A CANONICAL APPROACH TO HIGH ENERGY SCATTERING
IN MASSIVE QUANTUM ELECTRODYNAMICS

Lay-Nam Chang

The Enrico Fermi Institute

The University of Chicago, Chicago, Illinois, 60637

and

Ngee-Pong Chang

Physics Dep't., City College of the City University of New York

New York, New York, 10031

APRIL, 1971

Contract No. AT (11-1)-264

EFI 71-23

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Lay-Nam Chang *

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The University of Chicago, Chicago, Illinois 60637

Ngee-Pong Chang +

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ABSTRACT

By the canonical Hamiltonian approach to field theory we have studied the very high energy limit of massive Quantum Electrodynamics. We introduce a series of canonical transformations which bring the usual Hamiltonian of a neutral vector meson interaction with a conserved current to the effective Hamiltonian that operates at infinite energy.

The result we obtain for the S-matrix operator is that it factorizes at very high energies:

$$S = S_0 (U^\dagger T(\exp(-i \int_{-\infty}^{\infty} dt H_s(t))) U)$$

where S_0 is the S-matrix for the hard meson "core", first given by Cheng and Wu, Chang and Ma

$$S_0 = \exp \left\{ -i \frac{g^2}{(2\pi)^2} \int d^2k \frac{j_R(k_L) j_L(-k_L)}{k_L^2 + \mu^2} \right\}$$

and the second operator describes the contribution to scattering due to pionization effects. U is the dressing operator which clothes the hard meson "core" in a coherent state of virtual soft mesons and the T-product is obviously the time-evolution operator for soft meson dynamics.

*Supported in part by the U.S. Atomic Energy Commission.

+Supported in part by a grant from the City University Research Foundation.

Section 1.

Physics at very high energies is believed to be simple. This belief, until recently, was based more on physical intuition than on a complete theoretical framework such as that of field theory. However, recently,¹⁾ within the last two years, very much stimulated by the ideas of Feynman and Yang,²⁾ Cheng and Wu³⁾ and others^{3), 4)} have carried out an extensive investigation of field theory, summing various classes of Feynman diagrams and arriving at the infinite energy limit. The results of their investigations on the whole lend support to the parton ideas of Feynman¹⁾ and to the coherent droplet ideas of Yang.²⁾

In this paper we study the field theory of the massive Quantum Electrodynamics in the limit of very high energies from a Hamiltonian approach, rather than in terms of summing Feynman diagrams. Our analysis thus is very close in spirit to the talk that Feynman gave at Stony Brook 1969,¹⁾ except that we commit ourselves to a specific model field theory. The language we use is the noncovariant one, viz. with respect to the center-of-mass frame, and the picture we describe in this language is, similar, and we believe, adds to the one given by Cheng and Wu.

The picture we have for massive quantum electrodynamics at very high energies is this: An incident hadron, of momentum W along the z direction, is a fully dressed state with a "hard" meson "core" as well as a "soft" meson cloud.¹⁾ These are the "partons" in Feynman's language. The hard meson "core" is due to the virtual dissociation of a hadron of momentum $P_z = W$ to a hard meson ($p_z = xW, 1 > x > 0$) and a hadron ($p_z = (1-x)W$) and these correspond to the "wiggles" of Cheng and Wu.⁵⁾ The soft meson cloud is due to the virtual bremsstrahlung of a soft meson (momentum $\vec{k}, k_z \ll W$) by the hadron of momentum W . The soft meson cloud is in a coherent state,⁶⁾

and is attached to the "core", i.e. it is, in classical physics terms, a persistent field and not a radiation field of the moving particle.

One could either follow the motion of the core or that of the cloud. Let us first follow the motion of the cores. The right (R) and left (L) moving cores, if there were no soft meson clouds attached to them, would simply not interact to leading order in W since the energy cost is too great for a hard meson moving right to, in Feynman's words, jump the wagon.¹⁾ But there are soft meson clouds which overlap when the R and L cores meet and the resultant interaction Hamiltonian between the two cores is an effective current x current interaction; in particular it is the current for the right moving system interacting with the current for the left moving system as well as the current for soft (pionisation) particles, if they are present.

Now the current operator can connect between the initial dressed hadron state and any final physical multiparticle state. The momentum transfer carried by this current is soft, so that the final "fragmented" state carries the same large overall momentum W as the initial hadron. The effective Hamiltonian of interaction between the two cores may thus be called the fragmentation Hamiltonian. Were it not for pionisation, the resulting S-matrix element would simply be

$$S_0 = \exp \left(- \frac{ig^2}{(2\pi)^2} \int d^3k \underset{3), 4)}{j_R(k_1)} \underset{3)}{j_L(-k_1)} / (k_1^2 + \mu^2) \right) \quad (1.1)$$

a result first obtained by Cheng and Wu, Chang and Ma, and others. This S_0 corresponds to "black dots" in the impact picture of Cheng and Wu.⁵⁾ Matrix elements of j_R, j_L are their impact factors.⁵⁾ This form of the S-Matrix is consistent with the coherent - droplet model of Yang.²⁾ $j_R(\vec{k})$ and $j_L(\vec{k})$ are the Fourier transforms of the current operators. The effect of pionisation is to append on to S_0 the time evolution operator for the soft current

interaction with the right and left currents. (See (5.5))

To understand better the physical process involved in pionisation we should next follow the motion of the clouds. As we mentioned earlier, for the class of experiments where we do not or are not able to measure the fragmentation products, we can regard the initial hadrons as sources of the coherent state of soft virtual mesons. These soft virtual mesons will of course interact according to the time evolution operator appropriate to soft meson dynamics,

$$T\left(e^{-i\int_{-\infty}^{\infty} dt H_s(t)}\right) \quad (1.2)$$

Thus the full S-Matrix can be given by reading off from (1.2) in the coherent state basis rather than, as is usual, in the Fock space of definite numbers of meson. More precisely, if U is the dressing operator for the soft meson cloud, see eq. 4.5, then

$$S = S_0 U^\dagger T\left(e^{-i\int_{-\infty}^{\infty} dt H_s(t)}\right) U \quad (1.3)$$

