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A CORRELATION OF INTERACTION DATA WITH
CALCULATIONS USING THE GELLER MODEL

J. K. Fox

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**PHILLIPS
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ATOMIC ENERGY DIVISION

**NATIONAL REACTOR TESTING STATION
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PHILLIPS
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ABSTRACT

This report describes a simple model for calculating the critical separation of regular arrays of identical fissile units. The units must be separated mainly by air. The model used is one first derived by L. Geller. Use is made of the fact that for a critical system the material buckling is equal to the geometric buckling. The model assumes that the total interaction probability, ψ , is simply related to the ratio of the geometric buckling, B_g^2 , of the critical isolated unit to that of one unit in the array: $B_g^2 = B_1^2 (1 - \psi)$ where B_1^2 is the geometric buckling of a single unit in the array. The total interaction probability depends mainly upon the solid angle between two units and the total number of units in the array. Normalization to experiment is used to determine the proportionality constant relating the solid angle to the interaction probability. Thus, the calculation is reduced to a simple geometrical problem. To determine the total interaction probability as a function of number of units in an array a determinant is set up based on symmetry. A major difficulty with this method is in how to treat shadow effects. These become important for large arrays. Hence, extrapolations are used to estimate the coupling coefficients of large arrays.

The results of extensive comparisons of the model with data obtained at ORNL are presented. These results show that, in general, the calculated separations agree well with the data, except at very close spacings. The concept of a reflector saving applied in the calculation of the geometric buckling of units allows the estimation of the effect of thin reflectors around units and that of a reflecting vault around the array. Two methods are noted whereby conservatism may be introduced and its degree estimated. The fact that the same model gives good results for both dilute solutions and metal systems indicates that the interaction probability is essentially independent of the leakage neutron energy spectrum.

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A CORRELATION OF INTERACTION DATA WITH CALCULATIONS USING THE GELLER MODEL

1. INTRODUCTION

Several papers have been written on methods of analysis of neutron interaction effects between vessels containing fissile materials. The present report is limited to regular arrays separated by air. Several methods of approach to an understanding of interaction effects have been developed. Probably the most widely used concept is that of an interaction probability that is related to the solid angle between interacting units in an array. Several variations of this method are in use. In an early report, H. F. Henry derived a critical equation for two interacting units in terms of leakage and nonleakage probabilities[1]. Later, L. Geller[2,3] attempted to develop a model using multi-group diffusion theory. This proved to be too difficult for hand calculations. However, after several simplifying assumptions were made, Gellar arrived at a simple geometric model. In this method the interaction probability for a regular array of identical vessels is related to the solid angle between units and the ratio of the geometric buckling of one vessel in the array to that of an isolated critical vessel having the same fuel. In a variation of the solid angle method due to H. K. Clark[4], the interaction probability is expressed in terms of an albedo which, using a one-group model, Clark expresses as a reflector saving. J. A. Pond[5] and Henry et al[6, 7] have developed a formulation in which the K_{eff} of individual vessels in a critical array plus the total interaction probability gives a K_{eff} of unity for the system. A synthetic diffusion kernel is used to obtain single vessel K_{eff} values.

Another general approach is to use an empirical relation between the overall array density of fuel and that of the fuel in an individual unit to predict the critical number in a given array[8]. This has the disadvantage that two critical experiments are needed to establish the constants for each array type and fuel variation. Recently, Monte Carlo methods have been utilized for the machine solution of interaction problems. These offer a more exact treatment of the interaction probability. Unpublished results at ORNL indicate that the method gives very good agreement with experimental data.

The problem of interaction is one that exists wherever fissionable material is stored in multiple units. An example of this is the product cylinders at ICPP. These must be stored in rather large quantity before shipment. Hence, a need exists for the application of criticality data to these problems. In general, the experimental data were not obtained for the exact situation that may exist in the plant. Thus an analytical model is needed that agrees with experimental data over a wide range of conditions so that a given plant problem may be calculated with confidence. The author first became interested in interaction calculations while doing critical experiments at ORNL. It was felt that a simple hand method would be useful in shortening the number of trial separations in arriving at a just critical array. The Geller method proved to be quite helpful for this purpose. Later a decision was made to make an extensive comparison of the model with the available interaction data.

II. OUTLINE OF THE INTERACTION MODEL

The equation used in this model relates the total interaction probability to a ratio of geometric bucklings. This is based on the assumption that for a given fuel the ratio of leakage to total neutrons is directly proportional to the size or geometric buckling of unit vessels in a uniform critical array. For an isolated unit that is critical, all of the leakage neutrons are lost. Then if the unit size is decreased, the unit cannot remain critical unless leakage neutrons from other vessels contribute by interaction. Hence, the interaction probability must increase by an amount that compensates for the increased unit leakage, and this is proportional to the buckling ratio corresponding to the two unit sizes. For the method to be valid over a large range of fuel spectra requires that the leakage neutrons behave in a similar manner with regard to the interaction probability versus solid angle. The generally good agreement between the model and experimental data indicates that epithermal neutrons in air behave essentially the same over a wide range of energies in regard to solid angle versus interaction probability.

