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**MICROWAVE MEASUREMENTS OF 2XII RADIAL
AND LONGITUDINAL ELECTRON DENSITY PROFILES**

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MICROWAVE MEASUREMENTS OF 2XII RADIAL AND LONGITUDINAL ELECTRON DENSITY PROFILES

Abstract

Five microwave interferometers are used to measure radial and longitudinal electron density profiles versus time in the 2XII minimum B mirror containment experiment. The mean plasma radius determined from a radially offset interferometer agrees with electrostatic probe and laser measurements. The

longitudinal profile is consistent with a collisional ion angular distribution with a density at the mirrors of ~5% of the central density. After compression the plasma shape remains approximately constant. Plasma densities beyond the mirrors were less than 5% of the central density.

Introduction

We report measurements of electron density profiles in the 2XII mirror device^{1,2} by means of the array of five microwave interferometers depicted in Fig. 1. Comparisons are made with ruby laser Thomson scattering and electrostatic probe measurements. These measurements have been motivated to a large extent by the observation that the stability of negative energy ion cyclotron waves in mirror confined plasmas with loss cone ion energy distributions hinges critically on radial and longitudinal density scale lengths.³⁻⁶ In addition, density profile measurements as reported here provide indirect means of assessing plasma loss processes by enabling us to:

- Evaluate coulomb scattering rates.
- Detect expansions or contractions of the plasma which might arise

from instabilities, electric field drifts, or charge exchange.

- Calibrate other diagnostics such as microwave scattering systems.
- Estimate ion angular distributions.

This report describes the methods employed to make these density profile measurements and reports our results. Measurements over the wide range of plasma parameters possible in the 2XII device remain to be made. Furthermore, the measurements reported here were not made simultaneously nor under exactly identical machine conditions. They are, nevertheless, representative of 2 to 4 keV 2XII plasmas formed with:

- Seven 2-in. titanium washer source injection
- A dc magnetic guide field of 2 kG
- Pulsed central magnetic mirror field of 4.6 kG.

Fig. 1. Schematic view of 2XII microwave diagnostics.

Magnetic Field Profiles

Magnetic field profiles in 2XII have been measured with a magnetic pick-up loop movable along the z-axis and field lines have been calculated with the MAFCO II code.⁷ The profiles for a

2 kG dc base field with 4.6 kG central compression field are shown in Fig. 2. The calculations assume a uniform current distribution in the 20-cm-wide copper conductors. The actual distribution is

Fig. 2. Calculated and measured 2XII magnetic field profile.

Fig. 3. Calculated 2XII magnetic field line surface, after compression, side view. To obtain a top view rotate page 180° to interchange right and left sides.

concentrated on the inside of the turns, however, due to inductive effects. This shifts the mirror position from 62 cm to 57 cm and reduces the mirror ratio from 2:1 to 1.93:1. Figure 3 shows a calculated flux surface in two planes for similar magnetic parameters. This surface results from compression (from 2 to 6.6 kG)

of the maximum plasma injection diameter (35 cm) permitted in the vacuum chamber. The magnetic field lines do not strike the vacuum chamber side walls but extend 3 m beyond the mirror to the end flange. Furthermore a particle following any line having $r < 15$ cm will experience the full mirror ratio.

Injected Plasma Profile

The radial injected plasma profile has been measured with an array of 22 elec-

trostatic probes under a large variety of injector and guide magnetic field conditions.

Fig. 4. Injected radial density profiles for 10 kV injection with one and seven sources.

These probes are faraday cups spaced 1.25 cm apart each having a 0.25 mm diam collector opening. Radial profiles are determined from peak collector signals at approximately 30 μ sec. Two such profiles, both at 2 kG, are shown in Fig. 4 for seven- and one-source injection. The seven-source injection nearly fills the vacuum chamber whereas the single source produces a plasma column with a 15-cm FWHM. By

assuming single-particle adiabatic compression of such profiles the compressed plasma diameter can be determined and used to calculate electron density from the measured microwave interferometer line density. For the 3.3:1 compression ratio investigated, this adiabatic assumption is verified by means of a second microwave interferometer and ruby laser Thompson scatter measurement.