The approach we are taking in this report bears a great similarity to that used in studies on scalar field theories⁷⁾, and also, in the classic investigations on quantum electrodynamics⁸⁾. The basic Hamiltonian we start with is simple in structure, but leads to fairly complicated processes. Of these, only a certain class is expected to be important at high energies, and we can perform a canonical transformation on the original Hamiltonian so that this class is explicitly singled out. The analogy with QED is now clear. There, the initial Hamiltonian contains terms describing absorption and emission of mesons by free nucleons. These processes are forbidden if we impose energy conservation, and may be transformed away⁹⁾. The way to do this has already been pointed out by Schwinger and Tomonaga, and our canonical transformation is constructed to parallel theirs.

The plan of the paper is as follows. In Section 2 we set up our notation, and exhibit the relevant kinematics at high energies in the center-of-mass frame. In Section 2 we also introduced the concept of order of processes that occur at high energies in our model. With respect to such an order, we may expect certain processes to be suppressed relative to others. This is analogous to the case in QED where certain processes do not occur because of energy conservation. The concept of suppression by order is of course absent there, although the basic mechanisms involved is the same. We then construct, the appropriate canonical transformations (Sections 3 and 4) to exhibit these suppressions manifestly in the Hamiltonian. The

procedure is reminiscent of that used by Foldy and Wouthuysen¹⁰⁾ to exhibit the relevant non relativistic suppression factors for motion of electrons in an external field. In Section 5 we obtain our final expression for the S-matrix operator, and conclude with some remarks on the process of pionisation.

Section 2.

In this section we introduce more precisely the notation as well as the language that we will find to be convenient for our later discussion. This language is not a Lorentz covariant language but will refer exclusively to the Center-of-Mass frame. In this frame we picture the two incident hadrons as having momentum W in the positive and negative z -directions respectively, W being large. We will refer to particles moving in the C.M. frame as right, left, or "soft" in accordance with their P_z ranges:

$$\begin{aligned} \text{R:} & \quad \infty > p_z > x_0 W \\ \text{S:} & \quad x_0 W > p_z > -x_0 W \\ \text{L:} & \quad -x_0 W > p_z > -\infty \end{aligned} \quad (2.1)$$

In (2.1) x_0 is a small parameter, but not "wee" in the sense of Feynman, i.e. not of order $1/W$.

As a matter of fact, we will build this distinction into our field theory directly through the field operators. Thus, we write for the fermion field

$$\psi(\vec{x}) = \psi_R(\vec{x}) + \psi_S(\vec{x}) + \psi_L(\vec{x}) \quad (2.2)$$

where

$$\psi_R(\vec{x}) = \frac{1}{\sqrt{(2\pi)^3}} \int \frac{d^3p}{p_0} \sqrt{m} \left[u(\vec{p}, s) a(\vec{p}, s) e^{i\vec{p} \cdot \vec{x}} + v(\vec{p}, s) b^\dagger(\vec{p}, s) e^{-i\vec{p} \cdot \vec{x}} \right] \quad (2.3)$$

and similarly for $\psi_L(\vec{x})$ and $\psi_S(\vec{x})$. In (2.3) we have used the invariant normalization

$$\{ a(\vec{p}, s), a^\dagger(\vec{p}', s') \} = \delta_{ss'} p_0 \delta(\vec{p} - \vec{p}') \quad (2.4)$$

The advantage of (2.4) is that with respect to R(right) and L(left) particles, the normalization is independent of W .

Next we introduce the notion of an order with respect to W of an operator such as the free Hamiltonian. Consider the free Hamiltonian for the fermions

$$H_0 = \int d^3p \sum_i \{ a^\dagger(\vec{p}, i) a(\vec{p}, i) + b^\dagger(\vec{p}, i) b(\vec{p}, i) \} \quad (2.5)$$

The matrix elements of H_0 with respect to infinite energy states, normalized in accordance with (2.4) are of order W as $W \rightarrow \infty$. We refer to the operator H_0 as being of order W .

In general, we determine the order of any operator, by expressing it in terms of particle operators associated with invariantly normalized R , L and S states and to scale the R , L ranges in terms of the Feynman x -parameters

$$p_z = \pm x W \quad x_0 < x < \infty \quad (2.6)$$

If the operator is scale invariant, with respect to W , then it is of order 1, while if the operator has a scale factor W left then it is of order W , etc. An operator which involves only S states will be defined to be of order 1.

By this prescription, the usual mass renormalization term

$$\int d^3x \delta_m \bar{\Psi}(\vec{x}) \Psi(\vec{x}) = \delta_m \int d^3x \{ \bar{\Psi}_S(\vec{x}) \Psi_S(\vec{x}) + \bar{\Psi}_R(\vec{x}) \Psi_R(\vec{x}) + \bar{\Psi}_L(\vec{x}) \Psi_L(\vec{x}) \} \quad (2.7)$$

can be seen to be of order 1, $1/W$, $1/W$ respectively.

With respect to infinite energy states, mass renormalization term is of order $1/W$.