The general formula for the criticality of a regular array of identical vessels equally spaced may be stated as

$$B_g^2 = B_1^2 (1 - \psi) \quad (1)$$

where B_g^2 is the geometric buckling of an isolated critical vessel in air having the same fuel as that used in the array. B_1^2 is the geometric buckling of a single vessel in the critical array. ψ is the probability that a neutron leaking from any one vessel is absorbed in another vessel or is reflected back into the same vessel. This includes neutrons that may be scattered by a second vessel and are finally absorbed in a third one. In other words, it is a measure of the total coupling between vessels in the array. This goes to zero, as it should, for an isolated vessel in air. Thus, ψ is dependent upon the solid angle between two individual vessels, upon the number of vessels in the array, and upon the overall array shape. If we denote by C that part of the coupling which is dependent upon the number of vessels in the array and by W the probability of interaction between any two vessels, then

$$\psi = CW. \quad (2)$$

The probability, W , is mainly dependent upon the solid angle, Ω , subtended by a second vessel at points on the surface of the first. However, in planar arrays, neutrons leaking from surfaces more than ninety degrees away from the direction of any other vessel do not contribute appreciably. This is true of the end leakage of a planar array of cylinders or slabs. It is assumed that leakage is proportional to geometric buckling in a given direction, and if we denote the ratio of effective-to-total-leakage ratio by ℓ , then for a planar array of cylinders,

$$\ell = \frac{B_r^2}{B_r^2 + B_z^2} \quad (3)$$

Also, since the interaction probability is assumed to be proportional to the fractional solid angle, a proportionality constant, X , should be included; then

$$W = X\ell\Omega \quad (2')$$

for planar arrays. Henry *et al* have prepared curves showing the solid angle between two slabs as a function of slab size and spacing[9]. A similar set of curves for two parallel cylinders were prepared by Pond[10]. These curves have been used in this report because of the time required for analytical calculations. Since some simplifying assumptions were made in preparing the curves, the values are expected to be only approximate. Furthermore, the scale of the curves is such that accurate reading of the values is difficult. In order to estimate the proportionality constant X , the most convenient method is to normalize to Equation (1) using some of the best experimental data. Since the solid angle curves are used, the bias that may exist between curve values of Ω and the true interaction probability is included in the value of X obtained. Variation of X between various normalizing comparisons should be small and random if it is a true constant. However, some variation would be expected due to the inaccuracies outlined above. In the tabulated comparisons with experimental data discussed below, the values of X are shown that "force fit" or normalize the model to individual experiments. In general, except for close spacing, the constancy of the X values is quite good for parallel slabs. Here the solid angles are easier to evaluate than for cylinders. In the case of cylinders, there is more scatter in the values of X . However, no very definite trend in X values was noted due to vessel size or moderation. Because of this it is felt that, within the accuracy of the model, that X can be considered a constant. The average value of X for slabs is somewhat higher than that for cylinders. This is probably due to a difference in the methods of solid angle calculation. The best values of X are obtained by comparison of the model with data where only two or three vessels are present. This is due to certain approximations that must be used in determining values of C , the coupling coefficient, as the number in the array increase.

1. Determination of Coupling Coefficient

Several of the papers referred to previously give methods for evaluating the coupling coefficient for small planar arrays. For larger arrays, one approach is to set up a determinant with the neutron densities for units having like positions in the array as the unknowns. In some cases it is possible to calculate the coupling between vessels that are of unequal size, as for example, between parallel slabs of unequal thickness since the solid angles are independent of the thickness. Table I contains a rather complete list of coupling equations for the various types of arrays that have been tested experimentally. The method chosen to calculate the coupling coefficients is shown in Appendix A. Shadow effects are assumed to be compensated for by surface multiple scattering.

As the arrays become large, the assumption of no shadowing would be expected to make the C values too large. Figure 1 is a plot of coupling coefficients calculated by this method versus the number of units in each dimension of an array. For square planar arrays, it appears that up to a 5×5 array this relation is linear on a semilog plot. This fact has been used to extrapolate the C values to larger arrays under the assumption of continued linearity, and the extrapolated values are used in Table I. Figure 2 shows the experimental data for planar arrays of cylinders using semilog paper and the number of units in each dimension as the ordinate. Here the separation is on the linear abscissa scale

TABLE I
SOME INTERACTION EQUATIONS FOR ARRAYS
OF IDENTICAL VESSELS EQUALLY SPACED AND PARALLEL

<u>Vessel Configuration</u>	<u>Interaction Formula</u>
2	$B_g^2 = B_1^2 (1 - W)$
3 in a line	$B_g^2 = B_1^2 (1 - 1.415W)$
3 in a triangle	$B_g^2 = B_1^2 (1 - 2 W)$
4 in a line	$B_g^2 = B_1^2 (1 - 1.618 W)$
5 in a line	$B_g^2 = B_1^2 (1 - 1.733 W)$
6 in a line	$B_g^2 = B_1^2 (1 - 1.802 W)^{[a]}$
4 in a square	$B_g^2 = B_1^2 (1 - 2.6 W)^{[b]}$
7 in a hexagon	$B_g^2 = B_1^2 (1 - 4.6 W_1)^{[c]}$
3 x 3 planar square array	$B_g^2 = B_1^2 (1 - 4.9 W_1)^{[a]}$
4 x 4 planar square array	$B_g^2 = B_1^2 (1 - 6.8 W_1)^{[a]}$
19 in a planar hexagonal array	$B_g^2 = B_1^2 (1 - 8.25 W_1)^{[a]}$
5 x 5 planar square array	$B_g^2 = B_1^2 (1 - 8.12 W_1)^{[c]}$
6 x 6 planar square array	$B_g^2 = B_1^2 (1 - 9.3 W_1)^{[c]}$
7 x 7 planar square array	$B_g^2 = B_1^2 (1 - 10.25 W_1)^{[c]}$
8 x 8 planar square array	$B_g^2 = B_1^2 (1 - 11.1 W_1)^{[c]}$
9 x 9 planar square array	$B_g^2 = B_1^2 (1 - 11.85 W_1)^{[c]}$
10 x 10 planar square array	$B_g^2 = B_1^2 (1 - 12.3 W_1)^{[c]}$
2 x 2 x 2 cubic array	$B_g^2 = B_1^2 (1 - 5.2 W_1)^{[b]}$

