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THE RESPONSE OF MONOENERGETIC GAMMA RAYS
IN FINITE MEDIA

by

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ABSTRACT

The response parameters of monoenergetic gamma rays incident on various materials with finite geometries has been calculated on a CDC-3600 computer. The Monte Carlo technique was used to obtain the solution to this gamma-ray transport problem. The program permits calculations of various source geometries and detector shapes for incident gamma rays varying in energy from 5 keV to 12 MeV. Results are included for gamma rays normal to cylindrical germanium and silicon detectors. Also presented are the data representing the double and single escapes subsequent to positron annihilation occurring uniformly throughout the volume of the detector.

I. INTRODUCTION

The transport of radiation in matter is a phenomenon useful in the field of gamma-ray spectroscopy. The program described here uses the Monte Carlo method as a technique to obtain the solution to this transport problem. The theoretical energy-loss spectra^{1,2} and other detector-response parameters for gamma rays are useful, specifically, in the design of counter systems and in the analysis of complex spectra.³ These quantities are calculated for monoenergetic photons, incident to various materials in an energy range from 5 keV to 12 MeV. The materials relevant to this calculation may be specified as single elements or organic or inorganic compounds, by employing the appropriate cross sections and various numerical constants, such as binding energies, density, etc., which are peculiar to the material under consideration. Five materials are currently admitted, namely, sodium iodide, PILOT "B" (plastic material), cesium iodide, germanium, and silicon. The first three substances are materials relevant to scintillation counters; germanium and silicon are elements of solid-state radiation detectors. The predominant feature of solid-state detectors, as compared with scintillation detectors, is the fine energy resolution. The type of detector is of no consequence in this theoretical calculation because the physical processes simulated are the same for all substances; however, the transport of the radiation is affected by the total cross sections, which are intrinsic properties to the type of material considered.

This report is an extension and refinement of the work reported earlier.^{1,2} The algorithms used for describing Compton scattering and charged-particle transmission are somewhat the same; however, all other processes are treated differently. This report includes the former for completeness.

II. RADIATION TRANSPORT

Gamma rays produced by some process constitute the source. These rays enter the detector at some boundary and are diffused by the material. The radiation may be totally or partially absorbed by the medium. Primary and secondary radiations that are created by the interaction of gamma rays with the detector material are traced within the boundary of the detector to determine the amount of energy that escapes. This escape quantity leads to the solution of the transport problem. By generating numerous incident photons, we can produce a histogram in the form of a spectrum that represents the energy loss to the medium.

The complete history of an incident gamma ray and its subsequent radiation is simulated in the following way: The energy of the incident photon is determined from the monoenergetic source energy, E_0 . Its direction and point of entry are determined by random sampling from the appropriate source distribution. The direction and position of impact of the incident ray having been set, the position of the first interaction is chosen by random sampling from the exponential distribution using the total cross section. If the path reaches a boundary, the history is terminated and another source ray is generated; however, for photons not escaping, the type of first collision is determined by sampling the relative distribution of admissible events. A type event having been selected, the direction of scattering of the particle(s) and the associated energy are resolved by random sampling of the appropriate distributions. The further history of collision products is traced through the medium until absorption is complete. This includes all secondary, tertiary, etc., radiations greater than some threshold energy, E_t .

The energy-loss spectrum is in the form of a histogram over a given interval of the number of incident photons yielding an energy loss within a set of equal subintervals; that is, the energy range is divided by the set of energy points $E_1, E_2, \dots, E_n$, with $16 \leq n \leq 4096$, where $E_1 = E_0^\gamma/n$, and $E_n = E_0^\gamma$. The quantity $\bar{N}_i$ is defined as the number of incident photons of energy E_0^γ that lose, in the detector, an amount of energy in the interval (E_i, E_{i+1}) .

III. PHYSICAL PROCESSES

The collision processes, both primary and secondary, that affect the transport of gamma rays are Rayleigh and Compton scattering, pair production, and photoelectric effect. During these processes, secondary electrons, positrons, and photons are created. The secondary photons considered are the scattered gammas in Rayleigh and Compton scattering, the fluorescent radiation subsequent to the photoelectric effect, and photons created from positron annihilation. Charged particles accounted for are Compton-scattered electrons, photoelectrons, and positrons and electrons resulting from pair production. Positrons were taken as electrons because the mean ranges of each are equal in the energy range under consideration, except for the annihilation property of positrons. All other processes have been neglected because they occur with very small frequency at the energies treated herein.

Fig. 1. Gamma-ray Attenuation Coefficients for Germanium

The photoelectric effect is significant at low energies. It becomes more and more probable as the incident photon energy approaches zero. Pair production can only occur above $2m_0c^2$, the probability of occurrence increasing with energy. Rayleigh scattering occurs at low energies and is considered to vanish beyond a maximum energy $E_r^{\max}$. Compton effect is important throughout the entire energy range. (A typical example is shown in Fig. 1.)

A. Rayleigh Scattering

In Rayleigh scattering, the entire recoil momentum is absorbed by the atom as a whole; thus there is no scattered electron to account for. Due to momentum considerations, the atom can absorb very little energy from the photon. In the present work, the atom was treated as absorbing a very small amount of energy (less than 0.1 keV), and the photon was deflected through a small angle. Experimental results⁴ show that 75% of these photons are scattered by an angle less than

$$\theta_c = 2 \sin^{-1} \left(0.026 Z^{1/3} \frac{m_0 c^2}{E} \right),$$

where E is the kinetic energy of the incident photon and Z is the atomic number of the target material. The angle θ is chosen randomly and uniformly distributed between $\theta = 0$ and $\theta = \theta_c$, and ϕ is chosen uniformly on $(0, 2\pi)$.

B. Photoelectric Effect

The two secondary radiations that occur in a photoelectric collision are the photoelectron and the fluorescent X ray. In this interaction, the photon yields all its energy to a bound atomic electron, which is ejected with kinetic energy equal to the incident photon energy less the electron binding energy. The fluorescent X-ray yield varies for each material. The energy of the X ray is taken as the binding energy of the K-shell electron, and this X ray is emitted isotropically.

For energies less than 70 keV, the polar angle (θ) of scattering of the photoelectron is determined from Fisher's⁵ distribution,

$$d\phi = \frac{\sin^2 \theta}{\left[1 + \frac{E}{2m_0c^2} - \beta \cos \theta\right]^4},$$

and for energies greater than 70 keV, from Sauter's⁶ distribution,

$$d\phi = \beta^2 \sin^2 \theta \left\{ \frac{(1 - \beta^2)^{1/2}}{(1 - \beta \cos \theta)^4} - \frac{[1 - (1 - \beta^2)^{1/2}]}{2(1 - \beta^2)^{1/2} (1 - \beta \cos \theta)^3} + \frac{[1 - (1 - \beta^2)^{1/2}]^2}{2(1 - \beta^2)(1 - \beta \cos \theta)^3} \right\},$$

where E is the energy of the photoelectron in MeV, $d\phi$ is the differential cross section for the scattering process in which the photoelectron is scattered through the angle θ , and

$$\beta = \left[1 - \frac{1}{\left(\frac{E}{m_0c^2} + 1 \right)^2} \right]^{1/2}.$$

By sampling θ from the above distributions and ϕ uniformly from 0 to 2π , we have chosen a vector from the uniform distribution throughout the solid angle.

