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ACCELERATOR DEPARTMENT
Informal Report

SPACE CHARGE IN THE AGS AND SOME OF ITS CONSEQUENCES

J. Claus

August 6, 1973

NOTICE

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ABSTRACT

Preliminary results of calculations with respect to the self field of the beam in the AGS and its effect on the transverse motion of individual particles are presented. The effect appears to be severe at present intensities and means to minimize its consequences are discussed briefly.

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MASTER

ACCELERATOR DEPARTMENT
Informal Report

SPACE CHARGE IN THE AGS AND SOME OF ITS CONSEQUENCES

J. Claus
August 6, 1973

Summary

This note offers some preliminary results of transverse space charge calculations within the circulating beam in the AGS. To simplify the solution of Poisson's equation the true geometry of vacuum chamber and magnet poles is approximated by a rectangular one. The effect of the self-field of the beam on the motion of individual particles is considered and found to be severe for present intensities. It is proposed that the linear charge density be kept as low as possible during capture and early acceleration and that the injection process be modified to yield a more uniform or perhaps even dimpled density distribution over the available aperture to circumvent the problem and that the widths of the more important stopbands (1, $1/2$, $2/3$, $3/4$, $3/5$, $4/5$) be reduced as much as possible to alleviate it.

Introduction

The force field inside a beam of charged particles may be described as being due to the combination of the electric field associated with its charge distribution and the magnetic field associated with its current distribution. In bunched beams both fields are time dependent, yielding dc as well as ac components. We assume the frequencies of the ac components to be sufficiently low to permit calculation of the fields in a two dimensional geometry, neglecting the time dependent terms in Maxwell's equations. This assumption is valid as long as the relative change in source distribution over azimuthal distances of the order of the largest cross sectional dimension of the vacuum chamber are negligible.

We assume that the walls of the vacuum chamber have very high electrical conductivity, so that they form equipotentials for the electric field and force

the orthogonal component of the ac part of the magnetic field to zero on their inner surface. As is well known¹ the ac part of the force field is then proportional to the ac part of the electric field divided by γ^2 , where $\gamma = 1 + E/E_0$, with E the kinetic and E_0 the rest energy of the particles. The dc part of the magnetic field penetrates the vacuum chamber and is bounded by the pole iron of the accelerator magnets. We assume the permeability of the iron to be very high; if this is valid the pole surface forms a magnetic equipotential. Because the dc part of the electric field is bounded by the vacuum chamber rather than the pole iron, the proportionality between electric field and magnetic field are lost and the derivation of the dc part of the force field is somewhat more complex.

With respect to the consequences for the particle motion we limit ourselves to the motion of individual particles within a stationary source distribution since the collective motion of the distribution has been treated before.¹

Method

In a rectangular geometry Poisson's equation may be solved fairly easily by expanding the source distribution into a double periodic series. Therefore, we approximated the actual AGS vacuum chamber with one of a rectangular cross section and took the magnetic boundaries to form a rectangular aperture also. As shown in Fig. 1 the electric and magnetic apertures are coaxial and have coinciding symmetry axes; the width of the magnetic aperture is much (4x) larger than its height in order to simulate two infinitely wide pole faces.

Fig. 1

This method for finding the fields can be used only if the series for the source distribution has no constant term because else one experiences difficulties in meeting the boundary requirements. This causes no difficulties for the electric field but for the dc magnetic field it has to be forced to zero by addition of compensating current blocs as indicated in Fig. 1. These blocs (which carry identical currents, together equal and opposite to the dc part of the beam current) do not contribute to the field within the beam envelope.*

Although the method may be used for any source distribution within the boundaries we limited ourselves to

$$s = \hat{s} \cos\left(\frac{\pi}{2} \frac{x}{\xi}\right) \cos\left(\frac{\pi}{2} \frac{y}{\eta}\right) \quad \left|\frac{x}{\xi}\right| \text{ and } \left|\frac{y}{\eta}\right| \leq 1$$

$$s = 0 \quad \left|\frac{x}{\xi}\right| \text{ and/or } \left|\frac{y}{\eta}\right| > 1$$

for simplicity. This distribution is associated with properly centered uniformly populated emittance ellipses for the radial and axial directions. It is probably fairly realistic in the radial (x) direction as a consequence of our present injection method. For the same reason an exponential or a $(1 + \cos \pi \frac{y}{\eta})$ is probably better than the $\cos(\frac{\pi}{2} \frac{y}{\eta})$ used axially, though this is a good choice if we assume vertical stacking. 2ξ and 2η represent the radial and axial extends of the beam, x and y displacements with respect to the axis of the vacuum chamber.

Results

We programmed the 6600 to solve these potential problems. Using 60 x 60 harmonics a solution for either the electric or the magnetic case takes about 4.5 sec. Figures 2 and 3 give sample output for the listed conditions.

Vacuum Chamber Aperture	15 x 7.5 cm ²
Magnetic Aperture	30 x 7.5 cm ²
Beam Cross Section	10 x 6 cm ²
Beam Energy	200 MeV
Linear Charge Density	2.5 x 10 ¹⁰ protons/m.

