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Research carried out under this contract for several years has been concerned with the nonlinear behavior of unstable plasma in the threshold regime⁽¹⁾. Emphasis has been on carrying out calculations in realistic geometry and on making comparison with experiment. Earlier work concentrated on instabilities in weakly-ionized plasma⁽²⁻⁶⁾ because the application of fluid equations of motion was well justified and because some fairly clear experimental observations of stability thresholds were available (positive column instability, ExB instability, drift-dissipative instability). In the last two years, however, we have shifted to analysis of non-linear effects in fully ionized plasmas. Two "clean" sets of experiments have guided our initial choice of problems. One is the very clear threshold for onset of the flute instability in low-density, high-energy mirror machines⁽⁷⁻⁹⁾

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(DCX 1.5, Phoenix, Alice). Here the plasma is not only fully ionized but also collisionless. In addition, the instability shows explosive behavior near the threshold. Our analysis of this experiment is described below. A second set of experiments are those showing the onset of the drift instability (mostly in Q-machines)⁽¹⁰⁾. Here the plasma is fully ionized but collisional. Preliminary results are described below. Finally, we are beginning a study of the explosive nature of the mirror-flute instability. We hope to unravel the precise mechanism responsible for the effect.

1. Low Density Mirror Flute Instability: In this first application of the general method (for treating non-linear threshold instabilities) to a collisionless plasma, we ran across a number of new features. The most striking was the necessity to include particle source and loss terms in the equations of motion. Such terms are not at all important for evaluating the equilibrium or linear theory. However, no unique time-asymptotic solutions of the non-linear mode equations can be found unless these are included. The solution itself is actually independent of these terms. This result is entirely reminiscent of the need for dissipative terms in non-linear oscillation problems in order to obtain limit-cycle behavior. Another new problem is connected with

the existence of a large number of linear modes which are only slightly damped at the instability threshold. Any realistic analysis, of course, is limited to only a small number of these. It sometimes happens that harmonics created by non-linear beating among these few primary modes leads to a frequency very close to one of the linear modes which have not been included. In such a case, non-linear frequency shifts will produce "mode-locking" of the two, and one must be careful to exclude this harmonic in a consistent calculation.

Resolution of the problems noted above enabled us to carry through the threshold analysis of the mirror-flute instability⁽¹¹⁾. The most striking result was the discovery that explosive behavior above the threshold is possible only if one describes the plasma by use of a non-constant gravity⁽¹²⁾ rather than by the simpler constant gravity equations of Mikhailovskii⁽¹³⁾. Use of the later equations led to prediction of low-amplitude saturation of the flute-unstable mode, in direct disagreement with experiment. If we now use the more general set of equations for non-constant gravity, we obtain a very different state of affairs. In linear theory, the only difference is the addition of a slightly damped mode with frequency roughly double that of the threshold mode. In the non-linear regime however, this

upper mode is pumped by the lower. If the injected current is increased by a critical fractional amount Δ_c above its threshold value, the upper mode no longer saturates. Instead it increases without limit and also shuts off the lower mode. This behavior is in striking agreement with experiment.

The critical value Δ_c is quite sensitive to the assumed shape of the equilibrium density. We have tested various equilibria, each being a better approximation to the experiment and found that Δ_c decreases rapidly and becomes quite small compared to unity. One cannot make a better comparison at this time because the experiment only indicates a small value of Δ_c but does not pinpoint its value. We have suggested a number of additional experiments which could be tried. Details of the above will be published in the Physics of Fluids. Further details will be found in a thesis by Cheng-I. Weng⁽¹⁴⁾.

2. Collisional Drift Instability: We adopt this name to distinguish between the form of this instability in weakly-ionized plasma (drift-dissipative) and that occurring in collisionless plasma (universal or drift-instability). We are analyzing this instability using the general methods of reference 1. Because the basic equations of motion contain resistive terms (ion viscosity as well as electron resistivity

along the field direction) there is no need for source or loss terms as in the collisionless case. However, the introduction of viscosity led to its own new features. The resultant higher order equations of motion require additional boundary conditions for unique solutions. Neglect of this point led to indeterminate equations and an improper analysis of this problem by previous investigators⁽¹⁵⁾.

We have now completed a quasilinear analysis of this problem using a set of equations with constant electron temperature. The unstable mode is saturated at small amplitude, with the mean square amplitude $\langle n \rangle^2 = \alpha \Delta$ where α is a constant and Δ is the fractional increase of magnetic field above its critical value. The number α is one result which is directly comparable with experiment. The initial result for slab geometry and constant electron temperature is rather close to the experimental value. We are now repeating the calculation with a more general set of equations which include electron temperature variation. In addition, of course, we plan to compare our results with those carried out in cylindrical geometry. We expect to consult appropriate experimental groups in attempting a best fit. We also plan to extend the theory to the collisionless regime.

3. Origin of Explosive Instability for Mirror-Flute: We have begun a study of the explosive instability in the mirror-flute case, to see if we can unravel the basic cause. This is a non-linear instability and sets in only at a finite amplitude of the lower mode. We are investigating the conjecture that the saturated lower-mode instability sets up a modified equilibrium with a temperature gradient. The temperature gradient instability, in turn, is known to occur above a certain ratio of this gradient to the density gradient. It may be that this produces uncontrolled growth of the upper mode. Any knowledge of the mechanism, of course, would aid greatly in designing feedback systems for suppressing the effect or enhancing it, as desired.

4. Summary and Plans: The mirror-flute instability analysis has been carried through in both the quasilinear and non-linear limits in slab and cylindrical geometry. Explosive instability occurs only if one includes "non-constant" gravity in the equations of motion. Future work will concentrate on clarifying the origin of the explosive instability with application to feedback stabilization schemes. In addition we hope to explore non-linear aspects of the behavior below threshold in order to cast some light on the experimental observation of frequency jumps during the decay

process. We may do some further analysis of the mode-locking phenomena by extending the calculation to include one or more of the other near-threshold linear modes.

The analysis of the collisional drift instability has been carried through in both quasilinear and non-linear limits in slab geometry for constant electron temperature. We will now extend this to include electron temperature variation and cylindrical geometry. Emphasis will be on careful comparison with experimental observation. We plan to extend the theory to the collisionless case.

5. Personnel: The principal investigator has devoted 20% of his time to this project during the academic year. In addition, one summer month was spent on the project. Two graduate students were supported by this contract. Mr. Cheng I. Weng (Mechanical and Aerospace Sciences Department) is completing his thesis on the "Low Density Mirror Flute Instability". Mr. Don Monticello (Physics Department) is working on the collisional drift instability and will probably complete his work in the spring. Mr. V. G. Abhyanhan (Mechanical and Aerospace Sciences) has just begun working on the origins of the explosive nature of the flute instability.

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