315
20
LA-3833

MASTER

47 for Fast Reactor
Fammal
A Distr.
give back to Lab.
842

LOS ALAMOS SCIENTIFIC LABORATORY
of the
University of California
LOS ALAMOS • NEW MEXICO

Integral-Transform Lie-Series
Analysis of Transient Temperatures
in Reactor Coolant Channels

UNITED STATES
ATOMIC ENERGY COMMISSION
CONTRACT W-7405-ENG. 36

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

This report expresses the opinions of the author or authors and does not necessarily reflect the opinions or views of the Los Alamos Scientific Laboratory.

Printed in the United States of America. Available from
Clearinghouse for Federal Scientific and Technical Information
National Bureau of Standards, U. S. Department of Commerce
Springfield, Virginia 22151

Price: Printed Copy \$3.00; Microfiche \$0.65

LOS ALAMOS SCIENTIFIC LABORATORY
of the
University of California
LOS ALAMOS • NEW MEXICO

Report written: September 1967

Report distributed: March 12, 1968

Integral-Transform Lie-Series
Analysis of Transient Temperatures
in Reactor Coolant Channels

by

Roy Arthur Axford

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:
A. Makes any warranty, nor any representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or
B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.
As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, in the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

CONTENTS

Abstract	3
1. Introduction	3
2. Mathematical Formulation	3
3. Initial Steady-State Solutions	5
4. Hankel-Transform Reduction of Partial-Differential Energy Balances to a System of Ordinary Differential Equations	7
5. Lie-Series Computational Algorithm	9
6. Allowance for Heat-Transfer Coefficient Strongly Dependent on Coolant Speed	12
7. Function Subprograms Required for LIETRAN	14
Appendix - Fortran Listing of LIETRAN	15

INTEGRAL-TRANSFORM LIE-SERIES ANALYSIS
OF TRANSIENT TEMPERATURES
IN REACTOR COOLANT CHANNELS

by
Roy Arthur Axford*

ABSTRACT

A new method is developed for computing transient temperature fields in cylindrical reactor channels when the relative fuel power density, the coolant speed, and the channel inlet temperature are arbitrarily specified functions of time. Through the use of Hankel transforms, numerical instabilities inherent in finite difference approximations of spatial derivatives are avoided. Solutions of high order systems of ordinary differential equations in the time coordinate are efficiently obtained by a computational algorithm based upon the concept of Lie series.

1. Introduction

Preventing numerical instabilities in the temperature calculations can be the controlling factor in the efficiency of reactor dynamics calculations with feedbacks. This is particularly true as the temperature model becomes more detailed.

In this study, a method is developed for analyzing transient temperature fields in cylindrical reactor channels. Finite difference approximations to the spatial derivatives in the fundamental energy balances are not used, and inherent numerical instabilities are, thereby, avoided.

2. Mathematical Formulation

If axial heat conduction in the fuel is neglected and the fuel thermal properties are assumed constant, then the fuel-region temperature satisfies

$$\frac{1}{\alpha} \frac{\partial}{\partial t} v(r, z, t) = \frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial}{\partial r} v(r, z, t) \right] + \frac{1}{k} Q(r, z, t) \quad (1)$$

in the case of cylindrical fuel rods. Let $v_j(r, t)$

*The author, a professor at the Nuclear Engineering Laboratory, University of Illinois, Urbana, Illinois, is a consultant to the Los Alamos Scientific Laboratory.

be defined as the average fuel temperature over the j th axial segment with length Δz_j ; i.e.,

$$v_j(r, t) = \frac{1}{\Delta z_j} \int_{z_j}^{z_{j+1}} dz v(r, z, t). \quad (2)$$

When Eq. 1 is averaged over the j th axial segment, it follows that

$$\begin{aligned} \frac{1}{\alpha} \frac{\partial}{\partial t} v_j(r, t) \\ = \frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial}{\partial r} v_j(r, t) \right] + \frac{1}{k} \frac{1}{\Delta z_j} \int_{z_j}^{z_{j+1}} dz Q(r, z, t). \end{aligned} \quad (3)$$

If $\zeta(z)$ is the nondimensional relative power shape in the axial direction, so that the fuel power density takes the form,

$$Q(r, z, t) = Q(r, t) \zeta(z), \quad (4)$$

then the integral in Eq. 2 may be written as

$$\frac{1}{\Delta z_j} \int_{z_j}^{z_{j+1}} dz Q(r, z, t) = A_j Q(r, t), \quad (5)$$

in which the definition

$$A_j = \frac{1}{\Delta z_j} \int_{z_j}^{z_{j+1}} dz \zeta(z) \quad (6)$$

has been introduced. Accordingly, Eq. 3 reduces to

$$\frac{1}{\alpha} \frac{\partial}{\partial t} v_j(r, t) = \frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial}{\partial r} v_j(r, t) \right] + \frac{A_j}{k} Q(r, t). \quad (7)$$

Equation 7 is the energy balance for the average fuel temperature in the j th axial segment at radial position r and time t .

The axial power shape coefficient, A_j , defined by Eq. 6 may be interpreted as follows. If we assume that the active fuel region is defined by the closed interval, $-L \leq z \leq L$, so that $2L$ is the active fuel length, then the average-to-peak axial power ratio, P , is given by

$$P = \frac{1}{2L} \int_{-L}^L dz \zeta(z). \quad (8)$$

Since

$$\int_{-L}^L dz \zeta(z) = \sum_{j=1}^J \int_{z_j}^{z_{j+1}} dz \zeta(z), \quad (9)$$

in which J is the number of subintervals into which the active fuel interval is partitioned, it follows from Eqs. 8, 9, and 6 that

$$P = \frac{1}{2L} \sum_{j=1}^J A_j \Delta z_j. \quad (10)$$

Accordingly, the axial power shape coefficient, A_j , is the ratio of the average power generated in the j th axial segment to the peak power in the active fuel region.

In the case of a parabolic axial power shape symmetric about the center plane as defined by

$$\zeta(z) = 1 - (1 - \epsilon) \left(\frac{z}{L} \right)^2, \quad (-L \leq z \leq L), \quad (11)$$

in which ϵ is the axial edge-to-peak power ratio, the axial power shape coefficient takes the form

$$A_j = 1 - \frac{(1 - \epsilon)}{3} \left[\left(\frac{z_{j+1}}{L} \right)^2 + \left(\frac{z_{j+1}}{L} \right) \left(\frac{z_j}{L} \right) + \left(\frac{z_j}{L} \right)^2 \right]. \quad (12)$$

The axial positions, z_j , appearing in Eq. 12 are given by

$$z_j = -L + \frac{(j-1)}{J} 2L, \quad (j = 1, 2, \dots, J+1), \quad (13)$$

when the active fuel region is arbitrarily divided into J equal axial segments.

If axial heat conduction is neglected, the bulk mean coolant temperature, $T(z, t)$, satisfies

$$\begin{aligned} \rho_c c_{pc} A_F \frac{\partial}{\partial t} T(z, t) + \rho_c c_{pc} A_F u_0 u(t) \frac{\partial}{\partial z} T(z, t) \\ = -k2\pi a \frac{\partial}{\partial r} v(a, z, t), \end{aligned} \quad (14)$$

in which u_0 is the initial coolant speed and $u(t)$ is the relative coolant speed. In Eq. 14, A_F is the flow area. Let

$$T_j(t) = \frac{1}{\Delta z_j} \int_{z_j}^{z_{j+1}} dz T(z, t) \quad (15)$$

be the average bulk mean coolant temperature over the j th axial segment; then averaging Eq. 14 over the j th axial segment yields

$$\begin{aligned} \rho_c c_{pc} A_F \frac{d}{dt} T_j(t) \\ + \rho_c c_{pc} A_F u_0 u(t) \frac{1}{\Delta z_j} \int_{z_j}^{z_{j+1}} dz \frac{\partial}{\partial z} T(z, t) \\ = -2\pi a k \frac{\partial}{\partial r} v_j(a, t). \end{aligned} \quad (16)$$

The integral in Eq. 16 takes the form,

$$\frac{1}{\Delta z_j} \int_{z_j}^{z_{j+1}} dz \frac{\partial}{\partial z} T(z, t) = \frac{1}{\Delta z_j} [T(z_{j+1}, t) - T(z_j, t)], \quad (17)$$

in which

$T(z_{j+1}, t)$ = outlet temperature of j th axial segment,

and

$T(z_j, t)$ = inlet temperature of j th axial segment.

When the z -dependence of the bulk mean coolant temperature in the j th axial segment is approximated by a linear interpolation,

$$T(z, t) = \frac{(z_{j+1} - z)}{\Delta z_j} T(z_j, t) + \frac{(z - z_j)}{\Delta z_j} T(z_{j+1}, t), \quad (18)$$

for $z_j \leq z \leq z_{j+1}$, we find from Eq. 15 that

$$T(z_{j+1}, t) = 2T_j(t) - T(z_j, t). \quad (19)$$

From Eqs. 16, 17, and 19 it follows that

$$\begin{aligned} \rho_c c_{pc} A_F \frac{d}{dt} T_j(t) \\ + \rho_c c_{pc} A_F u(t) \frac{2u_0}{\Delta z_j} [T_j(t) - T(z_j, t)] \\ = -2\pi a k \frac{\partial}{\partial r} v_j(a, t). \end{aligned} \quad (20)$$

Equation 20 is the basic energy balance for the average bulk mean coolant temperature in the j th axial segment in terms of the inlet temperature to that segment, the relative coolant speed under variable coolant flow conditions, and the heat input to the coolant from the fuel.

The fuel energy balance, Eq. 1, and the coolant energy balance, Eq. 14, are coupled by the boundary condition,

$$-k \frac{\partial}{\partial r} v(a, z, t) = H[v(a, z, t) - T(z, t)]. \quad (21)$$

Averaging Eq. 21 over the j th axial segment gives

$$-k \frac{\partial}{\partial r} v_j(a, t) = H[v_j(a, t) - T_j(t)]. \quad (22)$$

In Eq. 22 H is an effective heat-transfer coefficient that takes into account the thermal resistances of the cladding wall and the thermal bond between the fuel region and the cladding. The effective heat transfer coefficient is, in general, a function of time, since the coolant speed is regarded as an arbitrary function of time. We will first assume that the effective heat-transfer coefficient is a weak function of the coolant speed as in the case of liquid metal coolants. It is discussed as a strong function of coolant speed in Section 6.