Next we write down the corresponding operator decomposition for the neutral vector meson field, $A_\mu(x)$.

$$A_\mu(\vec{x}) = A_\mu^R(\vec{x}) + A_\mu^S(\vec{x}) + A_\mu^L(\vec{x}) \quad (2.8)$$

where

$$A_\mu^R(\vec{x}) = \frac{1}{\sqrt{2(2\pi)^3}} \int \frac{d^3k}{\omega} (A_\mu(\vec{k}) e^{i\vec{k}\cdot\vec{x}} + A_\mu^*(\vec{k}) e^{-i\vec{k}\cdot\vec{x}}) \quad (2.9)$$

and similarly for A_μ^S and A_μ^L fields. ($A_\mu^* = (\vec{A}^\dagger, -A_4^\dagger)$)

The quantisation condition reads

$$[A_\mu(\vec{k}), A_\nu^*(\vec{k}')] = \delta_{\mu\nu} \omega \delta(\vec{k} - \vec{k}') \quad (2.10)$$

The spin content of $A_\mu(\vec{k})$ can be expressed by the relation

$$A_\mu(\vec{k}) = \sum_{\lambda=-1}^1 \epsilon_\mu(\vec{k}, \lambda) a(\vec{k}, \lambda) + \frac{k_\mu}{m_p} a(\vec{k}) \quad (2.11)$$

where

$$\sum_{\lambda=-1}^1 \epsilon_\mu(\vec{k}, \lambda) \epsilon_\nu^*(\vec{k}, \lambda) = (\delta_{\mu\nu} + \frac{k_\mu k_\nu}{m_p^2}) \quad (2.12)$$

and $a(\vec{k}, \lambda)$ and $a(\vec{k})$ are the annihilation operators for the spin 1 and spin 0 particles described by $A_\mu(\vec{k})$. Notice that $a(\vec{k})$ is not a physical particle operator since from (2.10)

$$[a(\vec{k}), a^\dagger(\vec{k}')] = -\omega \delta(\vec{k} - \vec{k}') \quad (2.13)$$

so that the spin 0 particle in A_μ is a ghost state.

If we had used a pure spin 1 vector field, the resulting interaction Hamiltonian, as is well known, has a normal dependent contact term.⁵⁾ For a conserved current interacting with the A_μ field, the two formulations are equivalent. For the sake of convenience as well as clarity we will focus our attention on the Stuckelberg formulation.

The usual vector meson interaction is thus

$$ig \int d^3x \bar{\Psi}(\vec{x}) \gamma_\mu \Psi(\vec{x}) A_\mu(\vec{x}) \quad (2.14)$$

which, in our language, can be split into various configuration for the virtual emission and absorption of neutral vector meson, viz.

$$H_{\text{int}} = H_b + H_d + H_s + H' \quad (2.15)$$

where

$$H_b = \int d^3x \left\{ ig \bar{\Psi}_R(\vec{x}) \gamma_\mu \Psi_R(\vec{x}) + ig \bar{\Psi}_L(\vec{x}) \gamma_\mu \Psi_L(\vec{x}) \right\} A_\mu^S(\vec{x}) \quad (2.16)$$

$$H_d = ig \int d^3x : \left\{ \bar{\Psi}_R(\vec{x}) \gamma_\mu \Psi_R(\vec{x}) A_\mu^R(\vec{x}) + \bar{\Psi}_L(\vec{x}) \gamma_\mu \Psi_L(\vec{x}) A_\mu^L(\vec{x}) \right\} : \quad (2.17)$$

$$H_s = ig \int d^3x : \bar{\Psi}_s(\vec{x}) \gamma_\mu \Psi_s(\vec{x}) A_\mu^S(\vec{x}) : \quad (2.18)$$

$$H' = ig \int d^3x : \left\{ \bar{\Psi}_R(\vec{x}) \gamma_\mu \Psi_R^L(\vec{x}) A_\mu^L(\vec{x}) + \bar{\Psi}_L(\vec{x}) \gamma_\mu \Psi_L(\vec{x}) A_\mu^R(\vec{x}) \right. \\ \left. + (\bar{\Psi}_R(\vec{x}) \gamma_\mu \Psi_L(\vec{x}) A_\mu^S(\vec{x}) + \bar{\Psi}_L(\vec{x}) \gamma_\mu \Psi_R(\vec{x}) A_\mu^S(\vec{x}) \right. \\ \left. + \bar{\Psi}_L(\vec{x}) \gamma_\mu \Psi_s(\vec{x}) A_\mu^L(\vec{x}) + \text{h.c.}) \right\} : \quad (2.19)$$

Now we observe that the different interaction pieces of the Hamiltonian can be classified in terms of their order with respect to W , in the sense that we have noted above. Thus, we find that the bremsstrahlung interaction for right moving particles is of order 1, since

$$ig \int d^3x : \bar{\Psi}_R(\vec{x}) \gamma_\mu \Psi_R(\vec{x}) : A_\mu^S(\vec{x}) \\ \xrightarrow{W \rightarrow \infty} \frac{1}{\sqrt{2(2\pi)^3}} \int \frac{d^3k}{\omega} \int_{x_0}^{\infty} \frac{dx}{x} \left\{ (a^\dagger(p_1+k_1, xW+k_2) a(p_1, xW) \right. \\ \left. - b^\dagger(p_1+k_1, xW+k_2) b(p_1, xW) A_-(\vec{k}) + \text{h.c.} \right\} \quad (2.20)$$

and matrix elements of (2.20) with respect to infinite energy states (normalized invariantly) clearly will no longer depend explicitly on W as $W \rightarrow \infty$. In (2.20) we have introduced the notation

$$A_\pm(\vec{k}) = A_3(\vec{k}) \mp iA_4(\vec{k}) \quad (2.21)$$

Similarly we find that the dissociation of right moving hadron into a hard vector meson and a hadron is of order $1/W$, viz.

