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INTERSECTING STORAGE ACCELERATOR NOTES

SMALL ANGLE ELASTIC SCATTERING IN THE ISA

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A reasonable extrapolation of pp elastic scattering cross sections to ISA energies suggests that the forward peak has a cross section of several millibarns. Consequently, with a potential luminosity of $\approx 10^{33} \text{ cm}^{-2}/\text{sec}$, one must expect that errors of a systematic nature will be much more serious than statistical fluctuations. Neglecting for the moment the question of nuclear-Coulomb interference, we can say roughly:

$$\frac{d\sigma}{dt} \approx \frac{2.6 \times 10^{-4}}{t^2} + 80e^{bt} \quad (1)$$

where the first term is due to Coulomb scattering and the second is due to nuclear scattering. [Throughout, the units of $\frac{d\sigma}{dt}$ will be $\text{mb}/(\text{GeV}/c)^2$ and those of t will be $(\text{GeV}/c)^2$.] The value of b is somewhat dependent on t , but a value of about $10 (\text{GeV}/c)^{-2}$ is typical at presently available energies and is sufficiently accurate for the analysis of possible uncertainties in measured cross sections. The nuclear and Coulomb cross sections are equal at about $|t| \approx .002$. From Eq. (1) we find that for $|t| \gg .002$

$$\left| \frac{\Delta \frac{d\sigma}{dt}}{\frac{d\sigma}{dt}} \right| \approx b \Delta t = (bt) \left(\frac{\Delta t}{t} \right) \quad (2)$$

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while for $t \ll .002$

$$\frac{\left| \frac{\Delta \frac{d\sigma}{dt}}{\frac{d\sigma}{dt}} \right|}{\left| \frac{d\sigma}{dt} \right|} \approx \frac{2\Delta t}{t} \quad (3)$$

Let us first consider the region $|t| \gg .002$, the nuclear scattering "diffraction peak". Since $\Delta t \propto \theta d\theta$ at a given momentum, unless b is a strong function of t the largest error will occur at large $|t|$. For example, at $|t| = 1$, setting $b = 10$, we get a 10% error in $\frac{d\sigma}{dt}$ for

$$\frac{\Delta t}{t} \approx 10^{-2}$$

or

$$\frac{\Delta \theta}{\theta} = 5 \times 10^{-3}$$

For this value of t , $\theta \approx 5$ mrad at 200 GeV so $\Delta \theta \approx 25$ μ rad. Assuming the same value of b this same accuracy in $\Delta \theta$ gives an uncertainty in the cross section at $t = .01$ of 1%. Of course, if one has detailed knowledge of the distribution of the incident beams as a function of θ , one can tolerate several times this spread, but since this is already an order of magnitude lower than the beam spread expected in the conditions of high luminosity, different beam conditions should be used for this experiment -- we note in passing that the expected momentum spread of the beam ($\lesssim .3\%$) produces a smaller effect on t than the angular spread.

One possible experimental setup is shown schematically in Figure 1. In order to reduce the angular divergence of the beams to an acceptable level, the diameter of the beam has been increased. A pair of quadrupoles (at each end of the intersection region) then focus the particles scattered at a small angle θ to rings of radius $r = f\theta$ at the respective focal planes. The dipoles are used to separate the two beams in order to avoid high luminosity near the detectors. In order to set the scale one should remember that if $f = 100$ meters, and $|t| = .002 \text{ (GeV/c)}^2$ ($\theta \approx 0.2$ mrad), $r = 2$ cms; since beam spots of a few mm should be attainable at a focus, this will be well outside any fringe of the beam. We have chosen a collinear intersection region so it is clearly necessary to reject scattering events in the fields of the quadrupoles and dipoles; the point of interaction can be determined via the time of flight of the two final state particles to about 30 cms which will permit the use of a long interaction length. With regard to the resolution of scattering angles, if we have detectors of 1 mm resolution in the focal plane (This can easily be achieved with proportional chamber or semiconductor counters.) for $f = 100$ meters this corresponds to $\approx 10 \text{ } \mu\text{rad}$ in θ , and ≈ 2 mrad in φ ; note that the momentum selection of the dipoles and quadrupoles will spread out background events from isobar production, etc., so that the elastic events, which will be determined by coplanarity and matching of the polar angles, should have little background in spite of the relatively poor resolution in φ . (It is comforting to realize that the large beam size enhances the use of the quadrupoles in the momentum selection!) The exact geometry