Let Q_0 be the maximum power density in the fuel region. Then the power density appearing in Eq. 7 becomes

$$Q(r, t) = Q_0 q(r, t), \quad (23)$$

in which $q(r, t)$ is the nondimensional relative power density at time t . In Eqs. 7, 20, and 22, replacing the time variable by the nondimensional time, at/a^2 , and the radial fuel coordinate by r/a gives for the energy balances the following forms in which the independent variables are nondimensional:

$$\frac{\partial}{\partial t} v_j(r, t) = \frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial}{\partial r} v_j(r, t) \right] + s_j q(r, t), \quad (24)$$

$$\begin{aligned} \frac{d}{dt} T_j(t) + u_j u(t) T_j(t) &= u_j u(t) T(z_j, t) \\ &+ f2\eta \frac{\rho_c p}{\rho_c c_{pc}} [v_j(1, t) - T_j(t)], \end{aligned} \quad (25)$$

and

$$-k \frac{\partial}{\partial r} v_j(1, t) = \eta [v_j(1, t) - T_j(t)]. \quad (26)$$

Equations 24-26 are valid for $j = 1, 2, \dots, J$ and for $0 \leq r \leq 1$. In Eqs. 24-26 the following definitions

have been introduced:

$$\eta \equiv aH/k, \quad (27)$$

$$s_j \equiv \frac{a^2 Q_0}{k} A_j, \quad (28)$$

$$u_j \equiv \frac{2a^2 u_0}{\alpha \Delta z_j}, \quad (29)$$

and

$$f \equiv \frac{\pi a^2}{A_F}. \quad (30)$$

Also, in Eq. 25 the gradient of the fuel temperature field evaluated at the outer edge of the fuel has been eliminated by means of Eq. 22. Equation 27 is the Biot number; Eq. 28 gives, in effect, the power generated in the j th axial segment of the fuel; and Eq. 30 is the fuel-to-coolant volume ratio.

3. Initial Steady-State Solutions

To specify the initial values of the unknown, axially averaged temperatures appearing in Eqs. 24 and 25, the solutions of these equations are required for an initially specified, time-independent relative power density, $q_0(r)$. The initial steady state counterpart of Eqs. 24-26 assumes the form

$$\frac{1}{r} \frac{d}{dr} \left[r \frac{d}{dr} v_j(r) \right] + s_j q_0(r) = 0, \quad (31)$$

$$u_j T_{j,0} = u_j T(z_j, 0) + f2\eta \frac{\rho_c p}{\rho_c c_{pc}} [v_j(1) - T_{j,0}], \quad (32)$$

and

$$-k \frac{d}{dr} v_j(1) = \eta [v_j(1) - T_{j,0}], \quad (33)$$

in which these definitions have been used:

$v_j(r)$ = average fuel temperature in the j th

axial segment at the radial position, r ,

$T_{j,0}$ = initial value of the average bulk mean coolant temperature in the j th axial segment,

and

$T(z_j, 0)$ = initial value of the coolant inlet

temperature to the j th axial segment.

Exact and approximate solutions of Eqs. 31-33 will be computed. The approximate solution based upon the method of zero-order Hankel transforms provides the initial values of the Hankel transform required in solving the transient system, Eqs. 24-26. The exact solution determines the accuracy of the Hankel transform solution of the initial steady-state problem as

a function of the number of terms retained in the Hankel transform inversion theorem.

The exact solution of Eq. 31 subject to Eq. 33 under the assumption that radial flux depression in the fuel is neglected, so the $q_0(r) = 1$, is found to be

$$v_j(r) = T_{j,0} + \frac{1}{2n} s_j + \frac{1}{4} s_j (1 - r^2), \quad (34)$$

for $j = 1, 2, 3, \dots, J$. From Eqs. 32 and 34 it follows that

$$T_{j,0} = T(z_j, 0) + f \frac{\rho_c p}{\rho_c c_{pc}} \frac{s_j}{u_j}, \quad (35)$$

for $j = 1, 2, 3, \dots, J$. Equation 35 gives the average bulk mean coolant temperature in the j th axial segment in terms of the inlet temperature to that segment. From Eqs. 19 and 35 it follows that the outlet temperature of the j th axial segment in the initial steady state is given by

$$T(z_{j+1}, 0) = T(z_j, 0) + 2f \frac{\rho_c p}{\rho_c c_{pc}} \frac{s_j}{u_j}, \quad (36)$$

for $J = 1, 2, \dots, J$.

Let

$$v_j(n_i) = \int_0^1 dr r J_0(n_i r) v_j(r) \quad (37)$$

be defined as the zero-order Hankel transform of the j th axial segment fuel temperature in the initial steady state. The constants, n_i , in Eq. 37 are yet to be defined. The zero-order Hankel transform of Eq. 31 yields

$$-n_i^2 v_j(n_i) + J_0(n_i) \frac{d}{dr} v_j(1) + n_i J_1(n_i) v_j(1) + s_j q_0(n_i) = 0, \quad (38)$$

in which $q_0(n_i)$ is the Hankel transform of the radially dependent relative power density in the fuel. Eliminating the gradient of the fuel temperature field evaluated at the outer edge of the fuel from Eq. 38 by means of Eq. 33 produces

$$-n_i^2 v_j(n_i) + n J_0(n_i) T_{j,0} + v_j(1) [n_i J_1(n_i) - n J_0(n_i)] + s_j q_0(n_i) = 0. \quad (39)$$

The requirement that the constants, n_i , are to be the zeros of the transcendental equation,

$$n_i J_1(n_i) - n J_0(n_i) = 0, \quad (40)$$

leads to the following result for the Hankel transform:

$$v_j(n_i) = \frac{n}{n_i^2} J_0(n_i) T_{j,0} + \frac{1}{n_i^2} s_j q_0(n_i). \quad (41)$$

Assume that the average fuel temperature in the j th axial segment may be represented by the Fourier-Bessel series,

$$v_j(r) = \sum_{i=1}^{\infty} A_{i,j} J_0(n_i r). \quad (42)$$

The zero-order Hankel transform of Eq. 42 is

$$v_j(n_k) = \sum_{i=1}^{\infty} A_{i,j} \int_0^1 dr r J_0(n_k r) J_0(n_i r). \quad (43)$$

Since

$$\int_0^1 dr r J_0(n_k r) J_0(n_i r) = \delta_{ik} \frac{1}{2n_i^2} (n^2 + n_i^2) J_0^2(n_i), \quad (44)$$

in which δ_{ik} is the Kronecker delta when the constants, n_i , are the zeros of Eq. 40, it follows from Eqs. 43 and 44 that

$$A_{i,j} = v_j(n_i) \frac{2n_i^2}{(n^2 + n_i^2) J_0^2(n_i)}. \quad (45)$$

Consequently, the Hankel-transform inversion theorem takes the form

$$v_j(r) = \sum_{i=1}^{\infty} \frac{2n_i^2}{(n^2 + n_i^2) J_0^2(n_i)} v_j(n_i) J_0(n_i r), \quad (46)$$

when the constants, n_i , are the roots of Eq. 40.

Combining Eqs. 41 and 46 produces

$$v_j(r) = T_{j,0} \sum_{i=1}^{\infty} \frac{2n}{(n^2 + n_i^2)} \frac{J_0(n_i r)}{J_0(n_i)} + s_j \sum_{i=1}^{\infty} \frac{2q_0(n_i)}{(n^2 + n_i^2)} \frac{J_0(n_i r)}{J_0^2(n_i)}. \quad (47)$$

A further simplification of Eq. 47 may be achieved by noting that the Fourier-Bessel series representation of unity is

$$1 = \sum_{i=1}^{\infty} \frac{2n}{(n^2 + n_i^2)} \frac{J_0(n_i r)}{J_0(n_i)}, \quad (48)$$

provided that the constants, n_i , are the zeros of Eq. 40. Thus, Eq. 47 reduces to

$$v_j(r) = T_{j,0} + s_j \sum_{i=1}^{\infty} \frac{2q_0(n_i)}{(n^2 + n_i^2)} \frac{J_0(n_i r)}{J_0^2(n_i)}, \quad (49)$$

which gives the fuel temperature field in the j th axial segment in terms of the zero-order Hankel transform of the radial variation of the relative power density in the fuel.

In comparing the exact result computed from Eq. 34 and the approximate result computed from Eq. 49 when a finite number, I , of terms are kept in the summation, it will be seen that, when $q_0(r) = 1$,

$$q_0(n_i) = \frac{1}{n_i} J_1(n_i), \quad (50)$$

so that, in view of Eq. 40, Eq. 49 specializes down to

$$v_j(r) = T_{j,0} + s_j \sum_{i=1}^I \frac{2\eta}{(n^2 + n_i^2)} \frac{J_0(n_i r)}{n_i^2 J_0(n_i)} \quad (51)$$

in the case of a radially flat fuel power density. Results for the average fuel temperature field in the j th axial segment computed from Eqs. 34 and 51 establish the number of terms required in the sum in Eq. 51 to obtain a specified number of significant digits in the approximate Hankel-transform solution for the initial steady-state fuel temperature.

Evaluating Eq. 51 at the outer edge of the fuel region and substituting the result in Eq. 32 gives the corresponding approximate solution,

$$T_{j,0} = T(z_j, 0) + 2\eta f \frac{\rho_c p}{\rho_c c_{pc}} \frac{s_j}{u_j} \sum_{i=1}^I \frac{2\eta}{(n^2 + n_i^2)} \frac{1}{n_i^2}, \quad (52)$$

for the axially averaged bulk mean coolant temperature in the j th axial segment to be compared with Eq. 35. From Eqs. 19 and 52 it follows that

$$T(z_{j+1}, 0) = T(z_j, 0) + 4\eta f \frac{\rho_c p}{\rho_c c_{pc}} \frac{s_j}{u_j} \sum_{i=1}^I \frac{2\eta}{(n^2 + n_i^2)} \frac{1}{n_i^2} \quad (53)$$

is the approximate result for the exit bulk mean coolant temperature from the j th axial segment which corresponds to the exact solution of Eq. 36.

From the steady-state counterpart of Eq. 19

$$T(z_{j+1}, 0) = 3T_{j,0} - T(z_j, 0), \quad (54)$$

it may be established by mathematical induction that

$$T(z_j, 0) = 2 \sum_{m=1}^{j-1} (-1)^{j+m+1} T_{m,0} + (-1)^{j-1} T(z_1, 0), \quad (55)$$

which is valid for $j = 2, 3, 4, \dots, J$, and in which $T(z_1, 0)$ is the initial value of the coolant inlet temperature to the channel. Equation 55 may be interpreted in two alternative ways. First, it may be thought of as giving the inlet temperature to the j th axial segment from the second segment on in terms of the inlet temperature to the entire channel and the average bulk mean coolant temperatures of the previous axial segments. If the index, $j = 2, 3, 4, \dots, J + 1$, then Eq. 55 is to be interpreted as providing the outlet temperature from the $(j - 1)$ st axial segment in terms of the channel inlet temperature and the average bulk mean temperatures of all previous axial segments including the segment for which the outlet bulk mean coolant temperature is being computed. An alternative consistency check on the coolant outlet temperatures from the various axial segments is afforded by Eq. 55.