$$H_d = \frac{1}{W} \frac{g}{\sqrt{2(2\pi)^3}} \int d^3k d^3p \left\{ \int_{x_0}^{\infty} dx \int_{x_0}^{x-x_0} dy \frac{1}{xy(x-y)} a^\dagger((x-y)W, p_1-k_1) \right. \\ \times \left[\frac{\sigma_3 \sigma_a}{2} \left(\sqrt{\frac{x-y}{x}} - \sqrt{\frac{x}{x-y}} \right) + \left(\sqrt{\frac{x}{x-y}} \frac{\sigma_1(p-k)_1 \sigma_a}{2m} + \sqrt{\frac{x-y}{x}} \frac{\sigma_a \sigma_1 p_1}{2m} \right) \right. \\ \left. \left. - \frac{k_a}{m} \sqrt{x(x-y)} / y \right] a(xW, p_1) A_a^\dagger(yW, k_1) + \text{h.c.} \right\} +$$

$$\begin{aligned}
& + \int_{2x_0}^{\infty} dy \int_{x_0}^{y-x_0} dx \frac{1}{xy(y-x)} a^\dagger((y-x)W, k_\perp - p_\perp) \left[-\frac{y}{2\sqrt{x(y-x)}} \sigma_a \right. \\
& \quad + \left(\sqrt{\frac{x}{y-x}} \frac{\sigma_\perp \cdot (p-k)_\perp \sigma_a \sigma_3}{2m} + \sqrt{\frac{y-x}{x}} \frac{\sigma_\perp \cdot p_\perp \sigma_a \sigma_3}{2m} + \frac{k_a}{m} \frac{\sqrt{x(y-x)}}{y} \right] \\
& \quad \times b^\dagger(xW, p_\perp) A_a(yW, k_\perp) + h.c. \Big\} \\
& + \frac{1}{W} \frac{g}{\sqrt{2(2\pi)^3}} \int d^2k d^2p \left\{ \int_{2x_0}^{\infty} dx \int_{x_0}^{x-x_0} dy \frac{1}{y\sqrt{x(x-y)}} a^\dagger((x-y)W, p_\perp - k_\perp) a(xW, p_\perp) a^\dagger(yW, k_\perp) \right. \\
& \quad \times \left[\frac{p_\perp^2 + m^2}{2x} - \frac{k_\perp^2 + \mu^2}{2y} - \frac{(p-k)_\perp^2 + m^2}{2(x-y)} \right] / (m\mu) + h.c. \\
& + \int_{2x_0}^{\infty} dy \int_{x_0}^{y-x_0} dx \frac{1}{y\sqrt{x(y-x)}} a^\dagger((y-x)W, k_\perp - p_\perp) b^\dagger(xW, p_\perp) a(yW, k_\perp) \\
& \quad \times \left[\frac{k_\perp^2 + \mu^2}{2y} - \frac{p_\perp^2 + m^2}{2x} - \frac{(k-p)_\perp^2 + m^2}{2(y-x)} \right] / (\mu m) + h.c. \Big\} \\
& \hspace{15em} (2.22)
\end{aligned}$$

$$a \leftrightarrow b$$

In (2.22), A_a^* is the particle operator for pure spin 1 field defined as

$$A_a^* = \sum_{\lambda} \epsilon_a(\vec{k}, \lambda) a^\dagger(\vec{k}, \lambda) \quad a = 1, 2 \quad (2.23)$$

Table I: a wavy line represents the neutral vector meson, a solid line the proton, say, while the $\pm$ charge will serve to distinguish between particle and antiparticle.

Interaction Vertex	Order of Operator	Energy Difference
	$0\left(\frac{1}{W}\right)$	$0\left(\frac{1}{W}\right)$
	$0(1)$	$0(1)$
	$0(1)$	$0(1)$
	$0(1)$	$0(W)$
	$0(1)$	$0(W)$
	$0\left(\frac{1}{\sqrt{W}}\right)$	$0(1)$
	$0(1)$	$0(W)$
	$0(1)$	$0(W)$
	$0\left(\frac{1}{W}\right)$	$0\left(\frac{1}{W}\right)$
	$0(1)$	$0(1)$

TABLE I

In this way, we can construct a table (Table 1) for the different pieces of the interaction. At the same time we shall note for each piece of the interaction the order with respect to W of the energy difference between the initial state and the final state after the interaction has acted.

In constructing Table 1, we have obviously omitted those interaction vertices that can be obtained by a mirror reflection. The role of the energy differences will be clear when we think in terms of the old fashioned perturbation theory. For the present we shall simply note that where the energy difference across the interaction vertex is of order 1 or smaller, we shall, following Feynman, refer to it as an energy matching interaction vertex, while for those cases where the energy difference across the vertex is of order W we refer to them as energy non-matching interaction vertices.

Section 3.

To prepare the groundwork for our discussion of field theory in the infinite energy limit, we present here a brief summary of the time-independent Lippman-Schwinger formalism for the description of scattering.¹⁸⁾

$$\text{Let } H = H_0 + V \quad (3.1)$$

and introduce eigenfunctions $\phi_\alpha, \psi_\alpha^\pm$ such that

$$\left. \begin{aligned} H_0 \phi_\alpha &= \epsilon_\alpha \phi_\alpha \\ H \psi_\alpha^\pm &= \epsilon_\alpha \psi_\alpha^\pm \end{aligned} \right\} \quad (3.2)$$

Then the scattering states $\psi_\alpha^\pm$ are related to ϕ_α by

$$\psi_\alpha^\pm = \phi_\alpha + \frac{1}{\epsilon_\alpha - H \pm i\epsilon} V \phi_\alpha \equiv \Omega_\pm \phi_\alpha \quad (3.3)$$

and the scattering matrix element will be given by

$$S_{\beta\alpha} = \delta_{\beta\alpha} - 2\pi i \delta(\epsilon_\beta - \epsilon_\alpha) (\psi_\beta^-, V \phi_\alpha) \quad (3.4)$$

$$\equiv \delta_{\beta\alpha} - 2\pi i \delta(\epsilon_\beta - \epsilon_\alpha) (\phi_\beta, T(\epsilon_\beta) \phi_\alpha) \quad (3.5)$$

$$\begin{aligned} \text{where } T(E) &= V + V \frac{1}{E - H_0 + i\epsilon} V \\ &+ V \frac{1}{E - H_0 + i\epsilon} V \frac{1}{E - H_0 + i\epsilon} V + \dots \end{aligned} \quad (3.6)$$

Equivalently, the S matrix operator may be expressed in terms of the $\Omega_\pm$ operators introduced in (3.3), viz.

$$S = \Omega_-^\dagger \Omega_+ \quad (3.7)$$

(3.6) is the so-called old-fashioned perturbation theory which is not manifestly covariant. The usefulness of this theory for infinite

momentum analysis was first pointed out by Weinberg.¹²⁾ However this was still for the case of total C.M. energy being finite, whereas we shall be interested in the limit as energy becomes large.

