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ESTIMATING PIPE RELIABILITY BY THE DISTRIBUTION OF TIME-TO-DAMAGE METHOD

S. A. WILSON

U. S. ATOMIC ENERGY COMMISSION
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**ESTIMATING PIPE RELIABILITY
BY THE
DISTRIBUTION OF TIME-TO-DAMAGE METHOD**

S. A. Wilson

Approved:

S. R. Vandenberg
S. R. Vandenberg
Project Engineer

Approved:

D. H. Imhoff
D. H. Imhoff, Manager
Development Engineering

Prepared for the
U. S. Atomic Energy Commission
Contract AT(04-3)-189
Project Agreement 37

ATOMIC POWER EQUIPMENT DEPARTMENT • GENERAL ELECTRIC COMPANY
SAN JOSE, CALIFORNIA 95114

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ABSTRACT

The "Distribution of Time-to-Damage Method" for reliability estimation is described. The method relies on knowledge of a damage prediction equation, and the uncertainties in the variables appearing in the equation. The uncertainties may be subjective judgments as well as proven or known distributions. For illustration purposes, the method has been used to predict the probability of growing a crack through the wall of a cylindrical pipe in a chosen time period, due to low-cycle fatigue. For this mechanism, the variables of materials, applied stress, and initial crack condition have been considered.

1. SCOPE, OBJECTIVE, AND CONCLUSIONS

1.1 SCOPE

This report covers a portion of the work done under Task A—Probability Study, of the Reactor Primary Coolant System Rupture Study (the "Pipe Rupture Study"), a reactor safety development program conducted for the Atomic Energy Commission by the Atomic Power Equipment Department, General Electric Company, San Jose, California, under Contract AT(04-3)-189, Project Agreement 37. The material has appeared in quarterly progress reports, and is presented here in unified form.

1.2 OBJECTIVES

The objective of the probability study task is to develop methods for:

- a. Estimation of the reliability of a reactor primary piping system in terms of leak or severance in a 40-year reactor service period.
- b. Identification of the principal sources of real variability or uncertainty in piping service life.

1.3 CONCLUSIONS

Development of the "distribution of time-to-damage method" has resulted in methods to do the following:

1. Systematize the general problem of estimating the reliability of a structure subject to damage from continuous damage mechanisms, by using uncertainties in the variables of the equation(s) governing the mechanism.
2. By use of uncertainty distributions, quantify individual judgment about differences between design and realized values of physical variables.
3. Use the low-cycle fatigue cycles to leak damage prediction equation in a probabilistic manner.
4. Estimate from test data the mean values and variances of the parameters in the low-cycle fatigue cycles to leak equation.
5. Identify what variables are most responsible for the probability in one tail of the distribution of a function.
6. Choose satisfactory weighting functions for use in Monte Carlo with importance sampling.
7. Estimate the probability of leakage at a piping location by low-cycle fatigue, using the foregoing methods.

2. INTRODUCTION

2.1 RELATED REPORTS

Details on estimating the parameters in the low cycle fatigue cycles to leak equation may be found in the following references:

Subsections 3.6 and 3.7, Reference 2-1

Subsection 3.4, Reference 2-2

Subsection 3.5 and Appendix A, Reference 2-3

Details on estimating pipe reliability considering both leakage and severance may be found in Section 3 of Reference 2-4.

All Pipe Rupture Study tasks and reports are listed in Appendix A. Further information on tasks referred to in this report may be found by consulting this listing.

2.2 ARRANGEMENT OF THIS REPORT

The first two sections of this report give summary information, references to related material, and a general discussion. Section 3 gives a brief description of the distribution of time-to-damage method. This section was originally written as a technical conference paper; it covers the essentials of the method and the low-cycle fatigue application, and serves as a guide to the following sections. Sections 4 to 9 provide the detail useful to the interpretation, extension, or application of the method to another damage mechanism. These sections are arranged in the logical sequence of the method. The steps to be followed were laid out very early in the program; it remained to carry them out analytically. Some specialized material is incorporated by reference.

2.3 ACKNOWLEDGMENTS

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J. D. Gilman
Dr. R. I. Post
R. Stijovic

S. W. Tagart
G. T. Vernon

The subsections for which each was author or collaborator are identified in the text.

2.4 GENERAL DISCUSSION

This subsection is intended to give perspective to the work described herein.

There are a number of approaches to reliability estimation, defining reliability as the probability that the life of the product will exceed its intended service period. One method is to run life tests in the laboratory on many copies of the product, inferring therefrom a time to failure distribution and associated probabilities. This method is widely used for small products. Another method is to document field experience, often used on operating equipment such as jet engines. Let us consider how useful these methods are to estimating the reliability of a piping system.

Piping failure may be by leakage or severance, the latter being considered as a sudden axial or circumferential opening releasing full flow. Such events are rare, piping systems are expensive, and any damage which does occur may be due to field conditions which would not have been duplicated in a laboratory, anyway. Thus, except for properties testing, laboratory life testing is infeasible. Field experience, however, systematically accumulated, would be extremely useful; but the available information, mainly from surveys, depends on recall or voluntary document search, and omits the vital ingredient of total life accumulated by the population. Moreover, some insight into the responsible practices, variables, etc., was desired. Thus, some other reliability estimating method was needed.

The method adopted, given the name "distribution of time-to-damage method" because it uses the distribution of time to a certain degree of damage such as leakage or severance, requires the ability to analytically predict time to damage by identifiable damage mechanisms, and requires—for all locations in the piping system—not only the design value of each participating variable, but also the distribution of the extent to which the actual, as-installed value of each variable might differ from the design value at that location. Such distributions are herein called uncertainty distributions. From this input the distribution of time to damage may be formed, or at least the fraction of the distribution lying less than the service period may be estimated. Broadly, this fraction will reflect the coincidence of rare, unfavorable values of the variables.

Because of its importance, and because a damage prediction equation could be proposed for it, the low-cycle fatigue damage mechanism was chosen for development of the method. The equation showed that number of cycles required to grow a crack from some initial depth, through the wall, depends on having an expression for stress intensity factor, ΔK , for the geometry and types of load encountered; on initial crack depth, wall thickness, system thermal expansion stress as influenced by modeling simplifications for stress analysis, unanticipated installation practices and defective geometries; on pressure stress and thus on pressure, diameter and wall thickness; and on two parameters which are believed to take on nominal values for a material, temperature and loading pattern (possibly also depending on cyclic frequency and environment).

A unique, one-dimensional equation for stress intensity factor range, ΔK , was developed by stress analysts; its features, along with discussion on stresses, appears in a later paragraph. Design values for the variables were available, and mean values for the parameters could be estimated from experimental data. It remained to arrive at the uncertainty distributions. For the two parameters, the variability of each among test specimens was estimated and could be reasonably represented by a normal distribution. Slope variability was as found; intercept variability was as found after fitting the data for all specimens to a common slope, the usual practice when applicable, as in this case. But uncertainty distributions for the physical variables had to be arrived at by engineering judgment. Factors considered were standard fabrication tolerances, nondestructive testing results, historical failure analyses, a study of "paths to failure," i.e., the kinds, probability, and possible magnitude of errors in design, fabrication, and operation; and other factors in the experience of two individuals. The uncertainty distributions were approached from the conditional probability viewpoint; that is, an error condition was identified which might affect that variable, the probability was judged with which it might occur, being specific to any particular categories if necessary (such as piping material or component type, for examples), and finally the distribution of values, relative to the design value, which might actually occur given that the error occurred, was judged. In order to standardize the distributions for use throughout the piping system, the scale used was actual value/design value; thus, scaling is normalized with respect to the design value, and the distributions are of coefficients on the design value, having values in the vicinity of 1.0. Underlying this approach is the assumption that individuals can make more correct judgments about the presence of the primary causes of damage (errors, stress levels, etc.), than about their combined effect on, say, cycles to leak.

Naturally, the question arises of the validity of the probability of damage estimate when some of the input is judgmental. Indeed, if the input distributions were known exactly, the probability of damage would be known exactly, except for the Monte Carlo portion to be described, and assuming that any imperfections in the damage prediction equation are reflected in the distributions of the parameters. Thus, from the viewpoint of method development, judgmental distributions are entirely satisfactory. But to arrive at the most valid results, use of the judgments of a number of persons is recommended. These should be comparably structured, and could be selected from at random—or in a weighted manner—in the Monte Carlo portion of the method. But quantified judgment appears to be the only source for the required broadly-scoped information. Even so, because a large number of variables enter the solution of the cycles to leak equation, only a substantial change in location or width of any one of them will bring about a marked change in the probability estimated. Finally, it is likely that the relative contribution by each variable to probability of damage is better estimated than is the magnitude of the probability estimate.

Special consideration was given stresses. The stress intensity factor equation accounted for four categories of stress: primary membrane, secondary membrane, primary bending, and secondary bending. It was judged that for the piping location studied, system thermal expansion stress would be oriented axially, tending to grow a circumferential crack. This stress, together with half the hoop stress, was treated as a primary membrane stress, in that the ΔK equation provided for no reduction in stress in this category as the crack grows through the wall. This behavior was confirmed in

prototype pipe testing under the Pipe Rupture Study (see Ref. 2-5) wherein load was found to not decrease until a crack had grown through the wall and part way around the circumference. Thus, while system thermal expansion stresses are regarded in paragraph NB-3213.9 Reference 2-6 as secondary, they do not decrease during the through-wall growth of a circumferential crack. Moreover, for one stress value, the primary membrane category led to fewest cycles to leak, and design stress values for the other categories could not be arrived at readily. Thus, only the primary membrane category was used for the piping location studied. It was further necessary to make an assumption about mean stress level, relative to stress range which appears in the ΔK equation, in order to select the value of intercept value $\log A$ correctly. It was assumed that mean stress is one-half of stress range; i.e., that piping cycles zero to tension due to thermal expansion and pressure stresses. Slightly lower values of $\log A$, and thus fewer cycles to leak, apply when mean stress is lower, as might result from shakedown, but no benefit was taken for this possibility.

The probability estimate has been made for that location found to have highest design stress in a reference piping system. The location is called a piping study section (a portion of a component) having a study surface of a determinable area. The area is required because the probability estimate involves the coincidence of a high stress and a crack depth. The probability distribution of the latter depends on a mean crack occurrence rate, the assumption that cracks have some distribution over the piping surface (Poisson was used), and a depth distribution, to arrive at the distribution of the deepest initial crack in an area of a stated size. It is assumed that it is the deepest crack in a surface of otherwise identical values of the variables which will grow through to leakage first. The area used is 1 inch along the pipe by 1/4 circumference, the former chosen arbitrarily, the latter to represent the portion of circumference in tension due to system thermal expansion.

One concept introduced which had not been encountered before was that of crack-specific mean $\log \Delta K$, used to achieve independence of the parameters in the cycles to leak equation. How this concept arises may be illustrated as follows. The crack growth rate equation is $da/dn = A \Delta K^B$, which linearizes as $\log da/dn = \log A + B(\log \Delta K)$, having the form $y = a + bx$. In the Monte Carlo portion the sampling procedure for a and b must be known; this is most straightforward when a and b are independent, which they are not in the form shown. But they may be made independent by writing the mean-adjusted form $y = c + b(x - \bar{x})$. What should be used for $\bar{x}$ (x -bar), the mean of $\log \Delta K$ in the crack growth equation? Ordinarily, in statistical regression analysis, this mean is that resulting from the experiment in which the parameters are estimated; but in this equation, because da appears on the left and a on the right (in $\log \Delta K$), any mean $\log \Delta K$ other than the one specific to a given crack has the effect of artificially inflating the variability of cycles to leak, and thus the probability of leakage. Mean $\log \Delta K$ was chosen to be the mean of the $\log \Delta K$'s of all the cycles required to grow the crack to leakage, and is thus called the crack-specific mean $\log \Delta K$. Using this concept not only permitted independent sampling from the distributions of the two parameters, but also resulted in permitting an equation for cycles to leak to be written which is linear in coefficients of functions of the major variables, replacing an integral equation. That equation had not theretofore lent itself to empirical approximation and was destined to be solved by five-point Gauss numerical integration in the Monte Carlo portion, a time-consuming procedure even though the best available.

The propagation of errors (variance) work on the integral equation for cycles to leak clearly revealed the negligible contributions of pressure, diameter, and wall thickness variations to the variance of the equation. Comparing the variance of the linear cycles-to-leak equation revealed the negligible contribution of the slope parameter when using crack-specific mean $\log \Delta K$, leaving uncertainties in initial crack depth, stress, and the intercept parameter as the principal contributors to probability of leak.

The method chosen to estimate the probability in the small lower tail of the distribution of the cycles-to-leak function was by Monte Carlo with importance sampling. This method requires no assumptions on the distribution of the function, and thus none on the form of the function nor on the distributions of the variables. It is sometimes proposed that the distribution of any function be assumed to be normal, a distribution which is fully characterized simply by the mean and variance of the function. Thus, when the mean and variance are found, respectively by solution using the means and by propagation of errors, probabilities in the tail below any ordinate may be found directly by reference to normal distribution tables. This method is completely correct when the function is the sum of random variables, with or without coefficients, each of which is normally distributed. It is approximately correct for some other simple functions, and at least symmetrical distributions for the variables, being less correct in the tails than in the body of the distribution. But for an involved function having no restrictions on the distribution of the variables and where a

small tail probability is to be estimated, it could not be expected to accurately account for the participating distributions. It became possible to systematically select weighting functions, and apply the Monte Carlo method with importance sampling, resulting in an 82% reduction in variance of estimate compared to ordinary Monte Carlo, equivalent to an 82% reduction in trials. Thus, it was possible to make a small probability estimate efficiently.

The estimate for the probability of leakage in a 40-year plant life due to low-cycle fatigue in the chosen small study surface and under a conservative assumption of "no checking or inspection" during design, fabrication, or operation is 0.0077. The value for an entire piping system is higher, and governed by the amount of pipe subject to high design stresses. A method for extending the estimate to the system is described.

2.5 REFERENCES

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3. BRIEF DESCRIPTION OF THE DISTRIBUTION OF TIME-TO-DAMAGE METHOD

3.1 INTRODUCTION

This section gives a brief description of the distribution of time-to-damage method, and its application to low-cycle fatigue crack growth. The section was originally prepared as a paper for the ASME First National Congress on Pressure Vessels and Piping, San Francisco, May 10-12, 1971.

3.2 OUTLINE OF THE METHOD

The subject method was devised because of lack of adequate historical failure rate information. It relies on design uncertainties. It is outlined on Figure 3-1 and is described as follows:

- Draw a probability diagram. One for piping is illustrated in Figure 3-2. The diagram shows a block for each stage of damage, such as crack initiation, leakage, and severance. Lines connect the blocks to show the logic of how one stage of damage can lead to another. Three ideas relate to each line: (1) a damage mechanism, such as low-cycle fatigue, (2) a time-to-damage prediction equation, such as for the number of cycles to grow a crack from initial depth through to leakage, and (3) the probability of this event in a chosen time such as 40 years. Thus many lines connect each block, one for each damage mechanism. See Section 4.
- Devise a damage prediction equation for each line, i.e., for each mechanism for each stage of damage. Figure 3-3 shows the basic damage prediction equation used for this illustration, for cycles to leak, n_t . See Section 5.
- Choose a piping study section, and assign a nominal value and an uncertainty distribution to each variable in the equation. Nominal values used, for this illustration, along with sources of nominal values and distributions, are shown in Table 3-1. Note that design values, results of experiments, and engineering judgment and experience were all drawn upon. Thus this method uses the best available information about how well the actual value of a variable is believed to agree with its nominal value. See Sections 6 and 7.
- Select only those variables making the major contributions to the variability of the equation, e.g., of n_t . This is done by a propagation of errors estimate of the variance of the equation shown in Subsection 3.4 for the case of independent variables. See Section 8.
- Form the distribution of the equation from the nominal values and distributions of its variables. Or only estimate the fraction of the equation distribution which lies less than 40 years, as illustrated in Figure 3.4. This fraction is the desired probability and will be estimated by Monte Carlo using importance sampling. See Section 9.

Table 3-1
NOMINAL VALUES USED, AND SOURCES OF NOMINAL VALUES AND DISTRIBUTIONS

Variable		Nominal Value Used	Source		
			Design Value	From Test Data	From assigned Uncertainty Distribution
Slope parameter, B	For SS 304 at R.T.	4.45		Nom. Distr.	
Parameter, A (Intercept is log A)		5.5×10^{-26}		Nom. Distr.	
Expansion stress, S_e , psi		27450	Nom.		Distr.
Wall Thickness, t , in.		0.432	Nom.		Distr.
Diameter, d , in.		6.624	Nom.		Distr.
Pressure, P , psig		1000	Nom.		Distr.
Initial crack depth ratio, a_0/t		*			Nom. and Distr.
Mean crack occurrence rate, λ , cracks/in ²		0.1			Value assg.

*0.036 without, 0.0146 with ultrasonic testing, arising from the distributions shown in Figures 3-8 and 3-9

Nom. = Shows source of nominal value used

Distr. = Shows source of uncertainty distribution used.

Figure 3-1. Distribution of Time-To-Damage Method

Figure 3-2. Piping Probability Diagram for One Piping Study Section and Damage Mechanism

Basic crack growth rate equation:

$$da/dn = A (\Delta K)^B \text{ inches/cycle}$$

Linearize for parameter estimation:

$$\log da/dn = \log A + B \log \Delta K$$

where da/dn = crack growth rate

ΔK = stress intensity factor range, a function of $\Delta\sigma_{PM}\sqrt{a}$ where $\Delta\sigma_{PM}$ is stress and a is crack depth

$\log A$ and B = intercept and slope parameters

Mean-adjusted form to make A and B independent:

$$\log da/dn = \log C + B [\log \Delta K - (\text{mean } \log \Delta K)] \text{ where } \log C \text{ is the intercept at mean } \log \Delta K. \text{ For simplicity, only } \log A \text{ is referred to in the text as the intercept.}$$

In original formulation:

$$da/dn = C (\Delta K / \bar{\Delta K})^B$$

where $\bar{\Delta K}$ is antilog mean $\log \Delta K$.

Invert and integrate for cycles to grow from

initial crack depth, a_0 , through wall

thickness, t , to leakage:

$$n_t = \frac{(\bar{\Delta K})^B}{C} \int_{a_0}^t \left(\frac{1}{\Delta K} \right)^B da, \text{ cycles to leak}$$

n_t = cycles to leak

a_0 = initial crack depth

t = wall thickness

Linear expression found to be nearly exact:

$$\begin{aligned} \log n_t = & 27.4687 - 1.0084 (\Delta \log C) \\ & - 4.5069 (\log \Delta\sigma_{PM}) - 6.2615 \sqrt{a_0/t} \\ & - 2.47 e^{-B} - 0.03 B, \end{aligned}$$

where $\Delta \log C$ is the random change in intercept from the nominal intercept for the material and temperature shown in Table 3-1 and $\Delta\sigma_{PM}$ is

primary membrane stress range appearing in ΔK ,

$$\Delta\sigma_{PM} = S_E \times S_E^* + \text{hoop stress}/2.$$

$\log$ is to base 10.

Figure 3-3. Low-Cycle Fatigue Crack Growth Rate Equation

Figure 3-4. Probability of Leakage in 360-Cycle Service Lifetime as the Tail of a Distribution of Cycles to Leak

Thus, in a systematic way, the "Distribution of Time-to-Damage" method calls for interrelations among stages of damage to be diagrammed, damage mechanisms to be identified, and design uncertainties to be carried ahead by a damage prediction equation to estimate the probability of moving from one stage of damage to the next.

There are additional benefits of this reliability estimation method. The time-to-damage equations form the basis for rational design equations. And the greatest contributors to lack of reliability, both in terms of damage mechanisms and variables within equations, can be identified for more exact definition.

The distribution of time-to-damage method is similar to methods known as "stress vs. strength" (see Reference 3-1) and "requirement vs. capability" (see Reference 3-2), in that areas under continuous probability distributions are used to estimate reliabilities.

The crack growth stage of low-cycle fatigue was chosen for first application of this method because of its importance, and the technical feasibility of proposing a cycles-to-leak equation.

The following subsections describe in more detail steps c, d, and e above on assigning uncertainty distributions, finding the equation variance, and making the Monte Carlo estimate.

3.3 NOMINAL VALUES AND DISTRIBUTIONS OF THE VARIABLES

The piping study section chosen was the one subject to highest design stress in a reference piping system. The surface area of the study section was 1 inch in length by 1/4 circumference, the latter chosen as the portion in tension from the expansion stress.

Table 3-1 shows notation and the source of each design value and distribution. Figures 3-5 to 3-9 show the distributions of those variables found by the variance solution—described in the next subsection—to make the substantial contributions to the variance of cycles to leak, and thus to the probability of leakage. The following paragraphs describe each variable.

Parameters A and B: The low-cycle fatigue testing from which the mean and variance of parameters A and B were estimated was conducted under another part of the program. The values estimated are for a particular material, temperature level, and loading pattern. The zero-to-tension loading pattern was chosen as most characteristic of piping loading between nonoperating and operating conditions. The distribution used for parameter A is shown in Figure 3-5.

S_E , t , d , and P : The assigned uncertainty distributions for S_E , t , d , and P were for the ratio: actual value/design value. Thus they are coefficients on the design values, and have values in the vicinity of 1. A distribution was sketched for each cause of uncertainty which was believed important, such as for expansion stress: stress analysis model error, computation error, installation arrangement error, restricted flexibility error, etc. Also, for each cause, a conditioning probability was assigned to reflect the probability with which each cause was judged to occur in a piping loop, joint, etc. Conditioning probabilities were assigned for two cases: with and without checking and inspection throughout the design and fabrication cycle. The distribution used for S_E is shown in Figure 3-6.

a_0/t : The assigned uncertainty distributions for initial crack depth ratio, a_0/t , are on a scale of that ratio, as shown in Figure 3-7. Two depth ratio distributions were considered: with and without ultrasonic testing. In addition, crack frequency was assumed to be Poisson distributed, with an assigned mean crack occurrence rate of λ per square inch. Since the deepest crack in a region of otherwise uniform properties will be the first to leak, we were interested in the distribution of maximum a_0/t in the chosen study surface, which can be derived from the Poisson and a_0/t distributions. The resulting distributions for maximum a_0/t are shown in Figures 3-8 and 3-9.

For both the variance solution and the Monte Carlo work it is desirable to write the n_t expression so that parameters A and B are independent. This was accomplished by the mean-adjusted form shown in Figure 3-3 which requires having the mean of the $\log \Delta K$'s for all the cycles to grow the crack from a_0 to leakage at t . This mean depends on slope B, and on initial and final $\log \Delta K$ values (which depend on a_0 , t , stress and the ΔK formulation); it does not depend on intercept $\log A$. Attention to the lack of independence between A and B in the standard formulation for n_t is important when studying the sensitivity of this function to these variables.

Figure 3-5. PDF and CDF of Intercept, $\text{Log } C (= \text{Log } A + 21.30)$

Figure 3-6. CDF and PDF of S_E^*

Figure 3-7. Judgmental Uncertainty Distributions for a_0/t , Initial Crack Depth Ratio

Figure 3-8. PDF and CDF of a_0/t Max Without U/S Test

Figure 3-9. PDF and CDF of a_o/t Max With U/S Test

It was found that a linear expression could be used for n_t shown in Figure 3-3, which simplified the Monte Carlo work compared to using numerical integration. The formulation for ΔK appears in Reference 3-3; it is a function of $\sigma\sqrt{a}$.

3.4 THE VARIANCE SOLUTION

In order to minimize computation time in the Monte Carlo work, it was desirable to retain only those variables whose distributions made large contributions to the variability of the equation. An approximate solution for the variance of the equation, often called the propagation of errors method, (see Reference 3-4), was appropriate. The general formulation for the variance of an equation is as follows:

$$\sigma_{n_t}^2 = \left(\frac{\partial n_t}{\partial A} \right)^2 \sigma_A^2 + \left(\frac{\partial n_t}{\partial B} \right)^2 \sigma_B^2 + \text{term for each variable}$$

It can be seen that a separate term appears for each variable. The formulation shown is for the case when all variables are independent, as applies here for the one study section. The percentage contribution found for each variable for both a_o/t distribution cases is tabulated in Table 3-2.

It can be seen that for the distributions assigned, the major contributor for the case of no ultrasonic testing is initial crack depth ratio, a_o/t ; lesser contributors are stress and intercept parameter $\log A$; and contributions by uncertainties in slope parameter B , pipe diameter, wall thickness, and operating pressure are negligible. Accordingly, the Monte Carlo work used only the distributions of A , stress, and a_o/t , with a_o/t identified for better definition. For the case of ultrasonic testing, the variance of $\log n_t$ is reduced to 31% in magnitude, and the contribution of the variance of a_o/t is reduced, leaving that of stress as the major contributor.

Table 3-2
CONTRIBUTIONS TO VARIANCE OF LOG n_t

	%	
	Without Ultrasonic Test	With Ultrasonic Test
a_o/t , Initial Crack Depth	78	42
Log A, Intercept	6	16
S_E , Expansion Stress	15	41
	99	99
B, t, d, P	1	1
	100	100
Magnitudes, Variance Log n_t	1	0.31

3.5 THE PROBABILITY ESTIMATE

If we could assume that the distribution of n_t , cycles to leak, was clearly related to the normal distribution, we could use the mean and variance values found in the preceding section to find the probability that n_t lies less than 360, the operating cycles in 40 years. But because the distribution of n_t is analytically inaccessible, the distributions of the variables are not standard, and we must estimate small probabilities, less than 10^{-2} , assuming a distribution for n_t appeared unwise. It was decided to estimate the required probability by Monte Carlo with importance sampling, a variance-reduction method (see References 3-5 and 3-6).

In ordinary Monte Carlo for estimating a tail probability of a function, a trial consists of selecting a value at random from the distribution of each independent variable, solving the function using those values, assigning 1 to the outcome if it lies in the tail region, 0 if it does not, and finally summing and dividing by number of trials to obtain the fraction of trials in the tail, which is an estimate of the tail probability. Clearly, if this true probability is small, a large number of trials is required for a precise estimate. Importance sampling overcomes this problem by directing the random choices to the region of each variable which leads more often to function solutions lying in the tail of interest. Then, rather than 1 being assigned to each such outcome, a weight is assigned which is the reciprocal of the density of the weighting function used to focus the random choices. This process is illustrated in Figure 3-10. Fortunately, n_t is monotonic in A, stress and a_o/t , so the region to sample from for each of these variables to give low n_t values is known.

The difficulty in using importance sampling lies in choosing appropriate weighting functions. In general, a weighting function should be uniformly distributed over that region of each variable which leads to function values in the tail of interest, and distributed in a manner falling away at first steeply and then more gradually over variable values leading to function values increasingly distant from the tail of interest. For this problem an ad hoc procedure was adopted whereby beta distributions having one parameter equal to 1 were systematically modified so as to be uniformly distributed over values in the equal-probability tail of each variable which, when chosen together, would always give n_t less than 360; and distributed in a decreasing exponential-appearing manner over the balance of the distribution, this portion being steeper (and so reflecting the other beta parameter value) if, for a variable, choosing values in it increasingly distant from the uniform portion would lead to n_t values rising more rapidly past 360.

A 3000-trial estimate of the probability that n_t is less than 360 cycles was made for the reference input conditions. The result, for the probability of leakage at the chosen 1 inch wide x 1/4 circumference study surface, in 40 years due to low cycle fatigue, was 0.0077, with 95% limits on the Monte Carlo portion alone at 0.0084 and 0.0112 assuming normality of estimate. Variance of estimate was reduced 82% by using importance sampling.

1. Choose 0-1 uniform random number, U^* .
2. Using weighting function having density f_2 and cumulative F_2 , transform U^* to F_1^* .
3. Using the distribution of X , having density f_1 and cumulative F_1 , transform F_1^* to x^* .
4. If $x^* \leq x_0$ assign weight $W = 1/f_2$; if $x^* > x_0$ assign $W = 0$.
5. Repeat for n trials.
6. Estimate $\Pr X \leq x_0 = (\sum W)/n$.
7. f_2 was chosen so $x^* \leq x_0$ usually results, and for other properties described in section 5.