C. Compton Scattering

In a Compton collision, the scattering of an incident energetic photon by an atomic electron that is initially at rest can be described as a two-particle collision if we neglect the small effects due to the binding of the electron to the nucleus. The energy and the momentum of the incident photon will be changed in the collision process, yielding energies for the scattered photon and the recoil electron determined by the conservation laws for each scattering angle. The photon will rebound at some angle θ_p with its initial direction. The electron will recoil at an angle θ_E with the

initial direction of the incident photon. The energy of the scattered photon E^P determined from the conservation laws is

$$E^P = \frac{E_0}{1 + \frac{E_0}{m_0 c^2} (1 - \cos \theta_P)}$$

The energy of the recoil electron is

$$e^- = E_0 - E^P$$

and

$$\tan \theta_E = \frac{E^P \sin \theta_P}{E_0 - E^P \cos \theta_P}$$

The azimuthal angle of scattering ϕ_E is sampled uniformly on the interval $(0, 2\pi)$.

For unpolarized radiation, the polar angle (θ_P) of scattering of the scattered photon is determined by random sampling from the Klein-Nishina distribution,

$$d\phi = \frac{R_0^2 (1 - \cos^2 \theta_P)}{2 \left[1 + \frac{E_0}{m_0 c^2} (1 - \cos \theta_P) \right]^2} \left\{ 1 + \frac{\left(\frac{E_0}{m_0 c^2} \right)^2 (1 - \cos \theta_P)^2}{(1 + \cos^2 \theta_P) \left[1 + \frac{E_0}{m_0 c^2} (1 - \cos \theta_P) \right]} \right\},$$

in which the incident photon of energy, E_0 , is scattered by a free electron through the angle θ_P into the solid angle with energy E^P ; the azimuthal angle, ϕ_P , is sampled from $(0, 2\pi)$ uniformly.

D. Pair Production

A photon with energy $\geq 2m_0 c^2$ in the vicinity of the nucleus can create an electron-positron pair. Mass will be created from energy in this process. If the energy of the photon is greater than $2m_0 c^2$, the excess energy is shared unequally between the positron and the electron in the form of kinetic energy. The selection technique for determining the amount of energy imparted to the electron and positron is calculated using the equation of Hough;⁷

$$H(x) = \psi_0 Z [1 + 0.135(\psi_0 - 0.52) Z(1 - Z^2)]$$

and

$$\psi_0 = (1 - \gamma) \left[\frac{1}{3} (4 - \gamma^2)(L - 1) - \gamma^2 \alpha (\alpha - 1) - \gamma^4 \alpha (L - \alpha) \right],$$

where

$$x = \frac{E_+ - 1}{k - 2}$$

and

$$Z = 2[x(1 - x)]^{1/2}.$$

The quantities E_+ and k are the positron and incident photon energies, respectively, in rest mass units. In the above distribution,

$$\gamma = 2/k,$$

$$L = \frac{2}{1 - \gamma^2} \ln \left(\frac{k}{2} \right),$$

and

$$\alpha = \frac{1}{(1 - \gamma^2)^{1/2}} \ln \left\{ \frac{k}{2} + \left[\left(\frac{k}{2} \right)^2 - 1 \right]^{1/2} \right\}.$$

To select the initial energies of the positron, two random numbers, r_1 and r_2 , are chosen uniform on $(0, 1)$ and accepted if $r_2 \leq H(r_1)$. Then

$$A = r_1 \left(\frac{E_0}{m_0 c^2} - 2 \right) + 1,$$

$$B = \frac{E_0}{m_0 c^2} - A,$$

and

$$E_+ = \frac{A}{A + B} (E_0 - 1.02), \quad E_- = E_0 - E_+,$$

where E_0 is the energy of the incident photon, E_- is the energy of the electron, and E_+ is the energy of the positron, all in MeV.

The average angle between the direction of motion of a created electron and the incident photon is approximately $\theta_0 = m_0c^2/E_0$. Therefore we see that the largest scattering angle,

$$\theta_0 = \frac{m_0c^2}{2m_0c^2} = \frac{1}{2} \text{ rad} = 28.65^\circ,$$

would occur at the threshold energy. Knowing θ_0 , we choose a scattering plane and, by the appropriate transformation of coordinates, obtain the direction cosines of the positron and electron vectors in the primary system. This is the coordinate system of the detector.

E. Annihilation Radiation

When the created positron traveling through the medium with initial energy $(E_+ + 1.02)$ MeV reaches zero velocity, it annihilates with an atomic electron. The positron collides with the atom, creating a vacant state into which an atomic electron may fall, releasing energy equal to $2m_0c^2$. This energy may be emitted in the form of a gamma pair, each with energy m_0c^2 , or instead transferred to a second electron, which escapes the atom. In this calculation, provision for only the former event is made due to the low probability of occurrence of the electron energy-transfer event. If the positron escapes the medium, no annihilation takes place. If, however, the positron is stopped in the medium, the gamma pair is emitted isotropically from the position of zero velocity, but oppositely directed.

F. Bremsstrahlung

Electrons and positrons are slowed by two processes, namely, radiative collisions with the fields of the nuclei, and inelastic collisions with bound electrons. The radiative losses or the energy imparted as bremsstrahlung photons is calculated first by determining the number of gamma-ray photons given off when an electron or positron is slowed in the medium. This is accomplished by solving^{8,9,10}

$$P(E, k) = \int_k^{E_0-1} dk \int_{k+1}^{E_0} \frac{\Phi(E, k) dE}{-\left(\frac{dE}{dx}\right)},$$

where $P(E, k)$ is the total number of photons created by an electron or positron with energy E_0 ,

$$\begin{aligned} \Phi(E, k) = \frac{4r_0^2 Z^2 N}{137k} & \left\{ \left[1 + \left(\frac{E-k}{E} \right)^2 \right] \left[\frac{\phi_1(\gamma)}{4} - \frac{1}{3} \ln Z \right] \right. \\ & \left. - \frac{2}{3} \left(\frac{E-k}{E} \right) \left[\frac{\phi_2(\gamma)}{4} - \frac{1}{3} \ln Z \right] \right\}, \end{aligned}$$

and

$$-\frac{dE}{dx} = \frac{2\pi r_0^2 N Z E^2}{E^2 - 1} \left\{ \ln \left[\frac{(E^2 - 1)(E - 1)}{2I^2} \right] + \left[\frac{E^2 + 9.0904E + 3.4548}{8E^2} \right] \right\},$$

where

$$\phi_1(\gamma) \simeq 5.089e^{-0.6587\gamma} + 15.661e^{-0.0481\gamma}$$

$$\phi_2(\gamma) \simeq \phi_1(\gamma) - 0.595e^{-3.04\gamma}$$

and

$$\gamma = \frac{100k}{E(E - k) Z^{1/3}}.$$

In the above equations, E is the energy of the incident electron or positron, k is the photon energy, Z is the atomic number of the material, N is the number of atoms per cubic centimeter, r_0 is the classical electron radius, and I is the mean excitation potential (see Table I).