Radial Electron Density Measurements

In the early 2XII operation two 4 mm microwave interferometers located at $z = 0$ were employed to measure electron line density. One interferometer⁸ was located on a plasma diameter while the second was located on a cord, offset 5.7 cm from the magnetic centerline. The 2.2 cm diam stainless steel microwave horns were separated by 30 cm. For these radial profile measurements plasma

was injected along a 2-kG-dc magnetic field and compressed to 6.6-kG central magnetic field value.

In order to evaluate the measured data we have calculated phase shifts for uniform and parabolic electron density profiles. As seen in Fig. 5 the measured density profile lies between these two profiles. In Appendix A we estimate that refractive corrections for this measurement are less than 15% for central densities well below cut-off $\hat{n}_e < \frac{1}{3} n_c = 2 \times 10^{13} \text{ cm}^{-3}$. Neglecting these refractive effects, we calculate the phase shift ϕ of an interferometer radially offset by an amount δ relative to the central interferometer as:

Uniform Profile

$$n_e(r) = \begin{cases} \hat{n}_e & 0 < r < r_0 \\ 0 & r > r_0 \end{cases} \quad (1)$$

$$R = \phi_{\text{offset}} / \phi_{\text{central}} = \sqrt{1 - \delta^2 / r_0^2} \quad (2)$$

Parabolic Profile

$$n_e(r) = \begin{cases} \hat{n}_e (1 - r^2 / r_0^2) & 0 < r < r_0 \\ 0 & r > r_0 \end{cases} \quad (3)$$

Fig. 5. Parabolic and uniform electron density profiles compared with measured profile.

$$R = \frac{P - \frac{1 - Q^2 P^2}{Q} \frac{1}{2} \ln \frac{1 + QP}{1 - QP}}{1 - \frac{1 - Q^2}{Q} \frac{1}{2} \ln \frac{1 + Q}{1 - Q}} \quad (4)$$

where

$$P = \sqrt{1 - \delta^2/r_0^2}$$

and

$$Q = \sqrt{\hat{n}_e/n_c}$$

The phase shift ratio R for both profiles is plotted vs δ/r_0 in Fig. 6 for several values of $\hat{n}_e/n_c$.

Data from seven source operation with a 23-cm aperture is shown in Fig. 7. From such data we conclude that the

Fig. 6. Phase shift ratio R vs interferometer offset normalized to plasma radius δ/r_0 for uniform and parabolic radial density distributions.

Fig. 7. Electron line density vs time measured by central and radially offset interferometers.

Fig. 8. Measured microwave phase shift ratio vs time for 12 kV injection with one and seven sources.

plasma shape is not changing substantially during the decay. Results from several such runs are summarized in Fig. 8 where the relative phase shift R is plotted versus time. From these curves at 400 μ sec we use Fig. 6 to estimate a range of plasma radii. Comparing these estimates with 22-channel probe measurements in Table I we find qualitative agreement.

Table 1. Measurement of plasma full width, $2r_0$.

Number of sources	Microwave measurements (cm)	22-channel probe (cm)
7	14-19	16
1	12-15	13

The data of Fig. 8 indicates small changes in plasma radial profile which cannot be accounted for by experimental

inaccuracies. We believe these changes result from a combination of several effects:

- Refractive effects, as discussed in Appendix A, lead to a small apparent decrease in plasma diameter as the peak density decreases.
- Charge-exchange erosion of the plasma boundary may influence the single source data. These measurements were made in the damaged high vacuum chamber at pressures

Fig. 9. Microwave electron line density measurements showing abrupt plasma loss during period of instability without substantial subsequent change in radial profile.

40 times higher than the 2×10^{-9} Torr normally achieved in the new chamber. No radial offset interferometer measurements have been made in the new chamber.

- Coulomb scattering flattens peaked density profiles.
- Instabilities can substantially change the plasma loss rate, as seen in Fig. 9, and thus could change the plasma radial profile.

Recently a 6 J ruby laser has been employed to make 90° Thomson scattering measurements of the central electron density. Results from the laser system are shown in Fig. 10 and compared with 4 mm microwave interferometer measurements. The reason for the discrepancy during compression ($0 < t < 400 \mu\text{sec}$) is not presently understood. In spite of the experimental uncertainty in the laser measurement we can conclude that the plasma fills only a 16 cm transverse region of the magnetic well rather than the entire 45 cm wide vacuum chamber.

Consequently we further conclude that the plasma does not directly contact the vacuum chamber walls and that the correct plasma diameter is being used to calculate the central electron density from the microwave interferometer line density measurement.