TABLE I (Cont.)

SOME INTERACTION EQUATIONS FOR ARRAYS
OF IDENTICAL VESSELS EQUALLY SPACED AND PARALLEL

Vessel Configuration	Interaction Formula
3 x 3 x 3 cubic array	$B_g^2 = B_1^2 (1 - 11.9 W_1)^{[b]}$
4 x 4 x 4 cubic array	$B_g^2 = B_1^2 (1 - 16.5 W_1)^{[c]}$
5 x 5 x 5 cubic array	$B_g^2 = B_1^2 (1 - 20.2 W_1)^{[c]}$

[a] Determinant solved by computer.

[b] Determinant solved by hand using solid angle ratios.

[c] Extrapolated values of C using linear extrapolation.

Fig. 1 Coupling coefficient "c" versus number of units in each dimension of array.

Fig. 2 Experimental critical separations for large planar and cubic arrays versus number of units in each dimension.

corresponding to values of C in Figure 1. The fact that most of the experimental curves are fairly linear in Figure 2 gives credence to using a linear extrapolation of C values in Figure 1.

III. COMPARISON OF THE MODEL WITH EXPERIMENT

Over a period of several years ORNL has performed a large number of critical experiments on interacting arrays. These cover many different types and sizes [11-18,20]. Those referred to here were all done with either fully enriched U-235 solution or metal. An extensive comparison of these data with the model outlined above is given below.

The geometric bucklings used in the calculations were obtained using a value of 2.6 cm for the extrapolation distance, δ , in air. Since vessel walls act as a reflector, depending upon their thickness, the δ values are increased proportionately for vessels thicker than 0.16 cm of aluminum. Plastic containers require a greater increase in δ value because of their better reflecting property. Some of the early experimental data were obtained inside of heavy metal tanks, and allowance should be made for this as reflector. Recent data indicate that this type of reflector is very effective in increasing the reactivity.

The accuracy of the Geller model depends very strongly upon having available an accurate value of the critical size of an isolated bare vessel of the same fuel as that used in the critical array. The expression of the interaction effect in terms of a buckling ratio in effect cancels out the diffusion or transport model otherwise needed for critical equations and allows a purely geometric model to be used. If for a given array, data are not available on the critical size of an isolated vessel of the same fuel, a calculated value may have to be used. Then the array calculations contain the additional errors that may be introduced due to the calculation of the isolated critical size. However, in the case of fully enriched solutions enough data exist to draw accurate B_g^2 versus concentration curves so that the B_g^2 for any concentration is available.

Table II [14-16] is a comparison of the model with data on parallel interacting slabs. First, the model was normalized to those experiments in which two bare slabs are involved so as to obtain an average X value. This value was $X_{\text{slab}} = 1.315$. Individual values vary from 1.27 to 1.36. Then the average value was inserted into the equation and the calculated separations obtained. In those cases where the interacting slabs were not all of the same thickness, the ℓ values were obtained using average values of the thickness buckling. In two sets of experiments the two slabs had reflector tanks against their outside faces. If an extrapolation distance only slightly less than that required for equal bucklings is used when comparing bare and reflected isolated vessels having the same fuel, agreement with experimental data is obtained. The δ values used were 5.6 and 5.7 cm; for isolated vessels, approximately 6.1 cm were required. This illustrates how the concept of reflector savings can be used for partially reflected systems where the units are separated mostly by air.

Included in the table is a comparison of the model with some recent data on enriched metal slabs in which the presence of any reflecting material was minimized [16]. The fact that even here there is fairly good agreement with the experimental data definitely proves that any dependence of the interaction probability on the leakage spectra is quite small.

