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***Stacking Spheres in a Cylinder
with Application for the Multi-
Hundred-Watt Heat Source***

J. S. Madachy

March 15, 1972

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Stacking Spheres in a Cylinder with Application for the Multi- Hundred-Watt Heat Source

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SUMMARY

The multi-hundred-watt heat source is to be a closed cylinder containing 24 radioactive spheres. Formulas giving the dimensions of a cylinder which will hold equal-diameter tangent spheres arranged peripherally around the inside of the cylinder and in close-packed tangent layers are derived. Modifications in the formulas for three and four spheres per layer are also given to allow spacing between the spheres in a layer and between layers. Calculations of the ratio of cylinder volume to total sphere volume show that there are minimum ratios for one sphere per layer and for four spheres per layer. Calculations are also included showing that six spheres per layer produce the minimum-weight cylinder.

INTRODUCTION

The multi-hundred-watt heat source is to be a closed cylinder containing 24 radioactive spheres. These spheres could be stacked on top of each other giving a configuration with the greatest volume of heat-producing material in a given cylinder volume. However, the result would be a long thin cylinder which would be unsuitable in a space vehicle. The spheres could be arranged in layers of several spheres each, producing a more convenient shorter cylinder. The calculations below indicate that a configuration of four spheres per layer offers a better combination of minimum cylinder dimensions and maximum heat output in a given cylinder volume than a lesser or greater number of spheres per layer.

Given a sphere with diameter S , determine the inside diameter, C , of a cylinder which will hold n such spheres arranged peripherally around the inside of the cylinder. Determine the height, H , of the cylinder given N layers of closely packed spheres, and the diameter, R , of a rod that will slip down the center of the cylinder and be tangent to all the spheres in each layer. Figure 1 shows a top view of a single layer of four spheres; Figure 2 is a side view which shows two layers of closely packed spheres with four spheres in each layer. The distance between the centers of the spheres in two successive layers is D .

Similar formulas are derived for three and for four spheres per layer, given the additional conditions that a gasket or spacer with thickness T_1 may be placed between the layers of spheres, and that the spheres in each layer may be separated by a gasket or spacer with thickness T_2 .

The cylinder for the multi-hundred-watt heat source will probably have grooves down the inside wall to hold the spheres in place. Figure 3 shows a top view of the grooved cylinder which would hold four spheres per layer. In this figure, C is the maximum distance, through the center of the cylinder, between opposite grooves.

RESULTS

From Figure 4, the radius of a cylinder with n spheres arranged peripherally inside is OV . $OV = OT + VT$, where $VT = S/2$, or the radius of a sphere; $OT = S/2\sin(\varphi/2)$, where $\varphi = 360^\circ/n$. Then

FIGURE 1 - Top view of single layer of spheres.

FIGURE 2 - Side view of two layers of spheres.

FIGURE 3 - Top view showing grooved walls of cylinder.

FIGURE 4 - Derivation of cylinder and rod diameters (see Results).

$$C = 20V = 2 \left[\frac{S}{2\sin(\varphi/2)} + \frac{S}{2} \right] = \frac{S[1 + \sin(180/n)]}{\sin(180/n)}, \quad (1)$$

$$R = C - 2S = \frac{S[1 - \sin(180/n)]}{\sin(180/n)}. \quad (2)$$

From Figure 5, the distance, D , between layers can be derived:

$$D = \frac{\sqrt{\sin(270/n)\sin(90/n)}}{\sin(180/n)}. \quad (3)$$

The height, H , from the bottom of the cylinder to the top of the N th layer of closely packed spheres is

$$H = S + (N - 1)D = \frac{S[\sin(180/n) + (N - 1)\sqrt{\sin(270/n)\sin(90/n)}]}{\sin(180/n)} \quad (4)$$

Table 1 gives the various formulas for C and D for one through six spheres per layer.

If a gasket or spacer with thickness T_L is placed between the layers and the spheres in a layer are separated by a space T_S , the formulas for three spheres per layer become:

$$C = \frac{2}{3} \sqrt{3(S^2 + 2ST_s + T_s^2)} + S, \quad (5)$$

FIGURE 5 - Derivation of layer and cylinder heights (see Results).

Table 1

FORMULAS FOR CYLINDER DIMENSIONS

Dimension \ n	Formulas for n Spheres per Layer					
	1	2	3	4	5	6
C	S	2S	$\frac{S(2\sqrt{3} + 3)}{3}$	$S(\sqrt{2} + 1)$	$\frac{S(\sqrt{50 + 10\sqrt{5}} + 5)}{5}$	3S
D	S	$\frac{S\sqrt{2}}{2}$	$\frac{S\sqrt{6}}{3}$	$\frac{S\sqrt{8}}{2}$	$\frac{S\sqrt{50 + 10\sqrt{5}}}{10}$	$S\sqrt{3} - 1$

$$R = C - 2S \quad H = S + (N - 1)D$$

C = Cylinder Diameter

D = Distance Between Layers

R = Rod Diameter

H = Height of N Layers of Spheres

$$R = C - 2S = \frac{2}{3} \sqrt{3(S^2 + 2ST_s + T_s^2)} - S, \quad (6)$$

$$D = \frac{1}{6} \sqrt{3(8S^2 + 12T_L^2 - T_s^2 + 24ST_L - 8ST_s)}, \quad (7)$$

$$H = S + (N - 1)D. \quad (8)$$

For four spheres per layer these formulas are:

$$C = \sqrt{2}(S + T_s) + S, \quad (9)$$

$$R = C - 2S = \sqrt{2}(S + T_s) - S, \quad (10)$$

$$D = \frac{1}{2} \sqrt{4T_L^2 - (T_s^2 + 2ST_s)(4 - 2\sqrt{2}) - 8S(T_s + T_L) + 2\sqrt{2}S^2}, \quad (11)$$

$$H = S + (N - 1)D. \quad (12)$$

Table 2 gives the factors, F_1 and F_2 , to calculate the cylinder diameter, C, and the distance, D, between two successive layers, respectively, for up to 10 spheres per layer. Then, with sphere diameter S,

$$C = F_1 S,$$

$$D = F_2 S,$$

$$R = F_1 S - 2S, \text{ and}$$

$$H = S + (N - 1)F_2 S.$$

Table 2

FACTORS, F_1 AND F_2 , FOR DETERMINING CYLINDER
DIAMETER AND LAYER HEIGHT, RESPECTIVELY

<u>n</u>	<u>F_1</u>	<u>F_2</u>
1	1.00000000	1.00000000
2	2.00000000	0.70710678
3	2.15470054	0.81649658
4	2.41421356	0.84089642
5	2.70130162	0.85065081
6	3.00000000	0.85559968
7	3.30476487	0.85847320
8	3.61312593	0.86029557
9	3.92380440	0.86152609
10	4.23606798	0.86239700

DISCUSSION

Table 3 gives the ratios of cylinder volume to total sphere volume for three, four, and five spheres per layer for various numbers of layers. The minimum ratio of 1.5 occurs when there is only one sphere per layer. If more than one sphere per layer is used, and the space at the center of the cylinder is not filled with spheres, then another minimum ratio of about 1.838 occurs when there are four spheres per layer.

Table 3

RATIOS OF CYLINDER VOLUME TO TOTAL SPHERE VOLUME

<u>Layers</u>	<u>Spheres Per Layer</u>		
	<u>3</u>	<u>4</u>	<u>5</u>
1	2.32136721	2.18566017	2.18910913
2	2.10837780	2.01178699	2.02563829
3	2.03738133	1.95382926	1.97114801
4	2.00188309	1.92485040	1.94390287
5	1.98058415	1.90746308	1.92755578
6	1.96638486	1.89587153	1.91665773
10	1.93798627	1.87268844	1.89486162
25	1.91242754	1.85182366	1.87524512
50	1.90390796	1.84486873	1.86870628
10^2	1.89964817	1.84139127	1.86543687
10^4	1.89543098	1.83794858	1.86220014
.	.	.	.
.	.	.	.
.	.	.	.
10^{11}	1.895388386	1.837913804	1.862167449

The maximum amount of fuel per unit cylinder volume is obtained when a one-sphere-per-layer geometry is used. However, this would require a tall slender cylinder if 24 spheres, for example, are used. This would probably pose problems in terms of heat output and maneuverability. If the spheres are 1.6 in. in diameter, Table 4 shows the various dimensions of a cylinder with up to seven spheres per layer, and the ratios of the cylinder volume to the total volume of the spheres in the cylinder. The table indicates a second fuel maximum per unit cylinder volume when a four-spheres-per-layer geometry is used.

If a minimum-weight cylinder is an important factor, then Table 4 indicates that a six-spheres-per-layer geometry, with four layers, produces a cylinder with minimum surface area (cylinder wall plus two ends). If other construction considerations are ignored, then a minimum-surface cylinder would imply a minimum-weight cylinder.

A possible compromise which would retain the minimum-weight advantage and even improve the maximum-fuel-per-unit-cylinder-volume factor can be suggested. Three spheres can be stacked in the center of a cylinder with four layers of six spheres arranged around the inside of the cylinder and set in close-packed layers. If the spheres are still assumed to be 1.6 in. in diameter, this would produce a cylinder

4.8 in. in diameter,
5.707 in. high,
103.269 in.³ in volume, and
122.249 in.² in surface area.

There would be 27 spheres with a total volume of 57.906 in.³, giving a cylinder volume to total sphere volume ratio of 1.783.

If 27 spheres, each 1.6 in. in diameter, produce too much heat, the spheres could be reduced in size. Twenty-seven spheres, each about 1.5384 in. in diameter, would have about the same volume as 24 1.6-in. diameter spheres. The dimensions of a cylinder with these 27 smaller spheres arranged as described in the previous paragraph are:

4.615 in. in diameter,
5.487 in. high,
91.796 in.³ in volume, and
113.017 in.² in area.

The ratio of cylinder volume to total sphere volume would still be 1.783.

Table 4

CYLINDER DIMENSIONS WITH UP TO SEVEN SPHERES PER LAYER

<u>Spheres per Layer</u>	<u>Layers</u>	<u>Number of Spheres</u>	<u>Cylinder Dimensions</u>				<u>Volume of Spheres (in.³)</u>	<u>Cylinder- to-Spheres Volume Ratio</u>
			<u>Diameter (in.)</u>	<u>Height (in.)</u>	<u>Volume (in.³)</u>	<u>Area (in.²)</u>		
1	24	24	1.600	38.400	77.208	197.041	51.472	1.500
2	12	24	3.200	14.045	112.957	157.282	51.472	2.195
3	8	24	3.448	10.745	100.299	135.042	51.472	1.949
4	6	24	3.863	8.327	97.584	124.488	51.472	1.896
5	5	25	4.322	7.044	103.349	124.991	53.617	1.928
6	4	24	4.800	5.707	103.269	122.249	51.472	2.006
7	4	28	5.288	5.721	125.620	138.947	60.050	2.092