Fig. 10. Comparison of Thomson scatter electron density measurement with microwave interferometer data.

Longitudinal Electron Density Measurements

A movable 8 mm microwave interferometer is used to measure electron line densities in the 2XII plasma fans. Figure 1 shows schematically how this double-pass interferometer uses the flat titanium-coated vacuum wall for a reflector. The interferometer can scan from the edge of the horizontal vacuum wall at $Z = 30$ cm to $Z = 130$ cm, and, in addition, can make radial scans of the plasma by pivoting about $z = 350$ cm. By using a 3-port microwave circulator, the phase of the reflected signal is measured and used to

calculate the electron line density. By accounting for the decreasing plasma thickness in the fan regions due to the magnetic field shape (see Fig. 11) we calculate the electron density from the measured line density. A potential difficulty associated with this measurement arises from the small transverse dimensions of the plasma fan. However, in Appendix B we conclude from a WKB analysis that 8 mm microwave reflections from typical 2XII density gradients are small and therefore do

Fig. 11. Variation of fan thickness normalized to central thickness vs longitudinal position.

not pose a serious problem for this measurement.

During operation of the 35 GHz system it has been observed that the reflected signal intensity varies with position, presumably from the local surface condition of the sand blasted titanium, and that beyond the mirror throats the system sensitivity is inadequate to make density measurements. Consequently the movable interferometer system is being modified to operate at 17.5 GHz. Within the mirror confinement regions, however, interferometer measurements have been made as discussed below.

In Fig. 12 the electron line density measured about halfway to the mirror ($Z = 32$ cm) is compared with the measured central electron line density. The primary conclusion to be drawn from this data is that the longitudinal plasma shape does not change dramatically during the decay. That the line density at $Z = 32$ cm is 25% of the central line density is accounted for by both a reduction in plasma shape (as shown in Fig. 11) and by a reduction in plasma density. From a series of such measurements, after taking

Fig. 12. Electron line density vs time measured with central and longitudinally offset microwave interferometers.

Fig. 13. Longitudinal electron density profile at 400 μ sec.

into account the changing plasma shape, the longitudinal density profile has been reconstructed as shown in Fig. 13. The horizontal error bars indicate microwave beam width and uncertainties in position while the vertical error bars indicate shot-to-shot variations and uncertainties in the phase measurement.

The curves shown in Fig. 13 have been calculated on the basis of two simple models for the plasma density inside and beyond the mirror. Inside the mirror,

the density distribution is calculated with an analytic approximation due to Cordey⁹ based on an assumed collisional normal mode angular distribution neglecting plasma losses and plasma potential,

$$\frac{n_e(z)}{n_e(0)} = \frac{\ln\left(\frac{1+S}{1-S}\right) - 2S}{\ln\left(\frac{1+S_0}{1-S_0}\right) - 2S_0} \quad Z < 57 \text{ cm} \quad (5)$$

where

$$S = \sqrt{1 - B(z)/B(L)}, S_0 = \sqrt{1 - B(0)/B(L)},$$

and

$$B(0)/B(L) = \frac{1}{2}.$$

Beyond the mirror the density is calculated under the assumption that the external plasma results entirely from small angle scattering into the loss cone¹⁰⁻¹² neglecting the effects of plasma potential:

$$\frac{n(z)}{n(0)} \approx \frac{1}{\tau} \frac{L}{v_{\parallel}(z)} \frac{B(z)}{B(0)} \quad (6)$$

where

$$v_{\parallel}(z) \approx v_{\theta i} \left(1 - B(z)/B(L) + 2L/\tau v_{\theta i} \right)^{1/2} \quad (7)$$

$\tau = 400 \mu\text{sec}$ measured plasma lifetime,
 $B(z)/B(0)$ = magnetic mirror ratio,
 $v_{\theta i}$ = ion thermal speed,
 $2L = 114 \text{ cm}$ distance between mirrors.

The moderately high density just outside the mirror results from ions escaping by small angle collisions and thereby leaving the containment region with little parallel velocity. This "traffic jam" coupled with

Fig. 14. Radial electron density profile in plasma fan at $400 \mu\text{sec}$.

the faster-than-classical loss of 2XII plasma results in the 5% density at the mirrors which drops rapidly as the ions fall down the magnetic hill. Inside the mirrors, the data qualitatively agrees with the simple models although a quantitative comparison must include measurements of the longitudinal plasma potential profile and more precise density measurements near and beyond the mirror throats.