The linear charge density chosen corresponds with 2×10^{13} protons in a dc beam, the beam dimensions are valid in the β_{av} points and correspond to those of a matched beam that just fills the available aperture. (Limited horizontally by inflector and extractor magnets and vertically to 7.2 cm.) The computer outputs cover the upper right hand quadrant of the beam cross section, the maps

*I am indebted to G. Parzen for noting this.

DEY/BX IN KVOLT/MH-02

Y (CM)	0.000	.250	.500	.750	1.000	1.250	1.500	1.750	2.000	2.250	2.500	2.750	3.000	3.250	3.500	3.750	4.000	4.250	4.500	4.750	5.000
13.0	13.7	13.6	13.4	13.1	12.5	12.7	12.1	11.4	10.6	9.7	8.9	7.5	6.2	5.0	3.9	2.9	2.0	1.1	.2	-1.0	-2.1
13.5	13.4	13.2	12.9	12.5	11.8	11.9	11.2	10.4	9.5	8.6	7.7	6.3	5.0	3.8	2.7	1.7	.8	0.0	-1.1	-2.2	-3.3
14.0	13.9	13.7	13.4	13.0	12.2	12.3	11.5	10.6	9.7	8.8	7.9	6.5	5.2	4.0	2.9	1.9	1.0	0.1	-1.0	-2.1	-3.2
14.5	14.4	14.2	13.9	13.5	12.6	12.7	11.8	10.8	9.9	9.0	8.1	6.7	5.4	4.2	3.1	2.1	1.2	0.3	-0.6	-1.7	-2.8
15.0	14.9	14.7	14.4	14.0	13.0	13.1	12.2	11.2	10.3	9.4	8.5	7.1	5.8	4.6	3.5	2.5	1.6	0.7	-0.2	-1.3	-2.4
15.5	15.4	15.2	14.9	14.5	13.4	13.5	12.5	11.5	10.6	9.7	8.8	7.3	6.0	4.8	3.7	2.7	1.8	0.9	0.0	-1.1	-2.2
16.0	15.9	15.7	15.4	15.0	13.8	13.9	12.8	11.7	10.8	9.9	9.0	7.5	6.2	5.0	3.9	2.9	2.0	1.1	.2	-0.9	-2.0
16.5	16.4	16.2	15.9	15.5	14.3	14.4	13.3	12.2	11.3	10.4	9.5	8.0	6.7	5.5	4.4	3.4	2.5	1.6	.7	-0.4	-1.5
17.0	16.9	16.7	16.4	16.0	14.7	14.8	13.6	12.5	11.6	10.7	9.8	8.2	6.9	5.7	4.6	3.6	2.7	1.8	.9	0.0	-1.1
17.5	17.4	17.2	16.9	16.5	15.2	15.3	14.0	12.8	11.9	11.0	10.1	8.5	7.2	6.0	4.9	3.9	3.0	2.1	1.2	.3	-0.8
18.0	17.9	17.7	17.4	17.0	15.6	15.7	14.3	13.1	12.2	11.3	10.4	8.8	7.5	6.3	5.2	4.2	3.3	2.4	1.5	.6	-0.5
18.5	18.4	18.2	17.9	17.5	16.1	16.2	14.7	13.5	12.6	11.7	10.8	9.1	7.8	6.6	5.5	4.5	3.6	2.7	1.8	.9	0.0
19.0	18.9	18.7	18.4	18.0	16.5	16.6	15.0	13.7	12.8	11.9	11.0	9.3	8.0	6.8	5.7	4.7	3.8	2.9	2.0	1.1	.2
19.5	19.4	19.2	18.9	18.5	16.9	17.0	15.3	14.0	13.1	12.2	11.3	9.5	8.2	7.0	5.9	4.9	4.0	3.1	2.2	1.3	.4
20.0	19.9	19.7	19.4	19.0	17.2	17.3	15.5	14.2	13.3	12.4	11.5	9.7	8.4	7.2	6.1	5.1	4.2	3.3	2.4	1.5	.6
20.5	20.4	20.2	19.9	19.5	17.6	17.7	15.8	14.5	13.6	12.7	11.8	10.0	8.7	7.5	6.4	5.4	4.5	3.6	2.7	1.8	.9
21.0	20.9	20.7	20.4	20.0	18.0	18.1	16.1	14.7	13.8	12.9	12.0	10.2	8.9	7.7	6.6	5.6	4.7	3.8	2.9	2.0	1.1
21.5	21.4	21.2	20.9	20.5	18.4	18.5	16.4	15.0	14.1	13.2	12.3	10.4	9.1	7.9	6.8	5.8	4.9	4.0	3.1	2.2	1.3
22.0	21.9	21.7	21.4	21.0	18.8	18.9	16.6	15.2	14.3	13.4	12.5	10.6	9.3	8.1	7.0	6.0	5.1	4.2	3.3	2.4	1.5
22.5	22.4	22.2	21.9	21.5	19.1	19.2	16.8	15.3	14.4	13.5	12.6	10.7	9.4	8.2	7.1	6.1	5.2	4.3	3.4	2.5	1.6
23.0	22.9	22.7	22.4	22.0	19.4	19.5	17.0	15.5	14.6	13.7	12.8	10.9	9.6	8.4	7.3	6.3	5.4	4.5	3.6	2.7	1.8
23.5	23.4	23.2	22.9	22.5	19.7	19.8	17.2	15.7	14.8	13.9	13.0	11.1	9.8	8.6	7.5	6.5	5.6	4.7	3.8	2.9	2.0
24.0	23.9	23.7	23.4	23.0	20.0	20.1	17.3	15.8	14.9	14.0	13.1	11.2	9.9	8.7	7.6	6.6	5.7	4.8	3.9	3.0	2.1
24.5	24.4	24.2	23.9	23.5	20.2	20.3	17.4	15.9	15.0	14.1	13.2	11.3	10.0	8.8	7.7	6.7	5.8	4.9	4.0	3.1	2.2
25.0	24.9	24.7	24.4	24.0	20.4	20.5	17.5	16.0	15.1	14.2	13.3	11.4	10.1	8.9	7.8	6.8	5.9	5.0	4.1	3.2	2.3
25.5	25.4	25.2	24.9	24.5	20.6	20.7	17.6	16.1	15.2	14.3	13.4	11.5	10.2	9.0	7.9	6.9	6.0	5.1	4.2	3.3	2.4
26.0	25.9	25.7	25.4	25.0	20.8	20.9	17.7	16.2	15.3	14.4	13.5	11.6	10.3	9.1	8.0	7.0	6.1	5.2	4.3	3.4	2.5
26.5	26.4	26.2	25.9	25.5	21.0	21.1	17.8	16.3	15.4	14.5	13.6	11.7	10.4	9.2	8.1	7.1	6.2	5.3	4.4	3.5	2.6
27.0	26.9	26.7	26.4	26.0	21.1	21.2	17.9	16.4	15.5	14.6	13.7	11.8	10.5	9.3	8.2	7.2	6.3	5.4	4.5	3.6	2.7