4. Hankel-Transform Reduction of Partial-Differential Energy Balances to a System of Ordinary Differential Equations

The system of equations embodied in the energy balances of Eqs. 24-26 comprises a coupled set of partial and ordinary differential equations for the average fuel and bulk mean coolant temperatures in the j th axial segment. By removing the radial spatial dependence from Eq. 24 by introducing the transient Hankel transform of the fuel temperature field in the j th axial segment, Eqs. 24-26 may be reduced to a system of pure differential equations together with subsidiary algebraic equations.

Let

$$v_j(n_i, t) = \int_0^1 dr \, r J_0(n_i r) v_j(r, t) \quad (56)$$

be the zero-order Hankel transform of the transient fuel temperature field in the j th axial segment. The Hankel transform of Eq. 24 is

$$\begin{aligned} \frac{d}{dt} v_j(n_i, t) = & -n_i^2 v_j(n_i, t) + J_0(n_i) \frac{\partial}{\partial r} v_j(1, t) \\ & + n_i J_1(n_i) v_j(1, t) + s_j q(n_i, t), \end{aligned} \quad (57)$$

in which

$$q(n_i, t) = \int_0^1 dr \, r J_0(n_i r) q(r, t) \quad (58)$$

is the Hankel transform of the radially dependent

relative power density. When the radial flux depression in the fuel region is neglected, Eq. 58 specializes to

$$q(n_i, t) = \frac{1}{n_i} J_1(n_i) q(t), \quad (59)$$

wherein $q(t)$ is the time-dependent relative power density such that $q(0) = 1$. The elimination of the radial gradient of the fuel temperature field in Eq. 57 by means of Eq. 26 brings Eq. 57 into the form,

$$\begin{aligned} \frac{d}{dt} v_j(n_i, t) = & -n_i^2 v_j(n_i, t) \\ & + v_j(1, t) [n_i J_1(n_i) - n J_0(n_i)] \\ & + n J_0(n_i) T_j(t) + s_j q(n_i, t). \end{aligned} \quad (60)$$

The second term on the right-hand side of Eq. 60 drops out, provided that the constants, n_i , are taken as the zeros of Eq. 40 as in the steady-state case. Accordingly, Eq. 60 can be simplified to

$$\begin{aligned} \frac{d}{dt} v_j(n_i, t) = & -n_i^2 v_j(n_i, t) + n J_0(n_i) T_j(t) + s_j q(n_i, t). \end{aligned} \quad (61)$$

In view of Eqs. 40 and 59, Eq. 61 becomes

$$\begin{aligned} \frac{d}{dt} v_j(n_i, t) = & -n_i^2 v_j(n_i, t) + n J_0(n_i) T_j(t) + s_j q(t) \frac{n}{n_i^2} J_0(n_i) \end{aligned} \quad (62)$$

for the case of a radially flat relative fuel power density.

Let $D_j(t)$ be defined as the temperature drop from the outer edge of the fuel region to the bulk mean coolant temperature in the j th axial segment; i.e.,

$$D_j(t) = v_j(1, t) - T_j(t). \quad (63)$$

Under transient conditions the Hankel-transform inversion theorem corresponding to Eq. 46 is

$$v_j(r, t) = \sum_{i=1}^{\infty} \frac{2n_i^2}{(n^2 + n_i^2) J_0^2(n_i)} v_j(n_i, t) J_0(n_i r). \quad (64)$$

With Eq. 64, Eq. 63 becomes

$$D_j(t) = \sum_{i=1}^{\infty} \frac{2n_i^2 v_j(n_i, t)}{(n^2 + n_i^2) J_0(n_i)} - T_j(t). \quad (65)$$

The series in Eq. 65 consists, in fact, of the sum of two series, which may be seen as follows.

Consider the situation in which the Hankel transform is changing slowly enough with time so that its derivative is approximately zero. Under these conditions Eq. 62 rearranges to

$$v_j(n_i, t) \triangleq \frac{n J_0(n_i)}{n_i^2} T_j(t) + s_j q(t) \frac{n}{n_i^2} J_0(n_i). \quad (66)$$

Combining Eqs. 65 and 66 gives

$$\begin{aligned} D_j(t) \triangleq & T_j(t) \sum_{i=1}^{\infty} \frac{2n}{n^2 + n_i^2} \\ & + s_j q(t) \sum_{i=1}^{\infty} \frac{2n}{(n^2 + n_i^2) n_i^2} - T_j(t) \end{aligned} \quad (67)$$

during slow transients. Of the two summations in Eq. 67, that in the first term on the right converges much less rapidly than that in the second term. This is particularly true for large values of the Biot number. However, the summation of the first term equals unity, so the first and third terms on the right cancel. Consequently, the temperature drop from the outer edge of the fuel to the bulk mean coolant temperature is, in fact, represented by the more rapidly converging series.

To cancel out the more slowly converging series when the Hankel transform is computed by solving Eq. 62, whether it is changing slowly or rapidly with time, the temperature drop from the outer edge of the fuel to the bulk mean coolant temperature is written as

$$D_j(t) = \sum_{i=1}^I \frac{2n_i^2 v_j(n_i, t)}{(n^2 + n_i^2) J_0(n_i)} - T_j(t) \sum_{i=1}^I \frac{2n}{n^2 + n_i^2}, \quad (68)$$

in which the fact that a finite number, I , of terms are retained in the Hankel transform inversion theorem is indicated.

In view of Eq. 63, the coolant energy balance of Eq. 25 becomes

$$\begin{aligned} \frac{d}{dt} T_j(t) + u_j u(t) T_j(t) \\ = u_j u(t) T(z_j, t) + f 2n \frac{\rho_c}{\rho_c c_{pc}} D_j(t). \end{aligned} \quad (69)$$

Equations 69 and 62 (or Eq. 61 if the radial flux depression in the fuel region is taken into account) are to be regarded as a system of $(I + 1)J$ ordinary,

first-order differential equations which govern the temporal behavior of the Hankel transforms, $v_j(n_i, t)$, of the fuel temperature field and the average bulk mean coolant temperature, $T_j(t)$, in the j th axial segment. When $j = 1$, Eq. 69 contains the time dependence of the inlet temperature to the channel, which is also an arbitrarily specified function of time, in addition to the relative coolant speed, $u(t)$, and relative power density, $q(t)$. There are an additional $2J$ unknown functions, namely, the bulk mean coolant outlet temperatures, $T(z_{j+1}, t)$, from each axial segment and the temperature drops, $D_j(t)$, from the outer edge of the fuel to the bulk mean coolant temperature in each axial segment. An additional $2J$ relations for these quantities are contained in Eqs. 19 and 68.

Accordingly, the time dependence of $v_j(n_i, t)$, $T_j(t)$, $D_j(t)$, and $T(z_{j+1}, t)$ is governed by a combined system of $(I + 3)J$ algebraic and first-order differential equations. The initial values of the average bulk mean coolant temperature in the j th axial segment are provided by Eq. 52; that of the exit bulk mean coolant temperature from the j th axial segment, by Eq. 53; and that of the Hankel transform of the fuel region temperature field, by Eq. 41, for an arbitrary radial variation of the relative power density in the fuel. The initial values of the temperature drops from the outer fuel edge to the coolant are computed from Eq. 49 or 51 for an arbitrary radial variation in the relative fuel power density or for a flat radial variation, respectively. For example, from Eq. 51

$$D_j(0) = s_j \sum_{i=1}^I \frac{2n}{(n^2 + n_i^2) n_i^2} \quad (70)$$

Note that the initial values of the unknown temporal functions are computed from the approximate Hankel-transform solution of the initial steady-state problem. The use of initial values calculated from the exact steady-state solution does not lead to a more accurate solution for the transient problem. The accuracy of the transient problem is controlled, in part, by the number of terms in the Hankel-transform inversion theorem, and further, by the convergence criterion placed on the Lie series.

5. Lie-Series Computational Algorithm

In the previous sections fundamental fuel region

and coolant energy balances, Eqs. 1 and 16, have been reduced to a system of first-order, ordinary, differential equations with subsidiary algebraic equations. The solution of this system is sought on the k th time interval such that $t_k \leq t \leq t_{k+1}$. To denote the fact that a computational algorithm is developed for the k th time interval, which is arbitrary, an additional subscript, k , will be added to all unknown functions. Consequently, the basic system of $(I + 3)J$ linear algebraic and ordinary differential equations under consideration is

$$\begin{aligned} \frac{d}{dt} v_{j,k}(n_i, t) &= -n_i^2 v_{j,k}(n_i, t) \\ &+ n J_0(n_i) T_{j,k}(t) + s_j q(t) \frac{n}{n_i^2} J_0(n_i), \end{aligned} \quad (71)$$

$$\begin{aligned} \frac{d}{dt} T_{j,k}(t) &+ u_j u(t) T_{j,k}(t) \\ &= u_j u(t) T_k(z_j, t) + f 2n \frac{\rho_c p}{\rho_c c_{pc}} D_{j,k}(t), \end{aligned} \quad (72)$$

$$D_{j,k}(t) = \sum_{i=1}^I \frac{2n_i^2 v_{j,k}(n_i, t)}{(n^2 + n_i^2) J_0(n_i)} - T_{j,k}(t) \sum_{i=1}^I \frac{2n}{n^2 + n_i^2}, \quad (73)$$

and

$$T_k(z_{j+1}, t) = 2T_{j,k}(t) - T_k(z_j, t), \quad (74)$$

all of which are valid for $j = 1, 2, 3, \dots, J$. Also, Eq. 71 holds for $i = 1, 2, 3, \dots, I$. A numerical computational algorithm to produce the Lie-series* representations of the solution of the above system will now be obtained.