Using the movable interferometer at a fixed longitudinal position, a radial scan of the plasma has been made. Results from this measurement are shown in Fig. 14. The radial distribution shown as $z = 0$ was determined from the 22-channel electrostatic probe and projected to $z = 47 \text{ cm}$ by means of Eq. 5 and magnetic field calculations. Although improvements in the accuracy of this measurement are in order, the data does nevertheless indicate a substantial fanning of the plasma as expected for the 2XII magnetic field configuration and, in agreement with Fig. 13, a higher density than expected is measured near the mirror.

This discrepancy may arise from the approach of fan thickness (~ 3 cm) near

the mirrors to typical ion cyclotron orbit dimensions.

End Region Electron Density Measurements

It has been recognized that plasma in the end regions can not only suppress MHD instabilities¹³ in mirror devices by line tying but can also limit the amplitude of microinstabilities⁴ and even, with sufficient density (much higher than those measured here), actually stabilize otherwise unstable mirror-contained plasmas.¹⁴ In addition, plasma in end regions may modify equilibrium parameters of the contained plasma such as ion and electron energies by thermal conduction. For these reasons the exterior electron line density in both ends of 2XII is continuously monitored with 3 cm microwave interferometers at $Z = \pm 160$ cm in an effort to uncover any correlation between the amount of exterior plasma and lifetime or other equilibrium parameters of the mirror-contained plasmas. To date no correlations have been found despite large variations in the measured exterior plasma density.

Figures 15 and 16 present measurements of exterior and central electron line densities. These measurements show that the exterior line densities are one to two orders of magnitude less than the central line density. Figure 15 shows a typical example where the exterior and contained plasma decay rates are comparable. Figure 16 represents a less frequently occurring example (case A in Ref. 1) where the exterior plasma decays very rapidly.

In order to determine end region plasma density from the line density

Fig. 15. Typical end region exterior electron line densities for well-gettered vacuum conditions. The central line density ($\times 1/10$) is plotted for comparison.

Fig. 16. Less frequently occurring example of end region exterior electron line density showing rapid decay. Central line density is plotted at $1/10$ scale.

measurements, the plasma thickness has been estimated from the measured number of fringes at microwave cut-off. A parabolic profile $n(r) = n(1 - r^2/r_0^2)$ at cut-off produces a phase shift $\phi = \beta_0 r_0$, and 3 to 4 fringes are typically measured, hence $9 < r_0 < 12$ cm. For a parabolic profile the peak density (for $n < 0.3n_c$) is determined from line density by dividing by $12 < 4r_0/3 < 16$ cm. This value is approximately equal to the 14 cm needed to convert the central line density measurements of Figs. 15 and 16 to peak density. Therefore the relative magnitude between exterior and central densities is approximately the same as for line densities shown in Figs. 15 and 16. The end region plasma thickness determined by the number of fringes at cutoff is compared with other characteristic transverse dimensions in Table 2. The plasma thickness determined by microwave cut-off is larger than the projected fan thickness but smaller than the injection diameter. Because of this range of dimensions it must be recognized that errors as large as a factor 2 are possible in determining end region densities from the measured microwave phase shifts.

The plasma line density resulting from end losses from the contained plasma is related by Eq. 6 to the central line density

$$\frac{n_e \ell(160)}{n_e \ell(0)} \approx \frac{L}{\tau v_{\theta i}} \frac{B(160)}{B(0)} \frac{\ell(160)}{\ell(0)}. \quad (8)$$

For a 4 keV deuterium plasma having a 400 μ sec lifetime this ratio is 2×10^{-4} and is typically two orders of magnitude less than the measured exterior line densities (e.g. Fig. 15). In fact, since

the minimum detectable electron line density for the 3 cm interferometers is approximately 5×10^{11} cm $^{-2}$, the interferometers are incapable of detecting densities due to end losses. The interferometer line densities for well-gettered vacuum conditions generally drop below this minimum detection level either slightly before or within several hundred microseconds following the end of magnetic compression depending on plasma injection and magnetic compression parameters. It is important to emphasize here that although the measured exterior densities are much larger than the end loss density they are more than an order of magnitude smaller than the central density of the confined plasma.

