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FINITE LARMOR ORBIT STABILIZATION

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FINITE LARMOR ORBIT STABILIZATION

by

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Abstract

The theory of the stabilizing effect of finite ion Larmor radius is applied to a plasma containing several species of ions. The resulting dispersion relation shows that this effect probably stabilizes Scylla and possibly also the plasma focus. Strictly, the theory applies only to a plasma of low β and with small gradients ($[\text{grad } \rho]/\rho$ small), thus our conclusions lack certainty.

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1. The Dispersion Relation

It has been shown¹ that the stabilizing effect of finite Larmor orbit radius can be understood from a purely hydromagnetic model. The effect is to introduce off-diagonal elements into the pressure tensor and thus produce a sort of nondissipative viscosity. If the fluid velocity is $\vec{u}$ and the z-axis is chosen in the direction of the magnetic field, the pressure tensor components are

$$\left. \begin{aligned} p_{xx} &= p_{\perp} - \rho v \left(\partial_x u_y + \partial_y u_x \right), \\ p_{yy} &= p_{\perp} + \rho v \left(\partial_x u_y + \partial_y u_x \right), \\ p_{zz} &= p_{\parallel}, \\ p_{xy} &= p_{yx} = \rho v \left(\partial_x u_x - \partial_y u_y \right), \\ p_{xz} &= p_{zx} = -2\rho v \left(\partial_y u_z + \partial_z u_y \right), \\ p_{yz} &= p_{zy} = 2\rho v \left(\partial_z u_x + \partial_x u_z \right), \end{aligned} \right\} \quad (1.1)$$

where

$$v = a^2 \Omega / 4 \quad (1.2)$$

and a , Ω are respectively the Larmor radius and frequency. Clearly only ions contribute to this effect. Throughout what follows a , Ω refer to the ions unless it is specifically stated otherwise. We can alternatively write Eq. (1.2) as

$$v = kT/2M\Omega. \quad (1.3)$$

We are interested in a plasma which may contain several ion types, all, however, at the same temperature; i.e.,

$$T_1 = T_2 = \dots = T_r = T. \quad (1.4)$$

Thus, as $M\Omega = eB/c$, and hence is independent of the mass, we have, on summing over ion types

$$\sum_{i=1}^r \rho_i v_i = v \sum_{i=1}^r n_i M_i = \rho v. \quad (1.5)$$

Thus Eqs. (1.1) apply to the total ion pressure if by p we mean

$$p = \sum_{i=1}^r p_i. \quad (1.6)$$

One can now combine moments of the Boltzmann equation in the usual ways to obtain the law of momentum conservation

$$\rho \frac{d\vec{u}}{dt} = -\nabla \cdot \vec{p} + \vec{J} \times \vec{B} + n_e \vec{F}_e + \sum_{i=1}^r n_i \vec{F}_i \quad (1.7)$$

and the "generalized Ohm's law"

$$\vec{E} + (\vec{u}/c) \times \vec{B} = [\vec{J} \times \vec{B} - \nabla p_e] / en_e +$$

$$[\vec{F}_e \sum n_i M_i - m \sum n_i \vec{F}_i] / e [n_e m + \sum n_i M_i] . \quad (1.8)$$

Here, $\vec{F}_e$, $\vec{F}_i$ represent nonelectrical forces acting on the particle types indicated. If these forces were purely gravitational, the last term on the right hand side of Eq. (1.8) would vanish identically. Note that in deriving Eq. (1.8) we have neglected collisional effects and have consistently ignored electron mass (and inertia) compared with ion mass. In the stability analysis, one does not use Eq. (1.8) directly, but rather takes its curl, obtaining in this fashion the equation of motion for $\vec{B}$:

$$\frac{\partial \vec{B}}{\partial t} = \text{curl} \left(\frac{\vec{u}}{c} \times \vec{B} \right) - \text{curl} \left(\frac{\rho}{en_e} \frac{d\vec{u}}{dt} \right) - \text{curl} \left(\frac{\nabla \cdot \vec{p}_i}{en_e} \right) + \text{curl} \left(\frac{\sum n_i \vec{F}_i}{en_e} \right) . \quad (1.9)$$

In obtaining Eq. (1.9) we have used Maxwell's curl $\vec{E}$ equation and Eq. (1.7).