On the k th time interval, the Hankel transform of the fuel temperature field is expanded in the form,

$$v_{j,k}(n_i, t) = v_{j,k}^0(n_i, t) + \sum_{v=1}^{\infty} v_{j,k}^v(n_i, t); \quad (75)$$

the average bulk mean coolant temperature in the j th axial segment, as

$$T_{j,k}(t) = T_{j,k}^0(t) + \sum_{v=1}^{\infty} T_{j,k}^v(t); \quad (76)$$

the temperature drop from the outer fuel edge to the coolant, as

*Wolfgang Gröbner, "Die Lie-Reihen und Ihre Anwendungen," Deutscher Verlag der Wissenschaften, Berlin, 1960.

$$D_{j,k}(t) = D_{j,k}^0(t) + \sum_{v=1}^{\infty} D_{j,k}^v(t); \quad (77)$$

and the outlet temperature of the j th axial segment, as

$$T_k(z_{j+1}, t) = T_k^0(z_{j+1}, t) + \sum_{v=1}^{\infty} T_k^v(z_{j+1}, t). \quad (78)$$

The first terms in Eqs. 75-78 are to be interpreted as the values of the unknown functions at the beginning of the k th time interval. That is,

$$v_{j,k}^0(n_i, t) = v_{j,k}(n_i, t_k), \quad (79)$$

$$T_{j,k}^0(t) = T_{j,k}(t_k), \quad (80)$$

$$D_{j,k}^0(t) = D_{j,k}(t_k), \quad (81)$$

and

$$T_k^0(z_{j+1}, t) = T_k(z_{j+1}, t_k). \quad (82)$$

The v th-order terms in Eqs. 75-78 are defined as

$$v_{j,k}^v(n_i, t) = \frac{(t - t_k)^v}{v!} \frac{d^v}{dt^v} v_{j,k}(n_i, t_k), \quad (83)$$

$$T_{j,k}^v(t) = \frac{(t - t_k)^v}{v!} \frac{d^v}{dt^v} T_{j,k}(t_k), \quad (84)$$

$$D_{j,k}^v(t) = \frac{(t - t_k)^v}{v!} \frac{d^v}{dt^v} D_{j,k}(t_k), \quad (85)$$

and

$$T_k^v(z_{j+1}, t) = \frac{(t - t_k)^v}{v!} \frac{d^v}{dt^v} T_k(z_{j+1}, t_k). \quad (86)$$

To determine the higher order terms in the Lie-series representations of Eqs. 75-78, the differential and algebraic relations of Eqs. 71-74 are transformed into a set of two-term recurrence relations. Differentiating Eq. 71 $v - 1$ times and evaluating the result at the beginning of the k th time interval gives

$$\begin{aligned} \frac{d^v}{dt^v} v_{j,k}(n_i, t_k) &= -n_i^2 \frac{d^{v-1}}{dt^{v-1}} v_{j,k}(n_i, t_k) \\ &+ n J_0(n_i) \frac{d^{v-1}}{dt^{v-1}} T_{j,k}(t_k) + s_j q_{v-1}(t_k) \frac{n}{n_i^2} J_0(n_i), \end{aligned} \quad (87)$$

in which $q_{v-1}(t_k)$ is the $(v - 1)$ st time derivative of the relative power density at the beginning of the k th time interval. Removing the Hankel trans-

form and bulk coolant temperature derivatives from Eq. 87 by Eqs. 83 and 84 yields the recurrence relation,

$$\begin{aligned} v_{j,k}^v(n_i, t) &= -n_i^2 \frac{(t - t_k)^v}{v} v_{j,k}^{v-1}(n_i, t) \\ &+ n J_0(n_i) \frac{(t - t_k)^v}{v} T_{j,k}^{v-1}(t) \\ &+ \frac{(t - t_k)^v}{v!} s_j \eta \frac{J_0(n_i)}{n_i^2} q_{v-1}(t_k), \quad v = 1, 2, 3, \dots, \end{aligned} \quad (88)$$

valid on the k th time interval. By the same procedure, the recurrence relations which are equivalent to Eqs. 73 and 74, respectively, are found to be

$$\begin{aligned} D_{j,k}^v(t) &= -T_{j,k}^v(t) \sum_{i=1}^I \frac{2\eta}{n_i^2 + n_i^2} \\ &+ \sum_{i=1}^I \frac{2n_i^2 v_{j,k}^v(n_i, t)}{(n_i^2 + n_i^2) J_0(n_i)}, \quad (v = 1, 2, 3, \dots) \end{aligned} \quad (89)$$

and

$$T_k^v(z_{j+1}, t) = 2T_{j,k}^v(t) - T_k^v(z_j, t), \quad (v = 1, 2, 3, \dots), \quad (90)$$

In deriving the recurrence relation which corresponds to Eq. 72, the fact that the relative coolant speed and the channel inlet temperature are both regarded as arbitrary functions of time must be considered. The $(v - 1)$ st derivative of Eq. 72 at $t = t_k$ is

$$\begin{aligned} \frac{d^v}{dt^v} T_{j,k}(t_k) &= -u_j \frac{d^{v-1}}{dt^{v-1}} [u(t) T_{j,k}(t)] \Big|_{t=t_k} \\ &+ u_j \frac{d^{v-1}}{dt^{v-1}} [u(t) T_k(z_j, t)] \Big|_{t=t_k} \\ &+ f 2\eta \frac{\rho_c p}{\rho_c c_{pc}} \frac{d^{v-1}}{dt^{v-1}} D_{j,k}(t_k). \end{aligned} \quad (91)$$

From Liebnitz's rule for differentiating the product of two functions, it follows that

$$\begin{aligned} \frac{d^{v-1}}{dt^{v-1}} [u(t) T_{j,k}(t)] \Big|_{t=t_k} \\ = \sum_{m=0}^{v-1} \frac{(v-1-m)!}{(v-1-m)! m!} \frac{d^{v-1-m}}{dt^{v-1-m}} T_{j,k}(t_k) u_m(t_k), \end{aligned} \quad (92)$$

in which $u_m(t_k)$ is the m th derivative of the relative coolant speed at the beginning of the k th time

interval. In view of Eq. 84, Eq. 92 reduces to

$$\begin{aligned} & \left. \frac{d^{v-1}}{dt^{v-1}} [u(t) T_{j,k}(t)] \right|_{t=t_k} \\ &= \sum_{m=0}^{v-1} \frac{(v-1)!}{m!} \frac{(t-t_k)^{m+1}}{(t-t_k)^v} T_{j,k}^{v-1-m}(t) u_m(t_k). \end{aligned} \quad (93)$$

In the same way it is found that

$$\begin{aligned} & \left. \frac{d^{v-1}}{dt^{v-1}} [u(t) T(z_j, t)] \right|_{t=t_k} \\ &= \sum_{m=0}^{v-1} \frac{(v-1)!}{m!} \frac{(t-t_k)^{m+1}}{(t-t_k)^v} T_k^{v-1-m}(z_j, t) u_m(t_k). \end{aligned} \quad (94)$$

With Eqs. 84, 85, 93, and 94 the coolant energy balance of Eq. 91 simplifies to the recurrence relation,

$$\begin{aligned} T_{j,k}^v(t) &= -u_j \sum_{m=0}^{v-1} \frac{1}{vm!} (t-t_k)^{m+1} T_{j,k}^{v-1-m}(t) u_m(t_k) \\ &+ u_j \sum_{m=0}^{v-1} \frac{1}{vm!} (t-t_k)^{m+1} T_k^{v-1-m}(z_j, t) u_m(t_k) \quad (95) \\ &+ 2nf \frac{\rho_c p}{\rho_c c_{pc}} \frac{(t-t_k)}{v} D_{j,k}^{v-1}(t), \quad (v = 1, 2, 3, 4, \dots), \end{aligned}$$

which is valid on the k th time interval. Equations 88, 95, 89, and 90 comprise two-term recurrence relations for consecutive terms of the Lie-Series representations of the unknowns governed by Eqs. 71-74, respectively. To incorporate the initial steady-state solutions on the first time interval and the values of the Hankel transform, bulk coolant temperatures, film temperature drops, and outlet temperatures of the j th axial segment at the beginning of the k th time interval specifically into the recurrence relation formulation of Eqs. 88, 95, 89, and 90, these equations are written out explicitly for the second and third terms in the Lie series, i.e., for $v = 1$ and $v = 2$.

The second terms of the Lie series of Eqs. 75-78 for the k th time interval are given by

$$\begin{aligned} v_{j,k}^1(n_i, t) &= -n_i^2 (t-t_k) v_{j,k}^1(n_i, t_k) \\ &+ n J_0(n_i) (t-t_k) T_{j,k}(t_k) + (t-t_k) s_j n \frac{J_0(n_i)}{n_i^2} q(t_k) \end{aligned} \quad (96)$$

and

$$\begin{aligned} T_{j,k}^1(t) &= -u_j (t-t_k) T_{j,k}(t_k) u(t_k) \\ &+ u_j (t-t_k) T_k(z_j, t_k) u(t_k) \\ &+ 2nf \frac{\rho_c p}{\rho_c c_{pc}} (t-t_k) D_{j,k}(t_k) \quad (97) \end{aligned}$$

together with Eqs. 89 and 90 with $v = 1$. The third terms of the Lie series of Eqs. 75-78 are

$$\begin{aligned} v_{j,k}^2(n_i, t) &= -n_i^2 \frac{(t-t_k)}{2} v_{j,k}^1(n_i, t) \\ &+ n J_0(n_i) \frac{(t-t_k)}{2} T_{j,k}^1(t) \quad (98) \\ &+ \frac{(t-t_k)^2}{2!} s_j n \frac{J_0(n_i)}{n_i^2} q_1(t_k), \end{aligned}$$

and

$$\begin{aligned} T_{j,k}^2(t) &= -u_j \frac{(t-t_k)}{2} T_{j,k}^1(t) u(t_k) \\ &+ u_j \frac{(t-t_k)}{2} T_k^1(z_j, t) u(t_k) \\ &- u_j \frac{(t-t_k)^2}{2} T_{j,k}(t_k) u_1(t_k) \\ &+ u_j \frac{(t-t_k)^2}{2} T_k(z_j, t_k) u_1(t_k) \\ &+ 2nf \frac{\rho_c p}{\rho_c c_{pc}} \frac{(t-t_k)}{2} D_{j,k}^1(t), \quad (99) \end{aligned}$$

along with Eqs. 89 and 90 with $v = 2$. For the higher order terms in the Lie series such that $v = 3, 4, 5, \dots$, the recurrence relations of Eqs. 88-90 are conveniently coded as they stand, but the coolant energy balance recurrence relation in Eq. 95 is most directly coded by splitting off the $m = 0$ and $m = v - 1$ terms under the summation, i.e., by coding

$$\begin{aligned}
T_{j,k}^v(t) = & -u_j \frac{(t-t_k)^v}{v} T_{j,k}^{v-1}(t) u(t_k) \\
& + u_j \frac{(t-t_k)^v}{v} T_k^{v-1}(z_j, t) u(t_k) \\
& - u_j \frac{(t-t_k)^v}{v!} T_{j,k}(t_k) u_{v-1}(t_k) \\
& + u_j \frac{(t-t_k)^v}{v!} T_k(z_j, t_k) u_{v-1}(t_k) \\
& - u_j \sum_{m=1}^{v-2} \frac{1}{vm!} (t-t_k)^{m+1} T_{j,k}^{v-1-m}(t) u_m(t_k) \\
& + u_j \sum_{m=1}^{v-2} \frac{1}{vm!} (t-t_k)^{m+1} T_k^{v-1-m}(z_j, t) u_m(t_k) \\
& + 2nf \frac{\rho_c p}{\rho_c c_{pc}} \frac{(t-t_k)}{v} D_{j,k}^{v-1}(t), \quad (v = 3, 4, 5, \dots)
\end{aligned} \tag{100}$$

As written, Eq. 100 explicitly contains the bulk coolant temperatures, the inlet temperatures to the various axial segments, and the coolant speed at the beginning of the k th time interval.