The exterior plasma density has been found to be related to the amount of plasma injected and to the titanium vacuum wall surface conditions. This last relation is illustrated in Fig. 17 which shows results from identical injection and magnetic field conditions but with one run

Fig. 17. End region electron line densities before and after titanium gettering.

taken without titanium gettering and the second taken after one cycle of gettering. In this example, where the exterior plasma line density at 900 μsec is reduced an order of magnitude by gettering, changes in plasma lifetime are observed late in time. However, in examining numerous decays with well-gettered vacuum walls no correlation has been found between exterior plasma density and contained plasma lifetimes.

Conclusions regarding the role of exterior plasma on the confined plasma are conjectural without measurements of the exterior plasma electron temperature and longitudinal plasma potential profiles in these end regions. Quantitative estimates of these unmeasured quantities are difficult due to the number of processes which presumably contribute to the exterior plasma formation: slow plasma streaming from titanium plasma sources, secondary electron as well as ion emission due to energetic ion wall bombardment, and ionization of sputtered

material and gas released by wall bombardment.¹⁵ In the absence of detailed examination of these processes we conjecture that this exterior plasma is relatively cold, $T_e \sim 1$ to 10 eV. In this event, early in the plasma decay – and especially during compression – the electron mean free path is less than the distance from mirror to end walls. Consequently classical resistivity and heat transport coefficients may be used to evaluate current and heat flow. Since the end region measurement is at a $B = 2$ kG axial location, however, the bulk of the cold exterior plasma probably does not penetrate into high field mirror regions due to combined effects of high 7:1 mirror ratio and plasma potential. Therefore, there also exists a collisions-free transition region¹¹ closer to the mirror. Such details, which are beyond the scope of this report, must be taken into account when examining the influence of end region plasma on electron temperature and microinstabilities of the confined plasma.

Table 2. End region transverse dimensions.

Method of determination	Thickness in cm
Projected plasma flux surface after compression	8
Fringes at microwave cutoff (3 to 4)	12 to 16
Maximum fan thickness at plasma injection	14
Microwave horn separation: injection end	56
far end	40

Summary

The principal results of this work include:

- Measurements of plasma radius are in agreement with single particle

adiabatic compression of injected plasma profiles.

- The radial density gradient scale length R_p is typically 3 to 8 times

the ion gyro radius a_i and hence represents a strong destabilizing force for drift-cone instability.

- No correlation between R_p/a_i and n_T may be made from the limited measurements reported here.
- Longitudinal density profile measurements are in qualitative agreement with collisional angular distribution.
- The plasma longitudinal scale length is typically $L/a_i \sim 15$ to 50 and hence is marginally stable to most finite length instability modes.
- Longitudinal density profiles required to evaluate quasi-classical scattering rates⁵ due to the convective loss cone instability have been measured.
- No evidence of rapid plasma expansion or contraction has been found.

- Exterior plasma density beyond the mirrors depends on vacuum wall surface conditions and on amount of injected plasma.
- Exterior plasma densities are one to two orders of magnitude less than central densities but are more than an order of magnitude above densities resulting from end losses of the mirror contained plasma.
- Exterior density is not correlated with contained n_T .

In future operation plasma diameters can be estimated from the microwave interferometer measurements of line density and the Thomson-scattering-laser observation of central density. End-region densities will continue to be monitored. The sensitivity of the movable interferometer has been increased to allow more accurate measurements of the low-density regions near the mirrors.

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Appendix A

Phase Shift Introduced by Refraction

PHASE SHIFT INTRODUCED BY REFRACTION

Since the plasma refractive index is less than unity it acts as a divergent cylindrical lens. A microwave beam passing through a plasma on axis diverges and is thereby attenuated^{A1,2} but, to first order, the phase shift is not modified.

A1. M. A. Heald and C. B. Wharton, Plasma Diagnostics with Microwaves (John Wiley and Son, New York, 1965), Ch. IV.

A2. H. Schneider, Effect of a Radially Inhomogeneous Plasma on Microwave Interferometer Measurements, Lawrence Livermore Laboratory, Rept. UCRL-14438 (1965).

An off-axis microwave detector, on the other hand, will intercept a beam following the path shown in Fig. A1. This deflected path results in two counterbalancing effects.