TABLE II

COMPARISON OF EXPERIMENTAL CRITICAL SEPARATIONS
OF PARALLEL INTERACTING SLABS WITH CALCULATED VALUES

A. Two 6-inch-thick slabs at $H/U-235 = 337$ ^[a]							
Experimental Critical Height, in. ^[b]	10.02 ^[b]	12.9	17.7	19.8	23.5	28.8	32.3
X Value for Experimental Fit	1.34	1.31	1.30	1.29	1.27	1.28	1.35
Experimental Separation, in.	2.0	6.0	15	20	30	48	66
Calculated Separation, in.	1.95	6.05	15.85	20.6	31.0	48.7	65.0
B. Three 3-inch-thick slabs at $H/U-235 = 337$ ^[c]							
Experimental Critical Height, in. ^[b]	13.57	23.14	33.67	42.27	47.4		
Experimental Separation, in.	1.0	3.0	4.5	5.5	6.0		
Calculated Separation, in.	1.36	3.17	4.3	5.3	5.4		
C. One 6-inch-thick slab and one 3-inch-thick slab at $H/U-235 = 337$							
Experimental Critical Height, in. ^[b]	12.75	17.55	25.9	36.4	44.8		
Experimental Separation, in.	2.0	6.0	15	30	48		
Calculated Separation, in.	2.16	5.95	14.8	29.7	46.1		
D. One 6-inch-thick slab centered between two 3-inch-thick slabs at $H/U-235 = 337$							
Experimental Critical Height, in. ^[b]	17.4	24.6	32.1				
Experimental Separation, in.	10	20	32				
Calculated Separation, in.	9.9	19.0	30.0				
E. Two 3-inch-thick slabs at $H/U-235 = 50$							
Experimental Critical Height, in. ^[c]	17.53	23.4					
X Value	1.33	1.36					
Experimental Separation, in.	1.0	2.0					
Calculated Separation, in.	0.95	1.7					
F. Two 6-inch-thick slabs reflected with 6-inch H_2O on back sides at $H/U-235 = 325$ ^[a]							
Experimental Critical Height, in. ^[c]	8.60	10.00	11.15	12.2			
Experimental Separation, in.	2.0	6.0	12	24			
Calculated Separation, in. ^[d]	2.16	6.25	12.3	24.4			
G. Two 6-inch-thick slabs reflected by 6-inch H_2O on back sides at $H/U-235 = 254$ ^[a]							
Experimental Critical Height, in. ^[c]	7.96	9.19	10.08	10.96	11.23		
Experimental Separation, in.	2.0	6.0	12	24	40		
Calculated Separation, in. ^[e]	1.98	6.3	12.3	25.0	39.9		
H. Two identical 8-inch x 10-inch metal slabs of varying thicknesses							
Experimental Thickness of slabs, in. ^[f]	2.375	2.625	2.875	3.125			
Experimental Separation, in.	1.26	2.04	3.14	4.90			
Calculated Separation, in.	1.25	1.96	2.86	4.55			

[a] One nominally 6-inch slab was actually 5.93 in. wide inside. The second slab was mocked up by clamping together two 3-inch slabs. Because of the aluminum plates at the center, the combination was assumed equivalent to the other slab.

[b] Data from ORNL-56-7-148.

[c] Data from graphs in ORNL-2389 and corresponding unreported tabulated data.

[d] Extrapolation distance for water reflector = 5.6 cm used.

[e] Extrapolation distance for water reflector = 5.7 cm used.

[f] Data from ORNL-3360.

In Table III[14, 18] some of the best data available on a single line of interacting cylinders in air are compared with the model. It was again necessary to normalize to the two-cylinder data to obtain an average X value (normalization constant). However, to obtain an overall X average for cylinders, some of the better data on triangular arrays were included since the coupling coefficient for these is also definite. Thus, an approximate value of $X_{cy} = 0.85$ was obtained. This average value was then used to obtain the calculated separations for all arrays of cylinders. The agreement of calculated values of separation with the experiment is quite good. However, there is more scatter than in the case of parallel slabs. The agreement is just about as good for the longer linear arrays as for those of two cylinders, indicating that the calculated coupling coefficients are accurate. There is no definite trend in agreement due to variation in H:U-235 atomic ratio.

Table IV[12, 18] continues the comparison of the model with data on triangular arrays of three, square arrays of four, and hexagonal arrays of seven cylinders. Some of the data in this table were obtained inside a large steel reflector tank[12]. Because of extra reflections from these walls the normalized X values run appreciably higher since no attempt was made to correct for this effect. Hence, using average X values the calculated values of the spacing for all data taken in the tank are nonconservative in critical separation distance. The agreement is generally good for data obtained with extraneous reflectors minimized.

TABLE III
COMPARISON OF EXPERIMENTAL CRITICAL SEPARATION OF
SINGLE LINE ARRAYS OF INTERACTING CYLINDERS WITH CALCULATED VALUES

Cylinder Diameter (in.)	H/U-235 Atomic Ratio	Number of Cylinders in Line	Experimental Critical Height (in.)	Experimental Separation (in.)	Calculated Separation (in.)	Normalized X Value
9.5 ^[a]	297	2	24.1	1.0	1.05	0.838
9.5	297	2	31.7	3.0	2.9	0.851
9.5	297	2	44.5	6.0	6.5	0.815
9.5	297	2	54.0	8.0	8.2	0.831
9.5	297	3	22.3	2.0	2.4	-----
9.5	297	3	33.2	6.0	7.0	-----
9.5	297	3	43.8	10	10.0	-----
9.5	297	3	60.1	15	13.8	-----
9.5	297	4	22.7	3.0	3.5	-----
9.5	297	4	30.0	6.0	6.8	-----
9.5	297	4	38.5	10	10.5	-----
9.5	297	5	21.7	3.0	3.4	-----
9.5	297	5	28.3	6.0	7.0	-----
9.5	297	5	36.2	10	10.0	-----
9.5	297	6	21.3	3.0	3.6	-----
9.5	297	6	34.8	10	10.0	-----
10.0 ^[b]	50	2	12.83	12	12.5	0.765
10.0	85.7	2	11.44	3.0	2.7	0.922
10.0	85.7	2	12.18	6.0	6.0	0.853
10.0	85.7	2	12.72	12	11.5	0.895
10.0	325	2	23.0	3.0	3.5	0.772
10.0	325	2	30.2	9.0	10.0	0.760
10.0	328	2	26.6	6.0	6.7	0.789
10.0	328	2	35.7	16	16.0	0.824

[a] Data from ORNL-2842.