In practice the maximum number of terms which can be retained in the Lie series is set at a given, fixed input number, say N . The inequalities,

$$\frac{\left| v_{j,k}^{N-1}(n_1, t_{k+1}) \right| + \left| v_{j,k}^N(n_1, t_{k+1}) \right|}{\left| v_{j,k}(n_1, t_{k+1}) \right|} \leq \epsilon, \tag{101}$$

$$\frac{\left| T_{j,k}^{N-1}(t_{k+1}) \right| + \left| T_{j,k}^N(t_{k+1}) \right|}{\left| T_{j,k}(t_{k+1}) \right|} \leq \epsilon, \tag{102}$$

$$\frac{\left| D_{j,k}^{N-1}(t_{k+1}) \right| + \left| D_{j,k}^N(t_{k+1}) \right|}{\left| D_{j,k}(t_{k+1}) \right|} \leq \epsilon \tag{103}$$

and

$$\frac{\left| T_k^{N-1}(z_{j+1}, t_{k+1}) \right| + \left| T_k^N(z_{j+1}, t_{k+1}) \right|}{\left| T_k(z_{j+1}, t_{k+1}) \right|} \leq \epsilon \tag{104}$$

comprise the convergence tests applied to all of the Lie series at the end of the k th time interval for a specified error, ϵ . The denominators in Eqs. 101-104

are the current values of the unknowns as computed with N terms in their Lie series. If the Lie series of any unknown in any axial segment fails to meet its convergence criterion in N terms, the width of the time interval is reduced until all unknowns in every axial segment do meet their convergence tests. The maximum number of terms, N , to be allowed in the Lie series for a given time increment is set primarily by consideration of underflow in the particular computer employed. For the IBM-7094, the inequality $15 \leq N \leq 20$ gives representative values of the maximum allowable number of terms which can and need be carried in the Lie series for a given time increment. When the convergence criteria of Eqs. 101-104 are satisfied in less than N terms, the computation is simply advanced to the next time interval with no change in time interval increment, which is reduced only if any of the unknown functions fails to converge within a maximum of N terms retained in their Lie series.

6. Allowance for Heat-Transfer Coefficient Strongly Dependent on Coolant Speed

The transient temperature field solutions so far obtained take into account arbitrary time variations in the relative power density, the coolant speed, and the channel inlet temperature under the restriction that the effective heat transfer coefficient is a weak function of the coolant speed. The time dependence of the coolant speed, relative power density, and channel inlet temperature may be specified either analytically or numerically provided that the required derivatives for the Lie series are known to sufficiently high order numerically.

When changes in the effective heat transfer coefficient are to be accounted for over the total time span during which the transient is followed, changes in the values of the roots of the transcendental relation, Eq. 40, have to be taken into account. Since relatively large changes in the effective heat transfer coefficient are required to produce small changes in the zeros of Eq. 40, it is not necessary to determine the zeros of Eq. 40 at every time step at which the transient temperatures are computed. Accordingly, the effective heat transfer coefficient may be assigned a constant value over each subinterval of a set into which the total time interval over which the transient is to be followed

is partitioned. That is, the actual time variation of the effective heat transfer coefficient is approximated by a set of step functions. Let n_h be the value of the Biot number on the h th time subinterval such that $\tau_h \leq t \leq \tau_{h+1}$ for $h = 1, 2, 3, 4, \dots$ and $\tau_1 = 0$. Note that subinterval partition in time for the effective heat transfer coefficient is not, in general, the same as the time subinterval partition which defines the points in time at which the transient temperature fields are computed. This latter partition is established primarily by the convergence criteria placed on the Lie-series representations of the transient temperature fields. Convergence of the Lie series is required regardless of the partition established for the effective heat transfer coefficient.

Since the effective heat-transfer coefficient is to be approximated by a set of step functions, it will be a discontinuous function in time. However, the transient temperature fields are continuous functions in time. Consequently, the key point in the approximate method under consideration is how to ensure time continuity in the transient temperature fields even though the heat-transfer coefficient is approximated discontinuously in time.

Let $n_i(k)$ be the i th root of Eq. 40 on the k th time interval, $t_k \leq t \leq t_{k+1}$, and let $n_i(k-1)$ be the i th root on the $(k-1)$ st time interval, $t_{k-1} \leq t \leq t_k$. Then, on the k th time interval, the transient fuel temperature field in the j th axial segment is given by

$$v_{j,k}(r,t) = \sum_{i=1}^{\infty} v_{j,k}[n_i(k), t] \frac{2n_i^2(k)}{[n_k^2 + n_i^2(k)]} \frac{J_0[n_i(k)r]}{J_0^2[n_i(k)]} \quad (105)$$

and on the $(k-1)$ st interval, by

$$v_{j,k-1}(r,t) = \sum_{i=1}^{\infty} v_{j,k-1}[n_i(k-1), t] \frac{2n_i^2(k-1)}{[n_{k-1}^2 + n_i^2(k-1)]} \frac{J_0[n_i(k-1)r]}{J_0^2[n_i(k-1)]} \quad (106)$$

Since the fuel temperature is continuous at time $t = t_k$, it follows that

$$v_{j,k}(r, t_k) = v_{j,k-1}(r, t_k). \quad (107)$$

By substituting Eqs. 105 and 106 into Eq. 107 and operating on the result with

$$\begin{aligned} & \int_0^1 dr \, r \, J_0[n_m(k)r], \text{ we find that} \\ & v_{j,k}[n_m(k), t_k] A_m(k) \int_0^1 dr \, r \, J_0^2[n_m(k)r] \\ & = \sum_{i=1}^{\infty} v_{j,k-1}[n_i(k-1), t_k] A_i(k-1) \\ & \int_0^1 dr \, r \, J_0[n_m(k)r] J_0[n_i(k-1)r], \end{aligned} \quad (108)$$

in which the definition,

$$A_m(k) = \frac{2n_m^2(k)}{n_k^2 + n_m^2(k)} \frac{1}{J_0^2[n_m(k)]} \quad (109)$$

has been introduced. By noting that

$$A_m(k) \int_0^1 dr \, r \, J_0^2[n_m(k)r] = 1, \quad (110)$$

and that

$$\begin{aligned} & \int_0^1 dr \, r \, J_0[n_m(k)r] J_0[n_i(k-1)r] = \\ & = \frac{1}{n_m^2(k) - n_i^2(k-1)} \left\{ n_m(k) J_0[n_i(k-1)] J_1[n_m(k)] \right. \\ & \quad \left. - n_i(k-1) J_1[n_i(k-1)] J_0[n_m(k)] \right\} \end{aligned} \quad (111)$$

we see that Eq. 108 reduces to

$$\begin{aligned} v_{j,k}[n_m(k), t_k] & = \sum_{i=1}^{\infty} v_{j,k-1}[n_i(k-1), t_k] A_i(k-1) \\ & \frac{1}{[n_m^2(k) - n_i^2(k-1)]} \times \left\{ n_m(k) J_0[n_i(k-1)] J_1[n_m(k)] \right. \\ & \quad \left. - n_i(k-1) J_1[n_i(k-1)] J_0[n_m(k)] \right\}. \end{aligned} \quad (112)$$

The following interpretation is to be placed on Eq. 112. At those points in time, t_k , at which the effective heat transfer coefficient is discontinuous, so that there is a change in value of the zeros of Eq. 40, the value of the Hankel transform at the beginning of the k th time subinterval is to be computed from the value of the Hankel transform at the end of the $(k-1)$ st time subinterval through

Eq. 112. Thus, Eq. 112 serves to initiate the Hankel transform for the next time step at those points in time at which the effective heat transfer coefficient changes value discontinuously. This initiation ensures that the computed transient fuel region temperature field is a continuous function with respect to time.

The Hankel transform is initiated at time points for which there is no change in value of the effective heat transfer coefficient through the relation,

$$v_{j,k}[n_m(k), t_k] = v_{j,k-1}[n_m(k-1), t_k] \quad (113)$$

in which

$$n_m(k) = n_m(k-1), \quad (114)$$

since at such points there is no change in the values of roots of Eq. 40.

Even at points in time at which the effective heat transfer coefficient becomes discontinuous, the values of all other unknown temporal functions appearing in the system of Eqs. 71-74 at the end of the $(k-1)$ st time subinterval are equal to the values of these functions at the beginning of the k th time subinterval.

7. Function Subprograms Required for LIETRAN

The Fortran listing of the main program of an IBM-7094 computer code, LIETRAN, developed to compute transient temperature fields in cylindrical reactor channels in accordance with the preceding analysis is contained in the Appendix. Since the relative power density in the fuel region, the relative coolant speed, and the inlet temperature have all been regarded as arbitrary functions of time, it is necessary for the user of LIETRAN to supply his own FUNCTION subprograms for the following quantities:

1. The relative power density in the fuel, $q(t_k)$, evaluated at the beginning of the k th time interval and called by POW(TIMEB).
2. The v th order derivative, $q_v(t_k)$, of the relative power density evaluated at the beginning of the k th time interval and called by POWD(TIMEB, NU).
3. The relative coolant speed, $u(t_k)$, evaluated at the beginning of the k th time interval and called by COOLS(TIMEB).
4. The v th order derivative, $u_v(t_k)$ of the relative coolant speed evaluated at the beginning of the

k th time interval and called by COOLSD(TIMEB, NU).

5. The channel inlet temperature, $T(z_1, t_k)$, evaluated at the beginning of the k th time interval and called by TEMIN(TIMEB).
6. The v th order term in the Lie-series representation of the channel inlet temperature, denoted by $T_k^v(z_1, t_k)$, evaluated at the beginning of the k th time interval and called by TEMIND(TIMEB, NU).

For example, let T_{in} be the channel inlet temperature at the beginning of the transient and let $f(t)$ be the relative channel inlet temperature, so that

$$T(z_1, t) = T_{in} f(t). \quad (115)$$

The Lie-series representation of the channel inlet temperature,

$$T_k(z_1, t) = T_k^0(z_1, t) + \sum_{v=1}^{\infty} T_k^v(z_1, t), \quad (116)$$

holds for the k th time interval. In Eq. 116

$$T_k^0(z_1, t) = T_{in} f(t_k), \quad (117)$$

and

$$T_k^v(z_1, t) = \frac{(t - t_k)^v}{v!} \frac{d^v}{dt^v} T(z_1, t) \Big|_{t=t_k}, \quad (v = 1, 2, 3, \dots). \quad (118)$$

Combining Eqs. 115 and 118 yields

$$T_k^v(z_1, t) = T_{in} f_v(t_k) \frac{(t - t_k)^v}{v!} \quad (119)$$

in which $f_v(t_k)$ is the v th order derivative of the relative channel inlet temperature evaluated at the beginning of the k th time interval.