27.5	27.4	27.2	26.9	26.5	21.3	21.4	18.0	16.5	15.6	14.7	13.8	11.9	10.6	9.4	8.3	7.3	6.4	5.5	4.6	3.7	2.8
28.0	27.9	27.7	27.4	27.0	21.4	21.5	18.1	16.6	15.7	14.8	13.9	12.0	10.7	9.5	8.4	7.4	6.5	5.6	4.7	3.8	2.9
28.5	28.4	28.2	27.9	27.5	21.6	21.7	18.2	16.7	15.8	14.9	14.0	12.1	10.8	9.6	8.5	7.5	6.6	5.7	4.8	3.9	3.0
29.0	28.9	28.7	28.4	28.0	21.7	21.8	18.3	16.8	15.9	15.0	14.1	12.2	10.9	9.7	8.6	7.6	6.7	5.8	4.9	4.0	3.1
29.5	29.4	29.2	28.9	28.5	21.9	22.0	18.4	16.9	16.0	15.1	14.2	12.3	11.0	9.8	8.7	7.7	6.8	5.9	5.0	4.1	3.2
30.0	29.9	29.7	29.4	29.0	22.0	22.1	18.5	17.0	16.1	15.2	14.3	12.4	11.1	9.9	8.8	7.8	6.9	6.0	5.1	4.2	3.3
30.5	30.4	30.2	30.0	29.5	22.1	22.2	18.6	17.1	16.2	15.3	14.4	12.5	11.2	10.0	8.9	7.9	7.0	6.1	5.2	4.3	3.4
31.0	30.9	30.7	30.4	30.0	22.2	22.3	18.7	17.2	16.3	15.4	14.5	12.6	11.3	10.1	9.0	8.0	7.1	6.2	5.3	4.4	3.5
31.5	31.4	31.2	30.9	30.5	22.3	22.4	18.8	17.3	16.4	15.5	14.6	12.7	11.4	10.2	9.1	8.1	7.2	6.3	5.4	4.5	3.6
32.0	31.9	31.7	31.4	31.0	22.4	22.5	18.9	17.4	16.5	15.6	14.7	12.8	11.5	10.3	9.2	8.2	7.3	6.4	5.5	4.6	3.7
32.5	32.4	32.2	31.9	31.5	22.5	22.6	19.0	17.5	16.6	15.7	14.8	12.9	11.6	10.4	9.3	8.3	7.4	6.5	5.6	4.7	3.8
33.0	32.9	32.7	32.4	32.0	22.6	22.7	19.1	17.6	16.7	15.8	14.9	13.0	11.7	10.5	9.4	8.4	7.5	6.6	5.7	4.8	3.9
33.5	33.4	33.2	32.9	32.5	22.7	22.8	19.2	17.7	16.8	15.9	15.0	13.1	11.8	10.6	9.5	8.5	7.6	6.7	5.8	4.9	4.0
34.0	33.9	33.7	33.4	33.0	22.8	22.9	19.3	17.8	16.9	16.0	15.1	13.2	11.9	10.7	9.6	8.6	7.7	6.8	5.9	5.0	4.1
34.5	34.4	34.2	33.9	33.5	22.9	23.0	19.4	17.9	17.0	16.1	15.2	13.3	12.0	10.8	9.7	8.7	7.8	6.9	6.0	5.1	4.2
35.0	34.9	34.7	34.4	34.0	23.0	23.1	19.5	18.0	17.1	16.2	15.3	13.4	12.1	10.9	9.8	8.8	7.9	7.0	6.1	5.2	4.3
35.5	35.4	35.2	34.9	34.5	23.1	23.2	19.6	18.1	17.2	16.3	15.4	13.5	12.2	11.0	9.9	8.9	8.0	7.1	6.2	5.3	4.4
36.0	35.9	35.7	35.4	35.0	23.2	23.3	19.7	18.2	17.3	16.4	15.5	13.6	12.3	11.1	10.0	9.0	8.1	7.2	6.3	5.4	4.5
36.5	36.4	36.2	35.9	35.5	23.3	23.4	19.8	18.3	17.4	16.5	15.6	13.7	12.4	11.2	10.1	9.1	8.2	7.3	6.4	5.5	4.6
37.0	36.9	36.7	36.4	36.0	23.4	23.5	19.9	18.4	17.5	16.6	15.7	13.8	12.5	11.3	10.2	9.2	8.3	7.4	6.5	5.6	4.7
37.5	37.4	37.2	36.9	36.5	23.5	23.6	20.0	18.5	17.6	16.7	15.8	13.9	12.6	11.4	10.3	9.3	8.4	7.5	6.6	5.7	4.8
38.0	37.9	37.7	37.4	37.0	23.6	23.7	20.1	18.6	17.7	16.8	15.9	14.0	12.7	11.5	10.4	9.4	8.5	7.6	6.7	5.8	4.9
38.5	38.4	38.2	37.9	37.5	23.7	23.8	20.2	18.7	17.8	16.9	16.0	14.1	12.8	11.6	10.5	9.5	8.6	7.7	6.8	5.9	5.0
39.0	38.9	38.7	38.4	38.0	23.8	23.9	20.3	18.8	17.9	17.0	16.1	14.2	12.9	11.7	10.6	9.6	8.7	7.8	6.9	6.0	5.1
39.5	39.4	39.2	38.9	38.5	23.9	24.0	20.4	18.9	18.0	17.1	16.2	14.3	13.0	11.8	10.7	9.7	8.8	7.9	7.0	6.1	5.2
40.0	39.9	39.7	39.4	39.0	24.0	24.1	20.5	19.0	18.1	17.2	16.3	14.4	13.1	11.9	10.8	9.8	8.9	8.0	7.1	6.2	5.3
40.5	40.4	40.2	39.9	39.5	24.1	24.2	20.6	19.1	18.2	17.3	16.4	14.5	13.2	12.0	10.9	9.9	9.0	8.1	7.2	6.3	5.4
41.0	40.9	40.7	40.4	40.0	24.2	24.3	20.7	19.2	18.3	17.4	16.5	14.6	13.3	12.1	11.0	10.0	9.1	8.2	7.3	6.4	5.5
41.5	41.4	41.2	40.9	40.5	24.3	24.4	20.8	19.3	18.4	17.5	16.6	14.7	13.4	12.2	11.1	10.1	9.2	8.3	7.4	6.5	5.6
42.0	41.9	41.7	41.4	41.0	24.4	24.5	20.9	19.4	18.5	17.6	16.7	14.8	13.5	12.3	11.2	10.2	9.3	8.4	7.5	6.6	5.7
42.5	42.4	42.2	41.9	41.5	24.5	24.6	21.0	19.5	18.6	17.7	16.8	14.9	13.6	12.4	11.3	10.3	9.4	8.5	7.6	6.7	5.8
43.0	42.9	42.7	42.4	42.0	24.6	24.7	21.1	19.6	18.7	17.8	16.9	15.0	13.7	12.5	11.4	10.4	9.5	8.6	7.7	6.8	5.9
43.5	43.4	43.2	42.9	42.5	24.7	24.8	21.2	19.7	18.8	17.9	17.0	15.1	13.8	12.6	11.5	10.5	9.6	8.7			