TABLE I. Mean Excitation Potentials¹¹

Element	Excitation Potential, I, eV	Element	Excitation Potential, I, eV
He	42.0	Cu	314.0
Li	38.0	Kr	381.0
Be	60.0	Ag	487.0
C	78.0	Sn	516.0
N	85.0	Xe	555.0
O	89.0	W	748.0
Ne	131.0	Au	797.0
Mg	156.0	Pb	826.0
Al	163.0	U	923.0
A	210.0		

A table of values of $P(E, k)$ was constructed for various initial electron energies (see Table II), and linear interpolation was used to determine the number of photons emitted for any intermediate initial electron energy. The double integrals, $P(E, k)$, for the tabular entries were determined by using the Romberg quadrature algorithm to evaluate the integral in the k direction and an alternate form, the Havie scheme, to evaluate the integral in the E direction with k fixed. Since $P(E, k)$ is not an integer, the number of photons N_p was taken as the integer part of $N_p = P(E_0, k) + R$, where

R is a random number uniform on the interval (0, 1). Knowing the number of bremsstrahlung photons created in the slowing down of a charged particle, we must determine the energy of each. This is accomplished by sampling from the normalized probability distribution,

TABLE II. Total Number of Bremsstrahlung Photons Emitted by a Charged Particle That Reaches Zero Velocity in the Medium

Initial Energy, MeV	Number of Photons	
	Germanium	Silicon
3.0	0.4902	0.1939
4.0	0.7769	0.3100
5.0	1.0821	0.4344
6.0	1.4003	0.5649
7.0	1.7277	0.6996
8.0	2.0620	0.8376
9.0	2.4030	0.9788
10.0	2.7494	1.1225
11.0	3.1006	1.2683
12.0	3.4554	1.4160

$$R = \frac{\int_{0.02}^k P(E_0, k) dk}{\int_{0.02}^{E_0-1} P(E_0, k) dk},$$

where R is a uniform random number on (0, 1) and k is the energy in MeV of the ith photon, $i = 1, 2, \dots, N_p$.

G. Charged Particles

The slowing of Compton-scattered electrons, photoelectrons, and pair-produced positrons and electrons constitute energy losses to the medium. In slowing, charged particles participate in very large numbers of interactions, each with relatively small energy loss per interaction, in contrast to the slowing down of gamma rays, which undergo rather few large-energy-loss interactions. Because of the large number of interactions it is too time-consuming to treat charged-particle slowing exactly. The approximation used here seems sufficient for the energy range under consideration. We assume that the electron or positron travels its mean distance and in a straight line in the direction determined by the appropriate angular distributions. The mean distance traveled by a charged particle was taken from the work of Berger and Seltzer^{12,13} (see Table III). According to their calculations, the mean distances traveled by the electron and positron were the same for the energy range considered here.

TABLE III. Mean Range of Electrons¹²

Energy, MeV	Range, in.		Energy, MeV	Range, in.	
	Germanium	Silicon		Germanium	Silicon
0.01	0.00004	0.00006	1.0	0.0474	0.0874
0.02	0.00011	0.00019	2.0	0.102	0.1924
0.03	0.00023	0.00038	3.0	0.1534	0.294
0.05	0.00054	0.00092	5.0	0.2474	0.4862
0.08	0.00118	0.00205	8.0	0.3722	0.7536
0.10	0.00171	0.00298	10.0	0.447	0.9204
0.20	0.0052	0.00923	20.0	0.7529	1.654
0.30	0.0096	0.0172	50.0	1.326	3.248
0.50	0.0197	0.0358	100.0	1.856	4.951
0.80	0.362	0.0664

Fig. 2. Various Radiation-source Geometries.

R_c = radius of collimation; H_p = height of point source from cylinder face; R = radius of cylinder; Y_B = center of side-beam displacement from X-Z plane; and Z_B = center of side-beam displacement from X-Y plane.

employing the appropriate shape. The current allowable detector shapes are a right circular cylinder, a parallelepiped, or a sphere.

IV. GEOMETRY

A. Radiation Source

The program permits calculations for the following source geometries (see Fig. 2):

1. Broad parallel beam
2. Collimated parallel beam
3. Point source on detector axis
4. Collimated point on detector axis
5. Disc source (disc center on the axis of the detector where the disc radius is less than or equal to the radius of the detector face)
6. Side beam

B. Detector Shape

The program can be set to use any detector geometry by em-

V. MONTE CARLO TECHNIQUES

A. Random Numbers

Fundamental to all rejection techniques is the use of pseudorandom numbers that are uniformly distributed and uncorrelated. These random numbers are generated by

$$R_n = R_{n-1} P(\text{MOD } 2^K),$$

where P is one of a unique family of multipliers, and K is determined by the word length of the machine ($K = 48$ for the CDC-3600).

B. Path-length Selection

The probability, $P(C)$, that a photon will undergo a collision after a path length between L and $L + dL$ is $\mu_T e^{-\mu_T L} dL$. Therefore,

$$P(C) = \mu_T \int_0^L e^{-\mu_T L} dL,$$

where μ_T is the total attenuation coefficient. We can represent the fraction that survives by

$$F = e^{-\mu_T L}$$

From a random number (R) uniform on the interval (0, 1), the determination is made of

$$\text{Path Length} = \frac{|\ln R|}{\mu_T}$$

C. Uniformly Distributed Random Angle

To select a random angle, θ , uniform on the interval $(0, 2\pi)$, consider the unit square and the inscribed unit circle, shown below.

Choose two random numbers, r_1 and r_2 , uniform on $(-1, 1)$. These numbers define a point inside the square, which must also lie inside the circle. The point (r_1, r_2) is accepted if $r_1^2 + r_2^2 \leq 1$. The sampling efficiency of this process is the ratio of the area of the circle to that of the square, or $\pi/4$. From (r_1, r_2) , $\sin \theta$ and $\cos \theta$ may be obtained as

$$\sin \theta = \frac{r_2}{(r_1^2 + r_2^2)^{1/2}}, \quad \cos \theta = \frac{r_1}{(r_1^2 + r_2^2)^{1/2}}$$

In order to eliminate the radical the equivalence of the double-angle distribution yields

$$\sin \theta = \frac{2r_1 r_2}{r_1^2 + r_2^2}, \quad \cos \theta = \frac{r_1^2 - r_2^2}{r_1^2 + r_2^2}$$

D. Selecting an Isotropic Direction

To select a vector uniform within the sphere, consider the unit cube and its inscribed unit sphere. Choose three random numbers, r_1 , r_2 , and r_3 , uniform on $(-1, 1)$, and reject unless

$$r_1^2 + r_2^2 + r_3^2 \leq 1.$$

The sampling efficiency of this procedure is $\pi/6$. Having selected a point in the sphere, define a vector with direction cosines

$$\cos \alpha = \frac{r_1}{P}, \quad \cos \beta = \frac{r_2}{P}, \quad \cos \gamma = \frac{r_3}{P},$$

where

$$P = (r_1^2 + r_2^2 + r_3^2)^{1/2}.$$

E. Sampling the Klein-Nishina Distribution

Let $d\phi$ denote the differential cross section for a scattering process in which the incident photon of energy E_0 is scattered through an angle θ into a solid angle $d\Omega$ with energy E_p . Then,

$$d\phi = E^2(E + E^{-1} - \sin^2 \theta) d\Omega, \quad d\Omega = \sin \theta d\theta d\phi,$$

where

$$E = \frac{1}{1 + \alpha\omega},$$

with

$$\alpha = \frac{E_0}{m_0 c^2}$$

and

$$\omega = 1 - \cos \theta,$$

$$P(\omega) = E^2[E + E^{-1} + \omega(\omega - 2)],$$

and r_1 and r_2 are chosen uniform on the interval $(0, 2)$. If $r_2 \leq P(r_1)$, r_1 is accepted as equal to ω , and $\cos \theta = 1 - \omega$. The azimuthal angle is then selected from the uniform distribution on $(0, 2\pi)$. The $\sin \phi$ and $\cos \phi$ define the plane in which the scattering takes place. By performing the appropriate

transformation of coordinates with the information known, we can determine the direction cosines of the scattered photon and electron in the coordinate system of the detector.