Appendix
Fortran Listing of LIETRA

LIETRA
DECKAA - EFN SOURCE STATEMENT - IFN(S) -

```

DIMENSION FACR(30)
DIMENSION ZND(31),Z(31),DELZ(30),ZM(30),
1 PSA(30),SAPD(30),ACSS(30)
DIMENSION ETEM(31),CTEM(30),FCLT(30),
1 OFET(30),CTEMA(30),ETEMA(31)
DIMENSION ZLN(31),ZMIN(30)
DIMENSION OFETX(30),OFETXX(30)
DIMENSION FCLTE(30)
DIMENSION TIMTER(30)
DIMENSION AK(50),BFA(50),BFB(50),
1T(360),AKS(50),ETAAK(50),ETABFA(50),
2BFBK(50),COEA(50),COEB(50),COEC(50),
3COED(50),AK4(50),COEE(50),HTB(30,50),
4HTE(30,50),CTEME(30),OFETA(30),ETEME(31),
5OFETE(30),VJ(30,50),TJ(30),STJ(30),
6TEE(30,31)
DIMENSION OFETAX(30),DB(30),DE(30),COEN(50),COEM(50)
COMMON TEMPIN,TIMTER,AAP,XTXTS
9500 FORMAT(I3)
9000 FORMAT(1H0,7X,2HJ0,14X,2HJ1,11X,
18HARGUMENT)
9001 FORMAT(1H ,3E16.8)
9002 FORMAT(1H ,8HFACR(I)=,E16.8,1X,6HFOR I=,I2)
9003 FORMAT(1H ,19X,F10.5,2E16.8)
9004 FORMAT(1H ,19X,F10.5,4E16.8,2X,E16.8)
9005 FORMAT(1H1,19X,1X,5HAXIAL,4X,1X,10HEXACT BULK,
15X,1X,11HAPPROX BULK,4X,1X,7HOUTSIDE,8X,1X,6HCENTER/
21H ,19X,1X,8HPOSITION,1X,1X,12HMEAN COOLANT,3X,1X,
312HMEAN COOLANT,3X,1X,9HFUEL EDGE,6X,1X,9HFUEL LINE/
41H ,19X,1X,9HIN INCHES,1X,11HTEMPERATURE,4X,1X,
511HTEMPERATURE,4X,1X,11HTEMPERATURE,4X,1X,11HTEMPERATURE///)
9006 FORMAT(1H1,2HAT,E16.8,1X,12HSECONDS THE ,
126HTRANSIENT TEMPERATURES ARE///)
9007 FORMAT(1H ,19X,F10.5,16X,E16.8)
9008 FORMAT(1H ,19X,F10.5,16X,3E16.8)
9009 FORMAT(1H0,17HBAD INPUT ON NLIE//)
9301 FORMAT(1H0,69X,F10.5,2X,6HOFETX=,E16.8)
9307 FORMAT(1H0,69X,F10.5,2X,7HOFETXX=,E16.8)
9308 FORMAT(1H0,///)
8765 FORMAT(1H ,5X,7HTIMTER(,I2,2H)=,E16.8)
1113 FORMAT(1H ,2HAT,F10.5,1X,3HDB=,E16.8,1X,11HAND OFETAX=,E16.8)
FACR( 1)=1.00000 000
FACR( 2)=5.00000 000E-01
FACR( 3)=1.66666 667E-01
FACR( 4)=4.16666 667E-02
FACR( 5)=8.33333 333E-03
FACR( 6)=1.38888 889E-03
FACR( 7)=1.98412 698E-04
FACR( 8)=2.48015 873E-05
FACR( 9)=2.75573 192E-06
FACR(10)=2.75573 192E-07
FACR(11)=2.50521 084E-08
FACR(12)=2.08767 570E-09
FACR(13)=1.60590 438E-10
FACR(14)=1.14707 456E-11

```

```

FACR(15)=7.64716 373E-13
FACR(16)=4.77947 733E-14
FACR(17)=2.81145 725E-15
FACR(18)=1.56192 070E-16
FACR(19)=8.22063 525E-18
FACR(20)=4.11031 762E-19
FACR(21)=1.95729 411E-20
FACR(22)=8.89679 139E-22
FACR(23)=3.86817 017E-23
FACR(24)=1.61173 757E-24
FACR(25)=6.44695 029E-26
FACR(26)=2.47959 626E-27
FACR(27)=9.18368 986E-29
FACR(28)=3.27988 924E-30
FACR(29)=1.13099 629E-31
FACR(30)=3.76998 763E-33
DO 9 NFACR=1,30
9 WRITE(9,9002) FACR(NFACR),NFACR
AK( 1)=1.9898
AK( 2)=4.7131
AK( 3)=7.6177
AK( 4)=10.6223
AK( 5)=13.6786
AK( 6)=16.7630
AK( 7)=19.8640
AK( 8)=22.9755
AK( 9)=26.0938
AK(10)=29.2169
AK(11)=32.3436
AK(12)=35.4727
AK(13)=38.6040
AK(14)=41.7367
AK(15)=44.8706
AK(16)=48.0055
AK(17)=51.1413
AK(18)=54.2777
AK(19)=57.4146
AK(20)=60.5521
AK(21)=63.6899
AK(22)=66.8282
AK(23)=69.9666
AK(24)=73.1054
AK(25)=76.2445

```

C
C TEST VALUES
C

```

ETA=5.0
I=10
WRITE(9,9000)
DO 10 IBF=1,I
XDF=AK(IBF)
NBF=0
CALL BESJN(XBF,NBF,T,B)
BFA(IRF)=B
NBF=1
CALL BESJN(XBF,NBF,T,B)
BFB(IBF)=B
WRITE(9,9001) BFA(IBF),BFB(IBF),XBF
10 CONTINUE
ETAS=ETA**2
DO 11 IA=1,I
AKS(IA)=AK(IA)**2
ETAAK(IA)=ETAS*AKS(IA)
ETABFA(IA)=ETA*BFA(IA)
BFBK(IA)=BFB(IA)/AK(IA)
COEA(IA)=2.0*AKS(IA)/(ETAAK(IA)*BFA(IA))
COEB(IA)=2.0/(AKS(IA)*ETAAK(IA))
AK4(IA)=AKS(IA)*AKS(IA)
COEC(IA)=ETABFA(IA)/AKS(IA)
COED(IA)=ETABFA(IA)/AK4(IA)
COEE(IA)=COEA(IA)/BFA(IA)
COEN(IA)=2.0*ETA/ETAAK(IA)

```

```

      COEM(IA)=COEN(IA)/BFA(IA)
11  CONTINUE
      SUMA=0.0
      DO 103 ISA=1,1
103  SUMA=SUMA+COEB(ISA)
      1 READ(10,9500) NLIE
C
C INPUT DATA VALUES FOR TESTING
C
      A=0.12500
      Q0=3.0E+06
      FK=4.3
      FRHO=684.0
      FCP=0.07
      SPEED=1.0
      CRHO=55.0
      CCP=0.31500000
      CH=15.0
      APEPS=0.50000000
      FCVR=1.5000000
      TEMPIN=500.0
      DTS=0.100
      NPRINT=5
      EPSLIE=0.00100000
      ENDT=25.00000000
      J=6
      AAPIS=0.1000
      AFT=A/12.0
      AQK=AFT**2*Q0/FK
      ALPHA=FK/(FRHO*FCP)
      SPEEDH=SPEED*3600.0
      XXXB=2.0*AFT**2*SPEEDH/ALPHA
      XXXC=FRHO*FCP/(CRHO*CCP)
      HCH=CH/24.00000000
      TXTT=ALPHA/(AFT**2)
      TXTTS=TXTT/3600.0000
      AAP=AAPIS/TXTTS
C STEADY STATE SOLUTIONS
C
      J1=J+1
      FJ=J
      F2J=2.0/FJ
      DO 100 JA=1,J1
      JAB=JA-1
      FJAB=FJAB
      ZND(JA)=-1.0+F2J*FJAB
100  Z(JA)=HCH*ZND(JA)
      XXXA=0.33333333*(1.0-APEPS)
      DO 101 JB=1,J
      DELZ(JB)=Z(JB+1)-Z(JB)
      ZH(JB)=Z(JB)+0.50000000*DELZ(JB)
      PSA(JB)=1.0-XXXA*(ZND(JB+1)**2+
1  ZND(JB+1)*ZND(JB)+ZND(JB)**2)
      SAPD(JB)=AQK*PSA(JB)
101  ACSS(JB)=XXXB/DELZ(JB)
      XXXD=XXXC*FCVR
      ETATWO=1.0/(2.0*ETA)
C EXACT STEADY STATE SOLUTION
      ETEM(1)=TEMPIN
      ETEMA(1)=TEMPIN
      DO 102 JE=1,J
      WX=XXXD*SAPD(JE)/ACSS(JE)
      CTEM(JE)=ETEM(JE)+WX
      JEA=JE+1
      ETEM(JEA)=ETEM(JE)+2.0*WX
      FCLT(JE)=CTEM(JE)+(0.25000000+
1  ETATWO)*SAPD(JE)
      OFET(JE)=CTEM(JE)+ETATWO*SAPD(JE)
C HANKEL TRANSFORM STEADY STATE SOLUTION
C WITH UNITY SUMMED OUT
      ETEMA(JE)=ETEM(JE)+WX*2.0*ETAS*SUMA
      ETEMA(JEA)=ETEM(JE)+WX*4.0*ETAS*SUMA