DEY/DY IN KVOLT/M*2

Y (CM)	0.000	.250	.500	.750	1.000	1.250	1.500	1.750	2.000	2.250	2.500	2.750	3.000	3.250	3.500	3.750	4.000	4.250	4.500	4.750	5.000
3.000	-13.1	-13.0	-12.9	-12.8	-12.7	-12.6	-12.5	-12.4	-12.3	-12.2	-12.1	-12.0	-11.9	-11.8	-11.7	-11.6	-11.5	-11.4	-11.3	-11.2	-11.1
2.750	-11.9	-11.8	-11.7	-11.6	-11.5	-11.4	-11.3	-11.2	-11.1	-11.0	-10.9	-10.8	-10.7	-10.6	-10.5	-10.4	-10.3	-10.2	-10.1	-10.0	-9.9
2.500	-9.8	-9.7	-9.6	-9.5	-9.4	-9.3	-9.2	-9.1	-9.0	-8.9	-8.8	-8.7	-8.6	-8.5	-8.4	-8.3	-8.2	-8.1	-8.0	-7.9	-7.8
2.250	21.4	21.3	21.2	21.1	21.0	20.9	20.8	20.7	20.6	20.5	20.4	20.3	20.2	20.1	20.0	19.9	19.8	19.7	19.6	19.5	19.4
2.000	32.7	32.6	32.5	32.4	32.3	32.2	32.1	32.0	31.9	31.8	31.7	31.6	31.5	31.4	31.3	31.2	31.1	31.0	30.9	30.8	30.7
1.750	43.6	43.5	43.4	43.3	43.2	43.1	43.0	42.9	42.8	42.7	42.6	42.5	42.4	42.3	42.2	42.1	42.0	41.9	41.8	41.7	41.6
1.500	54.2	54.1	54.0	53.9	53.8	53.7	53.6	53.5	53.4	53.3	53.2	53.1	53.0	52.9	52.8	52.7	52.6	52.5	52.4	52.3	52.2
1.250	64.4	64.3	64.2	64.1	64.0	63.9	63.8	63.7	63.6	63.5	63.4	63.3	63.2	63.1	63.0	62.9	62.8	62.7	62.6	62.5	62.4
1.000	74.1	74.0	73.9	73.8	73.7	73.6	73.5	73.4	73.3	73.2	73.1	73.0	72.9	72.8	72.7	72.6	72.5	72.4	72.3	72.2	72.1
0.750	83.2	83.1	83.0	82.9	82.8	82.7	82.6	82.5	82.4	82.3	82.2	82.1	82.0	81.9	81.8	81.7	81.6	81.5	81.4	81.3	81.2
0.500	91.7	91.6	91.5	91.4	91.3	91.2	91.1	91.0	90.9	90.8	90.7	90.6	90.5	90.4	90.3	90.2	90.1	90.0	89.9	89.8	89.7
0.250	99.6	99.5	99.4	99.3	99.2	99.1	99.0	98.9	98.8	98.7	98.6	98.5	98.4	98.3	98.2	98.1	98.0	97.9	97.8	97.7	97.6
0.000	104.7	104.6	104.5	104.4	104.3	104.2	104.1	104.0	103.9	103.8	103.7	103.6	103.5	103.4	103.3	103.2	103.1	103.0	102.9	102.8	102.7
0.000	113.2	113.1	113.0	112.9	112.8	112.7	112.6	112.5	112.4	112.3	112.2	112.1	112.0	111.9	111.8	111.7	111.6	111.5	111.4	111.3	111.2
0.000	118.8	118.7	118.6	118.5	118.4	118.3	118.2	118.1	118.0	117.9	117.8	117.7	117.6	117.5	117.4	117.3	117.2	117.1	117.0	116.9	116.8
0.000	123.6	123.5	123.4	123.3	123.2	123.1	123.0	122.9	122.8	122.7	122.6	122.5	122.4	122.3	122.2	122.1	122.0	121.9	121.8	121.7	121.6
0.000	127.6	127.5	127.4	127.3	127.2	127.1	127.0	126.9	126.8	126.7	126.6	126.5	126.4	126.3	126.2	126.1	126.0	125.9	125.8	125.7	125.6
0.000	131.8	131.7	131.6	131.5	131.4	131.3	131.2	131.1	131.0	130.9	130.8	130.7	130.6	130.5	130.4	130.3	130.2	130.1	130.0	129.9	129.8
0.000	134.4	134.3	134.2	134.1	134.0	133.9	133.8	133.7	133.6	133.5	133.4	133.3	133.2	133.1	133.0	132.9	132.8	132.7	132.6	132.5	132.4
0.000	134.8	134.7	134.6	134.5	134.4	134.3	134.2	134.1	134.0	133.9	133.8	133.7	133.6	133.5	133.4	133.3	133.2	133.1	133.0	132.9	132.8

0.000 .250 .500 .750 1.000 1.250 1.500 1.750 2.000 2.250 2.500 2.750 3.000 3.250 3.500 3.750 4.000 4.250 4.500 4.750 5.000
X (CM)

DEY/DY = 131.9-16.84*Y*2... (KV/M*2; Y IN CM)