[b] Data from ORNL-2389

TABLE IV

COMPARISON OF EXPERIMENTAL CRITICAL SEPARATIONS
OF VARIOUS PLANE ARRAYS OF CYLINDERS WITH CALCULATED VALUES

Cylinder Diameter (in.)	H/U-235 Atomic Ratio	Experimental Critical Height (in.)	Experimental Critical Separation (in.)	Calculated Critical Separation (in.)	Normalized X Value
A. Triangular arrays of 3 cylinders					
9.5 ^[a]	297	13.4	1.0	0.29	1.015
9.5 ^[a]	297	20.3	4.0	3.8	0.865
9.5 ^[a]	297	28.1	8.0	8.4	0.806
9.5 ^[a]	297	36.3	12	12.3	0.831
9.5 ^[a]	297	49.7	18	17.6	0.861
9.5 ^[a]	297	60.1	22	20.4	0.928
8.0 ^[b]	44	22.0	3.0	2.04	1.01
8.0 ^[b]	44	27.1	4.0	3.0	1.00
8.0 ^[b]	44	42±1	6.0	5.0	0.969
B. Square Array of 4 cylinders					
9.5 ^[a]	297	15.8	3.0	2.75	-----
9.5 ^[a]	297	27.2	10	10.0	-----
9.5 ^[a]	297	47.4	22	21.4	-----
9.5 ^[a]	297	62.5	30	26.6	-----
C. Hexagonal arrays of 7 cylinders					
6.0 ^[b]	44	20.3	2.0	1.3	-----
6.0 ^[b]	44	33±2	3.0	1.86	-----
6.0 ^[a]	309	22.4	1.0	0.75	-----
6.0 ^[a]	309	≈77	2.0	1.3	-----
8.0 ^[b]	44	8.5	1.0	0.80	-----
8.0 ^[b]	44	10.1	2.0	1.84	-----
8.0 ^[b]	44	11.7	3.0	2.7	-----
8.0 ^[b]	44	13.2	4.0	3.6	-----
8.0 ^[b]	44	16.5	6.0	5.5	-----
8.0 ^[b]	44	22±2	9.0	8.1	-----
8.0 ^[a]	309	11.3	1.0	0.92	-----
8.0 ^[a]	309	17.8	3.0	2.8	-----
8.0 ^[a]	309	35.4	6.0	5.8	-----
8.0 ^[a]	309	46.9	7.0	6.4	-----
9.5 ^[a]	297	12.1	3.0	3.5	-----
9.5 ^[a]	297	30.1	10	11.1	-----
9.5 ^[a]	297	32.9	22	22.8	-----

[a] Data obtained from ORNL-2482.

[b] Data obtained from ORNL-2367. Data obtained in a large steel tank.

The calculated separation distances for the seven cylinder hexagonal arrays were obtained using an average coupling coefficient of 4.6. This is the result if the coupling determinant is solved using solid angle ratios as follows:

$$\Omega_2/\Omega_1 \approx 0.47 \text{ and } \Omega_3/\Omega_1 \approx 0.37 \quad ,$$

where the subscripts refer to the three separation distances between individual cylinders. These ratios are approximate average values. It is possible to solve the coupling determinant in this case by hand without using solid angle ratios. Four cases were solved using this method and the results were in better agreement with the experiment. Other cases were not tried because of the time involved; however, the results indicated that use of solid angle ratios are an appreciable source of error.

For large planar arrays, most of the available data were obtained using polyethylene bottles as containers[17]. The data and comparisons with the model are shown in Table V. Two different bottles were used: A blown 5.38-inch OD one having a nonuniform ID (average 4.75 inch) and a uniform 5.63-inch OD bottle. To simplify the determination of B_f^2 for these arrays it was assumed that the thin polyethylene walls were equivalent to solution and the OD used. An average ID could have been used plus a reflector saving to account for the polyethylene. This would have been more difficult because of variations in thickness and uncertainties in the reflector saving. The calculated values were obtained using C values obtained by use of the determinant for arrays up to 5 x 5. For larger arrays the linear extrapolation used as shown in Figure 1 gives conservative results. This suggests that perhaps the C plot should curve slightly upward for large arrays. The nonconservative calculated separations for close separations (large Ω) are similar to most of the results for small arrays. A limited number of medium sized planar arrays of approximately 6-inch-diameter aluminum cylinders are also shown in Table V. The accuracy of these data is decreased by the fact that two slightly differing sizes of cylinders were used in each experiment. For these experiments four different heights were used. For some of these arrays, the calculated separations are conservative for short cylinders and nonconservative for tall ones. Since the solid angle values vary little and other factors are the same, it may be that the Pond curves do not give a good variation of solid angle with variations in the height-to-diameter ratios, λ , in the region from 3 to 10.