```

```

102 CONTINUE
C INITIAL VALUES OF THE HANKEL TRANSFORMS
  KJ=0
105 KJ=KJ+1
  DO 104 KI=1,1
104 HTB(KJ,KI)=CTEMA(KJ)*COEC(KI)+SAPD(KJ)*COED(KI)
  IF(KJ-J) 105,106,106
106 CONTINUE
C
C CALCULATION OF FUEL EDGE TEMPERATURE IN
C STEADY STATE WITH HANKEL TRANSFORM
C
  DO 833 JXP=1,J
  OFETA(JXP)=0.0
  DO 834 IXHT=1,1
834 OFETA(JXP)=OFETA(JXP)+COEA(IXHT)*
  1 HTB(JXP,IXHT)
833 CONTINUE
C OUTPUT OF STEADY STATE SOLUTIONS
  WRITE(9,9005)
  DO 107 JO=1,J
  ZIN(JO)=12.0*Z(JO)
  ZMIN(JO)=12.0*ZM(JO)
  WRITE(9,9003) ZIN(JO),ETEM(JO),ETEMA(JO)
  WRITE(9,9004) ZMIN(JO),CTEM(JO),CTEMA(JO),
  1 OFET(JO),FCLT(JO),OFETA(JO)
107 CONTINUE
  JOA=J+1
  ZIN(JOA)=12.0*Z(JOA)
  WRITE(9,9003) ZIN(JOA),ETEM(JOA),
  1 ETEMA(JOA)
C
C ALTERNATIVE EVALUATION OF OUTSIDE FUEL EDGE TEMPERATURE
  DO 9300 JZZ=1,J
  OFETX(JZZ)=0.0
  DO 9302 IZZ=1,1
9302 OFETX(JZZ)=OFETX(JZZ)+CTEMA(JZZ)*2.0*ETA/ETAAK(IZZ)+SAPD(JZZ)*
  1 ETA*COEB(IZZ)
9300 WRITE(9,9301) ZMIN(JZZ),OFETX(JZZ)
  WRITE(9,9308)
C
C SECOND ALTERNATIVE EVALUATION OF THE OUTSIDE FUEL EDGE TEMPERATURE
C
  DO 9305 JZZZ=1,J
  SZZZ=0.0
  DO 9306 IZZZ=1,1
9306 SZZZ=SZZZ-CTEMA(JZZZ)*2.0*ETA/ETAAK(IZZZ)+COEA(IZZZ)*HTB(JZZZ,
  1 IZZZ)
  OFETXX(JZZZ)=CTEMA(JZZZ)+SZZZ
9305 WRITE(9,9307) ZMIN(JZZZ),OFETXX(JZZZ)
C
C
C INITIAL STEADY STATE FILM DROP TEMPERATURE
C
  DO 1111 JABC=1,J
  DB(JABC)=0.0
  DO 1112 IABC=1,1
1112 DB(JABC)=DB(JABC)+COEA(IABC)*HTB(JABC,IABC)-CTEMA(JABC)*COEN(IABC)
  OFETAX(JABC)=CTEMA(JABC)+DB(JABC)
  WRITE(9,1113) ZMIN(JABC),DB(JABC),OFETAX(JABC)
1111 CONTINUE
C
C END OF STEADY STATE OUTPUT
  XXXX=2.0*ETA*XXXD
C SET UP OF TIME COUNTING AND POWERS OF THE
C NONDIMENSIONAL TIME INTERVAL BY NU FACTORIAL
C TO CONTROL UNDERFLOW
C
C RETURN TO THIS POINT IF TIME INTERVAL CHANGES
C
  DTH=DTS/3600.0000
  DTND=XTXT*DTH

```

```

      NOUT=0
      TIMEB=0.0
530  CONTINUE
      DTNDH=0.50000000*DTND
      TIMEE=TIMEB*DTND
C
      DO 160 NUTIME=1,30
      ZIP=DTND**NUTIME
      IF(ZIP-1.0E-17) 161,161,1606
1606  TIMTER(NUTIME)=ZIP*FACR(NUTIME)
      IF(TIMTER(NUTIME)-1.0E-33) 161,161,160
      161  CONTINUE
      DO 162 NUTIA=NUTIME,30
      162  TIMTER(NUTIA)=0.0
      160  CONTINUE
C
      DO 1607 NUTIME=1,30
      1607  WRITE(9,8765) NUTIME,TIMTER(NUTIME)
C
C
C TRANSIENT CALCULATIONS FOR FIRST AXIAL SEGMENT
C
C FIRST TERMS IN LIE SERIES
C RETURN TO THIS POINT FOR NEXT TIME STEP IF
C THERE IS NO TIME INTERVAL CHANGE
C
      795  CONTINUE
      DO 501 ITRA=1,I
      501  HTE(1,ITRA)=HTB(1,ITRA)
      CTEME(1)=CTEMA(1)
      DE(1)=DB(1)
      ETEME(2)=ETEMA(2)
C
C SECOND TERMS IN LIE SERIES
C
      DO 502 ITA=1,I
      502  VJ(1,ITA)=TIMTER(1)*(-AKS(ITA)*HTB(1,ITA)+
      1ETABFA(ITA)*CTEMA(1)+SAPD(1)*COEC(ITA)*POW(TIMEB))
      TJ(1)=TIMTER(1)*(COOLS(TIMEB)*ACSS(1)+
      1(TEMIN(TIMEB)-CTEMA(1))*XXXX*DB(1))
      STJ(1)=0.0
      DO 503 ITB=1,I
      503  STJ(1)=STJ(1)+COEA(ITB)*VJ(1,ITB)-TJ(1)*COEN(ITB)
      TEE(1,1)=TEMIND(TIMEB,1)
      TEE(1,2)=2.0*TJ(1)-TEE(1,1)
C
C THIRD TERMS IN LIE SERIES
C
      DO 504 ITC=1,I
      504  VJ(2,ITC)=DTNDH*(-AKS(ITC)*VJ(1,ITC)+
      1ETABFA(ITC)*TJ(1)+SAPD(1)*COEC(ITC)*
      2TIMTER(2)*POWD(TIMER,1)
      TJ(2)=DTNDH*(COOLS(TIMEB)*ACSS(1)+
      1(TEE(1,1)-TJ(1))*XXXX*STJ(1) )+
      2TIMTER(2)*ACSS(1)*COOLSD(TIMEB,1)+
      3(TEMIN(TIMEB)-CTEMA(1))
      STJ(2)=0.0
      DO 505 ITD=1,I
      505  STJ(2)=STJ(2)+COEA(ITD)*VJ(2,ITD)-TJ(2)*COEN(ITD)
      TEE(2,1)=TEMIND(TIMEB,2)
      TEE(2,2)=2.0*TJ(2)-TEE(2,1)
C
C HIGHER ORDER TERMS IN LIE SERIES
C
C SUM UP FIRST THREE TERMS IN LIE SERIES
C
      DO 512 ITG=1,I
      512  HTE(1,ITG)=HTE(1,ITG)+VJ(1,ITG)+VJ(2,ITG)
      CTEME(1)=CTEME(1)+TJ(1)+TJ(2)
      DE(1)=DE(1)+STJ(1)+STJ(2)
      ETEME(2)=ETEME(2)+TEE(1,2)+TEE(2,2)
      NU=3

```

```

C
C COME BACK TO THIS POINT IF MORE TERMS IN THE
C LIE SERIES ARE REQUIRED LATER
C
521 FNU=NU
    NUA=NU-1
    DO 515 ITI=1,I
515 VJ(NU,ITI)=TIMTER(1)*(-AKS(ITI)*
1   VJ(NUA,ITI)+ETABFA(ITI)*TJ(NUA))/FNU
2   *SAPD(1)*COEC(ITI)*TIMTER(NU)*POWD(TIMEB,NUA)
    STXA=0.0
    STXB=0.0
    NUB=NU-2
    NTX=0
516 NTX=NTX+1
    NUANTX=NUA-NTX
    CSPDER=COOLSD(TIMEB,NTX)
    STXA=STXA+TIMTER(NTX)*TJ(NUANTX)*CSPDER
    STXB=STXB+TIMTER(NTX)*TEE(NUANTX,1)*CSPDER
    IF(NTX-NUB) 516,517,517
517 CONTINUE
    TJ(NU)=
1ACSS(1)*TIMTER(1)*COOLS(TIMEB)*(TEE(NUA,1)
2   -TJ(NUA))/FNU+
3ACSS(1)*TIMTER(NU)*COOLSD(TIMEB,NUA)*(
4   TEMIN(TIMEB)-CTEMA(1))+
5ACSS(1)*TIMTER(1)*(STXB-STXA)/FNU+
6XXXX*TIMTER(1)*STJ(NUA)          /FNU
    STJ(NU)=0.0
    DO 518 ITJ=1,I
518 STJ(NU)=STJ(NU)+COEA(ITJ)*VJ(NU,ITJ)-TJ(NU)*COEN(ITJ)
    TEE(NU,1)=TEMIND(TIMEB,NU)
    TEE(NU,2)=2.0*TJ(NU)-TEE(NU,1)
    DO 601 KTA=1,I
601 HTE(1,KTA)=HTE(1,KTA)+VJ(NU,KTA)
    CTEME(1)=CTEME(1)+TJ(NU)
    DE(1)=DE(1)+STJ(NU)
    ETEME(2)=ETEME(2)+TEE(NU,2)
    IF(NU-NLIE) 599,600,600
599 CONTINUE
    NU=NU+1
    GO TO 521
600 CONTINUE
    NUC=NU-1
C
C LIE SERIES CONVERGENCE TEST FOR FIRST AXIAL SEGMENT
C
    DO 522 ITK=1,I
    TEST=ABS(VJ(NU,ITK))+ABS(VJ(NUC,ITK))
    TESTA=ABS(HTE(1,ITK))+TEST
    TESTB=TEST/TESTA
    IF(TESTB-EPSSLIE) 522,522,523
522 CONTINUE
    TEST=ABS(TJ(NU))+ABS(TJ(NUC))
    TESTA=ABS(CTEME(1))+TEST
    TESTB=TEST/TESTA
    IF(TESTB-EPSSLIE) 525,525,523
525 CONTINUE
    TEST=ABS(STJ(NU))+ABS(STJ(NUC))
    TESTA=ABS(DE(1))+TEST
    TESTB=TEST/TESTA
    IF(TESTB-EPSSLIE) 526,526,523
526 CONTINUE
    TEST=ABS(TEE(NU,2))+ABS(TEE(NUC,2))
    TESTA=ABS(ETEME(2))+TEST
    TESTB=TEST/TESTA
    IF(TESTB-EPSSLIE) 527,527,523
527 CONTINUE
C
C AT THIS POINT ALL LIE SERIES FOR THE
C FIRST AXIAL SEGMENT HAVE CONVERGED
    GO TO 531