In the context of the old-fashioned perturbation theory the relevance of the notions of the order of the operator and the energy matching across vertices becomes clear. Forgetting about the integration over intermediate states, the contribution to the T matrix element of an interaction vertex that, though of order 1, does not match the energy across the vertex will be suppressed by a factor of W relative to the contribution of a vertex that is of order 1 and matches the energy. Energy non-matching however is not the only consideration, for even where energy matching occurs, the relative size of the contribution to the T matrix element depends on the relative order (i.e. strength) of the interaction vertices.

But what about the integration over intermediate states? There are well known ultra-violet divergences that come from the $|\vec{p}| \rightarrow \infty$ end of the integration of the momenta of all on-shell intermediate particles. For a renormalizable theory, the divergences can be handled through mass and vertex renormalisation, in a systematic way. For our purposes, we simply note that where the divergences occur in the dx integrals after W has been scaled out, they occur in diagrams on which the renormalization procedure is to be carried out, and this procedure that makes the dx integrals well defined, does not depend on W . It is important here that x_0 , the lower limit of the dx integral, be small, but not "wee" in the sense of Feynman, i.e.

x_0 should not be of order $1/W$, since otherwise the remaining integral over dx will have a dependence on W which will spoil our notions of the relative size of contributions of various interaction vertices.

In this context, we should make a remark here about the s range integration. We have heretofore defined operators such as H_s to be of order 1 since it acted only on particles moving "slowly" in the C.M. frame. However the range of p_z extends to $x_0 W$, x_0 being small. There can be a dependence on W through the upper end of the "soft" integration, but this dependence by a usual power count can only be logarithmic: Our nomenclature of order 1 is thus to be taken as order 1 up to logarithms.⁵⁾

To proceed with the old fashioned perturbation theory we could write down the systematic rules for the time-ordered graphs and classify the graphs in terms of their limiting behavior as $W \rightarrow \infty$.¹³⁾ While this procedure may be well defined mathematically, a better physical insight is gained by examining the Hamiltonian itself.

Our terminology has been invented to guide us as to which piece of the interaction is effective for high energies. Our aim therefore should be to exhibit the effective Hamiltonian that operates at high energies. From the table, we see that H_s , H_d and H_b are all interactions that could contribute to the T matrix elements as $W \rightarrow \infty$. In H' however there are vertices that survive (i.e. of order 1) which do not match energy and vertices that match energy but whose strengths are relatively weak. By our previous discussion we expect that H' makes a relatively small contribution to the T-matrix element

as $W \rightarrow \infty$ although at the level of the Hamiltonian itself we cannot say that H' is negligible compared with H_b , H_d and H_s . (In fact H' is of order 1 as $W \rightarrow \infty$). To exhibit the suppression, we shall perform a canonical transformation which maps H' into an operator that is at most of order $\frac{1}{\sqrt{W}}$ and which does not match energy across the vertex.⁸⁾

Define the canonical transformation

$$U' = \exp \left\{ -i \int_{-\infty}^0 dt' e^{at'} H'(t') \right\} \quad a \rightarrow a_+ \quad (3.8)$$

$$\equiv e^{iF'} \quad (3.9)$$

where
$$H'(t) \equiv e^{iH_0 t} H' e^{-iH_0 t} \quad (3.10)$$

H' is the interaction Hamiltonian involving, from table I, energy mismatched vertices as well as vertices that match energy but are themselves weak as $W \rightarrow \infty$. We note that F' is an operator that is at most of order $1/\sqrt{W}$ as $W \rightarrow \infty$.

Now the transformed Hamiltonian is

$$\begin{aligned} U'^{\dagger} H U' &= H_0 + H_b + H_d + H_s + H' \\ &\quad - i [F', H_0] - \frac{1}{2!} [F', [F', H_0]] + \dots \\ &\quad - i [F', H_b + H_d + H_s + H'] - \frac{1}{2!} [F', [F', H_b + H_d + H_s + H']] + \dots \end{aligned} \quad (3.11)$$

but
$$[F', H_0] = -i H' \quad (3.12)$$

$$\begin{aligned} \therefore U'^{\dagger} H U' &= H_0 + H_b + H_d + H_s \\ &\quad - i [F', H_b + H_d + H_s + \frac{H'}{2}] \\ &\quad - \frac{1}{2} [F', [F', H_b + H_d + H_s + \frac{2}{3} H']] + \dots \end{aligned} \quad (3.13)$$

$$= H_0 + H_b + H_d + H_s + O\left(\frac{1}{\sqrt{W}}\right) \quad (3.14)$$

The last statement follows from the fact that F' is of order $\frac{1}{\sqrt{W}}$ and H_b, H_s are of order 1, (H_d is of order $1/W$) the commutator involves basic commutation relations (2.4) and (2.10) which are invariant under scaling in W and thus the commutators, $[F', H_b]$ etc., will not introduce additional powers of W . To be sure, however, that the operator $O(1/\sqrt{W})$ in (3.14) will not contribute to the transition matrix element we have checked that the energy difference across the effective vertex $O(1/\sqrt{W})$ in (3.14) cannot be of order $1/W$, so that the energy denominator cannot overcome the weak strength of the effective vertex.