Figure 3-10. Monte Carlo Using Importance Sampling to Estimate $\Pr [X \leq x_0]$

3.6 EXTENSIONS FOR SYSTEM RELIABILITY ESTIMATE

In principle, similar estimates are required at all other study sections of the piping system. This is manifestly infeasible, but it is possible to restrict attention to those study sections having sufficiently high design stress to govern the probability of leakage anywhere in the system. A simple empirical relationship between probability of leakage, nominal stress, and wall thickness is anticipated, permitting probability values to be obtained readily for the design values of any pipe study sections.

The probability of leakage anywhere in the system, in 40 years, is found by appropriately summing over the piping system the probabilities at each of the study sections; these probabilities are given by the structure of conditional probabilities dictated by the piping probability diagram. The summation problem must recognize lack of independence among actual stress levels at successive study sections; but all variables at one study section, and the other variables at successive study sections, appear to be independent.

Extensions are also required to both initiation and growth for other mechanisms, such as stress corrosion and fatigue-assisted stress corrosion; to more extensive critical size definitions for severance, and to knowledge of conditions favoring reaching critical size for severance without prior detectable leakage.

3.7 REFERENCES

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4. THE PIPING PROBABILITY DIAGRAM

4.1 INTRODUCTION

The piping probability diagram, introduced in this section, serves several purposes at once. It permits stages of damage to be identified, so that one may choose specifically the degree of damage for which the reliability is to be calculated. It requires that all important paths of proceeding from one stage of damage to the next be identified, including ways of interrupting those paths, such as by damage detection and repair. These paths may involve the operation of a continuous damage mechanism, such as low cycle fatigue, or the occurrence of a single damage event such as an earthquake. The probability, at any one piping study section, of proceeding from one stage of damage to the next by each mechanism is related to each line. The probability of reaching a certain stage of damage due to all mechanisms can be calculated.

The piping probability diagrams are presented in this section, together with several candidate damage prediction relationships which lend themselves to probability estimating by the distribution of time-to-damage method.

4.2 PIPING PROBABILITY DIAGRAMS FOR CONTINUOUS DAMAGE MECHANISMS AND SINGLE DAMAGE EVENTS

Leakage or severance in a particular piping study section (location) can occur from any one of several initial pipe conditions, and by several mechanisms; and severance can occur with or without prior leakage. To account for these diverse paths two "Piping Probability Diagrams" have been prepared, and are shown in Figures 4-1 and 4-2. Figure 4-1 accounts for leakage and severance by continuous or time-consuming damage mechanisms, such as low-cycle fatigue and stress corrosion; Figure 4-2 accounts for single event damage mechanisms, such as collision and earthquake. In Figure 4-1 a "Long Groove" or "Long Crack" is one of such a configuration to grow around to critical size for severance before leakage. A "Groove" is any notch, void, or inclusion acting as a stress concentrator.

Figures 4-1 and 4-2 show by lines possible paths from one condition to the next. For any one study section there is a probability associated with each line. These probabilities are of two types: (1) those designated by a letter are those associated with the size of the tail of the distribution curve of some time-consuming mechanism, such as cycles to initiate a crack, which lies below the number of operating cycles, and (2) those designated by a number are those not associated with a distribution, such as the probability that a leak will be detected. The sum of probabilities of all lines leaving a block must be 1.0. From the pattern of the lines, equations can be written for the probability of leakage, and of severance, at a study section. For Figures 4-1 and 4-2 these are Equations (4-1) through (4-4). Note the special meaning of the underscored letter terms and sums. They represent the probability that the actual operating time (or cycles) exceeds the total required operating time (or cycles) for leakage (or severance). See Equation (4-5).

4.3 PRIORITIES FOR DEVELOPMENT OF DAMAGE PREDICTION EQUATIONS

This subsection identifies those damage prediction equations appearing on Figure 4-1 which should be developed on a priority basis, reflecting their role in accounting for most of the probability of damage. Refer to subsection 3.3 of Reference 4-1.

4.4 CANDIDATE DAMAGE PREDICTION RELATIONSHIPS FOR INITIATION AND GROWTH THROUGH TO LEAKAGE

This subsection proposes candidate damage prediction relationships for all stages of damage and damage mechanisms, by way of suggesting the format in which such relationships might be developed so as to lend themselves to probability estimation by the distribution of time-to-damage method. Refer to subsection 3.3 of Reference 4-2.

4.5 CANDIDATE DAMAGE PREDICTION RELATIONSHIP FOR SEVERANCE

The material in this subsection, the application of the distribution of time to damage method to the severance problem, appearing in subsection 3.4 Reference 4-2, was implemented and is fully described in Section 3 of Reference 4-3 and the report identified in subsection 2.1.

LETTERS ARE PROBABILITIES OF EVENTS CAUSED BY TIME-CONSUMING (OR CONTINUOUS) DAMAGE MECHANISMS SUCH AS LOW-CYCLE FATIGUE.

NUMBERS ≤ 70 ARE PROBABILITIES OF EVENTS NOT ARISING FROM THE DISTRIBUTION OF A TIME-CONSUMING DAMAGE MECHANISM, SUCH AS THE PROBABILITY THAT A LEAK WILL BE DETECTED.

NUMBERS OVER 70 ARE PROBABILITIES OF ACHIEVING THE CONDITION SHOWN; "80" IS THE PROBABILITY THAT A SHORT CRACK IS INITIATED, BY ALL POSSIBLE PATHS.

67-01593

Figure 4-1. Piping Probability Diagram for Continuous Damage Mechanisms, at One Piping Study Section

67-01594

Figure 4-2. Probability Diagram for Single Damage Events for One Piping Study Section

PROBABILITY OF LEAK BY CONTINUOUS DAMAGE MECHANISMS

$$\begin{aligned}
100 = & 30 \left[\underline{(E+e')} + \underline{(I+i')} + \underline{(J+j')} + \underline{(K+k')} \right] \\
& + 40 \left[\underline{(D+d)} + \underline{(E+e)} + \underline{(F+f)} + \underline{(G+g)} + \underline{(H+h)} + \underline{(I+i)} + \underline{(J+j)} + \underline{(K+k)} \right] \\
& + 60 \left[\underline{D} + \underline{E} + \underline{F} + \underline{G} + \underline{H} + \underline{I} + \underline{J} + \underline{K} \right].
\end{aligned} \tag{4-1}$$

PROBABILITY OF SEVERANCE BY CONTINUOUS DAMAGE MECHANISMS

$$\begin{aligned}
200 = & (11 \times 12 + 14 \times 15 \times 16) \left\{ 30 \left[\underline{(e'+E+B)} + \underline{(e'+E+C)} + \underline{(i'+I+C)} + \underline{(j'+J+C)} + \underline{(k'+K+C)} \right] \right. \\
& + 40 \left[\underline{(d+D+A)} + \underline{(f+F+A)} + \underline{(e+E+B)} + \underline{(g+G+B)} + \underline{(d+D+C)} + \underline{(e+E+C)} + \underline{(f+F+C)} \right. \\
& \quad \left. + \underline{(g+G+C)} + \underline{(h+H+C)} + \underline{(i+I+C)} + \underline{(j+J+C)} + \underline{(k+K+C)} \right] \\
& + 60 \left[\underline{(D+A)} + \underline{(F+A)} + \underline{(E+B)} + \underline{(G+B)} + \underline{(D+C)} + \underline{(E+C)} + \underline{(F+C)} + \underline{(G+C)} + \underline{(H+C)} \right. \\
& \quad \left. + \underline{(I+C)} + \underline{(J+C)} + \underline{(K+C)} \right] \left\{ \right. \\
& + 17 \left\{ 50 \left[\underline{(l+L)} + \underline{(m+M)} + \underline{(n+N)} + \underline{(o+O)} + \underline{(p+P)} + \underline{(q+Q)} + \underline{(r+R)} + \underline{(s+S)} \right] \right. \\
& \quad \left. + 70 \left[\underline{L} + \underline{M} + \underline{N} + \underline{O} + \underline{P} + \underline{Q} + \underline{R} + \underline{S} \right] \right\}.
\end{aligned} \tag{4-2}$$

PROBABILITY OF LEAK BY SINGLE EVENT DAMAGE

$$\begin{aligned}
101 = & 1 \left[30 \times 31 + 40 \times 41 + (80+60)81 + 50 \times 51 + (90+70)91 \right] \\
& + 2 \left[30 \times 32 + 40 \times 42 + (80+60)82 + 50 \times 52 + (90+70)92 \right] \\
& + 3 \left[30 \times 33 + 40 \times 43 + (80+60)83 + 50 \times 53 + (90+70)93 \right] \\
& + 4 \left[30 \times 34 + 40 \times 44 + (80+60)84 + 50 \times 54 + (90+70)94 \right],
\end{aligned}$$

where

$$\begin{aligned}
80 = & 30(\underline{e'+i'+j'+k'}) + 40(\underline{d+e+f+g+h+i+j+k}) \\
90 = & 50(\underline{l+m+n+o+p+q+r+s}).
\end{aligned} \tag{4-3}$$

PROBABILITY OF SEVERANCE BY SINGLE EVENT DAMAGE

$$\begin{aligned}
201 = & 1 \left[30 \times 35 + 40 \times 45 + (80+60)85 + 100(12+16 \times 15)105 + 50 \times 55 + (90+70)95 \right] \\
& + 2 \left[30 \times 36 + 40 \times 46 + (80+60)86 + 100(12+16 \times 15)106 + 50 \times 56 + (90+70)96 \right] \\
& + 3 \left[30 \times 37 + 40 \times 47 + (80+60)87 + 100(12+16 \times 15)107 + 50 \times 57 + (90+70)97 \right] \\
& + 4 \left[30 \times 38 + 40 \times 48 + (80+60)88 + 100(12+16 \times 15)108 + 50 \times 58 + (90+70)98 \right], \quad (4-4)
\end{aligned}$$

where 80 and 90 are the same as for leakage, and 100 is the probability of a leak by continuous damage mechanisms.

NOTATION ON PROBABILITY EQUATIONS (4-1) THROUGH (4-4)

$100 = 100_i =$ Probability of leak by all continuous damage mechanisms, in 40 years, at ith cross section.

$40 = 40_i =$ Probability that ith cross section contains a short groove.

$$(\underline{E+e}) = (\underline{E+e})_i = \Pr \left[(E+e) - N \leq 0 \right] \quad (4-5)$$

where

$(E+e) =$ sum of cycles to initiate and to grow through a crack, by low-cycle fatigue, when initial groove is outside and circumferential, and

$N =$ operating cycles of ith section in 40 years.

4.6 REFERENCES

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5. DAMAGE PREDICTION EQUATION FOR GROWTH OF AN INITIATED CRACK BY LOW-CYCLE FATIGUE

5.1 INTRODUCTION

The fracture mechanics cycles-to-leak equation is presented. This damage prediction equation calculates the number of stress cycles required to grow a crack from some initial depth through a pipe wall by low-cycle fatigue at one piping study section. It is seen to require an equation for stress intensity factor (ΔK) for the geometry involved, and design or mean values and uncertainty (probability) distributions for expansion and pressure stresses, initial crack depth, wall thickness, and two crack growth parameters characteristic of a material, temperature, and mean stress. The concept of crack-specific mean $\log \Delta K$ is introduced for the analytical purpose of permitting independent sampling from the distributions of the two parameters, used in Section 9.

5.2 FRACTURE MECHANICS AND STRESS ANALYSIS BASES FOR CYCLES-TO-LEAK EQUATION

5.2.1 The Fracture Mechanics Cycles-to-Leak Equation

A crack growth rate equation arising from fracture mechanics concepts and now widely adopted, was put forward in 1962 in Reference 5-1. It is

$$da/dn = A(\Delta K)^B$$

crack growth rate is proportional to stress intensity factor range raised to an exponent B. It is hypothesized that parameters A and B are characteristic of a material at a temperature, and this was found to be so in testing under Tasks D-1 and D-2 as long as both parameters are considered random variables among specimens. To obtain number of cycles to leak, this equation is inverted and integrated, leading to

$$n_t = A^{-1} \int_{a_0}^t (\Delta K)^{-B} da$$

where a_0 and t are initial and final crack depths, respectively. t is piping wall thickness for a crack growing through the wall. An equation is required for stress intensity factor, ΔK , for the geometry to be considered. In general, ΔK varies as $\Delta \sigma \sqrt{a}$, stress range by square root of some linear crack dimension.

5.2.2 Tentative Equation for Stress Intensity Factor, ΔK , For Through-Wall Crack Growth (S. W. Tagart)

Surface cracks represent a "worst case" from the standpoint of fatigue design in pressure-containing members, in that bending-type stresses maximize at the wall surfaces, and a given depth crack at the surface has roughly 40% more stress intensity than one of equal depth at midthickness for the same nominal calculated stress. In addition, there is also a finite probability that surface cracks may be present in regions which already involve a surface stress concentration. In this case the crack growth rate will be magnified by the stress concentration until the crack grows out of the locally higher stress field. For these reasons it is desirable to develop an expression for the growth of surface cracks through the thickness of the member, which is more comprehensive than the one used in Appendix A of Reference 5-2. This expression can then be used in the failure path model to predict either the cycles required to grow through the piping wall from an original defect or from a crack which has been initiated because of low-cycle fatigue.

It is well known that biaxial cyclic strains are important to the mechanism of crack initiation. However, it is presently believed that only the stress components perpendicular to the crack face are of primary importance in the crack growth mechanism. For this reason only stress components normal to the plane of crack growth will be considered. It is desirable to deal with six idealized major stress categories: they are (1) primary bending, σ_{PB} (2) secondary bending, σ_{SB} (3) primary membrane, σ_{PM} (4) secondary membrane, σ_{SM} (5) bending and membrane stresses locally affected by a stress concentration factor, K_{σ} , and (6) localized self-equilibrating stresses, σ_L . These stresses are a convenient idealized set which are consistent with present design methods. It will be shown that by the use of several simplifications, the first five categories can be handled with an approximate crack growth equation.

The ideal primary-secondary stress categories are not easily defined except in the simple, more trivial cases. Therefore, some judgment must be exercised by the stress analyst in subdividing a given stress at a point into the stated ideal categories. However, a review of the bases of the proposed crack growth equation makes the subdivision objective. In addition, it may be possible to conservatively represent the sixth category as an equivalent secondary stress in the membrane or bending category even though it is not a shell bending or membrane stress. By conservative we mean here that such an assumption would appear to somewhat overemphasize the possible importance of the self-equilibrating type stresses and produce an analytic underestimation of true potential fatigue life of a pipe.

It will be assumed for this analysis that a slice out of the pipe wall (see Figure 5-1) perpendicular to the crack plane has applied to it a membrane force, T , per unit width and a bending force, M , per unit width.

Figure 5-1. Slice From Pipe Wall

The force resultants will be divided into two parts, a primary part which remains constant regardless of the depth of the crack and a secondary force which diminishes as the crack progresses through the thickness. As a first approximation it will be assumed that this set of secondary forces diminishes linearly with depth of crack as follows:

$$M = M_P + M_S \left(1 - \frac{a}{t}\right)$$

$$T = T_P + T_S \left(1 - \frac{a}{t}\right) \quad (5-1)$$

where M_P , M_S , T_P , T_S are the constant values of force when $a = 0$. The form of this force reduction can later be modified if necessary to make results agree acceptably with Task D-3 prototype experiments, reported in Reference 5-3. The corresponding nominal calculated stresses when $a = 0$ are

$$\begin{aligned}\sigma_{PB} &= \frac{6 M_P}{t^2} & \sigma_{SB} &= \frac{6 M_S}{t^2} \\ \sigma_{PM} &= \frac{T_P}{t} & \sigma_{SM} &= \frac{T_S}{t}\end{aligned}\quad (5-2)$$

Equations (5-1) and (5-2) relate forces transmitted through the crack region to nominally calculated design stresses which would exist if no crack were present. The total cyclic design stress, σ_T , perpendicular to the crack is the sum of its idealized parts

$$\sigma_T = K_\sigma (\sigma_{PB} + \sigma_{SB} + \sigma_{PM} + \sigma_{SM}) + \sigma_L, \quad (5-3)$$

where a single approximate concentration factor, K_σ , has been used for simplicity. It has already been shown in tests by Brothers that the linear elastic stress intensity factor, K , provides a quantitative method of correlating crack growth rate for a variety of geometric cracks through the simple equation

$$\frac{da}{dn} = A (\Delta K)^B, \quad (5-4)$$

where ΔK is the cyclic variation of stress intensity, n is cycles, and A and B are nearly constants for a material at a given temperature. The variation of A and B from true constants is covered by treating one or both as random variables in the reliability analysis. Although present materials tests have included no load reversals, such tests will be forthcoming so that quantitative effects of a steady mean stress intensity superimposed on the cyclic stress intensity will be available. For the present it appears appropriate to assume that the zero-to-tension crack growth tests can be used to correlate the crack growth rate by considering only the range and ignoring any effect of mean stress. Such an assumption appears to agree better with the available test data than the prediction by Krafft (pp. 6-4, 6-5, Reference 5-4). If the crack is known to close during part of the cycle because of mean stress it seems appropriate to simply relate only the observed range of K again, since that portion of the cycle where the crack is closed truly has a zero effect on cyclic stress intensity.

In the cases where both bending and membrane effects act simultaneously such as in Figure 5-1, we would expect that K factors can be superimposed. For reference the following cases for typical K factors are repeated in Figures 5-2 and 5-3. The equations representing the elastic calculated stresses for the penny-shaped crack of (b) in Figure 5-3 are listed to establish the definition of K used here.

$$\begin{aligned}\sigma_x &= \frac{K \cos \frac{\theta}{2}}{\sqrt{2 \pi r}} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2}\right) \\ \sigma_y &= \frac{K \cos \frac{\theta}{2}}{\sqrt{2 \pi r}} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2}\right) \\ \sigma_z &= \nu (\sigma_x + \sigma_y) \\ K &= 2\sigma_g \sqrt{\frac{a}{\pi}}\end{aligned}\quad (5-5)$$

(a) $K = 1.77 \sigma_g \sqrt{a}$

(b) $K = 0.722 \sigma \sqrt{a}$

(c) $K = 1.77 \sigma_N \sqrt{a}$

$$\sigma_N = \frac{6M}{(t-a)^2}$$

SHALLOW $a \ll t$

(d) $K = 0.722 \sigma_N \sqrt{t-a}$

$$\sigma_N = \frac{6M}{(t-a)^2}$$

DEEP $a \approx t$

Figure 5-2. Typical Stress Intensity Factors

Figure 5-3. Typical Stress Intensity Factors

The approximate shallow-deep K value in Figure 5-3 was obtained by fitting Bueckner's curve (Reference 5-5) with a simple hyperbolic function. See Figure 5-4. It is believed that the relation between G and K in the footnote of the reference is in error because K should not be a function of Poisson's ratio. Making this correction

$$K = \sigma_N \sqrt{(t - a) f\left(\frac{a}{t}\right)} \quad (5-6)$$

The function $f\left(\frac{a}{t}\right)$ can be approximated by

$$f\left(\frac{a}{t}\right) \approx 0.52 \tanh\left(5.5 \frac{a}{t}\right) \quad (\text{but see Subsection 5.3.4.2}). \quad (5-7)$$

A comparison of the actual $f\left(\frac{a}{t}\right)$ and the approximate expression is shown in Figure 5-4.

The limiting case comparisons when $a \ll t$ and $a \approx t$ from the approximate $f\left(\frac{a}{t}\right)$ are

$$a \ll t \quad K \text{ approximate} = 1.69 \sigma_N \sqrt{a},$$

which is 4% low in comparison with Figure 5-2(c), and

$$a \approx t \quad K \text{ approximate} = 0.722 \sigma_N \sqrt{t - a},$$

which is in exact agreement with (d) in Figure 5-2. From Figure 5-4 it can be seen that K will be at most about 5% high for any case between the limiting cases. The approximate K seems adequate to cover the cases of interest here. The cases (c) and (d) of Figure 5-3 can then be used to calculate the total cyclic ΔK for use in Equation (5-4).

Figure 5-4. Fit of Bueckner's Curve

$$\frac{da}{dn} = A \left\{ \alpha_N \sqrt{(t-a) 0.52 \tanh \left(5.5 \frac{a}{t} \right)} + \sigma_g \sqrt{2t \tan \left(\frac{\pi a}{2t} \right)} \right\}^B \quad (5-8)$$

Using the relations previously derived,

$$\alpha_N = \frac{6M}{(t-a)^2} ,$$

$$\sigma_g = \frac{T}{t} ,$$

$$M = M_P + M_S \left(1 - \frac{a}{t} \right) ,$$

$$T = T_P + T_S \left(1 - \frac{a}{t} \right) ,$$

$$\sigma_{PB} = \frac{6 M_P}{t^2} , \quad \sigma_{SB} = \frac{6 M_S}{t^2} ,$$

$$\sigma_{PM} = \frac{T_P}{t} , \text{ and } \sigma_{SM} = \frac{T_S}{t} ,$$

the following form of Equation (5-8) is produced:

$$\begin{aligned} \frac{da}{dn} = A \left\{ \left[\left(\frac{t}{t-a} \right)^2 \sigma_{PB} + \left(\frac{t}{t-a} \right)^1 \sigma_{SB} \right] \sqrt{0.52 (t-a) \tanh \left(5.5 \frac{a}{t} \right)} + \right. \\ \left. \left[\left(\frac{t}{t-a} \right)^0 \sigma_{PM} + \left(\frac{t}{t-a} \right)^{-1} \sigma_{SM} \right] \sqrt{2t \tan \left(\frac{\pi a}{2t} \right)} \right\}^B . \end{aligned} \quad (5-9)$$

The stress quantities in Equation (5-9) are the nominal stresses as if no crack were present. Equation (5-9) can be integrated to produce the cycles required to penetrate the wall (corresponding to the beginning of coolant leakage).

$$\begin{aligned} n_t = \frac{1}{A} \int_{a_0}^t \left\{ \left[\left(\frac{t}{t-a} \right)^2 \sigma_{PB} + \left(\frac{t}{t-a} \right)^1 \sigma_{SB} \right] \sqrt{0.52 (t-a) \tanh \left(5.5 \frac{a}{t} \right)} \right. \\ \left. + \left[\left(\frac{t}{t-a} \right)^0 \sigma_{PM} + \left(\frac{t}{t-a} \right)^{-1} \sigma_{SM} \right] \sqrt{2t \tan \left(\frac{\pi a}{2t} \right)} \right\}^{-B} da , \end{aligned} \quad (5-10)$$

where a_0 is the initial crack depth which is usually only known approximately as a random variable.

This equation covers the first four stress categories mentioned at the outset. The fifth category, that of localized concentration on the bending and membrane stress, was later dropped for crack growth purposes; see subsection 5.2.6.d.

5.2.3 Application Of The Cycles To Leak Equation In The Plastic Region (J. D. Gilman)

This subsection and the following two account for the change in stress with high strain in the plastic region of the cyclic stress-strain diagram, judged to be applicable only to secondary stresses. Refer to subsection 4.7.4 Reference 5-6.

5.2.4 Cyclic Stress-Strain Relationship (J. D. Gilman)

Refer to subsection 4.7.5 Reference 5-6.

5.2.5 Stress Modifications for the Plastic Region (J. D. Gilman)

Refer to subsection 4.7.6 Reference 5-6.

5.2.6 Final Equation for Stress Intensity Factor for Through-Wall Crack Growth

As a result of investigating the behavior of the equation for stress intensity factor ΔK given in subsection 5.2.2, the following equation was devised by J. D. Gilman, with new portions underlined. Uncertainty coefficients, the starred terms (*), have also been introduced.

$$\Delta K = \left\{ \underbrace{\left[C^{*2} \frac{\sigma_{PB}^* \sigma_{PB}}{\sigma_{PB}^* \sigma_{PB}} + 3 \frac{(C^*)^2 (E^*)^{3/2} \sigma_{PM}^*}{\sigma_{PM}^*} + C^* \frac{\sigma_{SB}^* \sigma_{SB}}{\sigma_{SB}^* \sigma_{SB}} + 3 C^* (E^*)^{3/2} \frac{\sigma_{SM}^* \sigma_{SM}}{\sigma_{SM}^* \sigma_{SM}} \right]}_{\text{bending portion}} \right. \\ \left. + \underbrace{\left[\sqrt{0.52 (t^* t - a) \tanh(5.5 E^*)} + \left[\sigma_{PM}^* + (1/C^*) \frac{\sigma_{SM}^* \sigma_{SM}}{\sigma_{SM}^* \sigma_{SM}} \right] \sqrt{2 t^* t \tan\left(\frac{\pi}{2} E^*\right)}}_{\text{membrane portion}} \right]} \right\} \\ \left[1 + 0.0325 (1 + \cos \pi E^*)^2 \right] \quad (5-11)$$

The corresponding cycles to leak equation is

$$n_t^* = \frac{1}{A^* A} \int_{\left(\frac{a_0}{t^* t}\right)^* t^* t}^{t^* t} (\Delta K)^{-B^* B} da \quad (5-12)$$

where

n_t^* = Distribution of number of cycles to completion of through-wall crack,	σ_{SB} = Secondary bending stress, psi,
$\left(\frac{a_0}{t^* t}\right)^* t^* t$ = Uncertainty distribution of a_0 , initial crack depth, inches,	σ_{SM} = Secondary membrane stress, psi,
t = Wall thickness, inches,	σ_{PB}^* = Coefficient uncertainty distribution of σ_{PB} ,
t^* = Coefficient uncertainty distribution of t ,	σ_{SB}^* = Coefficient uncertainty distribution of σ_{SB} ,
σ_{PB} = Primary bending stress, psi,	σ_{SM}^* = Coefficient uncertainty distribution of σ_{SM} ,

$$\sigma_{PM}^* = \frac{P^* (P - \Delta P^*) d^* d}{4 t^* t}$$

P = Pressure, psi,

ΔP^* = Difference uncertainty distribution of P, psi,

d = Diameter, inches,

P^* = Coefficient uncertainty distribution of P.

d^* = Coefficient uncertainty distribution of d,

A = Material parameter: log A is intercept of log (da/dn) versus log (ΔK),

A^* = Coefficient uncertainty distribution of A.

B = Material parameter: slope of log (da/dn) versus log (ΔK),

B^* = Coefficient uncertainty distribution of B.

$$C^* = \frac{t^* t}{t^* t - a}$$

$$E^* = \frac{a}{t^* t}$$

The changes in Equation (5-11) from Equation (5-10) can be summarized as follows:

- Membrane stresses, σ_{PM} and σ_{SM} , now appear in the "bending portion" of the equation as well as in the "membrane portion." This change recognizes that as a crack deepens, the loading of a membrane stress becomes eccentric, giving it an additional effect like that of a bending stress.
- Uncertainty coefficients have been assigned to each stress category, in place of having one such coefficient for all stress categories.
- The $(E^*)^{3/2}$ term and the cos coefficient were introduced, the latter to account for a back free surface effect.
- Stress concentration factor is omitted from this equation and will appear only in the crack initiation phase. The reason is that once a crack is initiated, its growth by low cycle fatigue does not depend on any stress concentration factor, only on total crack depth and stress field.

5.3 ANALYSIS OF THE CYCLES TO LEAK EQUATION

5.3.1 Characteristics of the Cycles to Leak Equation*

The "cycles to completion," predicted by Equation (5-11), has the following characteristics illustrated in Figures 5-5 to 5-8.

Figure No.	Characteristic
5-5	Cycles versus crack depth. The characteristic shape is maintained for a wide range of conditions, which shows that over 50% of cycles is required to increase crack depth to 10% of wall thickness.
5-6	Cycles to completion versus stress level. Shows characteristic shape of this curve, including operation of "plastic zone correction" for secondary stresses.
5-7	Cycles to completion versus stress level. Each stress category is considered separately, with the other stresses at 0, to observe the relative effect of each stress category.
5-8	Cycles to completion versus initial crack depth to wall thickness ratio, a_0/t . Curves added for different wall thicknesses reflect that cycles to completion decrease greatly for the same a_0/t fraction in heavier wall thicknesses, leading to the conclusion that cycles to completion depend much more on the value of a_0 than on wall thickness.