F. Choosing a Type Event

To determine a type event, a random number uniform on $(0, 1)$ is chosen, and the type event is made as follows:

where

μ_{pp} is the cross section for pair production,

μ_r is the cross section for Rayleigh scattering,

μ_c is the cross section for Compton scattering,

and

μ_T is the total cross section.

G. Selection of Incident Rays

The selection of incident rays depends on the detector shape. The following sampling procedures are peculiar to right circular cylinders. By various geometrical manipulations, these same procedures can be applied to any detector shape.

1. Broad Parallel Beam

It is assumed that the rays are uniformly distributed throughout the beam and that every ray in the beam strikes the face of the cylinder. The point of impact of the incident gamma ray is determined by selecting a random point on the face of the cylinder. The momentum of the gamma ray is taken as normal to the face at that point.

2. Collimated Parallel Beam

The incident ray is sampled in the same manner, except that the point of impact is restricted to an area defined by the radius of collimation.

3. Point Source on Detector Axis

The direction of emission is randomly chosen from vectors uniformly distributed in the solid angle subtended by the cylinder face. The point at which the gamma ray enters the cylinder face is determined from the direction cosines of the vector and the distance of the point source from the cylinder face. The fraction of the solid angle 2π that the cylinder face subtends is defined as G .

4. Collimated Point Source on Detector Axis

The incident ray is sampled as the ray for the point source, except that the point of impact is restricted to an area defined by the radius of collimation on the cylinder face.

5. Disc Source

The direction of emission is randomly chosen from vectors uniformly distributed in a solid angle that includes the entire cylinder face. The cone is a right circular cone with apex angle $2 \tan^{-1} (2R/H_p)$, where R is the radius of the cylinder and H_p is the height separating the disc and the cylinder face. Many of the rays emitted in this solid angle do not strike the cylinder face; however, many rays can be sampled extremely fast, and the fraction of these that strike the cylinder face can be easily computed. In this case, G is the product of this fraction with the fraction of the solid angle, 2π .

6. Side Beam

For the side beam, the incident ray is chosen similarly to 1 and 2, except that the beam is incident to the curved surface of the cylinder. The beam center can be positioned anywhere along the curved surface of the cylinder.

For a broad parallel beam and a side beam, $G = 1$. For a point source,

$$G = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\tan^{-1}(R/H_P)} \sin \theta d\theta d\phi \quad (\text{for } H_P > 0),$$

and for the disc source,

$$G = \frac{H}{2\pi} \int_0^{2\pi} \int_0^{\tan^{-1}(2R/H_P)} \sin \theta d\theta d\phi \quad (\text{for } H_P > 0),$$

where H is the fraction of those rays emitted in the designated cone that strike the cylinder face. If we designate the total number of photons emitted as N , the number that strike the cylinder as NG , the number that are completely absorbed, including all secondary radiation, as A , and the number of incident gamma rays that experience at least one interaction as I , then the efficiency is given by I/NG , the photofraction is A/I , and the photoefficiency as $A/(I+NG)$.

VI. COORDINATE SYSTEM

The coordinate system used is shown in Fig. 3; this is the primary or detector coordinate system. Various distributions are sampled in a secondary system. This system is defined by the incident photon (see Fig. 3). Having determined the appropriate scattering angles of the outgoing ray in the secondary coordinate system, we must rotate the ray back into the primary system. This is accomplished by using the following transformations:

$$\ell'' = (\cos \theta \cos \phi) \ell' - (\sin \phi) m' + (\sin \theta \cos \phi) n',$$

$$m'' = (\cos \theta \sin \phi) \ell' + (\cos \phi) m' + (\sin \theta \sin \phi) n',$$

and

$$n'' = (\cos \theta) n' - (\sin \theta) \ell',$$

where ℓ' , m' , and n' are the direction cosines of the incoming vector in the primary coordinate system, θ and ϕ are the polar and azimuthal angles of scattering with respect to the Z -axis, and ℓ'' , m'' , and n'' are the direction cosines of the outgoing vector in the primary system.

Fig. 3. Coordinate Systems

VII. CROSS SECTIONS

The gamma-ray cross sections for Rayleigh and Compton scattering, for the photoelectric effect, and for pair production are taken from the work of Davisson¹¹ (see Tables IV and V). These data are in the form of tables.

TABLE IV. Gamma-ray Cross Sections for Germanium* Cross Sections, in.⁻¹.

Energy, MeV	Photoelectric	Compton	Rayleigh	Pair Production
0.01	412.3759	2.3139	17.7662	
0.015	1287.9687	2.2731	10.0544	
0.02	578.4561	2.2357	7.1209	
0.03	183.0271	2.1586	4.0857	
0.04	78.2949	2.0902	2.6130	
0.05	40.4467	2.0293	1.8440	
0.06	23.9517	1.9729	1.2264	
0.08	10.2247	1.8682	0.7116	
0.1	5.3124	1.7809	0.4890	
0.15	1.6095	1.6038	0.23	
0.2	0.6913	1.4701	0.1514	
0.3	0.1971	1.278	0.066	
0.4	0.0908	1.1448	0.0354	
0.5	0.0519	1.0455	0.0225	
0.6	0.0339	0.9673	0.0148	
0.8	0.016	0.8497	0.0072	
1.0	0.0092	0.7633	0.0056	
1.3	0.0066	0.6776	0.0024	0.0019
1.5	0.0048	0.6205	0.0003	0.0056
2.0	0.0032	0.5295		0.0215
3.0	0.0009	0.4161		0.0599
4.0	0.0007	0.347		0.0971
5.0	0.0005	0.2994		0.1291
6.0	0.0003	0.2647		0.1563
8.0	0.0002	0.2165		0.2001
10.0	0.0001	0.1843		0.2365
15.0		0.1364		0.3068
20.0		0.1093		0.3566
30.0		0.0795		0.4284
50.0		0.0527		0.5131
100.0		0.0296		0.614

*in.⁻¹ x 8.851 = barns/atom; in.⁻¹ x 0.07345 = cm²/gram.

TABLE V. Gamma-ray Cross Sections for Silicon* Cross Sections, in.⁻¹.