DBX/DY IN MILLIGAUSS/M																					
Y (CM)	0.000	.250	.500	.750	1.000	1.250	1.500	1.750	2.000	2.250	2.500	2.750	3.000	3.250	3.500	3.750	4.000	4.250	4.500	4.750	5.000
3.000	-1405	-1400	-1396	-1392	-1389	-1387	-1383	-1379	-1373	-1368	-1366	-1362	-1358	-1354	-1350	-1346	-1342	-1337	-1331	-1325	-1319
2.750	-1359	-1355	-1353	-1352	-1350	-1348	-1346	-1343	-1339	-1334	-1332	-1328	-1324	-1320	-1316	-1312	-1308	-1303	-1297	-1291	-1285
2.500	-1314	-1310	-1309	-1307	-1305	-1304	-1302	-1299	-1295	-1290	-1288	-1284	-1280	-1276	-1272	-1268	-1264	-1259	-1253	-1247	-1241
2.250	-1269	-1265	-1264	-1263	-1261	-1260	-1258	-1255	-1251	-1246	-1244	-1240	-1236	-1232	-1228	-1224	-1220	-1215	-1209	-1203	-1197
2.000	-1224	-1220	-1219	-1218	-1216	-1215	-1213	-1210	-1206	-1201	-1199	-1195	-1191	-1187	-1183	-1179	-1175	-1170	-1164	-1158	-1152
1.750	-1179	-1175	-1174	-1173	-1171	-1170	-1168	-1165	-1161	-1156	-1154	-1150	-1146	-1142	-1138	-1134	-1130	-1125	-1119	-1113	-1107
1.500	-1134	-1130	-1129	-1128	-1126	-1125	-1123	-1120	-1116	-1111	-1109	-1105	-1101	-1097	-1093	-1089	-1085	-1080	-1074	-1068	-1062
1.250	-1089	-1085	-1084	-1083	-1081	-1080	-1078	-1075	-1071	-1066	-1064	-1060	-1056	-1052	-1048	-1044	-1040	-1035	-1029	-1023	-1017
1.000	-1044	-1040	-1039	-1038	-1036	-1035	-1033	-1030	-1026	-1021	-1019	-1015	-1011	-1007	-1003	-999	-995	-990	-984	-978	-972
0.750	-999	-995	-994	-993	-991	-990	-988	-985	-981	-976	-974	-970	-966	-962	-958	-954	-950	-945	-939	-933	-927
0.500	-954	-950	-949	-948	-946	-945	-943	-940	-936	-931	-929	-925	-921	-917	-913	-909	-905	-900	-894	-888	-882
0.250	-909	-905	-904	-903	-901	-900	-898	-895	-891	-886	-884	-880	-876	-872	-868	-864	-860	-855	-849	-843	-837
0.000	-864	-860	-859	-858	-856	-855	-853	-850	-846	-841	-839	-835	-831	-827	-823	-819	-815	-810	-804	-798	-792
0.000	-819	-815	-814	-813	-811	-810	-808	-805	-801	-796	-794	-790	-786	-782	-778	-774	-770	-765	-759	-753	-747
0.000	-774	-770	-769	-768	-766	-765	-763	-760	-756	-751	-749	-745	-741	-737	-733	-729	-725	-720	-714	-708	-702
0.000	-729	-725	-724	-723	-721	-720	-718	-715	-711	-706	-704	-700	-696	-692	-688	-684	-680	-675	-669	-663	-657
0.000	-684	-680	-679	-678	-676	-675	-673	-670	-666	-661	-659	-655	-651	-647	-643	-639	-635	-630	-624	-618	-612
0.000	-639	-635	-634	-633	-631	-630	-628	-625	-621	-616	-614	-610	-606	-602	-598	-594	-590	-585	-579	-573	-567
0.000	-594	-590	-589	-588	-586	-585	-583	-580	-576	-571	-569	-565	-561	-557	-553	-549	-545	-540	-534	-528	-522
0.000	-549	-545	-544	-543	-541	-540	-538	-535	-531	-526	-524	-520	-516	-512	-508	-504	-500	-495	-489	-483	-477
0.000	-504	-500	-499	-498	-496	-495	-493	-490	-486	-481	-479	-475	-471	-467	-463	-459	-455	-450	-444	-438	-432
0.000	-459	-455	-454	-453	-451	-450	-448	-445	-441	-436	-434	-430	-426	-422	-418	-414	-410	-405	-399	-393	-387
0.000	-414	-410	-409	-408	-406	-405	-403	-400	-396	-391	-389	-385	-381	-377	-373	-369	-365	-360	-354	-348	-342
0.000	-369	-365	-364	-363	-361	-360	-358	-355	-351	-346	-344	-340	-336	-332	-328	-324	-320	-315	-309	-303	-297
0.000	-324	-320	-319	-318	-316	-315	-313	-310	-306	-301	-299	-295	-291	-287	-283	-279	-275	-270	-264	-258	-252
0.000	-279	-275	-274	-273	-271	-270	-268	-265	-261	-256	-254	-250	-246	-242	-238	-234	-230	-225	-219	-213	-207
0.000	-234	-230	-229	-228	-226	-225	-223	-220	-216	-211	-209	-205	-201	-197	-193	-189	-185	-180	-174	-168	-162
0.000	-189	-185	-184	-183	-181	-180	-178	-175	-171	-166	-164	-160	-156	-152	-148	-144	-140	-135	-129	-123	-117