```



```

523 CONTINUE
    IF(NU-31) 528,529,529
528 CONTINUE
C
C AT THIS POINT LIE SERIES FAILED TO
C CONVERGE IN NLIE TERMS FOR THE GIVEN
C TIME INCREMENT INCREASE NLIE OR HALVE
C THE TIME INCREMENT
C
    TIMEB=TIMEE-DTND
    DTND=0.50000000*DTND
    GO TO 530
529 CONTINUE
C
C AT THIS POINT MORE TERMS IN LIE SERIES
C WERE CALLED FOR THAN AVAILABLE BAD INPUT
C ON NLIE
C
    WRITE(9,9009)
    GO TO 1
531 CONTINUE
C
C CONTINUATION TO NEXT AXIAL SEGMENT
C
    JX=1
650 CONTINUE
    JX=JX+1
    JXA=JX+1
C
C INITIALIZATION OF UNKNOWNNS FOR J-TH SEGMENT
C BY FIRST TERM IN LIE SERIES
C
    DO 535 IEA=1,I
535 HTE(JX,IEA)=HTB(JX,IEA)
    CTEME(JX)=CTEMA(JX)
    DE(JX)=DB(JX)
    ETEME(JXA)=ETEMA(JXA)
C
C SECOND LIE SERIES TERM IN J-TH SEGMENT
C
    DO 536 IEB=1,I
536 VJ(1,IEB)=TIMTER(1)*(-AKS(IEB)*HTB(JX,IEB)+
1 ETABFA(IEB)*CTEMA(JX)+SAPD(JX)*COEC(IEB)
2 *POW(TIMEB))
    TJ(1)=TIMTER(1)*(COOLS(TIMEB)*ACSS(JX)+
1 (ETEMA(JX)-CTEMA(JX))*XXXX* DB(JX))
    STJ(1)=0.0
    DO 537 IEC=1,I
537 STJ(1)=STJ(1)+COEA(IEC)*VJ(1,IEC)-TJ(1)*COEN(IEC)
    TEE(1,JXA)=2.0*TJ(1)-TEE(1,JX)
C
C THIRD LIE SERIES TERM IN J-TH SEGMENT
C
    DO 538 IEE=1,I
538 VJ(2,IEE)=DTNDH*(-AKS(IEE)*VJ(1,IEE)
1 +ETABFA(IEE)*TJ(1)+TIMTER(2)*SAPD(JX)+
2 COEC(IEE)*POWD(TIMEB,1)
    TJ(2)=DTNDH*(ACSS(JX)*COOLS(TIMEB)+
1 (TEE(1,JX)-TJ(1))*XXXX*
2 STJ(1) )+TIMTER(2)*ACSS(JX)+
3 COOLSD(TIMEB,1)*(ETEMA(JX)-CTEMA(JX))
    STJ(2)=0.0
    DO 539 IEF=1,I
539 STJ(2)=STJ(2)+COEA(IEF)*VJ(2,IEF)-TJ(2)*COEN(IEF)
    TEE(2,JXA)=2.0*TJ(2)-TEE(2,JX)
C
C SUMMATION OF FIRST THREE TERMS OF THE
C LIE SERIES ON THE J-TH AXIAL SEGMENT
C
    DO 540 IEH=1,I
540 HTE(JX,IEH)=HTE(JX,IEH)+VJ(1,IEH)
1+VJ(2,IEH)

```

```

      CTEME(JX)=CTEME(JX)+TJ(1)+TJ(2)
      DE(JX)=DE(JX)+STJ(1)+STJ(2)
      ETEME(JXA)=ETEME(JXA)+TEE(1,JXA)
      1+TEE(2,JXA)
C
C EVALUATION OF HIGHER ORDER TERMS IN LIE SERIES
C
      NU=3
548 FNU=NU
      NUA=NU-1
      DO 541 IEI=1,I
541 VJ(NU,IEI)=TIMTER(1)*(-AKS(IEI)*
1 VJ(NUA,IEI)+ETABFA(IEI)*TJ(NUA))/FNU
2 *TIMTER(NU)*SAPD(JX)*COFC(IEI)*
3 POWD(TIMEB,NUA)
      STXA=0.0
      STXB=0.0
      NUB=NU-2
      NTX=0
542 NTX=NTX+1
      NUA=NU-NTX
      CSPDER=COOLSD(TIMEB,NTX)
      STXA=STXA+TIMTER(NTX)*TJ(NUANTX)*CSPDER
      STXB=STXB+TIMTER(NTX)*TEE(NUANTX,JX)*CSPDER
      IF(NTX-NUB) 542,543,543
543 CONTINUE
      TJ(NU)=ACSS(JX)*TIMTER(1)*COOLS(TIMEB)*
1 (TEE(NUA,JX)-TJ(NUA))/FNU+
2 ACSS(JX)*TIMTER(NU)*COOLSD(TIMEB,NUA)*
3 (ETEMA(JX)-CTEMA(JX))+
4 ACSS(JX)*TIMTER(1)*(STXB-STXA)/FNU+
5 XXXX*TIMTER(1)*STJ(NUA) /FNU
      STJ(NU)=0.0
      DO 544 IEK=1,I
544 STJ(NU)=STJ(NU)+COEA(IEK)*VJ(NU,IEK)-TJ(NU)*COEN(IEK)
      TEE(NU,JXA)=2.0*TJ(NU)-TEE(NU,JX)
C
C ADDITION ON OF NU-TH TERM OF LIE SERIES
C
      DO 545 IEL=1,I
545 HTE(JX,IEL)=HTE(JX,IEL)+VJ(NU,IEL)
      CTEME(JX)=CTEME(JX)+TJ(NU)
      DE(JX)=DE(JX)+STJ(NU)
      ETEME(JXA)=ETEME(JXA)+TEE(NU,JXA)
      IF(NU-NLIE) 546,547,547
546 CONTINUE
      NU=NU+1
      GO TO 548
547 CONTINUE
C
C CONVERGENCE TEST ON LIE SERIES IN THE
C JH-AXIAL SEGMENT
C
      NUC=NU-1
      DO 549 IEM=1,I
      TEST=ABS(VJ(NU,IEM))+ABS(VJ(NUC,IEM))
      TESTA=TEST+ABS(HTE(JX,IEM))
      TESTB=TEST/TESTA
      IF(TESTB-EPSLIE) 549,549,550
549 CONTINUE
      TEST=ABS(TJ(NU))+ABS(TJ(NUC))
      TESTA=TEST+ABS(CTEME(JX))
      TESTB=TEST/TESTA
      IF(TESTB-EPSLIE) 551,551,550
551 CONTINUE
      TEST=ABS(STJ(NU))+ABS(STJ(NUC))
      TESTA=TEST+ABS(DE(JX))
      TESTB=TEST/TESTA
      IF(TESTB-EPSLIE) 552,552,550
552 CONTINUE
      TEST=ABS(TEE(NU,JXA))+ABS(TEE(NUC,JXA))
      TESTA=TEST+ABS(ETEME(JXA))

```

```

      TESTB=TEST/TESTA
      IF(TESTB-EPSTIE) 553,553,550
553 CONTINUE
C
C AT THIS POINT THE LIE SERIES FOR
C ALL THE UNKNOWN ON THE J-TH AXIAL
C SEGMENT HAVE CONVERGED
      GO TO 554
550 CONTINUE
C
C AT THIS ONE OF THE LIE SERIES DID NOT CONVERGE
C IN NLIE TERMS FOR THE GIVEN TIME INCREMENT
C INCREASE NLIE OR HALVE THE TIME INCREMENT
C
      IF(VU-31) 555,555,556
555 CONTINUE
      TIMEB=TIMEE-DTND
      DTND=0.50000000*DTND
C RESTART CALCULATION IN FIRST AXIAL SEGMENT
      GO TO 530
556 CONTINUE
      WRITE(9,9009)
      GO TO 1
554 CONTINUE
C
C GO TO NEXT AXIAL SEGMENT
C
      IF(JX-J) 650,559,559
559 CONTINUE
C
C AT THIS POINT ALL LIE SERIES ON ALL
C AXIAL SEGMENTS HAVE CONVERGED FOR THE
C GIVEN SAME TIME INTERVAL
C
C PRINT OUT CONTROL
C
      NOUT=NOUT+1
      IF(NOUT-NPRINT) 532,533,533
533 CONTINUE
C
C CALCULATE CURRENT TIME IN SECONDS
C RESET PRINT OUT CONTROL
C PRINT OUT RESULTS FOR ALL AXIAL SEGMENTS
C
      NOUT=0
      TIMES=TIMEE/XTXTS
      WRITE(9,9006) TIMES
      ETEME(1)=TEM1V(TIMEE)
      WRITE(9,9007) ZIN(1),ETEME(1)
C
C CALCULATE FUEL CENTER LINE TEMPERATURE
C
      OFETE(1)=CTEME(1)+DE(1)
      WSU=0.0
      DO 534 ITM=1,I
534 WSU=WSU+HTE(1,ITM)*COEE(ITM)-CTEME(1)*COEM(ITM)
      FCLTE(1)=CTEME(1)+WSU
      WRITE(9,9008) ZMIN(1),CTEME(1),
10FETE(1),FCLTE(1)
      WRITE(9,9007) ZIN(2),ETEME(2)
C
C COMPILES OUTPUT FOR FIRST AXIAL SEGMENT
C
C OUTPUT FOR REMAINING AXIAL SEGMENTS
C
      DO 785 JOP=2,J
      OFETE(JOP)=CTEME(JOP)+DE(JOP)
      WSU=0.0
      DO 786 IOUT=1,I
786 WSU=WSU+HTE(JOP,IOUT)*COEE(IOUT)-CTEME(JOP)*COEM(IOUT)
      FCLTE(JOP)=CTEME(JOP)+WSU
      WRITE(9,9008) ZMIN(JOP),CTEME(JOP),

```

```

      10FETE(JOP),FCLTE(JOP)
      JOPA=JOP+1
      WRITE(9,9007) ZIN(JOPA),ETEME(JOPA)
785 CONTINUE
C
C COMPLETES OUTPUT FOR THIS TIME
C
      532 CONTINUE
C INITIALIZE ALL UNKNOWNNS FOR NEXT TIME STEP
C FIRST AXIAL SEGMENT
C
      DO 513 ITH=1,I
513 HTB(1,ITH)=HTE(1,ITH)
      CTEMA(1)=CTEME(1)
      DB(1)=DE(1)
      ETEMA(2)=ETEME(2)
C
      DO 787 JIN=2,J
      DO 557 IEN=1,I
557 HTB(JIN,IEN)=HTE(JIN,IEN)
      CTEMA(JIN)=CTEME(JIN)
      DB(JIN)=DE(JIN)
      JINA=JIN+1
      ETEMA(JINA)=ETEME(JINA)
787 CONTINUE
C
C ADVANCE TIME COUNT
C CHECK TO SEE IF FINAL TIME HAS BEEN REACHED
C
      TIMES=TIMEE/XTXTS
      IF(TIMES-ENDT) 790,791,791
790 CONTINUE
      TIMEB=TIMEE
      TIMEE=TIMEB+DTND
      GO TO 795
791 CONTINUE
      GO TO 1
      END

```

TO : DTIE, OAK RIDGE, TENNESSEE

DATE: March 7, 1968

FROM : LASL, LOS ALAMOS, NEW MEXICO

SUBJECT: LA-3833

Enclosed are copies of a Fast Reactor Report for distribution in accordance with TID-4500, Standard Distribution List for Unclassified Reports. Also enclosed are 47 copies for distribution to the United Kingdom and EURATOM.