Energy, MeV	Photoelectric	Compton	Rayleigh	Pair Production
0.01	205.2251	1.1825	3.1727	
0.015	60.7097	1.1627	1.7408	
0.02	24.8118	1.1416	1.1020	
0.03	7.0608	1.1033	0.5860	
0.04	2.6923	1.0677	0.3577	
0.05	1.3066	1.0360	0.2442	
0.06	0.7391	1.0083	0.1663	
0.08	0.2904	0.9555	0.1003	
0.1	0.1439	0.9107	0.0633	
0.15	0.0396	0.8196	0.0304	
0.2	0.0158	0.7510	0.0172	
0.3	0.0049	0.6532	0.0079	
0.4	0.0022	0.5847	0.0040	
0.5	0.0012	0.5345	0.0026	
0.6	0.0008	0.4936	0.0013	
0.8	0.0004	0.4342		
1.0	0.0002	0.3907		
1.5	0.0001	0.3167		0.0012
2.0		0.2706		0.0046
3.0		0.2126		0.0133
4.0		0.1773		0.0218
5.0		0.1531		0.0292
6.0		0.1353		0.0358
8.0		0.1106		0.0467
10.0		0.0942		0.0560
15.0		0.0697		0.0736
20.0		0.0558		0.0865
30.0		0.0407		0.1045
50.0		0.0269		0.1268
80.0		0.0183		0.1453
100.0		0.0151		0.1542

*in.⁻¹ x 7.64 = barns/atom; in.⁻¹ x 0.164 = cm²/gram.

Each table is separated into energy intervals with corresponding cross sections. The cross sections are included for 21 elements, starting with hydrogen (1) and ending with uranium (92). A series of spline curves was fit to these data at each energy interval given to determine the cross sections for any element. The appropriate cross sections are input at run time. To obtain the cross section for any given energy, linear interpolation is done between the appropriate energy values in the stored table.

VIII. COMPARISON WITH EXPERIMENTAL RESULTS

Figure 4 compares the theoretical photoefficiencies $[A/(I+P)]$ with the experimental results of Freedman,¹⁴ Fig. 5 compares the theoretical photoefficiencies of Wood,¹⁵ and Fig. 6 compares the calculated double-escape efficiencies $[\text{number of double escapes}/(I+P)]$ with the experimental results of Blatchley.¹⁶ In all cases, the theoretical results are arbitrarily normalized to agree with the experimental curves. The theoretical points agree quite satisfactorily with the experimental curves in all cases represented here.

Fig. 4. Photoefficiencies for a Point Source Incident on a Cylindrical Germanium Detector with Thickness of 6 mm and Diameter of 22 mm. The source is located 15 mm above the detector face.

Fig. 5. Photoefficiencies for a Point Source Incident on a Cylindrical Germanium Detector with Thickness of 14 mm and Diameter of 17.5 mm. The source is located 25 mm above the detector face.

Fig. 6

Double-escape Efficiencies for a Side-beam Source Incident on a Cylindrical Germanium Detector with Thickness of 11.2 mm and Diameter of 20.8 mm. The source is a cylindrical parallel beam with a diameter of 9.525 mm whose axis is perpendicular to the detector axis at the center.

IX. RESULTS

Tables VI-IX show the total number of single and double escapes subsequent to positron annihilation for germanium and silicon for various detector sizes. The detector shape was in the form of a right circular cylinder, and the gamma pairs, each with energy equal to m_0c^2 , were emitted isotropically but oppositely directed from points uniformly distributed throughout the cylinder. In all cases shown, 5000 pairs were emitted.

TABLE VI. Total Number of Double Escapes for 5000 Gamma Pairs, Each with Energy Equal to m_0c^2 Emitted Uniformly throughout Various Right Circular Cylinders in Germanium.

Diameter, mm	Height, mm							
	5	10	15	20	25	30	35	40
5	4162	3999	3998	3955	3921	3917	3915	3940
10	3811	3533	3428	3321	3150	3180	3157	3204
15	3541	3127	2974	2780	2752	2680	2599	2652
20	3422	2886	2702	2480	2325	2293	2255	2250
25	3261	2698	2418	2208	2125	1968	1957	1899
30	3210	2575	2248	2074	1932	1753	1707	1703
35	3181	2463	2062	1885	1698	1605	1543	1455
40	3152	2463	2036	1721	1518	1429	1375	1271

TABLE VII. Total Number of Single Escapes for 5000 Gamma Pairs,
Each with Energy Equal to m_0c^2 Emitted Uniformly throughout
Various Right Circular Cylinders in Germanium.

Diameter, mm	Height, mm							
	5	10	15	20	25	30	35	40
5	39	74	82	95	84	93	83	77
10	97	161	155	200	220	234	228	226
15	140	230	289	337	340	369	454	393
20	171	313	360	451	529	496	570	556
25	193	340	447	572	593	677	668	706
30	211	401	532	621	695	798	824	822
35	241	415	617	689	794	898	906	912
40	247	447	619	763	925	931	965	1050

TABLE VIII. Total Number of Double Escapes for 5000 Gamma Pairs,
Each with Energy Equal to m_0c^2 Emitted Uniformly throughout
Various Right Circular Cylinders in Silicon.

Diameter, mm	Height, mm							
	5	10	15	20	25	30	35	40
5	4583	4520	4483	4435	4393	4442	4463	4445
10	4373	4158	4120	4046	4076	3963	3940	3982
15	4272	4008	3896	3729	3687	3626	3624	3528
20	4155	3787	3666	3491	3439	3368	3349	3323
25	4076	3732	3546	3323	3264	3151	3151	3105
30	3975	3610	3401	3173	3056	3058	2923	2801
35	3961	3520	3282	3036	2942	2846	2767	2713
40	3911	3482	3214	2969	2802	2648	2616	2550

TABLE IX. Total Number of Single Escapes for 5000 Gamma Pairs,
Each with Energy Equal to m_0c^2 Emitted Uniformly throughout
Various Right Circular Cylinders in Silicon.

Diameter, mm	Height, mm							
	5	10	15	20	25	30	35	40
5	0	1	4	4	4	3	0	1
10	1	3	1	1	1	2	3	5
15	1	2	7	10	9	16	4	11
20	2	5	5	5	11	13	9	18
25	2	11	9	11	18	10	21	15
30	6	10	14	22	17	23	28	21
35	6	9	16	18	26	33	35	26
40	5	8	20	26	23	33	51	43

Tables X-XVIII present the response parameters for monoenergetic gamma rays incident on right circular cylinders of germanium and silicon. The source geometry used in all cases was a broad parallel beam of radiation, where all the incident rays are normal to the cylinder face. The quantity I is the total number of primary gamma rays that interact within the detector at least once, A is the number of primary gamma rays that are completely absorbed, and P is the number of incident gamma rays that pass completely through the detector without interacting. The quantities labeled single and double escapes are the number of incident gamma rays that interact within the detector where all the incident energy is absorbed except m_0c^2 or $2m_0c^2$, created from positron annihilation subsequent to pair production, which escape. Also shown are the total number of positrons and electrons that escaped the detector, along with the distribution of primary type events and the distribution of all events that occurred in each case.