0.000	-144	-140	-139	-138	-136	-135	-133	-130	-126	-121	-119	-115	-111	-107	-103	-99	-95	-90	-84	-78	-72
0.000	-99	-95	-94	-93	-91	-90	-88	-85	-81	-76	-74	-70	-66	-62	-58	-54	-50	-45	-39	-33	-27
0.000	-54	-50	-49	-48	-46	-45	-43	-40	-36	-31	-29	-25	-21	-17	-13	-9	-5	0	4	8	12
0.000	-9	-5	-4	-3	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0.000	46	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70
0.000	101	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125
0.000	156	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
0.000	201	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225
0.000	250	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274
0.000	299	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323
0.000	344	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363	364	365	366	367	368
0.000	393	398	399	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417
0.000	444	449	450	451	452	453	454	455	456	457	458	459	460	461	462	463	464	465	466	467	468
0.000	493	498	499	500	501	502	503	504	505	506	507	508	509	510	511	512	513	514	515	516	517
0.000	544	549	550	551	552	553	554	555	556	557	558	559	560	561	562	563	564	565	566	567	568
0.000	593	598	599	600	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616	617
0.000	644	649	650	651	652	653	654	655	656	657	658	659	660	661	662	663	664	665	666	667	668
0.000	693	698	699	700	701	702	703	704	705	706	707	708	709	710	711	712	713	714	715	716	717
0.000	744	749	750	751	752	753	754	755	756	757	758	759	760	761	762	763	764	765	766	767	768
0.000	793	798	799	800	801	802	803	804	805	806	807	808	809	810	811	812	813	814	815	816	817
0.000	844	849	850	851	852	853	854	855	856	857	858	859	860	861	862	863	864	865	866	867	868
0.000	893	898	899	900	901	902	903	904	905	906	907	908	909	910	911	912	913	914	915	916	917
0.000	944	949	950	951	952	953	954	955	956	957	958	959	960	961	962	963	964	965	966	967	968
0.000	993	998	999	1000	1001	1002	1003	1004	1005	1006	1007	1008	1009	1010	1011	1012	1013	1014	1015	1016	1017
0.000	1044	1049	1050	1051	1052	1053	1054	1055	1056	1057	1058	1059	1060	1061	1062	1063	1064	1065	1066	1067	1068
0.000	1093	1098	1099	1100	1101	1102	1103	1104	1105	1106	1107	1108	1109	1110	1111	1112	1113	1114	1115	1116	1117
0.000	1144	1149	1150	1151	1152	1153	1154	1155	1156	1157	1158	1159	1160	1161	1162	1163	1164	1165	1166	1167	1168
0.000	1193	1198	1199	1200	1201	1202	1203	1204	1205	1206	1207	1208	1209	1210	1211	1212	1213	1214	1215	1216	1217
0.000	1244	1249	1250	1251	1252	1253	1254	1255	1256	1257	1258	1259	1260	1261	1262	1263	1264	1265	1266	1267	1268
0.000	1293	1298	1299	1300	1301	1302	1303	1304	1305	1306	1307	1308	1309	1310	1311	1312	1313	1314	1315	1316	1317
0.000	1344	1349	1350	1351	1352	1353	1354	1355	1356	1357	1358	1359	1360	1361	1362	1363	1364	1365	1366	1367	1368
0.000	1393	1398	1399	1400	1401	1402	1403	1404	1405	1406	1407	1408	1409	1410	1411	1412	1413	1414	1415	1416	1417
0.000	1444	1449	1450	1451	1452	1453	1454	1455	1456	1457	1458	1459	1460	1461	1462	1463	1464	1465	1466	1467	1468
0.000	1493	1498	1499	1500	1501	1502	1503	1504	1505	1506	1507	1508	1509	1510	1511	1512	1513	1514	1515	1516	1517
0.000	1544	1549	1550	1551	1552	1553	1554	1555	1556	1557	1558	1559	1560	1561	1562	1563	1564	1565	1566	1567	1568
0.000	1593	1598	1599	1600	1601	1602	1603	1604	1605	1606	1607	1608	1609	1610	1611	1612	1613	1614	1615	1616	1617
0.000	1644	1649	1650	1651	1652	1653	1654	1655	1656	1657	1658	1659	1660	1661	1662	166					