*With the collaboration of R. Stijovic

Figure 5-5. Characteristic Curve Shapes of Cycles versus Crack Depth in Low-Cycle Fatigue for Circumferential Crack Growth From Initial Depth, a_0 Through Wall Thickness, t

Figure 5-6. Cycles to Completion versus Uncertainty Coefficients on Each of the Four Stress Categories, Showing Effect of Plastic Stress Correction

EQUATION (5-11) VALUES:

σ_{SB} , σ_{PB} , σ_{PM} , σ_{SM} : THE STRESS FOR THE CURVE DESIGNATED IS ON THE ABSCISSA WITH OTHER STRESS CATEGORIES AT 0. EXAMPLE: FOR CURVE " σ_{SB} ", σ_{SB} INCREASES FROM 6000 TO 48 000 psi WITH $\sigma_{PB} = \sigma_{PM} = \sigma_{SM} = 0$.

$$a_0 = 0.010 \text{ inch}, t = 0.4 \text{ inch} \quad A = 1.66 \times 10^{-19} \quad B = 3.0$$

Figure 5-7. Cycles to Completion versus Value of One Stress Category

Figure 5-8. Cycles to Completion versus Ratio of Initial Crack Depth to Wall Thickness

All of the foregoing results for cycles to completion of a through-wall crack have been obtained using Simpson's Rule to approximate the integral. To ensure that the degree of approximation is adequate, the number of steps was repeatedly doubled until the relative change became less than 10^{-4} .

5.3.2 Sensitivity Testing of the Cycles to Leak Equation*

Figure 5-9 shows graphically the variation that can be expected in cycles to completion as the (a) design variables and (b) uncertainty coefficients vary over their expected ranges.

Run 1 is the reference run, using values given on Figure 5-9.

Runs 2 to 12 consider changes in the design variables.

Runs 2 to 5 show effects from material parameters A and B as they take on values in pairs over the ranges of values illustrated in Figure 5-10 for different materials and temperatures, as indicated by the Task D-1 and D-2 testing. Cycles to completion increase with increasing slope, B, when the line pivots at approximately the mean ΔK of the data range, because proportionately more cycles are consumed in the early, low ΔK , slow crack growth rate region, than in the high ΔK rapid crack growth rate region. Cycles to completion decrease with higher intercept log A, because higher crack growth rates apply for all ΔK values. (See subsection 5.3.5, where the effect of crack-specific mean log ΔK is considered).

Runs 6 to 7 show effects from selected combinations of t and d, where the constant values of a_o/tt^* and P used imply changes also in a_o and σ_{PM} as shown in parentheses. Changes in cycles to completion are due mainly to the resulting changes in a_o and σ_{PM} , rather than t or d directly. Runs 6A to 7A show the small effects from the same combinations of t and d for a_o and σ_{PM} held constant.

Runs 10 to 12 show that as each stress is decreased, cycles to completion increase.

Runs 13 to 35 consider changes in the uncertainty coefficients.

Runs 13 to 20 reflect the estimated uncertainty ranges of A^* (with any variability in B^* assigned to A^*), t^* (with a_o but not σ_{PM} held constant), ΔP^* , and P^*d^* . Any resulting changes in other variables are shown in parentheses. Changes in cycles to completion are nominal and as expected. Runs 15A and 16A show the small changes due to t^* with a_o and σ_{PM} held constant.

Runs 22 to 23 show the effect on cycles to completion of changes in a_o/tt^* , reflecting the pattern of Figure 5-8, which shows that large values of a_o will have great influence on cycles to completion, mainly independent of t. Runs 30 to 35 and 30A to 35A show that the effect of constant remaining ligaments of 0.1 and 0.3 inch is largest when the wall thickness is large, so that a_o is large.

Runs 24 to 29 show the effect of uncertainties on stress. Large unfavorable uncertainty in the secondary stresses materially reduce cycles to completion, even with the plastic stress correction.

This sensitivity analysis shows that for Equation (5-11), reductions in cycles to completion exceeding a factor of 10 or to below 10,000 cycles can come from unusually deep initial defects, or from high stresses, but not from design values or the uncertainty which can be foreseen in other variables.

*With the collaboration of R. Stijovic

Figure 5-9. Sensitivity Tests—Low-Cycle Fatigue Crack Growth Equation

Figure 5-10. A and B Relationships

5.3.3 Participation of the Stress Categories in the Cycles to Leak Equation*

To generate the distribution of cycles to leak, n_t , by the Monte Carlo method—i.e., by repeated solution of the equation using values selected at random from the distribution of each variable—a rapid method of solution of the equation is needed. In its present form, the equation can be solved only by numerical methods. To illustrate, if 10,000 solutions are needed for the Monte Carlo program, each requiring 500 solutions by a numerical method, then 5,000,000 solutions are required to form the distribution of cycles at one piping cross section, which is uneconomic. Some advantage is gained by forming only the lower tail of the cycles distribution, since the size of tail below the operating cycles is of interest. But understanding the behavior of the equation in detail, with the objective of achieving a form which may be computed rapidly, is important. This subsection, and the remainder of subsection 5.3, will achieve that objective.

The basic form of the low-cycle fatigue crack growth rate equation is as follows:

$$da/dn = A(\Delta K)^B \quad (5-13)$$

so

$$n_t = \frac{1}{A} \int_{a_0}^t (\Delta K)^{-B} da. \quad (5-14)$$

*With the collaboration of R. Stijovic.

For circumferential crack growth through the wall, the form for ΔK given in Equation (5-11), without the uncertainty coefficients, leads to

$$n_t = \frac{1}{A} \int_{a_0}^t \left\{ \left[\left(C^2 \sigma_{PB} + 3C^2 E^{3/2} \sigma_{PM} + C \sigma_{SB} + 3CE^{3/2} \sigma_{SM} \right) \cdot \sqrt{0.52 (t-a) \tanh 5.5E} \right. \right. \\ \left. \left. + \left(\sigma_{PM} + \frac{1}{C} \sigma_{SM} \right) \sqrt{2t \tan \frac{\pi}{2} E} \right] \left[1 + 0.0325 (1 + \cos \pi E)^2 \right] \right\}^{-B} da. \quad (5-15)$$

For symbols see Equation (5-11); for C and E, omit starred terms in C^* and E^* . Change variables by letting $x=a/t$, so $x_0=a_0/t$; and letting $z=(x-x_0)/(1-x_0)$. Then, using the notation that $x_0=1-x_0$ and $z=1-z$, one can obtain an expression involving a coefficient and a definite integral:

$$n_t = D \cdot \text{INT}$$

where

$$D = \frac{1}{A} \left(\frac{1}{\Sigma \sigma} \right)^B t^{1-\frac{B}{2}} (x_0)^{1+\frac{3B}{2}} (0.52)^{-\frac{B}{2}}. \quad (5-16)$$

$$\text{INT} = \int_0^1 \left\{ \left(\frac{\sigma_{PB}}{\Sigma \sigma} + \frac{\sigma_{SB} x_0 z}{\Sigma \sigma} \right) \frac{1}{z \sqrt{\frac{\tanh(x_0 + x_0 z)}{z}}} + \left(\frac{\sigma_{PM}}{\Sigma \sigma} + \frac{\sigma_{SM} x_0 z}{\Sigma \sigma} \right) \right. \\ \left. \left[3 \left(\frac{x_0 + x_0 z}{z} \right) \sqrt{\left(\frac{x_0 + x_0 z}{z} \right) \tanh(x_0 + x_0 z)} + x_0 \sqrt{\frac{2}{0.52} x_0 \tan \frac{\pi}{2} (x_0 + x_0 z)} \right] f \right\}^{-B} dz. \quad (5-17)$$

where

$$f = 1 + 0.0325 \left[1 + \cos \pi (x_0 + x_0 z) \right]^2. \quad (5-18)$$

The relationships among x_0 , x_0 , z , and z and the original terms a_0 and a , can be displayed as shown in Figure 5-11. Original variable was a ; new variable is z .

Note that the integral is now a function of only the following:

The ratio of each stress category to total stress: $\sigma_{PB}/\Sigma \sigma$, $\sigma_{SB}/\Sigma \sigma$, $\sigma_{PM}/\Sigma \sigma$, and $\sigma_{SM}/\Sigma \sigma$; $x_0 = a_0/t$; and B.

Thus it is possible to show the behavior of the ratio of the integral [when only one stress category is present and equal to some value, say S], to [the integral when all four stresses are equal to S/4 so that $\Sigma \sigma = S$], as a function of a_0/t and of B, but independent of the value of S. That is, this ratio of integrals will be the same for any value of S. Such ratios appear in Figure 5-12. It can be seen that for small a_0/t values the ratio for all stress categories is near 1; that is,

Figure 5-11. Relationships Among Variables, Equation (5-17)

Figure 5-12. Ratio of Integrals versus a_0/t

the different way in which each stress participates in this ΔK formula is not important when a_o/t is small. As a_o/t increases the ratios differ more, but in a predictable way depending only on a_o/t and B .

Figure 5-12 also shows that for a given set of values, σ_{PM} will lead to fewest cycles to grow through, followed by σ_{PB} (which is surprisingly close to all four stresses equal), σ_{SM} , and σ_{SB} . The σ_{SB} is least effective because its initial stress level decreases because of both its secondary and bending characteristics.

Note that coefficient D is a function of

Total stress, $\Sigma\sigma$;
Both $x_o = a_o/t$ and t ; and
Both A and B .

Coefficient D can be solved directly. It is this coefficient that introduces the effect of A , the effect of t separate from a_o/t , and the level of stress as given by $\Sigma\sigma$.

5.3.4 Transformations to Bound the Cycles-to-Leak Equation (G. T. Vernon)

5.3.4.1 Introduction to Subsection 5.3.4 (G. T. Vernon)

The modified through-wall circumferential crack growth equation, first introduced as Equation (5-11), may be put in the following form:

$$n_t = \frac{1}{A} \int_{a_o}^t \left\{ \left| \left(\frac{1}{1-a/t} \right)^2 \cdot \left(\sigma_{PB} + \left(1 - \frac{a}{t} \right) \cdot \sigma_{SB} \right) + 3 \left(\frac{a}{t} \right)^{3/2} \cdot \left(\frac{1}{1-a/t} \right)^2 \right. \right. \\ \left. \left. \cdot \left(\sigma_{PM} + \left(1 - \frac{a}{t} \right) \cdot \sigma_{SM} \right) \right| \cdot \sqrt{t \left(1 - \frac{a}{t} \right) f \left(\frac{a}{t} \right)} + \left[\sigma_{PM} + \left(1 - \frac{a}{t} \right) \cdot \sigma_{SM} \right] \cdot \sqrt{t g \left(\frac{a}{t} \right)} \cdot h \left(\frac{a}{t} \right) \right\}^{-B} da, \quad (5-19)$$

(without the uncertainty coefficients), with f , g , and h defined as follows:

$$f \left(\frac{a}{t} \right) = 0.52 \tanh \left(5.5 \frac{a}{t} \right),$$

$$g \left(\frac{a}{t} \right) = 2 \tan \left(\frac{\pi}{2} \cdot \frac{a}{t} \right),$$

$$h \left(\frac{a}{t} \right) = 1 + 0.0325 \left(1 + \cos \left(\pi \frac{a}{t} \right) \right)^2.$$

A more accurate functional fit to Bueckner's data:

$$f \left(\frac{a}{t} \right) = \frac{\pi}{6} \cdot \left(1 - e^{-6 \frac{a}{t}} \right) \quad (\text{in place of the above}) \quad (a/t \text{ is in exponent.})$$

is discussed in the next subsection.

A transformed version of the above Equation (5-19) is presented in the section following that, which is suitable for rapid evaluation by Gauss' rule. Tested against most of the same values of variables as in Figure 5-9, the transformed equation gives less than 0.15% error when a second of computation time is consumed.

5.3.4.2 Bueckner Coefficient Modification (G. T. Vernon)

To take into account in the crack growth equation the change in the stress intensity factor quantified by H. F. Bueckner as the crack grows from the shallow crack, $a \ll t$, to the deep crack, $a \approx t$, S. W. Tagart in Equation (5-6), required a function satisfying the following properties:

$$a \ll t \quad \sqrt{(t-a) f\left(\frac{a}{t}\right)} \approx 1.77 \sqrt{a} \quad (5-20)$$

$$a \approx t \quad \sqrt{(t-a) f\left(\frac{a}{t}\right)} \approx 0.72 \sqrt{t-a} \quad (5-21)$$

The function chosen there, Equation (5-7), and used in the intervening subsections has been:

$$f\left(\frac{a}{t}\right) = 0.52 \tanh\left(5.5 \frac{a}{t}\right) \text{ giving: } \left\{ \begin{array}{l} 1.69 \sqrt{a} \quad \text{in (5-20)} \\ 0.72 \sqrt{t-a} \quad \text{in (5-21)} \end{array} \right\}$$

Since, however, the accuracy in Equation (5-20) plays a greater role in determining the number of cycles, n_t , [because of the weighting of $x^{-B/2}$ in Equation (5-23) below], a new approximation has been made as follows:

$$f\left(\frac{a}{t}\right) = \frac{\pi}{6} \left(1 - e^{-6 \frac{a}{t}}\right) \text{ giving: } \left\{ \begin{array}{l} 1.77 \sqrt{a} \quad \text{in (5-20)} \\ 0.72 \sqrt{t-a} \quad \text{in (5-21)} \end{array} \right\} \quad (5-22)$$

A comparison with Reference 5-7 shows this representation to be closer to the calculated results for the lower values of a/t (see also Figure 5-13):

$\frac{a}{t}$	.05	.1	.2	.3	.4	.5	.6
$\sqrt{f\left(\frac{a}{t}\right)} \cdot (\text{Bueckner})$	.36	.49	.60	.66	.69	.72	.73
$\sqrt{0.52 \tanh\left(5.5 \frac{a}{t}\right)}$	.373	.51	.645	.695	.712	.719	.72
$\sqrt{\frac{\pi}{6} \left(1 - e^{-6 \frac{a}{t}}\right)}$	.368	.486	.605	.661	.690	.705	.714

When used in place of $f(x)$ in Equation (5-27) below, the number of cycles, n_t , for the reference run of Figure 5-9 increases from 178,500 to 188,500 cycles. Equation (5-22) will be adopted.

Figure 5-13. Fitting the Bueckner Coefficient

5.3.4.3 Numerical Solution of the n_t Equation (G. T. Vernon)

By substituting:

$$x = \frac{a}{t}$$

$$x_0 = \frac{a_0}{t}$$

$$dx = \frac{1}{t} da$$

in Equation (5-19) above, we find:

$$n_t = \frac{1}{A \cdot t^{(B/2)-1}} \int_{x_0}^1 \left\{ \left[\left(\frac{1}{1-x} \right)^2 \cdot (\sigma_{PB} + (1-x) \cdot \sigma_{SB}) + 3x^{3/2} \cdot \left(\frac{1}{1-x} \right)^2 \cdot (\sigma_{PM} + (1-x) \cdot \sigma_{SM}) \right] \sqrt{(1-x)f(x)} + (\sigma_{PM} + (1-x) \cdot \sigma_{SM}) \sqrt{g(x)} \right\} \cdot h(x) dx$$

$$= \frac{1}{A \cdot t^{(B/2)-1}} \cdot \int_{x_0}^1 \left\{ x^{-B/2} (1-x)^{3B/2} \right\} \cdot \left\{ \left[S_B(x) + 3x^{3/2} \cdot S_M(x) \right] \cdot \sqrt{f(x)/x} \right. \\ \left. + (1-x) \cdot S_M(x) \cdot \sqrt{\frac{1-x}{x}} \cdot g(x) \right\} \cdot h(x) \Bigg\}^{-B} dx \quad (5-23)$$

where

$$S_B(x) = \sigma_{PB} + (1-x) \cdot \sigma_{SB}$$

$$S_M(x) = \sigma_{PM} + (1-x) \cdot \sigma_{SM}$$

in which primary stresses σ_{PB} and σ_{PM} will predominate, per subsection 5.2.7.

It will become clear that Equation (5-23) is a useful rearrangement if we show that the large curly bracketed expression is bounded away from zero for any combinations of stresses. To that end let us investigate the limiting values of its components as x approaches both 0 and 1:

$x \rightarrow 0$:

$$\frac{f(x)}{x} \approx \frac{\pi}{6} \cdot \left| 1 - (1 - 6x) \right| / x = \pi$$

$$\frac{1-x}{x} \cdot g(x) \approx \frac{1-x}{x} \cdot \left| 2 \cdot \frac{\pi}{2} \cdot x \right| \approx \pi$$

$$h(x) \approx 1 + 0.0325 \cdot \left[1 + \left(1 - \frac{\pi^2 x^2}{2} \right) \right]^2 \approx 1.13$$

$$S_B(x) \approx \sigma_{PB} + \sigma_{SB}$$

$$S_M(x) \approx \sigma_{PM} + \sigma_{SM}$$

$$\left\{ * \right\} \approx (1.13\sqrt{\pi} \approx 2.0) \cdot \left[(\sigma_{PB} + \sigma_{SB}) + (\sigma_{PM} + \sigma_{SM}) \right], \quad * \text{ from (5-23)}$$

$x \rightarrow 1$: (or $y = 1 - x \rightarrow 0$)

$$\frac{f(1-y)}{1-y} \approx (1+y) \cdot \left[\frac{\pi}{6} (1 - e^{-6}) \right] \approx 0.52$$

$$\begin{aligned}\frac{y}{1-y} \cdot g(1-y) &= \frac{2y}{1-y} \cdot \tan\left(\frac{\pi}{2} - \frac{\pi}{2} \cdot y\right) \\ &= \frac{2}{1-y} \cdot \frac{y}{\tan\left(\frac{\pi}{2} \cdot y\right)} \approx \frac{4}{\pi}\end{aligned}$$

$$\begin{aligned}h(1-y) &= 1 + 0.0325 \cdot \left[1 + \cos(\pi - \pi y)\right]^2 \\ &= 1 + 0.0325 \cdot \left[1 - \cos(\pi y)\right]^2 \approx 1\end{aligned}$$

$$S_B(x) \approx \sigma_{PB}$$

$$S_M(x) \approx \sigma_{PM}$$

$$\left\{ * \right\} \approx \sqrt{0.52} \left[\sigma_{PB} + 3\sigma_{PM} \right], \quad * \text{ from (5-23)}$$

The large curly bracketed expression is positive and nearly constant for all values of x and any combinations of stress with the primary stresses predominating. The problem of evaluating the n_t Equation (5-23) is almost equivalent to that of evaluating the equation:

$$I = \int_{x_0}^1 x^{-B/2} (1-x)^{3B/2} dx \quad (5-24)$$

The term $x^{-B/2}$ in the above gets very large for small values of x . However, the two transformations:

$$z = \left(\frac{1}{x}\right)^{(B/2)-1} \quad (5-25)$$

$$z = \left(\frac{1-x}{x}\right)^{(B/2)-1} \quad (5-26)$$

are available to keep the integrand in Equation (5-24) bounded and may each be used in Equation (5-23) to evaluate the n_t equation numerically. Transformation (5-25) has given better numerical results when tested against direct solution by Simpson's Rule (with exhaustive iterations) in most of the runs described in Figure 5-9. However, both transformations yield very accurate results using 1 second of computation time in the 10-point Gauss' quadrature.

We make the transformation, (5-25), valid for $B \neq 2$:

$$\begin{aligned}x &= \left(\frac{1}{z}\right)^{2/(B-2)} \\ dx &= \frac{-2}{B-2} \left(\frac{1}{z}\right)^{2 + [2/(B-2)] - 1} dz = -\frac{2}{B-2} \left(\frac{1}{z}\right)^{B/(B-2)} dz\end{aligned}$$

for $B > 2$:

$$n_t = \frac{1}{((B/2)-1) \cdot A \cdot t^{(B/2)-1}} \int_1^{x_0^{(B/2)-1}} \left[F\left(\left\{\frac{1}{z}\right\}^{2/(B-2)}\right) \right]^B dz, \quad (5-27)$$

for $B < 2$:

$$n_t = \frac{t^{1-B/2}}{(1-B/2) \cdot A} \int_{x_0^{1-B/2}}^1 \left[F\left(z^{2/(2-B)}\right) \right]^B dz, \quad (5-28)$$

where

$$F(x) = (1-x)^{3/2} \left\{ * \right\}^{-1}, \quad *, \text{ from 5-23}.$$

For $B = 2$ we make the transformation:

$$\begin{aligned} z &= \ln x \\ dz &= \frac{1}{x} dx \\ n_t &= \frac{1}{A} \int_{\ln x_0}^0 \left[F(e^z) \right]^B dz \end{aligned} \quad (5-29)$$

The function $F(x)$ is indeed well behaved for any combinations of stresses. A plot is made, Figure 5-14, of F with each stress acting alone and having the value $\sigma = 1$ and of F with every stress having the value $\sigma = 1/4$.

5.3.5 Sources of Cyclic Stress in the Piping System (J. D. Gilman)

The stresses of primary importance in crack initiation and crack propagation studies are the thermal expansion stresses, S_E , which have been calculated, and the pressure stresses S_p , which are readily calculated from nominal pipe dimensions. These stress cycles are associated with reactor startup and shutdown, and may number a few hundred. Weight stresses and possible earthquake stresses have also been calculated, but consideration of these stresses is appropriate to analysis of the later phases of possible piping failure, rather than the crack initiation and fatigue crack propagation phases. Only the stresses S_E and S_p will be included in further discussion here, and will be assumed to occur simultaneously in cyclic loading.

A technique for accounting for the plastic portion of the cyclic stress-strain diagram, applicable to secondary stresses, was developed in subsections 5.2.3–5.2.5. As indicated in the following subsection, however, it became feasible to restrict attention to the primary stress categories.

Figure 5-14. Function $F(x)$ From $n_t = \frac{1}{A \cdot t^{(B/2)-1}} \int_{x_0}^1 x^{-B/2} [F(x)]^B dx$, Equation (5-23), Is Well Behaved

5.3.6 Stress Categories in the Cycles to Leak Equation (J. D. Gilman)

Four categories of stress in a pipe wall have been identified in the analysis of fatigue crack propagation through the pipe wall. These are:

- a. Primary membrane,
- b. Primary bending
- c. Secondary membrane
- d. Secondary bending

The terms membrane and bending refer to stresses which act normally to the plane of the crack and are uniformly distributed and linearly distributed, respectively, through the pipe wall thickness. The secondary stress categories are, by definition, those nominal stresses which are reduced in magnitude as the crack depth increases, as a result of increased local flexibility because of the presence of the crack. The primary stress categories are load dependent and do not decrease with increasing crack depth.

The proper classification of stresses in a piping component into primary and secondary categories requires the stress analysis of the component with the equivalent crack flexibility included. This type of analysis is not yet practicable. But, more importantly, there is evidence from preliminary calculations that the primary versus secondary distinction will not be necessary in most cases for application of the crack growth equation. If the initial crack depth is small compared to the pipe wall thickness, then a very large proportion of the total number of cycles to grow through the wall is consumed while the crack is short and the primary versus secondary distinction is not significant. Furthermore, it is estimated that primary stresses predominate in most of the piping components. It will be expedient, conservative, and apparently quite accurate, then, to omit the secondary stress categories from the crack growth equation, with the possible exception of special cases of local thermal stress which can be considered separately.

5.3.7 Simplified Interpretation of Piping Stresses (J. D. Gilman)

Some simplifying assumptions regarding the orientation of the thermal expansion stresses in straight and curved components are necessary to permit superposition of expansion stresses and pressure stresses. These simplifications will also permit a potential fatigue crack to be identified as either circumferentially or axially oriented, and thus avoid the complexities of a consideration of skew-oriented cracks.

An effective stress has been calculated at all joints identified for stress analysis purposes in a reference piping system. These stresses are reported in Reference 5-8; the calculation method is described in Appendix A therein. Each stress was calculated without system pressure and therefore is comprised of torsional and system bending contributions due to thermal expansion.

In the straight pipe section, the thermal expansion stress, S_E , is the maximum difference of principal stresses resulting from system bending and torsional moments in the pipe. These principal stresses may actually be oriented at an angle to the axis of the pipe, as a result of the torsional component; but it will be assumed that S_E is due to system bending only and represents a uniaxial stress parallel to the axis of the pipe.

In the curved pipe section, S_E is an estimate, based on Piping Code rules, of the effective stress for fatigue crack initiation resulting from a combination of in-plane, out-of-plane, and torsional moments in the pipe. The actual stress distribution in the piping elbow is more complex than is implied by the method of calculation of S_E , but it will be assumed that S_E is the stress resulting from bending to oval shape of the elbow cross section. The S_E is oriented in the transverse, or circumferential, direction.

The stresses to be used in the crack growth equation may now be defined, using the following nomenclature:

- σ_{PM} = Primary membrane stress
- σ_{PB} = Primary bending stress
- S_E = Calculated expansion stress
- S_p = Calculated pressure hoop stress = (pr/t)

For the straight pipe section, use the larger of

$$\sigma_{PM} = S_E + \frac{1}{2} S_p \text{ (effective in growing a crack having a circumferential orientation)} \quad (5-30)$$

or

$$\sigma_{PM} = S_p \text{ (effective in growing a crack having an axial orientation)} \quad (5-31)$$

with

$$\sigma_{PB} = 0.$$

For the piping elbow, $\sigma_{PM} = S_p$, $\sigma_{PB} = S_E$, and the crack is oriented in the axial (longitudinal) direction.

5.3.8 Crack-Specific Mean Log ΔK . Linear Form of the Cycles to Leak Equation

In the basic fracture mechanics crack growth rate equation,

$$da/dn = A \Delta K^B \quad (5-32)$$

the sensitivity of the function to A and ΔK may be investigated independently, but not the sensitivity to B, which is not independent of A. This may be seen when logs are taken to linearize the equation, giving

$$\log da/dn = \log A + B \log \Delta K \quad (5-33)$$

This equation is of the form

$$y = a + bx, \quad (5-34)$$

in which, in general, slope is not independent of intercept a . Such an equation implies a far greater sensitivity of y to changes in b than is in fact the case. Random cracks, described by values of $\log A$ and B drawn independently at random from their respective distributions, along with the nominal crack, are illustrated by Figure 5-15. Obviously, lines 1 and 2 miss entirely the region of observed cracks. This problem is remedied by writing Equation (5-43) in mean-adjusted form,

$$y = c + b(x - \bar{x}). \quad (5-35)$$

Now when b is changed, the line pivots at $x = \bar{x}$, at value c on the y scale; previously it pivoted at $x = 0$, at value a on the y scale. Now changes in b may be made, independent of c , giving the results illustrated in Figure 5-16 where random cracks vary within the envelope of experience.

The problem is made more complicated for the crack growth rate equation because of how x must be chosen. If Equation (5-33) is modified by including the $\bar{x}$ term, it becomes, say,

$$\log da/dn = \log C + B (\log \Delta K - \overline{\log \Delta K}), \quad (5-36)$$

where $\overline{\log \Delta K}$ is mean $\log \Delta K$, $\log C = \log A + B \overline{\log \Delta K}$.

Ordinarily, $\bar{x}$ is the mean of x values in an experiment in which parameters a and b are estimated, resulting in an equation for predicting a y value at a given x value, the uncertainty in a predicted y reflecting the variances of estimate of a and b . But in the crack growth rate problem we are not interested in predicting a crack growth rate value at some stress intensity factor but, rather, in the number of cycles for a crack to grow from an initial depth a_0 through wall thickness t ; that is between starting and stopping ΔK values denoted $\Delta K(a_0, \sigma)$ and $\Delta K(t, \sigma)$, along a line defined by B and $\log C$ at $\overline{\log \Delta K}$ being subjected to a slightly larger da/dn value on each cycle. How much the number of cycles to leak might vary depends on the variability in parameters A and B , seen experimentally *among specimens*, that is, among *different* crack growth lines, and these variabilities have been used. But the appropriate $\overline{\log \Delta K}$ is not the one which happened to occur among such lines as fixed by some experiments, but is characteristic of each crack *grown*, i.e., the combination of a_0 , t , σ and B chosen to represent the crack and its location. It was appropriate to find $\overline{\log \Delta K}$ for this crack, as if an experiment had been run *under* these conditions. For a random crack, $\log \Delta K$ was found for every cycle, and these values were averaged to find $\overline{\log \Delta K}$. In practice, $\overline{\log \Delta K}$ was found iteratively, starting at the mean of the means of the Schenectady testing. $\log \Delta K$ depends on a_0 , σ and B , very slightly on t , and not at all on A .