- an increase in phase due to the longer vacuum path, $\mu = 1$
- a decrease in phase due to the larger plasma path, $\mu < 1$

The net phase shift arising from these two effects is calculated below by means of the geometric optics approximation^{A1} which may be employed since:

- the plasma diameter (~ 12 cm) is much larger than the microwave wavelength (0.4 cm)

Fig. A1. Refraction of an offset microwave beam through a uniform plasma of radius r_0 .

- the antenna is located far from the plasma ($D/\lambda \approx 40$)
- the antenna is large compared to a wavelength ($\times 5$)

Using the notation of Fig. A1 we calculate the phase shift measured by the central interferometer as

$$\Delta\phi = 2\beta_0 r_0 (1 - \mu). \quad (A1)$$

Likewise the phase of the offset interferometer is

$$\Delta\phi' = 2\beta_0 r_0 \left[d - \mu \cos \theta_2 - \frac{d - \cos \theta_2}{\cos(\theta_2 - \theta_1)} \right] \quad (A2)$$

where $d = D/r_0 > 1$ and $\sin \theta_1 = \mu \sin \theta_2$.

The phase shift ratio

$$R = \Delta\phi' / \Delta\phi = \frac{d - \mu \cos \theta_2 - (d - \cos \theta_2) / \cos(\theta_2 - \theta_1)}{1 - \mu} \quad (A3)$$

Fig. A2. Variation, due to refraction, in phase shift ratio R vs electron density of a uniform plasma.

is plotted in Fig. A2 for typical 2XII parameters: $\delta = 5.7$ cm, $r_0 = 8$ cm and $D = 22$ cm. It is seen that for $n_e/n_c < 0.3$ the correction due to refraction is less than 15% and that the net effect of ray bending is to indicate an enlarged plasma diameter.

Appendix B

Calculations of Reflection Errors for a Double-Pass Interferometer

CALCULATIONS OF REFLECTION ERRORS FOR A DOUBLE-PASS INTERFEROMETER

A potential difficulty in the use of a double-pass interferometer to measure electron line densities in plasmas with sharp density gradients arises due to reflections from the plasma^{B1} as well as from the reflectors. In this appendix we estimate the magnitude of the plasma reflections and conclude that for 2XII experimental parameters the return signal accurately represents the phase of the signal transmitted through the plasma.

The interferometer electric field is polarized parallel to the magnetic containment field. For such a wave, the effective plasma dielectric constant is^{A1}:

$$K(x) = 1 - \omega_p^2(x)/\omega^2 = 1 - n(x)/n_c \quad (B1)$$

where $n_c = m\epsilon_0\omega^2/e^2$ is the critical density. For the 35 GHz operating frequency, $n_c = 1.52 \times 10^{13} \text{ cm}^{-3}$. By considering a density profile of the form

$$n(x) = \hat{n}_e f(x;W) \quad (B2)$$

having a characteristic width W , the reflection from the plasma may be calculated using WKB methods.^{B2} These WKB re-

sults are then valid for $W \gg \lambda_0$,

$\lambda_0 = 8.6 \text{ mm}$ is the free-space wavelength.

For inhomogeneous media, the wave equation

$$\frac{d^2 E(x)}{dx^2} + \frac{\omega^2}{c^2} K(x) E(x) = 0 \quad (B3)$$

has a zero-order solution

$$E(x) = E_0 K^{-1/4}(x) \exp \left(-j \frac{\omega}{c} \int_{x_0}^x [K(x')]^{1/2} dx' \right). \quad (B4)$$

A first order solution which takes into account reflections due to the inhomogeneity is

$$E(x) = E_0 K^{-1/4}(x) \times \exp \left(-j \frac{\omega}{c} \int_{x_0}^x [K(\xi)]^{1/2} d\xi \right) + E_1(x) \exp \left(+j \frac{\omega}{c} \int_{x_0}^x [K(\xi)]^{1/2} d\xi \right). \quad (B5)$$

$E_1(x)$ is calculated by substituting B5 into Eq. B3 and equating terms of equal order.^{B2} The resulting reflection coefficient is

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$$R_1 = \left| \frac{E_1/E_0}{[K(x)]^{1/4}} \right|$$

$$= \frac{\lambda_0}{8\pi} \left| \int_{x_1}^x \frac{1/2}{[K(\xi)]^{1/4}} \frac{d}{d\xi} \left(\frac{dK(\xi)/d\xi}{[K(\xi)]^{5/4}} \right) \right. \\ \left. \times \exp \left(-2j \frac{W}{c} \int_{x_0}^{\xi} [K(\eta)]^{1/2} d\eta \right) d\xi \right|. \quad (B6)$$

