-Thermodynamics Department, Sandia Corporation.

where

$$k_1 = \frac{\rho_o A}{4\beta m \sin \Gamma} \left[C_D - C_{L_\alpha} + \left(C_{M_q} + C_{M_{\dot{\alpha}}} \right) \left(\frac{\ell}{\sigma} \right)^2 \right]$$

and

$$k_2 = \frac{\rho_o A}{2\beta^2 m \sin^2 \Gamma} \left[- C_{M_\alpha} \left(\frac{\ell}{\sigma^2} \right) + C_{L_\alpha} \beta \sin \Gamma \right].$$

The resulting envelope is shown in Figure 22 where no divergent motion is seen to occur during the period of dynamic instability (indicated in Figure 21) or at any other time in the trajectory.

Dynamic Motion

Another convergence criterion has been developed by Sommer and Tobak¹³ which determines whether the envelope of the maximum amplitude of oscillations is convergent or divergent at each point in the trajectory. The expression is of the form,

$$K < -1 + \left(\frac{W}{C_D A} \right) \sin \Gamma \left[\frac{1}{q} + \frac{\beta}{g\rho} + \frac{1}{g\rho \sin \Gamma} \frac{C'_{M_\alpha}(s)}{C_{M_\alpha}(s)} \right],$$

where

$$K = \frac{1}{C_D} \left[- C_{L_\alpha} + \left(\frac{\ell}{\sigma} \right)^2 \left(C_{M_q} + C_{M_{\dot{\alpha}}} \right) \right]$$

and

$$C'_{M_\alpha}(s) = \frac{1}{V} \dot{C}_{M_\alpha}(t).$$

The numerical values of the above inequality are given in Figure 23, where the motion of the vehicle is seen to be convergent over the entire trajectory.

Perturbation Damping

The above investigations of the dynamic motion indicate that the RFD-2 vehicle undergoes a stable convergent motion over the entire trajectory but says nothing about the rate of recovery from a perturbation.

An expression is developed by Rowe¹⁴ which gives the time it takes for a vehicle to damp to one-half of the amplitude of an arbitrary (but small) perturbation. The equation,

$$T_{1/2} = -5.544 I / \rho V A^2 \left(C_{M_q} + C_{M_{\dot{\alpha}}} \right),$$

was evaluated for the RFD-2 vehicle over the entire trajectory, and the results are given in Figure 24.

The applied two-dimensional theories indicate that no specific problem areas exist for the RFD-2 vehicle, although the rate of decay of a perturbation was less than desirable for altitudes about 100,000 feet. However, this theory is for steady-state conditions only, and the transient nature of the trajectory does not allow these long decay times to exist.

Roll-Pitch Coupling

The possibility of a roll-pitch coupling problem arose from the fact that the vehicle re-entered with a nominal roll rate of 2-1/2 revolutions per second and was despun at 350,000 feet to a rate of 1/4 revolution per second. The purpose of the initial high spin rate was to maintain stability during the time previous to the ejection of the fourth-stage motor from the vehicle.

The natural pitch frequency of an aerodynamic vehicle is given implicitly by the expression,¹⁴

$$C_{M_{\alpha}} = - \frac{1}{57.3} \left[\frac{4I\pi^2}{P^2 q A \ell} + \frac{1}{q A \ell} \left(\frac{0.693}{T_{1/2}} \right)^2 \right] \approx - \frac{1.754 I}{d^2 \ell \rho V^2 P^2}$$

or, since $P = 1/\omega$,

$$\omega \approx \left[- \frac{C_{M_{\alpha}} q d^2 \ell}{0.877 I} \right]^{1/2}.$$

The roll rate and calculated pitch frequency are compared in Figure 25. It is evident that no problem areas occur, even though the roll and pitch frequencies are equal at one point, since the pitch frequency is rapidly increasing and there is not sufficient time for a divergent motion to develop.