Section 4.

The Hamiltonian that concerns us thus at ∞ energy then is

$$H = H_0 + H_d + H_b + H_s + O\left(\frac{1}{\sqrt{W}}\right) \quad (4.1)$$

The physical significance of each piece of the Hamiltonian is clear.

Notice, in particular, the $H_0 + H_d$ can be rewritten as $H_R + H_L$,

where

$$H_R = \int_{p_z > 0} d^3p \left[a^\dagger(\vec{p}, s) a(\vec{p}, s) + b^\dagger(\vec{p}, s) b(\vec{p}, s) \right] + \int_{k_z > 0} d^3k A_\mu^*(\vec{k}) A_\mu(\vec{k}) \\ + ig \int d^3x : \bar{\Psi}_R(\vec{x}) \gamma_\mu \Psi_R(\vec{x}) : A_\mu^R(\vec{x}) \quad (4.2)$$

and H_L is similarly defined for the left moving system. Since

$$[H_R, H_L] = 0 \quad (4.3)$$

the eigenstates of H_R and H_L are not confused with each other so that the eigenstates of $H_R + H_L$ are direct products of R and L states. Physically H_R and H_L generate the time evolution of right and left moving hadrons minus their virtual soft meson cloud.

The structure of the virtual soft meson cloud can of course be deduced from the full Hamiltonian (4.1); it is however easier to exhibit the virtual soft meson cloud by analogy with quantum electrodynamics⁸⁾ and perform the following canonical transformation.

Let

$$U \equiv e^{iF} \quad (4.4)$$

$$= \exp\left(-i \int_{-\infty}^0 H_b(t) e^{\alpha t} dt\right) \quad \alpha \rightarrow 0_+ \quad (4.5)$$

where

$$H_b(t) \equiv e^{i(H_R+H_L)t} H_b e^{-i(H_R+H_L)t} \quad (4.6)$$

Then

$$\begin{aligned} U^\dagger U'^\dagger H U' U &= H_R + H_L - \frac{i}{2} [F, H_b] - \frac{1}{3} [F[F, H_b]] + \dots \\ &+ H_s - i [F, H_s] - \frac{1}{2} [F[F, H_s]] + \dots \end{aligned} \quad (4.7)$$

At this point we make use of the vectorial nature of the basic interaction to observe that, as $W \rightarrow \infty$,

$$\begin{aligned} H_b &= g \int d^3x : \bar{\psi}_R(\vec{x}) \gamma_+ \psi_R(\vec{x}) : A_-^s(\vec{x}) \\ &- g \int d^3x : \bar{\psi}_L(\vec{x}) \gamma_- \psi_L(\vec{x}) : A_+^s(\vec{x}) + O\left(\frac{1}{W}\right) \end{aligned} \quad (4.8)$$

$$\gamma_{\pm} \equiv (\gamma_4 \pm i\gamma_3)/2 \quad (4.9)$$

$$A_{\pm}^s(\vec{x}) \equiv A_3^s(\vec{x}) \mp i A_4^s(\vec{x}) \quad (4.10)$$

so that

$$F = - \int_{-\infty}^0 dt \int d^3x g \left\{ \bar{\psi}_R(\vec{x}, t) \gamma_+ \psi_R(\vec{x}, t) A_-^S(\vec{x}, t) - \bar{\psi}_L(\vec{x}, t) \gamma_- \psi_L(\vec{x}, t) A_+^S(\vec{x}, t) \right\} + O\left(\frac{1}{\omega}\right) \quad (4.11)$$

A very important property of the $A_{\pm}^S$ fields is the fact that

$$[A_{\pm}^S(\vec{x}, t), A_{\pm}^{S\dagger}(\vec{y}, t)] = 0 \quad (4.12)$$

while

$$[A_{\pm}^S(\vec{x}, t), A_{\mp}^{S\dagger}(\vec{y}, t')] = 2 \Delta^S(\vec{x} - \vec{y}, t - t') \quad (4.13)$$

$$\Delta^S(\vec{x}, t) = \frac{1}{2(2\pi)^3} \int \frac{d^3k}{\omega} (e^{ik \cdot x} - e^{-ik \cdot x}) \quad (4.14)$$

This property follows directly from the quantisation rule (2.11).

Another important fact to observe is that involving the commutators of currents. For this, it is convenient to work with the Fourier transforms

$$\bar{\psi}_R(\vec{x}) \gamma_+ \psi_R(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k} \cdot \vec{x}} j_R(\vec{k}) \quad (4.15)$$

$$\bar{\psi}_L(\vec{x}) \gamma_- \psi_L(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k} \cdot \vec{x}} j_L(\vec{k}) \quad (4.16)$$

then, to order $1/W$, F may be given in terms of J_R, J_L as follows:

$$F = \frac{ig}{\sqrt{2}(2\pi)^3} \int_S \frac{d^3k}{\omega} \left\{ \frac{J_R(\vec{k}) A_-(\vec{k})}{k_z - \omega} + \frac{J_L(\vec{k}) A_+(\vec{k})}{k_z + \omega} - \text{h.c.} \right\} \quad (4.17)$$

(4.17) follows from the commutation rule

$$[H_R, J_R(\vec{k})] = k_z J_R(\vec{k}) + O\left(\frac{1}{W}\right) \quad (4.18)$$

In terms of the operators $J_R(\vec{k})$, a very important property that emerges is the commutator.

$$[J_R(\vec{k}), J_R(\vec{k}')] = O\left(\frac{1}{W}\right) \quad \vec{k}, \vec{k}' \in S \quad (4.19)$$

Because of (4.19), the transformed Hamiltonian (4.7) terminates if we are only interested in $O(1)$ operators.