We are interested in the variance of cycles to leak. To find, first, an equation for cycles to leak, take antilogs of Equation (5-36), resulting in

$$da/dn = C (\Delta K / \widetilde{\Delta K})^B, \quad (5-37)$$

where $\widetilde{\Delta K}$ is the antilog of $\overline{\log \Delta K}$, which, as before, is the mean of the $\log \Delta K$'s of all the cycles to which the crack is subjected. Inverting, and integrating with respect to crack depth, gives

$$n_t = \frac{1}{C} \int_{a_0}^t \left(\frac{\Delta K}{\widetilde{\Delta K}} \right)^B da \quad (5-38)$$

Cracks for different values of slope only, adjusted for mean $\log \Delta K$, are illustrated in Figure 5-17. Mean $\log \Delta K$ versus initial crack depth, stress, and slope are illustrated in Figures 5-18 through 5-20. Finally, the variations of log cycles to leak, $\log n_t$, as functions of initial crack depth, stress, slope, and intercept are given in Figures 5-21 through 5-24. The most important observation is that there is now little dependence of n_t on B .

Figure 5-15. Nominal and Two Random Cracks, Showing Improper Pivoting at Log $\Delta K = 0$

Figure 5-16. Nominal and Two Random Cracks, Showing Proper Pivoting at Log $\Delta K = \text{Mean Log } \Delta K$

Figure 5-17. Crack Growth Lines of Cracks Having Fixed Intercept and Random Slope Values, Showing that Line Pivoting Follows Mean Log ΔK

Figure 5-18. Initial Crack Depth (as $\sqrt{a_0/t}$) versus Mean Log ΔK

Figure 5-19. Stress (as $\text{Log } \sigma_{PM}$) versus Mean $\text{Log } \Delta K$

Figure 5-20. Slope B, versus Mean Log ΔK

Figure 5-21. Cycles to Grow Through, n_t , versus Initial Crack Depth (as $\sqrt{a_0/t}$)

Figure 5-22. Cycles to Grow Through, n_t versus Stress (as Log σ_{PM})

Figure 5-23. Cycles to Grow Through, n_t , versus Random Component of Intercept, $\Delta \text{Log } C$

Figure 5-24. Cycles to Grow Through, n_t versus Slope B

It can be seen by comparison of Figures 5-15 and 5-16 that a reduction in variance n_t results from using mean $\log \Delta K$, for the same variability in B . This was anticipated, as it appeared that any choice of value on the x scale at which to pivot the random line, other than the appropriate x , would artificially inflate the variability of n_t .

Since all lines in Figures 5-21 through 5-24 are parallel, the variables are independent; and since they are straight lines, with the exception of n_t versus B , n_t may be written as an equation linear in coefficients, as shown below, for B fixed at 4.45. Inherent in this equation is the shifting of mean $\log \Delta K$ with changes in initial crack depth, final crack depth, and stress specific to a particular crack. Having such a linear expression will speed the Monte Carlo solutions. B will not participate as a variable at this time; probability values so obtained will be slightly low.

A linear equation for n_t , for $B = 4.45$, is

$$\log n_t = 27.3064 - 1.0084 (\Delta \log c) - 4.5069 (\log \sigma_{PM}) - 6.2615 \sqrt{a_o/t} \quad (5-39)$$

To include B , the following expression should be added:

$$0 = 0.1623 - 2.47 e^{-B} - 0.03 B, \quad (5-40)$$

resulting in the following equation for n_t in terms of all four major variables:

$$\log n_t = 27.4687 - 1.0084 (\Delta \log C) - 4.5069 (\log \sigma_{PM}) - 6.2615 \sqrt{a_o/t} - 2.47 e^{-B} - 0.03 B, \quad (5-41)$$

where

n_t = Cycles to leak at the selected piping study section.

$\Delta \log C$ = Random deviation from the nominal intercept $\log C$, at some $\log \Delta K$; the equation is written for $\log C = -3.962$ at $\log \Delta K = 4.786$, the grand mean $\log \Delta K$ of the testing in Tasks D-1 and D-2.

$\log \sigma_{PM}$ = Primary membrane stress per Equation (5-11), and values which will be selected in Table 6-1.

$$\sigma_{PM} = S_{exp} \times S_{exp}^* + S_{hoop}/2$$

S_{exp} = expansion stress

S_{exp}^* = uncertainty coefficient on expansion stress

$$S_{hoop} = Pd/2t = 1000 \times 6.624/(2 \times 0.432) = 7667 \text{ psi}$$

a_o/t = Initial crack depth ratio wall thickness.

B = Slope parameter

$\log \equiv \log_{10}$

This equation is valid for a_o/t values chosen from the distributions of (a_o/t) max with or without ultrasonic testing.

5.4 REFERENCES

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- 5-8 English, W. F., and Gilman, J. D., *BWR Primary Piping Stresses for System Reliability Analysis*, September 1968, (GEAP-5724).
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6. UNCERTAINTY DISTRIBUTIONS OF VARIABLES

6.1. INTRODUCTION

The probability distribution, herein called uncertainty distribution, which reflects the ratio of the actual value to the design value of a variable is given in this section for each variable in the cycles-to-leak equation. Distributions for the two crack growth parameters were obtained from reduction of laboratory data. Distributions for the physical variables, however, such as stress and initial crack depth, were drawn from engineering experience and judgment. Many of these are conditional distributions related to some identifiable cause. That is, a probability is assigned that a certain type of error is made; then, given the occurrence of the error, a distribution of ratio values is judged to result. Further remarks on the use of judgmental input appear in the introduction, subsection 2.4, General Discussion.

6.2 THE UNCERTAINTY DISTRIBUTIONS*

The principal purpose of this section is to give structure to the problem of quantifying uncertainties in the parameters governing low-cycle fatigue. Table 3-2 in Reference 6-1 listed a number of causes of real variation or uncertainty in the parameters governing failure by low-cycle fatigue. Such variation needs to be expressed quantitatively as an uncertainty distribution, usually a probability distribution of the ratio of realized values/planned values. Tentative formulations of these distributions related to the growth to through-wall crack stage of damage are given on the following pages. All causes listed in Table 3-2 of Reference 6-1 are included and referenced below, except line 21 "Other material properties."

The values chosen and the width and shape of the distributions reflect preliminary engineering judgment and experience. In fact, throughout the following presentation of conditioning probabilities and uncertainty distributions, the phrase "is judged to be" can be read into each claim, as illustrated in cause 1.1.

Most of the uncertainty distributions are conditional; i.e., the probability that there might be a difference between planned and realized values is first stated, and then the distribution of such differences is proposed, given that such a difference occurred. Unless otherwise stated, the distributions apply to all materials, sizes, and types of components.

PARAMETER 1.0: STRESS

Cause 1.1: Arrangement Error

	<u>Planned Values</u>	<u>Realized Values</u>	<u>Reference Line Number**</u>
B.	Configuration Design Stage	E. Shop and Field Fabrication Stage	1
1.0	Routing based on:	2.0 Configuration correct; right parts in right places	
1.1	Function - performance	7.0 Accurate alignment on runs	
1.2	Function - measurement		
2.0	Routing based on:		
2.1	Interconnection locations		
2.2	Ease in field fabrication		

*With the collaboration of S. W. Taggart.

**The "Reference Line Number" is the sequence number of the line in Table 3-2 Reference 6-1 which listed the cause shown.

The sequence of events between design and fabrication is shown schematically in Figure 6-1. Any error in the sequence so that the arrangement of installed piping, spring hangers, or welded anchors does not match the stress analysis model is included in the following. The errors included here are in arrangement only. Wall thickness, diameter, and material errors are treated later. Such errors might be caused by failure of the stress analysis model to match the design line drawings, or the configuration drawings to match the design line drawings, or the piping as fabricated in shop or field to match the configuration drawings.

Figure 6-1. Sequence of Events

The conditioning probability of a loop having such an event occur is judged to be 10^{-2} without checking or inspection at any step, and 10^{-5} with checking or inspection at all steps. The broad scope and implications of "checking and inspection" are discussed in subsection 3.3 of Reference 6-1. The distribution of stresses among components in loops with such errors is shown in Figure 6-2.

Figure 6-2. Conditional Stress Uncertainty Distribution for Cause 1.1: Arrangement Error

The foregoing should be interpreted as follows. Assume the probability of occurrence of a stress ratio exceeding 5.0 is of interest, and that the conditional probability of such an event is approximately 0.05, the area under the curve between stress ratios of 5.0 and 10.0, and given that an error has been made. The probability of an error being made, and that the error results in a stress ratio of between 5.0 and 10.0 in a component, for the case without checking or inspection at any step, is the product of the conditional and conditioning probabilities,

Probability of stress ratio of between 5.0 and 10.0 in a component for the case chosen

$$= (0.05)(10^{-2}) = 5 \times 10^{-4}, \text{ or } 5 \text{ in } 10,000.$$

* p.d.f. = probability density function. The probability density function takes on values along this scale so that the area under the curve between any two values of the variable is the probability that a value in that range occurs, and the probability (area) under the entire curve is 1.0. Thus the p.d.f. scale values are different for each diagram, and for each curve on diagrams with several curves.

Cause 1.2: Calculation Method Error

<u>Planned Values</u>		<u>Realized Values</u>	<u>Reference Line Number</u>
B.	Configuration Design Stage	F. Operating Conditions Stage	2
3.0	Calculation Methods	10.0 Shakedown	
		40.0 Operation	

Calculation method errors are unconditional, and arise from the degree of approximation of the stress analysis model to the real piping system, and failure to identify special configuration requirements because of stress, such as nozzle reinforcements and piping support design. From experience solutions are accurate "within $\pm 10\%$," but conservative features of the methods are believed to lengthen the lower tail. The distribution in Figure 6-3 applies to individual components.

Figure 6-3. Unconditional Stress Uncertainty Distribution for Cause 1.2: Calculation Method Error

Cause 1.3: Calculation Error

<u>Planned Values</u>		<u>Realized Values</u>	<u>Reference Line Number</u>
B.	Configuration Design Stage	F. Operating Conditions Stage	2
3.0	Calculation Methods	10.0 Shakedown	
		40.0 Operation	

Calculation errors arise whenever the wrong answer is obtained using correct input and methods. The two conditional probability distributions in Figure 6-4 apply to individual components without checking. One distribution is for desk calculation, the other for computer calculation. The distributions are seen to be based on the size of the problem in terms of the square of the number of simultaneous equations involved, as a reference for comparison. With review checking the probability so found should be multiplied by 10^{-1} ; with completely independent checking it should be multiplied by 10^{-3} .

Results which are above anticipated levels will often be recognized and lowered if appropriate, thus decreasing the probability of an actual calculated stress ratio in the region below 1.0; results below anticipated levels will be less suspect, thus increasing probability of a ratio in the region above 1.0. The distributions in Figure 6-5 describe probabilities of the stress ratios occurring, for problems of increasing size, given that an error was made.

Figure 6-4. Conditioning Probability of Cause 1.3: Calculation Error

Figure 6-5. Conditional Stress Uncertainty Distribution for Cause 1.3: Calculation Error

For example, following the pattern of cause 1.1, the probability of a stress ratio between 2.0 and 4.0 occurring to a component, for the case of 100 input values, computer calculation and review checking—taking the conditional probability under the curve as 0.2—is:

Probability of stress ratio of between 2.0 and 4.0 in a component for the case chosen

$$= (0.2)(0.6)(10^{-1}) = 12 \times 10^{-3}.$$

Cause 1.4: Loss of Flexibility

Planned Values		Realized Values		Reference Line Number
B.	Design Configuration Stage	E.	Shop and Field Fabrication Stage	8
4.11	Pipe not touching other pipe, equipment or structures	11.0	Piping not touching other equipment, pipe or structures	
		F.	Operating Conditions Stage	
		52.0	Piping not touching other pipes, equipment or structures	

Flexibility can be decreased by the installation of equipment or structures immediately adjacent to piping. The effect of such installation is more severe when shorter pipe lengths are involved. Such installations could be made on a prestart path to failure, i.e., during original installation of equipment, before startup, or on a poststart path, i.e., by the customer after plant operation. The conditioning probability of a compromise of flexibility of a loop on a prestart path to failure is 10^{-2} without inspection, and 10^{-4} with trained inspection. The conditioning probability of such a compromise on a poststart path is 10^{-5} ; made up 5×10^{-2} , the probability that a customer modifies equipment or structure adjacent to a loop (not the loop itself) during the 40-year life of a reactor; and 2×10^{-4} , the probability that he does so in such a way as to reduce flexibility. Given in either case that flexibility is impaired, the distribution of stresses among components is as in Figure 6-6.

Figure 6-6. *Conditional Stress Uncertainty Distribution for Cause 1.4: Loss of Flexibility*

Cause 1.5: Loop Redesign Error

				Reference Line Number
<u>Planned Values</u>		<u>Realized Values</u>		
B.	Configuration Design Stage	E.	Shop and Field Fabrication Stage	13
11.0	Redesigns Required	F.	Operating Conditions Stage	

Redesign required on a prestart path to failure, while the plant is still under designer control, is considered as part of each applicable heading. For example, under arrangement error the probabilities given apply for the first design attempt, and for any modifications where redesign is required. However, on a poststart path, under customer control, the conditioning probability that a loop will require redesign during 40-year life is 10^{-2} , and that a mismatch between line drawings, stress analysis model, and construction is 10 times that applicable to a prestart path, i.e., 10^{-1} when there is no checking or inspection at any step, including omission of stress analysis, and 10^{-4} when all steps are checked or inspected by trained individuals. Given that poststart redesign is required, and that a mismatch occurs, the possible distribution of resulting stresses in a component in the affected loop is the same as for the arrangement error cause.

Cause 1.6: Spring Support and Welded Anchor Error

<u>Planned Values</u>		<u>Realized Values</u>	<u>Reference Line Number</u>	
B.	Configuration Design Stage	E.	Field Fabrication Stage	12
10.0	Support designs and locations	8.0	Support designs and locations for engineering-designed or field located supports	

Errors in spring support design are included in calculation method error; errors in spring support location are included in arrangement error; and errors in welded anchor location are included in arrangement error.

Cause 1.7: Reinforcement Error

	<u>Planned Values</u>	<u>Realized Values</u>	<u>Reference Line Number</u>
B.	Configuration Design	F. Operating Conditions	10
	7.0 Reinforcements	40.0 Operations	

Uncertainty in stress on account of this cause is included in the calculation method cause

Cause 1.8: Shock Loading

	<u>Planned Values</u>	<u>Realized Values</u>	<u>Reference Line Number</u>
B.	Configuration Design Stage	F. Operating Conditions Stage	7
	4.9 Related equipment shock (hammer, water slug, earthquake)	46.0 Shock, valve hammer, earth- quake, pipe or plant shifts position, water slug formation	

The effect of hammer is on operating pressure; therefore its distribution is presented under that cause.

The effect of earthquakes must present the following information:

- Distribution of frequency of earthquakes for the site selected.
- Distribution of severity of earthquakes in Richter scale units, for the site selected.
- Distribution of lateral forces in g's and number of cycles as related to severity of earthquakes.
- Distribution of lateral stresses as related to distribution of lateral forces.

Typical distributions will be presented later.

The effect of a water slug is that of a single-event impact load, and will be treated under that failure mechanism.

PARAMETER 2.0: INITIAL CRACK DEPTH

Note: The difference distributions of this parameter are given for all those causes of uncertainty identified for stress concentration factor in Table 3-2 Reference 6-1.

Cause 2.1: Defects in Material

	<u>Planned Values</u>	<u>Realized Values</u>	<u>Reference Line Number</u>
B.	Configuration Design Stage	D. Component Manufacture Stage	15
	14.0 Defects in ingots or manu- factured parts	12.0 Ingot defects	
		25.0 Finished parts defects	

Surface and internal defects in piping materials can be detected by ultrasonic testing. Production sensitivity to defects extending radially from inner or outer surfaces is approximately 10% of the wall thickness. Defects in planes normal or parallel to the axis can be detected depending on the direction of ultrasonic energy insertion. Circumferential defects can be detected at the same time, with higher sensitivity percentage. The presence of defects in the material has the effect of bypassing the initiation phase of crack growth in low-cycle fatigue; thus the size of such defects forms a distribution initial crack depth, a_0 , in the through-wall crack propagation equation. In Figure 6-7 is a possible distribution of defect sizes before and after ultrasonic testing. The distributions of the number and orientation of defects per unit volume or surface of metal is to be displayed later.

Figure 6-7. Conditional Initial Crack Size Uncertainty Distribution as Affected by Ultrasonic Testing

Cause 2.2: Defects in Welds

	Planned Values	Realized Values	Reference Line Number
B.	Configuration Design Stage	D.	Component Manufacture Stage
C.	Material Selection Stage	E.	Shop and Field Fabrication Stage
B9.0	Weld prep configuration	D30.0	Weld prep configuration
		E4.0	Field weld prep configuration
C2.0	Weld filler metal selection	E5.3	Filler metal correct; coating correct
C3.0	Coating and flux selection	E5.3	Filler metal correct; coating correct
		E5.1	Code qualified welders
		E5.2	Special qualified welders
		E5.5	Configuration of field welds is acceptable especially on instrumentation runs to main line
		E6.0	Accurate alignment for welding

Lack of compliance with any of the foregoing characteristics can lead to voids or cracks in welds, or to sharp rooted junctions such as when weld fusion is incomplete between two mating faces. All can be considered to form a distribution of a_0 , initial crack depth for growth. For defects having a leading edge shape other than a sharp-edged crack, calculating stress intensity factor

$$K = 2\sigma \sqrt{\frac{a}{\pi}} \quad (\text{By reference 6-2})$$

where

σ = gross stress,

a = radius of sharp-edged crack $\approx$ crack depth,

using the radius of a spherical void, for example, as " a " would lead to conservative results for cycles to failure since the profile of the void at 90 degrees to a plane chosen for " a " does not reveal a sharp crack, but one with a finite radius. This conservatism for the case of voids and lack of fusion is not reflected in the distribution shown in Figure 6-8.

Figure 6-8. Conditional Initial Crack Size Uncertainty Distribution as Affected by Radiography and Dye Penetrant Testing

The reduction in undetected defect depth for radiography and dye penetrant above appears to be much greater than given previously for ultrasonic testing for defects in material. The sensitivity level was taken as 2.5% of wall thickness for radiography, and 10% of wall thickness for ultrasonic testing. But it should be noted that ultrasonic testing will detect no-void radial cracks which would not be detected by radiography. As in the previous discussion of defect in materials the distributions of orientation and number of defects per unit length of weld are to be made later.

Cause 2.3: Inside or Outside Surface Finish

<u>Planned Values</u>		<u>Realized Values</u>	Reference Line Number	
B.	Configuration Design Stage	D.	Component Manufacture Stage	16
15.0	Surface finish of manufactured parts	35.0	Inside and outside surface finish	

Surface finish specifications are typically given in microns as the root mean square of deviations from a mean surface (Figure 6-9). Grinding will produce a 32- micron finish or better; forgings and sand castings will have 500- to 1000-micron finishes. As with voids in welds, profiles of surface imperfections will be round-bottomed rather than sharp as assumed for cracks. Nevertheless, such imperfections form a distribution for initial crack depth, a_0 ; cycles to failure so found will be conservative.

Figure 6-9. Frequency Distribution of Piping Surface Finish

PARAMETER 3.0: MATERIAL PROPERTIES: CRACK GROWTH RATE VERSUS STRESS INTENSITY FACTOR SLOPE AND INTERCEPT

Cause 3.1: Material Actually Used is Different from What was Designed

<u>Planned Values</u>		<u>Realized Values</u>	<u>Reference Line Number</u>
B.	Configuration Design Stage	D.	Component Manufacture Stage
C.	Material Selection Stage	E.	Shop and Field Fabrication Stage
B12.0	Manufactured parts identification system	D32.0	Manufactured parts identification system
		E1.0	Manufactured parts identification system
		E3.0	Correct material of parts
C1.0	Material selection		17
C Criteria 1.0	Strength at room temperature	D21.0	Yield and ultimate strengths
		/ 22.0	at room and operating temperatures
C Criteria 2.0	Strength at operating temperature		
		E5.4	Weld technique optimizes metallurgical properties
			23

The above characteristics identify ways in which material used could have different properties from those designed. In the crack growth equation for cycles to failure the material properties which appear are the slope and antilog intercept of crack growth rate versus stress intensity factor. The conditioning probability that unplanned or unsuitable material will be used for individual components is judged to be 10^{-2} without checking of drawings or material inspection at the shop or site; and 10^{-4} with drawing checking, and inspection at the shop and site for correct material. If unplanned or unsuitable material is used, the probability distributions of the above slope and intercept are required. These distributions will be proposed when the behavior of these parameters among the possible materials which could be substituted is better known.

The foregoing relates to uncertainty distributions because of substitution of wrong materials. But these are parameters for which real variability distributions are also being formed, as a result of the Task D-1 laboratory fatigue testing. Statistical analysis of the data for each specimen leads to estimates of the required mean and variance of B, the slope, and A, the antilog of the intercept, for graphs of log crack growth rate versus log stress intensity factor. Techniques will be found for combining the uncertainty and real variability distributions.

PARAMETER 4.0: STRESS/STRAIN CONCENTRATION FACTOR

Cause 4.1: Uncertainty in Component Geometry

<u>Planned Values</u>	<u>Realized Values</u>	<u>Reference Line Number</u>
B. Configuration Design Stage	F. Operating Conditions Stage	(Added)

With respect to low-cycle fatigue the following applies during crack initiation only, not during growth where a crack is present having a stress intensity factor. For strain-hardening materials, such as the principal three materials being investigated in this study (A-106B, A335 P22, and A-312 TP304), strain concentration factors effective in the plastic region are approximately equal to the calculated elastic stress concentration factors. The probability distributions shown in Figure 6-10 apply to individual components, conditional only upon the geometry of the component. Components with more complex geometries have wider uncertainty distributions for this factor.

For any particular application it must be determined whether other materials selected strain harden or strain soften. The probability distributions shown in Figure 6-11 apply to strain softening materials, again dependent only upon geometry.

Figure 6-10. Unconditional Strain Uncertainty Distribution for Strain-Hardening Materials

Figure 6-11. Unconditional Strain Uncertainty Distribution for Strain-Softening Materials

PARAMETER 5.0: PRESSURE

Cause 5.1: Operating Conditions Different from Design Conditions

<u>Planned Values</u>	<u>Realized Values</u>	<u>Reference Line Number</u>
A. Operating Requirements Stage	F. Operating Conditions Stage	3
B. Configuration Design Stage	12.0/ Magnitude and pattern of operating pressure	
A3.1 Magnitude and pattern of operating pressure	42.0	
B4.1.1 Magnitude and pattern of operating pressure		

Displayed in Figure 6-12 is the unconditional probability distribution of difference between planned and realized operating pressure judged to apply to loops. The +200 psi bias represents the difference of design pressure minus operating pressure for loops whose operating pressure is 1000 psi; the variability is in operating pressure representing the maximum pressure in the loop in an annual operating cycle shutdown to shutdown.

Figure 6-12. Unconditional Distribution of Difference Between Planned and Realized Operating Pressure

Cause 5.2: Water Hammer

<u>Planned Values</u>		<u>Realized Values</u>	<u>Reference Line Number</u>
B.	Configuration Design Stage	F. Operating Conditions Stage	7
4.9	Related equipment shock (hammer, etc.)	46.0 Shock; valve hammer, etc.	

The distribution in Figure 6-13 deals with water hammer, which is characterized by a sudden overpressure, for example, because of a quick valve closing. Hammer can occur because of failure of the design to recognize conditions for its occurrence, or because of malfunction of components in service. The conditioning probability of such an event occurring in a loop is judged to be 10^{-3} . If hammer occurs, the distribution of pressure ratio is judged to apply to loops, based in part on p. 3-9, reference 6-3.

Figure 6-13. Conditional Pressure Uncertainty Distribution for Cause 5.2: Water Hammer

PARAMETER 6.0: TEMPERATURE

Cause 6.1: Difference Between Design and Actual Temperatures; and Local Thermal Effects

<u>Planned Values</u>		<u>Realized Values</u>	<u>Reference Line Number</u>
A.	Operating Requirements Stage		5
4.1	Operating temperature - magnitude and pattern		
B.	Configuration Design Stage	F. Operating Conditions Stage	
4.2.1	Temperature - magnitude and pattern	43.1 Temperature - magnitude and pattern	

Operating temperature for all loops has been tabulated. In all cases the highest temperatures which the loop is expected to see have been shown; thus the calculated thermal expansion stresses are conservative. The possible distribution of range of temperature from shutdown to operating is interesting only when the range of thermal stress is high. This is true of only a limited number of components, and in special cases where there is mixing of liquid streams of widely different temperatures, inducing high local thermal stresses. Both of these cases will be investigated where they arise.

PARAMETER 7.0: NUMBER OF PRESSURE CYCLES

PARAMETER 8.0: NUMBER OF TEMPERATURE CYCLES

Causes 7.1 and 8.1: Actual Cycles Different from Planned Cycles

<u>Planned Values</u>		<u>Realized Values</u>	<u>Reference Line Number</u>
A.	Operating Requirements Stage		
3.2	Operating pressure - No. of cycles		4
3.2	Operating temperature - No. of cycles		4
8.0	Service Life		
B.	Configuration Design Stage	F.	Operating Conditions Stage
4.1.2	Pressure - No. of cycles	42.0	Pressure - No. of cycles
4.2.2	Temperature - No. of cycles	43.2	Temperature - No. of cycles

The number of full thermal cycles for primary steam lines in a 40-year reactor service period is estimated at 360: 160 normal startups and 200 scrams. This figure is conservative, but will be adopted at present for this study as a constant for critical cycles, rather than a mean surrounded by an uncertainty range as has been done for the parameters governing failure, because it appears that the uncertainty range on operating cycles must be small compared to the uncertainty range on cycles to failure.

Lifetime cycles for loops other than the primary steam lines will be treated in a similar way. Pressure cycles have less effect, but will also be taken as 360 in 40 years for the primary steam lines, and corresponding values for other loops.

PARAMETER 9.0: WALL THICKNESS

Cause 9.1: Wall Thickness Used Different from That Designed

<u>Planned Values</u>		<u>Realized Values</u>	<u>Reference Line Number</u>
B.	Configuration Design Stage	D.	Component Manufacture Stage
5.0	Wall Thickness	27.0	Wall Thickness

Entire components might be used which are of less than design wall thickness. Since all such components are subjected to hydrostatic test at a minimum of 1-1/2 times the working pressure (on ≤ 14 -inch nominal diameter pipe, see ASTM A-106, A-312, and A-335), and this pressure is limited to the lower of 2/3 yield strength or 1/3 ultimate strength (S_m in ASME Section III Code), a minimum wall thickness can be found which would be expected to fail hydrostatic test. Where S_m is 1/3 ultimate strength, test pressure will be $1\text{-}1/2 \times 1/3 = 1/2$ ultimate strength for correct wall thickness pipe. Thus, where wall thickness is 1/2 design thickness, ultimate strength should be reached in the hydrostatic test, and the pipe should fail. If the allowable stress, S_m , caused by pressure is limited by 2/3 yield strength rather than 1/3 ultimate, test pressure will induce stresses which will be less than 1/2 ultimate strength, indicating wall thickness may be less than 1/2 design - say 1/3 design - before stress equal to ultimate strength would be reached in the hydrostatic test, and the pipe would fail.