Digital Computer Studies

Since the employed theories are restricted to planar motion, the encouraging results cannot be considered to guarantee stability of the vehicle. To further check the above results, a 6-degree-of-freedom digital computer study was conducted. The computer study gave no evidence of instability at any point in the trajectory and verified the theoretical analysis.

Conclusions

3 A theoretical and digital computer aerodynamic stability analysis was conducted for the RFD-2 re-entry vehicle. The trajectory considered was based on (1) a prediction of the disassembly of the generator mounted on the nose of the vehicle and (2) aerodynamic coefficients obtained from wind tunnel tests. The wind tunnel stability data exhibited considerable scatter in the hypersonic Mach number regime, but it is believed that the results are conservative.

As a result of the theoretical analysis and computer studies using the wind tunnel data, it was concluded that the stability of the RFD-2 vehicle over the considered trajectory is assured.

The RFD-2 flight test took place on October 9, 1964. A theoretical trajectory using the aerodynamic coefficients and stability derivatives reported here, but with the actual flight initial conditions, was calculated. The resulting trajectory was compared with the actual trajectory (derived from test data) and a maximum discrepancy of 400 feet of distance or 1 second of flight time was found to exist between the two trajectories.

Figure 1. Complete Re-entry Vehicle and Generator

(a) Complete Generator
(Configuration I)

(b) Generator Partially
Disassembled as a
Result of Aerodynamic
Heating
(Configuration II)

(c) Nose Section of Re-entry
Vehicle After Complete
Disassembly of Generator
(Configuration III)

Figure 2.

Figure 3. Theoretical Drag Coefficient of the RFD-2 Vehicle Based on the RFD-1 Trajectory

Figure 4. Subtraction of Cones to Apply Conical Theory to Cone Frustrums

Figure 5. RFD-2 Theoretical Normal Force Coefficient Derivative versus Mach Number

Figure 6. RFD-2 Theoretical Pitching Moment Coefficient Derivative versus Mach Number

Figure 7. Comparison of RFD-1 Mach - Reynolds Numbers Relation During Flight (with data test points)

Figure 8. Comparison of RFD-1 Mach Number - Viscous Interaction Parameter During Flight (with data test points)

Figure 9. RFD-2 Experimental Drag Coefficient

Figure 10. RFD-2 Experimental Normal Force Coefficient Derivative versus Mach Number

Figure 11. RFD-2 Experimental Pitching Moment Coefficient Derivative versus Mach Number

Figure 12. RFD-2 Experimental Center of Pressure Variation with Mach Number

REFERENCE AREA = BASE AREA

REFERENCE LENGTH BASE DIAMETER

$\frac{X_{CG}}{L}$ REFERENCE FROM STATION 73.750 IN PERCENT OF 73.750"

Figure 13. RFD-2 Experimental Dynamic Damping Coefficient versus Mach Number

Figure 14. RFD-2 Theoretical Altitude and Velocity versus Time (including configuration changes)

Figure 15. RFD-2 Theoretical Mach Number versus Altitude
 (including configuration changes)

Figure 16. RFD-2 Variation of Physical Characteristics with Time

Figure 17. RFD-2 Drag Coefficient Variation with Time (including configuration changes)

Figure 18. RFD-2 Pitching Moment Coefficient Derivative Variation with Time
(including configuration changes)

Figure 19. RFD-2 Normal Force Coefficient Derivative Variation with Time (including configuration changes)

Figure 20. RFD-2 Center of Pressure and Center of Gravity Variation with Time (including configuration changes)

Figure 21. RFD-2 Dynamic Damping Coefficient Derivative Variation with Time (including configuration changes)

Figure 22. RFD-2 Maximum Amplitude Envelope

Figure 23. RFD-2 Convergence Criteria Variation with Time During Re-entry (including configuration changes)

Figure 24. RFD-2 Time to Damp to One-Half Amplitude From a Perturbation

Figure 25. RFD-2 Roll Rate and Pitch Frequency as a Function of Time During Re-entry (including configuration changes)

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