We now follow reference (1) in treating a low β plasma in a uniform magnetic field. Nothing varies in the z -direction (the direction of $\vec{B}$) and u_z is taken to vanish. Thus we have

$$\left. \begin{aligned} \frac{\partial \vec{B}}{\partial t} &= 0 \\ \text{curl } (\vec{u} \times \vec{B}) &= -\vec{B} \text{ div } \vec{u} . \end{aligned} \right\} \quad (1.10)$$

We take all the $\vec{F}$'s to be parallel and define

$$\left. \begin{aligned} \vec{F}_+ &= \left(\sum n_i \vec{F}_i \right) / n_+ , \\ n_+ &= \sum n_i . \end{aligned} \right\} \quad (1.11)$$

We also assume that all n 's have the same spatial and temporal dependence and, as in reference (1), we assume T to be uniform. Then we obtain

$$\text{curl } \left(\nabla p_i / en_e \right) = 0 , \quad (1.12)$$

where p_i is the scalar part of the ion pressure tensor, and

$$\text{curl} \left(n_+ \vec{F}_+ / n_e \right) = \left(n_+ / n_e \right) \text{curl} \vec{F}_+ . \quad (1.13)$$

As $\text{curl} \vec{F}_+$ vanishes for a uniform field, and also for the centrifugal force field of particles moving uniformly on a circle, we shall set it zero. We thus find that Eq. (1.9) reduces to

$$\vec{b}(\nabla \cdot \vec{u}) + \frac{1}{\Omega} \text{curl} \frac{d\vec{u}}{dt} - \frac{\nu}{\Omega} \text{curl} (\vec{\lambda}/\rho) = 0 , \quad (1.14)$$

where

$$\Omega \stackrel{\text{def}}{=} eBn_e/\rho c \quad (1.15)$$

and $\vec{\lambda}$ is the vector, written out in full in reference (1), arising from the nondiagonal elements of the ion pressure tensor. We can write more simply

$$\Omega = eB/Mc , \quad (1.16)$$

where e/M now denotes a proper average of this quantity over the ion types, namely

$$e/M = \left(e \sum n_i Z_i \right) / \sum n_i M_i . \quad (1.17)$$

Our Eqs. (1.7) and (1.14) are identical with the corresponding equations of reference (1) provided we make the identifications of Eq. (1.15) and

$$\vec{g} = (n_e \vec{F}_e + n_+ \vec{F}_+) / \rho . \quad (1.18)$$

Thus we recover the dispersion relation of reference (1)

$$\omega^2 + (2\eta\nu k + gk/\Omega)\omega + \eta g = 0 . \quad (1.19)$$

As in reference (1)

$$\left. \begin{aligned} \eta &= -\frac{1}{\rho_0} \frac{\partial \rho_0}{\partial x} , \\ k &= \text{wave number of the disturbance} , \end{aligned} \right\} \quad (1.20)$$

and g , Ω , ν have the meanings given then by Eqs. (1.18), (1.15), and (1.3). The stability condition is that

$$(2\nu\eta k + gk/\Omega)^2 > 4g\eta , \quad (1.21)$$

2. Application to Scylla

In Scylla there are two destabilizing forces, both of them centrifugal. The first of these² arises from the rotation of the electrons which carries the pressure balancing current. The second destabilizing force arises from the motion of particles along the curved magnetic lines. The first of these forces is obviously small. The second is also small if the discharge is long compared with its radius. We take up these forces in turn.