may be completed by imaging in the planes of symmetry. The values found are directly proportional with the linear charge density and appear to increase about proportionally with decreasing linear beam dimensions.

We chose to print out the gradients of the fields $\partial E_x/\partial x$, $\partial B_y/\partial x$, $\partial E_y/\partial y$, $\partial B_x/\partial y$ because these are the quantities entering into the equations of motion for the particles; in addition this offers a simple check on the correctness of the calculation because for all x and all y :

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = - \frac{\rho(x,y)}{\epsilon_0}$$

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = - \mu_0 s(x,y) .$$

Applying this test one finds that for $x = 0$, $y = 0$ the magnetic field appears 4% low for as yet unresolved reasons.

Consequences For the Particle Motion

Limiting ourselves to motion in the planes of symmetry and neglecting the possible effects of the AG structure (reflected in the beam dimensions via the beta functions) we consider

$$x'' + \left(\frac{\nu}{R}\right)^2 x - ax - bx^3 = 0$$

where ν is the nominal betatron frequency, R the radius of the accelerator and $ax + bx^3$ the contribution of the self field to the focusing of individual particles, $|a + bx^2| \ll (\nu/R)^2$. In first order approximation we have for the space charge induced depression of the betatron frequency of a specific particle

$$\frac{\Delta\nu}{\nu} = - \frac{1}{2} a \left(\frac{R}{\nu}\right)^2 \left[1 + \frac{3}{4} \frac{b}{a} \frac{x^2}{R^2} \left\{ 1 - a \left(\frac{R}{\nu}\right)^2 \right\} \right]$$

where x represents the maximum excursion of the particle under consideration. We evaluated the coefficients a and b by fitting parabolae to the calculated distributions of the field gradients in the symmetry planes. These parabolae appear in the printout. The field coefficients have to be multiplied with $1/(E_0 \beta^2 \gamma)$ resp. $\beta c/(E_0 \beta^2 \gamma)$, ($\beta = v/c = \sqrt{1 - 1/\gamma^2}$, $\gamma = 1 + E/E_0$) to obtain the contributions of the electric and magnetic fields to the force field coefficients a and b . Numerically we find for $E = 200$ MeV:

	a_e	a_m	b_e	b_m
x	1.414×10^{-4}	-0.851×10^{-4}	-9.795×10^{-6}	3.47×10^{-6}
y	3.688×10^{-4}	-0.681×10^{-4}	-46.07×10^{-6}	16.45×10^{-6}