The probability of a component being supplied to the site with wall thickness between 1/3 and 1 times design is 5×10^{-3} . With site inspection, where the probability that a wrong wall thickness component will pass inspection is 10^{-2} , the probability of supplying such a component to the place of fabrication is 5×10^{-5} . If such a component is supplied, the distribution of its wall thickness is as shown in Figure 6-14. The probability of such a component's being welded into place, because of mismatch of its inside diameters, is shown in Figure 6-15.

Figure 6-14. Conditional Uncertainty Distribution on Wall Thickness Less Than Specification

Figure 6-15. Conditional Probability of Use of Piping Having Wall Thickness Less Than Specification (For Greater Than Specification, Invert Bottom Scale)

For example, the probability of using a pipe having between 1/3 and 1/2 design wall thickness, with site inspection, is thus $(5 \times 10^{-5})(0.15)(0.1) = 7.5 \times 10^{-7}$, where 0.15 is approximately the area between 0.33 and 0.5 wall thickness ratio in Figure 6-14, and 0.1 is approximately the average probability for the same wall thickness ratio range.

Pipe of less than 1/3 design wall thickness could be supplied to the site, having never been hydrostatic tested, or which passed hydrostatic test with a small margin. The probability of this happening is 10^{-3} ; the probability of such a part passing site inspection if presented is 10^{-2} ; the probability of such a part being welded if presented is 10^{-2} . Thus the probability of pipe of less than 1/3 design wall thickness actually being used is 10^{-5} without site inspection, and 10^{-7} with inspection.

Components can have regions of thin wall as well as the entire wall being thin. The probability of a component being supplied and used with a thin region is 10^{-3} ; any site inspection practice or observation by the welder is not detailed enough to detect such defects. If such a component is used, the distribution of its wall thickness in the thin region is the same as shown in Figure 6-14. It is judged that 10^{-2} of surface is in thin regions, i.e., 10^{-2} of cross-sections are thin.

Components with greater than design wall thickness can also be supplied and used in error. The probabilities of supply and use are the same as for components between 1/3 and 1 times design wall thickness. If such a component is used, its wall thickness is distributed as shown in Figure 6-16.

Figure 6-16. Conditional Uncertainty Distribution on Wall Thickness Greater Than Specification

PARAMETER 10.0: DIAMETER

Cause 10.1: Diameter Used Different from That Designed

<u>Planned Values</u>		<u>Realized Values</u>		<u>Reference Line Number</u>
B.	Configuration Design Stage	D.	Component Manufacture Stage	(Added)
6.0	Inner Diameter	26.0	Inner Diameter	

An individual component could be supplied to the site and used which is of wrong diameter with probability 10^{-3} without site inspection, and 10^{-5} with site inspection. If such a wrong component were used, the distribution of its diameter is shown in Figure 6-17. Since wall thicknesses exceed 10% of the diameter only rarely, two components with diameters differing by more than 20% will not assemble. The probability that an entire line of the wrong diameter would be installed, because wrong end connections were also available, is 10^{-4} without site fabrication inspection, and 10^{-6} with site fabrication inspection.

Figure 6-17. Conditional Diameter Uncertainty Distribution

The foregoing covers uncertainties in constant diameter; uncertainties in ovality are discussed in subsection 6.4, cause 1.13.

6.3 UNCERTAINTY DISTRIBUTIONS ON STRESS DUE TO DIMENSIONAL VARIATIONS*

The following uncertainty distributions are supplemental to those which appeared under the heading of **PARAMETER 1.0: STRESS** in subsection 6.2. The following distributions on stress are attributable to each of the causes shown, in that each not only appears directly as a parameter in the equation for cycles to completion of a through-wall crack, but also has a separate effect on stress. In each case subsection 6.2 gives conditioning probabilities for the cause shown, and the distribution of stress ratios resulting when the cause does occur is given by each of the following diagrams. As in the above referenced section, the stipulation applies that: "The...width and shape of the distributions reflect preliminary judgment and experience."

PARAMETER 1.0: STRESS (CONTINUED FROM SUBSECTION 6.2)

Cause 1.9: Wall Thickness less than Specification (throughout) (Reference: Cause 9.1, subsection 6.2)

* Probability density function.

Probability density function.

Figure 6-18. Conditional Stress Uncertainty Distribution Resulting from Figure 6-14 (Throughout)

Cause 1.10: Wall Thickness less than Specification (in isolated regions) (Reference: Cause 9.1, subsection 6.2)

Figure 6-19. Conditional Stress Uncertainty Distribution Resulting from Figure 6-14 (Isolated Regions)

* With the collaboration of S. W. Tagart.

Cause 1.11: Wall Thickness greater than Specification (Reference: Cause 9.1, subsection 6.2)

Figure 6-20. Conditional Stress Uncertainty Distribution Resulting from Figure 6-16

Cause 1.12: Diameter outside Specification, but constant (Reference: Cause 10.1, subsection 6.2)

Figure 6-21. Conditional Stress Uncertainty Distribution Resulting from Figure 6-17

Cause 1.13: Ovality (Diameter 90° Apart, Above and Below Specification) (J. D. Gilman)

Estimation of the effect of an initial ovality or out-of-roundness was deferred. A judgment-based estimate of the stress ratio uncertainty caused by this effect is shown in Figure 6-22, having conditioning probability at 1.0, and effective only in growing a crack with an axial orientation.

Figure 6-22. Conditional Stress Uncertainty Distribution for Cause 1.13: Ovality

6.4 CAUSES OF UNCERTAINTY RESTRICTED TO EXPANSION STRESS (J.D. Gilman)

Some of the causes of uncertainty on stress given in subsection 6.2 relate only to thermal expansion stresses, S_e , and have no bearing on the pressure stresses, S_p . Specifically, these causes are:

- 1.1 Arrangement error
- 1.3 Calculation error
- 1.4 Loss of flexibility
- 1.5 Loop redesign error
- 1.6 Spring support and welded anchor error
- 6.1 Temperature different from design temperature.

6.5 COMBINING UNCERTAINTY DISTRIBUTIONS ON STRESS*

The nature of most of the uncertainty distributions by cause is that they are conditional, that is, each cause is judged to occur among piping components with some low probability; if it does occur, the uncertainty distribution reflects the results. It is desirable to combine the distributions by cause into a single distribution for each parameter, so that distribution can represent the parameter in the equation for the variance of cycles to completion. The pattern for doing this will now be presented.

To discuss for example, the distributions for stress, note that they are in terms of stress ratio: actual stress/calculated stress. Thus, when there is perfect prediction, the stress ratio is 1.0; we are particularly concerned about stress ratios exceeding 1.0. The effect of all the causes will be the product of the stress ratios of all the n causes; thus, where r_c is the combined stress ratio of all the n causes,

$$r_c = \prod_{i=1}^n r_i$$

But all the causes are not always effective; allowance must be made for the conditioning probabilities of each cause. The analytical method for doing this is described in the following subsection. Details on the application to the distribution of the product of stress uncertainty distributions appear in subsection 3.3 of both References 6-4 and 6-5. The application included the first four causes of uncertainty in stress listed in Table 6-1. These are listed in decreasing order of conditioning probability for the "without checking or inspections" case, and therefore are likely to also be in decreasing order of contribution to a combined distribution. The first four causes account for nearly all of the combined distribution. The individual conditional distributions by cause are shown in Figure 6-23; the resulting combined probability density function is designated level 4 in Figure 6-24, with its corresponding cumulative distribution function shown in Figure 6-25.

6.6 AN APPROXIMATE METHOD FOR FORMING THE DISTRIBUTION OF THE PRODUCT OF RANDOM VARIABLES*

The process used to form product distributions is briefly described below.

The distribution of the product of only two distributions such as $r_1 r_2$, here generalized to xy , is formed at one time.

1. For input distributions in x and y find the values $x_1, x_2, \dots, x_n$, and $y_1, y_2, \dots, y_m$, corresponding to selected cumulative probability values $F(x_1), F(x_2), \dots, F(x_n)$, and $F(y_1), f(y_2), \dots, F(y_m)$.

If input and output probability intervals are not the same, redivide each input distribution to match the output probability points, assuming the input distributions are uniformly distributed between the input probability points given. Each probability increment need not have the same value; for example, tails may be more finely divided.

*With the collaboration of Dr. R. I. Post.

Table 6-1

CAUSES OF UNCERTAINTY IN STRESS RANKED BY CONDITIONING PROBABILITY, WITHOUT INSPECTION

Rank by Conditioning Probability Without Inspection	* Cause No.	Cause Name	Minimum SR** Value	Maximum SR** Value	Conditioning Probabilities		Basis	Remarks
					Without Inspections	With Inspections		
1	1.2	Calculation Method Error	0.5	1.1	1.0	1.0	Components	Take cumulation probability due to problem size as 1.0 (large problem), and diagram for $n^2=1000$.
2	1.3	Calculation Error	0.5	7.5	10^{-1}	10^{-3}	Components	
3	1.1	Arrangement Error	0.5	10.0	10^{-2}	10^{-5}	Loops	
4	1.4	Loss of Flexibility						
5		Pre-Start Path	1.0	10.0	10^{-2}	10^{-4}	Loops	
5	1.9	Wall Thickness less than Specification (throughout)	1.0	3.3	5×10^{-3}	5×10^{-5}	Components	Apply also Figure 6-15: use the reciprocal of the bottom scale for stress ratio.
(The case of wall thickness less than 1/3 design is omitted because of the small conditioning probabilities judged to apply: 10^{-5} and 10^{-7} without and with inspection, respectively.)								
6	1.11	Wall Thickness greater than Specification (throughout)	0.33	1.0	5×10^{-3}	5×10^{-5}	Components	Apply also Figure 6-15: use the bottom scale as is for stress ratio.
	1.5	Loop Redesign Error						Covered by cause 1.1 Arrangement Error.
		Pre-Start Path						
7		Post-Start Path	0.5	10.0	10^{-3}	10^{-6}	Loops	Figure 6-15 does not apply.
8	1.10	Wall Thickness less than Specification (isolated regions)	1.0	2.0	10^{-3}	10^{-3}	Components	
9	1.12	Diameter outside Specification	0.85	1.15	10^{-3}	10^{-5}	Components	
	1.4	Loss of Flexibility						No reduction is taken in conditioning probability for inspection.
10		Post-Start Path	1.0	10.0	10^{-5}	10^{-5}	Loops	

* See subsection 6.2

** stress ratio

Figure 6-23. Individual Uncertainty Distributions, $f(r_i)$

Figure 6-24. Probability Density Functions of Combined Uncertainty Distributions

Figure 6-25. Cumulative Distribution Functions of Combined Uncertainty Distributions

2. To form the $z=xy$ distribution, find the four products given by all combinations of both end points of the first cell of the x and y distributions; for example, x_1y_1 , x_1y_2 , x_2y_1 , and x_2y_2 . Select the maximum and minimum products, and assign the probability found as the product of the two cell probabilities uniformly to the range on the z scale between these two products. If x_1y_1 and x_2y_2 are minimum and maximum, let $z_1 = x_1y_1$ and $z_2 = x_2y_2$, and distribute the product of the x and y cell probabilities, $[F(x_2) - F(x_1)][F(y_2) - F(y_1)]$, uniformly between z_1 and z_2 . If there are w output probability points required, there will be w^2 such uniform distributions on the z scale, whose areas will sum to 1.0.
3. Form the discrete-appearing z density distribution by summing the contributions of all the w^2 uniform distributions.
4. Find the z values corresponding to the w cumulative probabilities chosen to describe the output distribution. Referring to Equation (4) of Reference 6-5, for example, these z values must be the same for the r_1r_2 and r_1 distributions, and will correspond for the r_c distribution. Thus, with $w F(z)$ values known for each contributing distribution, applying the weights p_2 and $1-p_2$ and summing at each z value gives $F(r_c)$ for each z value. For illustration, such summed values for $F(r_c)$ at level 4 are tabulated in column 5 of Table 3-3, Reference 6-5.

This procedure introduces two elements of approximation: one is in step 1 where intervals between cumulative probabilities given to describe the input distributions are assumed to be uniform. Smaller intervals reduce the error introduced by this approximation. The second approximation is in step 2 where the probability product of each cell pair is uniformly distributed between maximum and minimum end-point products for the cells. This simplification—in effect saying that the product of two uniform distributions is uniform—introduces only a small error, because there are w^2 such uniform distributions to be merged to form the z distribution, which is then described by only w probability points. Note that the probability contribution of each such uniform distribution is known exactly; only its shape and location are approximated.

6.7 REFERENCES

- 6-1 *Reactor Primary Coolant System Rupture Study, Quarterly Progress Report No. 5*, April-June 1966, (GEAP-5192).
- 6-2 Paris, P. C., and Sih, G. C., "Stress Analysis of Cracks," Equation (37); in "Fracture Toughness Testing and Applications," ASTM STP No. 381, 1965.
- 6-3 Kilsby, E. R. Jr., *A Survey of Water Reactor Primary System Conditions Pertinent to the Study of Pipe Rupture*, January 1964, (GEAP-4445).
- 6-4 *Reactor Primary Coolant System Rupture Study, Quarterly Progress Report No. 7*, October-December 1966, (GEAP-5427).
- 6-5 *Reactor Primary Coolant System Rupture Study, Quarterly Progress Report No. 10*, July-September 1967, (GEAP-5554).

7. SELECTION OF FIRST PIPING STUDY SECTION FOR CALCULATION OF PROBABILITY OF LEAKAGE. NOMINAL VALUES OF VARIABLES

7.1 INTRODUCTION

The probability estimating method described in this report is applied to specific locations in the piping system, called piping study sections, so that design or mean values may be assigned to the variables in a damage prediction equation, and so that the applicable uncertainty distributions may be selected. The piping study section having highest design stress in a reference reactor primary piping system was used for method development. The piping study surface, over which the value of each variable was assumed to be constant, was chosen to have area 1 inch in length by 1/4 circumference of 6-inch diameter pipe. Values for all the variables in the cycles-to-leak equation are given in this section, together with references to their uncertainty distributions.

7.2 PIPING STUDY SECTION AND PIPING STUDY SURFACE

Use of the cycles-to-leak equation [Equation (5-11)] requires matching the stress with an initial defect of some depth a_0 oriented 90 degrees to the stress, in a particular piping location which will be called a "piping study section." This is a length of pipe having a unique nominal value for each variable. Typically, it is the length to which is assigned the design stress found at one stress analysis node or "joint," usually less than the length of one component. A change in nominal value of some other variable might also define the extent of the piping study section, however, such as a change in mean crack occurrence rate. The distribution of initial cracks is characterized by a size frequency distribution, and by a mean rate of occurrence per square inch with the assumption that cracks are Poisson distributed on a per square inch basis. We become interested in the distribution of the maximum depth crack in the study surface in a region of otherwise constant values of the variables, on the basis that the deepest such crack will be the first to leak. This distribution depends on the area of surface chosen. Thus, it is necessary to choose a "piping study surface," a portion of the study section. For the case of axial stress due to system bending from thermal expansion, a portion of the surface is in tension while the portion 180 degrees away is in compression—in the absence of other important stresses. Thus, for axial tensile stress due to expansion, which will operate to grow a circumferential crack, it is reasonable to restrict attention to 1/4 circumference. It is shown in subsection 7.4 that with respect to analytical considerations a piping study surface may be arbitrary in size; it must merely be within a study section as defined above. To make the first estimate of the probability of leakage, a piping study surface of 1/4 circumference by 1 inch in axial length was chosen, as illustrated in Figure 7-1.

7.3 NOMINAL VALUES AND UNCERTAINTY DISTRIBUTIONS OF FIRST STUDY SECTION

A particular piping study section was chosen at which to calculate the probability of leakage in 10 to 40 years from low-cycle fatigue, starting with an initial defect. The material is Type-304 stainless steel, operating temperature is 150°F, so the room temperature A value will be used. It will be assumed that the cyclic loading is sufficiently close to zero-to-tension to warrant using the A and B estimates from testing by Tasks D-1 and D-2. This cross section was chosen because it has the highest design stress of any in the primary system of the reference plant design. See Reference 7-1. The probability to be calculated is "E" in the Piping Probability Diagram, Figure 4-1. Equation (4-5) will be used. The governing case is for axial stress, circumferential crack growth, see σ_{PM} entry in Table 7-1. The design values and uncertainty distributions for this cross section are listed in Table 7-1. However, as a result of the sensitivity testing displayed in Figure 5-9, only the following uncertainty distributions will be used for the initial evaluation of probability "E":

$$(\log A)^*, B^*, \Delta P^* = 200 \text{ psi in } S_p^*, S_E^* \text{ and } a_0/tt^*.$$

Values of all other * terms will be fixed at 1.0.

7.4 DEMONSTRATION THAT PROBABILITY OF LEAKAGE IN A PIPING LOOP IS INDEPENDENT OF THE AREA CHOSEN FOR EACH INDIVIDUAL PIPING STUDY SURFACE

The conclusion from this demonstration, that one reaches the same probability of leakage in a loop comprising a certain total surface area, regardless of the size of individual surface areas chosen, facilitates extending the probability estimate at one study surface to the entire loop per subsection 9.13. Refer to subsection 3.5, Reference 7-2.

Table 7-1
DESIGN VALUES AND UNCERTAINTY DISTRIBUTIONS

Design Values	Uncertainty Distributions
$\text{Log A} = -25.260$ $\text{Log A} = 5.495 \times 10^{-26}$	$(\text{Log C})^*$ is normally distributed; $\text{Var}(\log C) = \text{Var}(\log A) = 0.138$
$\text{Log C} = -3.962$ $\text{Log C} = 1.091 \times 10^{-4}$	$\text{at Log } \Delta K = 0$ $\text{at Log } \Delta K = 4.786 (\Delta K = 6.109 \times 10^4)$
$B = 4.45$	B^* is normally distributed; $\text{Var } B = 1.016$
$t = 0.432 \text{ in.}$	Values from Reference 5-10
$d = 6.624 \text{ in. (outside)}$	d^* : Per cause 10.1, subsection 6.2
$\sigma_{PB} = 0$	See subsection 5.3.7
$\sigma_{SB} = 0$	See subsection 5.3.6
$\sigma_{SM} = 0$	See subsection 5.3.6
$\sigma_{PM} = S_E S_E^* + (1/2) S_p^*$	See Equation (5-30) Circumferential crack case governs.
where	
$S_E = \text{Expansion stress}$ $= 27,450 \text{ psi}$	
$S_p^* = \text{Hoop stress from pressure}$ $= \frac{P^*(P - \Delta P^*) d d^*}{2 t t^*}$	S_E^* : Curve 4, Figure 6-20. This curve complies with subsection 6.4. The comparable "with inspection" case may also be needed to test sensitivity. The uncertainty distribution on stress effect of ovality also in this section is judged to apply to all components (conditioning probability = 1.0), but only when stresses favor the axial crack case.
$P = 1200 \text{ psi}$	P^* : Per cause 5.2, subsection 6.2 ΔP^* : Per cause 5.1, subsection 6.2
	$\frac{a_o}{t t^*}$: Use of the curve "without testing" for cause 2.1, subsection 6.2.
	Assume the frequency of defects in welds is Poisson distributed with parameter 1 per 10 sq. in. Only 1/4 of circumference is assumed subject to crack growth, per subsection 7.2. Credit is not taken for radiography because of possible crack shapes not detected; see cause 2.1 discussion subsection 6.2.
$N \text{ for } 10 \text{ years} = 90 \text{ operating cycles}$ $N \text{ for } 20 \text{ years} = 180 \text{ operating cycles}$ $N \text{ for } 30 \text{ years} = 270 \text{ operating cycles}$ $N \text{ for } 40 \text{ years} = 360 \text{ operating cycles}$	

Figure 7-1. Piping Steady Surface, 1" x 1/4 Circumference

7.5 REFERENCES

- 7-1. *Reactor Primary Coolant System Rupture Study, Quarterly Progress Report No. 4*, January-March 1966, Joint 27, (GEAP-5147), Figure 3-2.
- 7-2. *Reactor Primary Coolant System Rupture Study, Quarterly Progress Report No. 20*, January-March 1970, (GEAP-11069).

8. DISPERSION OF CYCLES TO LEAK: THE VARIANCE OF LOG n_t

8.1 INTRODUCTION

In Equations (5-27), (5-28), and (5-29), we transformed the equation for the number of cycles, n_t , required for a crack to grow through a pipe wall to a form suitable for rapid numerical evaluation. This was preparatory to conducting a Monte Carlo simulation of the distribution of n_t under the hypothesized distributions of the physical and equation parameters.

In this subsection, the input distributions come under scrutiny. It is in fact possible—with the aid of the same transformation given in the Equation (5-25)—to estimate what spread of values of n_t to expect under given mean reactor conditions. Not only will a measure of dispersion, $\text{var } \ln n_t$, be given to good approximation, but the contribution of each of the causes of uncertainty to the total dispersion will be estimated. Thus, by eliminating the negligible contributors, it should be possible to characterize the lower tail of the n_t distribution with greater accuracy for a given outlay of resources in the Monte Carlo simulation.

The technique to be used for finding the variance of the natural log n_t function is often called "propagation of errors." To use it, the partial derivative of the function with respect to each variable is required, together with the variance of each variable. These terms are derived from the corresponding uncertainty distributions in a carefully detailed description of this technique given in the references below. But first, the equation for $\ln n_t$ will be developed.

In the basic crack growth rate equation, (5-13), the symbol A has been used both as a scaling constant and as a random parameter. Subsequently, it was found (see subsection 5.3.8) that it is convenient to use separate symbols and write (5-13) as

$$\frac{da}{dn} = C \cdot \left(\frac{\Delta K}{\bar{\Delta K}} \right)^B \quad (8-1)$$

where $\bar{\Delta K}$ is the grand mean of $\log \Delta K$'s from analysis of testing under Tasks D-1 and D-2. This should have the effect of minimizing the correlation between B and C so that the experimental determination of B and C can best be used for prediction. Equation (8-1) may be integrated over $a/t = x$ for the number of cycles to grow through:

$$n_t = \frac{t}{C} \cdot \int_{\frac{a_0}{t}}^1 \left(\frac{\bar{\Delta K}}{\Delta K} \right)^B dx, \quad (8-2)$$

which may be expressed:

$$n_t = \frac{(\bar{\Delta K})^B}{C \cdot t^{(B/2) - 1}} \cdot \int_{\frac{a_0}{t}}^1 \left[x^{-1/2} F(S_B, S_M; x) \right]^B dx, \quad (8-3)$$

with F defined as following Equation (5-28):

$$\begin{aligned} F(S_B, S_M; x) = & (1-x)^{3/2} \left(\left\{ [S_B(x) + 3x^{3/2} S_M(x)] \cdot \sqrt{\frac{f(x)}{x}} \right. \right. \\ & \left. \left. + (1-x) \cdot S_M(x) \cdot \sqrt{\frac{1-x}{x}} \cdot g(x) \right\} \cdot h(x) \right)^{-1} \end{aligned} \quad (8-4)$$

where it was found to be well behaved for any combinations of stresses, σ_{PB} and σ_{SB} , and σ_{PM} and σ_{SM} . Notation is in Table 8-1.

Table 8-1
NOTATION FOR CYCLES-TO-LEAK EQUATION

a	Crack depth
n	Number of cycles
n_t	Number of cycles to grow through the wall
A	Scaling constant and random parameter
B	Equation parameter
ΔK	Cyclic variation of stress intensity
t	Pipe wall thickness
$S_B(x) = \sigma_{PB} + (1 - x) \cdot \sigma_{SB}$	Bending stress
$S_M(x) = \sigma_{PM} + (1 - x) \cdot \sigma_{SM}$	Membrane stress
σ_{PB}	Primary bending stress
σ_{PM}	Primary membrane stress
σ_{SB}	Secondary bending stress
σ_{SM}	Secondary membrane stress
$f(x) = \pi/6 \cdot (1 - e^{-6x})$	Bending coefficient (Bueckner)*
$g(x) = 2 \tan(\pi x/2)$	Membrane coefficient
$h(x) = 1 + 0.0325 [1 + \cos(\pi x)]^2$	Front free surface correction factor**
C	Random parameter
$\bar{\Delta K}$	Grand mean of log ΔK values in experimental runs
$S_p = 1/2 \cdot pd/t$	Hoop pressure stress
S_E	Calculated expansion stress
p	Pressure
d	Pipe diameter
a_0	Initial crack depth

*See subsection 5.3.4.2.

**Reference 8-2.

In subsection 3.4.4, Reference 8-1, we give some justification for using—with the first expression alone on the right-hand side retained—Equation (14) of Reference 8-1:

$$\text{var } \phi(X_1, \dots, X_I) \approx \sum_{i=1}^I \left[\frac{\partial \phi}{\partial x_i} (EX_1, \dots, EX_I) \right]^2 \cdot \text{var } X_i \quad (8-5)$$

for $\phi = \ln n_t$ with the X_i identified as the variables, B , $\ln C$, t , etc. This approximate equation, (8-5), is known as the propagation of errors approximation since uncertainties in the X_i reveal themselves as contributors to the uncertainty in $\phi(X_1, \dots, X_I)$. It is widely used to compare the relative importance of the uncertainties in the X_i so as, for example, to reduce the uncertainty in $\phi(X_1, \dots, X_I)$ most efficiently through possible reductions in the uncertainties of the X_i .

From Equation (8-3), we obtain

$$\ln n_t = (\ln \tilde{\Delta K}) \cdot B - \ln C - \left(\frac{B}{2} - 1\right) \cdot \ln t + \ln \int_{\frac{a_0}{t}}^1 \left[x^{-1/2} F(S_B, S_M; x) \right]^B dx, \quad (8-6)$$

for which the approximate variance will be found by Equation (8-5).

We have broken down the causes of uncertainty in the n_t equation so as to identify what independent influences are at work. If, subsequently, it appears that any of these uncertainties do not arise independently, then an attempt will be made to evaluate the covariance between these for use in Equation (13), Reference 8-1, or rearrange the n_t equation to obviate the difficulty. However, we feel that there are intuitive grounds for believing the uncertainties have minimal interdependence.

This variance solution is based on the assumption of a constant $\tilde{\Delta K}$, the standard assumption regarding the mean of the independent variable when parameters of a linear equation are estimated by regression from experimental work. However, when the concept of the crack-specific $\tilde{\Delta K}$ (mean log ΔK) was introduced in subsection 5.3.8, $\tilde{\Delta K}$ and B were no longer independent. This change is reflected in subsection 8.9 below. Precautions in the use of the variance results which will appear in Tables 8-2 and 8-3 are given therein.

8.2 STATISTICAL CONCEPTS (G. T. Vernon)

Refer to subsection 3.4.2, Reference 8-1.

8.3 THE APPROXIMATE VARIANCE OF A FUNCTION OF RANDOM VARIABLES BY PROPAGATION OF ERRORS (G. T. Vernon)

Refer to subsection 3.4.3, Reference 8-1.

8.4 APPLICATION OF VARIANCE APPROXIMATION TO MEASURING THE DISPERSION OF CYCLES TO LEAK (G. T. Vernon)

Refer to subsection 3.4.4, Reference 8-1.

8.5 VARIANCE OF THE RANDOM VARIABLES IN THE CYCLES TO LEAK EQUATION (G. T. Vernon)

Refer to subsection 3.4.5, Reference 8-1.

8.5.1 Wall Thickness, t (G. T. Vernon)

Refer to subsection 3.4.5.1, Reference 8-1.

8.5.2 Initial Crack Depth Ratio, a_o/t , Without Ultrasonic Testing (G. T. Vernon)

Refer to subsection 3.4.5.2, Reference 8-1.

8.5.3 Diameter, d (G. T. Vernon)

Refer to subsection 3.4.5.3, Reference 8-1.

8.5.4 Pressure, P (G. T. Vernon)

Refer to subsection 3.4.5.4, Reference 8-1.

8.5.5 Expansion Stress, S_E , Other than from P , d and t (G. T. Vernon)

Refer to subsection 3.4.5.5, Reference 8-1.

8.5.6 Other Sources of Uncertainty in the n_t Equation (G. T. Vernon)

Refer to subsection 3.4.5.6, Reference 8-1.

8.5.7 Comment on Independence of Uncertainties (G. T. Vernon)

Refer to subsection 3.4.5.7, Reference 8-1.