From (4.17), (4.19) and (4.13), it is easy to see that the resulting Hamiltonian is

$$\begin{aligned} U^\dagger U'^\dagger H U' U &= H_R + H_L + \frac{2g^2}{(2\pi)^3} \int_S d^3k \frac{J_R(\vec{k}) J_L(-\vec{k})}{k_z^2 + \mu^2} \\ &+ H_S + \frac{2g^2}{(2\pi)^3} \int_S d^3k \frac{J_R(\vec{k}) J_-(-\vec{k})}{k_z^2 + \mu^2} \\ &+ \frac{2g^2}{(2\pi)^3} \int_S d^3k \frac{J_+^2(\vec{k}) J_L(-\vec{k})}{k_z^2 + \mu^2} + O\left(\frac{1}{\sqrt{W}}\right) \end{aligned} \quad (4.20)$$

Section 5.

We have finally then an effective Hamiltonian that for ∞ energy states has the form

$$\mathcal{H} = H_R + H_L + V + O\left(\frac{1}{\sqrt{W}}\right) \quad (5.1)$$

where H_R and H_L describe the dynamical evolution of the right and left moving hadron systems minus their virtual soft meson clouds. Or in a more picturesque language it would be as if H_R and H_L describe the motion of the hard meson "core" and that in this picture the soft meson cloud always follows the "core". V is the effective Hamiltonian that gives the interaction between the hard meson "cores". Of course, this interaction came about through the overlap of the virtual soft meson clouds.

Now a good basis for the application of Perturbation Theory is to use as H_0 the full $H_R + H_L$. This would be the basis where we recognise that the appropriate incoming states are not bare particles but rather, in the language of field theory, dressed particles where the dressing is governed by $H_R + H_L$.

To obtain the S-matrix operator from this final form of H as given in (5.1) it is easiest now to work in the interaction picture. In this picture

$$V(t) = e^{i(H_R+H_L)t} V e^{-i(H_R+H_L)t} \quad (5.2)$$

$$\begin{aligned} &= \frac{2g^2}{(2\pi)^3} \int d^3k e^{2ik_z t} \frac{j_R(\vec{k}) j_L(-\vec{k})}{k_1^2 + \mu^2} + H_S(t) \\ &+ \frac{2g^2}{(2\pi)^3} \int d^3k e^{ik_z t} \frac{[j_R(\vec{k}) j_L^S(-\vec{k}, t) + j_L^S(\vec{k}, t) j_R(-\vec{k})]}{k_1^2 + \mu^2} \\ &+ O\left(\frac{1}{\sqrt{W}}\right) \end{aligned} \quad (5.3)$$

The S-matrix operator that follows from this is

$$S = T \left(e^{-i \int_{-\infty}^{\infty} dt V(t)} \right) \quad (5.4)$$

$$= e^{-i \frac{g^2}{(2\pi)^2} \int d^3k \frac{j_R(k_L) j_L(-k_L)}{k_L^2 + \mu^2}} \times T \left(e^{-i \int_{-\infty}^{\infty} H_S(t) dt} \cdot e^{-i \frac{g^2}{(2\pi)^2} \int dt \int d^3k e^{ik_L t} \frac{[j_R(\vec{k}) j_L^S(-\vec{k}, t) + j_L^S(\vec{k}, t) j_R(-\vec{k}, t)]}{k_L^2 + \mu^2}} \right) \quad (5.5)$$

If soft current interactions were absent, the result (5.5) would correspond to the infinite energy limit first obtained by Cheng and Wu^{3),4),5)} in their summation of ladder diagrams for massive Quantum Electrodynamics. The simple exponential nature of S_0 , for the elastic scattering matrix element, translates into a simple eikonal form.⁽⁵⁾ There are, however, differences.

Cheng and Wu,³⁾ and others,⁴⁾ have considered the ladder diagrams with point γ_μ vertices and found an exponentiation neglecting the H_S interaction. In our case we have the operators $j_R(\vec{k})$ and $j_L(\vec{k})$ which can connect between different eigenstates of $H_R + H_L$ and thus in principle include the possibility of fragmentation of initial

particles. In the language of Cheng-Wu,^{3), 5)} our S_0 includes the dissociation of hadrons into its component states.

S_0 also coincides with the coherent droplet model proposed by Yang and co-workers,²⁾ when expressed in the operator formulation as given by Lee.⁴⁾

Our major result however is to express how pionisation affects the basic S_0 at infinite energy. By pionisation we mean the production of soft mesons in the C.M. frame.

An important feature that we note at this point is the fact that the effective Hamiltonian at infinite energy does not allow for physical bremsstrahlung of soft mesons. This is easy to understand physically since at high energies the interaction Hamiltonian is finite and the momentum W of the incident hadron is changed only by a finite amount. The hadron thus maintains its initial straight line path in the limit as $W \rightarrow \infty$ and to leading order the hadron will not radiate physical soft mesons. This is easy to see also from the Feynman diagram for bremsstrahlung, as was already noted by Cheng and Wu.¹⁶⁾

Pionisation however is a different matter. The virtual soft mesons in a coherent state⁶⁾ act as a semi-classical field which however can produce soft mesons or pions.

An alternate physical understanding of the pionisation result is through the following: The infinite energy hadron carries with it, as we already know, a virtual soft meson cloud, in a coherent state.⁶⁾ Instead of describing the scattering by following the motion of the hard meson "core" we can alternately think in terms of following the motion of the virtual soft meson clouds.