TABLE X. Response Parameters for Monoenergetic Gamma Rays Incident on Right Circular Cylinder of 1-cm-diam, 1-cm-thick Germanium

Energy, MeV	Interacting Primary Gamma Rays, I	Absorbed Primary Gamma Rays, A	Incident Gamma Rays, P	Single Escapes	Double Escapes	Positron Escapes	Electron Escapes	Primary Events				Total Events				
								Pair Production	Compton	Rayleigh	Photo-electric	Pair Production	Compton	Rayleigh	Photo-electric	X Rays
0.05	10,000	9,764	0	0	0	0	16	0	443	432	9,125	0	508	674	14,675	4,923
0.10	10,000	8,941	551	0	0	0	36	0	2,346	678	6,976	0	2,829	1,052	13,413	4,457
0.20	10,000	4,871	6,661	0	0	0	87	0	6,355	656	2,989	0	8,017	1,010	7,281	2,351
0.30	10,000	2,548	11,968	0	0	0	82	0	8,270	438	1,292	0	10,275	746	3,893	1,303
0.40	10,000	1,604	15,748	0	0	0	131	0	8,984	270	746	0	11,058	474	2,452	802
0.50	10,000	1,106	17,969	0	0	0	194	0	9,320	213	467	0	11,304	374	1,741	586
0.60	20,000	1,711	40,499	0	0	0	467	0	19,026	307	667	0	23,203	608	2,678	875
0.70	20,000	1,371	44,839	0	0	0	694	0	19,197	256	547	0	23,146	501	2,209	732
0.80	20,000	1,118	48,359	0	0	0	880	0	19,443	174	383	0	23,379	356	1,819	596
1.00	20,000	865	55,584	0	0	0	1,233	0	19,600	156	244	0	23,356	316	1,448	476
2.50	30,000	424	136,781	58	1,247	637	6,173	2,406	27,453	0	141	2,443	32,370	207	1,922	631
5.00	30,000	64	161,745	130	2,540	3,607	11,658	8,953	21,026	0	21	9,073	26,944	495	9,534	3,137
7.50	30,000	8	168,394	103	1,575	7,488	16,325	13,747	16,238	0	15	13,900	21,907	574	11,206	3,690
10.00	50,000	0	278,979	56	1,047	18,532	33,305	27,932	22,041	0	27	28,234	29,876	820	15,638	5,150
12.00	50,000	0	269,615	19	372	22,872	36,487	30,747	19,230	0	23	31,007	25,873	719	13,823	4,554

TABLE XI. Response Parameters for Monoenergetic Gamma Rays Incident on Right Circular Cylinder of 2-cm-diam, 2-cm-thick Germanium

Energy, MeV	Interacting Primary Gamma Rays, I	Absorbed Primary Gamma Rays, A	Incident Gamma Rays, P	Single Escapes	Double Escapes	Positron Escapes	Electron Escapes	Primary Events				Total Events				
								Pair Production	Compton	Rayleigh	Photo-electric	Pair Production	Compton	Rayleigh	Photo-electric	X Rays
0.05	10,000	9,789	0	0	0	0	5	0	480	375	9,145	0	541	611	14,647	4,871
0.10	10,000	9,417	20	0	0	0	11	0	2,398	638	6,967	0	3,000	1,002	14,013	4,614
0.20	10,000	6,290	1,926	0	0	0	28	0	6,373	643	2,984	0	9,130	1,196	9,420	3,129
0.30	10,000	3,778	4,199	0	0	0	40	0	8,342	407	1,251	0	12,281	867	5,656	1,869
0.40	10,000	2,687	5,823	0	0	0	57	0	9,006	266	728	0	13,090	657	4,013	1,311
0.50	10,000	2,069	6,935	0	0	0	88	0	9,296	178	526	0	13,494	542	3,111	1,008
0.60	10,000	1,635	8,060	0	0	0	110	0	9,546	140	314	0	13,526	449	2,489	843
0.70	10,000	1,392	8,899	0	0	0	142	0	9,594	137	269	0	13,626	392	2,151	729
0.80	10,000	1,214	9,957	0	0	0	222	0	9,729	79	192	0	13,645	350	1,871	626
1.00	10,000	1,013	11,847	0	0	0	297	0	9,798	76	126	0	13,721	320	1,610	548
2.50	10,000	455	20,586	64	358	105	958	854	9,115	0	31	870	12,800	153	1,214	386
5.00	20,000	245	49,168	387	1,837	1,197	3,896	6,096	13,887	0	17	6,249	23,340	766	11,680	3,880
7.50	20,000	134	52,044	359	1,588	2,350	5,381	9,113	10,875	0	12	9,321	22,534	1,101	18,199	6,004
10.00	40,000	104	102,344	503	2,506	7,194	13,326	22,495	17,486	0	19	22,981	43,069	2,843	46,124	15,204
12.00	40,000	48	99,177	347	1,767	8,942	15,253	24,681	15,299	0	20	25,195	42,516	3,266	53,948	17,800

TABLE XII. Response Parameters for Monoenergetic Gamma Rays Incident on Right Circular Cylinder of 3-cm-diam, 3-cm-thick Germanium

Energy, MeV	Interacting Primary Gamma Rays, I	Absorbed Primary Gamma Rays, A	Incident Gamma Rays, P	Single Escapes	Double Escapes	Positron Escapes	Electron Escapes	Primary Events				Total Events				
								Pair Production	Compton	Rayleigh	Photo-electric	Pair Production	Compton	Rayleigh	Photo-electric	X Rays
0.05	10,000	9,818	2	0	0	0	2	0	426	413	9,161	0	497	650	14,736	4,958
0.10	10,000	9,520	1	0	0	0	14	0	2,317	668	7,015	0	2,969	1,055	14,180	4,672
0.20	10,000	7,149	688	0	0	0	21	0	6,379	670	2,951	0	9,788	1,361	10,709	3,557
0.30	10,000	4,745	1,947	0	0	0	35	0	8,341	439	1,220	0	13,593	1,082	7,152	2,390
0.40	10,000	3,616	2,837	0	0	0	37	0	8,968	264	768	0	15,012	846	5,395	1,767
0.50	10,000	2,830	3,657	0	0	0	66	0	9,312	201	487	0	15,452	708	4,282	1,433
0.60	10,000	2,462	4,297	0	0	0	73	0	9,503	145	352	0	15,637	597	3,674	1,194
0.70	10,000	2,163	4,889	0	0	0	112	0	9,623	127	250	0	15,627	540	3,273	1,085
0.80	10,000	1,968	5,586	0	0	0	141	0	9,759	83	158	0	15,769	500	2,974	984
1.00	10,000	1,532	6,783	0	0	0	175	0	9,821	60	119	0	15,506	370	2,324	761
2.50	10,000	824	12,318	106	284	61	664	809	9,135	0	56	851	14,682	256	1,989	633
5.00	20,000	649	30,408	673	1,445	751	2,581	6,021	13,962	0	17	6,224	29,285	1,160	15,442	5,140
7.50	20,000	365	31,222	712	1,523	1,504	3,572	9,067	10,917	0	16	9,405	29,770	1,659	24,121	8,072
10.00	30,000	258	46,150	902	1,846	3,262	6,401	16,691	13,293	0	16	17,252	46,169	3,325	48,865	16,101
12.00	30,000	182	44,644	742	1,394	4,345	7,219	18,434	11,547	0	19	19,051	47,893	3,975	59,215	19,485

TABLE XIII. Response Parameters for Monoenergetic Gamma Rays Incident on Right Circular Cylinder of 4-cm-diam, 4-cm-thick Germanium