The available data on cubic arrays are compared with the model in Table VI[13, 20]. All of these data were obtained using arrays of near equal height-to-diameter cylinders. It should be noted that the semilog plot of critical separations versus array size for cubic arrays in Figure 2 curves toward wider separations for large arrays. This indicates that the linear extrapolation of the two calculated C values in Figure 1 give results that are too low. An increase in C for the 5 x 5 x 5 array would improve the agreement with the experiment. A difficulty arises in the calculations for the cubic arrays because cylinders were used in the experiments. Here the vertical interaction is of the slab type; whereas, in the two horizontal directions it is cylindrical and in the inclined directions it is of both types combined. The chosen approach to a solution of this difficulty was to divide the interaction into two parts: one cylindrical and the other slab type. On the average there is twice as much radial interaction as there is vertically and the two components are so weighted. The general equation is:

TABLE V

COMPARISON OF EXPERIMENTAL CRITICAL SEPARATION
OF LARGE PLANE ARRAYS OF CYLINDERS WITH CALCULATED VALUES

Fuel Solution: $\text{UO}_2(\text{NO}_3)_2$ at 410 g/l U-235^[a]
H: U-235 Atomic Ratio ≈ 60

A. Square Arrays of Type A (5.38 in. OD) Polyethylene Cylinders

Array Size	Solution Ht. = 22-1/2 in.		Solution Ht. = 44-1/4 in.	
	Expt. Separation	Calc. Separation, in.	Expt. Separation	Calc. Separation, in.
3 x 3	1.18	0.89	1.75	1.37
4 x 4	2.16	2.15	3.32	3.34
5 x 5	3.00	3.23	4.55	4.68
6 x 6	3.58	4.04	5.64	5.92
8 x 8	4.43	5.10	----	----
9 x 9	-----	-----	7.79	8.07
10 x 10	5.04	5.65	----	----

B. Square Arrays of Type B (5.63 in. OD) Polyethylene Cylinders

Array Size	Solution Ht. = 22-1/2 in.	
	Expt. Separation	Calc. Separation, in.
3 x 3	1.43	1.10
4 x 4	2.60	2.53
6 x 6	4.25	4.55
8 x 8	5.32	5.63

C. Type C Arrays of Aluminum Cylinders (Approximately 6-in. diameter)

3 x 3 Square Arrays:

Solution Ht., in.	Expt. Separation, in.	Calc. Separation, in.
21.67	1.70	1.53
30.66	2.30	1.90
40.35	2.70	2.23
49.6	3.00	2.44

4 x 4 Square Arrays:

Solution Ht., in.	Expt. Separation, in.	Calc. Separation, in.
20	2.58	2.95
30	3.77	3.80
40	4.50	4.22
50	5.0	4.52

19 Cylinder Hexagonal Arrays:

Solution Ht., in.	Expt. Separation, in.	Calc. Separation, in.
20	3.51	4.17
30	4.95	5.23
40	5.94	5.62
50	6.64	6.24

[a] Data from ORNL-3193.

$$B_g^2 = B_1^2 (1 - C \times \ell \Omega), \text{ or}$$

$$C \times \ell \Omega = 1 - \frac{B_g^2}{B_1^2} = K (\text{const.})$$

Then, separating and weighting the components the result is:

$$\frac{2 (X_{c c c} \ell \Omega_c) + (X_{s s s} \ell \Omega_s)}{3} = \frac{K}{C}$$

In this equation the c subscripts refer to cylinder interaction and the s to slab interaction. It is important to note that when the components are separated, the ℓ factors apply just as if a planar array were used. If a cubic array of

TABLE VI

COMPARISON OF EXPERIMENTAL CRITICAL SEPARATIONS
OF CUBIC ARRAYS OF SHORT CYLINDERS WITH CALCULATED VALUES

A. Plexiglas 5 liter cylinders: 8.0 in. O.D. x 7.5 in. Height ^[a]		
<u>Array Size</u>	<u>Experimental Critical Separation (in.)</u>	<u>Calculated Critical Separation (in.)</u>
2 x 2 x 2	0.56	0.30
3 x 3 x 3	2.55	2.95
4 x 4 x 4	4.20	4.37
5 x 5 x 5	5.67	5.45
B. Plexiglas 5 liter cylinders in 6 in. paraffin vault ^[a]		
<u>Array Size</u>	<u>Experimental Critical Separation (in.)</u>	<u>Vault Extrapolation Distance, δ, cm</u>
2 x 2 x 2	3.54	6.0
3 x 3 x 3	6.51	5.75
C. Enriched metal cylinders: Diameter = 4.53 in., Height = 4.24 in. ^[b]		
<u>Array Size</u>	<u>Experimental Critical Separation (in.)</u>	<u>Calculated Critical Separation (in.)</u>
2 x 2 x 2	0.89	0.82
3 x 3 x 3	2.51	2.60
D. Enriched metal cylinders: Diameter = 4.53 in., Height = 5.3 in. ^[b]		
<u>Array Size</u>	<u>Experimental Critical Separation (in.)</u>	<u>Calculated Critical Separation (in.)</u>
2 x 2 x 2	1.36	1.05
3 x 3 x 3	3.34	3.25
E. Enriched metal cylinders: Diameter = 4.53 in., Height = 4.24 in. ^[b] In 6 inch thick paraffin vault		
<u>Array Size</u>	<u>Experimental Critical Separation (in.)</u>	<u>Vault Extrapolation Distance, δ, cm</u>
2 x 2 x 2	4.72	4.5
3 x 3 x 3	7.54	4.43

[a] Data from ORNL-TM-719 for H:U-235 = .59

[b] Data from ORNL-3499

spheres were used, the above separation device would not be needed and the value of ℓ would be unity. The above formula was used in calculating the separation values in Table VI.