For this picture we may think of the incident hadrons as being the classical source for soft mesons. The soft mesons interact in accordance with the Hamiltonian H_s and the corresponding S-matrix would be simply

$$T \left(e^{-i \int_{-\infty}^{\infty} dt H_s(t)} \right) \quad (5.6)$$

Now our picture would have it that instead of reading off matrix elements for initial states with definite number of soft meson going into final states with another definite number of mesons, we would read off it for initial and final states that are coherent states.^{6), 17)}

In terms of the operator U , defined on (4.5), we have then

$$S = U^\dagger S_0 T \left(e^{-i \int_{-\infty}^{\infty} dt H_s(t)} \right) U \quad (5.7)$$

Since S_0 commutes with U , to leading order on W , the resulting S-matrix operator is

$$S = S_0 U^\dagger T \left(e^{-i \int_{-\infty}^{\infty} dt H_s(t)} \right) U \quad (5.8)$$

This form of the S-matrix operator agrees with the form (5.5) obtained earlier.

A detailed study of the amplitude when one evaluates the S-matrix operator between coherent states is now under way, and we will come back to the pionisation process elsewhere.

We have thus obtained the S-matrix operator by utilising the transformation properties of the Hamiltonian of the theory. It should perhaps be recognised that our method is, in principle, applicable to other field theoretic models, like those that possess chiral symmetry. In practice, however, internal symmetry operators tend to make the manipulations more involved. The method, in any case, provides a good complementary approach in studying high energy limits to those that exist in the literature.

Finally, we make a comment about the non-unique choice of x_0 . Our implicit and intuitive assumption is that physical results do not depend sensitively on x_0 , provided x_0 is small enough. The reason for this observation is that if we set $x_0 = 1$, or .5, or even .1, and then proceed to transform away the "braking" vertex $\psi_R \gamma_\mu \psi_S A_\mu^R$, we would be missing a large piece of the dissociation vertex which by a mere quirk of definition has been shifted into the braking vertex. But as we decrease x_0 and make it smaller and smaller we would be including more and more of the dissociation vertex. Transforming away then the braking vertex will not mean dropping a large part of the dissociation vertex. A posterior experience with the Cheng-Wu calculations indicates that, order by order, the dependence on x_0 is a weak logarithmic one.

APPENDIX A

In this appendix we give a simple derivation of S_0 using old fashioned perturbation theory.⁽⁸⁾ For this derivation we ignore the possibility of H_s and deal directly with a Hamiltonian

$$H = H_0 + H_b + H_d \quad (A.1)$$

$$\equiv H_R + H_L + H_b \quad (A.2)$$

Calling $H_R + H_L$ the free Hamiltonian, the problem is to find the S-matrix operator for (A.2) as $W \rightarrow \infty$. Perform the canonical transformation, U , as defined in (4.5), and obtain

$$U^\dagger H U = H_0 + H_b + \frac{2g^2}{(2\pi)^2} \int d^3k \frac{d_R(\vec{k}) d_L(-\vec{k})}{k^2 + \mu^2} \quad (A.3)$$

$$= H_R + H_L + V \quad (A.4)$$

If we define

$$V \equiv \int d^3Q V(\vec{Q}) \quad (A.5)$$

an important property is that, to leading order of W ,

$$[V(\vec{Q}), V(\vec{Q}')] = O\left(\frac{1}{W}\right) \quad (A.6)$$

and also

$$[H_R + H_L, V(\vec{Q})] = 2Q_z V(\vec{Q}) + O\left(\frac{1}{W}\right) \quad (A.7)$$

Because of (A.7), the Moller operator defined through

$$\psi_a^+ = \Omega_+ \varphi_a = \varphi_a + \frac{1}{E_a - H_R - H_L + i\epsilon} V \psi_a^+ \quad (\text{A.8})$$

can be solved by direct iteration or by verification to be

$$\Omega_+ = e^{\int \frac{d^3 Q}{2Q_0 + i\epsilon} V(\vec{Q})} \quad (\text{A.9})$$

Similarly

$$\Omega_- = e^{\int \frac{d^3 Q}{2Q_0 - i\epsilon} V(\vec{Q})} \quad (\text{A.10})$$

so that

$$\begin{aligned} S_0 &= \Omega_-^\dagger \Omega_+ = e^{-i\pi \int d^3 Q V(Q_1)} \\ &= e^{-\frac{i g^2}{(2\pi)^3} \int d^3 k \frac{f_R(k_1) f_L(-k_1)}{k_1^2 + \mu^2}} \quad (\text{A.11}) \end{aligned}$$

APPENDIX B

The physical significance of U is clearly exhibited when we suppress H_s . It is the dressing operator for a one-particle state in the absence of soft meson dynamics. Turning on H_s would of course, cause further "dressing" of the coherent state.

Consider the one-particle eigenstate of H as given in (A.1)

$$H |\vec{p}\rangle_d = \left(W + \frac{p_L^2 + m^2}{2W} + O\left(\frac{1}{W^3}\right) \right) |\vec{p}\rangle_d \quad (B.1)$$

To exhibit the soft meson cloud, we do a canonical transformation

$$|\vec{p}\rangle_d = U |\vec{p}\rangle \quad (B.2)$$

If U is chosen to be (4.5), then the vector $|\vec{p}\rangle$ will satisfy a new equation

$$(H_0 + H_d + V) |\vec{p}\rangle = \left(W + \frac{p_L^2 + m^2}{2W} \right) |\vec{p}\rangle \quad (B.3)$$

But V acting on $|\vec{p}\rangle$ is zero since, if $\vec{p}$ is right moving, it will have no overlap with a left moving current and thus V annihilates $|\vec{p}\rangle$.

Therefore we have $(H_0 + H_d) |\vec{p}\rangle = \left(W + \frac{p_L^2 + m^2}{2W} \right) |\vec{p}\rangle$

which shows that the structure in $|\vec{p}\rangle$ comes entirely from the virtual dissociation process, and all the soft meson structure has been obtained by the U transformation.

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ACKNOWLEDGMENT

One of us (N.P.C.) wishes to acknowledge the warm hospitality of Prof. E.C.G. Sudarshan at the Center for Particle Theory, University of Texas, where this work was started.