where subscripts e and m stand for electric and magnetic. It follows that we have for the dc part of the force field

	a_{dc}	b_{dc}
x	0.563×10^{-4}	-6.325×10^{-6}
y	3.007×10^{-4}	-29.62×10^{-6}

and for its ac part

	a_{ac}	b_{ac}
x	0.961×10^{-4}	-6.65×10^{-6}
y	2.506×10^{-4}	-31.30×10^{-6}

Correspondingly we have for the tune shifts in the dc beam

$$\Delta v_x = -0.053 (1 - 0.0965 \hat{x}^2)$$

i.e. $-0.053 \leq \Delta v_x \leq 0.063$ since

$$0 \leq \hat{x} \leq 5 \text{ cm and } v_x \sim 8.75$$

$$\Delta v_y = -0.283 (1 - 0.070 \hat{y}^2)$$

i.e. $-0.283 \leq \Delta v_y \leq -0.105$ since

$$0 \leq \hat{y} \leq 3 \text{ cm and } v_y \sim 8.75$$

The motion for the vast majority of particles that move with respect to both the radial and axial planes of symmetry is rather more difficult to assess. Because the two betatron frequencies differ only by a relatively small amount (0.1 or 0.2 in 8.75) it is a spiral motion around and concentric with the beam axis on the surface of a cylinder of elliptic cross section. The ellipse changes its proportions continuously and more or less periodically, degenerating twice per cycle to a straight line through the axis. In these last conditions

the particle experiences space charge forces that are much stronger than when the ellipse is "fat". Thus the betatron frequencies must be expected to be modulated with twice the difference of the horizontal and vertical betatron frequencies, the span in the frequency shifts being larger than that for motion in either of the planes of symmetry. As yet we have made no attempt to analyze this interesting matter at all.

In bunched beams the frequency shifts are determined by the sum of the dc part and the local value (local with respect to the bunch) of its ac part. If we take for that last value the difference of the local linear density and the dc component we can estimate the shifts. If the local density is $n \times$ average we have

$$\begin{array}{ll} \overset{a}{x} \left\{ 0.563 + (n-1) 0.961 \right\} 10^{-4} & \overset{b}{\left\{ -6.375 - (n-1) 6.65 \right\} \times 10^{-6}} \\ \overset{a}{y} \left\{ 3.007 + (n-1) 2.506 \right\} 10^{-4} & \overset{b}{\left\{ -29.62 - (n-1) 31.30 \right\} \times 10^{-6}} . \end{array}$$

For large values of n this becomes

$$\begin{array}{ll} \overset{a}{x} & 0.961 n \times 10^{-4} \\ \overset{b}{-6.65 n \times 10^{-6}} \\ \overset{a}{y} & 2.506 n \times 10^{-4} \\ \overset{b}{-31.30 n \times 10^{-6}} . \end{array}$$

For motion in the planes of symmetry and $n = 2$ we have

$$\begin{array}{l} \Delta v_x = - 0.1435 (1 - 0.053 \hat{x}^2) \\ \Delta v_y = - 0.519 (1 - 0.099 \hat{y}^2) . \end{array}$$

For large n , and motion of small amplitude ($\hat{x} \sim 0$, $\hat{y} \sim 0$):

$$\begin{array}{l} \Delta v_x = - 0.09 n \\ \Delta v_y = - 0.236 n . \end{array}$$

Discussion

Our calculations show that at 2×10^{13} protons in the AGS the self field of the beam affects the motion of individual particles noticeably, in particular in the axial direction. Since the dimensions of the circulating beam were taken to be the largest that would fit in the vacuum chamber these numbers are optimistically low. During and immediately after injection the beam height is about

2.5 cm, not 6, so much larger tune shifts must then exist. Some improvement may be expected from vertical stacking since that method causes a more uniform population of the available aperture. Conversely, any injection method attempting to reduce the beam cross section while maintaining the same beam current will cause the frequency shifts to increase.

We have seen that even moderate amounts of bunching cause large increases in the tune shifts. Therefore it may well be that we loose particles along resonances out of betatron phase while bunching. Thus methods to circumvent the space charge problem would include injection of a low current high brightness linac beam, using vertical stacking, in such a fashion that the eventual beam is as uniformly populated and as large as possible. The uniform population should help to keep the frequency spreads small, the large beam size to keep the frequency shifts small. A bright low current linac beam is not an asset from space charge standpoint, but is probably tolerable, and should reduce the losses at injection: $\text{emittance} < \text{partial acceptance}$. Rf capture and bunching might be directed towards avoidance of high local longitudinal densities until the beam has been sufficiently accelerated. The problem would be alleviated by attacking the real culprit, particle losses due to resonances. By reducing the stopband widths through compensation it should be possible to decrease the growth rates.²

By making use of Parzen's suggestion of incorporating multipole magnets in the accelerator lattice and controlling such multipoles via the PDP-10 it should be possible to tackle one stopband at a time without disturbing any of the others.

We may have to strike a delicate balance with respect to the rate of bunching and early acceleration. If we do things too slowly, we lose because a few stopbands have a long time to act, if too fast we lose again because there are many (due to the increased bunching factor).

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