For the comparisons with the experiments using plexiglas containers[13] it was found that to obtain reasonable agreement with experimental data the plastic cannot be assumed equivalent to solution, as was done for cylinders. For calculating B_1^2 , a extrapolation distance of 3.0 cm was used. This is in line with Figure 3 based on data in K-643 on the variation of critical size with water reflector thickness[19]. Then in measuring the Ω values the solution volumes were taken as the interacting units. This is a crude approximation since the plastic is quite different from air. However, for the more widely spaced arrays this error should be small. There was an additional experimental error in these arrays because considerable support material between units was used in the experiments.

In some of the experiments the effect of enclosing the bare array in a 6-inch-thick paraffin vault was investigated. Here we have an array that should behave just like a bare array as far as interchange of neutrons between units is concerned. But in considering overall leakage neutrons from the array that would be lost for a bare array, it is evident that a thick, closed vault would reflect a large fraction of these into the array for another chance at causing fissions. Hence, the system behaves as if reflected in regard to overall leakage. This suggests that the present model could be used for this case by the addition of a reflector saving in calculating the unit geometric buckling. For the two comparisons using solutions shown in Table VI, the extrapolation distance required to fit the experiment was 6.0 and 5.75 cm. These values are only slightly lower than that required to equate critical bare and reflected single unit bucklings of this fuel concentration. This value is between 6.0 and 6.3 cm.

Some recent data have been obtained on cubic arrays of enriched metal cylinders[20]. Here the complication due to containers is not present as in the case of solutions. For the two array sizes compared the agreement with the experiment is quite good as shown in Table VI. Two cases were also compared where the arrays were enclosed in a 6-inch-thick paraffin vault. The required extrapolation distances were calculated as previously. The values obtained of 4.5 and 4.43 cm are reasonable. If data are used for a bare- and water-reflected enriched metal sphere[21] and a bare extrapolation of 2.6 cm, the extrapolation distance required is 4.5 for equal bucklings. For metal isolated slabs[22], the corresponding value is 4.7 cm. Hence, the method again gives reasonable results.

Fig. 3 Extrapolation distance versus water reflector thickness.

IV. APPLICATIONS

In formulating a working model for the solution of practical interaction problems, it has become customary to insert conservative conditions in an effort to assure that safe spacings are predicted. Often it is hard to determine quantitatively the effect of these conditions. In the above model no such effort was made. Instead the aim was to obtain the best general agreement possible with the experiment. Then the effect of varying the constants X_C and X_S was investigated. Table VII is a part of Table IV expanded to include results of calculating the critical separations of hexagonal arrays of cylinders using $X_C = 1.0$ and 1.1 . If the normalized average $X_C = 0.85$ is used, some of the results tend to be slightly nonconservative even for the very clean experiments. However, using $X_C = 1.0$, only one result was nonconservative -- and this was for an array of tall cylinders in the large steel tank with no allowance for reflections. Results using $X_C = 1.1$ were 20 to 30 percent conservative in nearly all cases. As the value of X_C used is increased further, the degree of conservatism would also increase. As yet, no generally accepted rules are available for spacing safety factors.

For slab arrays, a value of $X_S = 1.40$ is sufficient to make all of the calculated results conservative when compared with the experiment. A value of $X_S = 1.6$ should give spacings that are about 50 percent greater than the critical separations.

Since reflections near an array are so important, they must be accounted for if calculated results are to be reliable. Note that critical arrays in a vault are spaced two to three times as far apart as the same arrays bare^[13]. Also, in the case of plane arrays of cylinders in a steel tank the normalized values of X_C average is higher than for experiments done outside the tank. Another illustration of the effect of wall reflections is an experiment in which an otherwise bare critical slab parallel to a concrete wall was moved away stepwise and the increase in critical height measured^[14]. It was found that the effect of the wall was measurable at a separation of 90 inches. From this it is obvious that in practice the effect of wall and floor reflections must always be evaluated. This suggests that another approach to conservatism is to use a reflector saving when calculating bucklings of individual units.