of the detectors will depend somewhat on the minimum size of the beam pipe at the intersection region compatible with the beam size at low momentum. We have discussed this point with members of the Accelerator Department who are now studying this question. Hopefully, the whole region can be studied in one setup, but perhaps a different arrangement will be needed for very small $|t|$. With regard to rate, the differential cross section at $|t| = 1$ is probably about 1/1000 of that at $t = 0$. Consequently, we estimate a cross section of $4 \times 10^{-30} \text{ cm}^2$ in a region $\Delta t = .05$ at $|t| = 1$. Since a counting rate of 1/minute is quite respectable, integral luminosities of $\approx 10^{28} \text{ cm}^{-2}/\text{sec}$ are usable if this is necessary to achieve the required angular resolution. In addition, the resolution of the detectors will be sufficiently good that an independent study of the relative angle of the incident beam particles can be made, should the spread be uncomfortably large.

Now, back to the question of nuclear-Coulomb interference which would provide a measurement of the real part of the nuclear scattering amplitude. If the real nuclear amplitude is very large, then violent changes in the shape of the cross section will be observed in the interference region. However, our experience with measurements of this type at up to 26 GeV/c protons on a stationary target indicates that if the real amplitude is comparable to that observed to date, a precise absolute differential cross section determination is necessary; if the scale error is $\approx 1\%$ an error in $\alpha = \frac{\text{Re } F}{\text{Im } F}$ of $\Delta\alpha \approx .02$ will result for $\alpha \approx 0.2$. Probably an error of this size is not too significant since the necessary assumption

of spin independence is not easily defended so let's use this absolute precision as a lower limit. We have two obvious possibilities, a good determination of the luminosity and the normalization to a known cross section. At the 1% level let's try to avoid the former for the time being. The obvious known cross section is that for very small angle single Coulomb scattering. Let us therefore examine the consequences of Eq. (3). For dominance of Coulomb scattering over the as-yet-unknown nuclear scattering, we need $|t| \ll .002$. Being optimistic we may take $|t| = .001$, or $\theta \approx .15$ mrad for the 200 GeV ISA. If we demand 1% for $\frac{d\sigma}{dt}$ this means $\frac{\Delta t}{t} \approx \frac{1\%}{2}$ or $\frac{\Delta\theta}{\theta} \approx \frac{1\%}{4}$. [N.B. At this point we should also begin to worry about $\frac{\Delta p}{p}$.] This gives $\Delta\theta \approx 0.4$ μ rad! At this point, we feel that the determination of the luminosity is more likely to provide a solution to the problem of normalization. In particular, with the setup described above the values of t are sufficiently well measured in the region $|t| \approx .01$ that a single absolute determination of the cross section with any beam conditions which can yield a good measure of luminosity will permit the absolute matching of the cross sections and so allow a determination of the real amplitude in pp scattering (still depending on the assumption of spin independence, of course). Incidentally, again within the assumptions of spin independence, this measurement will perhaps provide the most accurate determination of the p-p total cross section.

In conclusion, measurements of diffraction scattering can be made at the ISA with relatively simple experimental arrangements to a

precision equal to that obtained with conventional machines of lower energy. The measurements should extend into the region of nuclear-Coulomb interference so that a separate determination of the absolute differential elastic cross section, perhaps over a limited range of t with special machine operation, can be used to normalize the data in order to yield an accurate determination of the phase and magnitude of the forward scattering amplitude.

FIG. 1. POSSIBLE EXPERIMENTAL ARRANGEMENT