8.6 COEFFICIENTS ON VARIANCES FOR APPROXIMATION OF VAR $\ln n_t$ (G. T. Vernon)

Refer to subsection 3.4.6, Reference 8-1.

8.6.1 Evaluation of Partial Derivatives for Var $\ln n_t$ Approximation (G. T. Vernon)

Refer to subsection 3.4.6.1, Reference 8-1.

8.7 APPLICATION OF RESULTS TO A PARTICULAR PIPING STUDY SECTION:

DISPERSION OF n_t WITHOUT ULTRASONIC TESTING (G. T. Vernon)

In Equation (8-5) we found:

$$\text{var } \ln n_t \approx \sum_{i=1}^I \left[\frac{\partial \ln n_t}{\partial x_i} \right]_{(EX_1, EX_2, \dots, EX_I)}^2 \cdot \text{var } X_i \quad (8-7)$$

The coefficients, $\frac{\partial \ln n_t}{\partial x_i}$, are evaluated at $(EX_1, \dots, EX_I)$. To compute the expected values,

EX_i , we need to know the plan values, since EX_i is the product of one of these with EU, the respective bias in uncertainty. Similarly, in subsection 3.4.5 of Reference 8-1 we evaluated $\text{var } X_i$ for each i as a multiple of the square of the respective plan value.

The coefficients in the approximate equation (8-7), have been evaluated for the chosen piping study section. These coefficients—together with distribution parameters of the uncertainty coefficients under their cause code numbers (from subsection 6.2), design values, plan values, expected values, variances and contributions to the linear part of $\text{var } \ln n_t$ [given in the approximate equation, (8-7)]—are listed in Table 8-2.

It should be noted that each term in the approximate equation, (8-7), may be found either as the product of

$$\left(\frac{\partial \ln n_t}{\partial x_i} \right)^2 \text{ with } \text{var } X_i \text{ or as the product of } \left[(\text{plan})^2 \cdot \left(\frac{\partial \ln n_t}{\partial x_i} \right)^2 \right]$$

Table 8-2
CONTRIBUTION TO DISPERSION OF n_t
AT THE CHOSEN STUDY SECTION
FOR a_0/t WITHOUT ULTRASONIC TESTING

	Symbol	Cause Code No.	Design	Δ Design	Plan	$E \ell n U_i \times 10^{-3}$	Expected Value	$\text{var } \ell n U_i \times 10^{-3}$	$\text{var } U \times 10^{-3}$	Variance	$\frac{\partial \ell n n_t}{\partial x_i}$	$\left(\frac{\partial \ell n n_t}{\partial x_i}\right)^2$	$(\text{plan})^2 \cdot \left(\frac{\partial \ell n n_t}{\partial x_i}\right)^2$	Contribution to $\text{var } \ell n n_t$	Contribution to $\text{var } (n_t)$ as percentage
Equation parameter	B	--	--	--	--	--	4.45	--	--	1.016	1.808	3.27		3.3231	17.49
Equation parameter	log C	--	--	--	--	--	-3.96	--	--	0.138	-2.303	5.30		0.7318	3.85
Thickness, in.	$\left. \begin{matrix} U_1 \\ U_2 \end{matrix} \right\}$	$\left\{ \begin{matrix} 1.9 \\ 1.11 \\ 9.1 \end{matrix} \right\}$	$\left\{ \right\}$			$\begin{matrix} -0.158 \\ 0 \end{matrix}$		$\begin{matrix} 0.443 \\ 2.012 \end{matrix}$							
	$\begin{matrix} U \\ t \end{matrix}$	$\begin{matrix} \\ -- \end{matrix}$	$\begin{matrix} 0.432 \\ -- \end{matrix}$	$\begin{matrix} -- \\ -- \end{matrix}$	$\begin{matrix} 0.432 \\ -- \end{matrix}$	$\begin{matrix} -0.158 \\ -- \end{matrix}$	$\begin{matrix} 0.432 \\ -- \end{matrix}$	$\begin{matrix} 2.455 \\ -- \end{matrix}$	$\begin{matrix} 2.454 \\ -- \end{matrix}$	$\begin{matrix} 0.46E-3 \\ -- \end{matrix}$	$\begin{matrix} 8.324 \\ -- \end{matrix}$	$\begin{matrix} 69.29 \\ -- \end{matrix}$	$\begin{matrix} 12.9 \\ -- \end{matrix}$	$\begin{matrix} 0.0317 \\ -- \end{matrix}$	$\begin{matrix} 0.17 \\ -- \end{matrix}$
Initial crack depth	$\frac{a_0}{t}$	2.2	--	--	--	--	0.0364	--	--	0.007	-43.992	1935.30		13.5595	71.39
Diameter, in.	$\left. \begin{matrix} U_1 \\ U_2 \\ U_3 \end{matrix} \right\}$	$\left\{ \begin{matrix} 1.12 \\ 10.1 \end{matrix} \right\}$	$\left\{ \begin{matrix} -- \\ -- \\ -- \end{matrix} \right\}$	$\begin{matrix} -- \\ -- \\ -- \end{matrix}$	$\begin{matrix} 1 \\ 1 \\ 1 \end{matrix}$	$\begin{matrix} -0.0003 \\ -0.0033 \\ 0 \end{matrix}$		$\begin{matrix} 0.000 \\ 0.003 \\ 0.025 \end{matrix}$	$\begin{matrix} 0.000 \\ 0.003 \\ 0.025 \end{matrix}$		$\begin{matrix} -4.821 \\ 3.640 \\ 7.870 \end{matrix}$	$\begin{matrix} 23.24 \\ 13.25 \\ 61.93 \end{matrix}$	$\begin{matrix} 23.2 \\ 13.2 \\ 61.9 \end{matrix}$	$\begin{matrix} 0.0000 \\ 0.0000 \\ 0.0015 \end{matrix}$	$\begin{matrix} 0.00 \\ 0.00 \\ 0.01 \end{matrix}$
	$\begin{matrix} U \\ d \end{matrix}$	$\begin{matrix} \\ -- \end{matrix}$	$\begin{matrix} 6.624 \\ -- \end{matrix}$	$\begin{matrix} -- \\ -- \end{matrix}$	$\begin{matrix} 6.624 \\ -- \end{matrix}$	$\begin{matrix} -0.004 \\ -- \end{matrix}$	$\begin{matrix} 6.624 \\ -- \end{matrix}$	$\begin{matrix} 0.029 \\ -- \end{matrix}$	$\begin{matrix} 0.029 \\ -- \end{matrix}$	$\begin{matrix} 1.5E-3 \\ -- \end{matrix}$	$\begin{matrix} 0.029 \\ -- \end{matrix}$	$\begin{matrix} 0.029 \\ -- \end{matrix}$	$\begin{matrix} 0.029 \\ -- \end{matrix}$	$\begin{matrix} 0.0016 \\ -- \end{matrix}$	$\begin{matrix} 0.01 \\ -- \end{matrix}$
Pressure, lb/in. ⁻²	$\begin{matrix} U_1 \\ U_2 \\ U \end{matrix}$	$\begin{matrix} 5.1 \\ 5.2 \end{matrix}$				$\begin{matrix} -0.134 \\ 0.033 \\ -0.101 \end{matrix}$		$\begin{matrix} 0.267 \\ 0.002 \\ 0.269 \end{matrix}$	$\begin{matrix} 0.268 \\ 0.268 \end{matrix}$						
	$\begin{matrix} U \\ P \end{matrix}$	$\begin{matrix} -- \\ -- \end{matrix}$	$\begin{matrix} 1200 \\ -- \end{matrix}$	$\begin{matrix} -200 \\ -- \end{matrix}$	$\begin{matrix} 1000 \\ -- \end{matrix}$	$\begin{matrix} -- \\ -- \end{matrix}$	$\begin{matrix} 1000 \\ -- \end{matrix}$	$\begin{matrix} -- \\ -- \end{matrix}$	$\begin{matrix} -- \\ -- \end{matrix}$	$\begin{matrix} 268 \\ -- \end{matrix}$	$\begin{matrix} -0.0006 \\ -- \end{matrix}$	$\begin{matrix} 35E-8 \\ -- \end{matrix}$	$\begin{matrix} 0.35 \\ -- \end{matrix}$	$\begin{matrix} 0.0001 \\ -- \end{matrix}$	$\begin{matrix} 0.00 \\ -- \end{matrix}$
Expansion stress (other than from p, d, and t), lb/in. ⁻²	$\begin{matrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U \end{matrix}$	$\begin{matrix} 1.1 \\ 1.2 \\ 1.3 \\ 1.4 \\ 1.5 \end{matrix}$				$\begin{matrix} 5.4 \\ -199.3 \\ 44.1 \\ 8.9 \\ 0.5 \end{matrix}$		$\begin{matrix} 6.95 \\ 30.85 \\ 50.45 \\ 10.77 \\ 0.70 \end{matrix}$							
	$\begin{matrix} U \\ S \end{matrix}$	$\begin{matrix} \\ -- \end{matrix}$	$\begin{matrix} 27450 \\ -- \end{matrix}$	$\begin{matrix} -- \\ -- \end{matrix}$	$\begin{matrix} 27450 \\ -- \end{matrix}$	$\begin{matrix} -140.5 \\ -- \end{matrix}$	$\begin{matrix} 25042 \\ -- \end{matrix}$	$\begin{matrix} 99.7 \\ -- \end{matrix}$	$\begin{matrix} 75.3 \\ -- \end{matrix}$	$\begin{matrix} 57E-6 \\ -- \end{matrix}$	$\begin{matrix} 1.5E-4 \\ -- \end{matrix}$	$\begin{matrix} 2.4E-8 \\ -- \end{matrix}$	$\begin{matrix} 17.9 \\ -- \end{matrix}$	$\begin{matrix} 1.3476 \\ -- \end{matrix}$	$\begin{matrix} 7.09 \\ -- \end{matrix}$
Total Variance	$\ell n n_t$													18.9944	100.00

*Using a fixed value for mean log ΔK . These entries are superseded by Table 8-5 for mean log ΔK pertinent to the values of the parameters of the individual crack, which shows a negligible contribution by the variance of B.

with var U, the variance of the uncertainty coefficient. But var U measures the spread of X_i in ratio form, irrespective of the original units of X_i or the value of the expected value of X_i in these units, and this allows us to compare var U

between the random variables, X_i . Then the size of $\left[(\text{plan})^2 \cdot \left(\frac{\partial \ln n_t}{\partial x_i} \right)^2 \right]$

determines how strongly the functional form of the n_t equation will transmit uncertainty in a particular X_i into uncertainty in the number of cycles to grow through, n_t . For instance, in computing the contribution of uncertainty in expansion stress (other than from p, d, and t) to the uncertainty in n_t , from (8-7), we may either multiply $57 \text{ E}6 \text{ lb}^2 \text{ in}^{-4}$ by $2.4 \text{ E}8 \text{ c}^2 \text{ lb}^{-2} \text{ in}^4$ or multiply 0.0753 by 17.9 c^2 , reading from Table 8-2; the latter, however, tells us, in scanning the respective columns in the table, that stress contributes to the uncertainty in n_t rather because what stress is realized is highly uncertain than because the n_t equation is sensitive to changes in stress.

The expected value of n_t for this cross section is computed to be 1.08×10^6 cycles, with spread characterized from Table 8-2, $(\sqrt{\text{total variance}})$, as a factor of $\exp(\sqrt{18.9944}) = 78$ —an indication that 65 to 70% of tests under similar physical conditions would have fatigue lives lying between 14000 and 84×10^6 cycles. Table 8-2 also shows that the contribution to the total var $\ln n_t$ of initial crack depth, a_o/t , is predominant. Thus, we must expect the skewness of the distribution of a_o/t to skew the distribution of $\ln n_t$. This must be taken to indicate, at least for this welded joint, that finding a method for generating the tail distribution is mandatory for any approximation of the probability of leakage within the number of operating cycles expected in 40 years.

The negligible contributions of the variances of wall thickness, diameter, and pressure justify omitting these variables from the Monte Carlo work. Parameter log C, initial crack depth, and stress must be retained, however. Disposition of B will be made in Table 8-5.

8.8 UNCERTAINTY DISTRIBUTIONS FOR MAXIMUM INITIAL CRACK DEPTH

Subsection 7.3 describes the first piping cross section at which the probability of leakage by low-cycle fatigue in 40 years is being estimated. Within that 1/4-circumference surface of the 1-inch-wide cross section that is subject to the maximum axial stress (due to bending and pressure), it is assumed that the initial defect that will leak first is the deepest one, designated $(a_o/t)_{\max}$. The distribution of this variable is specified by the distribution of a_o/t by depth, and the occurrence rate λ , cracks per square inch. Two judgmental distributions of a_o/t —without and with ultrasonic testing—were given in subsection 6.2, but to see the distributions of $(a_o/t)_{\max}$ for chosen values of λ , see Figures 8-1 and 8-2. The distribution being used in the first reliability estimate is without ultrasonic testing (Figure 8-1) for $\lambda = 0.1$ crack per square inch. It is seen that distribution of the deepest starting crack concentrates increasingly over the large values as higher occurrence rates are considered. The same would be true as larger surfaces are considered, since surface area and occurrence rate appear as a product ($\alpha\lambda$) in the probability density function of $(a_o/t)_{\max}$. It is also seen that the distributions of $(a_o/t)_{\max}$ take on values at 0, and then over the same continuous range as the distributions for a_o/t : from 0.01 to 0.8 for the case without ultrasonic testing and from 0.01 to 0.12 with ultrasonic testing, since for this case it is assumed that defects exceeding approximately 10% of the wall thickness are detected. The probability of 0 initial defect is seen to increase as the crack occurrence rate, λ , decreases, and is given by Equation (8-16) (shown later) as $F_{\max X}(x=0) = e^{-\alpha\lambda}$. Probability at 0 plus area under curve equals 1.

Specifically, Figures 8-1 and 8-2 display the p.d.f. of $(a_o/t)_{\max}$ which is the derivative with respect to x of Equation (8-16) (shown later), which is

$$f_{\max X}(x) = \frac{dF_{\max X}(x)}{dx} = \frac{d}{dx} e^{-\alpha\lambda} [1 - F_X(x)] \quad \text{or} \quad \frac{d}{dx} e^{-\alpha\lambda} [1 - F_{\log X}(x)] \quad (8-8) (8-9)$$

$$= \alpha\lambda F_{\max X}(x) \left[\frac{dF_X(x)}{dx} = f_X(x) \right] \quad \text{or} \quad \alpha\lambda F_{\max X}(x) \left[\frac{dF_{\log X}(x)}{dx} = f_X(x) \right] \quad (8-10) (8-11)$$

Figure 8-1. Probability Density Functions of Maximum a_0/t for Selected λ 's, Without Ultrasonic Testing

It is more convenient to write $F_X(x)$ as $F_{\log X}(z)$, where X is the random variable a_0/t , and z is a dummy scale designator for $\log a_0/t$, since the judgmental distributions for a_0/t were given on a $\log a_0/t$ scale. Log a_0/t may be either common or natural log; for convenience, the differential given below is found for the natural log. The required equations are stated as follows.

	$1 - F_{\log X}(z)$	$f_{\max X}(x)$
Without ultrasonic testing	Equation, mid. p. 3-20, Reference 8-1	Table 9-3, $f_1 X(x)$ for Figure 9-11. Equation at top p. 3-20, Reference 8-1, is dF/dz not dF/dx , i.e., density on log x scale.
With ultrasonic testing	Equation (8-15)	Table 9-3, $f_1 X(x)$ for Figure 9-12. Equation (8-12) is dF/dz .

Figure 8-2. Probability Density Functions of Maximum a_o/t for Selected λ 's With Ultrasonic Testing

Thus, the two density functions for $(a_o/t)_{max}$ can be compiled, per Equation (8-10), and plotted in Figures 8-1 and 8-2. Note that $\alpha\lambda$ appears as a product, as in the Poisson distribution by which it is assumed initial cracks are distributed over the pipe surface. Thus, the Poisson parameter is in effect $\alpha\lambda$. The specific α value used for Figures 8-1 and 8-2 is for the chosen pipe cross section, as follows:

$$\alpha = \frac{\ell C}{4} = \frac{\ell \pi d}{8} = \frac{1 \pi 6.624}{8} = 5.2025 \text{ in.}^2$$

λ and a_o/t are as shown on the figures.

8.9 DISPERSION OF n_t WITH ULTRASONIC TESTING (G. T. Vernon)

A method was described in subsections 8-1 to 8-7 for measuring the dispersion of n_t at one piping study section by approximating the variance of the natural log of n_t , $\text{var } \ln n_t$. The contribution of each variable depends on the distribution used for each. Because of the predominating contribution of a_o/t and because a conservative (wide) a_o/t distribution had been used—for the case of no ultrasonic testing having been performed—it was decided to check the contribution of a_o/t using the narrower distribution for the case of ultrasonic testing having been performed. Reference distributions for both of these cases appear under cause 2.1, subsection 6.2.

The probability density function for a_o/t with ultrasonic testing is read from the graph on the above-mentioned page, to be:

$$\frac{dF_{\log X}(z)}{dz} = \begin{cases} 0 & z < -\log 100 \\ y.01 \frac{\log 0.8 - z}{\log 0.8 + \log 100} & -\log 100 \leq z \leq -\log 15 \\ y.01 \frac{\log 0.8 + \log 15}{\log 0.8 + \log 100} \cdot \frac{\log 0.12 - z}{\log 0.12 + \log 15} & -\log 15 \leq z \leq \log 0.12 \\ 0 & z \geq \log 0.12 \end{cases} \quad (8-12)$$

where

X is a_o/t .

z is a value on the $\log a_o/t$ scale, and

$y.01$ is the value of this p.d.f. at $z = \log 0.01 = -\log 100$, and may be evaluated as

$$y.01 = \frac{2(\log 0.8 + \log 100)}{(\log 0.8 + \log 100)^2 - (\log 0.8 + \log 15) \cdot (\log 0.8 - \log 0.12)} \quad (8-13)$$

Let us make the definition:

$$k = 1 / [(\log 0.8 + \log 100)^2 - (\log 0.8 + \log 15) \cdot (\log 0.8 - \log 0.12)] \quad (8-14)$$

then we may integrate the p.d.f. from the right to obtain:

$$1 - F_{\log X}(y) = \begin{cases} 0 & y \geq \log 0.12 \\ k \cdot \frac{\log 0.8 + \log 15}{\log 0.12 + \log 15} \cdot (\log 0.12 - y)^2 & \log 0.12 \geq y \geq -\log 15 \\ k \cdot [(\log 0.8 - y)^2 - (\log 0.8 + \log 15) \cdot (\log 0.8 - \log 0.12)] & -\log 15 \geq y \geq -\log 100 \\ 1 & y \leq -\log 100 \end{cases} \quad (8-15)$$

and—as in subsection 8.5.2—we obtain:

$$F_{\max X}(x) = e^{-\alpha\lambda} [1 - F_{\log X}(\log x)] \text{ for } x \geq 0 \quad (8-16)$$

where

α is the surface area of pipe under consideration, and

λ is the average number of cracks per unit area. (See subsection 8.5.2 for details.)

Equation (8-12) is the cumulative distribution of the fractional depth of the largest crack in the pipe cross section, given that the cross section has survived the ultrasonic test.

We may use Equations (33) and (34) of subsection 8.5.2 to compute the expected value and variance, $E X_{\max}$ and $\text{var } X_{\max}$, of the variable $X_{\max}$, distributed according to Equation (8-10), from the p.d.f. of X given by Equation (8-12) (with ultrasonic testing) and find

$$\begin{aligned}
 E X_{\max} = & 1 - \left\{ \int_0^{0.01} e^{-\alpha \lambda} dx + \int_{0.01}^{1/15} e^{-\alpha \lambda \cdot k} \cdot [(\log 0.8 - \log x)^2 \right. \\
 & - (\log 0.8 + \log 15) \cdot (\log 0.8 - \log 0.12)] dx \\
 & + \int_{1/15}^{0.12} e^{-\alpha \lambda \cdot k} \cdot \frac{\log 0.8 + \log 15}{\log 0.12 + \log 15} \cdot (\log 0.12 - \log x)^2 dx + \left. \int_{0.12}^{1.0} dx \right\} \\
 = & 0.0146,^*
 \end{aligned} \tag{8-17}$$

and by a similar method obtain

$$\text{var } X_{\max} = 0.531 \times 10^{-3},^*$$

These compare with 0.0364 and 7.0×10^{-3} , respectively, computed in subsection 8.5.2 under the assumption of no ultrasonic testing of the weld.

The contributions to the variance of $\ln n_t$ for the case with ultrasonic testing are displayed in Table 8-3. As to be expected, the contribution of a_0/t has dropped away from its previous dominant position. The expected value of n_t is higher at 4.1×10^6 cycles. As in subsection 8.7, 65 to 70% of tests under similar physical conditions would have fatigue lives lying between 13000 and 129×10^6 cycles, that is, within a factor of $\exp(\sqrt{11.9005}) = 31.5$ of 4.1×10^6 cycles.

8.10 DISPERSION OF n_t CONSIDERING CRACK-SPECIFIC MEAN LOG ΔK

The contributions of the variances of the important variables to the variance of $\log n_t$, considering crack-specific mean $\log \Delta K$, can be found by applying the propagation of errors method to the linear equation for $\log n_t$, Equation (5-41). Reapplying Equation (8-5), for any equation

$$Y = g(X_1, X_2, \dots, X_n) \tag{8-18}$$

where the X_i , $1 \leq i \leq n$, are independent random variables, the variance equation is as follows, the E notation indicating that each partial derivative is to be evaluated at the expected value (mean) of the variable:

$$\text{Var } Y \triangleq \sum_{i=1}^n \left(\left. \frac{\partial Y}{\partial X_i} \right|_{EX_1, EX_2, \dots, EX_n} \right)^2 \text{Var } X_i. \tag{8-19}$$

For the case at hand where the function $g(\cdot)$ is a simple linear sum (of independent random variables),

$$g(\cdot) = \sum_{i=1}^n a_i X_i, \tag{8-20}$$

* Evaluated with $\alpha = 5.2025 \text{ in.}^2$, $\lambda = 0.1 \text{ crack/in.}^2$ per Table 7-1.

Table 8-3
CONTRIBUTIONS TO DISPERSION OF n_t
AT THE CHOSEN STUDY SECTION
(a_o/t with Ultrasonic Testing)

	Symbol	Cause Code No.	Design	Δ Design	Plan	$E \ln U_i \times 10^{-3}$	Expected Value	$\text{var } \ln U_i \times 10^{-3}$	$\text{var } U \times 10^{-3}$	Variance	$\frac{\partial \ln n_t}{\partial x_i}$	$\left(\frac{\partial \ln n_t}{\partial x_i}\right)^2$	$(\text{plan})^2 \cdot \left(\frac{\partial \ln n_t}{\partial x_i}\right)^2$	Contribution to $\text{var } \ln n_t$	Contribution to $\text{var } \ln n_t$ as percentage
Equation parameter	B	--	--	--	--	--	4.45	--	--	1.016	2.242*	5.02*		5.1051*	42.90*
Equation parameter	log C	--	--	--	--	--	-3.96	--	--	0.138	-2.303	5.30		0.7318	6.15*
Thickness, in.	$\left. \begin{matrix} U_1 \\ U_2 \\ U \end{matrix} \right\}$	$\left\{ \begin{matrix} 1.9 \\ 1.11 \\ 9.1 \end{matrix} \right\}$	$\left\{ \begin{matrix} \\ \\ \end{matrix} \right\}$			$\begin{matrix} -0.158 \\ 0 \\ -0.158 \end{matrix}$		$\begin{matrix} 0.443 \\ 2.012 \\ 2.455 \end{matrix}$							
	t	--	0.432	--	0.432	--	0.432	--	--	0.46E-3	8.324	69.29	12.9	0.0317	0.27*
Initial crack depth	$\frac{a_o}{t}$	2.2	--	--	--	--	0.0146*	--	--	0.531E-3*	-93.934*	8823.59*		4.6826*	39.35*
Diameter, in.	$\left. \begin{matrix} U_1 \\ U_2 \\ U_3 \\ U \end{matrix} \right\}$	$\left\{ \begin{matrix} 1.12 \\ \\ 10.1 \end{matrix} \right\}$	$\left\{ \begin{matrix} -- \\ -- \\ -- \end{matrix} \right\}$	--	1	$\begin{matrix} -0.0003 \\ -0.0033 \\ 0 \\ -0.004 \end{matrix}$		$\begin{matrix} 0.000 \\ 0.003 \\ 0.025 \\ 0.029 \end{matrix}$	$\begin{matrix} 0.000 \\ 0.003 \\ 0.025 \\ 0.029 \end{matrix}$		$\begin{matrix} -4.821 \\ 3.640 \\ 7.870 \end{matrix}$	$\begin{matrix} 23.24 \\ 13.25 \\ 61.93 \end{matrix}$	$\begin{matrix} 23.2 \\ 13.2 \\ 61.9 \end{matrix}$	$\begin{matrix} 0.0000 \\ 0.0000 \\ 0.0015 \end{matrix}$	$\begin{matrix} 0.00 \\ 0.00 \\ 0.01 \end{matrix}$
	d		6.624		6.624		6.624			1.5E-3				0.0016	0.01
Pressure, lb/in. ⁻²	$\left. \begin{matrix} U_1 \\ U_2 \\ U \end{matrix} \right\}$	$\begin{matrix} 5.1 \\ 5.2 \end{matrix}$				$\begin{matrix} -0.134 \\ 0.033 \\ -0.101 \end{matrix}$		$\begin{matrix} 0.267 \\ 0.002 \\ 0.269 \end{matrix}$	$\begin{matrix} \\ \\ 0.268 \end{matrix}$						
	P	--	1200	-200	1000	--	1000	--	--	268	-0.0006	35E-8	0.35	0.0001	0.00
Expansion stress (other than from p, d, and t), lb/in. ⁻²	$\left. \begin{matrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U \end{matrix} \right\}$	$\begin{matrix} 1.1 \\ 1.2 \\ 1.3 \\ 1.4 \\ 1.5 \end{matrix}$				$\begin{matrix} 5.4 \\ -199.3 \\ 44.1 \\ 8.9 \\ 0.5 \\ -140.5 \end{matrix}$		$\begin{matrix} 6.95 \\ 30.85 \\ 50.45 \\ 10.77 \\ 0.70 \\ 99.7 \end{matrix}$	$\begin{matrix} \\ \\ \\ 75.3 \end{matrix}$						
	S	--	27450	--	27450	--	25042	--	--	57E 6	1.5E-4	2.4E-8	17.9	1.3476	11.32*
Total Variance	$\ln n_t$													11.9005*	100.00

* Changes from Table 8-2.

**Using a fixed value for mean log ΔK. These entries are superseded by Table 8-5 for mean log ΔK pertinent to the values of the parameters of the individual crack, which shows a negligible contribution by the variance of B.

Equation (8-19) leads to

$$\text{Var } Y = \sum_{i=1}^n a_i^2 \text{Var } X_i \quad (8-21)$$

Equation (8-21) may be used to find the variance of Equation (5-41).

Contributions to $\text{Var } \log n_t$ will be in proportion to those to $\text{Var } n_t$, and numerically smaller by a factor of approximately n_t^2 by Equation (8-19).

Using Equation (8-21), the variance of Equation (5-41) is

$$\begin{aligned} \text{Var } \log n_t &\doteq 1.0084^2 \text{Var } \Delta \log C \\ &+ 4.5069^2 \text{Var } \log \sigma_{PM} \\ &+ 6.2615^2 \text{Var } \sqrt{a_o/t} \\ &+ 2.47^2 \text{Var } e^{-B} \\ &+ 0.03^2 \text{Var } B. \end{aligned} \quad (8-22)$$

Using Equation (8-19), the required variance expressions are as follows:

$$\text{Var } \log \sigma_{PM} \doteq (\log e)^2 \text{Var } \sigma_{PM} / (E\sigma_{PM})^2 \quad (8-23)$$

$$\text{Var } \sqrt{a_o/t} \doteq \text{Var } (a_o/t) / 4E(a_o/t) \quad (8-24)$$

$$\text{Var } e^{-B} \doteq e^{-2EB} \text{Var } B \quad (8-25)$$

The required values for variances and expected values are shown in Table 8-4, from the sources indicated.