TABLE VII

COMPARISON OF EXPERIMENTAL WITH CALCULATED
CRITICAL SEPARATIONS USING HEXAGONAL SEVEN CYLINDER ARRAYS

Effect of Varying X_c Value					
Cylinder Diameter (in.)	H:U-235 Atomic Ratios	Experimental Critical Separation (in.)	Calculated Critical Separations (in.) as X_c varies		
			$X_c = 0.85$	$X_c = 1.0$	$X_c = 1.1$
6.0	44	2.0 ^[a]	1.3	2.04	2.58
6.0	44	3.0 ^[a]	1.86	2.82	3.54
6.0	309	1.0 ^[b]	0.75	1.35	1.80
6.0	309	2.0 ^[b]	1.3	2.2	2.88
8.0	44	1.0 ^[a]	0.80	1.4	1.8
8.0	44	2.0 ^[a]	1.84	2.6	3.1
8.0	44	3.0 ^[a]	2.7	3.5	4.1
8.0	44	4.0 ^[a]	3.6	4.55	5.2
8.0	44	6.0 ^[a]	5.5	6.8	7.5
8.0	44	9.0 ^[a]	8.1	10.0	10.8
8.0	309	1.0 ^[b]	0.92	1.6	2.05
8.0	309	3.0 ^[b]	2.8	3.8	4.55
8.0	309	6.0 ^[b]	5.8	7.2	8.15
8.0	309	7.0 ^[b]	6.4	8.0	9.2
9.5	297	3.0 ^[b]	3.5	4.45	5.1
9.5	297	10.0 ^[b]	11.1	13.3	14.4
9.5	297	22.0 ^[b]	22.8	25.7	27.6

[a] Experiments done in large steel tank.

[b] Experiments done outside tank.

V. CONCLUSIONS

Calculations and comparisons with experimental data have been done for a wide variety of cases to show that the method can be used for many types of interaction problems. As might be expected, the results are not accurate for close spacings. However, in most practical cases storage separations are far from contact. For separations greater than about 3 inches, the agreement between calculated and experimental separations is quite good. The trends in degree of agreement suggest that more precise values of the solid angles would help. The general agreement is good enough to show that the assumptions made in deriving the formula are, at least, approximately correct. The method has considerable practical value in fuel storage problems. The ease and speed with which calculations can be made is quite an advantage in this respect. The fact that the model can also be used for metal systems shows that the exact leakage spectrum is not important in regard to the interaction probability.

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APPENDIX DETERMINATION OF COUPLING COEFFICIENTS

The method used to obtain the coupling coefficients for the larger arrays may be illustrated by an example. If a regular square array of nine cylinders in a plane is used, as shown in Figure 4, then from the symmetry of the array there are identical neutron densities in the four corner cylinders and also in the four centered in each outside row. The central cylinder is unique and has the highest neutron density while the corner cylinders have the lowest. The form of the coupling equation depends upon how shadowed effects are treated. Here it is assumed that all vessels interact with all others, or that shadow effects are equal and opposite in their effect to surface scattering where neutrons are deflected into third vessels. Hence, for this array there are five different distances between interacting vessels. These correspond to five values of W : W_1 , W_2 , W_3 , W_4 , and W_5 . If N_1 , N_2 , and N_3 refer to the differing neutron densities, and B_g^2 and B_1^2 are as defined previously, then the three coupled neutron balance equations are:

$$N_1 B_g^2 = B_1^2 (N_1 - 4W_1 N_2 - 4W_2 N_3)$$

$$N_2 B_g^2 = B_1^2 (N_2 - W_1 N_1 - 2W_1 N_3 - 2W_2 N_2 - 2W_4 N_3 - W_5 N_2)$$

$$N_3 B_g^2 = B_1^2 (N_3 - W_2 N_1 - 2W_1 N_2 - 2W_3 N_3 - 2W_4 N_2 - W_5 N_3)$$

Since in a given problem $X\ell$ is a constant, the equations may be greatly simplified if the ratios of Ω_1/Ω_2 , Ω_1/Ω_3 , etc., are obtained so as to have the equations all in terms of Ω_1 . These ratios as obtained from the solid angle curves are not strictly constant with increasing array spacing. However, except at close spacing, they vary only a few percent. The approximate values of these ratios for a square array are:

$$\Omega_2 \approx 0.62 \Omega_1$$

$$\Omega_3 \approx 0.34 \Omega_1$$

$$\Omega_4 \approx 0.27 \Omega_1$$

$$\Omega_5 \approx 0.17 \Omega_1$$

Then, defining $B_g^2/B_1^2 - 1 = X$ the equations are as follows:

$$XN_1 + 4W_1 N_2 + 2.48W_1 N_3 = 0$$

$$W_1 N_1 + (X + 1.58W_1)N_2 + 2.54W_1 N_3 = 0$$

$$0.62W_1 N_1 + 2.5W_1 N_2 + (X + 0.85W_1)N_3 = 0$$

Let $X/W_1 = C$, then the determinant of the coefficients is:

$$\begin{vmatrix} C & 4 & 2.48 \\ 1 & C + 1.58 & 2.54 \\ 0.62 & 2.54 & C + 0.85 \end{vmatrix} = 0.$$

The above determinant can be evaluated to give $C \approx 4.9$ and, hence, $B_g^2 = B_1^2 (1 - 4.9W_1)$ for the 3×3 planar array.

If instead of obtaining approximate solid angle ratios as used above, a more refined matrix formulation had been used, as reported by Czerniejewski in HW-80213 to obtain overall interaction probabilities, better agreement with the experiment would have resulted. However, it would have required machine calculations.

A few comparisons with recent unpublished data on arrays of five percent enriched solution indicate that for these more closely spaced arrays of larger cylinders the shadow effect is more important. For example, a 5×5 planar array calculation agrees with the experiment when the coupling coefficient is reduced from 8.15 to about 7.2. This illustrates the difficulty of obtaining the true total interaction probability for a given array. The arbitrary elimination of all vessels that are geometrically shadowed from the array center reduces couplings too much since it neglects scattering effects.

Fig. 4 Planar square arrays of nine cylinders equally spaced.

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