For the current carrying electrons we have

$$v_{\theta} = -cJ/en = -p'/nM\Omega, \quad (2.1)$$

whence we obtain

$$g = rM\left(\frac{p'}{nM\Omega}\right)^2, \quad (2.2)$$

where r is the radius of the discharge. Now

$$p'/n = \frac{n'}{n}kT \approx kT/r \quad (2.3)$$

and

$$kT = \frac{1}{2} M \Omega^2 a^2, \quad (2.4)$$

whence

$$g = \frac{m}{4rM} \left(\frac{a}{r} \right)^2 \Omega^2 a^2. \quad (2.5)$$

Setting in addition

$$\eta = 1/r$$

and

$$(2.6)$$

$$k = 1/r,$$

the stability condition becomes

$$\left[1 + \frac{m}{4M} \left(\frac{a}{r} \right)^2 \right]^2 > 4m/M \quad (2.7)$$

which is clearly satisfied.

In estimating the second case, let us consider a cigar shaped plasma as shown in Fig. 1. The length is $2L$ and the radius is r ; R is radius of curvature of the B-lines owing to the finite length of the

Figure 1. Plasmoid Geometry

discharge. We have

$$R = (L^2 + r^2)/2r \approx L^2/2r . \quad (2.8)$$

Again, η and k are as given by Eq. (2.6). The destabilizing force is now given by

$$F = mv_{||}^2/R = kT/R , \quad (2.9)$$

whence

$$\left. \begin{aligned} g &= \alpha kT / MR , \\ \alpha &= \left(T_e / T_+ + n_+ / n_e \right) . \end{aligned} \right\} \quad (2.10)$$

Normally α will lie between 1 and 2 depending on the electron temperature. However, α can be less than unity provided the electrons are cold and there are multiply charged ions. The stability condition now becomes

$$\frac{kT}{M\Omega^2 r^2} > 8\alpha \frac{r^2}{L^2} \quad (2.11)$$

if we assume $r^2/L^2 \ll 1$. This condition can be rewritten in terms of the Larmor radius a , as

$$a/r > (4r/L)\sqrt{\alpha} . \quad (2.12)$$

Thus, for a reasonably large aspect ratio, we can expect stability.

3. The Plasma Focus

For the plasma focus we again have g given by Eq. (2.10), but now $r \approx R$. In this case the condition for stability becomes

$$kT(1 + \alpha)^2 / M^2 r^2 > 4\alpha \quad (3.1)$$

or, in terms of the Larmor radius,

$$a/r > 2\sqrt{2\alpha}/(1 + \alpha) . \quad (3.2)$$

This requires a to be of the same size as r , and in this case all our approximations break down. In deriving the condition (3.2) we have assumed an $m = 1$ mode, which really does not make sense for a plasma no longer than broad. For the $m = 2$ mode, $k = 2/r$, and the condition becomes

$$a/r > \sqrt{2\alpha}/(1 + \alpha) . \quad (3.3)$$

It is clear that high m modes are suppressed, but the low mode numbers lie outside the range of the theory. As condition (3.3) is approached (but not satisfied) we should expect that the plasma focus at least has the growth rate of its instability appreciably slowed by finite Larmor orbit effects. A much more sophisticated theory would be necessary to indicate stability or otherwise with any confidence.

4. Conclusions

It would seem likely that the Scylla pinch is stabilized by the

effects of finite ion Larmor orbit radius. As regards the plasma focus, a stabilizing effect is possible, but conditions in this device are well outside the realm of the theory.

In fact, the Scylla devices are also somewhat outside the realm of the theory of references (1) and (2). The theory has been developed only in the limit $p \ll B^2/8\pi$ so that the magnetic field, both in equilibrium and in the disturbed configuration, is uniform. Moreover it was assumed that all gradients in the direction of $\vec{g}$ were small. As the dispersion relation, Eq. (1.19), indicates that strong gradients in the $\vec{g}$ direction are stabilizing, it would be nice to remove these restrictions on the theory.

References

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2. M. Rosenbluth, N. A. Krall, and N. Rostaker, Nuclear Fusion Suppl., Pt. I, p. 143 (1962).