Table 8-4
EXPECTED VALUES AND VARIANCES

Variable	Expected Value	Source	Variance	Source	Remarks
$\Delta \log C$	(0)		0.138	(1) (2)	
σ_{PM}	25042	(1) (2)	57×10^6	(1) (2)	
a_o/t max.	0.0364	(1)	0.007	(1)	Without ultrasonic testing.
a_o/t max.	0.0146	(2)	0.000531	(2)	With ultrasonic testing.
B	4.45	(1) (2)	1.016	(1) (2)	

(1) Table 8-2

(2) Table 8-3

Substituting these values into Equations (8-23) to (8-25), and into Equation (8-22), gives in Table 8-5 the contributions of the variances of the variables to that of $\log n_t$, for the initial crack size distributions assumed, with and without ultrasonic testing.

Table 8-5
CONTRIBUTIONS TO VARIANCE OF $\log n_t$

	Without Ultrasonic Test		With Ultrasonic Test	
		Percent		Percent
$\Delta \log C$, material property intercept, change from nominal	0.1403	5.9	0.1403	16.5
σ_{PM} , primary membrane stress	0.3488	14.7	0.3488	41.2
a_o/t max., initial crack depth	1.8830	79.3	0.3568	42.1
B, material property, slope	0.0018	0.1	0.0018	0.2
Total Var Log n_t	2.3739	100.0	0.8477	100.0
Var $\ln n_t$ (1)	12.5864		4.4945	
Replaces Value in Ref. shown	18.9944	(2)	11.9005	(3)

(1) $\ln n_t = (\ln 10) (\log n_t)$
 $\text{Var } \ln n_t = (\ln 10)^2 \text{Var } \log n_t$, by Equation (8-11).

(2) Table 8-2

(3) Table 8-3

A 5000-trial Monte Carlo estimate of the variance contribution of the two B terms alone verified exactly the amount found by finding the variance of Equation (5-40) using Equation (8-19).

Thus it is seen that, using the uncertainty distributions assumed, variability in slope B now makes a negligible contribution to the overall variance, whereas, without ultrasonic testing, initial crack depth makes the major contribution, and with ultrasonic testing makes an equal contribution with stress, reducing the variance of the function about 2/3 for this case. The foregoing implies that a reduction of some percentage of the variance assumed in the maximum initial crack depth distribution would reduce the variance of the function more than the same percent reduction in the variance of the other variables. And further, while the relationship is not exact due to different shapes for the uncertainty distributions of the variables, such a reduction would also reduce the probability of leakage more than a comparable reduction for the other variables. Finally, it is seen at the bottom of Table 8-5 that consideration of crack-specific mean $\log \Delta K$ has resulted in a considerable reduction in the variance of $(\ln) n_t$. The leakage probabilities given in Table 8-4 are based on the lower values.

8.11 REFERENCES

- 8-1. *Reactor Primary Coolant System Rupture Study, Quarterly Progress Report No. 14*, July-September 1968, (GEAP-5716).
- 8-2. Paris, P. C. and Sih, G. C., *Fracture Toughness Testing and Applications*, 1965; Equation (4) in *Stress Analysis of Cracks*, (ASTM STP No. 381).

9. ESTIMATING THE PROBABILITY OF LEAKAGE BY MONTE CARLO WITH IMPORTANCE SAMPLING

9.1 INTRODUCTION

It has been mentioned that Monte Carlo simulation methods would be needed to make estimates of the lower tails of time to damage distributions (by lower tail, we mean those times to damage which are unusually low and would occur infrequently). In this section we will show how we hope to generate certain tail probabilities by these methods. The initial application will be on estimating the probability of leakage by low-cycle fatigue at the chosen piping study detailed in subsection 7.3. Before describing the Monte Carlo method we are planning to adopt, however, some general orientation might be of help. After definitions and an introduction of ordinary (or crude) Monte Carlo technique, we will show how a transformation of the original problem can be more efficient.

9.2 PROBABILITY NOTATION

In probability work, some common notation takes on specific meanings. This notation is described in the following paragraph.

A "random variable" is a variable which may take on different values under apparently identical circumstances, merely by chance. The name of the random variable, such as "expansion stress," is designated by a capital letter, as X . Values which X may take on are designated by lower-case letters, such as x (also $x_1, x_2, \dots, x_n$). The frequency distribution of X —known as the probability density function, PDF—is designated $f_X(x)$, "the PDF of random variable X evaluated at x ;" or simply $f(x)$, "little f of x ." The cumulative frequency distribution of X , known as the cumulative distribution function, CDF, is designated $F_X(x)$, or simply $F(x)$, "capital F of x ." Areas under $f(x)$ are probabilities, so $\Pr[x_1 \leq X \leq x_2]$ is the probability that X may take on values between x_1 and x_2 ; the total area under $f(x)$ is 1; any function for which this is true can be a PDF, and it must be true for a function to be a PDF. Thus, $f(x)$ values may cover any range, depending on the scale of X , but must be positive or 0. But $F(x)$ values themselves are probabilities, $\Pr[X \leq x_1]$, thus, they are "lower tail" probabilities. $F(x)$ values range from 0 to 1. $F(x)$ and $f(x)$ are related by

$$F(x) = \int_{-\infty}^x f(t)dt.$$

9.3 ORDINARY MONTE CARLO (G. T. Vernon)

The ordinary Monte Carlo method is to simulate the physical situation. Each physical variable in the model is simulated by one or more random numbers. The random numbers interact according to the equations of the model, yielding random numbers which simulate one possible outcome of the physical "test" (or experiment). By statistical techniques we may reduce the outcomes of several such iterations to a characterization of the distribution of possible outcome values. The procedure is analogous to laboratory experimentation, where some interesting aspect of the physical world is simulated under closely controlled laboratory conditions to test certain physical equations of a model. In Monte Carlo simulation, on the one hand, the model must simulate all physical processes by mathematical equations, and introduces distributions of the variables to account for the many perturbations to be found in nature; while in the laboratory, on the other hand, these perturbations are eliminated or failing that, compensations are made. In the laboratory, the complexity of the physical world is reduced to a situation where mathematical equations apply; in Monte Carlo simulation, the mathematical equations are augmented by distributions on the physical variables to approximate the complexity of the world more closely.

In a single Monte Carlo iteration (loop), a certain number of random numbers (which themselves might be random or depend on the random numbers) are generated to be substituted for random variables in the model equations. Let us call the random variables in a Monte Carlo simulation $X_1, X_2, \dots, X_l$ which take on random number values $x_1, x_2, \dots, x_l$ according to a model distribution $F(x_1, x_2, \dots, x_l)$.*

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* Defined to be $\Pr \{X_1 \leq x_1, X_2 \leq x_2, \dots, X_l \leq x_l\}$.

After various manipulations of these random values, we generate the outcome values. It may be clear that the set of instructions by which the outcome is attained is a procedure or—in mathematical terminology—a function of $x_1, x_2, \dots, x_I, \phi(x_1, x_2, \dots, x_I)$. This function may not have a large range of possible values; in fact, the values may just be the logical values TRUE (represented by 1 sometimes) to show that some condition occurs, and FALSE that it does not (0 in the above representation)—in which case the question “how often?” is being asked in the simulation. After several iterations, a set of outcome values is built up from which inductive inferences may be drawn by the usual statistical techniques.

In subsection 8.2, we introduced many of the necessary concepts and definitions for this subsection. The “expected (or average) value” and “variance” of a function of random variables were defined and were noted to be analogous to the center of mass and second moment about the center of mass. It was not, however, justified how the terminology “average” would also be appropriate. By the “Law of Large Numbers”* under mild assumptions, which hold throughout this report, the arithmetic average of a sequence of iterations of $\phi(X_1, X_2, \dots, X_I)$ lies within a given closeness of the expected value, $E \phi(X_1, X_2, \dots, X_I)$, with higher probability as the number of iterations increases (when, of course, the variables, $X_1, X_2, \dots, X_I$, take on values drawn from the same distribution at each iteration). This means that the arithmetic average of $\phi(X_1, X_2, \dots, X_I)$ over several iterations is an estimate of $E \phi(X_1, X_2, \dots, X_I)$ which improves as the number of iterations grows larger. The variance, $\text{var } \phi(X_1, X_2, \dots, X_I)$, turns out to be useful for showing how good this improvement will be, as we shall show later in this subsection.

9.4 APPLICATION OF MONTE CARLO TO ESTIMATING LEAKAGE PROBABILITY (G. T. Vernon)

We now demonstrate the application of the ordinary Monte Carlo simulation method to generating the probability that the number of fatigue cycles, n_t , will be less than some critical level, say n_c , the number of thermal cycles expected in the operation of a boiling water reactor in say 10 (or 40) years.

First, we build the model. This consists of the n_t equation, a numerical solution of which was included in subsection 5.3.4, together with the distribution function of the physical variables, $a_o/t, B, S_E \log C, t, d$, and p , each of which may have several uncertainty coefficients. The total number, I , of random numbers to be drawn would be the total number of uncertainty coefficients, or variables, whose distributions are thought to contribute significantly to the distribution of n_t . The distributions for the model we have been using were proposed in the subsection 6.2, or were derived from data supplied by Tasks D-1 and D-2 (see subsection 2.1). In subsection 3.4 of the same Quarterly we showed a way of choosing the significant contributors. Further, the distributions are thought to arise from independent causes, so that the model distribution is separable:

$$F(x_1, x_2, \dots, x_I) = F_{X_1}(x_1) \cdot F_{X_2}(x_2) \cdot \dots \cdot F_{X_I}(x_I) \quad (9-1)$$

Second, we find a way of generating random numbers according to the distributions of the model. Such methods will be discussed in another context in subsection 9.6.

Third, we make numerous iterations of the Monte Carlo simulation to generate from the random numbers by the numerical solution of the n_t equation a collection of random numbers of cycles. If the fraction of these which lie below n_c is calculated, then we would feel that such a fraction would be typical of the frequency of leaks to be expected in 10 (or 40) years, provided that the model was realistic.

As we hinted earlier, another equivalent way of describing the above simulation is to let the function $\phi(x_1, x_2, \dots, x_I)$ take on the value TRUE if n_t , calculated from the random numbers $x_1, x_2, \dots, x_I$ (each of which corresponds with one uncertainty coefficient or variable), is less than n_c , and FALSE if not. Representing TRUE as 1 and FALSE as 0, the probability of n_t being less than n_c is then just the expected value, $E \phi(X_1, X_2, \dots, X_I)$, and the estimate of this is the average value of $\phi(X_1, X_2, \dots, X_I)$ over the numerous iterations of the simulation. The latter assertion can be verified readily, while the former must hold true by the Law of Large Numbers if the latter is to be a good estimate and can be verified by methods of the Calculus of Probability. (It is analogous to finding the mass of part of a body of varying specific gravity; see subsection 8.2.) For ease of conceptualization, we designate this function:

* See Reference 9-1.

$$I_{\{n_t < n_c\}}(x_1, x_2, \dots, x_I) = \begin{cases} 1 & \text{if } n_t \text{ evaluated at } x_1, x_2, \dots, x_I \text{ is less than } n_c \\ 0 & \text{otherwise} \end{cases} \quad (9-2)$$

Putting the strands together—when N is the number of iterations—the Monte Carlo simulation generates an estimate:

$$\frac{1}{N} \sum_{\text{over all } N \text{ iterations}} I_{\{n_t < n_c\}}(X_1, X_2, \dots, X_I) \quad (9-3)$$

$$\text{of } \Pr\{n_t < n_c\} = E I_{\{n_t < n_c\}}(X_1, X_2, \dots, X_I)$$

$$= \iiint \dots \int I_{\{n_t < n_c\}}(x_1, x_2, \dots, x_I) dF_{X_1}(x_1) \cdot dF_{X_2}(x_2) \cdot \dots \cdot dF_{X_I}(x_I) \quad (9-4)$$

The main drawback of the ordinary Monte Carlo method, when a very small probability such as $\Pr\{n_t < n_c\}$ must be estimated, is the enormous number of calculations that must be made to be reasonably sure of getting even a single iteration where n_t is less than n_c . Similar difficulties arise in the use of Monte Carlo simulations to assist in solving radiation shielding problems. A transformation of the original problem will now be displayed, which can be simulated with many fewer iterations and still will give answers more exact as the number of iterations increases.

9.5 MONTE CARLO WITH IMPORTANCE SAMPLING (G. T. Vernon)

Importance sampling is a Monte Carlo simulation method that allows one to guide the random numbers to be generated into a region of importance and yet still obtain results similar to those of the ordinary Monte Carlo method. For example, in the analysis of radiation shielding efficiency, mentioned previously, certain random variables may represent the physical coordinates of one or more types of particle on the hot side of a shield, and the region of importance might comprise any combination of the physical coordinates which would be likely to impel the particle (or particles) through the shielding; the other particles may be in the vast majority, but hardly worth simulating, especially in proportion to their numbers.

It is possible, with the random variables $X'_1, X'_2, \dots, X'_I$ taking on random number values $x'_1, x'_2, \dots, x'_I$ under the importance weighted distribution function $F'(x'_1, x'_2, \dots, x'_I)$, to use the estimate:

$$\frac{1}{N} \sum_{\text{over all } N \text{ iterations}} \frac{I_{\{n_t < n_c\}}(X'_1, X'_2, \dots, X'_I)}{\partial F'(x_1, x_2, \dots, x_I) / \partial x_1 \partial x_2 \dots \partial x_I} \cdot \frac{\partial F(x_1, x_2, \dots, x_I)}{\partial x_1 \partial x_2 \dots \partial x_I} \quad (9-5)$$

of $\Pr\{n_t < n_c\}$ in place of the estimate [Equation (9-3)] for our application, when the partial derivatives exist for all argument values (the partial derivatives being evaluated at the random numbers taken on by $X'_1, X'_2, \dots, X'_I$). What is written above is incomplete, however, until we display a distribution function, $F'(x'_1, x'_2, \dots, x'_I)$ which is, in some way, better for estimating $\Pr\{n_t < n_c\}$ than ordinary Monte Carlo simulation would be, and we will deal with this aspect next.

9.6 EFFICIENT CALCULATING PROCEDURES (G. T. Vernon)

Before we spell out in detail the methods available for finding an importance weighted distribution function $F'(x'_1, x'_2, \dots, x'_I)$, we will return to the ordinary Monte Carlo method estimate [Equation (9-3)] of $\Pr\{n_t < n_c\}$,

given in Equation (9-4). We stated that we were to find a way of generating random numbers according to the distribution function of the model. As a matter of fact, just about all methods of generating random numbers according to a distribution function on an electronic computer start from a selection of one or more random numbers from the interval (0, 1), each number being drawn independent of the others and with a density that is uniform in the interval. Let us investigate how these numbers are transformed into our desired distribution.

Fortunately, it will be sufficient to consider only distributions with the following properties: separability as given by Equation (9-1), and one-one individual distribution functions $F_{X_1}(x_1), F_{X_2}(x_2), \dots, F_{X_I}(x_I)$. These properties allow us to make the convenient transformation:

$$y_1 = F_{X_1}(x_1) \quad dy_1 = dF_{X_1}(x_1) \quad x_1 = F_{X_1}^{-1}(y_1)$$

$$y_2 = F_{X_2}(x_2) \quad dy_2 = dF_{X_2}(x_2) \quad x_2 = F_{X_2}^{-1}(y_2)$$

$$\vdots \quad \vdots \quad \vdots$$

$$y_I = F_{X_I}(x_I) \quad dy_I = dF_{X_I}(x_I) \quad x_I = F_{X_I}^{-1}(y_I)$$

of Equation (9-4) to yield

$$\Pr \{n_t < n_c\} = \int_0^1 \int_0^1 \dots \int_0^1 1_{\{n_t < n_c\}} \left[F_{X_1}^{-1}(y_1), F_{X_2}^{-1}(y_2), \dots, F_{X_I}^{-1}(y_I) \right] dy_1 dy_2 \dots dy_I \quad (9-6)$$

If we call S the set of all coordinate values $(y_1, y_2, \dots, y_I)$ such that n_t , when evaluated at

$$x_1 = F_{X_1}^{-1}(y_1), x_2 = F_{X_2}^{-1}(y_2), \dots, x_I = F_{X_I}^{-1}(y_I),$$

yields a number of cycles below n_c , then the integrand above becomes merely $1_S(y_1, y_2, \dots, y_I)$, (the function which takes on the value 1 on S and 0 outside S); so that Equation (9-4) is now written

$$\Pr \{n_t < n_c\} = \int_0^1 \int_0^1 \dots \int_0^1 1_S(y_1, y_2, \dots, y_I) dy_1 dy_2 \dots dy_I \quad (9-7)$$

Suppose the random variables $Y_1, Y_2, \dots, Y_I$ take on random number values, drawn from the uniform distribution function on (0, 1): $F_Y(y) = y, dF_Y(y) = dy$, then analogously to Equations (9-3) and (9-4) we find that

$$\frac{1}{N} \sum_{\text{over all } N \text{ iterations}} 1_S(Y_1, Y_2, \dots, Y_I) \quad (9-8)$$

generates, as the number of iterations N increases, an improving estimate of

$$\begin{aligned} & \int_0^1 \int_0^1 \dots \int_0^1 1_S(y_1, y_2, \dots, y_I) dF_Y(y_1) \cdot dF_Y(y_2) \cdot \dots \cdot dF_Y(y_I) \\ &= \int_0^1 \int_0^1 \dots \int_0^1 1_S(y_1, y_2, \dots, y_I) dy_1 dy_2 \dots dy_I = \Pr \{n_t < n_c\} \end{aligned} \quad (9-9)$$

The following procedure for evaluating the estimate [Equation (9-8)] on an electronic computer suggests itself:

Put SUM = 0

*Draw I random numbers, using the random number generator.

For each i evaluate, at the ith random number value, y_i , the inverse distribution function, $F_{X_i}^{-1}(y_i) = x_i$.

Evaluate n_t at $x_1, x_2, \dots, x_I$.

If $n_t < n_c$ go back to * for next iteration.

Otherwise put SUM = SUM + 1 and go back to *.

When N such iterations have been performed, divide SUM by N to obtain the estimate [Equation (9-8)] of $\Pr\{n_t < n_c\}$ [Equation (9-9)].

In this estimate [Equation (9-8)], each iteration is given equal importance and, as we indicated earlier, this will require millions of iterations to attain a reasonable confidence in our estimate of the very small probability, $\Pr\{n_t < n_c\}$. On the other hand, to weight the iterations effectively, we need some criterion to compare one system of weights with another. A statistical tool that is of prime importance for this and other such computations is the Central Limit Theorem,** which we will describe briefly in the following paragraph.

9.7 EFFICIENT ESTIMATING PROCEDURES (G. T. Vernon)

Let $\phi(y_1, y_2, \dots, y_I)$ be a function of the random numbers taken on by the random variables $Y_1, Y_2, \dots, Y_I$ from a distribution function $F(y_1, y_2, \dots, y_I)$ [for example, the function $I_S(y_1, y_2, \dots, y_I)$ in the above paragraph]. Define Φ_n to be the random variable taking the value $\phi(Y_1, Y_2, \dots, Y_I)$ at the nth iteration. Suppose also the $E\Phi_n = E\phi(Y_1, Y_2, \dots, Y_I)$ and $\text{var } \Phi_n = \text{var } \phi(Y_1, Y_2, \dots, Y_I)$ are finite real numbers; then, by the Central Limit Theorem, the larger the number of iterations, N, the closer is the distribution function of

$$\frac{1}{N} \sum_{n=1}^N \Phi_n$$

to a normal distribution (i.e., with a bell-shaped density curve) with central location parameter $E\Phi_n$ and dispersion parameter $\text{var } \Phi_n/N$. This is a powerful result, since we can put rough confidence bands around any Monte Carlo simulation estimate of a reasonably large size (N) when we have a rough estimate of $\text{var } \Phi_n$ (from tabled values of the normal distribution). Furthermore, if two estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ have the same expected value θ , then broadly the one with the lowest $\text{var } \Phi_n$ gives the smallest confidence bands at any confidence level. However, one must exercise a degree of caution for bands of high confidence, since one estimate may converge to a normal distribution at a much faster rate than another. This, however, is not generally a problem, when a simulation is expected to contain thousands of iterations, which is usually possible without excessive cost on today's high speed electronic computers. Further, a test for normality may be made on the results for groups of a hundred or so iterations, in case there is doubt about the convergence to the normal distribution of any estimate selected. We are now ready to search for the best importance weighted estimate.

9.8 PROPERTIES OF WEIGHTING FUNCTIONS (G. T. Vernon)

In performing the importance weighting of the estimate (9-3) to yield the estimate (9-5) of the probability (9-4), we used the partial derivative with respect to an unknown distribution function. Here, any strictly positive weighting

** See Reference 9-1.

function, $w(y_1, y_2, \dots, y_I)$, applied to the estimate (9-8) of the probability (9-9) may be converted into a distribution function; in fact, we merely make the definition

$$F(y_1, y_2, \dots, y_I) = \int_0^{y_1} \int_0^{y_2} \dots \int_0^{y_I} w'(z_1, z_2, \dots, z_I) dz_1 dz_2 \dots dz_I, \quad (9-10)$$

where

$w'(z_1, z_2, \dots, z_I)$ is equal to $K \cdot w(z_1, z_2, \dots, z_I)$ for an appropriate constant K , which scales the weighting function, so that the equation $F(1, 1, \dots, 1) = 1$ is satisfied. Therefore, without any loss of generality, we may assume that the weighting function integrates to a distribution function.

Equation (9-10) may now be written in the form:

$$\Pr\{n_t < n_c\} = \int_0^1 \int_0^1 \dots \int_0^1 \frac{1_S(y_1, y_2, \dots, y_I)}{w(y_1, y_2, \dots, y_I)} w(y_1, y_2, \dots, y_I) dy_1 dy_2 \dots dy_I, \quad (9-11)$$

which is of the form, $E\phi(Y_1, Y_2, \dots, Y_I)$, when the random numbers taken on by the random variables $Y_1, Y_2, \dots, Y_I$ are drawn from the distribution (9-10) and when we define:

$$\phi(y_1, y_2, \dots, y_I) = \frac{1_S(y_1, y_2, \dots, y_I)}{w(y_1, y_2, \dots, y_I)}, \quad (9-12)$$

so that

$$\frac{1}{N} \sum_{\text{over all } N \text{ iterations}} \phi(Y_1, Y_2, \dots, Y_I) \quad (9-13)$$

is an improving estimate of $\Pr\{n_t < n_c\}$ as the number of iterations increases. And, since this gives us an estimate for each strictly positive weighting function, we will now search for a weighting function which minimizes $\text{var } \phi(Y_1, Y_2, \dots, Y_I)$ —our criterion for efficiency by the Central Limit Theorem.

The solution to this problem is relatively simple. Defining $\theta = \Pr\{n_t < n_c\}$, then, over the strictly positive weighting functions $w(y_1, y_2, \dots, y_I)$, we are to minimize:

$$\int_0^1 \int_0^1 \dots \int_0^1 \left[\frac{1_S(y_1, y_2, \dots, y_I)}{w(y_1, y_2, \dots, y_I)} - \theta \right]^2 w(y_1, y_2, \dots, y_I) dy_1 dy_2 \dots dy_I. \quad (9-14)$$

But if $w(y_1, y_2, \dots, y_I)$ is put approximately equal to $1/\theta$ on S and a fraction above zero elsewhere, this will imply both $F(1, 1, \dots, 1) = 1$ since $1_S(y_1, y_2, \dots, y_I)$ integrates to θ , and that the variance is near zero. However, things are not quite so simple in practice because it is very difficult to find the shape of the set S , and if we did know it exactly, we would no doubt be able to find its volume by numerical integration to yield the exact value of $\Pr\{n_t < n_c\} = \theta$. So, instead, we must have the following properties for $w(y_1, y_2, \dots, y_I)$:

- It must have a value near $1/\theta$, where we are sure the coordinate values lie in S .
- It must have a value near zero, where we are sure the coordinate values lie outside S .
- It should have intermediate values reflecting our uncertainty elsewhere.
- It must be a density of a distribution function.

Practical aspects of this importance weighted transformed Monte Carlo procedure are described in the following subsection. Numerical results will be obtained with the use of crude importance weighting functions and approximations to the inverse distribution functions, $F_{X_1}^{-1}(y_1)$, $F_{X_2}^{-1}(y_2)$, ..., $F_{X_I}^{-1}(y_I)$ —in estimate (9-13). A preliminary estimate of $\theta = \Pr\{n_t < n_c\}$ will, of course, be necessary for constructing a better importance weighting function.

9.9 ILLUSTRATING MONTE CARLO WITH IMPORTANCE SAMPLING

Attention is restricted to the problem of estimating the fraction of a distribution which lies less than some chosen value, as compared to describing the entire distribution. We illustrate the single variable case. Consider the random variable X . We wish to estimate $\Pr[X \leq x_0]$. In this elementary illustration, the distribution of X is known, so no doubt this probability could be calculated analytically. But let us illustrate the Monte Carlo approach, first by "ordinary Monte Carlo," then by importance sampling. Theory for both appears in subsections 9.1 to 9.8 and in Reference 9-2. We could sample at random n times from the distribution of X , assign 1 to each trial where $x \leq x_0$, and 0 otherwise. Then an estimate of $\Pr[X \leq x_0]$ is the sum of 1's and 0's, divided by n . But if this probability is small, such as 10^{-4} , then on the average only one result in 10,000 will be assigned 1, and a large number of trials is required to achieve a satisfactory confidence interval on the estimate; indeed, to discover that $\Pr[X \leq x_0]$ is not 0 with 0 variance. We require a procedure for making a more correct estimate with fewer trials; such procedures are called "variance reduction procedures," of which importance sampling is one. Before illustrating importance sampling, let us note how one draws random samples from a distribution.

Random values of X may be chosen by solving $F(x)$ for x , resulting in the inverse cumulative designated $x = F^{-1}(x)$, selecting random numbers uniformly distributed from 0 to 1 for $F(x)$, and finding the corresponding x values.

This procedure is illustrated in Figure 9-1(a)2 where a right triangular density function, $f_1(x)$, is shown in dotted lines, with cumulative $F_1(x)$ shown in solid lines, for $0 \leq x \leq 10$. If one enters the figure on the $F_1(x)$ scale with uniformly distributed random numbers, $0 \leq U \leq 1$, the x values given by the $F_1(x)$ line are randomly drawn in proportion to their frequency of occurrence in the distribution of X .

Suppose that we wish to estimate a lower-tail probability, say $\Pr[X \leq x_0]$. We may introduce a weighting function on the random $0 \leq U \leq 1$ values. Since for this illustration we know $f(x)$, we may also know that $\Pr[X \leq x_0] = 0.2$. We could make a perfectly correct estimate of this probability with every random draw by introducing a weighting function $f_2[F_1(x)]$, uniformly distributed over $0 \leq F_1(x) \leq 0.2$, as shown by dotted lines in Figure 9-1(e)1, $f_2[F_1(x)] = 5$ at all values of $F_1(x)$ in this range. We show the CDF, $F_2[F_1(x)]$, of this weighting function in solid lines also in Figure 9-1(e)1. Now we choose random values of U , uniformly distributed as before $0 \leq U \leq 1$, assign them to $F_2[F_1(x)]$, choose the corresponding $F_1(x)$ values, and then the corresponding X values. All such values of X will fall less than x_0 ; but instead of assigning 1 to these outcomes, we assign a weight which is $1/f_2[F_1(x)]$. This is $1/5 = 0.2$ for all possible U values. Thus, $\Pr[X \leq x_0]$ is estimated exactly correctly as 0.2 on every trial.