Energy, MeV	Interacting Primary Gamma Rays, I	Absorbed Primary Gamma Rays, A	Incident Gamma Rays, P	Single Escapes	Double Escapes	Positron Escapes	Electron Escapes	Primary Events				Total Events				
								Pair Production	Compton	Rayleigh	Photo-electric	Pair Production	Compton	Rayleigh	Photo-electric	X Rays
0.05	10,000	9,809	0	0	0	0	4	0	454	398	9,148	0	513	636	14,648	4,860
0.10	10,000	9,555	0	0	0	0	7	0	2,298	630	7,072	0	2,873	1,043	14,254	4,707
0.20	10,000	7,682	285	0	0	0	31	0	6,423	621	2,956	0	10,408	1,420	11,446	3,757
0.30	10,000	5,730	953	0	0	0	29	0	8,264	419	1,317	0	14,686	1,230	8,517	2,778
0.40	10,000	4,414	1,629	0	0	0	45	0	9,031	279	690	0	16,446	1,046	6,577	2,153
0.50	10,000	3,587	2,107	0	0	0	50	0	9,340	196	464	0	16,836	890	5,303	1,707
0.60	10,000	3,167	2,566	0	0	0	62	0	9,535	147	318	0	17,377	738	4,776	1,597
0.70	10,000	2,884	2,922	0	0	0	68	0	9,582	123	295	0	17,428	694	4,280	1,389
0.80	10,000	2,534	3,369	0	0	0	94	0	9,735	89	176	0	17,546	587	3,838	1,293
1.00	10,000	2,208	4,213	0	0	0	128	0	9,813	86	101	0	17,551	569	3,357	1,132
2.50	10,000	1,224	8,433	177	209	50	494	834	9,132	0	34	891	16,614	336	2,766	904
5.00	10,000	562	10,239	501	581	266	914	3,030	6,963	0	7	3,187	17,300	761	9,335	3,081
7.50	10,000	322	10,763	526	626	547	1,236	4,568	5,424	0	8	4,813	18,260	1,092	14,394	4,760
10.00	20,000	407	21,345	924	1,069	1,602	3,077	11,338	8,657	0	5	11,783	39,496	2,951	39,854	13,110
12.00	20,000	275	20,601	774	835	2,010	3,427	12,353	7,632	0	15	12,900	42,269	3,345	48,319	15,990

TABLE XIV. Response Parameters for Monoenergetic Gamma Rays Incident on Right Circular Cylinder of 5-cm-diam, 5-cm-thick Germanium

Energy, MeV	Interacting Primary Gamma Rays, I	Absorbed Primary Gamma Rays, A	Incident Gamma Rays, P	Single Escapes	Double Escapes	Positron Escapes	Electron Escapes	Primary Events				Total Events				
								Pair Production	Compton	Rayleigh	Photo-electric	Pair Production	Compton	Rayleigh	Photo-electric	X Rays
0.05	10,000	9,817	1	0	0	0	6	0	448	420	9,132	0	520	648	14,693	4,907
0.10	10,000	9,617	0	0	0	0	14	0	2,351	644	7,005	0	2,967	1,043	14,421	4,825
0.20	10,000	8,161	109	0	0	0	15	0	6,388	587	3,025	0	10,586	1,446	12,207	4,052
0.30	10,000	6,263	499	0	0	0	19	0	8,313	411	1,276	0	15,609	1,342	9,333	3,083
0.40	10,000	5,109	908	0	0	0	22	0	8,982	270	748	0	17,637	1,149	7,618	2,512
0.50	10,000	4,395	1,277	0	0	0	40	0	9,308	209	483	0	18,389	984	6,550	2,149
0.60	10,000	3,896	1,549	0	0	0	57	0	9,516	162	322	0	18,925	937	5,837	1,939
0.70	10,000	3,528	1,849	0	0	0	63	0	9,609	117	274	0	19,410	765	5,291	1,752
0.80	10,000	3,323	2,155	0	0	0	72	0	9,739	80	181	0	19,289	741	4,985	1,645
1.00	10,000	2,807	2,703	0	0	0	105	0	9,813	63	124	0	19,370	680	4,217	1,388
2.50	10,000	1,663	5,781	208	153	30	352	842	9,111	0	47	886	18,557	489	3,611	1,174
5.00	10,000	864	7,419	628	418	164	720	2,956	7,029	0	16	3,120	20,172	951	10,764	3,510
7.50	10,000	503	7,748	720	509	404	949	4,552	5,444	0	4	4,795	21,769	1,313	16,650	5,514
10.00	10,000	314	8,046	627	460	599	1,205	5,660	4,338	0	2	5,963	24,022	1,734	22,741	7,451
12.00	10,000	255	7,474	569	354	767	1,338	6,197	3,797	0	6	6,548	25,948	2,165	27,652	9,099

TABLE XV. Response Parameters for Monoenergetic Gamma Rays Incident on Right Circular Cylinder of 2-cm-diam, 2-cm-thick Silicon

Energy, MeV	Interacting Primary Gamma Rays, I	Absorbed Primary Gamma Rays, A	Incident Gamma Rays, P	Single Escapes	Double Escapes	Positron Escapes	Electron Escapes	Primary Events				Total Events			
								Pair Production	Compton	Rayleigh	Photo-electric	Pair Production	Compton	Rayleigh	Photo-electric
0.05	10,000	7,218	1,520	0	0	0	11	0	4,051	938	5,011	0	5,276	1,274	7,229
0.10	10,000	2,270	7,177	0	0	0	12	0	8,076	548	1,376	0	10,877	844	2,279
0.20	10,000	422	11,761	0	0	0	22	0	9,575	218	207	0	12,431	346	428
0.30	10,000	206	14,568	0	0	0	49	0	9,810	105	85	0	12,465	175	212
0.40	10,000	93	16,647	0	0	0	82	0	9,893	72	35	0	12,417	135	95
0.50	20,000	117	37,965	0	0	0	288	0	19,848	113	39	0	24,625	218	122
0.60	20,000	103	41,588	0	0	0	414	0	19,897	63	40	0	24,517	131	110
0.70	20,000	81	44,690	0	0	0	544	0	19,916	58	26	0	24,174	108	83
0.80	30,000	94	72,933	0	0	0	1,098	0	29,887	87	26	0	36,105	154	103
1.00	30,000	85	82,716	0	0	0	1,559	0	29,970	0	30	0	35,982	62	92
2.50	30,000	20	136,675	3	569	269	5,838	1,072	28,928	0	0	1,088	34,054	58	402
5.00	40,000	2	259,151	7	1,577	2,620	16,342	6,557	33,443	0	0	6,661	42,062	454	4,581
7.50	40,000	0	296,351	1	712	5,765	23,365	10,916	29,084	0	0	11,031	37,263	442	4,048
10.00	50,000	0	398,027	3	262	12,463	35,702	18,592	31,408	0	0	18,754	39,588	427	3,730
12.00	50,000	0	403,401	0	92	15,960	38,581	21,302	28,698	0	0	21,485	36,250	384	3,386

TABLE XVI. Response Parameters for Monoenergetic Gamma Rays Incident on Right Circular Cylinder of 3-cm-diam, 3-cm-thick Silicon