In Eq. B6, x is the point of observation, x_0 is an arbitrary phase reference, and x_1 is a finite reference point beyond which the integrand is negligible or zero. Equation B6 has this simple physical interpretation: A wave is launched from x in the $+x$ direction. Due to the inhomogeneity, there are incremental reflections of the wave at every point ξ . These incremental reflections are weighted by their respective phase factors,

$$\exp \left(-2j \frac{W}{c} \int_{x_0}^{\xi} [K(\eta)]^{1/2} d\eta \right),$$

the exponent being the phase change the wave undergoes as it propagates from the phase reference x_0 to the reflection point ξ and back to x_0 . These weighted increments are summed at x to give the total reflection from the slab. Calculations of Eq. B6, using three different density profiles are described below.

Rectangular Profile

The reflection coefficient for a rectangular profile can be calculated exactly, so we do not use the WKB approach for such a slab.

$$|R_1| = |\rho| \left[\frac{2 + 2 \cos \left(4 \frac{W}{c} \sqrt{K_0} W \right)}{1 + \rho^4 + 2\rho^2 \cos \left(4 \frac{W}{c} \sqrt{K_0} W \right)} \right]^{1/2} \quad (B7)$$

$$\rho = \frac{\sqrt{K_0} - 1}{\sqrt{K_0} + 1} \quad \text{and} \quad K_0 = 1 - n_0/n_c.$$

Due to the cosine terms the magnitude of $|R|$ is a rapidly oscillating function of W for $W \gg \lambda_0$. We suppress this behavior by taking the worst case, which is $4 \frac{W}{c} \sqrt{K_0} = 2\pi m$, for which

$$|R_1| = \frac{2\rho}{1 + \rho^2}. \quad (B8)$$

Fig. B1. Maximum reflection coefficient R_1^2 of 8 mm microwave power vs density for three different 5-cm-wide density profiles.

Since the reflection from a rectangular profile exceeds that from any other distribution having the same peak density it serves as an upper bound on the reflection coefficient for a given density.

Parabolic and Gaussian Profiles

Equation B6 must be evaluated numerically for parabolic and Gaussian density profiles. It also contains a rapidly varying phase factor which we wish to suppress. We note that Eq. B6 is of the form:

$$|R_1| = \frac{\lambda_0}{8\pi} \left| \int_{x_1}^x f(\xi) e^{-j\phi(\xi)} d\xi \right| \quad (B9)$$

where

$$f(\xi) = \frac{1/2}{[K(\xi)]^{1/4}} \frac{d}{d\xi} \left(\frac{d}{d\xi} K(\xi) \right)$$

$$\phi(\xi) = 2 \frac{\omega}{c} \int_{x_0}^{\xi} [K(\eta)]^{1/2} d\eta.$$

Here $f(\xi)$ varies slowly, on scale lengths of order W . The phase factor, $e^{-j\phi(x)}$ varies greatly over a wavelength or less. However, we may conclude

$$|R_1| \leq \frac{\lambda_0}{8\pi} \int_{x_1}^x |f(\xi)| d\xi. \quad (B10)$$

Equation B10 thus gives us an upper bound on $|R_1|$ which, if small enough, will be adequate for our purpose. This integral was evaluated numerically for $W = 2.5$ cm using Simpson's parabolic rule, for both the parabolic and Gaussian profiles. The results together with those for the rectangular profile (Eq. B8) are shown in Fig. B1. Clearly, errors introduced by these reflections alone will be small, especially for the Gaussian profile. This is the most realistic profile, in view of its smoothness in all derivatives.

In conclusion, these calculations demonstrate that, except near cutoff, plasma reflection introduces negligible error. If strong attenuation is present in the transmission path through the plasma, however, then it is necessary to consider the relative magnitudes of the reflected signal and the signal transmitted back and forth through the plasma. Because of the finite beam widths of the small microwave horns the received signals will be down by a factor approximately equal to the square of the ratio of the transmission path lengths. In the 2XII, this ratio is around 5 and thus, as seen in Fig. B1, reflections are unimportant (for $n_e < 1/2 n_c$) unless double pass attenuation exceeds 10 dB.

NOTICE

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