In real problems, however, X may be a complicated physical equation, or the sum of production and delay times of a part through a shop, or conditional probabilities in a network of events leading to a system reliability estimate, etc. Its distribution is not known, and cannot be found readily—or at all—because the distributions of the participating variables cannot readily be combined analytically in accordance with the function. This is the case for the n_t equation. Thus, it may be possible to arrive only roughly at a satisfactory shape of weighting function for each variable; only the correct end of the distribution of each variable to be weighted may be known, if the function is monotonic in that variable; or the weighted end may change for one or more variables, depending on values of other variables chosen. While, in principle, multidimensional weighting functions would be appropriate, it may be necessary to weight each variable separately. Weighting functions should be high in the region of favorable outcome, and low over the remaining region. Illustrative results for other one-dimensional weighting functions are illustrated in Figure 9-1, and tabulated in Table 9-1. Figure 9-1(b)1 shows a weak function, Figure 9-1(c)1 a very strong function, and Figure 9-1(d)1 a tapered step function. Note in each illustration how the weighting function governs what value of $F_1(x)$ is selected for the same uniform random value, and thus what value of X is selected. Also note how the variance of estimate of the $\Pr[X \leq x_0]$ is 0 for Figure 9-1(e)1 and increases to Figure 9-1(a)1 where there was no weighting, except for Figure 9-1(c)1 which shows that weighting can be excessive. This pattern was evident with only 30 trials.

Figure 9-1. Weighting Functions for Importance Sampling in a Monte Carlo Estimate of $\Pr[X \leq x]$ (See Table 9-1)

Table 9-1
DETAILS FOR WEIGHTING FUNCTIONS ILLUSTRATED IN FIGURE 9-1

Random Variable K: $f_1(x) = 0.02x$ for $0 \leq x \leq 10$
 $F_1(x) = 0.01x^2$ for $0 \leq x \leq 10$
 $x = F_1^{-1}(x) = 10\sqrt{F_1}$ for $0 \leq x \leq 10$

When $\Pr[X \leq x_0] \equiv F_1(x) = 0.2$, $x = 4.476$.

Description of Weighting PDF Figure 9-1	Weighting PDF $f_2[F_1(x)] =$	Weighting CDF $F_2[F_1(x)] =$	Weighting Inverse CDF $F_2^{-1}[F_1(x)] =$	Pr[X ≤ x ₀]	
				Estimate	Variance of Estimate in 30 Trials Using Identical Random Numbers U = F ₂
a. No Weighting: Uniform, "Ordinary Monte Carlo"	(1)	(F ₁)	(F ₂)	0.133	0.00385
b. Weak Weighting: Right Triangular plus Small Uniform	$1.9 - 1.8 F_1$	$1.9 F_1 - 0.9 F_1^2$	$\frac{1.9 - \sqrt{3.61 - 3.6 F_2}}{1.8}$	0.1297	0.001911
c. Strong Weighting: 1/F ₁ ² plus Small Uniform	$\frac{0.01}{(F_1 + 0.01)^2} + 0.009901$	$1 - \frac{0.01}{F_1 + 0.01} + 0.009901 F_1$	$\frac{(F_2 - 1.0009901) + \sqrt{(1.0009901 - F_2)^2 + 0.000396 F_2}}{0.019802}$	0.1999	0.01076
d. Tapered Step Weighting: $0 \leq F_1 \leq 0.15$ $0 \leq F_2 \leq 0.744$ $0.15 \leq F_1 \leq 0.25$ $0.744 \leq F_2 \leq 0.9925$ $0.25 \leq F_1 \leq 1.0$ $0.9925 \leq F_2 \leq 1.0$	4.96 $12.385 - 49.5 F_1$ 0.01	$4.96 F_1$ $-0.556 + 12.385 F_1 - 24.75 F_1^2$ $0.99 + 0.01 F_1$	$F_2/4.96$ $\frac{12.385 - \sqrt{98.344 - 99 F_2}}{49.5}$ $(F_2 - 0.99)/0.01$	0.1954	0.0001042
e. Exact Step Weighting: $0 \leq F_1 \leq 0.2$ $0 \leq F_2 \leq 1$	5	$5 F_1$	$F_2^{1/5}$	0.2	0

9.10 WEIGHTING FUNCTIONS FOR THE VARIABLES IN THE CYCLES-TO-LEAK EQUATION

In the low-cycle fatigue equation for n_t we have three variables: initial crack depth, expansion stress, and parameter $\log C$. It has not been possible to devise a four-dimensional function, but separate one-dimensional functions have been applied to the uncertainty distribution for each variable. Now the weight on 1 in the event $n_t \leq n_c$ for, say, 40 years is the product of the weights found for each variable separately.

Weighting functions will be used which are based on the beta distribution, which may assume a wide variety of shapes on the scale zero to one. Since $0 \leq F_1(x) \leq 1$, the beta distribution is convenient.

The beta CDF, known as the incomplete beta function, is

$$F(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^x t^{\alpha-1} (1-t)^{\beta-1} dt, \quad 0 \leq x \leq 1 \quad (9-15)$$

If $\beta = 1$, then CDF becomes simply

$$[F(x) \text{ for } \beta = 1] = x^\alpha \quad (9-16)$$

so the cumulative inverse, $F^{-1}(x)$, for this case is

$$x = [F(x)]^{1/\alpha} \quad (9-17)$$

and the PDF is

$$[f(x) \text{ for } \beta = 1] = \alpha x^{\alpha-1} \quad (9-18)$$

Characteristic shapes of the beta distribution CDF and PDF for $\alpha = 2, 3, 5, 10, 20$, and 40 are illustrated in Figures 9-2 through 9-7. The distribution will weight the low rather than high values of $F_1(x)$ by reversing the roles of α and β .

We wish to estimate $\Pr[n_t \leq n_c]$ where n_c is a critical number of cycles chosen to represent 40 years of service at the pipe cross section chosen. Investigation of the behavior of the n_t function indicates that minimum n_t values are always given by maximum values of a_0/t , S_E , and C . Therefore, these distributions should always be sampled near their maximum ends.

To arrive at weighting functions which are high and constant over most of the region of the variable which gives results in the region of the function for which the probability is desired, low and constant in most of the remaining region, and tapered between the two, the following approach was used:

- Find the value of each variable such that the CDF value $F_1(x)$ for each variable is the same, and $n_t = 360$ cycles, reference number of cycles for 40 years service.
- Find the value of each variable in turn which, when changed separately, give $n_t = 2000$, the other two variables remaining at the values found in Step a.
- Find the α value in the beta distribution [in Equations (9-15) to (9-18)] such that the CDF $F_2[F_1(x)]$ of the beta weighting function equals 0.05 at the value of the variable giving $n_t = 2000$ found in Step b.
- Remove the tip, on the density scale, of the PDF $f_2[F_1(x)]$ of the beta weighting function so found and standardize the area back to 1.

This process resulted in the three weighting functions shown in Figures 9-8 through 9-10, with their parameters shown in Table 9-2. They are drawn to scales identical to Figures 9-2 through 9-7.

Figure 9-2. Beta Distribution for $\alpha = 2$, $\beta = 1$

Figure 9-3. Beta Distribution for $\alpha = 3$, $\beta = 1$

Figure 9-4. Beta Distribution for $\alpha = 5, \beta = 1$

Figure 9-5. Beta Distribution for $\alpha = 10, \beta = 1$

Figure 9-6. Beta Distribution for $\alpha = 20, \beta = 1$

Figure 9-7. Beta Distribution for $\alpha = 40, \beta = 1$

Figure 9-8. Weighting Function for a_0/t Max

Figure 9-9. Weighting Function for S_E^*

Figure 9-10. Weighting Function for $\Delta \log C$ Table 9-2
WEIGHTING FUNCTION PARAMETERS

Figure Number	Random Variable	Common $F_1(x)$ for $n_t = 360$	Corresponding x Value	Beta Distribution α Value for $F_2[F_1(x)] = 0.05$ and $n_t = 2000$ (Each Variable Separately)	Corresponding x Value (Prior to Truncation)	Area of Beta Density after Truncation, before Readjustment to 1
9-8	$a_o/t \text{ max}$	0.94647	0.1924	27.70	0.1085	0.5591
9-9	S_E^*	0.94647	2.0020	34.09	1.3207	0.4488
9-10	$\Delta \log C$	0.94647	+0.5989	2.88	- 0.1356	0.9925

NOTE: See Figures 9-8 through 9-10.

9.11 UNCERTAINTY DISTRIBUTIONS FOR THE VARIABLES IN THE CYCLES-TO-LEAK EQUATION—SUMMARY

The uncertainty distributions for the three variables are given here in common format for comparison in Figures 9-11 through 9-14 for a_0/t MAX, S_E^* , and $\log C$, respectively. Table 9-3 gives the pertinent references and the inverse cumulative distributions. Note that trial graphical weighted random sampling is facilitated by the $F_1(x)$ scales for the weighting functions—Figures 9-8 through 9-10—being drawn to the same scale as those for the uncertainty distributions—Figures 9-11 through 9-14, except 9-13.

9.12 THE PROBABILITY ESTIMATE AT ONE STUDY SECTION

The weighting functions and uncertainty distributions just described, together with the linear equation of $\log n_t$ without B terms shown as Equation (5-39), were used to make estimates of the probability that cycles to leak, n_t , are less than 90, 360, and 1000 cycles, using the Monte Carlo method with importance sampling. Ten years of service are represented by 90 cycles, 40 years by 360 cycles. Three runs of 1000 trials each were made for each of the three numbers of cycles. The results are tabulated in Table 9-4, and for 360 cycles are displayed graphically in Figure 9-15. The overlapping of the confidence intervals shown in the figure for each of the 1000 trials is evidence of the stability of the method using the weighting functions adopted.

The most pertinent results appear in Table 9-4 on the line for runs 1-3, 3000 trials, 360 cycles. The overall estimate of probability of leakage in 40 years, using the uncertainty distributions found by judgment or measurement, for the chosen pipe study surface having area 5.2025 square inches, was 0.00769, with two standard deviation confidence limits due to the Monte Carlo work alone of 0.00633 to 0.00905. The variance of this estimate was 0.461, whereas with crude Monte Carlo, for the same number of trials, it would have been expected to be 2.543, indicating a reduction to 18.1% of the crude method variance by using importance sampling. Uncertainty contributed by the judgmental nature of uncertainty distributions for initial crack depth, and stress, is not included.

Figure 9-11. PDF and CDF of a_0/t Max Without U/S Test (Conditions in Table 9-3)

Figure 9-12. PDF and CDF of $a_0/t \text{ Max}$ With U/S Test (Conditions in Table 9-3)

Figure 9-13. Original and Fitted CDF's of S_E^* , Uncertainty Coefficient on Expansion Stress (Conditions in Table 9-3)

Figure 9-14. PDF and CDF of Log C (Conditions in Table 9-3)

Table 9-3
DETAILS FOR UNCERTAINTY DISTRIBUTIONS SHOWN IN FIGURES 9-11 TO 9-14
FOR VARIABLES IN LOW CYCLE FATIGUE CRACK GROWTH EQUATION

Figure	Random Variable	Original Display (See Also Table 7-1)	Density Function $f_{1X}(x) =$	Cumulative $F_{1X}(x) =$	Inverse Cumulative $F_{1X}^{-1}(x) =$
9-11	Maximum Initial Crack Depth Ratio, $X = a_o/t$ MAX ($Z = a_o/t$)	Cause 2.1, subsection 6.2, for Z, "Without U/S Testing," with $\alpha = \pi d/4$ for $d = 6.624$ in., and $\lambda = 0.1$ cracks/in. ² $\alpha = 5.2025$ in. ² $t = 0.432$ in.	$\frac{2\alpha\lambda}{x} e^{-\alpha\lambda [1-F_Z(x)]} \frac{(\ln 0.8 - \ln x)}{(\ln 0.8 + \ln 100)^2}$ [This is $dF_1(x)/dx$, the density on the x scale, as in Figure 9-11. For the density on log x scale, find $dF_1(x)/d \log x$.] $1 - F_Z(x) = \left(\frac{\log 0.8 - \log x}{\log 0.8 + \log 100} \right)^2$ per subsection 8.5.2	$e^{-\alpha\lambda [1-F_Z(x)]}$ per subsection 8.5.2	$0.8 \left[0.0125 (-\ln F_{1X}(x)/\alpha\lambda)^{1/2} \right]$
9-12	Maximum Initial Crack Depth Ratio, $X = a_o/t$ MAX ($Z = a_o/t$)	Cause 2.1, subsection 6.2, for Z, "With U/S Testing," with $\alpha = \pi d/4$ for $d = 6.624$ in., and $\lambda = 0.1$ cracks/in. ² $\alpha = 5.2025$ in. ² $t = 0.432$ in.	For $0.01 \leq x \leq 0.0667$: $\frac{2\alpha\lambda k}{x} (\ln 0.8 - \ln x) e^{-\alpha\lambda [1-F_Z(x)]}$ $1 - F_Z(x) = k[(\ln 0.8 - \ln x)^2 - (\ln 0.8 + \ln 15)(\ln 0.8 - \ln 0.12)]$ For $0.0667 \leq x \leq 0.12$: $\frac{2\alpha\lambda k}{x} (\ln 0.12 - \ln x) \left(\frac{\ln 0.8 + \ln 15}{\ln 0.12 + \ln 15} \right) e^{-\alpha\lambda [1-F_Z(x)]}$ $1 - F_Z(x) = k \left(\frac{\ln 0.8 + \ln 15}{\ln 0.12 + \ln 15} \right) (\ln 0.12 - \ln x)^2$ Where $k = 1/[(\ln 0.8 + \ln 100)^2 - (\ln 0.8 + \ln 15)(\ln 0.8 - \ln 0.12)]$ $1 - F_Z(x)$ per Equation (8-15)	$e^{-\alpha\lambda [1-F_Z(x)]}$ per Equation (8-16)	---
9-13	Uncertainty Coef. on Expansion Stress, S_E	PDF: Curve 4, Figure 6-2. CDF: Curve 4, Figure 6-3 Nominal expansion stress = 27,450 psi	---	---	(See Supplement 1 Table 9-3)
9-14	Intercept Parameter, Log C	A log C, random deviation from (mean log C = -3.962); nominal distribution, mean = 0, variance = 0.13802	Normal Distribution: $\frac{1}{\sigma\sqrt{2\pi}} \exp \left[-1/2 \left(\frac{x-\mu}{\sigma} \right)^2 \right]$	Normal Distribution: $\int_{-\infty}^x f(x) dx$	(See Supplement 2 to Table 9-3)
	Slope Parameter, B	Treated as constant: $B = 4.45$	---		

Supplement 1 to Table 9-3. Inverse Cumulative for S_E^*

$$0 \leq F_1 \leq 0.89917 \quad (F^{-1} \leq S_E^*)$$

$$0.45 \leq F^{-1} \leq 1.0471:$$

$$F^{-1} = \frac{2.2990 + \sqrt{0.0876 + 10.205 F_1}}{5.1024}$$

In Figure 9-13, Curve P

$$0.89917 < F_1 \leq 1 - 9.0 \times 10^{-4}:$$

$$1.0471 < F^{-1} \leq 8.1128:$$

$$F^{-1} = \frac{-\ln(1 - F_1) - 1.6}{0.6631}$$

Curve L1

$$1 - 9.0 \times 10^{-4} < F_1 \leq 1 - 2.2 \times 10^{-5}:$$

$$8.1128 < F^{-1} \leq 33.755:$$

$$F^{-1} = \frac{-\ln(1 - F_1) - 5.8}{0.1454}$$

Curve L2

$$1 - 2.2 \times 10^{-5} < F_1 \leq 1:$$

$$33.755 < F^{-1} \leq \sim 100$$

$$F^{-1} = \frac{-\ln(1 - F_1) - 7.4}{0.098}$$

Curve L3

Supplement 2 to Table 9-3

Rational Approximation for x_p (normal inverse CDF) where $Q(x_p) = p$

$$0 < p \leq .5$$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt = \int_x^\infty Z(t) dt$$

Equation 26.2.22, Reference 9-3:

$$x_p = t - \frac{a_0 + a_1 t}{1 + b_1 t + b_2 t^2 + \epsilon(p)}, \quad t = \sqrt{\ln \frac{1}{p^2}}$$

$$|\epsilon(p)| < 3 \times 10^{-3}$$

$$a_0 = 2.30753 \quad b_1 = .99229$$

$$a_1 = .27061 \quad b_2 = .04481$$

Table 9-4
ESTIMATES OF PROBABILITY OF LEAKAGE
(At One Piping Study Surface Having Area 5.2025 Square Inches)

Run No.	Trials	n_c = Cycles Shown	Estimate of Probability of Leakage: $P_r [n_t \leq n_c]$	Variance of Estimate, All $\times 10^{-6}$	2 σ Confidence Limits on Estimate		Variance of Same Estimate by Crude Monte Carlo (Binomial), All $\times 10^{-6}$	Reduction in Variance: $\frac{V \text{ (Importance Sampling)}}{V \text{ (Crude Monte Carlo)}}$
					Lower	Upper		
1†	1000	90	0.00535	0.5481	0.00386	0.00683	5.320	0.103
2	1000	90	0.00425	0.4075	0.00297	0.00553	4.232	0.0963
3	1000	90	0.00398	0.1469	0.00321	0.00475	3.960	0.0371
1-3	3000	90	0.00453	0.123	0.00383	0.00523	1.503	0.0818
1	1000	360	0.00867	2.109	0.00576	0.01157	8.594	0.246
2	1000	360	0.00714	0.970	0.00517	0.00911	7.087	0.137
3	1000	360	0.00727	1.072	0.00520	0.00934	7.220	0.148
1-3	3000	360	0.00769	0.461	0.00633	0.00905	2.543	0.181
1	1000	1000	0.01099	2.180	0.00804	0.01394	10.870	0.201
2	1000	1000	0.00917	1.032	0.00714	0.01121	9.086	0.114
3	1000	1000	0.00930	1.153	0.00715	0.01144	9.214	0.125
1-3	3000	1000	0.00982	0.485	0.00843	0.01121	3.241	0.150

NOTE: See Subsection 9-12 for discussion.

† Run 1 was performed simultaneously for $n_c = 90, 360$, and 1000, using the same random numbers and n_t solutions. Runs 2 and 3 were performed in the same way, using entirely different random numbers.

Figure 9-15. Probability of Leakage at One Study Surface in 40 Years, Due to Low-Cycle Fatigue

9.13 EXTENDING THE PROBABILITY ESTIMATE TO THE PIPING SYSTEM

Extending the above estimating method to the entire piping system requires the following steps:

1. Make assumptions about the independence of uncertainty distribution values among adjacent piping study sections. It would be reasonable to assume that initial crack depth and log C values are independent, but stress values are not. That is, if some error has caused a high stress in one study section, some high level of stress will be present in adjacent study sections. It would be reasonable, and conservative, to assign the same stress uncertainty coefficient to all design stresses in the loop.
2. Because of the nonindependence of stress uncertainty coefficients just cited, the probability of leakage in a loop is found by performing the importance sampling Monte Carlo work simultaneously at all piping study sections in a loop at which the design stress is high enough to make a significant contribution to the loop leakage probability, applying the nonzero weight to all outcomes less than 360 in the trial (if any), and retaining the highest such weight among the study sections. The probability estimate is found as described previously in this section, by summing the zeros and weights over the trials, and dividing by number of trials. The variance of the estimate is also found as before, by dividing the variance among the zeros and weights by the number of trials.
3. Assuming that there is no dependence among loops of any of the variables affecting leakage, the probability of leakage among n loops is

$$1 - \prod_{i=1}^n (1 - p_i) ,$$

where the probability of leakage in the i^{th} loop is p_i , as found in Step 2.

9.14 REFERENCES

- 9-1. Parzen, E., *Modern Probability Theory and Its Applications*, John Wiley and Sons, New York, 1960, pp. 371 ff.
- 9-2. Hammersley, J. M. and Handscomb, D. C., *Monte Carlo Methods*, John Wiley and Sons, New York, 1964.
- 9-3. Abramowitz, M. and Stegun, I. A., *Handbook of Mathematical Functions*, USGPO, Washington, DC, 1964, NBS AMS-55.

APPENDIX A. TASKS AND REPORTS OF THE PIPE RUPTURE STUDY

The following tasks have been conducted under the Pipe Rupture Study:

Task A	Probability Study
Task B	Stress Analysis
Task B-1	Computer Code Development
Task B-2	Problem Solutions
Task B-3	Experimental Stress Analysis
Task C	Fracture Mechanics
Task D	Low-Cycle Fatigue
Task D-1	Crack Growth Studies
Task D-2	Crack Initiation in the Presence of Stress Concentration
Task D-3	Prototype Pipe Fatigue Tests
Task D-4	Cyclic Strain Aging
Task D-5	Fatigue Data for Piping Code
Task K	Effect of BWR Environment

Work of these tasks is reported in the following progress reports, and is summarized in the topical reports listed thereafter.

PROGRESS REPORTS (QUARTERLY EXCEPT AS SHOWN)

GEAP-4911	Reactor Primary Coolant System Rupture Study Quarterly Progress Report No. 1, April-June 1965.
GEAP-4964	No. 2, July-September 1965.
GEAP-5082	No. 3, October-December 1965.
GEAP-5147	No. 4, January-March 1966.
GEAP-5192	No. 5, April-June 1966.
GEAP-5279	No. 6, July-September 1966.
GEAP-5427	No. 7, October-December 1966.
GEAP-5474	No. 8, January-March 1967.
GEAP-5512	No. 9, April-June 1967.
GEAP-5554	No. 10, July-September 1967.
GEAP-5587	No. 11, October-December 1967.
GEAP-5637	No. 12, January-March 1968.
GEAP-5680	No. 13, April-June 1968.
GEAP-5716	No. 14, July-September 1968.
GEAP-5770	No. 15, October-December 1968.
GEAP-10024	No. 16, January-March 1969.
GEAP-10072	No. 17, April-June 1969.
GEAP-10120	No. 18, July-September 1969.

GEAP-10143	No. 19, October—December 1969.
GEAP-11069	No. 20, January—March 1970.
GEAP-10207	No. 21, April—June 1970.
GEAP-10207-22	No. 22, July—September 1970.
GEAP-10207-23	No. 23, October—December 1970.
GEAP-10207-24	No. 24, January—June 1971 (six months).

Topical Reports and Tasks Covered

GEAP-4445	—	A Survey of Water Reactor Primary System Conditions Pertinent to the Study of Pipe Rupture, January 1964.
GEAP-4446	—	A Review of Fracture Modes as Related to Reactor Primary Coolant Pipe Rupture, May 1964.
GEAP-4474	—	Experimental and Analytical Program Recommendations—Reactor Pipe Rupture Study, January 1965.
GEAP-4574	—	Survey of Piping Failures for the Reactor Primary Coolant Pipe Rupture Study, May 1964.
GEAP-4578	—	Mechanical Design Considerations in Primary Nuclear Piping, March 1964.
GEAP-4678	C	Fracture Mechanics and the Stability of Engineering Structures, September 1965.
GEAP-5014	C	Fracture Toughness of Some Engineering Alloys, March 1966.
GEAP-5410	D-2	Low-Cycle Fatigue Strength Reduction in Notched Flat Plates, January 1967.
GEAP-5471	B-1	PAPA—Structural Analysis of Plates and Shells Using Trapezoidal and Triangular Plate Elements, March 1967.
GEAP-5553	B-1	PAPA—The Digital Computer Program for the Static Analysis of Structures Made up of Plate and Panel Elements, October 1967.
GEAP-5557	B-2	Stress Intensity for a Circumferential Through-Wall Crack in a Straight Pipe, January 1968.
GEAP-5607	D-1	Fatigue Crack Growth in Nuclear Reactor Piping Steels, March 1968.
T&AM-677	B-3	A Three-Dimensional Photoelastic Study of Stress Near Cracks in Shell Structures, University of Illinois, March 1968.
GEAP-5620	C	Failure Behavior in ASTM A 106-B Pipes Containing Axial Through-Wall Flaws, April 1968.
GEAP-5622	C	A Failure Diagram for Axially Flawed Pipes, April 1968.
GEAP-5653	—	Status of Pipe Rupture Study at General Electric—II, July 1968.
GEAP-5714	D-2	Notched High-Strain Fatigue Behavior of Three Low-Strength Structural Steels, September 1968.
GEAP-5724	B-2	BWR Primary Piping Stresses for System Reliability Analysis, September 1968.
GEAP-5726	D-2	Influence of Stress/Strain Concentration and Mean Stress on the Low-Cycle Fatigue Behavior of Three Structural Steels at Room Temperature, September 1968.

GEAP-10452

GEAP-10007	B-1	Restart for the PAPA Program, March 1969.
GEAP-10090	D-2	The 550°F Notched High-Strain Fatigue Behavior of Three Low-Strength Structural Steels, August 1969.
GEAP-10135	D-3	Low-Cycle Fatigue of Prototype Piping, January 1970.
GEAP-10140	D-4	The Effect of Cyclic Strain Aging on the Embrittlement of a Plain Carbon Steel Plate, July 1970.
GEAP-10170	D-2	The Effect of Stress Concentration on the Low-Cycle Fatigue of Three Low-Strength Structural Steels at Room Temperature and 500°F, March 1970.
GEAP-10181	D-1	Low Cycle, Strain Controlled Fatigue Crack Propagation in Nuclear Piping Steels, May 1970
GEAP-10205	—	Status of Pipe Rupture Study at General Electric Company, Part III, June 1970.
GEAP-10236	C	Failure Behavior of Flawed Carbon Steel Pipes and Fittings, October 1970.
GEAP-10452	A	Estimating Pipe Reliability by the Distribution of Time-to-Damage Method, March 1972.

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United Kingdom Atomic Energy Authority Reactor Materials Laboratory Wigshaw Lane, Culcheth Warrington, Lancs. England Attn: R.W. Nichols	1	United Engineers and Constructors, Inc. 1401 Arch Street Philadelphia, Pennsylvania 19105 Attn: Mr. John Crowley	1
Atomic Energy of Canada Limited Chalk River Nuclear Laboratories Chalk River, Ontario, Canada Attn: Dr. George Pon	1	Argonne National Laboratory 9700 South Cass Avenue Argonne, Illinois Attn: Mr. Craig Cheng	1
Mr. Robert D. Wylier Department of Materials Engineering Southwest Research Institute 8599 Culebra Road San Antonio, Texas 78228	1	AEG Telefunken AEC Hochhaus Sued 6 Frankfurt/Main 70 West Germany Attn: Mr. Dieter Ewers	1
O.A. Kellerman Institute Fur Reaktorsicherheit Der Technischen Uberwachungs Vereine, e.V. 5 KOLN 1 Glockengasse 2 West Germany	1	Division of Reactor Development & Technology U.S. Atomic Energy Commission Washington, D.C. 20545 Attn: J. R. Hunter Chief, Reactor Vessels Branch	1
C. A. G. Phillips U.K.E.A. Safeguards Division Authority Health & Safety Branch Risley, Warrington, Lancashire England	1	Asst. Director, Instruction Division U.S. Atomic Energy Commission Chicago Operations Office 9800 South Cass Avenue Argonne, Illinois 60439 Attn: R. M. Moser	1
Knolls Atomic Power Laboratory P. O. Box 1072 Schenectady, New York 12301 Attn: Dr. Robert A. Barnes	1	Mr. T. R. Mager Westinghouse Electric Company PWR Systems Division P. O. Box 355 Pittsburgh, Pennsylvania 15230	1
Reactor Materials Branch Metallurgy Division Naval Research Laboratory Washington, D. C. 20545 Attn: Mr. L. E. Steele	1	Hans A. Maurer Commission of the European Communities RUE DE LA LOI 200 1040 Brussels, Belgium	1
Sargent and Lundy 140 So. Dearborn Street Chicago, Illinois 60603 Attn: Mr. E. B. Branch	1	Atomic Energy Commission Division of Reactor Development & Technology Washington, D.C. 20545 Attn: R. W. Cochran	1
Metals Research Chicago Bridge and Iron Company 901 W. 22nd Street Oak Brook, Illinois 60521 Attn: Mr. Robert M. Bruscato	1	Batelle Memorial Institute 505 King Avenue Columbus, Ohio 43201 Attn: Mr. E. Rodabaugh	1
		Bechtel Corporation 50 Beale Street San Francisco, California 94105 Attn: Mr. R. P. Schmitz	1