Energy, MeV	Interacting Primary Gamma Rays, I	Absorbed Primary Gamma Rays, A	Incident Gamma Rays, P	Single Escapes	Double Escapes	Positron Escapes	Electron Escapes	Primary Events				Total Events			
								Pair Production	Compton	Rayleigh	Photo-electric	Pair Production	Compton	Rayleigh	Photo-electric
0.05	10,000	7,881	521	0	0	0	6	0	3,996	921	5,083	0	5,589	1,351	7,888
0.10	10,000	2,675	3,515	0	0	0	5	0	8,216	544	1,240	0	12,357	964	2,679
0.20	10,000	599	6,511	0	0	0	20	0	9,579	215	206	0	14,123	413	602
0.30	10,000	288	8,394	0	0	0	36	0	9,809	111	80	0	14,000	251	291
0.40	10,000	164	10,124	0	0	0	61	0	9,877	77	46	0	13,689	169	171
0.50	10,000	113	11,114	0	0	0	91	0	9,919	59	22	0	13,746	125	115
0.60	10,000	103	12,544	0	0	0	107	0	9,951	30	19	0	13,580	107	106
0.70	10,000	68	13,816	0	0	0	174	0	9,973	23	4	0	13,405	90	70
0.80	20,000	133	29,426	0	0	0	476	0	19,914	75	11	0	26,485	182	139
1.00	20,000	104	33,853	0	0	0	675	0	19,983	0	17	0	26,161	80	112
2.50	30,000	59	86,478	4	525	168	3,969	1,076	28,924	0	0	1,102	37,297	115	491
5.00	30,000	9	123,824	10	1,280	1,215	8,297	4,747	25,253	0	0	4,843	36,938	658	5,400
7.50	30,000	3	143,798	5	801	2,945	11,740	8,126	21,874	0	0	8,268	34,016	738	5,876
10.00	40,000	4	205,697	5	751	6,786	20,228	14,955	25,045	0	0	15,173	40,704	913	6,982
12.00	40,000	1	210,126	4	503	8,867	23,280	17,167	22,833	0	0	17,167	37,751	912	6,470

TABLE XVII. Response Parameters for Monoenergetic Gamma Rays Incident on Right Circular Cylinder of 4-cm-diam, 4-cm-thick Silicon

Energy, MeV	Interacting Primary Gamma Rays, I	Absorbed Primary Gamma Rays, A	Incident Gamma Rays, P	Single Escapes	Double Escapes	Positron Escapes	Electron Escapes	Primary Events				Total Events			
								Pair Production	Compton	Rayleigh	Photo-electric	Pair Production	Compton	Rayleigh	Photo-electric
0.05	10,000	8,212	193	0	0	0	5	0	3,945	1,006	5,049	0	5,810	1,518	8,217
0.10	10,000	3,138	2,152	0	0	0	2	0	8,197	546	1,257	0	13,638	1,187	3,139
0.20	10,000	824	4,268	0	0	0	14	0	9,587	238	175	0	15,726	550	828
0.30	10,000	390	5,530	0	0	0	20	0	9,813	113	74	0	15,510	311	392
0.40	10,000	279	6,509	0	0	0	45	0	9,895	65	40	0	15,376	211	280
0.50	10,000	202	7,331	0	0	0	66	0	9,925	53	22	0	15,287	169	203
0.60	10,000	145	8,349	0	0	0	95	0	9,954	26	20	0	15,032	125	145
0.70	10,000	130	9,206	0	0	0	135	0	9,955	28	17	0	14,662	120	130
0.80	10,000	102	9,982	0	0	0	177	0	9,965	29	6	0	14,568	111	106
1.00	10,000	80	11,729	0	0	0	237	0	9,991	0	9	0	14,460	74	81
2.50	20,000	73	42,083	8	304	80	1,931	696	19,304	0	0	725	26,907	101	445
5.00	20,000	25	59,696	18	811	649	4,238	3,190	16,810	0	0	3,274	27,803	533	4,526
7.50	30,000	19	104,269	19	917	2,230	9,001	8,098	21,902	0	0	8,313	40,736	1,110	7,867
10.00	30,000	8	112,770	22	790	3,635	11,386	11,133	18,867	0	0	11,325	38,149	1,203	7,791
12.00	30,000	0	114,387	20	654	4,927	13,085	12,848	17,152	0	0	13,091	36,991	1,235	7,964

TABLE XVIII. Response Parameters for Monoenergetic Gamma Rays Incident on Right Circular Cylinder of 5-cm-diam, 5-cm-thick Silicon

Energy, MeV	Interacting Primary Gamma Rays, I	Absorbed Primary Gamma Rays, A	Incident Gamma Rays, P	Single Escapes	Double Escapes	Positron Escapes	Electron Escapes	Primary Events				Total Events			
								Pair Production	Compton	Rayleigh	Photo- electric	Pair Production	Compton	Rayleigh	Photo- electric
0.05	10,000	8,462	66	0	0	0	2	0	3,996	966	5,038	0	5,997	1,511	8,465
0.10	10,000	3,773	1,269	0	0	0	7	0	8,098	610	1,292	0	14,880	1,374	3,781
0.20	10,000	1,117	2,759	0	0	0	6	0	9,578	226	196	0	17,566	650	1,118
0.30	10,000	580	3,687	0	0	0	18	0	9,810	117	73	0	17,383	381	584
0.40	10,000	375	4,478	0	0	0	39	0	9,901	55	44	0	16,885	250	377
0.50	10,000	302	5,371	0	0	0	59	0	9,925	54	21	0	16,782	222	306
0.60	10,000	235	5,942	0	0	0	77	0	9,965	22	13	0	16,305	162	236
0.70	10,000	198	6,608	0	0	0	82	0	9,973	19	8	0	16,194	133	200
0.80	10,000	229	7,362	0	0	0	151	0	9,968	21	11	0	15,995	146	230
1.00	10,000	173	8,740	0	0	0	213	0	9,994	0	6	0	15,760	99	175
2.50	20,000	140	31,166	7	280	62	1,584	724	19,276	0	0	749	29,037	154	541
5.00	20,000	41	46,059	28	777	483	3,225	3,198	16,802	0	0	3,297	31,576	735	5,279
7.50	20,000	26	53,734	28	633	1,150	4,731	5,505	14,495	0	0	5,660	31,483	954	6,291
10.00	30,000	15	87,474	34	854	2,993	9,082	11,226	18,774	0	0	11,508	45,595	1,557	9,846
12.00	30,000	12	88,595	30	675	3,823	10,498	12,818	17,182	0	0	13,108	45,987	1,633	10,227

In all cases presented, the gamma-ray cutoff energy E_t was 5 keV and the electron cutoff energy was 15 keV. This means that all gamma rays or electrons that were less than this cutoff energy were considered to be absorbed at the point of creation. Other parameters used for these calculations were as follows:

	<u>Germanium</u>	<u>Silicon</u>
Density, g/cc	5.36	2.4
K-binding energy, MeV	0.0125	0.0018
X-ray energy, MeV	0.01	-
Fluorescent yield	49%	-

No fluorescent X rays were traced for silicon because these X rays are less than the gamma-ray cutoff energy of 5 keV.

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